CH5 Suggested Problem Solutions

P- 5.3

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) MAKE A TABLE:

n	<0	0	1	2	3	4	5	6	7	≥ 8
$x[n]$	0	1	2	3	2	1	1	1	1	1
$y[n]$	0	2	1	2	-1	2	3	1	1	1 .....

$$y[0] = 2x[0] - 3x[-1] + 2x[-2] = 2(1) = 2$$

$$y[1] = 2x[1] - 3x[0] + 2x[-1] = 2(2) - 3(1) = 1$$

$$y[2] = 2x[2] - 3x[1] + 2x[0] = 2(3) - 3(2) + 2(1) = 2$$

$$y[3] = 2(2) - 3(3) + 2(2) = -1$$

$$y[4] = 2(1) - 3(2) + 2(3) = 2$$

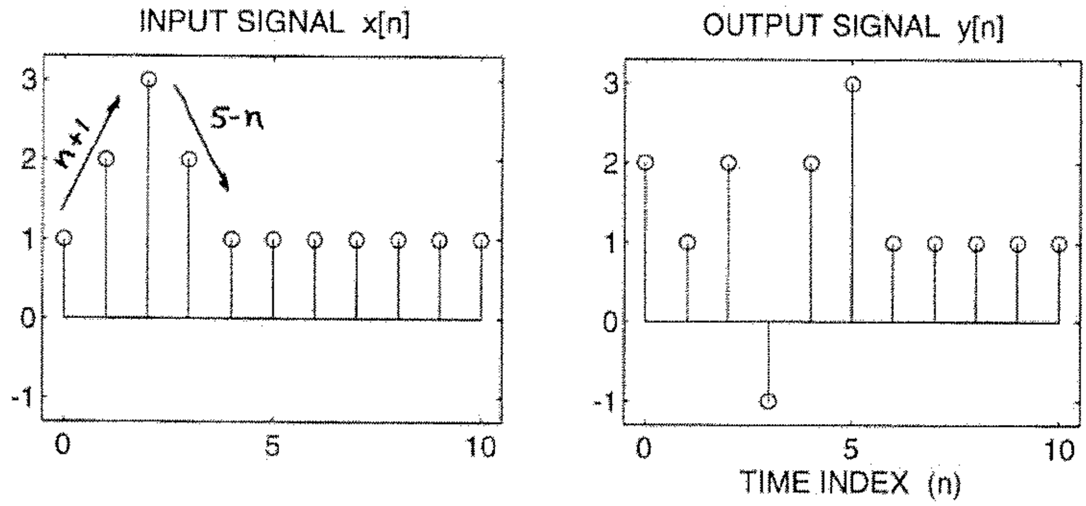
$$y[5] = 2(1) - 3(1) + 2(2) = 3$$

$$y[6] = 2(1) - 3(1) + 2(1) = 1$$

$$y[7] = 2(1) - 3(1) + 2(1) = 1$$

$$y[8] = 2(1) - 3(1) + 2(1) = 1$$

(b)



(c) Impulse Response

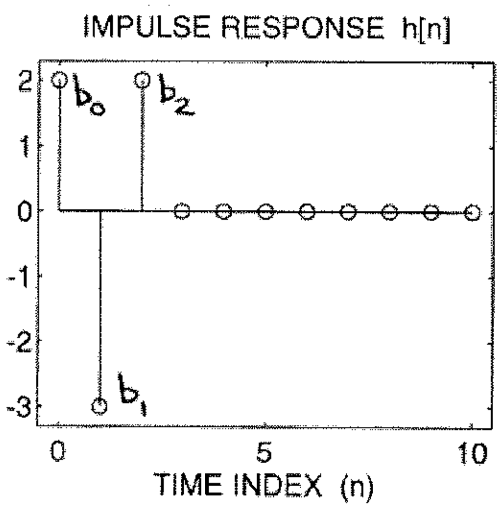
$h[0] = 2(1) - 3(0) + 2(0) = 2$

$h[1] = 2(0) - 3(1) + 2(0) = -3$

$h[2] = 2(0) - 3(0) + 2(0) = 2$

Notice $h[n]$ just "reads out" the filter coefficients:

i.e., $h[n] = b_n$



P5.8

Use convolution

n:	...	-2	-1	0	1	2	3	4	....
$x[n]$:		0	1	0	1	0	1	0	....
$h[n]$:				13	-13	13			
	....	0	13	0	13	0	13	0	....
	....	-13	0	-13	0	-13	0	-13	....
	....	0	13	0	13	0	13	0	.....
		-13	26	-13	26	-13	26	-13	

$\uparrow$ $n=0$

$$\Rightarrow \quad y[n] = \begin{cases} -13 & \text{for } n \text{ even} \\ 26 & \text{for } n \text{ odd} \end{cases}$$

P-5.9

Linearity?

(a) YES.

Let $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$ $\qquad x_1[n] \to y_1[n]$, $\; x_2[n] \to y_2[n]$

$$\Rightarrow y[n] = (\alpha_1 x_1[n] + \alpha_2 x_2[n]) \cos(0.2\pi n)$$

$$= \alpha_1 \underbrace{x_1[n]\cos(0.2\pi n)}_{y_1[n]} + \alpha_2 \underbrace{x_2[n]\cos(0.2\pi n)}_{y_2[n]}$$

(b) YES.

$$y[n] = (\alpha_1 x_1[n] - \alpha_2 x_2[n]) - (\alpha_1 x_1[n-1] + \alpha_2 x_2[n-1])$$

$$= \alpha_1 \underbrace{(x_1[n] - x_1[n-1])}_{y_1[n]} + \alpha_2 \underbrace{(x_2[n] - x_2[n-1])}_{y_2[n]}$$

(c) NO.

Let $x_1[n] = \delta[n]$ and $x_2[n] = -2\delta[n]$.

$\hookrightarrow y_1[n] = \delta[n]$ $\qquad \hookrightarrow y_2[n] = |x_2[n]| = 2\delta[n]$

Let $x[n] = x_1[n] + x_2[n] = \delta[n] - 2\delta[n] = -\delta[n]$

$\hookrightarrow y[n] = |x[n]| = \delta[n] \longleftrightarrow y_1[n] + y_2[n] = \delta[n] + 2\delta[n] = 3\delta[n]$

NOT EQUAL!

(d) NO! if $B \neq 0$

if $x_1[n] \to y_1[n]$, test $2x_1[n] \to 2y_1[n]$.

$$A(2x_1[n]) + B = 2(Ax_1[n] + B) - B \neq 2y_1[n]$$

TIME-INVARIANT?

(a) NO!

Let $x[n] = \delta[n]$, then $y[n] = \delta[n]\cos(0.2\pi n) = \delta[n]$

EVAL AT $n=0$

Try $x[n-1] = \delta[n-1]$, then output is

$$\delta[n-1]\cos(0.2\pi n) = \cos(0.2\pi)\,\delta[n-1].$$

But $\cos(0.2\pi)\,\delta[n-1] \neq y[n-1] = \delta[n-1]$

5.9 cont...

TIME-INVARIANT?

(b) Yes.

If $x[n] \longrightarrow y[n]$, Let $v[n] = x[n-n_o]$

OUTPUT $= v[n] - v[n-1] = x[n-n_o] - x[n-n_o-1]$

This is the same as $y[n-n_o] = x[n-n_o] - x[n-n_o-1]$

(c) YES.

Output depends only on x[ ] at "n", so $y[n-n_o] = |x[n-n_o]|$

(d) Yes

$y[n-n_o] = Ax[n-n_o] + B$ is always true.

CAUSAL?

(a) YES.

y[n] at $n=n_o$ depends only on x[n] at $n=n_o$, and not on past <u>or</u> future values.

(b) Yes.

y[n] at $n=n_o$ depends only on x[n] at $n=n_o$ & $n=n_o-1$ so it only uses the "present" and the "past."

(c) YES

y[n] at $n=n_o$ depends only on x[n] at $n=n_o$.

$$y[n_o] = |x[n_o]|$$

(d) YES

y[n] at $n=n_o$ depends only on x[n] at $n=n_o$.

$$y[n_o] = Ax[n_o] + B$$

P_5.12

$x_1[n] = u[n] \longrightarrow y_1[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$

$x_2[n] = 3u[n] - 2u[n-4]$

Use linearity and time-invariance:

$$y_2[n] = 3y_1[n] - 2y_1[n-4]$$

$$= 3\delta[n] + 6\delta[n-1] - 3\delta[n-2] - 2\delta[n-4] - 4\delta[n-5] + 2\delta[n-6].$$

List of values:

n	<0	0	1	2	3	4	5	6	≥ 7
$y_2[n]$	0	3	6	-3	0	-2	-4	2	0

P.5.14

(a) $h[n] = \delta[n-2] \Rightarrow$ filter is a delay by 2

$y[n] = u[n-3] - u[n-6]$

To find $x[n]$ we need to "un-delay" $y[n]$.

$\Rightarrow \quad x[n] = u[n-1] - u[n-4]$

(b) First-difference FIR $\Rightarrow h[n] = \delta[n] - \delta[n-1]$

The first-difference filter has a nonzero output at n when $x[n]$ & $x[n-1]$ are not equal.

If $y[n] = \delta[n] - \delta[n-4]$, then the input $x[n]$ changes value at $n=0$ and $n=4$. At $n=0$, it jumps up by one; at $n=4$, it jumps down.

$$\Rightarrow \quad x[n] = \underbrace{u[n]}_{\text{jump up by one}} - \underbrace{u[n-4]}_{\text{jump down}}$$

(c) 4-pt averager: $y[n] = \frac{1}{4}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$

If $y[n] = -5\delta[n] - 5\delta[n-2]$

$y[0] = -5 = \frac{1}{4}\left(x[0] + x[-1] + x[-2] + x[-3]\right)$

** if we assume $x[n] = 0$ for $n < 0$, then $x[0] = -20$

$y[1] = 0 = \frac{1}{4}\left(x[1] + x[0] + x[-1] + x[-2]\right) = \frac{1}{4}x[1] - 5$

$\Rightarrow x[1] = 20$

$$\left.\begin{aligned} y[2] = -5 &= \tfrac{1}{4}\left(x[2] + x[1] + x[0] + x[-1]\right) \\ &= \tfrac{1}{4}\left(x[2] + 20 - 20 + 0\right) = \tfrac{1}{4}x[2] \end{aligned}\right\} x[2] = -20$$

$y[3] = 0 = \frac{1}{4}\left(x[3] + x[2] + x[1] + x[0]\right) \Rightarrow x[3] = -20$

$$\Rightarrow \quad x[n] = \begin{cases} 0 & \text{for } n < 0 \\ -20 & \text{for } n \text{ even} \\ 20 & \text{for } n \text{ odd} \end{cases}$$

P-5-17

(a) $h_1[n] = \delta[n] - \delta[n-1]$

$h_2[n] = \delta[n] + \delta[n-2]$

$h_3[n] = \delta[n-1] + \delta[n-2]$

(b) The overall $h[n]$ is the convolution of the $h_i[n]$.

$$h[n] = h_1[n] * h_2[n] * h_3[n]$$

$$h_1[n] * h_2[n] = \big(\delta[n] - \delta[n-1]\big) * \big(\delta[n] + \delta[n-2]\big)$$

$$= \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$$

Now convolve with $h_3[n]$

$$\begin{array}{cccccc}
1 & -1 & 1 & -1 & & \\
0 & 1 & 1 & & & \\
\hline
0 & 0 & 0 & 0 & & \\
 & 1 & -1 & 1 & -1 & \\
 & & 1 & -1 & 1 & -1 \\
\hline
0 & 1 & 0 & 0 & 0 & -1 \\
n=0 & \uparrow n=1 & & & & \uparrow n=5
\end{array}$$

$$\boxed{h[n] = \delta[n-1] - \delta[n-5]}$$

(c) $y[n] = h[n] * x[n]$

$= \big(\delta[n-1] - \delta[n-5]\big) * x[n]$

$y[n] = x[n-1] - x[n-5]$