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CS353 Section 2

HW5

Q1)

a)  $A \rightarrow B$  is violated because in the 1st row there is (a1, b1) but in the 2nd row there is (a1, b2).

b)  $B \rightarrow C$  is not violated because for every same B value the C values are the same.

c)  $C \rightarrow A$  is violated because in the 1st row there is (c1, b1) but in the 3rd row there is (c1, b3).

d)  $AB \rightarrow C$  is not violated as there are no two rows with the same A and B values.

e)  $AC \rightarrow B$  is violated because in the 3rd row there is (a2, c1, b3) but in the 5th row there is (a2, c1, b1).

f)  $BC \rightarrow A$  is violated because in the 1st row there is (b1, c1, a1) but in the 5th row there is (b1, c1, a2).

Q2)

a)  $R1 \cap R2 = A$  and  $A \rightarrow R1 = AB$  holds since  $A \rightarrow B$ , hence the decomposition is lossless.

b)

R:

A	B	C
a1	b1	c1
a1	b1	c2
a2	b1	c3

R1:

A	B
a1	b1
a2	b1

R2:

B	C
b1	c1
b1	c2
b1	c3

R1  $\bowtie$  R2:

A	B	C
a1	b1	c1
a1	b1	c2
a1	b1	c3
a2	b1	c1
a2	b1	c2
a2	b1	c3

R is a subset of R1  $\bowtie$  R2, hence the decomposition is not lossless.

Q3)

a) C is not implied by other attributes, so C is a part of any candidate key.

$C^+$  -> using the algorithm for computing the closure of an attribute

result = c -> terminates after 1st turn of while loop because there is no change in result, **C is not a candidate key.**

**(AC)<sup>+</sup>:**

result = AC

1st while:

result = result  $\cup$  D (ACD)

result = result  $\cup$  AB (ABCD)

2nd while:

result = result  $\cup$  D (ABCD)

result = result  $\cup$  E (ABCDE)

result = result  $\cup$  AB (ABCDE)

No change in 3rd while, so terminate.

**(AC)<sup>+</sup> = ABCDE  $\Rightarrow$  AC is a superkey. It is also minimal, so it is a candidate key.**

**(BC)<sup>+</sup>:**

result = BC

1st while:

result = result  $\cup$  E (BCE)

2nd while:

result = result  $\cup$  E (BCE)

No change in 2nd while, so terminate.

**(BC)<sup>+</sup> = BCE  $\Rightarrow$  BC is not a candidate key.**

**(DC)<sup>+</sup>:**

result = DC

1st while:

result = result  $\cup$  AB (ABCD)

2nd while:

result = result  $\cup$  D (ABCD)

result = result  $\cup$  E (ABCDE)

result = result  $\cup$  AB (ABCDE)

No change in 3rd while, so terminate.

**(DC)<sup>+</sup> = ABCDE  $\Rightarrow$  DC is a superkey. It is also minimal, so it is a candidate key.**

**(EC)<sup>+</sup>:**

result = EC

**No change in 1st while, so terminate. EC is not a candidate key.**

**AC and DC are candidate keys.**

b) R is not in BCNF, as in the nontrivial functional dependency  $A \rightarrow D$ , A is not a superkey.

c) R is not in 3NF, as in the nontrivial functional dependency  $BC \rightarrow E$ , BC is not a superkey and  $E - BC = E$  is not part of a candidate key either.

Q4)

a)

- Using the decomposition rule, we can decompose  $A \rightarrow BC$  in G as  $A \rightarrow B$  and  $A \rightarrow C$ , so  $A \rightarrow B$  in F can be inferred from G.
- Using the augmentation rule, we can augment  $A \rightarrow BC$  in G as  $AB \rightarrow BC$ , then with the decomposition rule we can decompose it into  $AB \rightarrow B$  and  $AB \rightarrow C$ , hence  $AB \rightarrow C$  in F can be inferred from G.
- Using the decomposition rule, we can decompose  $D \rightarrow AE$  in G as  $D \rightarrow A$  and  $D \rightarrow E$ . We can also decompose  $A \rightarrow BC$  as  $A \rightarrow B$  and  $A \rightarrow C$ . As  $D \rightarrow A$  and  $A \rightarrow C$ , we can use transitivity to show  $D \rightarrow C$ . Since we now have both  $D \rightarrow A$  and  $D \rightarrow C$ , we can say  $D \rightarrow AC$ , so  $D \rightarrow AC$  in F can be inferred from G.
- Using the decomposition rule, we can decompose  $D \rightarrow AE$  in G as  $D \rightarrow A$  and  $D \rightarrow E$ . Hence,  $D \rightarrow E$  in F can be inferred from G.

As a result, all of the functional dependencies in F can be inferred from G and G covers F.

b) F doesn't cover G, as the functional dependency  $E \rightarrow B$  in G cannot be obtained by the dependencies in F, E doesn't appear on the left hand side in any of the dependencies in F.

c) As F doesn't cover G, F and G are not equivalent.

Q5)

a)  $F = \{A \rightarrow BD, CD \rightarrow B, C \rightarrow D, B \rightarrow D\}$

There are no functional dependencies with the same left hand side, so no replacement is done.

**Eliminating extraneous attributes on the lefthand side:**

for  $CD \rightarrow B$ :

for C:

$D^+$  under F:

result = D

no changes to result in the first turn of the while loop, so  $D^+ = D$

$D^+ = D$  does not contain B, so C is not extraneous in  $CD \rightarrow B$ .

for D:

$C^+$  under F:

result = C

1st while loop:

result = result  $\cup$  D (CD)

2nd while loop:

result = result  $\cup$  B (BCD)

no changes to result in the 3rd while loop, so  $C^+ = BCD$

$C^+ = BCD$  contains B, so D is extraneous in  $CD \rightarrow B$ .

After eliminating extraneous attribute on LHS,  $F' = \{ A \rightarrow BD, C \rightarrow B, C \rightarrow D, B \rightarrow D \}$  and if we unite  $C \rightarrow B$  and  $C \rightarrow D$ ,  $F' = \{ A \rightarrow BD, C \rightarrow BD, B \rightarrow D \}$

#### **Eliminating extraneous attributes on the righthand side:**

for  $A \rightarrow BD$ :

for B:

Compute  $A^+$  under  $\{ A \rightarrow D, C \rightarrow BD, B \rightarrow D \}$

result = A

1st while loop:

result = result  $\cup$  D (AD)

no changes to result in the 2nd while loop, so  $A^+ = AD$

$A^+ = AD$  does not contain B, so B is not extraneous in  $A \rightarrow BD$

for D:

Compute  $A^+$  under  $\{ A \rightarrow B, C \rightarrow BD, B \rightarrow D \}$

result = A

1st while loop:

result = result  $\cup$  B (AB)

result = result  $\cup$  D (ABD)

no changes to result in the 2nd while loop, so  $A^+ = ABD$

$A^+ = ABD$  contains D, so D is extraneous in  $A \rightarrow BD$ .

$F'' = \{ A \rightarrow B, C \rightarrow BD, B \rightarrow D \}$

for  $C \rightarrow BD$ :

for B:

Compute  $C^+$  under  $\{ A \rightarrow B, C \rightarrow D, B \rightarrow D \}$

result = C

1st while loop:

result = result  $\cup$  D (CD)

no changes to result in the 2nd while loop, so  $C^+ = CD$

$C^+ = CD$  does not contain B, so B is not extraneous in  $C \rightarrow BD$ .

for D:

Compute  $C^+$  under  $\{ A \rightarrow B, C \rightarrow B, B \rightarrow D \}$

result = C

1st while loop:

result = result  $\cup$  B (BC)

result = result  $\cup$  D (BCD)

no changes to result in the 2nd while loop, so  $C^+ = BCD$

$C^+ = BCD$  contains D, so D is extraneous in  $C \rightarrow BD$ .

$F''' = \{ A \rightarrow B, C \rightarrow B, B \rightarrow D \} = F_c$  (no other eliminations)

b) A and C don't appear on the right side of any functional dependency, so they must be part of any candidate key.

**(AC)<sup>+</sup>:**

result = AC

1st while:

result = result  $\cup$  BD (ABCD)

There is no change to result in the second while, so algorithm terminates.

(AC)<sup>+</sup> = ABCD, so AC is a superkey. It is also minimal, so it is a candidate key.

R is not in 3NF, as in the nontrivial functional dependency  $A \rightarrow BD$ , A is not a superkey and the attributes in BD - A = BD are not part of any candidate key.

**Lossless and dependency preserving decomposition:**

$F_c = \{ A \rightarrow B, C \rightarrow B, B \rightarrow D \}$

Set of decomposed relation: AB, CB, BD

None of the relations contains a candidate key, so add one more relation which contains the attributes of a candidate key: AB, CB, BD, AC

Decomposed relations: AB, BC, BD, AC