Efe Beydogan
21901548
IE400 HW1

Subject :

Date :......./......./.......

Q1) a) Decision variables:

x: x coordinate of the warehouse
y: y coordinate of the warehouse

Parameters:

x_i: x coordinate of client i
y_i: y coordinate of client i

$i \in \{1,2,3,4,5\}$

Model:

$$\min \sum_{i=1}^{5} (c_i + k_i)$$

s.t. $c_i \geq x_i - x \quad \forall i = 1...5$

$c_i \geq -(x_i - x) \quad \forall i = 1...5$

$k_i \geq y_i - y \quad \forall i = 1...5$

$k_i \geq -(y_i - y) \quad \forall i = 1...5$

x, y urs

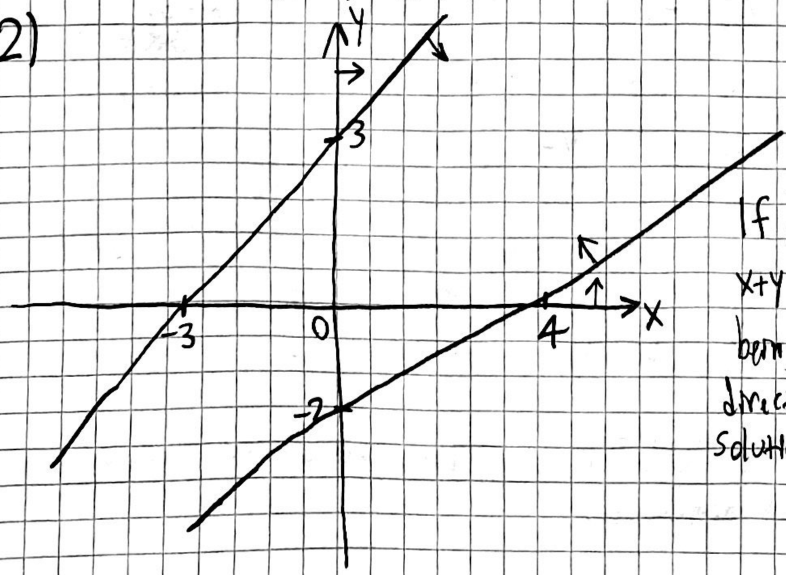
b) Let $z = \max_{1 \leq i \leq 5} \{c_i + k_i\}$, minimize z and add new constraint

min z

s.t. $z \geq c_i + k_i \quad \forall i = 1...5 \rightarrow$ new constraint (so $z = \max_{1 \leq i \leq 5} \{c_i + k_i\}$)

other constraints from (a) will remain

Q2)



If $\boxed{c_1 = 1 \text{ and } c_2 = -1}$, the line x+y will touch (0,0) when being pushed in the [-1 -1] direction. The unique optimal solution will be 0.

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Subject : Date :....../....../......

Q3) a) Make the coefficient of x_2 "0" in row 0. So x_2 will enter. x_5 will exit.

BV	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	1	12	0	3	20	0	0	0	120
x_2	0	$\frac{7}{4}$	1	$-\frac{3}{4}$	-1	$\frac{1}{4}$	0	0	2
x_6	0	$-\frac{11}{4}$	0	$\frac{11}{4}$	-2	$-\frac{1}{4}$	1	0	2
x_7	0	$\frac{37}{4}$	0	$-\frac{9}{4}$	-4	$\frac{3}{4}$	0	1	9

b) Make the coefficient of x_4 "0" in row 0. Since x_4 has negative coefficients in every other row, it can't enter the basis. The value of x_4 can be increased arbitrarily but the optimal value won't change. There is only one bfs but infinitely many solutions.

c) Make the coefficient of x_2 "-1" in row 0. x_2 will enter, x_5 exits.

BV	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	1	55/4	0	9/4	19	1/4	0	0	122
x_2	0	7/4	1	-3/4	-1	1/4	0	0	2
x_6	0	-11/4	0	11/4	-2	-1/4	1	0	2
x_7	0	37/4	0	-9/4	-4	3/4	0	1	9

d) Make the coefficient of x_3 "-1" in row 0. x_3 enters, x_6 exits.

BV	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	1	23/2	11/2	0	37/2	0	1/2	0	122
x_5	0	11/2	11/2	0	-17/2	1	3/2	0	14
x_3	0	$-\frac{1}{2}$	$\frac{1}{2}$	1	$-\frac{3}{2}$	0	$\frac{1}{2}$	0	2
x_7	0	4	-3	0	-1	0	0	1	3

e) Make the RHS of row 1 "0". This way, basic variable x_5 takes on a zero value and the bfs is degenerate.

Subject :　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　Date :......./......./.......

Q4) a) Decision variables

$$X_i = \begin{cases} 1, & \text{if plane } i \text{ used} \\ 0, & \text{otherwise} \end{cases} \qquad i = 1, 2, \ldots, 20$$

e_i = # of economy seats in plane i 　　　$i = 1, 2, \ldots, 20$

b_i = # of business seats in plane i

$$Y = \begin{cases} 1, & \text{if penalty incurred} \\ 0, & \text{otherwise} \end{cases}$$

Parameters

T = threshold, S = penalty amount, p = economy seat price

Model

$$\max \sum_{i=1}^{20} x_i(e_i p + 2b_i p) - YS$$

$$\text{s.t.} \quad \sum_{i=1}^{20} x_i = 8$$

$$e_i + 1.5b_i \leq 200 \quad \forall i = 1 \ldots 20$$

$$\begin{cases} T - \sum_{i=1}^{20} e_i x_i \leq Tz \\ 1 - Y \leq T(1-z) \end{cases}$$

$$e_i, b_i \geq 0, \quad x_i \in \{0,1\}, \quad Y, z \in \{0,1\}$$

$i = 1 \ldots 20$, e_i, b_i integer

$$\sum_{i=1}^{20} e_i x_i < T \Rightarrow Y = 1$$

$$\sum_{i=1}^{20} x_i e_i \geq T \text{ or } Y = 1 \ (Y \geq 1)$$

$$T - \sum_{i=1}^{20} x_i e_i \leq 0 \qquad 1 - Y \leq 0$$

$$T - \sum_{i=1}^{20} x_i e_i \leq Mz \qquad 1 - Y \leq M(1-z)$$

Choose $M = T$, constraints become:

$$T - \sum_{i=1}^{20} x_i e_i \leq Tz \qquad 1 - Y \leq T(1-z)$$

b) new model:

$$\min z \longrightarrow z = \max_{1 \leq i \leq 20} \{b_i\}$$

$$\text{s.t.} \quad \left.\begin{array}{l} \sum_{i=1}^{20} x_i(e_i p + 2b_i p) \geq G \\ z \geq b_i x_i \quad \forall i = 1 \ldots 20 \end{array}\right\} \text{additional constraints}$$

z integer

Subject :   Date :......./......./.......

c) $X_i = \begin{cases} 1, & \text{if plane } i \text{ used} \\ 0, & \text{otherwise} \end{cases}$ $i = 1..20$

i) $x_1 + x_5 \leq x_6 + x_8 + 1$

ii) $1 - x_4 \leq x_9 + x_{10}$

iii) $x_6 + x_9 + x_{10} \leq 2$ or $x_1 + x_4 + x_5 + x_{13} + x_{15} \geq 4$

$x_6 + x_9 + x_{10} - 2 \leq 0$ or $4 - (x_1 + x_4 + x_5 + x_{13} + x_{15}) \leq 0$

$x_6 + x_9 + x_{10} - 2 \leq Mt$ $4 - (x_1 + x_4 + x_5 + x_{13} + x_{15}) \leq M(1-t)$

Choose $M = 4$, constraints become:

$x_6 + x_9 + x_{10} - 2 \leq 4t$

$4 - (x_1 + x_4 + x_5 + x_{13} + x_{15}) \leq 4(1-t)$

$t \in \{0,1\}$

iv) $\underbrace{x_{14} + x_{17} + x_{19} \geq 1}_{A} \Rightarrow x_{15} + x_{20} = 1$

$(x_{15} + x_{20} = 1) \equiv (\underbrace{x_{15} + x_{20} \leq 1}_{B}$ and $\underbrace{x_{15} + x_{20} \geq 1}_{C})$

$A \Rightarrow (B \text{ and } C) \rightarrow -A \text{ or } (B \text{ and } C) \rightarrow (-A \text{ or } B) \text{ and } (-A \text{ or } C)$

$-A$ or B = $x_{14} + x_{17} + x_{19} \leq 0$ or $x_{15} + x_{20} - 1 \leq 0$

$x_{14} + x_{17} + x_{19} \leq Mq$ $x_{15} + x_{20} - 1 \leq M(1-q)$

choose $M = 3$, $x_{14} + x_{17} + x_{19} \leq 3q$ and $x_{15} + x_{20} - 1 \leq 3(1-q)$

$-A$ or C = $x_{14} + x_{17} + x_{19} \leq 0$ or $1 - x_{15} - x_{20} \leq 0$

$x_{14} + x_{17} + x_{19} \leq Mk$ $1 - x_{15} - x_{20} \leq M(1-k)$

choose $M = 3$, $x_{14} + x_{17} + x_{19} \leq 3k$ and $1 - x_{15} - x_{20} \leq 3(1-k)$

Constraints

$x_{14} + x_{17} + x_{19} \leq 3q$

$x_{15} + x_{20} - 1 \leq 3(1-q)$

$x_{14} + x_{17} + x_{19} \leq 3k$

$1 - x_{15} - x_{20} \leq 3(1-k)$

$q, k \in \{0,1\}$