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IE400 HW2

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Q1)

Decision variables

$$x_i = \begin{cases} 1, & \text{if ingredient } i \text{ is included in drug} \\ 0, & \text{otherwise} \end{cases} \qquad i \in \{1,2,3,4\}$$

Model

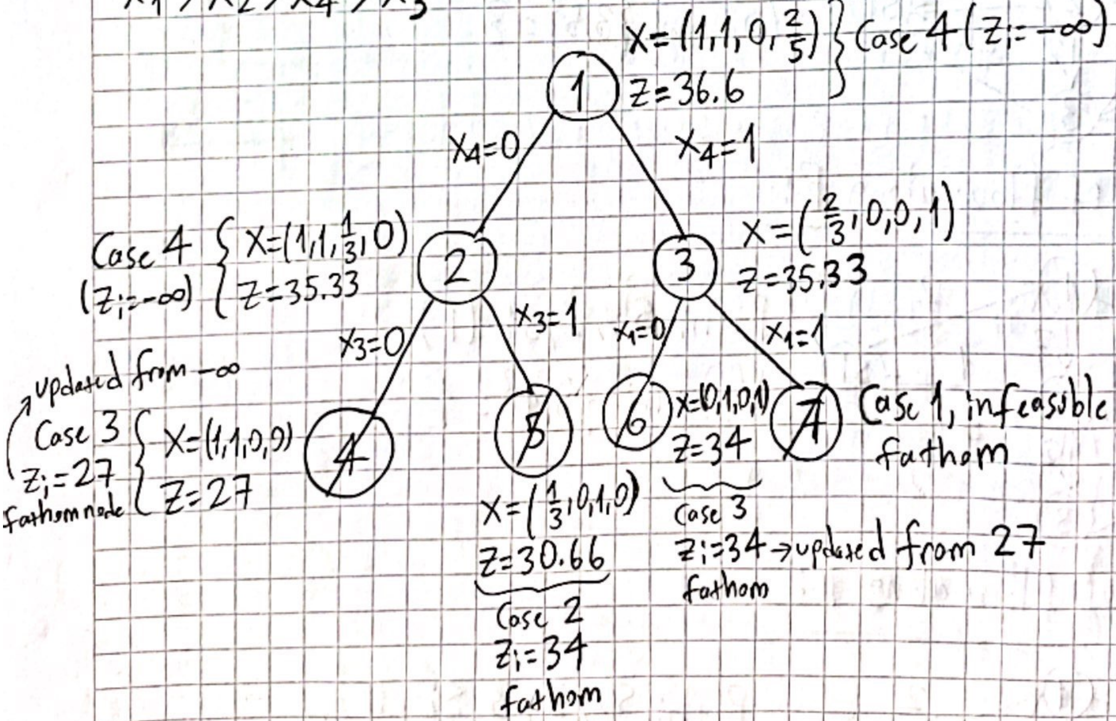
$$\max\ 17x_1 + 19x_2 + 25x_3 + 24x_4$$

$$\text{s.t. } 3x_1 + 2x_2 + 6x_3 + 5x_4 \le 7$$

$$x_1, x_2, x_3, x_4 \in \{0,1\}$$

$x_1 \to \frac{17}{3}$, $x_2 \to 5$, $x_3 \to \frac{25}{6}$, $x_4 \to \frac{24}{5}$ (utility/ml)

$x_1 > x_2 > x_4 > x_3$



<u>Optimal solution:</u> $x^* = (0,1,0,1)$

<u>Optimal value:</u> $Z = 34$

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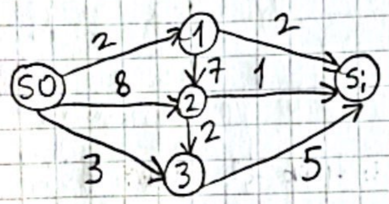
Q2) Using Ford-Fulkerson algorithm to find max-flow, considering costs as capacities:

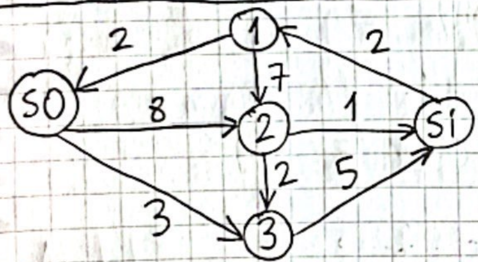
Create $G(x)$:



Path: so, 1, si (P)

$\delta(P) = \min\{2,2\} = 2$

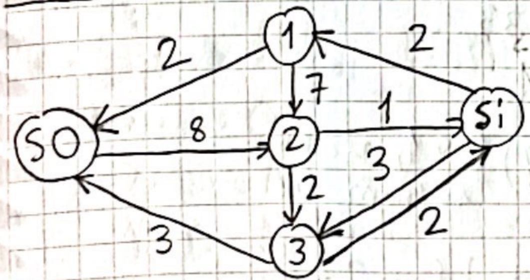
Send 2 units of flow along P:



Path: so, 3, si (P)

$\delta(P) = \min\{3,5\} = 3$

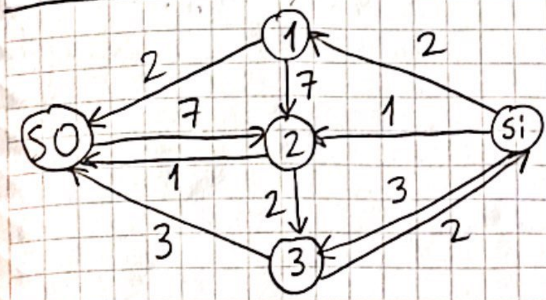
Send 3 units of flow along P:



Path: so, 2, si (P)

$\delta(P) = \min\{8,1\} = 1$

Send 1 unit of flow along P:



Path: so, 2, 3, si (P)

$\delta(P) = \min\{7,2,2\} = 2$

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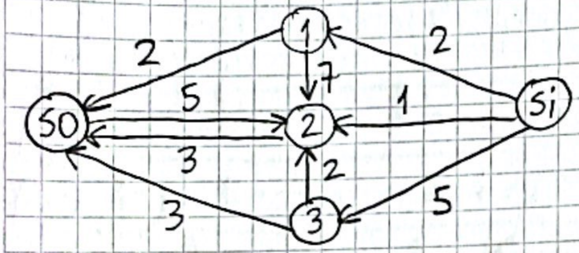
Send 2 units of flow along P:



There is no more directed path from SO to Si, so algorithm ends

Nodes reachable from SO in final G(x): {2, SO}

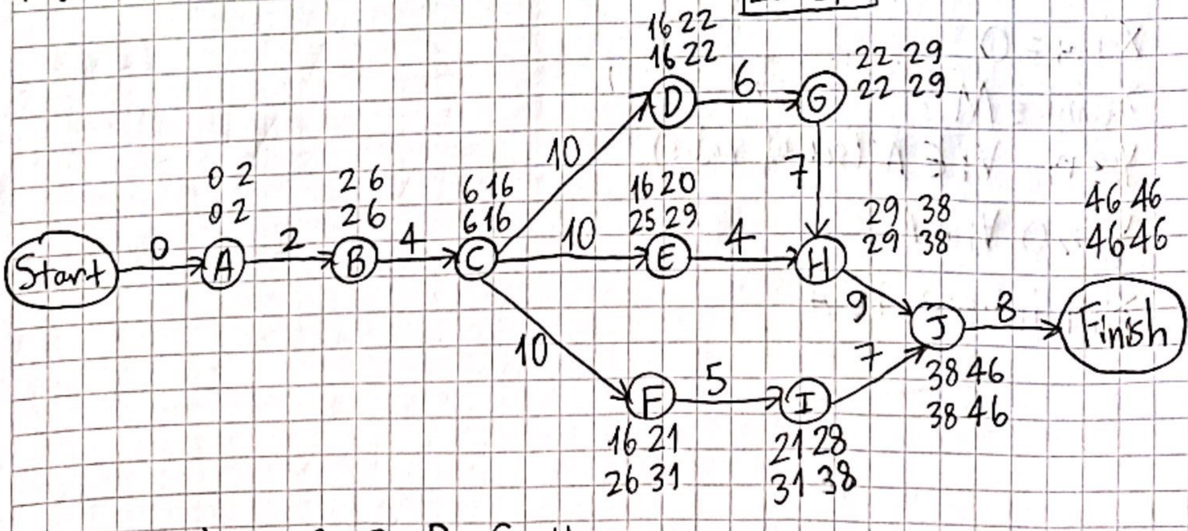
Capacity of cut: 8

Max-flow is equal to min-cut in optimal solution:

Min cost to block: 8

Arcs chosen to be blocked: {(SO,1), (SO,3), (2,3), (2,Si)}

(Q3) a) Following notation is used on top of nodes → $\boxed{\begin{matrix} ES & EF \\ LS & LF \end{matrix}}$



Critical path: A→B→C→D→G→H→J

Project duration: 46 days

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b) E can be delayed 9 days.
F can be delayed 10 days,
I can be delayed 10 days.

c) Decision Variables

X_i: Start time of activity i

Y_i: amount of days activity i is reduced

Parameters

C_i: cost per day of reducing the duration of activity i

r_i: maximum possible reduction in duration (days) of activity i

d_i: duration of activity i

Model E → set of edges, A → set of nodes (including start & finish)

$\min \sum_i C_i Y_i$

s.t. $X_j \geq X_i + d_i - Y_i \quad \forall (i,j) \in E$ (set of edges)

$X_{start} = 0$

$X_{finish} \leq M$

$Y_i \leq r_i \quad \forall i \in A$ (set of nodes)

$Y_i \geq 0 \quad \forall i \in A$

X_i, Y_i integer, $i \in A$

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Q4)

t	$V^t_{(1)}$	$V^t_{(2)}$	$V^t_{(3)}$	$V^t_{(4)}$	$V^t_{(5)}$	$V^t_{(6)}$	d(1)	d(2)	d(3)	d(4)	d(5)	d(6)
0	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$						
1	0	8	6	$+\infty$	$+\infty$	$+\infty$		1	1			
2	0	8	6	8	9	$+\infty$				3	2	
3	0	8	6	8	9	16						5
4	0	8	6	8	9	16						

No change after $t=4$, the algorithm terminates

Shortest path tree: