

2.1, 2.2, 2.4, 2.10, 2.14, 2.17.

(1)

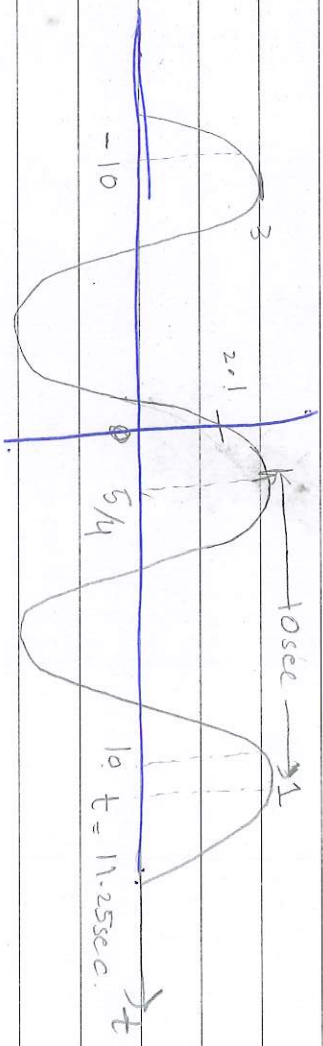
P-2.1 $x(t) = 3 \cos(\omega t - \pi/4)$

$\omega \sim \pi/5 \quad -10 \leq t \leq 20$

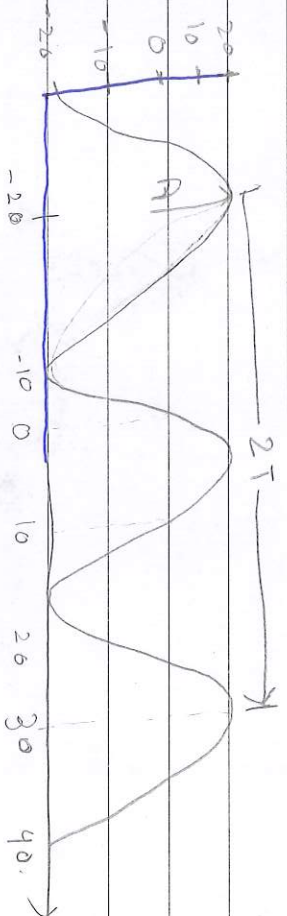
Solution: $2\pi f \sim \pi/5 \Rightarrow f = \frac{1}{10} \Rightarrow T = 10 \text{ sec (Period)}$

$\phi = -2\pi \frac{t}{T} \Rightarrow t_1 = \frac{-\phi}{T} = \frac{\pi/4}{2\pi} \times 10 = \frac{5}{4} = 1.25 \text{ sec}$
at $t = 0$

$x(t) = 3 \cos(-\pi/4) = \frac{3\sqrt{2}}{2} = 2.1$



P-2.2



Period: $2T = (30 - (-20)) \text{ msec} = 50 \text{ msec}$

$\Rightarrow T = 25 \text{ msec}$

Frequency: $\omega = 2\pi/T = 2\pi \left(\frac{1}{2.5 \times 10^{-3}} \right) = 2\pi (400) \text{ rad}$

$f = 40 \text{ Hz}$

Amplitude: $A = 20$

Time shift: $t_m = t + 5 \text{ msec}$

Phase:

$$\phi = -\omega t_m = -2\pi (40) \times 5 \times 10^{-3}$$

$$\phi = -2\pi (0.2) = -0.4\pi$$

$$x(t) = 20 \cos(80\pi t - 0.4\pi)$$

$$P_{2.4}:- e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \dots$$

$$= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} + \dots$$

Separate the real and imaginary parts.

$$e^{j\theta} = \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots \right) + j \left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots \right)$$

$$\underbrace{\hspace{10em}}_{\cos \theta} \quad \underbrace{\hspace{10em}}_{\sin \theta}$$

$$e^{j\theta} = \cos \theta + j \sin \theta$$

which proves Euler's formula

(3)

P2-10:

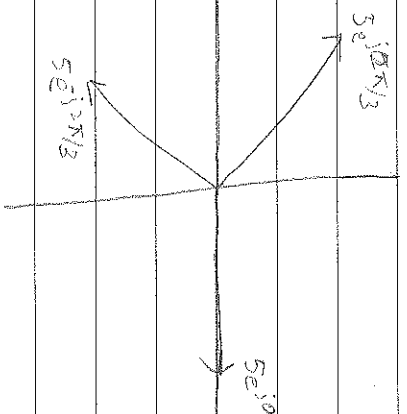
$$x(t) = 5 \cos(\omega t) + 5 \cos(\omega t + 120^\circ) + 5 \cos(\omega t - 120^\circ)$$

Use Phasors:

$$5 \cos(\omega t) \rightarrow 5e^{j0} = 5 + j0$$

$$5 \cos(\omega t + 120^\circ) \rightarrow 5e^{j2\pi/3} = -\frac{5}{2} + j\frac{5\sqrt{3}}{2}$$

$$5 \cos(\omega t - 120^\circ) \rightarrow 5e^{-j2\pi/3} = -\frac{5}{2} - j\frac{5\sqrt{3}}{2}$$



Vector Sum:

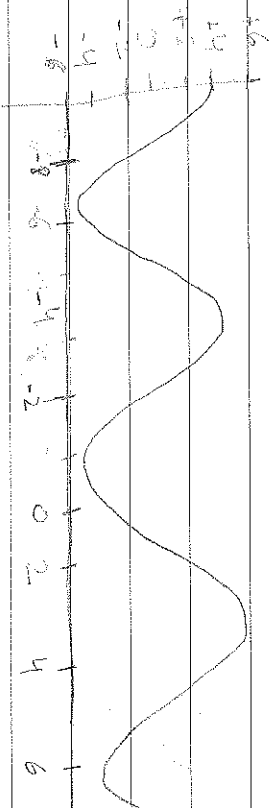
$$5 + \left(-\frac{5}{2} + j\frac{5\sqrt{3}}{2}\right) + \left(-\frac{5}{2} - j\frac{5\sqrt{3}}{2}\right)$$

$$= \left(5 - \frac{5}{2} - \frac{5}{2}\right) + j\left(\frac{5\sqrt{3}}{2} - \frac{5\sqrt{3}}{2}\right) = 0$$

Thus, $x(t) = 0$.

2.2.14.

positive peak at $t = -4 \text{ msec}$, value = 5
negative peak at $t = 6 \text{ msec}$



There are $1\frac{1}{2}$ periods from $t = -4 \text{ ms}$ to $t = 6 \text{ ms}$.

$$(1\frac{1}{2})T = 10 \text{ msec.}$$

$$\Rightarrow T = \frac{20 \text{ msec}}{3} = 6\frac{2}{3} \text{ msec}$$

$$\omega_b = \frac{2\pi}{T} = 2\pi / (20/3000) = 300\pi \text{ rad/sec}$$

$$\text{Phase: } \phi = -2\pi \left(\frac{t_1}{T} \right)$$

$$= -2\pi \left(\frac{-4}{20/3} \right) = \frac{12\pi}{5} = 1.2\pi$$

$$x(t) = 5 \cos(300\pi t + 1.2\pi)$$

For the complex notation $\tilde{x} = M e^{j\phi} e^{j\omega t}$

$$\tilde{x} = 5 e^{j1.2\pi}$$

$$x(t) = \text{Re} \{ 5 e^{j1.2\pi} e^{j300\pi t} \}$$

Q 17: $x(t) = 5 \cos(\omega_0 t + 3\pi/2) + 4 \cos(\omega_0 t + \pi/3) +$

(a) Express $x(t)$ in the form $x(t) = A \cos(\omega_0 t + \phi)$

$$z_1 = 5e^{j3\pi/2} = 0 - 5j$$

$$z_2 = 4e^{j2\pi/3}$$

$$z_3 = 4e^{j\pi/3} = 2 + j3.46$$

$$\left. \begin{array}{l} z_1 = 0 - 5j \\ z_2 = 2 + j3.46 \\ z_3 = 2 + j3.46 \end{array} \right\} = 2 + j1.928 = 1.928e^{j\pi/2}$$

$$\therefore x(t) = 1.928 \cos(\omega_0 t + \pi/2)$$

