EEE391 Analytical Assignment 1
Fall 2022

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CS

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1) a) $3e^{j\frac{\pi}{3}} + 4e^{-j\frac{\pi}{6}} = 3\left(\cos\frac{\pi}{3} + j\sin\frac{\pi}{3}\right) + 4\left(\cos\left(-\frac{\pi}{6}\right) + j\sin\left(-\frac{\pi}{6}\right)\right)$

$= 3\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 4\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) = \frac{3}{2} + j\frac{3\sqrt{3}}{2} + \frac{4\sqrt{3}}{2} - \frac{4j}{2}$

$= \frac{3}{2} + 2\sqrt{3} + j\left(\frac{3\sqrt{3}}{2} - 2\right)$

b) $(1-j)^2 = 1 - 2j + j^2 = 1 - 2j + 1 = -2j = 2e^{j\left(-\frac{\pi}{2}\right)}$

c) $(\sqrt{3} - 3j)^{10}$ $\quad x = \sqrt{3}$, $y = -3$ $\quad r = \sqrt{x^2 + y^2} = \sqrt{3+9} = 2\sqrt{3}$

$\theta = \arctan\left(\frac{-3}{\sqrt{3}}\right) = \arctan(-\sqrt{3}) = -\frac{\pi}{3}$

$\downarrow$

$\left(2\sqrt{3}e^{-j\frac{\pi}{3}}\right)^{10} = 248832\, e^{-j\frac{10\pi}{3}}$

d) $\frac{(\sqrt{2} + \sqrt{2}j)}{(1 + \sqrt{3}j)}$

$(\sqrt{2} + \sqrt{2}j) \rightarrow x = \sqrt{2}, y = \sqrt{2} \rightarrow r = \sqrt{2+2} = 2, \theta = \arctan(1) = \frac{\pi}{4} \Rightarrow 2e^{j\frac{\pi}{4}}$

$(1 + \sqrt{3}j) \rightarrow x = 1, y = \sqrt{3} \rightarrow r = \sqrt{1+3} = 2, \theta = \arctan(\sqrt{3}) = \frac{\pi}{3} \Rightarrow 2e^{j\frac{\pi}{3}}$

$= \frac{2e^{j\frac{\pi}{4}}}{2e^{j\frac{\pi}{3}}} = e^{-j\frac{\pi}{12}}$

e) $\text{Re}\{je^{-j\frac{\pi}{3}}\}$ $\quad je^{-j\frac{\pi}{3}} = j\left(\cos\left(-\frac{\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)\right) = j\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$

$= \frac{j}{2} + \frac{\sqrt{3}}{2} \Rightarrow \text{Re}\{je^{-j\frac{\pi}{3}}\} = \frac{\sqrt{3}}{2}$

f) $j(1-j) = j - j^2 = 1 + j \Rightarrow x = 1, y = 1 \Rightarrow r = \sqrt{1+1} = \sqrt{2}, \theta = \arctan(1) = \frac{\pi}{4}$

$j(1-j) = \sqrt{2}e^{j\frac{\pi}{4}}$

g) $(\sqrt{3} - 3j)^{-1}$ $\quad x = \sqrt{3}$, $y = -3$ $\quad r = \sqrt{3+9} = 2\sqrt{3}$

$\theta = \arctan\left(\frac{-3}{\sqrt{3}}\right) = \arctan(-\sqrt{3}) = -\frac{\pi}{3}$

$\downarrow$

$\left(2\sqrt{3}e^{-j\frac{\pi}{3}}\right)^{-1} = \frac{1}{2\sqrt{3}e^{-j\frac{\pi}{3}}} = \frac{e^{j\frac{\pi}{3}}}{2\sqrt{3}} = \frac{\sqrt{3}}{6}e^{j\frac{\pi}{3}}$

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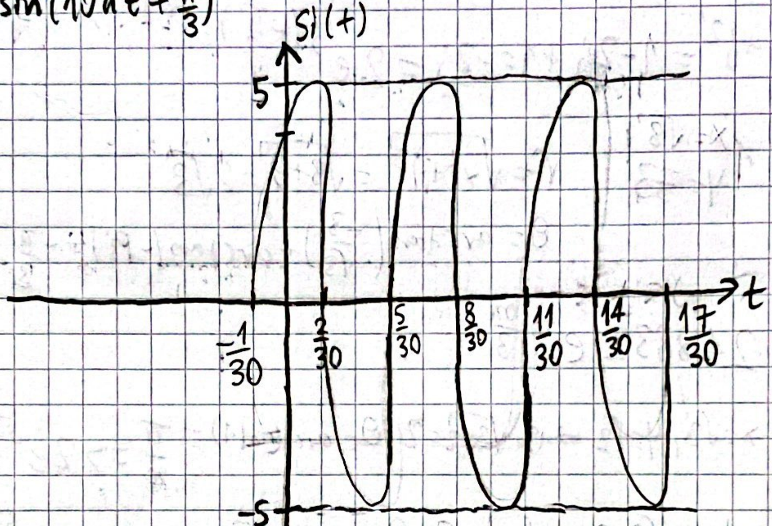
2) $s(t) = 5e^{j\frac{\pi}{3}} e^{j10\pi t} = 5e^{j(10\pi t+\frac{\pi}{3})}$

$= 5(\cos(10\pi t+\frac{\pi}{3}) + j\sin(10\pi t+\frac{\pi}{3}))$

$= 5\cos(10\pi t+\frac{\pi}{3}) + 5j\sin(10\pi t+\frac{\pi}{3})$

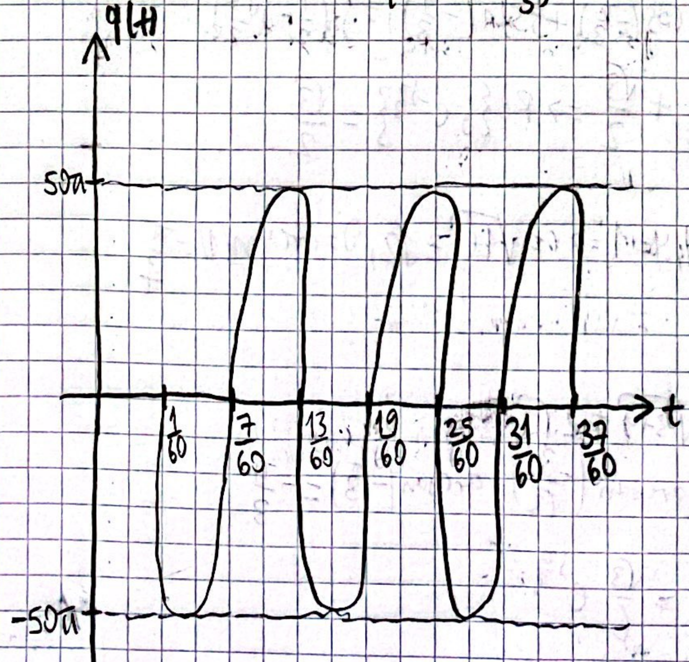
$s_i(t) = 5\sin(10\pi t+\frac{\pi}{3})$



b) $\dot{s}(t) = -5\sin(10\pi t+\frac{\pi}{3})10\pi + 5j\cos(10\pi t+\frac{\pi}{3}).10\pi$

$= -50\pi\sin(10\pi t+\frac{\pi}{3}) + 50\pi j\cos(10\pi t+\frac{\pi}{3})$

$\mathrm{Im}\{\dot{s}(t)\} = q(t) = 50\pi\cos(10\pi t+\frac{\pi}{3})$



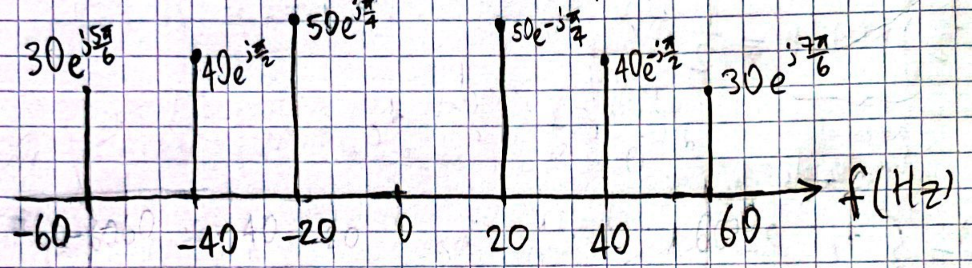
3) $x(t) = 100\cos(40\pi t - \frac{\pi}{4}) + 80\sin(80\pi t) - 60\cos(120\pi t + \frac{\pi}{6})$

$$= 50e^{j40\pi t}e^{-j\frac{\pi}{4}} + 50e^{-j40\pi t}e^{j\frac{\pi}{4}} - j40e^{j80\pi t} + j40e^{-j80\pi t}$$

$$- 30e^{j120\pi t}e^{j\frac{\pi}{6}} - 30e^{-j120\pi t}e^{-j\frac{\pi}{6}}$$

$$= 50e^{j40\pi t}e^{-j\frac{\pi}{4}} + 50e^{-j40\pi t}e^{j\frac{\pi}{4}} + 40e^{j80\pi t}e^{-j\frac{\pi}{2}} + 40e^{-j80\pi t}e^{j\frac{\pi}{2}} + 30e^{j120\pi t}e^{j\frac{7\pi}{6}} + 30e^{-j120\pi t}e^{+j\frac{5\pi}{6}}$$

a)



b) $x(t) = 100\cos(\underbrace{40\pi t - \frac{\pi}{4}}_{f=20Hz}) + 80\sin(\underbrace{80\pi t}_{f=40Hz}) - 60\cos(\underbrace{120\pi t + \frac{\pi}{6}}_{f=60Hz})$

GCD(20,40,60) = 20 => $x(t)$ is periodic with $f = 20Hz$, $T = \frac{1}{20} = 0.05s$

c) $y(t) = x(t) + 90\cos(\underbrace{60\pi t + \frac{\pi}{6}}_{f=30Hz}) = 45e^{j\frac{\pi}{6}}e^{j60\pi t} + 45e^{-j\frac{\pi}{6}}e^{-j60\pi t} + x(t)$

In the spectrum: at 30 Hz there will be $45e^{j\frac{\pi}{6}}$

at -30Hz there will be $45e^{-j\frac{\pi}{6}}$

GCD(20,30,40,60) = 10 => $x(t)$ is periodic with $f = 10Hz$, $T = \frac{1}{10} = 0.1s$

d) $w(t) = x(t) + 10\cos(\underbrace{280t + \frac{\pi}{2}}_{f=\frac{140}{\pi}Hz}) = x(t) + 5e^{j\frac{\pi}{2}}e^{j280t} + 5e^{-j\frac{\pi}{2}}e^{-j280t}$

In the spectrum: at $\frac{140}{\pi}$ Hz there will be $5e^{j\frac{\pi}{2}}$

at $-\frac{140}{\pi}$ Hz there will be $5e^{-j\frac{\pi}{2}}$

GCD(20, 40, $\frac{140}{\pi}$, 60) is undefined, so $x(t)$ is not periodic

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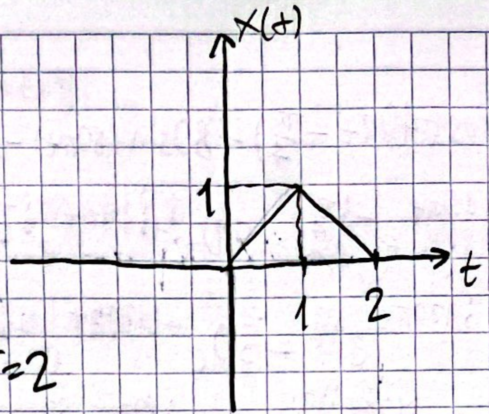
4) $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \end{cases}$



$T = 2$

a) $a_0 = \frac{1}{2} \underbrace{\int_0^2 x(t)\,dt}_{\text{area of the triangle}=1} = \frac{1}{2}$

b) $\frac{dx(t)}{dt} = \begin{cases} 1, & 0 \le t \le 1 \\ -1, & 1 \le t \le 2 \end{cases}$

$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\left(\frac{2\pi}{T_0}\right)kt}\,dt = \frac{1}{2} \int_0^2 x(t) e^{-j\pi k t}\,dt$

$= \frac{1}{2}\left(\int_0^1 e^{-j\pi k t}\,dt - \int_1^2 e^{-j\pi k t}\,dt \right)$

$= \frac{1}{2}\left(\frac{e^{-j\pi k t}}{-j\pi k}\Big|_0^1 - \frac{e^{-j\pi k t}}{-j\pi k}\Big|_1^2 \right) = \frac{1}{2}\left(\frac{j}{\pi k}\left(e^{-j\pi k} - 1\right) - \frac{j}{\pi k}\left(e^{-2j\pi k} - e^{-j\pi k}\right) \right)$

$= \frac{1}{2}\frac{j}{\pi k}\left(e^{-j\pi k} - 1 - e^{-2j\pi k} + e^{-j\pi k}\right) = \frac{1}{2}\frac{j}{\pi k}\left(2e^{-j\pi k} - \underbrace{e^{-2j\pi k}}_{1} - 1\right)$

$= \frac{1}{2}\frac{j}{\pi k}\left(2e^{-j\pi k} - 2\right) = \frac{j}{\pi k}\left(e^{-j\pi k} - 1\right) = \frac{j}{\pi k}\left((-1)^k - 1\right)$

$a_0 = \frac{1}{2}\int_0^2 x(t)\,dt = \frac{1}{2}\left(\int_0^1 1\,dt + \int_1^2 (-1)\,dt\right) = \frac{1}{2}(1-1) = 0$

$a_k = \begin{cases} 0 & \text{if } k \text{ even} \\ \frac{-2j}{\pi k} & \text{if } k \text{ odd} \end{cases}$

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c) $b_k = \begin{cases} 0 & \text{if } k \text{ even} \\ \frac{-2j}{\pi k} & \text{if } k \text{ odd} \end{cases}$

$b_k = (jk\omega_0)a_k \Rightarrow \frac{-2\cancel{j}}{\pi k} = \cancel{j}k\frac{2\pi}{\cancel{2}}a_k \Rightarrow a_k = \frac{-2}{\pi^2 k^2}$

$a_k = \begin{cases} 0 & \text{if } k \text{ even} \\ \frac{-2}{\pi^2 k^2} & \text{if } k \text{ odd} \end{cases}$

5) $x[n] = 2.2\cos(0.3\pi n - \frac{\pi}{3})$ $f_s = 6000$ samples/sec

$\hat{\omega} = \frac{\omega}{f_s}$ $0.3\pi = \frac{\omega}{6000} \Rightarrow \omega = 1800\pi$

$f = \frac{1800\pi}{2\pi} = 0.9\,kHz$

1) $x(t) = 2.2\cos(1800\pi t - \frac{\pi}{3})$

2) $x_1[n] = 2.2\cos(0.3\pi n + 2\pi n - \frac{\pi}{3}) = 2.2\cos(2.3\pi n - \frac{\pi}{3})$

$2.3\pi = \frac{\omega}{6000} \Rightarrow \omega = 13800\pi$

$f = 6900\,Hz = 6.9\,kHz$

$x_1(t) = 2.2\cos(13800\pi t - \frac{\pi}{3})$

3) $x_2[n] = 2.2\cos(-0.3\pi n + 2\pi n + \frac{\pi}{3}) = 2.2\cos(1.7\pi n + \frac{\pi}{3}) \Rightarrow$ folded alias

$1.7\pi = \frac{\omega}{6000}$ $\omega = 10200\pi$

$f = 5100\,Hz = 5.1\,kHz$

$x_2(t) = 2.2\cos(10200\pi t + \frac{\pi}{3})$

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6) a) $y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$

$= h[-1]x[n+1] + h[1]x[n-1] = 2x[n+1] + 2x[n-1]$

$= 2(\delta[n+1] + 2\delta[n] - \delta[n-2]) + 2(\delta[n-1] + 2\delta[n-2] - \delta[n-4])$

$= 2\delta[n+1] + 4\delta[n] - 2\delta[n-2] + 2\delta[n-1] + 4\delta[n-2] - 2\delta[n-4]$

$= 2\delta[n+1] + 4\delta[n] + 2\delta[n-2] + 2\delta[n-1] - 2\delta[n-4]$

b) $y_2[n] = x[n+2] * h[n]$

$= \sum_{k=-\infty}^{\infty} h[k]x[(n+2)-k] \Rightarrow y_2[n] = y_1[n+2]$ since system is LTI

$y_2[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n] + 2\delta[n+1] - 2\delta[n-2]$

c) $y_3[n] = x[n] * h[n+2]$

$= \sum_{k=-\infty}^{\infty} h[k+2]x[n-k] \Rightarrow \left.\begin{matrix} k+2 = k' \\ k = k'-2 \end{matrix}\right\} y_3[n] = \sum_{k'=-\infty}^{\infty} h[k']x[(n+2)-k']$

$\Rightarrow y_3[n] = y_1[n+2]$ since system is LTI

$y_3[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n] + 2\delta[n+1] - 2\delta[n-2]$

7) a) $H(e^{j\hat{\omega}}) = \sum_{k=0}^{2} b_k e^{-j\hat{\omega}k} = 2 - 3e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}}$

$= e^{-j\hat{\omega}}(2e^{j\hat{\omega}} - 3 + 2e^{-j\hat{\omega}})$

$= (4\cos\hat{\omega} - 3)e^{-j\hat{\omega}}$

b) $H(e^{j\hat{\omega}}) = 2 - \underbrace{3e^{-j\hat{\omega}}}_{T_1 = 2\pi} + \underbrace{2e^{-j2\hat{\omega}}}_{T_2 = \pi}$

$LCM(2\pi, \pi) = 2\pi \Rightarrow \boxed{T = 2\pi}$

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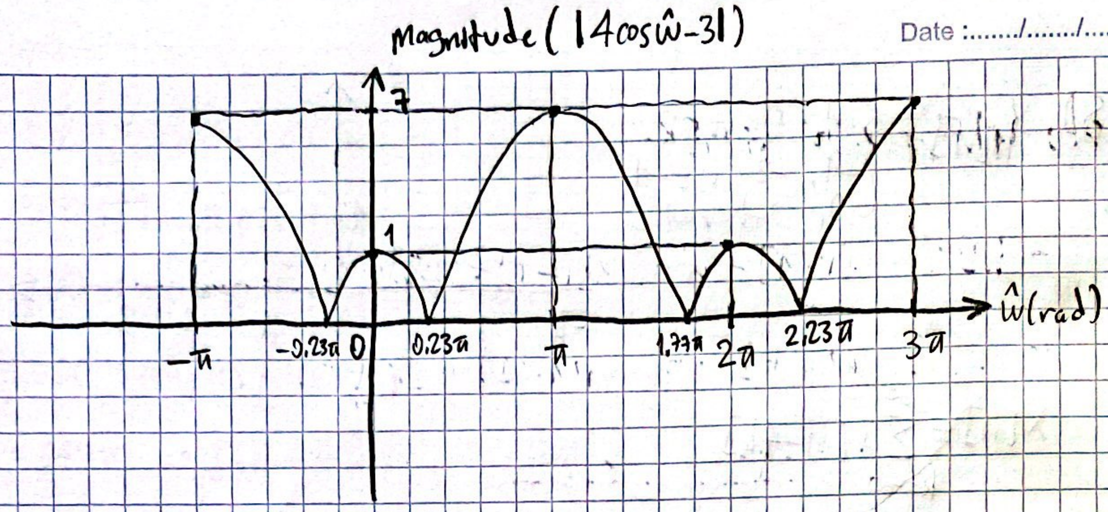
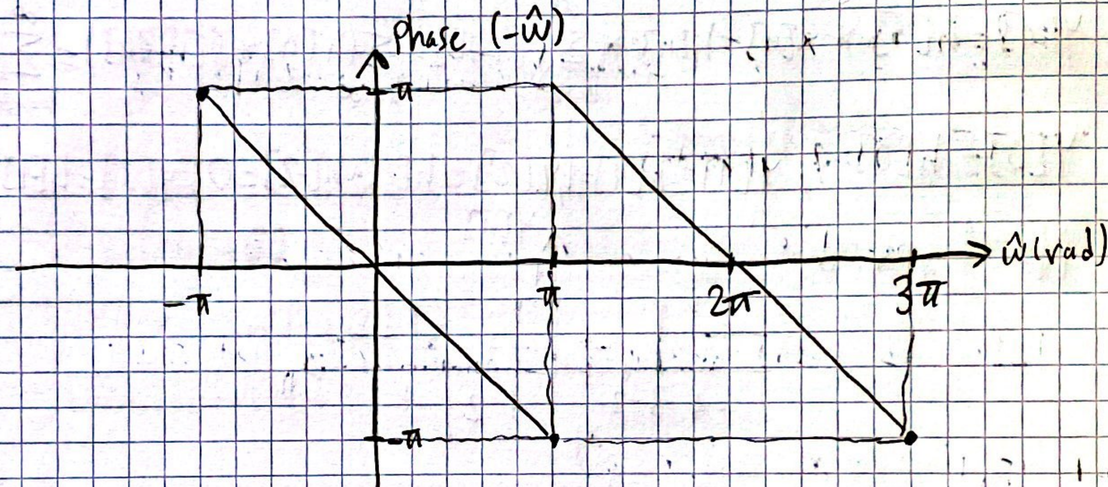
c)





d) $H(e^{j\hat{w}}) = (4\cos\hat{w}-3)\, e^{-j\hat{w}} = 0 \Rightarrow 4\cos\hat{w}-3=0$

$\cos\hat{w} = \frac{3}{4}$

$\Rightarrow \hat{w} = \arccos\left(\frac{3}{4}\right) + 2\pi l$

or $\qquad l = 0, \pm1, \pm2, \ldots$

$\hat{w} = -\arccos + 2\pi l$

e) $\sin\left(\frac{\pi}{13}n\right) = \frac{1}{2j} e^{j\frac{\pi n}{13}} - \frac{1}{2j} e^{-j\frac{\pi n}{13}}$, $H\left(\frac{\pi}{13}\right) = 0{,}8838\, e^{-j0{,}077\pi}$

$y[n] = H\left(\frac{\pi}{13}\right)\frac{1}{2j} e^{j\frac{\pi n}{13}} - H\left(-\frac{\pi}{13}\right)\frac{1}{2j} e^{-j\frac{\pi n}{13}}$

$= \frac{1}{2j}\left(0{,}8838\, e^{j\left(\frac{\pi n}{13} - 0{,}077\pi\right)} - 0{,}8838\, e^{-j\left(\frac{\pi n}{13} - 0{,}077\pi\right)}\right)$

$= 0{,}8838 \sin\left(\frac{\pi n}{13} - 0{,}077\pi\right) = 0{,}8838 \cos\left(\frac{\pi n}{13} - 0{,}577\pi\right) = y[n]$

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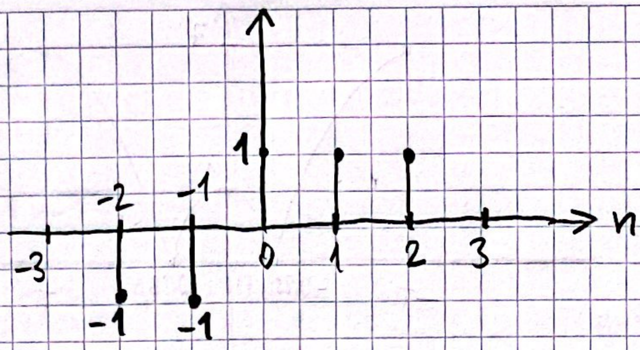
8) $h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ -1, & -2 \le n \le -1 \\ 0, & \text{otherwise} \end{cases}$

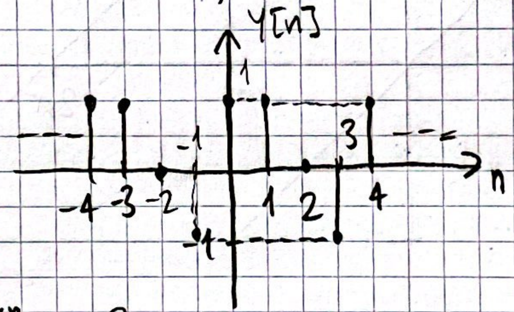


$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

$$y[n] = h[n] * x[n] = h[n] * \sum_{k=-\infty}^{\infty} \delta[n-4k] = \sum_{k=-\infty}^{\infty} (h[n] * \delta[n-4k]) = \sum_{k=-\infty}^{\infty} h[n-4k]$$

$y[0] = h[0] = 1$, $y[1] = h[1] = 1$, $y[2] = h[2] + h[-2] = 0$, $y[3] = h[-1] = -1 \ldots$

$N = 4$ period



$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} y[n] e^{-j\frac{2\pi k n}{N}} = \frac{1}{4} \sum_{0}^{3} y[n] e^{-j\frac{2\pi k n}{4}} = \frac{1}{4}\left(1 + e^{-j\frac{\pi k}{2}} + 0 + (-1) e^{-j\frac{3\pi k}{2}}\right)$$

$$= \frac{1}{4}\left(1 + \underbrace{e^{-j\pi k}}_{(-1)^k}\left(e^{\frac{j\pi k}{2}} - e^{-\frac{j\pi k}{2}}\right)\right) = \frac{1}{4}\left(1 + (-1)^k \cdot 2j \sin\left(\frac{\pi k}{2}\right)\right)$$

$$a_k = \frac{1}{4} + \frac{j}{2} (-1)^k \sin\left(\frac{\pi k}{2}\right)$$

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9) a) $x(t) = 10\cos(880\pi t + \phi)$

$X[n] = 10\cos(880\pi nT_s + \phi)$

$T_s = 0{,}0001 \text{ sec} = 10^{-4} \text{ sec}$

$f_s = 10000$ samples/sec

$2\pi f_0 = 880\pi \Rightarrow f_0 = 440 Hz, \; T_0 = \frac{1}{440} s$

$\frac{10000}{440} = \frac{1000}{44} \cong 22{,}72 \Rightarrow 22$ intervals

$\Rightarrow$ There are 23 samples in one period

b) $y(t) = 10\cos(w_0 t + \phi)$

$y(nT_s) = 10\cos(w_0 nT_s + \phi)$

$y[n] = 10\cos(w_0 nT_s + \phi)$

$\frac{880\pi}{10000} + 2\pi = \frac{20000\pi + 880\pi}{10000} = \frac{20880\pi}{10000} = \hat{w}_y$

$\hat{w}_y = \frac{20880\pi}{10000} = \frac{w_0}{10000} \quad w_y = w_0 = 20880\pi$

c) $f_s = 10000$ samples/sec

$T_0 = \frac{1}{10440} s$

$\frac{10000}{10440} \cong 0.9578$

$\Rightarrow$ 1 sample in one period

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10) a) $Y[n] = x[-n]$

Linearity: $W[n] = \alpha x_1[-n] + \beta x_2[-n]$

$\alpha x_1[n] + \beta x_2[n] \mapsto \alpha_1 x_1[-n] + \beta x_2[-n]$

System is linear

Time Invariance: $x[n] \xrightarrow{delay} x[n-n_0] \mapsto W[n] = x[-(n)-n_0] = x[-n-n_0]$

$Y[n-n_0] = x[-(n-n_0)] = x[-n+n_0] \neq W[n]$

System is not time invariant

Causality: If $n<0$, system looks to the future, so not causal.

b) $Y[n] = x[n-2] - 2x[n-8]$

Linearity:

$x_1[n] \mapsto \overbrace{x_1[n-2] - 2x_1[n-8]}^{Y_1[n]}$

$x_2[n] \mapsto \underbrace{x_2[n-2] - 2x_2[n-8]}_{Y_2[n]}$

$W[n] = \alpha Y_1[n] + \beta Y_2[n]$
$= \alpha x_1[n-2] - 2\alpha x_1[n-8]$
$+ \beta x_2[n-2] - 2\beta x_2[n-8]$

$\alpha x_1[n] + \beta x_2[n] \mapsto \alpha x_1[n-2] + \beta x_2[n-2] - 2(\alpha x_1[n-8] + \beta x_2[n-8])$

$= \alpha x_1[n-2] - 2\alpha x_1[n-8] + \beta x_2[n-2] - 2\beta x_2[n-8]$

$=$

System is linear

Time Invariance: $x[n] \xrightarrow{\text{delay by } n_0} x[n-n_0] \mapsto W[n] = x[n-n_0-2] - 2x[n-n_0-8]$

$Y[n-n_0] = x[n-n_0-2] - 2x[n-n_0-8]$

$W[n] = Y[n-n_0]$

System is time invariant

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Causality: The output doesn't depend on future inputs, <u>system is causal</u>.

c) $Y[n] = Even\{x[n-1]\} = \dfrac{x[n-1] + x[-n+1]}{2}$

Linearity:

$$\left.\begin{array}{l} x_1[n] \mapsto \dfrac{x_1[n-1] + x_1[-n+1]}{2} \\ x_2[n] \mapsto \dfrac{x_2[n-1] + x_2[-n+1]}{2} \end{array}\right\} W[n] = \frac{\alpha x_1[n-1] + \alpha x_1[-n+1] + \beta x_2[n-1] + \beta x_2[-n+1]}{2}$$

$$//$$

$$\alpha x_1[n] + \beta x_2[n] \mapsto \frac{\alpha x_1[n-1] + \beta x_2[n-1] + \alpha x_1[-n+1] + \beta x_2[-n+1]}{2} = W[n]$$

<u>System is linear</u>

Time Invariance:

$$x[n] \xrightarrow{delay} x[n-n_0] \mapsto \frac{x[n-n_0-1] + x[-n-n_0+1]}{2} = W[n]$$

$$Y[n-n_0] = \frac{x[n-n_0-1] + x[-n+n_0+1]}{2} \neq W[n]$$

<u>System is not time invariant</u>

Causality: If $x<0$, output depends on future inputs, <u>so not causal</u>

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d) $Y[n] = n x[n]$

Linearity: $\left.\begin{array}{l} x_1[n] \mapsto n x_1[n] \\ x_2[n] \mapsto n x_2[n] \end{array}\right\} W[n] = \alpha n x_1[n] + \beta n x_2[n]$

$//$

$\alpha x_1[n] + \beta x_2[n] \mapsto n(\alpha x_1[n] + \beta x_2[n]) = W[n]$

System is Linear

Time Invariance: $x[n] \xrightarrow{delay} x[n-n_0] \mapsto n\, x[n-n_0] = W[n]$

$Y[n-n_0] = (n-n_0)\, x[n-n_0] \neq W[n]$

System is not time Invariant

Causality: System doesn't depend on future inputs, it is causal

(at $n = n_0$, system depends on $x[n_0]$)

e) $Y[n] = x[4n+1]$

Linearity: $\left.\begin{array}{l} x_1[n] \mapsto x_1[4n+1] \\ x_2[n] \mapsto x_2[4n+1] \end{array}\right\} W[n] = \alpha x_1[4n+1] + \beta x_2[4n+1]$

$\alpha x_1[n] + \beta x_2[n] \mapsto \alpha x_1[4n+1] + \beta x_2[4n+1] = W[n]$

System is linear

Time Invariance: $x[n] \xrightarrow{delay} x[n-n_0] \mapsto x[4n+1-n_0] = W[n]$

$Y[n-n_0] = x[4n-4n_0+1] \neq W[n]$

System is not time invariant

Causality: System depends on future inputs, so it is not causal.

$(n=5,\ Y[5] = x[21])$