

EE391 Analytical Assignment 2  
Fall 2022

Subject :

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Date : ..... / ..... / .....

$$1) Y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

$$a) h[n] = \frac{1}{4}(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3])$$

$$b) H(e^{j\omega}) = \frac{1}{4} + \frac{1}{4}e^{-j\hat{\omega}} + \frac{1}{4}e^{-2j\hat{\omega}} + \frac{1}{4}e^{-3j\hat{\omega}}$$

$$c) H(e^{j\hat{\omega}}) = \frac{1}{4} e^{-\frac{3}{2}j\hat{\omega}} \left( e^{\frac{3}{2}j\hat{\omega}} + e^{\frac{3}{2}j\hat{\omega}} + e^{-\frac{3}{2}j\hat{\omega}} + e^{-\frac{3}{2}j\hat{\omega}} \right)$$

$$= \frac{1}{4} e^{-\frac{3}{2}j\hat{\omega}} \left( 2 \cos \frac{\hat{\omega}}{2} + 2 \cos \frac{3\hat{\omega}}{2} \right)$$

$$= \frac{1}{2} e^{-\frac{3}{2}j\hat{\omega}} \left( \cos \frac{\hat{\omega}}{2} + \cos \frac{3\hat{\omega}}{2} \right)$$

1,539

$$H[0] = \frac{1}{2} \cdot (1+1) = 1$$

$$H[0, 2\pi] = \frac{1}{2} e^{-\frac{3}{2}j0.2\pi} (\cos 0, 1\pi + \cos 0, 3\pi) \\ \approx 0,769 e^{-0.3\pi j}$$

$$H[0.5\pi] = \frac{1}{2} e^{-\frac{3}{2}j0.5\pi} (\cos 0, 25\pi + \cos 0, 75\pi) = 0$$

$$Y[n] = H[0.5 + 4|H[0, 2\pi]| \cos(0, 2\pi n + \angle H[0, 2\pi])]$$

$$= 5 + 4 \cdot (0,769) \cos(0,2\pi n - 0,3\pi) = 5 + 3,076 \cos(0,2\pi n - 0,3\pi)$$

d)  $x[n] = 0$  for  $n < 0$

$$Y_1[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

when  $n \geq 3$ ,  $Y_1[n] = Y[n]$  in (c)

$$2) a) \delta[n+5] \xrightarrow{z} z^5$$

$$b) \delta[n-5] \xrightarrow{z} z^{-5}$$

$$c) \delta[n-1] \xrightarrow{z} z^{-1}$$

$$d) x(z) = 2 + 4z^{-1} + 6z^{-2} + 4z^{-3} + 2z^{-4}$$

$$X[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$

$$e) x(z) = 1 - 2z^{-1} + 3z^{-3} - z^{-5}$$

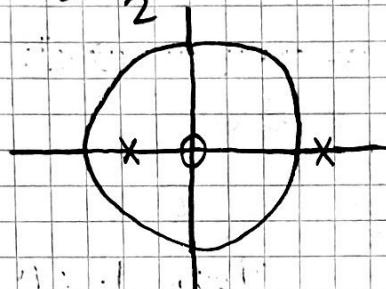
$$X[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-3] - \delta[n-5]$$

$$3) Y[n] = Y[n-1] + Y[n-2] + X[n-1]$$

$$Y(z) = z^{-1}Y(z) + z^2Y(z) + z^{-1}X(z)$$

$$Y(z)(1 - z^{-1} - z^2) = z^{-1}X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{(1 - z^{-1} - z^2)} \cdot \frac{z^2}{z^2} \\ = \frac{z}{z^2 - z - 1}$$

Zeros:  $z=0$ , poles:  $z=\frac{1 \pm \sqrt{5}}{2}$

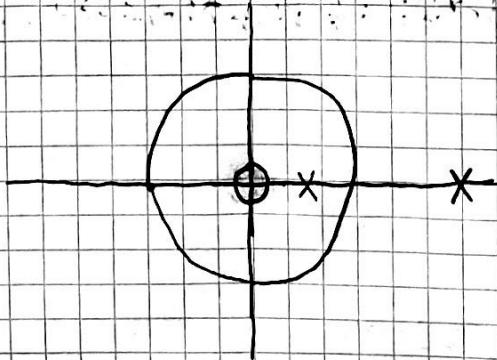


$$Y[n-1] - \frac{5}{2}Y[n] + 4Y[n+1] = X[n]$$

$$z^{-1}Y(z) - \frac{5}{2}Y(z) + 2Y(z) = X(z)$$

$$Y(z)(z^{-1} - \frac{5}{2} + z) = X(z) \Rightarrow H(z) = \frac{1}{z^{-1} + z - \frac{5}{2}} = \frac{z}{z^2 + \frac{5}{2}z + 1}$$

Zeros:  $z=0$ , poles:  $z=2, z=-\frac{5}{2}$



a)  $h[n] = \delta[n-2] \rightarrow \text{delay by two}$

$$y[n] = u[n-3] - u[n-6]$$

$$x[n] = u[n-1] - u[n-4]$$

b) first difference;  $h[n] = \delta[n] - \delta[n-1]$

$x[n]$  is nonzero at  $n=0$  and  $n=4$  ( $n=0 \Rightarrow x[n]=1$ ,  $n=4 \Rightarrow x[4]=-1$ )

$$x[n] = u[n] - u[n-4]$$

c) 4 point average;  $y[n] = \frac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$

$$y[n] = -5\delta[n] - 5\delta[n-2]$$

Assuming  $x[n]=0$  for  $n < 0$

$$y[0] = -5 = \frac{1}{4}(x[0] + x[-1] + x[-2] + x[-3]) \Rightarrow x[0] = -20$$

$$y[1] = 0 = \frac{1}{4}(x[1] + \underbrace{x[0]}_{-20} + x[-1] + x[-2]) = \frac{1}{4}x[1] - 5$$

$$x[1] = 20$$

$$y[2] = -5 = \frac{1}{4}(x[2] + x[1] + x[0] + x[-1]) = \frac{1}{4}(x[2] + 20 - 20 + 0) = \frac{1}{4}x[2]$$

$$x[2] = -20$$

$$y[3] = 0 = \frac{1}{4}(x[3] + x[2] + x[1] + x[0]) = \frac{1}{4}(x[3] - 20 + 20 - 20) = \frac{1}{4}x[3] - 5$$

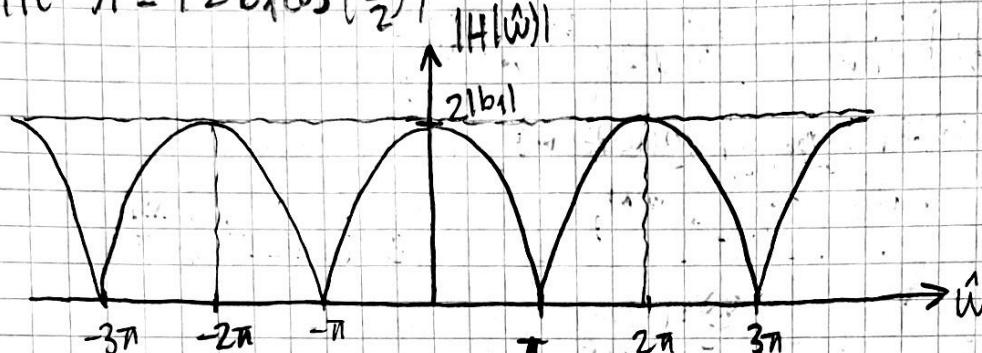
$$x[3] = 20$$

$$x[n] = \begin{cases} 0 & \text{if } n < 0 \\ -20 & \text{if } n \text{ even} \\ 20 & \text{if } n \text{ odd} \end{cases}$$

$$5) H(e^{j\hat{\omega}}) = b_0 + b_1 e^{-j\hat{\omega}} + b_2 e^{-j2\hat{\omega}} + b_3 e^{-j3\hat{\omega}}$$

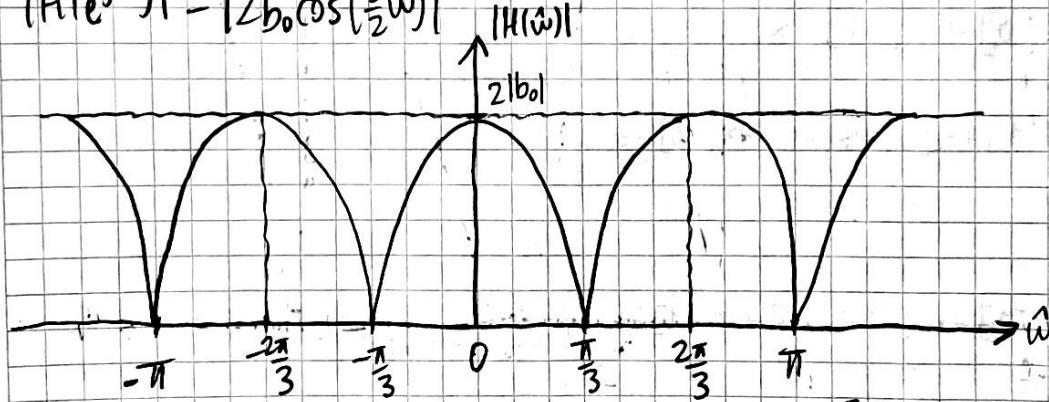
$$\text{a) } H(e^{j\hat{\omega}}) = b_1(e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}}) = b_1 e^{-j\frac{3}{2}\hat{\omega}} \left( e^{j\frac{1}{2}\hat{\omega}} + e^{-j\frac{1}{2}\hat{\omega}} \right) = 2b_1 \cos\left(\frac{\hat{\omega}}{2}\right) e^{-j\frac{3}{2}\hat{\omega}}$$

$$|H(e^{j\hat{\omega}})| = 12b_1 \cos\left(\frac{\hat{\omega}}{2}\right)$$



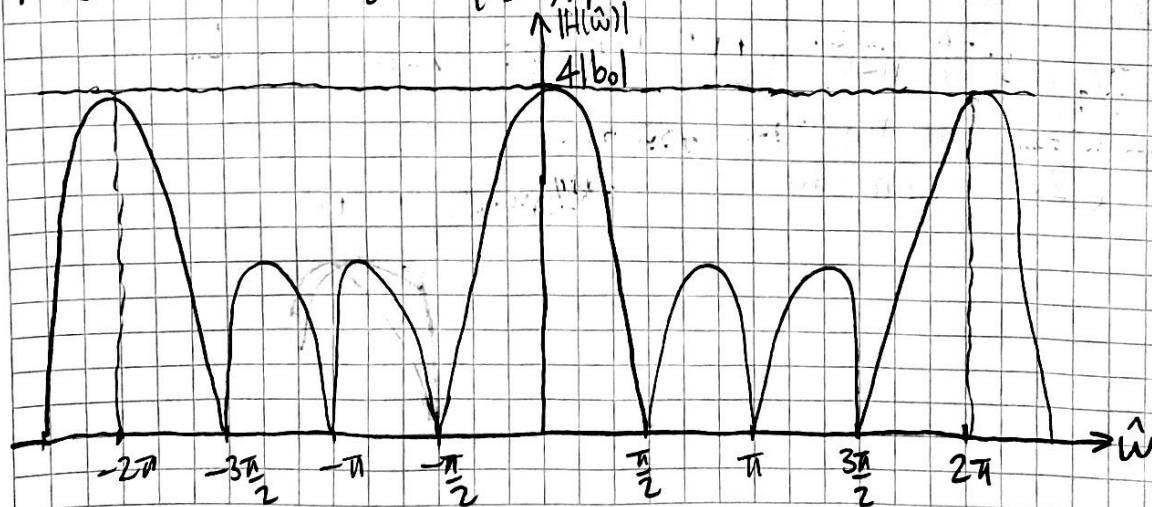
$$\text{b) } H(e^{j\hat{\omega}}) = b_0(1 + e^{-j3\hat{\omega}}) = b_0 e^{-j\frac{3}{2}\hat{\omega}} \left( e^{j\frac{3}{2}\hat{\omega}} + e^{-j\frac{3}{2}\hat{\omega}} \right) = 2b_0 \cos\left(\frac{3}{2}\hat{\omega}\right) e^{-j\frac{3}{2}\hat{\omega}}$$

$$|H(e^{j\hat{\omega}})| = 12b_0 \cos\left(\frac{3}{2}\hat{\omega}\right)$$



$$\text{c) } H(e^{j\hat{\omega}}) = b_0 e^{-j\frac{3}{2}\hat{\omega}} \left( 2\cos\left(\frac{\hat{\omega}}{2}\right) + 2\cos\left(\frac{3}{2}\hat{\omega}\right) \right) = 2b_0 e^{-j\frac{3}{2}\hat{\omega}} \left( \cos\left(\frac{\hat{\omega}}{2}\right) + \cos\left(\frac{3}{2}\hat{\omega}\right) \right)$$

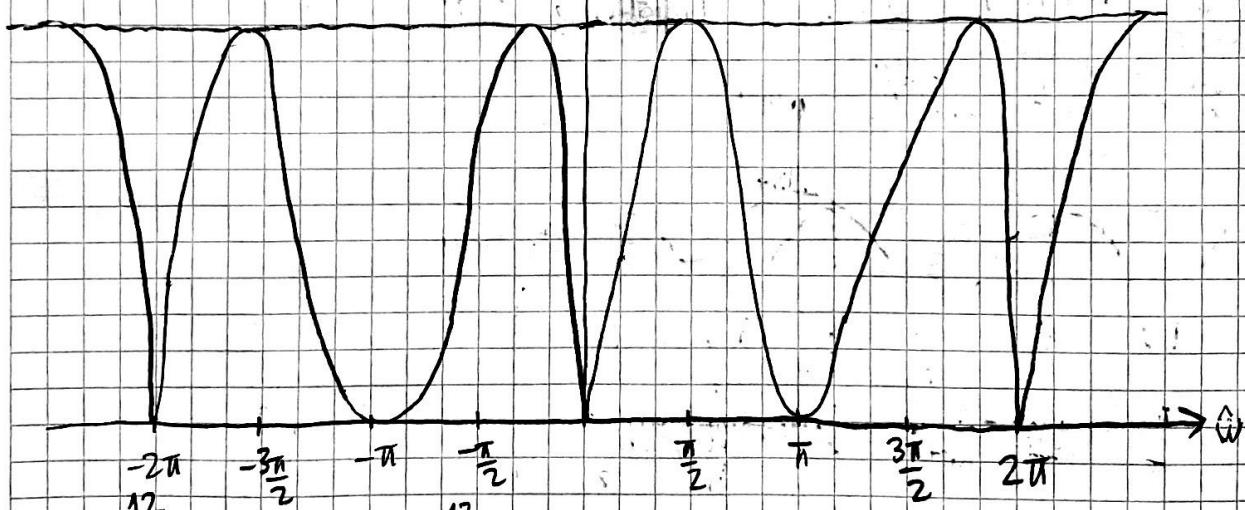
$$|H(e^{j\hat{\omega}})| = 12b_0 \left( \cos\left(\frac{\hat{\omega}}{2}\right) + \cos\left(\frac{3}{2}\hat{\omega}\right) \right)$$



$$\begin{aligned}
 d) H(e^{j\hat{\omega}}) &= -b_1 + b_1 e^{-j\hat{\omega}} - b_3 e^{-j2\hat{\omega}} + b_3 e^{-j3\hat{\omega}} \\
 &= b_1(e^{-j\hat{\omega}} - 1) + b_3(e^{-j3\hat{\omega}} - e^{-j2\hat{\omega}}) \\
 &= b_1(e^{-j\hat{\omega}} - 1 + e^{-j3\hat{\omega}} - e^{-j2\hat{\omega}}) \\
 &= b_1 e^{-j\frac{3}{2}\hat{\omega}} (e^{j\frac{1}{2}\hat{\omega}} - e^{j\frac{3}{2}\hat{\omega}} + e^{-j\frac{3}{2}\hat{\omega}} - e^{-j\frac{1}{2}\hat{\omega}}) \\
 &= b_1 e^{-j\frac{3}{2}\hat{\omega}} (2j \sin(\frac{\hat{\omega}}{2}) + 2j \sin(\frac{3\hat{\omega}}{2})) \\
 &= 2j b_1 e^{-j\frac{3}{2}\hat{\omega}} (\sin(\frac{\hat{\omega}}{2}) + \sin(\frac{3\hat{\omega}}{2})) = 2b_1 e^{-j\frac{3}{2}\hat{\omega}} e^{j\frac{\pi}{2}} (\sin(\frac{\hat{\omega}}{2}) + \sin(\frac{3\hat{\omega}}{2}))
 \end{aligned}$$

$$|H(e^{j\hat{\omega}})| = |2b_1(\sin(\frac{\hat{\omega}}{2}) + \sin(\frac{3\hat{\omega}}{2}))|$$

$|H(\hat{\omega})|$



$$\begin{aligned}
 6) a) \int_{-\infty}^{\infty} |e^{-0.1(t-2)}| dt &= \int_{-\infty}^{\infty} e^{-0.1t} e^{0.2} dt = e^{0.2} \left[ \frac{e^{-0.1t}}{-0.1} \right]_{-\infty}^{\infty} \\
 &= -\frac{e^{0.2}}{0.1} (e^{-1.2} - e^{0.2}) = -10(e^{-1} - 1) < \infty \Rightarrow \text{system is stable}
 \end{aligned}$$

b) System is causal because  $h(t) = 0$  for  $t < 0$ .

$$c) Y(z) = \int_{-\infty}^{\infty} (t+2) h(t) z^{-t} dt = h(t+2) = e^{-0.1(t+4)} (u(t+4) - u(t-4))$$

$$\text{Q1) } h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

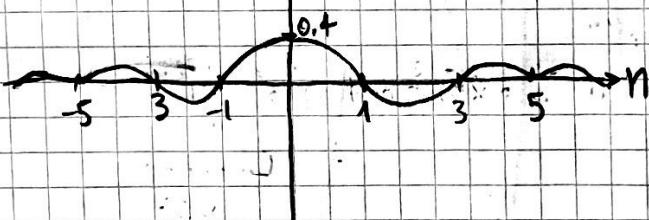
$$h[n] = \frac{1}{2\pi} \left[ \int_{-\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c} e^{j\omega n} d\omega + \int_{\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c} e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi} \left[ \left[ \frac{e^{j\omega n}}{jn} \right]_{-\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c} + \left[ \frac{e^{j\omega n}}{jn} \right]_{\frac{\pi}{2}-w_c}^{\frac{\pi}{2}+w_c} \right] = \frac{1}{2\pi} \left( \frac{1}{jn} (e^{jn(\omega_c - \frac{\pi}{2})} - e^{-jn(\omega_c + \frac{\pi}{2})}) + \frac{1}{jn} (e^{jn(\omega_c + \frac{\pi}{2})} - e^{-jn(\omega_c - \frac{\pi}{2})}) \right)$$

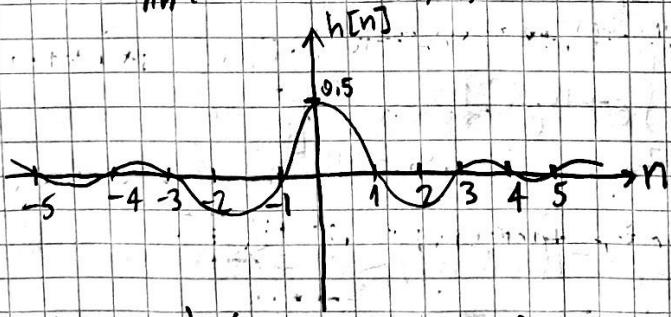
$$= \frac{1}{\pi n} \left( \frac{e^{jn(\omega_c - \frac{\pi}{2})} - e^{-jn(\omega_c - \frac{\pi}{2})}}{2j} + \frac{e^{jn(\omega_c + \frac{\pi}{2})} - e^{-jn(\omega_c + \frac{\pi}{2})}}{2j} \right)$$

$$= \frac{1}{\pi n} (\sin(n(\omega_c - \frac{\pi}{2})) + \sin(n(\omega_c + \frac{\pi}{2}))$$

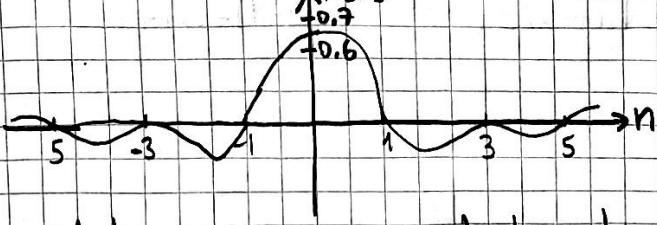
a)  $\omega_c = \frac{\pi}{5} \Rightarrow h[n] = \frac{1}{\pi n} (\sin(-\frac{3\pi}{10}n) + \sin(\frac{7\pi}{10}n))$



b)  $\omega_c = \frac{\pi}{4} \Rightarrow h[n] = \frac{1}{\pi n} (\sin(-\frac{\pi}{4}n) + \sin(\frac{3\pi}{4}n))$



c)  $\omega_c = \frac{\pi}{3} \Rightarrow h[n] = \frac{1}{\pi n} (\sin(-\frac{\pi}{6}n) + \sin(\frac{5\pi}{6}n))$



As  $\omega_c$  is increased,  $h[n]$  gets more concentrated about the origin.

8) a)  $y(t) = x(t-2) + x(2-t)$

Linearity:  $x_1(t) \rightarrow x_1(t-2) + x_1(2-t) \xrightarrow{\alpha}$   
 $x_2(t) \rightarrow x_2(t-2) + x_2(2-t) \xrightarrow{\beta} \xrightarrow{\alpha\beta} \alpha x_1(t-2) + \alpha x_1(2-t) + \beta x_2(t-2) + \beta x_2(2-t)$

Time Invariance:  $x(t) \xrightarrow{\text{delay}} x(t-t_0) \rightarrow x(t-2-t_0) + x(2-t+t_0)$   
 $y(t-t_0) = x(t-t_0-2) + x(2-t+t_0) \xrightarrow{\text{Time invariance}}$

Stability:  $h(t) = f(t-2) + f(2-t)$

$$\int_{-\infty}^{\infty} |f(t-2) + f(2-t)| dt = 2 < \infty \Rightarrow \boxed{\text{Stable}}$$

Causality: If  $t=-1 \Rightarrow y(-1) = x(-3) + x(3) \Rightarrow$  depends on future input, Not causal

b)  $y(t) = \cos(\beta t) x(t)$

Linearity:  $x_1(t) \rightarrow \cos(\beta t) x_1(t) \xrightarrow{\alpha}$   
 $x_2(t) \rightarrow \cos(\beta t) x_2(t) \xrightarrow{\beta} \xrightarrow{\alpha\beta} \alpha \cos(\beta t) x_1(t) + \beta \cos(\beta t) x_2(t)$

Time Invariance:  $x(t) \xrightarrow{\text{delay}} x(t-t_0) \rightarrow \cos(\beta t) x(t-t_0)$   
 $y(t-t_0) = \cos(\beta t - \beta t_0) x(t-t_0)$  Not time invariant

Stability: If we assume  $x(t)$  is bounded, then  $y(t)$  will also be bounded as it is the multiple of two bounded functions, it will be stable.

Causality: Doesn't depend on future input, hence causal

Subject :

$$C) Y(t) = \int_{-\infty}^{2t} x(2) dZ$$

Linearity:  $x_1(t) \mapsto \int_{-\infty}^{2t} x_1(Z) dZ \xrightarrow{\alpha} 2t$

$x_2(t) \mapsto \int_{-\infty}^{2t} x_2(Z) dZ \xrightarrow{\beta} 2t$

$\left. \begin{aligned} & \int_{-\infty}^{2t} (\alpha x_1(Z) + \beta x_2(Z)) dZ \\ &= \int_{-\infty}^{2t} \alpha x_1(Z) dZ + \int_{-\infty}^{2t} \beta x_2(Z) dZ \end{aligned} \right) \Rightarrow \boxed{\text{Linear}}$

Time Invariance:  $W(t) = \int_{-\infty}^{2t} x(Z-t_0) dZ \rightarrow \tau = Z-t_0$

$d\tau = dt$

$= \int_{-\infty}^{2t-t_0} x(\tau) d\tau \neq Y(t-t_0) = \int_{-\infty}^{2t-t_0} x(Z) dZ$

$\boxed{\text{Time variant}}$

Stability: As lower limit is  $-\infty$ , sum can grow infinitely large without bound, hence  $\boxed{\text{Unstable.}}$

Causality:  $\boxed{\text{Not causal}}$ , depends on future input ( $2t$ ),

$$d) Y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t-2) & t \geq 0 \end{cases} = (x(t) + x(t-2)) u(t)$$

Linearity:  $x_1(t) \mapsto (x_1(t) + x_1(t-2)) u(t) \xrightarrow{\alpha}$

$x_2(t) \mapsto (x_2(t) + x_2(t-2)) u(t) \xrightarrow{\beta} \alpha(x_1(t) + x_1(t-2)) u(t) + \beta(x_2(t) + x_2(t-2)) u(t)$

$\left. \begin{aligned} & \alpha x_1(t) + \beta x_2(t) \mapsto u(t)(\alpha x_1(t) + \beta x_2(t) + \alpha x_1(t-2) + \beta x_2(t-2)) \\ & x_1(t) \xrightarrow{\alpha} \alpha x_1(t) + \beta x_2(t) \mapsto u(t)(\alpha x_1(t) + \beta x_2(t) + \alpha x_1(t-2) + \beta x_2(t-2)) \end{aligned} \right) \Rightarrow \boxed{\text{Linear}}$

Time Invariance:  $x(t) \xrightarrow{\text{delay}} x(t-t_0) \mapsto (x(t-t_0) + x(t-t_0-2)) u(t) \quad \text{not equal}$

$y(t-t_0) = (x(t-t_0) + x(t-t_0-2)) u(t-t_0)$

$\boxed{\text{Time variant}}$

Stability:  $\boxed{\text{Stable}}$  if  $x(t)$  is bounded.

Causality: Doesn't depend on future input,  $\boxed{\text{causal.}}$

$$e) Y(t) = \begin{cases} 0 & x(t) < 0 \\ (x(t) + x(t-2)) & x(t) \geq 0 \end{cases} = (x(t) + x(t-2)) u(x(t))$$

Linearity:  $x_1(t) \mapsto (x_1(t) + x_1(t-2)) u(x_1(t)) \xrightarrow{x_1}$   
 $x_2(t) \mapsto (x_2(t) + x_2(t-2)) u(x_2(t)) \xrightarrow{x_2} \alpha(x_1(t) + x_1(t-2)) u(x_1(t)) + \beta(x_2(t) + x_2(t-2)) u(x_2(t))$

$\neq$  **Not Linear**

$$x_1(t) \xrightarrow{\alpha x_1} \alpha x_1(t) + \beta x_2(t) \mapsto (\alpha x_1(t) + \beta x_2(t) + \alpha x_1(t-2) + \beta x_2(t-2)) u(x_1(t) + \beta x_2(t))$$

Time Invariance:  $x(t) \xrightarrow{\text{delay}} x(t-t_0) \mapsto (x(t-t_0) + x(t-t_0-2)) u(x(t-t_0))$

$$Y(t-t_0) = (x(t-t_0) + x(t-t_0-2)) u(x(t-t_0)) \quad \boxed{\text{Time Invariant}}$$

Stability: **Stable** if  $x(t)$  is bounded.

Causality: Doesn't depend on future input, **Causal**.

$$f) Y(t) = x\left(\frac{t}{3}\right)$$

Linearity:  $x_1(t) \mapsto x_1\left(\frac{t}{3}\right) \xrightarrow{\alpha x_1}$   
 $x_2(t) \mapsto x_2\left(\frac{t}{3}\right) \xrightarrow{\beta x_2} \alpha x_1\left(\frac{t}{3}\right) + \beta x_2\left(\frac{t}{3}\right) \quad \boxed{\text{Linear}}$

$$\alpha x_1(t) + \beta x_2(t) \mapsto \alpha x_1\left(\frac{t}{3}\right) + \beta x_2\left(\frac{t}{3}\right)$$

Time Invariance:  $x(t) \xrightarrow{\text{delay}} x(t-t_0) \mapsto x\left(\frac{t-t_0}{3}\right)$

$$Y\left(\frac{t-t_0}{3}\right) = x\left(\frac{t-t_0}{3}\right) \quad \boxed{\text{Time Variant}}$$

Stability: If input is bounded, output will also be bounded, hence **Stable**.

Causality: If  $t = -3 \Rightarrow Y(-3) = x(-1) \Rightarrow$  future input, **Not Causal**

$$g) Y(t) = \frac{dx(t)}{dt}$$

Linearity:  $x_1(t) \mapsto \frac{dx_1(t)}{dt} \xrightarrow{\alpha \frac{dx_1(t)}{dt} + \beta \frac{dx_2(t)}{dt}}$   
 $x_2(t) \mapsto \frac{dx_2(t)}{dt} \xrightarrow{\alpha \frac{dx_1(t)}{dt} + \beta \frac{dx_2(t)}{dt}}$

$$\alpha x_1(t) + \beta x_2(t) \mapsto \frac{d(\alpha x_1(t) + \beta x_2(t))}{dt} \quad \boxed{\text{Linear}}$$

Subject :

Time Invariance:  $x(t) \xrightarrow{\text{delay}} x(t-t_0) \Leftrightarrow \frac{dx(t-t_0)}{dt} = \boxed{\text{Time Invariant}}$

$$y(t-t_0) = \frac{dx(t-t_0)}{dt}$$

Stability: Unstable as for a discontinuous signal, derivative at point of discontinuity becomes unbounded.

Causality:  $\frac{dx(t)}{dt} = \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h}$  future input, not causal

9)  $y[n] = -y[n-5] + x[n]$

a)  $y(z) = -z^{-5}y(z) + x(z)$

$$y(z) + z^{-5}y(z) = x(z) \Rightarrow y(z)(1+z^{-5}) = x(z)$$

$$H(z) = \frac{y(z)}{x(z)} = \frac{1}{1+z^{-5}} = \frac{z^5}{z^5+1}$$

b)  $H(z) = \frac{z^5}{z^5+1} \Rightarrow \boxed{5 poles}$ :  $z^5 = -1 \Rightarrow z^5 = j^2 \Rightarrow z = j^{\frac{2}{5}}$   
5 zeros

$$z = r e^{j\theta} = r(\cos\theta + j\sin\theta) \Rightarrow z^5 = (r e^{j\theta})^5 = r^5 e^{j5\theta} = r^5 (\cos 5\theta + j\sin 5\theta)$$

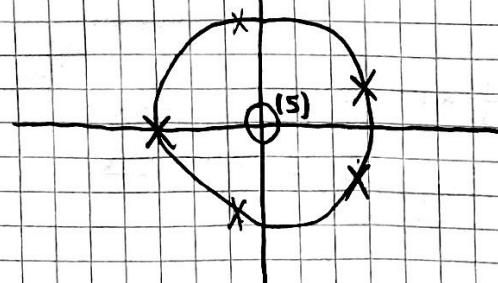
$$j = \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} \Rightarrow j^{\frac{2}{5}} = \left( \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} \right)^{\frac{2}{5}} = \left( \cos(2\pi k + \frac{\pi}{2}) + j \sin(2\pi k + \frac{\pi}{2}) \right)^{\frac{2}{5}} \\ = \cos\left(\frac{4\pi k}{5} + \frac{\pi}{5}\right) + j \sin\left(\frac{4\pi k}{5} + \frac{\pi}{5}\right)$$

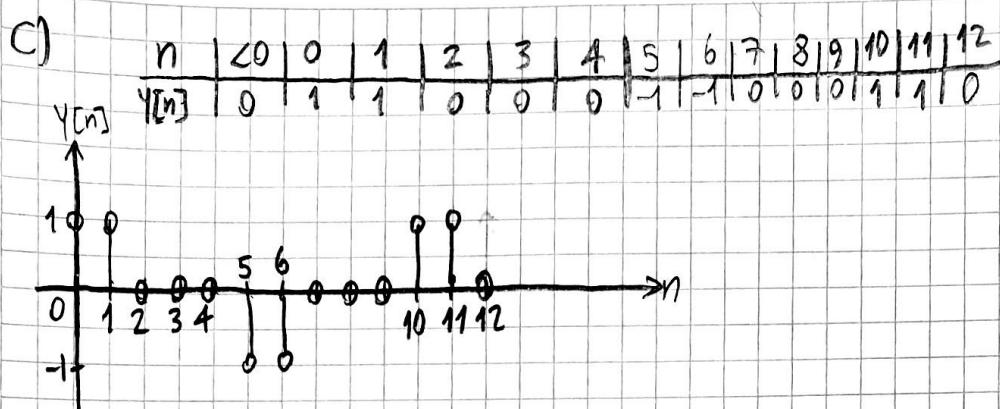
$$k=0 \Rightarrow z = \cos\left(\frac{\pi}{5}\right) + j \sin\left(\frac{\pi}{5}\right) = e^{j\frac{\pi}{5}} \quad | \quad k=1 \Rightarrow z = \cos\left(\frac{4\pi}{5} + \frac{\pi}{5}\right) + j \sin\left(\frac{4\pi}{5} + \frac{\pi}{5}\right) = \cos\pi + j \sin\pi = -1$$

$$k=2 \Rightarrow z = \cos\left(\frac{8\pi}{5} + \frac{\pi}{5}\right) + j \sin\left(\frac{8\pi}{5} + \frac{\pi}{5}\right) \stackrel{\uparrow}{=} \cos\left(\frac{\pi}{5}\right) - j \sin\left(\frac{\pi}{5}\right) = e^{-j\frac{\pi}{5}}$$

$$k=3 \Rightarrow z = \cos\left(\frac{13\pi}{5}\right) + j \sin\left(\frac{13\pi}{5}\right) = \cos\frac{3\pi}{5} + j \sin\left(\frac{3\pi}{5}\right) = e^{j\frac{3\pi}{5}}$$

$$k=4 \Rightarrow z = \cos\left(\frac{17\pi}{5}\right) + j \sin\left(\frac{17\pi}{5}\right) = \cos\frac{3\pi}{5} - j \sin\left(\frac{3\pi}{5}\right) = e^{-j\frac{3\pi}{5}}$$





d)  $T=10$ , as can be observed from the plot in part (c).

$$10) Y(t) = x_1(t) * x_2(t) \xrightarrow{\text{time convolution}} Y(jw) = X_1(jw) \times X_2(jw)$$

Since  $X_1(jw) = 0$  for  $-1000\pi < w < 1000\pi$ , frequency of  $Y(t)$  is  $W_y = 1000\pi$

$$f_y = 500 \text{ Hz}$$

FT  $\downarrow$   $Y_p(t) = a_0 + \sum_{n=-\infty}^{\infty} Y(nT) S(t-nT)$

$$Y_p(jw) = a_0 + \sum_{n=-\infty}^{\infty} Y(nT) S(w-nT) = 2\pi a_0 S(w) + \sum_{n=-\infty}^{\infty} Y(nT) S(w-nT)$$

We must have  $f_s \geq 2f_y$  to recover the signal completely

$$\Rightarrow f_s \geq 2 \cdot 500 = 1000$$

$$\Rightarrow \frac{1}{T_s} \geq 1000 \Rightarrow T_s \leq 10^{-3} \Rightarrow T_s \leq 1 \text{ ms}$$

$T_s \in [0, 1] \text{ ms}$  so that  $Y(t)$  is recoverable from  $Y_p(t)$ .