Кр по теорверу

rqu 3V9K

№6

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim N\left(\begin{pmatrix} 8 \\ -1 \\ 3 \end{pmatrix}, \underbrace{\begin{pmatrix} 7 & 2 & 5 \\ 2 & 6 & 3 \\ 5 & 3 & 10 \end{pmatrix}}_{=R} \right)$$

$$\vec{y} = \begin{pmatrix} 2X_1 - 3X_2 \\ X_2 + X_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & -3 & 0 \\ 0 & 1 & 1 \end{pmatrix}}_{=A} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

$$\Rightarrow \vec{y} \sim N\left(A \begin{pmatrix} 8 \\ -1 \\ 3 \end{pmatrix}, ARA^T \right)$$

$$AR = \begin{pmatrix} 2 & -3 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 7 & 2 & 5 \\ 2 & 6 & 3 \\ 5 & 3 & 10 \end{pmatrix} = \begin{pmatrix} 8 & -14 & 1 \\ 7 & 9 & 13 \end{pmatrix}$$

$$ARA^T = \begin{pmatrix} 8 & -14 & 1 \\ 7 & 9 & 13 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -3 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 58 & -13 \\ -13 & 22 \end{pmatrix}$$

$$A \begin{pmatrix} 8 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 19 \\ 2 \end{pmatrix}$$

$$\Rightarrow y \sim N\left(\begin{pmatrix} 19 \\ 2 \end{pmatrix}, \begin{pmatrix} 58 & -13 \\ -13 & 22 \end{pmatrix} \right)$$

№3

$\xi_0, \xi_1, \varkappa$ – нез , $q = 1-p$

$\xi_n \sim Gamma(1, n+1)$, $n = 0, 1$

$\varkappa \sim Be(p)$, $\eta = \xi_\varkappa = ?$

$$\varphi_{\xi_n}(t) = \left(1-(n+1)it\right)^{-1} = \frac{1}{1-(n+1)it}$$

$$\varphi_{\xi_n}(t) = \mathbb{E}\exp(it\xi_n) = \mathbb{E}\left[\exp(it\xi_0)q + \exp(it\xi_1)p\right] =$$

$$= q\underbrace{\mathbb{E}\exp(it\xi_0)}_{\varphi_{\xi_0}(t)} + p\underbrace{\mathbb{E}\exp(it\xi_1)}_{\varphi_{\xi_1}(t)} = \underbrace{\frac{q}{1-it} + \frac{p}{1-2it}}$$

№4

$$\eta_n = \sum_{k=1}^{n}\frac{1}{10^k}\xi_k, \quad \xi_k - \begin{cases} 0, & p = \frac{1}{10} \\ \vdots & \\ 9, & p = \frac{1}{10} \end{cases}$$

тд: $\varphi_{\eta_n}(t) \to \varphi_\delta(t)$, $\delta \sim U[0,1]$

$$\exists\ \alpha_k = \frac{1}{10^k}\xi_k \Rightarrow \varphi_{\alpha_k}(t) = \mathbb{E}e^{it\alpha_k} =$$

$$= \mathbb{E}\exp\left(\frac{1}{10^k}it\xi_k\right) = \sum_{j=0}^{9} e^{\frac{it}{10^k}\cdot j}\cdot\frac{1}{10} = \frac{1}{10}\sum_{j=0}^{9} e^{\frac{it}{10^k}j}$$

$$\varphi_{\eta_n}(t) = \varphi_{\alpha_1}(t) \cdot \ldots \cdot \varphi_{\alpha_n}(t) =$$

$$= \frac{1}{10^n}\left(e^{it/10}+1\right)\left(\ \ldots\ \right)\left(e^{it/10^n}+1\right) =$$

$$= \frac{1}{10^n} \frac{\left(e^{it/2}+1\right)\left(\ \ldots\ \right)\left(e^{it/10^n}+1\right)\left(e^{it/10^n}-1\right)}{e^{it/10^n}-1} =$$

$$= \frac{1}{10^n} \frac{e^{it}-1}{e^{it/10^n}-1} = \frac{1}{10^n} \frac{e^{it}-1}{1+\frac{it}{10^n}+o\left(\frac{1}{10^n}\right)-1} =$$

$$= \frac{e^{it}-1}{it+o(1)} = \varphi_\delta(t), \quad \underline{\delta \sim U[0;0]}$$