

$$1) \text{ a) } 3e^{j\frac{\pi}{3}} + 4e^{-j\frac{\pi}{6}} = 3(\cos\frac{\pi}{3} + j\sin\frac{\pi}{3}) + 4(\cos(-\frac{\pi}{6}) + j\sin(-\frac{\pi}{6})) \\ = 3(\frac{1}{2} + j\frac{\sqrt{3}}{2}) + 4(\frac{\sqrt{3}}{2} - j\frac{1}{2}) = \frac{3}{2} + j\frac{3\sqrt{3}}{2} + \frac{4\sqrt{3}}{2} - j\frac{4}{2} \\ = \frac{3}{2} + 2\sqrt{3} + j(\frac{3\sqrt{3}}{2} - 2)$$

$$\text{b) } (1-j)^2 = 1 - 2j + j^2 = 1 - 2j + 1 = -2j = 2e^{j(-\frac{\pi}{2})}$$

$$\text{c) } (\sqrt{3} - 3j)^{10} \quad \begin{array}{l} x = \sqrt{3} \\ y = -3 \end{array} \quad \begin{array}{l} r = \sqrt{x^2 + y^2} = \sqrt{3+9} = 2\sqrt{3} \\ \theta = \arctan\left(\frac{-3}{\sqrt{3}}\right) = \arctan(-\sqrt{3}) = -\frac{\pi}{3} \end{array} \\ \downarrow \\ (2\sqrt{3} e^{-j\frac{\pi}{3}})^{10} = 248832 e^{-j\frac{10\pi}{3}}$$

$$\text{d) } \frac{(2+j\sqrt{2})}{(1+\sqrt{3}j)} \rightarrow \begin{array}{l} x = \sqrt{2}, y = \sqrt{2} \rightarrow r = \sqrt{2+2} = 2, \theta = \arctan(1) = \frac{\pi}{4} \Rightarrow 2e^{j\frac{\pi}{4}} \\ x = 1, y = \sqrt{3} \rightarrow r = \sqrt{1+3} = 2, \theta = \arctan(\sqrt{3}) = \frac{\pi}{3} \Rightarrow 2e^{j\frac{\pi}{3}} \end{array} \\ = \frac{2e^{j\frac{\pi}{4}}}{2e^{j\frac{\pi}{3}}} = e^{-j\frac{\pi}{12}}$$

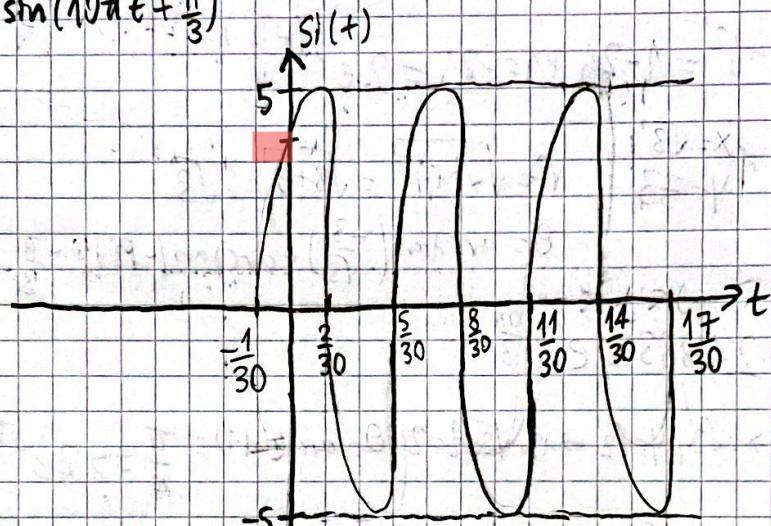
$$\text{e) } \operatorname{Re}\{je^{-j\frac{\pi}{3}}\} \quad je^{-j\frac{\pi}{3}} = j(\cos(-\frac{\pi}{3}) + j\sin(-\frac{\pi}{3})) = j(\frac{1}{2} - j\frac{\sqrt{3}}{2}) \\ = \frac{j}{2} + \frac{\sqrt{3}}{2} \Rightarrow \operatorname{Re}\{je^{-j\frac{\pi}{3}}\} = \frac{\sqrt{3}}{2}$$

$$\text{f) } j(1-j) = j - j^2 = 1 + j \Rightarrow x = 1, y = 1 \Rightarrow r = \sqrt{1+1} = \sqrt{2}, \theta = \arctan(1) = \frac{\pi}{4} \\ j(1-j) = \sqrt{2} e^{j\frac{\pi}{4}}$$

$$\text{g) } (\sqrt{3} - 3j)^{-1} \quad \begin{array}{l} x = \sqrt{3} \\ y = -3 \end{array} \quad \begin{array}{l} r = \sqrt{3+9} = 2\sqrt{3} \\ \theta = \arctan\left(\frac{-3}{\sqrt{3}}\right) = \arctan(-\sqrt{3}) = -\frac{\pi}{3} \end{array} \\ \downarrow \\ (2\sqrt{3} e^{-j\frac{\pi}{3}})^{-1} = \frac{1}{2\sqrt{3} e^{-j\frac{\pi}{3}}} = \frac{e^{j\frac{\pi}{3}}}{2\sqrt{3}} = \frac{\sqrt{3}}{6} e^{j\frac{\pi}{3}}$$

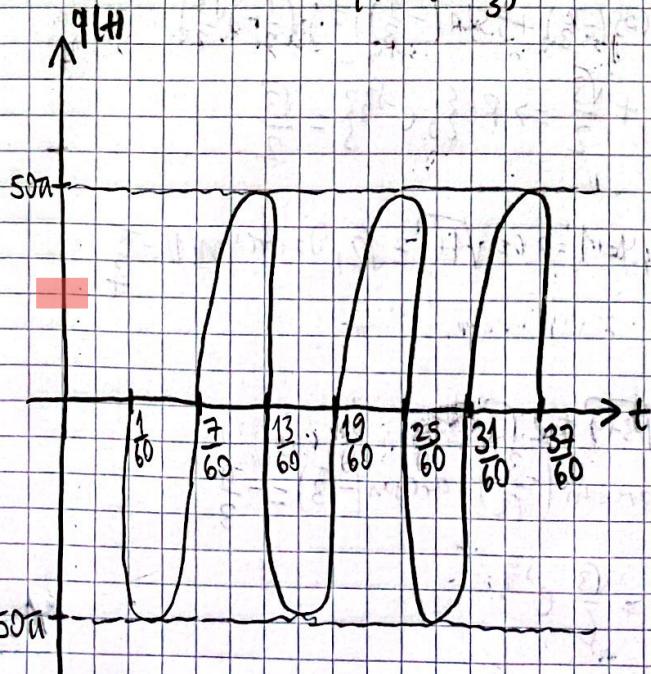
$$\begin{aligned}
 2) s(t) &= 5e^{j\frac{\pi}{3}} e^{j10at} = 5e^{j(10at + \frac{\pi}{3})} \\
 &= 5(\cos(10at + \frac{\pi}{3}) + j\sin(10at + \frac{\pi}{3})) \\
 &= 5\cos(10at + \frac{\pi}{3}) + 5j\sin(10at + \frac{\pi}{3})
 \end{aligned}$$

$$s_1(t) = 5\sin(10at + \frac{\pi}{3})$$



$$\begin{aligned}
 b) \dot{s}(t) &= -5\sin(10at + \frac{\pi}{3}) 10a + 5j\cos(10at + \frac{\pi}{3}), 10a \\
 &= -50a \sin(10at + \frac{\pi}{3}) + 50a j \cos(10at + \frac{\pi}{3})
 \end{aligned}$$

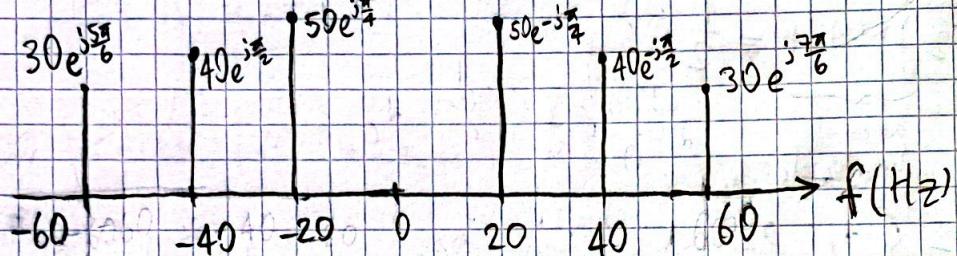
$$|m\{\dot{s}(t)\}| = q(t) = 50a \cos(10at + \frac{\pi}{3})$$



3)

$$\begin{aligned}
 X(t) &= 100\cos(40\pi t - \frac{\pi}{4}) + 80\sin(80\pi t) - 60\cos(120\pi t + \frac{\pi}{6}), \\
 &= 50e^{j40\pi t} e^{-j\frac{\pi}{4}} + 50e^{-j40\pi t} e^{j\frac{\pi}{4}} + 40e^{j80\pi t} + j40e^{-j80\pi t} \\
 &\quad - 30e^{j120\pi t} e^{j\frac{\pi}{6}} - 30e^{-j120\pi t} e^{-j\frac{\pi}{6}} \\
 &= 50e^{j40\pi t} e^{-j\frac{\pi}{4}} + 50e^{-j40\pi t} e^{j\frac{\pi}{4}} + 40e^{j80\pi t} e^{-j\frac{\pi}{2}} + 40e^{-j80\pi t} e^{j\frac{\pi}{2}} + 30e^{j120\pi t} e^{j\frac{7\pi}{6}} + 30e^{-j120\pi t} e^{-j\frac{5\pi}{6}}
 \end{aligned}$$

a)



b)

$$\begin{aligned}
 X(t) &= 100\cos(40\pi t - \frac{\pi}{4}) + 80\sin(80\pi t) - 60\cos(120\pi t + \frac{\pi}{6}) \\
 &\quad \underbrace{f=20\text{Hz}} \quad \underbrace{f=40\text{Hz}} \quad \underbrace{f=60\text{Hz}}
 \end{aligned}$$

$\text{GCD}(20, 40, 60) = 20 \Rightarrow X(t)$ is periodic with $f=20\text{Hz}$, $T=\frac{1}{20}=0.05\text{s}$

$$\begin{aligned}
 Y(t) &= X(t) + 90\cos(60\pi t + \frac{\pi}{6}) = 45e^{j\frac{\pi}{6}} e^{j60\pi t} + 45e^{-j\frac{\pi}{6}} e^{-j60\pi t} + X(t) \\
 &\quad \underbrace{f=30\text{Hz}}
 \end{aligned}$$

In the spectrum: at 30 Hz there will be $45e^{j\frac{\pi}{6}}$
at -30 Hz there will be $45e^{-j\frac{\pi}{6}}$

$\text{GCD}(20, 30, 40, 60) = 10 \Rightarrow X(t)$ is periodic with $f=10\text{Hz}$, $T=\frac{1}{10}=0.1\text{s}$

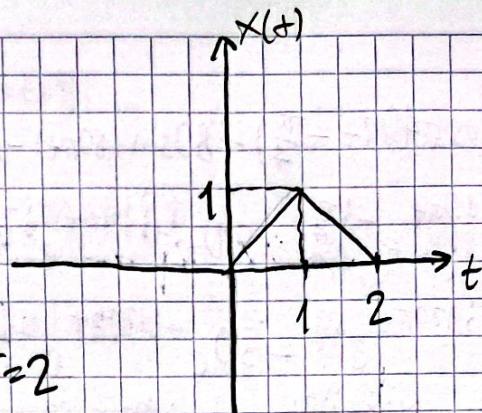
$$\begin{aligned}
 W(t) &= X(t) + 10\cos(280t + \frac{\pi}{2}) = X(t) + 5e^{j\frac{\pi}{2}} e^{j280t} + 5e^{-j\frac{\pi}{2}} e^{-j280t} \\
 &\quad \underbrace{f=\frac{140}{\pi}\text{Hz}}
 \end{aligned}$$

In the spectrum: at $\frac{140}{\pi}\text{ Hz}$ there will be $5e^{j\frac{\pi}{2}}$

at $-\frac{140}{\pi}\text{ Hz}$ there will be $5e^{-j\frac{\pi}{2}}$

$\text{GCD}(20, 40, \frac{140}{\pi}, 60)$ is undefined, so $X(t)$ is not periodic

4) $x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t, & 1 \leq t \leq 2 \end{cases}$



a) $a_0 = \frac{1}{2} \int_0^2 x(t) dt = \frac{1}{2}$

area of
the triangle = 1

b) $\frac{dx(t)}{dt} = \begin{cases} 1, & 0 \leq t \leq 1 \\ -1, & 1 \leq t \leq 2 \end{cases}$

$$\begin{aligned} a_k &= \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\left(\frac{2\pi}{T_0}\right)kt} dt = \frac{1}{2} \int_0^2 x(t) e^{-j\pi kt} dt \\ &= \frac{1}{2} \left(\int_0^1 e^{-j\pi kt} dt - \int_1^2 e^{-j\pi kt} dt \right) \\ &= \frac{1}{2} \left(\frac{e^{-j\pi kt}}{-j\pi k} \Big|_0^1 - \frac{e^{-j\pi kt}}{-j\pi k} \Big|_1^2 \right) = \frac{1}{2} \left(\frac{j}{\pi k} (e^{-j\pi k} - 1) - \frac{j}{\pi k} (e^{-2j\pi k} - e^{-j\pi k}) \right) \\ &= \frac{1}{2} \frac{j}{\pi k} (e^{-j\pi k} - 1 - e^{-2j\pi k} + e^{-j\pi k}) = \frac{1}{2} \frac{j}{\pi k} (2e^{-j\pi k} - e^{-2j\pi k} - 1) \end{aligned}$$

$$= \frac{1}{2} \frac{j}{\pi k} (2e^{-j\pi k} - 2) = \frac{j}{\pi k} (e^{-j\pi k} - 1) = \frac{j}{\pi k} ((-1)^k - 1)$$

$$a_0 = \frac{1}{2} \int_0^2 x(t) dt = \frac{1}{2} \left(\int_0^1 1 dt + \int_1^2 (-1) dt \right) = \frac{1}{2} (1 - 1) = 0$$

$$a_k = \begin{cases} 0 & \text{if } k \text{ even} \\ \frac{-2j}{\pi k} & \text{if } k \text{ odd} \end{cases}$$

$$C) b_k = \begin{cases} 0 & \text{if } k \text{ even} \\ -\frac{2}{\pi k} & \text{if } k \text{ odd} \end{cases}$$

$$b_k = (jkw_0)a_k \Rightarrow -\frac{2j}{\pi k} = jk \cdot \frac{\pi}{2} a_k \Rightarrow a_k = -\frac{2}{\pi^2 k^2}$$

$$a_k = \begin{cases} 0 & \text{if } k \text{ even} \\ -\frac{2}{\pi^2 k^2} & \text{if } k \text{ odd} \end{cases}$$

$$5) X[n] = 2.2 \cos(0.3\pi n - \frac{\pi}{3}) \quad f_s = 6000 \text{ samples/sec.}$$

$$\hat{w} = \frac{w}{f_s} \quad 0.3\pi = \frac{w}{6000} \Rightarrow w = 1800\pi$$

$$f = \frac{1800\pi}{2\pi} = 9 \text{ kHz}$$

$$1) X(t) = 2.2 \cos(1800\pi t - \frac{\pi}{3})$$

$$2) X_1[n] = 2.2 \cos(0.3\pi n + 2\pi n - \frac{\pi}{3}) = 2.2 \cos(2.3\pi n - \frac{\pi}{3})$$

$$2.3\pi = \frac{w}{6000} \Rightarrow w = 13800\pi$$

$$f = 6900 \text{ Hz} = 6.9 \text{ kHz}$$

$$X_1(t) = 2.2 \cos(13800\pi t - \frac{\pi}{3})$$

$$3) X_2[n] = 2.2 \cos(-0.3\pi n + 2\pi n + \frac{\pi}{3}) = 2.2 \cos(1.7\pi n + \frac{\pi}{3}) \Rightarrow \text{folded alias}$$

$$1.7\pi = \frac{w}{6000} \quad w = 10200\pi$$

$$f = 5100 \text{ Hz} = 5.1 \text{ kHz}$$

$$X_2(t) = 2.2 \cos(10200\pi t + \frac{\pi}{3})$$

$$\begin{aligned}
 6) a) y_1[n] &= x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k] \times [n-k] \\
 &= h[-1] \times [n+1] + h[1] \times [n-1] = 2x[n+1] + 2x[n-1] \\
 &= 2(\delta[n+1] + 2\delta[n] - \delta[n-2]) + 2(\delta[n-1] + 2\delta[n-2] - \delta[n-4]) \\
 &= 2\delta[n+1] + 4\delta[n] - 2\delta[n-2] + 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-4] \\
 &= 2\delta[n+1] + 4\delta[n] + 2\delta[n-2] + 2\delta[n-1] - 2\delta[n-4]
 \end{aligned}$$

$$\begin{aligned}
 b) y_2[n] &= x[n+2] * h[n] \\
 &= \sum_{k=-\infty}^{\infty} h[k] \times [(n+2)-k] \Rightarrow y_2[n] = y_1[n+2] \text{ since system is LTI}
 \end{aligned}$$

$$y_2[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n] + 2\delta[n+1] - 2\delta[n-2]$$

$$\begin{aligned}
 c) y_3[n] &= x[n] * h[n+2] \\
 &= \sum_{k=-\infty}^{\infty} h[k+2] \times [n-k] \Rightarrow k+2 = k' \quad \left\{ \begin{array}{l} y_3[n] = \sum_{k=k'-2}^{\infty} h[k'] \times [(n+2)-k'] \\ k'= -\infty \end{array} \right.
 \end{aligned}$$

$$\Rightarrow y_3[n] = y_1[n+2] \text{ since system is LTI}$$

$$y_3[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n] + 2\delta[n+1] - 2\delta[n-2]$$

$$\begin{aligned}
 7) a) H(e^{j\hat{\omega}}) &= \sum_{k=0}^2 b_k e^{-j\hat{\omega}k} = 2 - 3e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} \\
 &= e^{-j\hat{\omega}} (2e^{j\hat{\omega}} - 3 + 2e^{-j\hat{\omega}}) \\
 &= (4\cos\hat{\omega} - 3) e^{-j\hat{\omega}}
 \end{aligned}$$

$$\begin{aligned}
 b) H(e^{j\hat{\omega}}) &= 2 \underbrace{-3e^{-j\hat{\omega}}}_{T \geq 2\pi} + 2 \underbrace{e^{-j2\hat{\omega}}}_{T \geq \pi}
 \end{aligned}$$

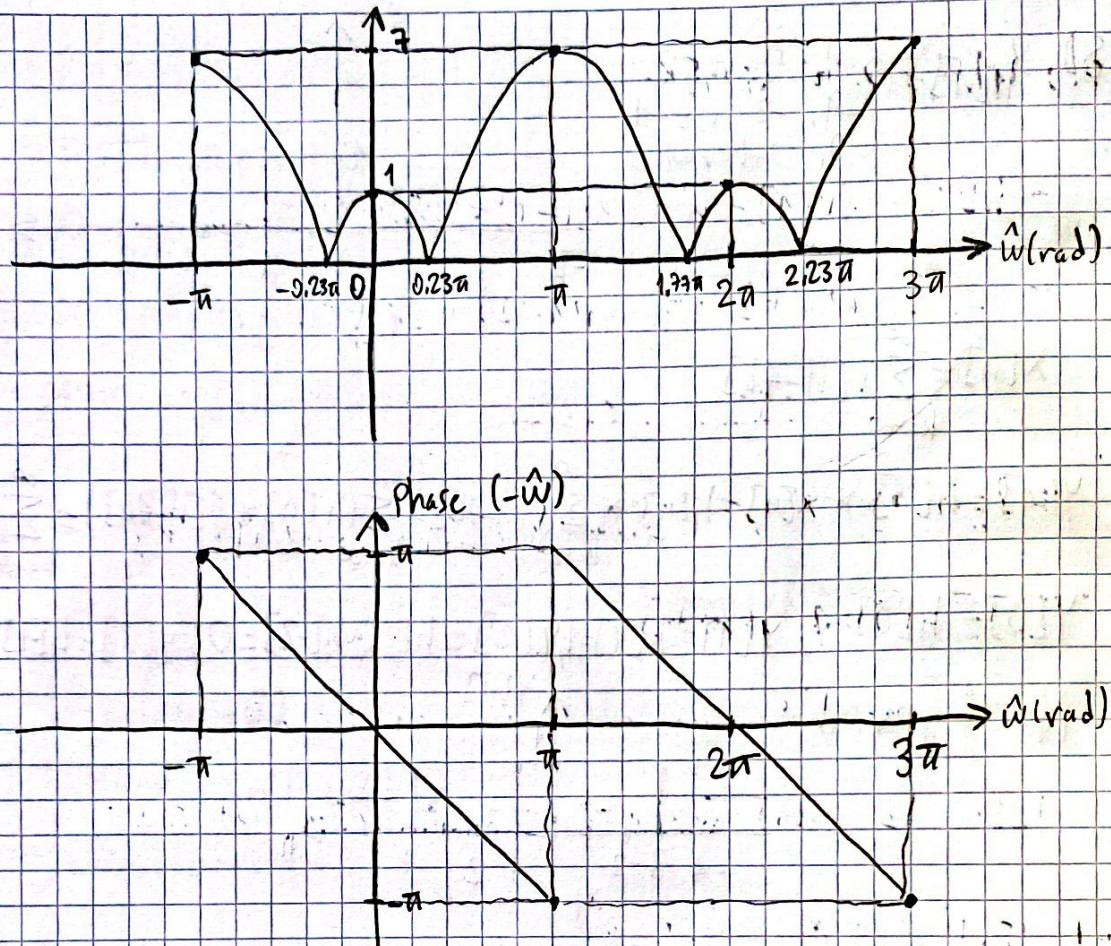
$$\text{LCM}(2\pi, \pi) = 2\pi \Rightarrow T = 2\pi$$

Subject :

Magnitude ($|4\cos\hat{\omega} - 3|$)

Date : / /

C)



$$d) H(e^{j\hat{\omega}}) = (4\cos(\hat{\omega}) - 3)e^{j\hat{\omega}} = 0 \Rightarrow 4\cos\hat{\omega} - 3 = 0$$

$$\cos\hat{\omega} = \frac{3}{4}$$

$$\Rightarrow \hat{\omega} = \arccos\left(\frac{3}{4}\right) + 2\pi l$$

$$\text{or } \hat{\omega} = -\arccos\left(\frac{3}{4}\right) + 2\pi l$$

$$l = 0, \pm 1, \pm 2, \dots$$

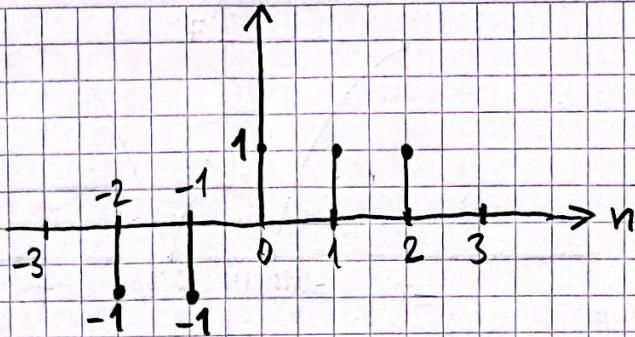
$$e) \sin\left(\frac{\pi}{13}n\right) = \frac{1}{2j} e^{j\frac{\pi n}{13}} - \frac{1}{2j} e^{-j\frac{\pi n}{13}}, \quad H\left(\frac{\pi}{13}\right) = 0,8838 e^{-j0,077\pi}$$

$$y[n] = H\left(\frac{\pi}{13}\right) \frac{1}{2j} e^{j\frac{\pi n}{13}} - H\left(-\frac{\pi}{13}\right) \frac{1}{2j} e^{-j\frac{\pi n}{13}}$$

$$= \frac{1}{2j} \left(0,8838 e^{j(\frac{\pi n}{13} - 0,077\pi)} - 0,8838 e^{-j(\frac{\pi n}{13} - 0,077\pi)} \right)$$

$$= 0,8838 \sin\left(\frac{\pi n}{13} - 0,077\pi\right) = 0,8838 \cos\left(\frac{\pi n}{13} - 0,577\pi\right) = y[n]$$

8) $h[n] = \begin{cases} 1, & 0 \leq n \leq 2 \\ -1, & -2 \leq n \leq -1 \\ 0, & \text{otherwise} \end{cases}$

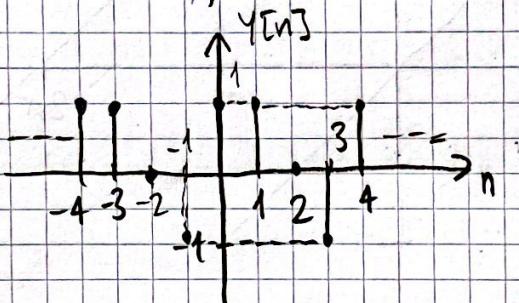


$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

$$y[n] = h[n] * x[n] = h[n] * \sum_{k=-\infty}^{\infty} \delta[n-4k] = \sum_{k=-\infty}^{\infty} (h[n] * \delta[n-4k]) = \sum_{k=-\infty}^{\infty} h[n-4k]$$

$$Y[0] = h[0] = 1, Y[1] = h[1] = 1, Y[2] = h[2] + h[-2] = 0, Y[3] = h[3] = -1 \dots$$

$N=4$ period



$$\begin{aligned} a_k &= \frac{1}{N} \sum_{n=0}^{N-1} y[n] e^{-j \frac{2\pi k n}{N}} = \frac{1}{4} \sum_{n=0}^{3} y[n] e^{-j \frac{2\pi k n}{4}} = \frac{1}{4} \left(1 + e^{-j \frac{\pi k}{2}} + 0 + (-1) e^{-j \frac{3\pi k}{2}} \right) \\ &= \frac{1}{4} \left(1 + e^{-j \pi k} \left(e^{j \frac{5\pi k}{2}} - e^{-j \frac{5\pi k}{2}} \right) \right) = \frac{1}{4} \left(1 + (-1)^k \cdot 2j \sin\left(\frac{\pi k}{2}\right) \right) \end{aligned}$$

$$a_k = \frac{1}{4} + \frac{3}{2} (-1)^k \sin\left(\frac{\pi k}{2}\right)$$

$$9) \text{ a) } x(t) = 10 \cos(880\pi t + \phi)$$

$$x[n] = 10 \cos(880\pi n T_s + \phi)$$

$$T_s = 0.0001 \text{ sec} = 10^{-4} \text{ sec}$$

$$f_s = 10000 \text{ samples/sec}$$

$$2\pi f_0 = 880\pi \Rightarrow f_0 = 440 \text{ Hz}, T_0 = \frac{1}{440} \text{ s}$$

$$\frac{10000}{440} = \frac{1000}{44} \approx 22.72 \Rightarrow 22 \text{ intervals}$$

\Rightarrow There are 23 Samples in one period

$$\text{b) } y(t) = 10 \cos(\omega_0 t + \phi)$$

$$y(n T_s) = 10 \cos(\omega_0 n T_s + \phi)$$

$$y[n] = 10 \cos(\omega_0 n T_s + \phi)$$

$$\frac{880\pi}{10000} + 2\pi = \frac{20000\pi + 880\pi}{10000} = \frac{20880\pi}{10000} = \hat{\omega}_y$$

$$\hat{\omega}_y = \frac{20880\pi}{10000} = \frac{\omega_0}{10000}, \quad \omega_y = \omega_0 = 20880\pi$$

$$\text{c) } f_s = 10000 \text{ samples/sec}$$

$$T_0 = \frac{1}{10440} \text{ s}$$

$$\frac{10000}{10440} \approx 0.9578$$

\Rightarrow 1 sample in one period

10) a) $y[n] = x[-n]$

Linearity: $w[n] = \alpha x_1[n] + \beta x_2[-n]$

$$\alpha x_1[n] + \beta x_2[-n] \mapsto \alpha x_1[-n] + \beta x_2[-n]$$

System is linear

Time Invariance: $x[n] \xrightarrow{\text{delay}} x[n-n_0] \mapsto w[n] = x[-(n)-n_0] = x[-n-n_0]$

$$y[n-n_0] = x[-(n-n_0)] = x[-n+n_0] \neq w[n]$$

System is not time invariant

Causality: If $n < 0$, system looks to the future, so not causal.

b) $y[n] = x[n-2] - 2x[n-8]$

Linearity: $\left. \begin{array}{l} x_1[n] \mapsto x_1[n-2] - 2x_1[n-8] \\ x_2[n] \mapsto x_2[n-2] - 2x_2[n-8] \end{array} \right\} w[n] = \alpha y_1[n] + \beta y_2[n]$

$$\begin{aligned} &= \alpha x_1[n-2] - 2\alpha x_1[n-8] \\ &\quad + \beta x_2[n-2] - 2\beta x_2[n-8] \\ \alpha x_1[n] + \beta x_2[n] \mapsto & \alpha x_1[n-2] + \beta x_2[n-2] - 2(\alpha x_1[n-8] + \beta x_2[n-8]) \\ &= \alpha x_1[n-2] - 2\alpha x_1[n-8] + \beta x_2[n-2] - 2\beta x_2[n-8] \end{aligned}$$

System is linear

Time Invariance: $x[n] \xrightarrow{\text{delay by } n_0} x[n-n_0] \mapsto w[n] = x[n-n_0-2] - 2x[n-n_0-8]$

$$y[n-n_0] = x[n-n_0-2] - 2x[n-n_0-8]$$

$$w[n] = y[n-n_0]$$

System is time invariant

Causality: The output doesn't depend on future inputs, System is causal.

$$c) Y[n] = \text{Even}\{x[n-1]\} = \frac{x[n-1] + x[-n+1]}{2}$$

Linearity:

$$\left. \begin{aligned} x_1[n] &\mapsto \frac{x_1[n-1] + x_1[-n+1]}{2} \\ x_2[n] &\mapsto \frac{x_2[n-1] + x_2[-n+1]}{2} \end{aligned} \right\} \quad \left. \begin{aligned} w[n] &= \alpha x_1[n-1] + \alpha x_1[-n+1] + \beta x_2[n-1] + \beta x_2[-n+1] \\ &= \frac{\alpha x_1[n-1] + \beta x_2[n-1] + \alpha x_1[-n+1] + \beta x_2[-n+1]}{2} \end{aligned} \right\} = w[n]$$

System is linear

Time Invariance:

$$x[n] \xrightarrow{\text{delay}} x[n-n_0] \mapsto \frac{x[n-n_0-1] + x[-n+n_0+1]}{2} = w[n]$$

$$y[n-n_0] = \frac{x[n-n_0-1] + x[-n+n_0+1]}{2} \neq w[n]$$

System is not time invariant

Causality: If $x < 0$, output depends on future inputs, so not causal

d) $y[n] = n x[n]$

Linearity: $\left. \begin{array}{l} x_1[n] \mapsto n x_1[n] \\ x_2[n] \mapsto n x_2[n] \end{array} \right\} w[n] = \alpha n x_1[n] + \beta n x_2[n]$

//

$$\alpha x_1[n] + \beta x_2[n] \mapsto n(\alpha x_1[n] + \beta x_2[n]) = w[n]$$

System is Linear

Time Invariance: $x[n] \xrightarrow{\text{delay}} x[n-n_0] \mapsto n x[n-n_0] = w[n]$

$$y[n-n_0] = (n-n_0) x[n-n_0] \neq w[n]$$

System is not time invariant

Causality: System doesn't depend on future inputs, it is causal
 (at $n=n_0$, system depends on $x[n_0]$)

e) $y[n] = x[4n+1]$

Linearity: $\left. \begin{array}{l} x_1[n] \mapsto x_1[4n+1] \\ x_2[n] \mapsto x_2[4n+1] \end{array} \right\} w[n] = \alpha x_1[4n+1] + \beta x_2[4n+1]$

$$\alpha x_1[n] + \beta x_2[n] \mapsto \alpha x_1[4n+1] + \beta x_2[4n+1] = w[n]$$

System is linear

Time Invariance: $x[n] \xrightarrow{\text{delay}} x[n-n_0] \mapsto x[4n+1-n_0] = w[n]$

$$y[n-n_0] = x[4n-4n_0+1] \neq w[n]$$

System is not time invariant

Causality: System depends on future inputs, so it is not causal.
 ($n=5, y[5]=x[21]$)