# EEE 391: Basics of Signals and Systems

## Homework 1

Due:

1) Convert the ones in Cartesian form to polar form and the ones in polar form to Cartesian form.

a) $3e^{j\pi/3} + 4e^{-j\pi/6}$
b) $(1-j)^2$
c) $(\sqrt{3} - j3)^{10}$
d) $(\sqrt{2} + j\sqrt{2}) / (1 + j\sqrt{3})$
e) $\mathrm{Re}\{ je^{-j\pi/3}\}$
f) $j(1-j)$
g) $(\sqrt{3} - j3)^{-1}$

2) Define the following complex exponential signal:

$s(t) = 5e^{j\pi/3}e^{j10\pi t}$

a)Make a plot of $s_i(t) = \mathrm{Im}\{s(t)\}$. Pick a range of values for t that will include exactly three periods of the signal.
b)Make a plot of $q(t) = \mathrm{Im}\{\dot{s}(t)\}$, where the dot mean differentiation with respect to time t. Again plot three cycles of the signal.

3) A signal composed of sinusoids is given by the equation

$x(t) = 100\cos(40\pi t - \pi / 4) + 80\sin(80\pi t) - 60\cos(120\pi t + \pi / 6)$

a) Sketch the spectrum of this signal indicating the complex size of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex phasor value at the appropriate frequency.
b) Is x(t) periodic? If so, what is the period?
c) Now consider a new signal $y(t) = x(t)+90 \cos(60\pi t +\pi/6)$. How is the spectrum changed? Is y(t) periodic? If so, what is the period?
d) Finally, consider another new signal $w(t) = x(t)+10 \cos(280t+\pi/2)$. How is the spectrum changed? Is w(t) periodic? If so, what is the period? If not, why not?

4) Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \end{cases}$$

be a periodic signal with fundamental period T =2 and Fourier coefficients ak.

a) Determine the value of $a_0$.
b) Determine the Fourier series representation of dx(t)/dt.
c) Use the result part (b) and differentiation property of continuous – time Fourier series to help determine the Fourier series coefficients of x(t).

5) Suppose that a discrete- time signal x[n] is given by the formula

$$x[n] = 2.2 \cos(0.3\pi n - \pi / 3)$$

and that it was obtained by sampling a continuous- time signal x(t)= $A\cos(2\pi f_0 t + Ø)$ at a sampling rate of $f_s$ = 6000 samples/ sec. Determine three different continuous-time signals that could have produced x[n]. All these continuous time signals should have a frequency less than 8kHz. Write the mathematical formula for all three.

6) Let

$$x[n] = \delta[n] + 2\,\delta[n-1] - \delta[n-3] \text{ and } h[n]= 2\,\delta[n+1] + 2\,\delta[n-1].$$

Compute each of the following convolutions.

a) $y_1[n] = x[n] * h[n]$ b) $y_2[n] = x[n + 2] * h[n]$ c) $y_3[n] = x[n] * h[n+2]$

7) A linear time-invariant system is described by the difference equation

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

a) Find the frequency response $H(e^{jw})$; then express it as a mathematical formula, in polar (magnitude and phase)
b) $H(e^{jw})$ is a periodic function of w; determine the period.
c) Plot the magnitude and phase of $H(e^{jw})$ as a function of w for $-\pi<w<3\pi$.
d) Find all frequencies w, for which the output response to the input $e^{jwn}$ is zero.
e) When the input to the system is $x[n] = \sin(\pi n/13)$, determine the output signal and express it in form $y[n] = A(\cos w_0 n + Ø)$.

8) Consider a discrete time LTI system with impulse response

$$h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ -1, & -2 \le n \le -1 \\ 0, & \text{otherwise} \end{cases}$$

Given that the input to the system is

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$$

determine the Fourier series coefficients of the output y[n].

9) Consider the cosine wave

$$x(t) = 10\cos(880\pi t + \emptyset)$$

Suppose that we obtain a sequence of numbers by sampling the waveform at equally spaced time instants nTs. In this case the resulting sequence would have the values

$$x[n] = n(nTs) = 10\cos(880\pi nTs + \emptyset)$$

for $-\infty < n < \infty$ Suppose that Ts= 0.0001 sec.

a) How many samples will be taken in one period of the cosine wave ?
b) Now consider another wave form y(t) such that

$$y(t) = 10\cos(w_0 t + \emptyset)$$

Find a frequency $w_0 > 880\pi$ such that $y(nT_s) = x(nT_s)$ for all integers n.
Hint: Use the fact that $\cos(\theta + 2\pi n) = \cos(\theta)$ if n is an integer.

c) For the frequency found in (b), what is the total number of samples taken in one period of x(t)?

10) For each of the following systems, determine whether or not the systems is linear, time-invariant and causal.

a) $y[n] = x[-n]$
b) $y[n] = x[n-2] - 2x[n-8]$
c) $y[n] = \text{Even}\{x[n-1]\}$
d) $y[n] = nx[n]$
e) $y[n] = x[4n+1]$