EEE391 Analytical Assignment 2

Fall 2022

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CS

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1) $Y[n]=\frac{1}{4}(x[n]+x[n-1]+x[n-2]+x[n-3])$

a) $h[n]=\frac{1}{4}(\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3])$

b) $H(e^{j\hat{\omega}})=\frac{1}{4}+\frac{1}{4}e^{-j\hat{\omega}}+\frac{1}{4}e^{-2j\hat{\omega}}+\frac{1}{4}e^{-3j\hat{\omega}}$

c) $H(e^{j\hat{\omega}})=\frac{1}{4}e^{-\frac{3}{2}j\hat{\omega}}\left(e^{\frac{3}{2}j\hat{\omega}}+e^{\frac{j}{2}\hat{\omega}}+e^{-\frac{j}{2}\hat{\omega}}+e^{-\frac{3}{2}j\hat{\omega}}\right)$

$=\frac{1}{4}e^{-\frac{3}{2}j\hat{\omega}}\left(2\cos\frac{\hat{\omega}}{2}+2\cos\frac{3\hat{\omega}}{2}\right)$

$=\frac{1}{2}e^{-\frac{3}{2}j\hat{\omega}}\left(\cos\frac{\hat{\omega}}{2}+\cos\frac{3\hat{\omega}}{2}\right)$

$H(0)=\frac{1}{2}\cdot(1+1)=1$ $\quad H(0{,}2\pi)=\frac{1}{2}e^{-\frac{3}{2}j0{,}2\pi}\overbrace{(\cos 0{,}1\pi+\cos 0{,}3\pi)}^{1{,}539}$

$\cong 0{,}769\,e^{-0.3\pi j}$

$H(0{,}5\pi)=\frac{1}{2}e^{-\frac{3}{2}j0{,}5\pi}(\cos 0{,}25\pi+\cos 0{,}75\pi)=0$

$Y[n]=H(0).5+4|H(0{,}2\pi)|\cos(0{,}2\pi n+\angle H(0{,}2\pi))$

$=5+4.(0{,}769)\cos(0{,}2\pi n-0{,}3\pi)=5+3{,}076\cos(0{,}2\pi n-0.3\pi)$

d) $x[n]=0$ for $n<0$

$Y_1[n]=\frac{1}{4}(x[n]+x[n-1]+x[n-2]+x[n-3])$

when $n\geq 3$, $Y_1[n]=Y[n]$ in (c)

2) a) $\delta[n+5]\xrightarrow{z} z^{5}$

b) $\delta[n-5]\xrightarrow{z} z^{-5}$

c) $\delta[n-1]\xrightarrow{z} z^{-1}$

d) $x(z)=2+4z^{-1}+6z^{-2}+4z^{-3}+2z^{-4}$

$x[n]=2\delta[n]+4\delta[n-1]+6\delta[n-2]+4\delta[n-3]+2\delta[n-4]$

e) $x(z)=1-2z^{-1}+3z^{-3}-z^{-5}$

$x[n]=\delta[n]-2\delta[n-1]+3\delta[n-3]-\delta[n-5]$

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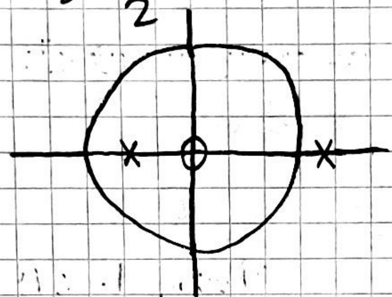
3) $y[n] = y[n-1] + y[n-2] + x[n-1]$

$Y(z) = z^{-1}Y(z) + z^{-2}Y(z) + z^{-1}X(z)$

$Y(z)(1 - z^{-1} - z^{-2}) = z^{-1}X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{(1-z^{-1}-z^{-2})} \cdot \frac{z^2}{z^2}$

$= \frac{z}{z^2 - z - 1}$

Zeros: $z = 0$, poles: $z = \frac{1 \mp \sqrt{5}}{2}$

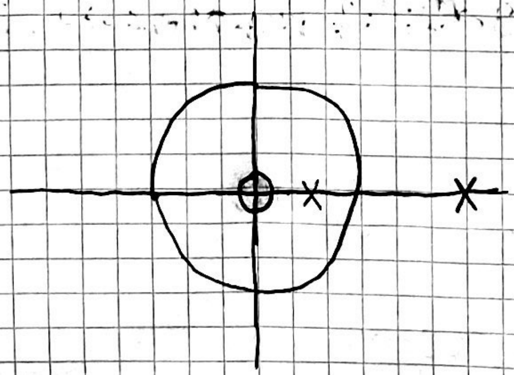


$y[n-1] - \frac{5}{2}y[n] + y[n+1] = x[n]$

$z^{-1}Y(z) - \frac{5}{2}Y(z) + zY(z) = X(z)$

$Y(z)(z^{-1} - \frac{5}{2} + z) = X(z) \Rightarrow H(z) = \frac{1}{z^{-1} + z - \frac{5}{2}} = \frac{z}{z^2 - \frac{5}{2}z + 1}$

Zeros: $z = 0$, poles: $z = 2$, $z = \frac{1}{2}$



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4) a) $h[n] = \delta[n-2] \rightarrow$ delay by two

$Y[n] = U[n-3] - U[n-6]$

$x[n] = U[n-1] - U[n-4]$

b) first difference: $h[n] = \delta[n] - \delta[n-1]$

$x[n]$ is nonzero at $n=0$ and $n=4$ ($n=0 \Rightarrow x[n]=1$, $n=4 \Rightarrow x[4]=-1$)

$x[n] = U[n] - U[n-4]$

c) 4 point averager: $Y[n] = \frac{1}{4}(x[n]+x[n-1]+x[n-2]+x[n-3])$

$Y[n] = -5\delta[n] - 5\delta[n-2]$

Assuming $x[n]=0$ for $n<0$

$Y[0] = -5 = \frac{1}{4}(x[0]+x[-1]+x[-2]+x[-3]) \Rightarrow x[0] = -20$

$Y[1] = 0 = \frac{1}{4}(x[1]+\underbrace{x[0]}_{-20}+x[-1]+x[-2]) = \frac{1}{4}x[1] - 5$

$x[1] = 20$

$Y[2] = -5 = \frac{1}{4}(x[2]+x[1]+x[0]+x[-1]) = \frac{1}{4}(x[2]+20-20+0) = \frac{1}{4}x[2]$

$x[2] = -20$

$Y[3] = 0 = \frac{1}{4}(x[3]+x[2]+x[1]+x[0]) = \frac{1}{4}(x[3]-20+20-20) = \frac{1}{4}x[3] - 5$

$x[3] = 20$

$$x[n] = \begin{cases} 0 & \text{if } n<0 \\ -20 & \text{if } n \text{ even} \\ 20 & \text{if } n \text{ odd} \end{cases}$$

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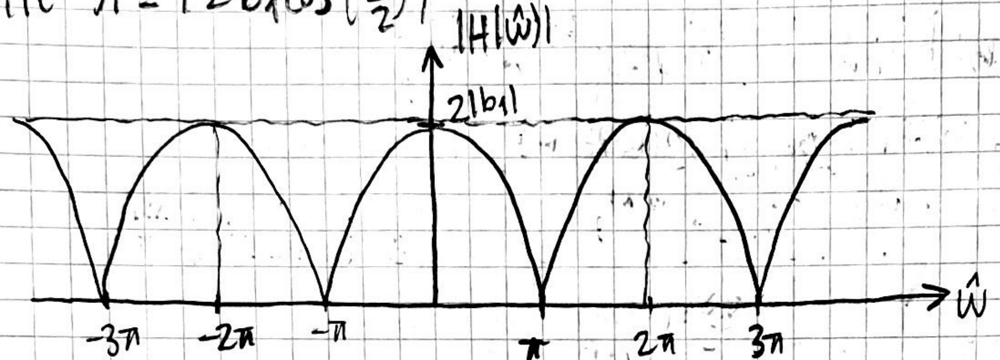
5) $H(e^{j\hat{\omega}}) = b_0 + b_1 e^{-j\hat{\omega}} + b_2 e^{-j2\hat{\omega}} + b_3 e^{-j3\hat{\omega}}$

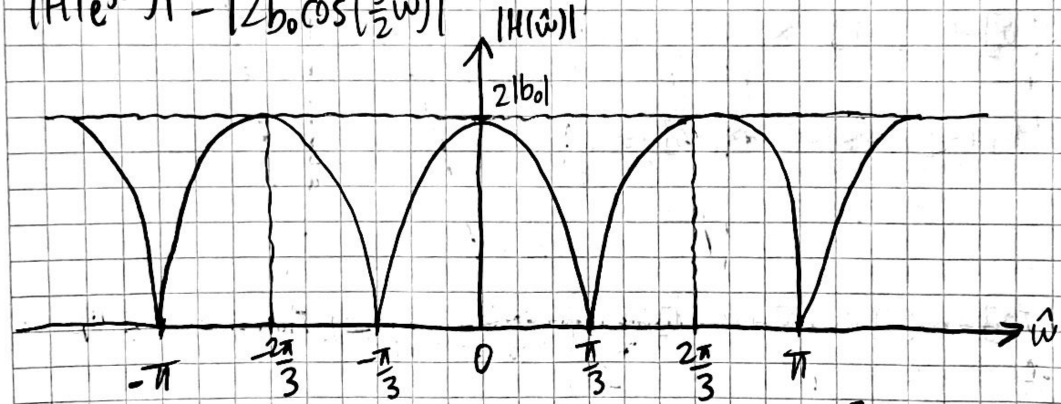
a) $H(e^{j\hat{\omega}}) = b_1(e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}}) = b_1 e^{-j\frac{3}{2}\hat{\omega}}(e^{j\frac{1}{2}\hat{\omega}} + e^{-j\frac{1}{2}\hat{\omega}}) = 2b_1\cos(\frac{\hat{\omega}}{2})\, e^{-j\frac{3}{2}\hat{\omega}}$

$|H(e^{j\hat{\omega}})| = |2b_1\cos(\frac{\hat{\omega}}{2})|$



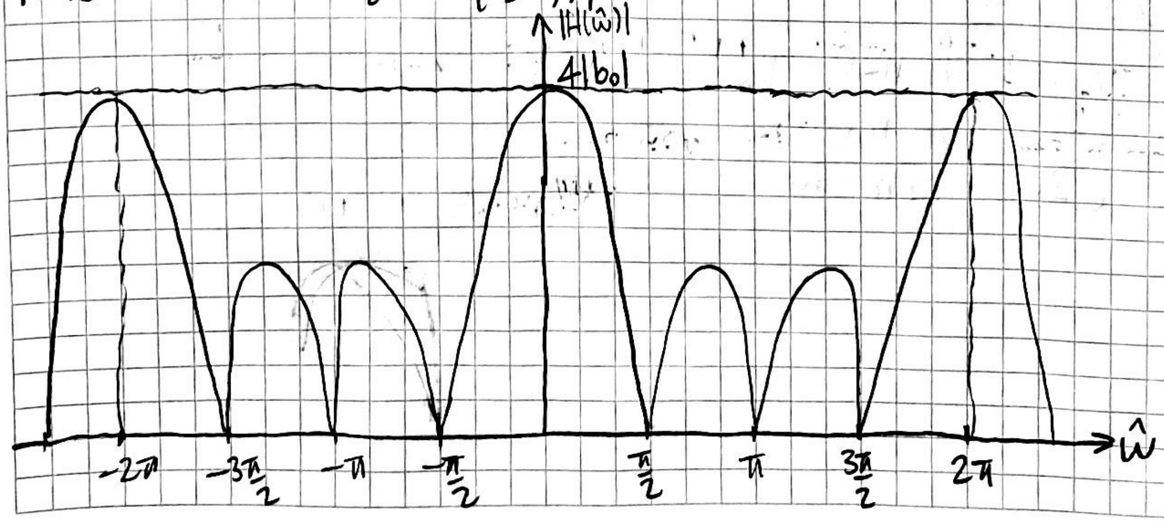

b) $H(e^{j\hat{\omega}}) = b_0(1 + e^{-j3\hat{\omega}}) = b_0 e^{-j\frac{3}{2}\hat{\omega}}(e^{j\frac{3}{2}\hat{\omega}} + e^{-j\frac{3}{2}\hat{\omega}}) = 2b_0\cos(\frac{3}{2}\hat{\omega})\, e^{-j\frac{3}{2}\hat{\omega}}$

$|H(e^{j\hat{\omega}})| = |2b_0\cos(\frac{3}{2}\hat{\omega})|$




c) $H(e^{j\hat{\omega}}) = b_0 e^{-j\frac{3}{2}\hat{\omega}}(2\cos(\frac{\hat{\omega}}{2}) + 2\cos(\frac{3}{2}\hat{\omega})) = 2b_0 e^{-j\frac{3}{2}\hat{\omega}}(\cos(\frac{\hat{\omega}}{2}) + \cos(\frac{3}{2}\hat{\omega}))$

$|H(e^{j\hat{\omega}})| = |2b_0(\cos(\frac{\hat{\omega}}{2}) + \cos(\frac{3}{2}\hat{\omega}))|$



d) $H(e^{j\hat{\omega}}) = -b_1 + b_1 e^{-j\hat{\omega}} - b_3 e^{-j2\hat{\omega}} + b_3 e^{-j3\hat{\omega}}$

$= b_1(e^{-j\hat{\omega}} - 1) + b_3(e^{-j3\hat{\omega}} - e^{-j2\hat{\omega}})$

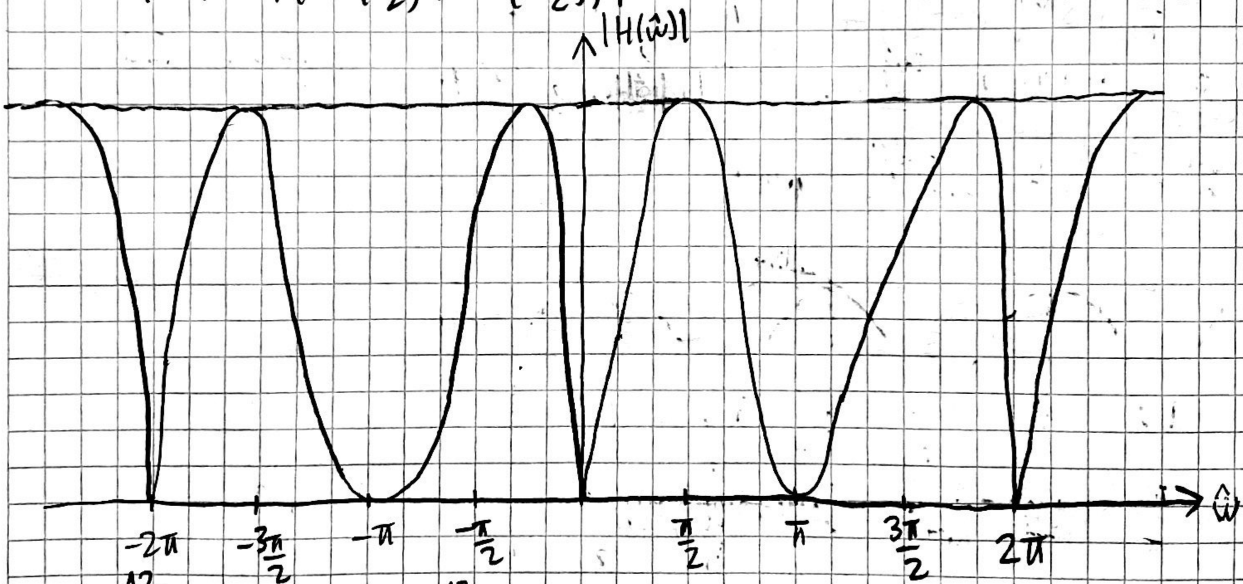
$= b_1(e^{-j\hat{\omega}} - 1 + e^{-j3\hat{\omega}} - e^{-j2\hat{\omega}})$

$= b_1 e^{-j\frac{3}{2}\hat{\omega}}(e^{j\frac{1}{2}\hat{\omega}} - e^{j\frac{3}{2}\hat{\omega}} + e^{-j\frac{3}{2}\hat{\omega}} - e^{-j\frac{1}{2}\hat{\omega}})$

$= b_1 e^{-j\frac{3}{2}\hat{\omega}}(2j\sin(\frac{\hat{\omega}}{2}) + 2j\sin(\frac{3}{2}\hat{\omega}))$

$= 2jb_1 e^{-j\frac{3}{2}\hat{\omega}}(\sin(\frac{\hat{\omega}}{2}) + \sin(\frac{3}{2}\hat{\omega})) = 2b_1 e^{-j\frac{3}{2}\hat{\omega}} e^{j\frac{\pi}{2}}(\sin(\frac{\hat{\omega}}{2}) + \sin(\frac{3}{2}\hat{\omega}))$

$|H(e^{j\hat{\omega}})| = |2b_1(\sin(\frac{\hat{\omega}}{2}) + \sin(\frac{3\hat{\omega}}{2}))|$




6) a) $\int_2^{12} |e^{-0,1(\tau-2)}| d\tau = \int_2^{12} e^{-0,1\tau} e^{0,2} d\tau = \frac{e^{0,2}}{(-0,1)}\left[e^{-0,1\tau}\right]_2^{12}$

$= \frac{-e^{0,2}}{0,1}(e^{-1,2} - e^{-0,2}) = -10(e^{-1} - 1) < \infty \Rightarrow$ system is stable

b) System is causal because $h(t) = 0$ for $t < 0$.

c) $y(t) = \delta(t-2) * h(t) = h(t-2) = e^{-0,1(t-4)}(u(t-4) - u(t-14))$

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7) $h[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\hat{w}})e^{j\hat{w}n}\,d\hat{w}$

$$h[n]=\frac{1}{2\pi}\left[\int_{-\frac{\pi}{2}-w_c}^{-\frac{\pi}{2}+w_c} e^{j\hat{w}n}\,d\hat{w}+\int_{\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c} e^{j\hat{w}n}\,d\hat{w}\right]$$

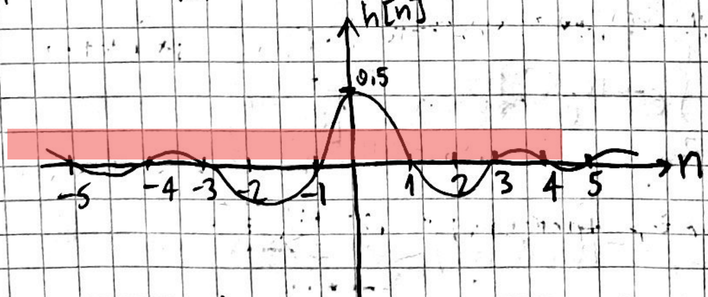
$$=\frac{1}{2\pi}\left[\left[\frac{e^{j\hat{w}n}}{jn}\right]_{-\frac{\pi}{2}-w_c}^{-\frac{\pi}{2}+w_c}+\left[\frac{e^{j\hat{w}n}}{jn}\right]_{\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c}\right]=\frac{1}{2\pi}\left(\frac{1}{jn}\left(e^{jn(w_c-\frac{\pi}{2})}-e^{-jn(w_c+\frac{\pi}{2})}\right)+\frac{1}{jn}\left(e^{jn(w_c+\frac{\pi}{2})}-e^{jn(\frac{\pi}{2}-w_c)}\right)\right)$$

$$=\frac{1}{\pi n}\left(\frac{e^{jn(w_c-\frac{\pi}{2})}-e^{-jn(w_c-\frac{\pi}{2})}}{2j}+\frac{e^{jn(w_c+\frac{\pi}{2})}-e^{-jn(w_c+\frac{\pi}{2})}}{2j}\right)$$

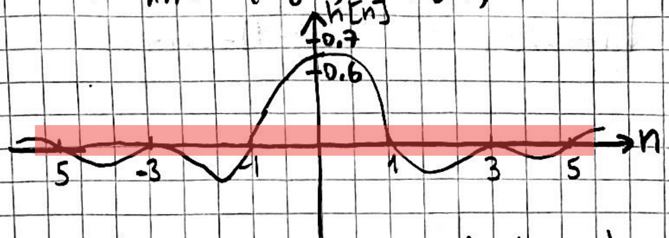
$$=\frac{1}{\pi n}\left(\sin\left(n\left(w_c-\frac{\pi}{2}\right)\right)+\sin\left(n\left(w_c+\frac{\pi}{2}\right)\right)\right)$$

a) $w_c=\frac{\pi}{5}\Rightarrow h[n]=\frac{1}{\pi n}\left(\sin\left(-\frac{3\pi n}{10}\right)+\sin\left(\frac{7\pi}{10}n\right)\right)$


b) $w_c=\frac{\pi}{4}\Rightarrow h[n]=\frac{1}{\pi n}\left(\sin\left(-\frac{\pi}{4}n\right)+\sin\left(\frac{3\pi}{4}n\right)\right)$




c) $w_c=\frac{\pi}{3}\Rightarrow h[n]=\frac{1}{\pi n}\left(\sin\left(-\frac{\pi}{6}n\right)+\sin\left(\frac{5\pi}{6}n\right)\right)$




As $w_c$ is increased, $h[n]$ gets more concentrated about the origin.

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8) a) $y(t) = x(t-2) + x(2-t)$

**Linearity:** $x_1(t) \mapsto x_1(t-2) + x_1(2-t) \xrightarrow{\times\alpha}$
$x_2(t) \mapsto x_2(t-2) + x_2(2-t) \xrightarrow{\times\beta}$ $\xrightarrow{+} \alpha x_1(t-2) + \alpha x_1(2-t) + \beta x_2(t-2) + \beta x_2(2-t)$

$x_1(t) \xrightarrow{\times\alpha}$, $x_2(t) \xrightarrow{\times\beta}$ $\xrightarrow{+} \alpha x_1(t) + \beta x_2(t) \mapsto \alpha x_1(t-2) + \beta x_2(t-2) + \alpha x_1(2-t) + \beta x_2(2-t)$

// $\boxed{\text{Linear}}$

**Time invariance:** $x(t) \xrightarrow{\text{delay}} x(t-t_0) \mapsto x(t-2-t_0) + x(2-t+t_0)$

$y(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$ $\Rightarrow$ = $\rightarrow \boxed{\text{Time invariant}}$

**Stability:** $h(t) = \delta(t-2) + \delta(2-t)$

$$\int_{-\infty}^{\infty} |\delta(\tau-2) + \delta(2-\tau)| d\tau = 2 < \infty \Rightarrow \boxed{\text{Stable}}$$

**Causality:** If $t=-1 \Rightarrow y(-1) = x(-3) + x(3) \Rightarrow$ depends on future input, $\boxed{\text{not causal}}$

b) $y(t) = \cos(3t)\, x(t)$

**Linearity:** $x_1(t) \mapsto \cos 3t\, x_1(t) \xrightarrow{\times\alpha}$
$x_2(t) \mapsto \cos 3t\, x_2(t) \xrightarrow{\times\beta}$ $\xrightarrow{+} \alpha \cos 3t\, x_1(t) + \beta \cos 3t\, x_2(t)$

// $\boxed{\text{Linear}}$

$x_1(t) \xrightarrow{\times\alpha}$, $x_2(t) \xrightarrow{\times\beta}$ $\xrightarrow{+} \alpha x_1(t) + \beta x_2(t) \mapsto \cos 3t(\alpha x_1(t) + \beta x_2(t))$

**Time invariance:** $x(t) \xrightarrow{\text{delay}} x(t-t_0) \mapsto \cos 3t\, x(t-t_0)$ ?
$y(t-t_0) = \cos(3t-3t_0)\, x(t-t_0)$ $\Big\}$ $\boxed{\text{Not time invariant}}$

**Stability:** If we assume $x(t)$ is bounded, then $y(t)$ will also be bounded as it is the multiple of two bounded functions, it will be $\boxed{\text{Stable}}$.

**Causality:** Doesn't depend on future input, hence $\boxed{\text{Causal}}$.

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c) $Y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$

Linearity: $x_1(t) \mapsto \int_{-\infty}^{2t} x_1(\tau)d\tau \xrightarrow{\times\alpha}$

$x_2(t) \mapsto \int_{-\infty}^{2t} x_2(\tau)d\tau \xrightarrow{\times\beta}$ $+ \rightarrow \int_{-\infty}^{2t} \alpha x_1(\tau)d\tau + \int_{-\infty}^{2t} \beta x_2(\tau)d\tau$

$x_1(t) \xrightarrow{\times\alpha}$

$x_2(t) \xrightarrow{\times\beta}$ $+ \rightarrow \alpha x_1(t) + \beta x_2(t) \mapsto \int_{-\infty}^{2t} \alpha x_1(\tau) + \beta x_2(\tau)\,d\tau$ $\Big\} = \rightarrow$ $\boxed{\text{Linear}}$

Time Invariance: $w(t) = \int_{-\infty}^{2t} x(\tau - t_0)\,d\tau \rightarrow \sigma = \tau - t_0$, $d\sigma = d\tau$ $\boxed{\text{Time variant}}$

$= \int_{-\infty}^{2t-t_0} x(\sigma)\,d\sigma \neq Y(t-t_0) = \int_{-\infty}^{2t-2t_0} x(\tau)\,d\tau$

Stability: As lower limit is $-\infty$, sum can grow infinitely large without bound, hence $\boxed{\text{unstable.}}$

Causality: $\boxed{\text{Not causal}}$, depends on future input $(2t)$.

d) $Y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t-2) & t \geq 0 \end{cases} = \big(x(t) + x(t-2)\big)\,u(t)$

Linearity: $x_1(t) \mapsto (x_1(t) + x_1(t-2))\,u(t) \xrightarrow{\times\alpha}$

$x_2(t) \mapsto (x_2(t) + x_2(t-2))\,u(t) \xrightarrow{\times\beta}$ $+ \rightarrow \alpha(x_1(t) + x_1(t-2))u(t) + \beta(x_2(t) + x_2(t-2))u(t)$

$// \rightarrow$ $\boxed{\text{Linear}}$

$x_1(t) \xrightarrow{\times\alpha}$

$x_2(t) \xrightarrow{\times\beta}$ $+ \rightarrow \alpha x_1(t) + \beta x_2(t) \mapsto u(t)\big(\alpha x_1(t) + \beta x_2(t) + \alpha x_1(t-2) + \beta x_2(t-2)\big)$

Time Invariance: $x(t) \xrightarrow{\text{delay}} x(t-t_0) \mapsto (x(t-t_0) + x(t-t_0-2))\,u(t)$ $\Big\}$ not equal

$Y(t-t_0) = (x(t-t_0) + x(t-t_0-2))\,u(t-t_0)$ $\boxed{\text{Time variant}}$

Stability: $\boxed{\text{Stable}}$ if $x(t)$ is bounded.

Causality: Doesn't depend on future input, $\boxed{\text{causal.}}$

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e) $y(t)=\begin{cases}0 & x(t)<0\\ x(t)+x(t-2) & x(t)\geq 0\end{cases} = [x(t)+x(t-2)]\,u(x(t))$

Linearity: $x_1(t) \mapsto (x_1(t)+x_1(t-2))\,u(x_1(t)) \xrightarrow{\times\alpha}$
$x_2(t) \mapsto (x_2(t)+x_2(t-2))\,u(x_2(t)) \xrightarrow{\times\beta}$ $+ \to \alpha(x_1(t)+x_1(t-2))u(x_1(t))+\beta(x_2(t)+x_2(t-2))u(x_2(t))$

$\neq$ $\boxed{\text{Not linear}}$

$x_1(t) \xrightarrow{\times\alpha}$
$x_2(t) \xrightarrow{\times\beta}$ $+ \to \alpha x_1(t)+\beta x_2(t) \mapsto (\alpha x_1(t)+\beta x_2(t)+\alpha x_1(t-2)+\beta x_2(t-2))\,u(\alpha x_1(t)+\beta x_2(t))$

Time invariance: $x(t) \xrightarrow{\text{delay}} x(t-t_0) \mapsto (x(t-t_0)+x(t-t_0-2))\,u(x(t-t_0))$
$y(t-t_0)=(x(t-t_0)+x(t-t_0-2))\,u(x(t-t_0))$ $\boxed{\text{Time invariant}}$

Stability: $\boxed{\text{Stable}}$ if $x(t)$ is bounded.

Causality: Doesn't depend on future input, $\boxed{\text{Causal}}$

f) $y(t)=x\left(\frac{t}{3}\right)$

Linearity: $x_1(t) \mapsto x_1\left(\frac{t}{3}\right) \xrightarrow{\times\alpha}$
$x_2(t) \mapsto x_2\left(\frac{t}{3}\right) \xrightarrow{\times\beta}$ $+ \to \alpha x_1\left(\frac{t}{3}\right)+\beta x_2\left(\frac{t}{3}\right)$ $\boxed{\text{Linear}}$
$\alpha x_1(t)+\beta x_2(t) \mapsto \alpha x_1\left(\frac{t}{3}\right)+\beta x_2\left(\frac{t}{3}\right)$

Time invariance: $x(t) \xrightarrow{\text{delay}} x(t-t_0) \mapsto x\left(\frac{t}{3}-t_0\right)$
$y(t-t_0)=x\left(\frac{t-t_0}{3}\right)$ $\neq$ $\boxed{\text{Time Variant}}$

Stability: If input is bounded, output will also be bounded, hence $\boxed{\text{stable}}$

Causality: If $t=-3 \Rightarrow y(-3)=x(-1) \Rightarrow$ future input, $\boxed{\text{not causal}}$

g) $y(t)=\frac{dx(t)}{dt}$

Linearity: $x_1(t) \mapsto \frac{dx_1(t)}{dt}$
$x_2(t) \mapsto \frac{dx_2(t)}{dt}$ $\to \alpha\frac{dx_1(t)}{dt}+\beta\frac{dx_2(t)}{dt}$
$\alpha x_1(t)+\beta x_2(t) \mapsto \frac{d(\alpha x_1(t)+\beta x_2(t))}{dt}$ $= \to \boxed{\text{Linear}}$

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Time invariance: $x(t) \xrightarrow{delay} x(t-t_0) \mapsto \frac{dx(t-t_0)}{dt}$ $\Rightarrow$ $\boxed{\text{Time invariant}}$

$y(t-t_0) = \frac{dx(t-t_0)}{dt}$

Stability: $\boxed{\text{Unstable}}$ as for a discontinuous signal, derivative at point of discontinuity becomes unbounded.

Causality: $\frac{dx(t)}{dt} = \lim_{h \to 0} \frac{x(t+h)-x(t)}{h}$ future input, $\boxed{\text{not causal}}$

9) $y[n] = -y[n-5] + x[n]$

a) $Y(z) = -z^{-5}Y(z) + X(z)$

$Y(z) + z^{-5}Y(z) = X(z) \Rightarrow Y(z)(1+z^{-5}) = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+z^{-5}} = \frac{z^5}{z^5+1}$

b) $H(z) = \frac{z^5}{z^5+1} \Rightarrow \boxed{5\text{poles}}$: $z^5 = -1 \Rightarrow z^5 = j^2 \Rightarrow z = j^{\frac{2}{5}}$

5zeros

$z = x+jy = re^{\theta} = r(\cos\theta + j\sin\theta) \Rightarrow z^5 = (x+jy)^5 = r^5 e^{5\theta} = r^5(\cos 5\theta + j\sin 5\theta)$

$j = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} \Rightarrow j^{\frac{2}{5}} = (\cos\frac{\pi}{2} + j\sin\frac{\pi}{2})^{\frac{2}{5}} = (\cos(2\pi k + \frac{\pi}{2}) + j\sin(2\pi k + \frac{\pi}{2}))^{\frac{2}{5}}$

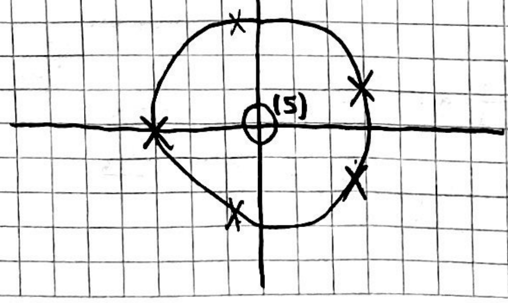
$= \cos(\frac{4\pi k}{5} + \frac{\pi}{5}) + j\sin(\frac{4\pi k}{5} + \frac{\pi}{5})$

$k=0 \Rightarrow z = \cos(\frac{\pi}{5}) + j\sin(\frac{\pi}{5}) = e^{j\frac{\pi}{5}}$ | $k=1 \Rightarrow z = \cos(\frac{4\pi}{5}+\frac{\pi}{5}) + j\sin(\frac{4\pi}{5}+\frac{\pi}{5}) = \cos\pi + j\sin\pi = -1$

$k=2 \Rightarrow z = \cos(\frac{8\pi}{5}+\frac{\pi}{5}) + j\sin(\frac{8\pi}{5}+\frac{\pi}{5}) \overset{\text{subtract } 2\pi}{=} \cos(\frac{\pi}{5}) - j\sin(\frac{\pi}{5}) = e^{-j\frac{\pi}{5}}$

$k=3 \Rightarrow z = \cos(\frac{13\pi}{5}) + j\sin(\frac{13\pi}{5}) = \cos\frac{3\pi}{5} + j\sin(\frac{3\pi}{5}) = e^{j\frac{3\pi}{5}}$

$k=4 \Rightarrow z = \cos(\frac{17\pi}{5}) + j\sin(\frac{17\pi}{5}) = \cos\frac{3\pi}{5} - j\sin\frac{3\pi}{5} = e^{-j\frac{3\pi}{5}}$

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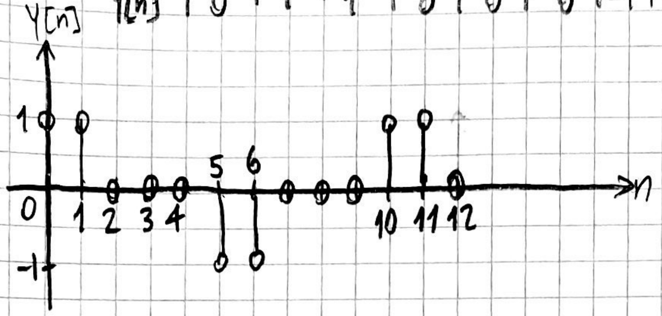
c)

| $n$ | $<0$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y[n]$ | 0 | 1 | 1 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 |




d) $T=10$, as can be observed from the plot in part (c).

10) $y(t) = x_1(t) * x_2(t) \xrightarrow[\text{convolution}]{\text{time}} Y(jw) = X_1(jw)X_2(jw)$

Since $X_1(jw)=0$ for $-1000\pi < w < 1000\pi$, frequency of $y(t)$ is $w_y = 1000\pi$

$f_y = 500\ Hz$

$$y_p(t) = a_0 + \sum_{n=-\infty}^{\infty} y(nT)\,\delta(t-nT)$$

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$$Y_p(jw) = \cancel{2\pi a_0 \delta(w) +} \sum_{n=-\infty}^{\infty} Y(w-nw_s) = 2\pi a_0\,\delta(w) + \sum_{n=-\infty}^{\infty} Y(w-w_s n)$$

We must have $f_s \geq 2f_y$ to recover the signal completely

$\Rightarrow f_s \geq 2 \cdot 500 = 1000$

$\Rightarrow \frac{1}{T_s} \geq 1000 \Rightarrow T_s \leq 10^{-3} \Rightarrow T_s \leq 1\ ms$

$T_s \in [0,1]\ ms$ so that $y(t)$ is recoverable from $y_p(t)$.

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