EEE 391

HW 2

SOLUTION

1) a) $h[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$

b) $H(\hat{\omega}) = \frac{1}{4}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}})$

$= \frac{1}{4} e^{-j\frac{3}{2}\hat{\omega}} (e^{j\frac{3}{2}\hat{\omega}} + e^{j\frac{1}{2}\hat{\omega}} + e^{-j\frac{1}{2}\hat{\omega}} + e^{-j\frac{3}{2}\hat{\omega}})$

$= \frac{1}{2} e^{-j\frac{3}{2}\hat{\omega}} (\cos(\frac{3}{2}\hat{\omega}) + \cos(\frac{1}{2}\hat{\omega}))$

c) $x[n] = 5 + 4\cos(0.2\pi n) + 3\cos(0.5\pi n + \pi/4)$

Need $H(0)$ Need $H(0.2\pi)$ $H(0.5\pi)$

$H(0) = 1$

$H(0.2\pi) = 0.769\, e^{-j0.3\pi}$ ANGLE $= -54° = -0.942$ rad

$H(0.5\pi) = 0$

$\Rightarrow y[n] = 5 + \underbrace{4(0.769)}_{3.078} \cos(0.2\pi n - 0.3\pi)$

d) $x_1[n] = 0$ for $n < 0$

$y_1[n] = \frac{1}{4}(x_1[n] + x_1[n-1] + x_1[n-2] + x_1[n-3])$

since $x_1[n] = x[n]$ for $n \geq 0$, the filtered outputs will be the same when $n-3 \geq 0$

$\Rightarrow n \geq 3$ is the region where $y_1[n] = y[n]$

Here's a table of the first few values:

n	-1	0	1	2	3	4 ...
$y[n]$	4.029	6.809	7.927	7.927	6.809	5 ...
$y_1[n]$	0	2.780	4.309	5.338	6.809	5 ...

2 a) $\delta[n+5]$

$X(z) = z^{5}$

b) $\delta[n-5]$

$X(z) = z^{-5}$

c) $\delta[n-1]$

$X(z) = z^{-1}$

d) $X(z) = 2 + 4z^{-1} + 6z^{-2} + 4z^{-3} + 2z^{-4}$

$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$

e) $X(z) = 1 - 2z^{-1} + 3z^{-3} - z^{-5}$

$x[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-3] - \delta[n-5]$

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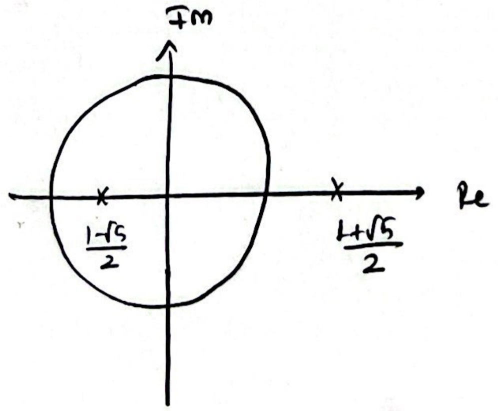
$H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{z^{-1}}{1 - z^{-1} - z^{-2}}$ → for $y[n] = y[n-1] + y[n-2] + x[n-1]$

Poles of $H(z)$ are at $z = (1/2) \pm (\sqrt{5}/2)$. $H(z)$ has a zero at $z=0$.
$h[n]$ is causal, the ROC for $H(z)$ has to be $|z| > (1/2) + (\sqrt{5}/2)$

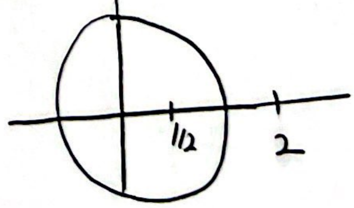


$H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1}{z - \frac{5}{2} + z^{-1}} = \dfrac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$ → for $y[n-1] - 5/2\, y[n] + y[n+1] = x[n]$

$H(z) = \dfrac{-2/3}{1 - \frac{1}{2}z^{-1}} + \dfrac{2/3}{1 - 2z^{-1}}$

$z = 1/2$ $\quad z = 2$

$\boxed{\text{zeros} \rightarrow 1/2, 2}$



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a) $h[n] = \delta[n-2] \Rightarrow$ filter is a delay by 2

$y[n] = u[n-3] - u[n-6]$

To find $x[n]$ we need to "un-delay" $y[n]$.

$\Rightarrow x[n] = u[n-1] - u[n-4]$

b) First difference FIR $\Rightarrow h[n] = \delta[n] - \delta[n-1]$

The first-difference filter has a nonzero output at n when $x[n]$ & $x[n-1]$ are not equal.

If $y[n] = \delta[n] - \delta[n-4]$, then the input $x[n]$ changes value at $n=0$ and $n=4$. At $n=0$, it jumps up by one; at $n=4$, it jumps down.

$\Rightarrow x[n] = \underbrace{u[n]}_{\text{jump up by one}} - \underbrace{u[n-4]}_{\text{jump down}}$

c) 4 pt averager : $y[n] = \frac{1}{4}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$

If $y[n] = -5\delta[n] - 5\delta[n-2]$

$y[0] = -5 = \frac{1}{4}\left(x[0] + x[-1] + x[-2] + x[-3]\right)$

** If we assume $x[n] = 0$ for $n < 0$, then $x[0] = -20$

$y[1] = 0 = \frac{1}{4}\left(x[1] + x[0] + x[-1] + x[-2]\right) = \frac{1}{4}x[1] - 5$

$\Rightarrow x[1] = 20$

$$\left.\begin{aligned} y[2] = -5 &= \frac{1}{4}\left(x[2] + x[1] + x[0] + x[-1]\right) \\ &= \frac{1}{4}\left(x[2] + 20 - 20 + 0\right) = \frac{1}{4}x[2] \end{aligned}\right\} x[2] = -20$$

$y[3] = 0 = \frac{1}{4}\left(x[3] + x[2] + x[1] + x[0]\right) \Rightarrow x[3] = -20$

$$\Rightarrow x[n] = \begin{cases} 0 & \text{for } n < 0 \\ -20 & \text{for } n \text{ even} \\ 20 & \text{for } n \text{ odd} \end{cases}$$

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a) $b_0 = b_3 = 0, \quad b_1 = b_2$

$$H(e^{j\omega}) = b_1 e^{-j\omega} + b_2 e^{-2j\omega} = 2b_1 e^{-j3\omega/2} \cos(\omega/2)$$

$$|H(e^{j\omega})| = 2|b_1|\,|\cos(\omega/2)|$$

b) $b_1 = b_2 = 0, \quad b_0 = b_3$

$$H(e^{j\omega}) = b_0 + b_3 e^{-3j\omega} = 2b_0 e^{-j3\omega/2} \cos(3\omega/2)$$

$$|H(e^{j\omega})| = 2|b_0|\,|\cos(3\omega/2)|$$

c) $b_0 = b_1 = b_2 = b_3$

$$H(e^{j\omega}) = b_0 + b_1 e^{-j\omega} + b_2 e^{-j2\omega} + b_3 e^{-j3\omega} = 2b_0 e^{-j3\omega/2} \cos(\omega)\cos(\omega/2)$$

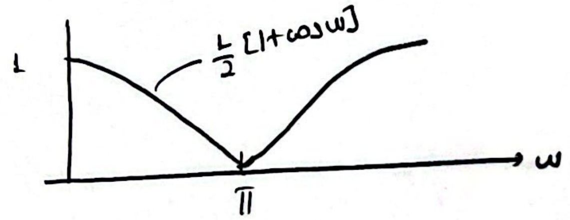
$$|H(e^{j\omega})| = 2|b_0|\,|\cos(\omega)|\,|\cos(\omega/2)|$$

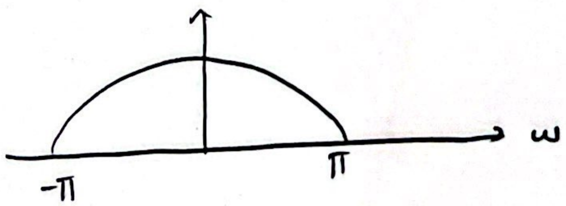
d) $b_0 = -b_1 = b_2 = -b_3$

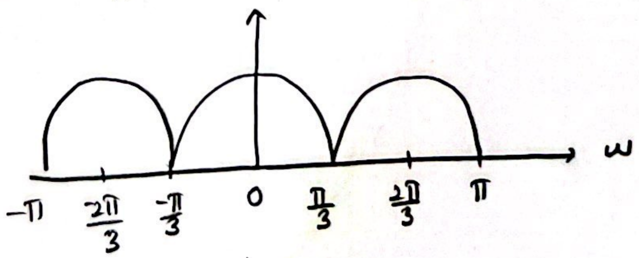
$$H(e^{j\omega}) = b_0 + b_1 e^{-j\omega} + b_2 e^{-j2\omega} + b_3 e^{-j3\omega} = -2b_0 e^{-j3\omega/2} \sin(\omega)\sin(\omega/2)$$

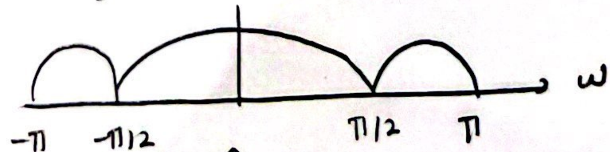
$$|H(e^{j\omega})| = 2|b_0|\,|\sin(\omega)|\,|\sin(\omega/2)|.$$

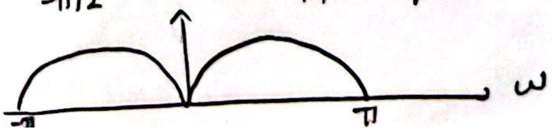
Graphs:











6 $h(t) = e^{-0.1(t-2)}\left(u(t-2) - u(t-12)\right)$

a) The system is stable because $\int |h(t)|\,dt < \infty$

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{2}^{12} |e^{-0.1(t-2)}|\,dt < \int_{2}^{12} dt = 10 < \infty$$

b) The system is causal because $h(t) = 0$ for $t < 0$

A plot of $h(t)$ starts at $t=2$

c) $x(t) = \delta(t-2)$

$$\Rightarrow y(t) = \delta(t-2) * h(t)$$

$$= h(t-2)$$

7 $$H(e^{j\omega}) = \frac{1}{2\pi}\left(H_1(e^{j\omega}) * \{2\pi\delta(\omega - \pi/2) + 2\pi\delta(\omega + \pi/2)\}\right)$$

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| < \pi \end{cases}$$

using properties of Fourier transform, we obtain

$$h[n] = h_1[n][2\cos(\pi n/2)]$$

where

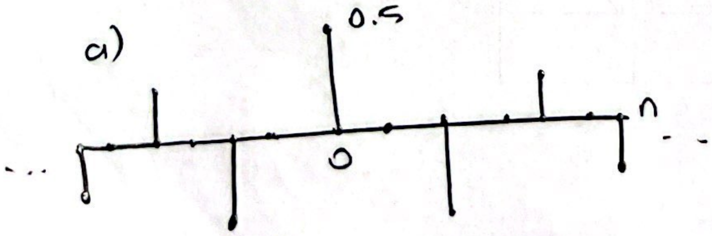
$$h_1[n] = \frac{\sin(\omega_c n)}{\pi n}$$

a) when $\omega_c = \pi/5$, $h[n] = 2\,\dfrac{\sin(\pi n/5)}{\pi n}\cos(\pi n/2)$

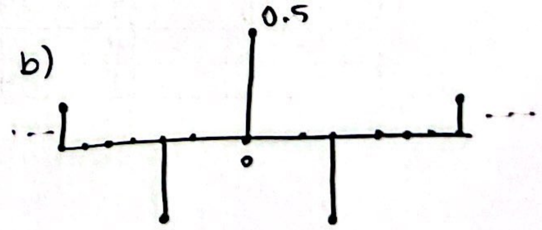
b) when $\omega_c = \pi/4$, $h[n] = \dfrac{2\sin(\pi n/4)}{\pi n}\cos(\pi n/2)$

c) when $\omega_c = \pi/3$, $h[n] = \dfrac{2\sin(\pi n/3)}{\pi n}\cos(\pi n/2)$

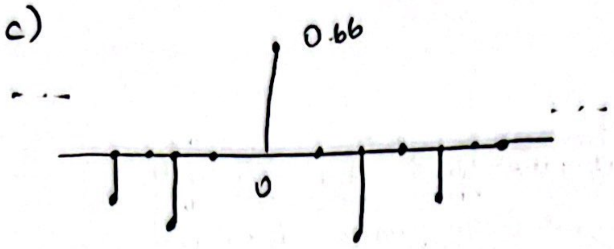
a)



b)



c)



8)

a) Linear, stable

b) memoryless, linear, causal, stable

c) Linear

d) Linear, causal, stable

e) Time invariant, Linear, causal, stable

f) Linear, stable

g) Time invariant, linear, causal

9)

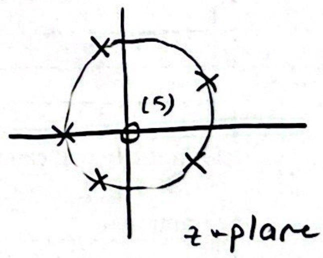
a) $H(z) = \dfrac{1}{1+z^{-5}}$

b) FIVE POLES - find roots of

$z^5+1=0$

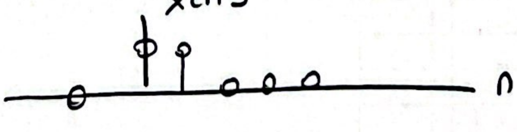
$z = e^{j\pi/5}, e^{j3\pi/5}, e^{j\pi}, e^{-j\pi/5}, e^{-j3\pi/5}$

36° 108°

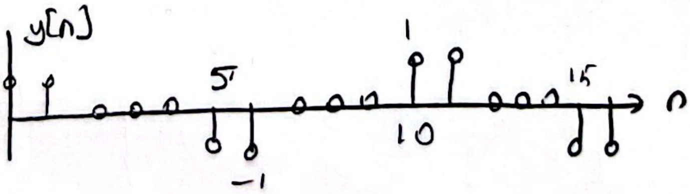


z-plane

c) x[n]



n	<0	0	1	2	3	4	5	6	7	8	9	10	11	12
y[n]	0	1	1	0	0	0	-1	-1	0	0	0	1	1	0



d) PERIOD = 10

which can be determined from the plot above

10) Using the properties of Fourier Transform, we obtain

$$Y(j\omega) = X_1(j\omega)\, X_2(j\omega)$$

Therefore $Y(j\omega) = 0$ for $|\omega| > 1000\pi$. This implies that the Nyquist rate for $y(t)$ is $2 \times 1000\pi = 2000\pi$.

Therefore the sampling period T can at most be $2\pi/(2000\pi) = 10^{-3}$ sec. Therefore we have to use $T < 10^{-3}$ sec in order to be able to recover $y(t)$ from $y_p(t)$.