IE 400: Principles of Engineering Management
Homework 1 Solutions

Spring 2022-2023

Question 1.

(a) **Parameters:**
x_i: location of client i in x-coordinate, $i \in \{1, 2, ..., 5\}$
y_i: location of client i in y-coordinate, $i \in \{1, 2, ..., 5\}$

Decision Variables:
x: location of warehouse in x-coordinate
y: location of warehouse in y-coordinate
tx_i: distance between client i and the warehouse in x-coordinate
ty_i: distance between client i and the warehouse in y-coordinate

Objective:
Minimizing the sum of the distances from the warehouse to all clients.

Model:

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{i=1}^{5}(tx_i + ty_i) \\ \text{subject to} \quad & tx_i = |x - x_i| \ , \quad i \in \{1, 2, ..., 5\} \\ & ty_i = |y - y_i| \ , \quad i \in \{1, 2, ..., 5\} \end{aligned}$$

Since the above model has non-linear terms, we rewrite it as:

$$
\begin{array}{lll}
\text{minimize} & \sum_{i=1}^{5}(tx_i + ty_i) & \\
\\
\text{subject to} & tx_i \geq x - x_i \ , & i \in \{1, 2, ..., 5\} \\
& tx_i \geq x_i - x \ , & i \in \{1, 2, ..., 5\} \\
& ty_i \geq y - y_i \ , & i \in \{1, 2, ..., 5\} \\
& ty_i \geq y_i - y \ , & i \in \{1, 2, ..., 5\} \\
& tx_i, ty_i \geq 0 \ , & i \in \{1, 2, ..., 5\}
\end{array}
$$

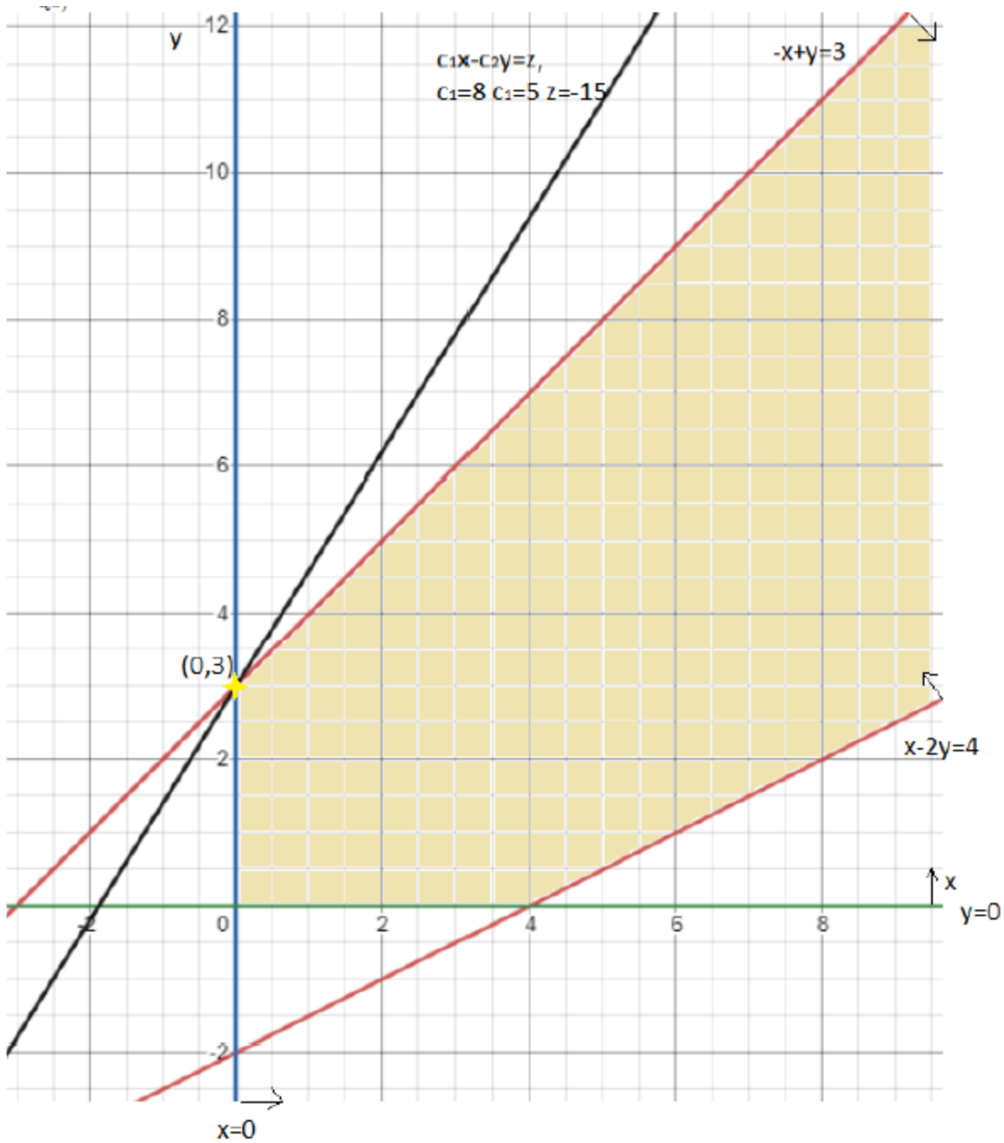
(b)

$$
\begin{array}{lll}
\text{minimize maximum}_{i \in \{1,2,...,5\}} & (tx_i + ty_i) & \\
\\
\text{subject to} & tx_i \geq x - x_i \ , & i \in \{1, 2, ..., 5\} \\
& tx_i \geq x_i - x \ , & i \in \{1, 2, ..., 5\} \\
& ty_i \geq y - y_i \ , & i \in \{1, 2, ..., 5\} \\
& ty_i \geq y_i - y \ , & i \in \{1, 2, ..., 5\} \\
& tx_i, ty_i \geq 0 \ , & i \in \{1, 2, ..., 5\}
\end{array}
$$

Since the above model has non-linear terms, we rewrite it as:

$$
\begin{array}{lll}
\text{minimize} & t & \\
\\
\text{subject to} & tx_i \geq x - x_i \ , & i \in \{1, 2, ..., 5\} \\
& tx_i \geq x_i - x \ , & i \in \{1, 2, ..., 5\} \\
& ty_i \geq y - y_i \ , & i \in \{1, 2, ..., 5\} \\
& ty_i \geq y_i - y \ , & i \in \{1, 2, ..., 5\} \\
& tx_i, ty_i \geq 0 \ , & i \in \{1, 2, ..., 5\} \\
& t \geq tx_i + ty_i \ , & i \in \{1, 2, ..., 5\} \\
& \text{t} \geq 0 &
\end{array}
$$

Question 2.



Question 3.

(a) The bfs is optimal and there exists an alternate optimal bfs.
Any one of the following circled numbers can be changed to zero to answer this part.

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	1	12	5	3	20	0	0	0	120
x_5	0	7	4	-3	-4	1	0	0	8
x_6	0	-1	1	2	-3	0	1	0	4
x_7	0	4	-3	0	-1	0	0	1	3

(b) The bfs is optimal and there exists alternate optimal solutions but only one bfs which is optimal.
This is only possible if the entering variable is the following circled one, so the circled number should be changed to zero. Unlike part a), there is a unique choice for this part.

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	1	12	5	3	20	0	0	0	120
x_5	0	7	4	-3	-4	1	0	0	8
x_6	0	-1	1	2	-3	0	1	0	4
x_7	0	4	-3	0	-1	0	0	1	3

(c) The bfs is not optimal and x_5 will become nonbasic after the next pivot. So the unique choice is to make the following circled element -1 (or any negative value).

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*RHS*
	1	12	5	3	20	0	0	0	120
x_5	0	7	4	-3	-4	1	0	0	8
x_6	0	-1	1	2	-3	0	1	0	4
x_7	0	4	-3	0	-1	0	0	1	3

(d) The bfs is not optimal and x_3 will become basic after the next pivot. So we should change the circled value to -1 (or any negative value).

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*RHS*
	1	12	5	3	20	0	0	0	120
x_5	0	7	4	-3	-4	1	0	0	8
x_6	0	-1	1	2	-3	0	1	0	4
x_7	0	4	-3	0	-1	0	0	1	3

(e) The bfs is degenerate.
Any one of the following circled elements could be changed to zero to achieve this situation.

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*RHS*
	1	12	5	3	20	0	0	0	120
x_5	0	7	4	-3	-4	1	0	0	8
x_6	0	-1	1	2	-3	0	1	0	4
x_7	0	4	-3	0	-1	0	0	1	3

Question 4.

(a)

$$\begin{aligned}
\text{maximize} \quad & 2p\sum_{i=1}^{20} y_{ib} + p\sum_{i=1}^{20} y_{ie} - Sz \\
\text{subject to} \quad & \sum_{i=1}^{20} x_i = 8 \\
& y_{ie} + 1.5y_{ib} \le 200x_i & i \in \{1, 2, ..., 20\} \\
& T - \sum_{i=1}^{20} y_{ie} \le Tz \\
& y_{ie}, y_{ib} \ge 0 \text{ integer} & i \in \{1, 2, ..., 20\} \\
& x_i \in \{0, 1\} & i \in \{1, 2, ..., 20\} \\
& z \in \{0, 1\}
\end{aligned}$$

(b) Define new decision variables

u = maximum number of business class seats sold in any plane

Model:

$$\begin{aligned}
\text{minimize} \quad & u \\
\text{subject to} \quad & u \ge y_{ib} & i \in \{1, 2, ..., 20\} \\
& 2p\sum_{i=1}^{20} y_{ib} + p\sum_{i=1}^{20} y_{ie} \ge G
\end{aligned}$$

+ all constrainsts of part (a) except the T penalty case

(c)

i. Alternative 1:

$$x_1 + x_5 \le 1 + x_1 + x_5$$

Alternative 2:

$$x_1 + x_5 \le 1 + z$$
$$x_6 + x_8 \ge z$$
$$z \in \{0, 1\}$$

ii. Alternative 1:

$$x_4 + x_9 + x_{10} \ge 1$$

Alternative 2:

$$x_9 + x_{10} \ge 1 - z$$
$$x_4 \ge z$$
$$z \in \{0, 1\}$$

iii. $x_6 + x_9 + x_{10} \le 2 + z$
$x_1 + x_4 + x_5 + x_{13} + x_{15} \ge 4z$
$z \in \{0, 1\}$

iv. $x_4 + x_7 + x_{20} \le 3z$
$x_{15} + x_{20} \le 2 - z$
$x_{15} + x_{20} \ge z$
$z \in \{0, 1\}$