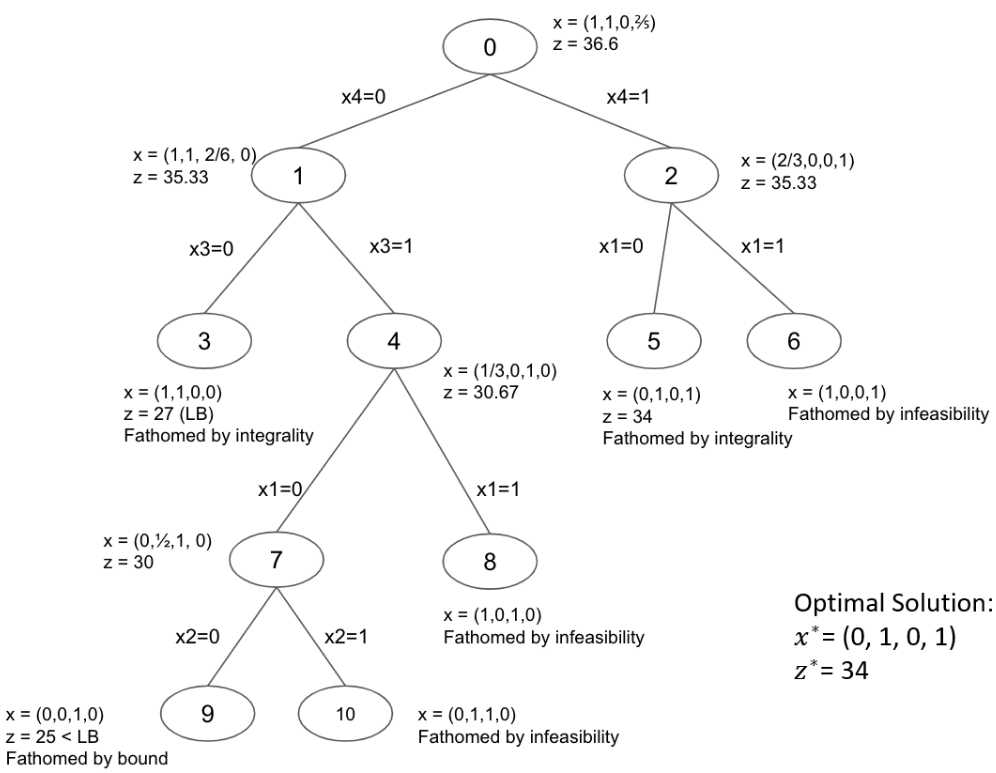
IE 400: Principles of Engineering Management
Homework 2 Solutions

Spring 2022-2023

Question 1.

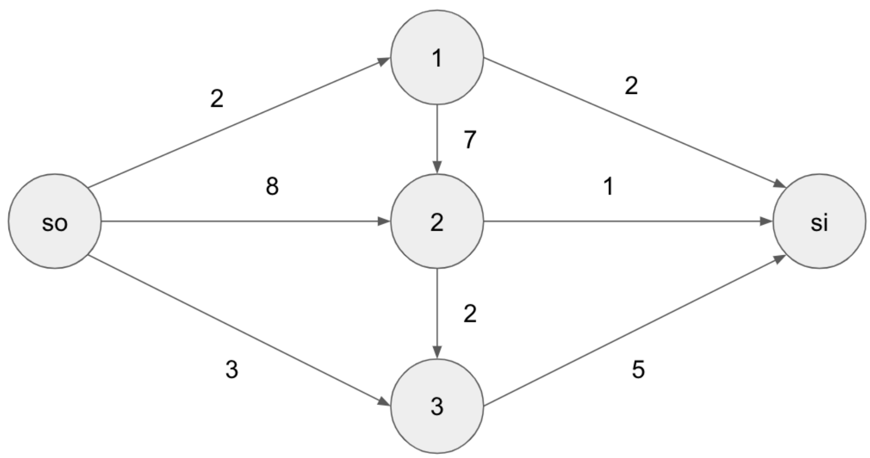
$$\begin{aligned} \text{maximize} \quad & 17x_1 + 10x_2 + 25x_3 + 24x_4 \\ & \text{s.t. } 3x_1 + 2x_2 + 6x_3 + 5x_4 \leq 7 \\ & x_i \in \{0,1\}, \forall i \end{aligned}$$

$$\text{order} \quad x_1 > x_2 > x_4 > x_3$$

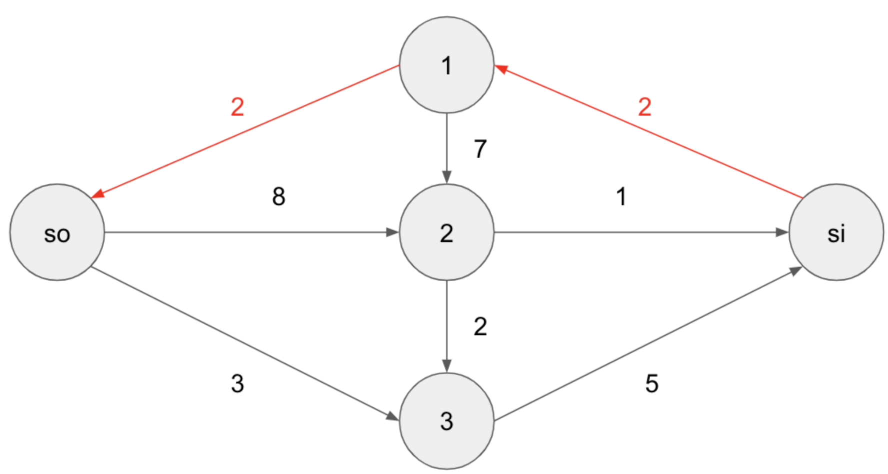


Question 2.

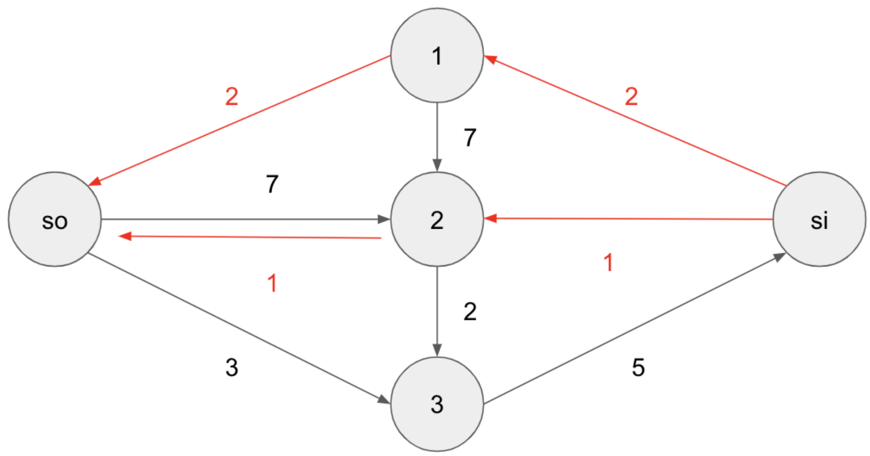
We can apply Ford-Fulkerson algorithm to find the max flow of below networks. At the beginning the residual network is the same with the original network.



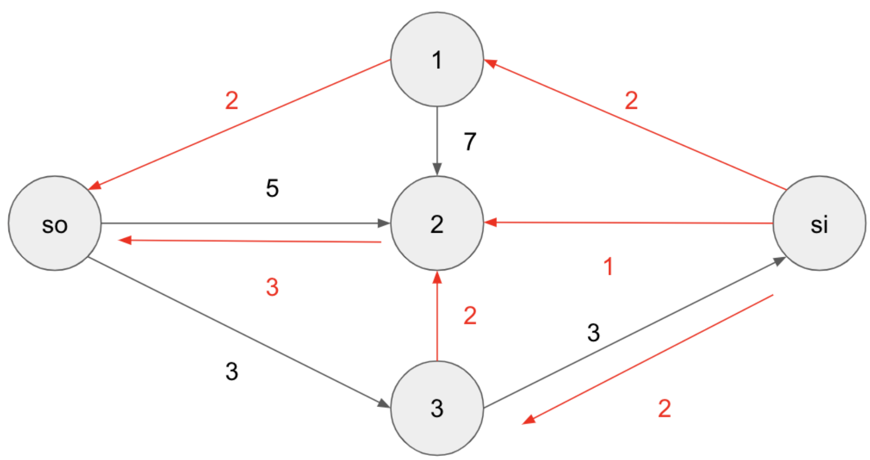
Step 1) Send 2 units on the path so-1-si since along the path minimum capacity is 2 i.e, min{2,2}=2



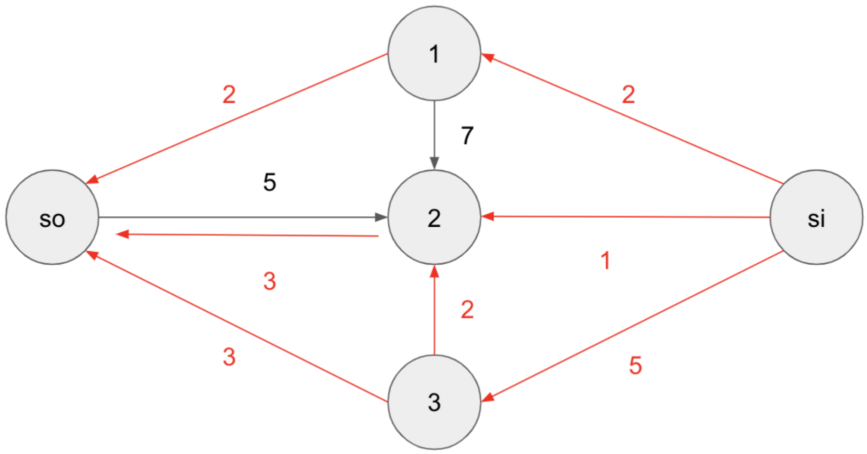
Step 2) Send 1 unit on the path so-2-si since along the path minimum capacity is 1 i.e, min{8,1}=1



Step 3) Send 2 unit on the path so-2-3-si since along the path minimum capacity is 2 i.e, min{7,2,5}=2



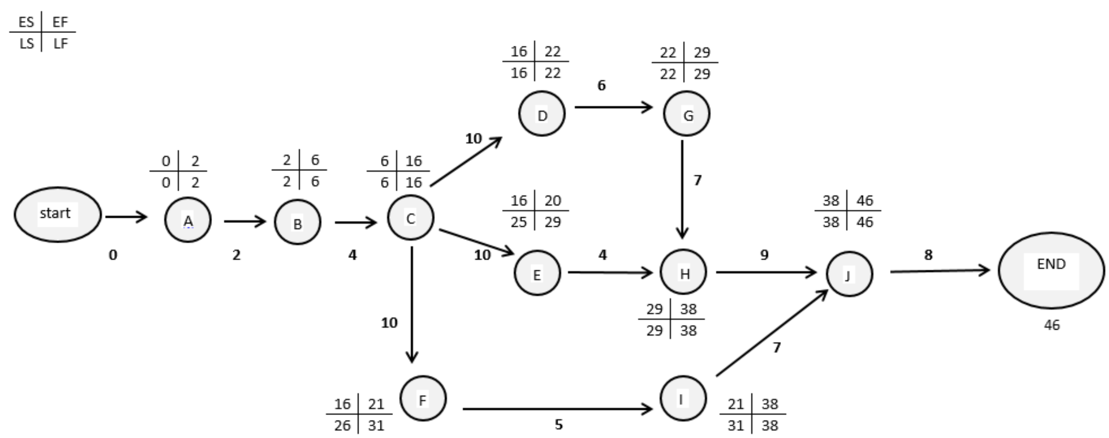
Step 4) Send 3 unit on the path so-2-4-si since along the path minimum capacity is 3 i.e, min{3,3}=3



There is no path from source to sink in the residual network. Then the max flow value is 8. The minimum cut can be constructed. Let S be the set of nodes reachable from the source in the residual network and T be the set of remaining nodes. S={so, 2} and T={1, 3, si}. (S, T) cut has a capacity of 8 which is equal to max flow value that we found. Strong duality states that if a flow x and a cut [S, T) has the same value, then they are individually optimal.

Question 3.

a)



Activities	Slack	Activities	Slack
A	0	F	10
B	0	G	0
C	0	H	0
D	0	I	10
E	9	J	0

Critical path: A - B - C - D - G - H - J
Duration: 46

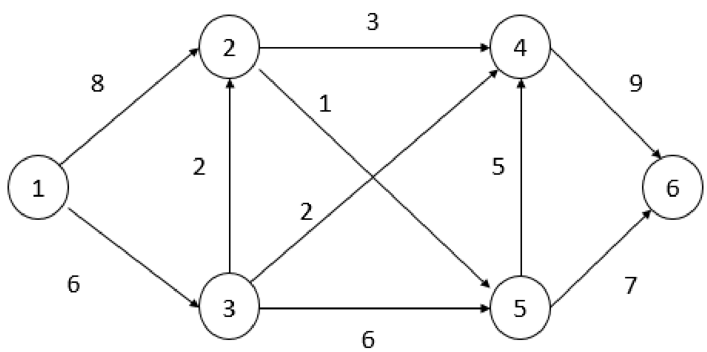
b) E can be delayed for 9 days.
F can be delayed for 10 days.
I can be delayed for 10 days.

c) **Parameters:**
c_j: cost of reducing the duration of activity j
d_i: duration of activity i
r_j: maximum possible reduction in duration of activity j
Decision Variables:
x_j: start time of activity j
y_j: number of days duration of activity j is reduced

Model:

$$
\begin{array}{ll}
\text{minimize} & \sum_j c_j y_j \\
& \\
\text{s.t.} & y_j \leq r_j,\ \forall j \\
& x_j \geq x_i + d_i \text{ - } y_j\ ,\ \forall i, j \\
& x_{start} = 0 \\
& x_{end} \leq \text{M} \\
& y_j \geq 0\ \&\ \text{integer},\ \forall j
\end{array}
$$

Question 4.



v[1] = 0	
v[2] = 8	d[2] = 1
v[3] = 6	d[3] = 1 <-- Permanent
v[4] = v[5] = v[6] = ∞	

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v[2] = 8	d[2] = 1
v[4] = 8	d[4] = 3 <-- Permanent
v[5] = 12	d[5] = 3
v[6] = ∞	

---

v[2] = 8	d[2] = 1 <-- Permanent
v[5] = 12	d[5] = 3
v[6] = 17	d[6] = 4

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v[5] = 9	d[5] = 2 <-- Permanent
v[6] = 17	d[6] = 17

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v[6] = 16	d[6] = 5 <-- Stop