EEE 391: Basics of Signals and Systems

Homework 2

Due:

1) An LTI filter is described by the difference equation

$y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$

a) What is the impulse response h[n] of this system ?
b) Obtain an expression for the frequency response of this system.
c) Suppose that the input is

$x[n] = 5 + 4\cos(0.2\pi n) + 3\cos(0.5\pi n)$ for $-\infty<n<\infty$. Obtain an expression for the output in the form $y[n] = A + B\cos(w_0 n + \varnothing_0)$.

d) Suppose that the input is

$x[n] = [5 + 4\cos(0.2\pi n) + 3\cos(0.5\pi n)]\, u[n]$ where u[n] is the unit step sequence. For what values of n will the output $y_1[n]$ be equal to the output y[n] in c?

2) Determine the z transform for a,b and c.Determine inverse z transform for d and e.
a) $\delta[n+5]$
b) $\delta[n-5]$
c) $\delta[n-1]$
d) $X(z) = 2+4z^{-1}+6z^{-2}+4z^{-3}+2z^{-4}$
e) $X(z) = 1-2z^{-1}+3z^{-3}-z^{-5}$

3) Two causal LTI systems are described by the difference equations.

$$y[n] = y[n-1] + y[n-2] + x[n-1].$$

$$y[n-1]-5/2y[n] + y[n+1] = x[n].$$

Find the system functions H(z) = Y(z)/X(z) for both systems. Plot their poles and zeros of H(z).

4) One form of deconvolution process starts with the output signal and the filter's impulse response, from which it should be possible to find the input signal.

a) If the output of an FIR Filter with $h[n] = \delta[n - 2]$ is

$y[n] = u[n-3] – u[n-6]$,

determine the input signal, x[n].

b) If the output of a first difference FIR filter is

$y[n] = \delta[n] – \delta[n-4]$,

determine the input signal, x[n].

c) If the output of four point averager is

$y[n] = -5\delta[n] - 5\delta[n-2]$,

determine the input signal, x[n].

5) Consider a four point, moving average, discrete – time filter for which the difference equation is

$$y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + b_3x[n-3]$$

Determine and sketch the magnitude of the frequency response for each of the following cases:

a) $b_0 = b_3 = 0,\ b_1 = b_2$
b) $b_1 = b_2 = 0,\ b_0 = b_3$
c) $b_0 = b_1 = b_2 = b_3$
d) $b_0 = -b_1 = b_2 = -b_3$

6) The impulse response of a linear time – invariant system is

$$h(t) = \begin{cases} e^{-0.1(t-2)} & 2 \le t < 12 \\ 0 & \text{otherwise} \end{cases}$$

a) Is the system stable ? Justify your answer.
b) Is the system causal ? Justify your answer.
c) Find the output y(t) when the input is $x(t) = \delta(t-2)$.

7) Consider an ideal bandpass filter whose frequency response in the region $-\pi \le w \le \pi$ is specified as

$$H(e^{jw}) = \begin{cases} 1, & \pi/2 - w_c \le |w| \le \pi/2 + w_c \\ 0, & \text{otherwise} \end{cases}$$

Determine and sketch the impulse response h[n] for this filter when

a) $w_c = \pi/5$
b) $w_c = \pi/4$
c) $w_c = \pi/3$

As w_c increased, does h[n] get more or less concentrated about the origin?

8) For each of the following systems, determine whether or not the systems is linear, time-invariant , stable, causal. In each example, y(t) denotes the system output and x(t) denotes the system input.

a) $y(t) = x(t-2) + x(2-t)$
b) $y(t) = [\cos(3t)]x(t)$
c) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$
d) $y(t) = \begin{cases} 0, & t<0 \\ x(t) + x(t-2) & t \ge 0 \end{cases}$

e) $y(t) = \{ 0, \quad x(t)<0$
$\quad x(t) + x(t-2) \quad x(t)\geq 0\}$
f) $y(t) = x(t/3)$
g) $y(t) = dx(t) /dt$

9) Given an IIR filter defined by the difference equation

$y[n] = -y[n-5] + x[n]$

a) Determine the system function $H(z)$
b) How many poles does the system have? Compute and plot the pole locations.
c) When the input to the system is the two point pulse signal:

$y(t) = \{ +1, \quad \text{when } n= 0,1$
$\quad 0 , \quad \text{when } n\neq 0,1$

determine the output signal $y[n]$, so that you can make a plot of its general form. Assume that the output signal is zero for $n<0$.

d) The output signal is periodic for $n>0$. Determine the period.

10) The signal $y(t)$ is generated by convolving a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$, that is,

$y(t) = x_1(t) * x_2(t)$
where
$X_1(jw) = 0 \quad \text{for, } |w| > 1000\pi$
$X_2(jw) = 0 \quad \text{for, } |w| > 2000\pi$
Impulse train sampling is performed on $y(t)$ to obtain

$$yp(t) = a_0 + \sum_{n=-\infty}^{\infty} y(nT)\delta(\mathrm{t}-\mathrm{nT})$$

Specify the range of values for the sampling period T which ensures that $y(t)$ is recoverable from $yp(t)$.