Reliability-based design optimization of automotive body structures under crashworthiness constraints

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M. Moustapha
Clermont Université, Institut Francais de Mécanique Avancée, Institut Pascal & PSA Peugeot Citroën

B. Sudret
ETH Zürich, Institute of Structural Engineering, Chair of Risk, Safety & Uncertainty Quantification

J.-M. Bourinet
Clermont Université, Institut Francais de Mécanique Avancée, Institut Pascal

B. Guillaume
PSA Peugeot Citroën

ABSTRACT: The use of RBDO in industrial problems is limited because of its greed in terms of computational power and the numerous numerical difficulties related to the solution of the nested optimizations problem it requires. It is proposed here to investigate a surrogate-based RBDO approach where the probabilistic constraints are evaluated in terms of quantiles. The nested optimizations problem, hence decoupled, is easier to solve while the surrogate-model addresses the issue of time as expensive finite element simulations are replaced by easy-to-evaluate analytical functions. The application is aimed to be the lightweight design of automotive body structures under crashworthiness constraints. These constraints have shown to be very noisy as uncertainties in the input parameters amplify and propagate to the outputs. They are approximated here by Kriging and support vector regression. In this paper, a reduced car model which exhibits the same level of noise as in a full car finite element model is investigated (the so-called *sidemember subsystem*). The methodology is first validated on an analytical example. Then application on the sidemember subsystem has shown potential weight savings and improved reliability levels of the optimal designs. However the computation of the quantile with metamodels without adaptive design is shown to lack sufficient accuracy.

1 INTRODUCTION

The lightweight design of automotive body structures has become a necessary and convenient task in the global reduction of cars CO_2 emissions. This has been made possible thanks to the development of high-fidelity finite element models for the assessment of multi-disciplinary constraints *e.g.* NVH (noise, vibration and harshness), side or frontal impact. However, the associated simulations are time-consuming (up to 20 hours in clustered PCs for frontal impact). The only alternative for doing optimization in an acceptable time-lapse is the recourse to *surrogate models*. In such an approach, a model $\mathscr{M}(\boldsymbol{x})$ is considered as a black-box function only known pointwise: $\mathscr{D} = \left\{\left(\boldsymbol{x}^{(i)}, y_i\right), i \in \{1,\dots,n\}, \forall \boldsymbol{x}^{(i)} \in \mathbb{X} \subset \mathbb{R}^s, y_i \in \mathbb{Y} \subset \mathbb{R}\right\}$, where $\boldsymbol{x}^{(i)}$ is an s-dimensional input and y_i its corresponding output. With prior assumptions about the nature of $\mathscr{M}$, the dependency structure in $\mathscr{D}$ is then learned to build a global emulator $\widehat{\mathscr{M}}$, known as *surrogate model* or *metamodel*. While the use of metamodels solves the matter of time, it does not suffice as the constraints to emulate are noisy when it comes to crash simulation. This noise is especially due to the chaotic behavior of a crash. That is, uncertainties in the initial

conditions are dramatically amplified during the crash, leading to noticeably different solution paths. This noise is taken into account by propagating the uncertainties to the outputs through a *reliability-based design optimization* (RBDO). The paper is organized as follows. The first part introduces two types of metamodeling techniques: *Kriging* and *support vector regression*. The second part is devoted to RBDO and the third one to the application to an analytical function and to the sidemember subsystem.

2 SURROGATE MODELS

2.1 *Kriging*

Kriging or Gaussian process model (Santner et al., 2003) is a surrogate that has the interesting property of providing not only a prediction but also an associated variance. This variance is a quantification of the *epistemic* uncertainty related to the construction of the model, mainly due to the lack of knowledge (sparsity of data). In this sense, the unknown function to approximate is considered as a realization of a Gaussian random variable. The forms taken by the deterministic part give rise to various types of Kriging. We focus here on *universal* Kriging for which the deterministic part is considered as a linear combination of regression functions:

$$Y(\boldsymbol{x}) = \sum_{j=1}^{p} \beta_j f_j(\boldsymbol{x}) + Z(\boldsymbol{x}) \tag{1}$$

where $\mu(\boldsymbol{x}) = \sum_{j=1}^{p} \beta_j f_j(\boldsymbol{x})$ is the deterministic part approximating the mean trend. The departure from this trend is considered as a zero-mean Gaussian process $Z(\boldsymbol{x})$ of auto-covariance function $\mathrm{Cov}[Z(\boldsymbol{x}), Z(\boldsymbol{x'})] = \sigma^2 R(\boldsymbol{x}, \boldsymbol{x'})$, where σ^2 is the variance and $R(\boldsymbol{x}, \boldsymbol{x'})$ is a stationary auto-correlation function.

The *best linear unbiased prediction* or universal Kriging for an unknown point $Y_0 = \widehat{\mathscr{M}}\left(\boldsymbol{x}^{(0)}\right)$ is then a Gaussian variable:

$$\widehat{Y_0} \sim \mathscr{N}\left(\mu_{\widehat{Y_0}}, \sigma^2_{\widehat{Y_0}}\right): \qquad \begin{cases} \mu_{\widehat{Y_0}} = \boldsymbol{f}\left(\boldsymbol{x}^{(0)}\right)^T \widehat{\boldsymbol{\beta}} + \boldsymbol{r}_0^T \boldsymbol{R}^{-1}\left(\boldsymbol{y} - \boldsymbol{F}\widehat{\boldsymbol{\beta}}\right) \\ \sigma^2_{\widehat{Y_0}} = \widehat{\sigma^2}\left(1 - \boldsymbol{r}_0^T \boldsymbol{R}^{-1} \boldsymbol{r}_0 + \boldsymbol{u}^T (\boldsymbol{F}^T \boldsymbol{R}^{-1} \boldsymbol{F})^{-1} \boldsymbol{u}\right) \end{cases} \tag{2}$$

where $\widehat{\boldsymbol{\beta}} = \left(\boldsymbol{F}^T \boldsymbol{R}^{-1} \boldsymbol{F}\right)^{-1} \boldsymbol{F}^T \boldsymbol{R}^{-1} \boldsymbol{y}$ and $\widehat{\sigma^2} = \frac{1}{n} (\boldsymbol{y} - \boldsymbol{F}\boldsymbol{\beta})^T \boldsymbol{R}^{-1} (\boldsymbol{y} - \boldsymbol{F}\boldsymbol{\beta})$ are respectively the *least-square estimates* of the weights and variance introduced in Eq. (1) and $\boldsymbol{u} = \boldsymbol{F}^T \boldsymbol{R}^{-1} \boldsymbol{r_0} - \boldsymbol{f}\left(\boldsymbol{x}^{(0)}\right)$. In these equations, $\boldsymbol{F} = \left\{F_{ij} = f_j\left(\boldsymbol{x}^{(i)}\right), i = 1, \ldots, n, j = 1, \ldots, p\right\}$, $\boldsymbol{R} = \left\{r_{ik} = R\left(\boldsymbol{x}^{(i)}, \boldsymbol{x}^{(k)}\right), i = 1, \ldots, n, k = 1, \ldots, n\right\}$ and $\boldsymbol{r_0} = \left\{R(\boldsymbol{x}^{(0)}, \boldsymbol{x}^{(i)}), i = 1, \ldots, n\right\}$.

The construction of the Kriging predictor supposes the choice of an auto-correlation function which provides the dependency properties. It is usually chosen within a family of multivariate stationary auto-correlation functions which write as a product of univariate functions (Rasmussen and Williams, 2005). A critical step is then the proper choice of the auto-correlation parameters $\boldsymbol{\theta}$. In this paper, anisotropy is assumed for Kriging. That is, the sought parameters can be different in each direction. Among the various techniques, the most widely used in computer experiments is the *maximum likelihood estimation* (Koehler and Owen, 1996). It results in a not easy-to-solve optimization problem which writes:

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta} \in \mathbb{R}^d} \widehat{\sigma}^2(\boldsymbol{\theta}) \det \boldsymbol{R}(\boldsymbol{\theta})^{\frac{1}{n}} \tag{3}$$

where d is the number of parameters of the auto-correlation function.

This optimization problem, which turns out to be unstable, can be regularized by the so-called *nugget effect*. It consists in the addition of a term σ_ζ^2 in the diagonal of the auto-correlation function and translates by a discontinuity at its origin (Matheron, 1971). This nugget may also, in some cases, be interpreted as an additional variance in the presence of noisy data. In such a case, the predictor does not interpolate the training points anymore.

2.2 *Support vector regression*

Support vector machine (SVM) is a machine learning technique developed by Vapnik (1995). It is based on *structural risk minimization* principles where the learning problem is addressed by minimizing a bound on the *true risk*. This is in opposition to other learning techniques (*e.g.* neural networks) which rely on the minimization of the *empirical risk*, hence avoiding overfitting problems. SVM was first developed for classification and then adapted to solve regression problems (Drucker et al., 1997).

In support vector regression, a linear regressor that minimizes a so-called *loss function* is sought. One of the most widely used is the ε-*insensitive* loss function. It consists in penalizing deviations from the learning points only if they are beyond a given threshold (ε-insensitivity). This penalization might be linear (L_1-SVR) or quadratic (L_2-SVR). Only the latter case is considered here. When the data are highly non-linear and cannot be regressed linearly, they are mapped into a higher dimensional space known as *feature space*. Operations in this space are carried directly in the original space thanks to a *kernel* function k. This kernel can be linked to the Kriging auto-correlation function. In this paper, we use Matérn 5/2 for the two surrogates. However contrary to Kriging, only the isotropic case is considered in SVR.

The problem is solved in its dual form as a convex quadratic optimization and reads:

$$\begin{aligned} \arg\min_{\boldsymbol{\alpha},\boldsymbol{\alpha}^*} \quad & \frac{1}{2}(\boldsymbol{\alpha}-\boldsymbol{\alpha}^*)^T\,\tilde{\boldsymbol{K}}\,(\boldsymbol{\alpha}-\boldsymbol{\alpha}^*)+\sum_{i=1}^{n}(\varepsilon+y_i)\alpha_i+\sum_{i=1}^{n}(\varepsilon-y_i)\alpha_i^* \\ \text{subject to} \quad & \sum_{i=1}^{n}(\alpha_i-\alpha_i^*)=0, \quad \alpha_i,\alpha_i^*\geq 0, \quad i=\{1,\ldots,n\} \end{aligned} \tag{4}$$

where $\tilde{\boldsymbol{K}}=\boldsymbol{K}+(1/C)\boldsymbol{I}$ with $\boldsymbol{K}$ and $\boldsymbol{I}$ being respectively the matrix of size $n\times n$ defined by $K_{jk}=k(\boldsymbol{x}^{(j)},\boldsymbol{x}^{(k)})$ and the identity matrix. C is a penalty term and $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}^*$ are vectors gathering introduced Lagrange multipliers.

Solving Eq. (4) provides us with the values of the coefficients α_i and α_i^* used to express the prediction at a new point $\boldsymbol{x}^{(0)}$:

$$\widehat{y}\left(\boldsymbol{x}^{(0)}\right)=\sum_{i=1}^{n}\left(\alpha_i-\alpha_i^*\right)k\left(\boldsymbol{x}^{(i)},\boldsymbol{x}^{(0)}\right)+b \tag{5}$$

where the offset parameter b is also retrieved from the optimization results.

As with Kriging, the hyper-parameters of the model play a crucial role in the fitting quality of SVR. The traditional approach to tune the parameters is to minimize the generalization error through a cross-validation procedure. A special case is the *leave-one-out* error, which is used here. As it is too expensive to evaluate, the error to minimize is approximated by a bound that depends on the parameters. One of them is the so-called *span bound* (Chapelle, 2002). This bound is obtained as a post-processing of the model and thus does not require any additional computation beside a matrix inversion of size $n\times n$. However, it is a noisy discontinuous function and a special care should be given to its optimization. Vapnik and Chapelle (1999) smoothed it in order to use a gradient-based algorithm. Here, we adopted cross-entropy (Rubinstein and Davidson, 1999), a first-order stochastic algorithm, which allows us to directly use the span bound as it stands.

3 MULTI-CONSTRAINED OPTIMIZATION IN THE PRESENCE OF UNCERTAINTIES

3.1 *Reliability-based design optimization*

Structural optimization seeks to minimize a cost while satisfying some performance functions. In *deterministic design optimization* (DDO), the optimal design is usually found at the boundary of the admissible space. Considering uncertainties in this configuration, the optimal design will likely not perform safely. To avoid this situation, uncertainties are taken into consideration through so-called *robust design optimization* (RDO) or *reliability-based design optimization*

(RBDO), among others. The former looks for a design that is less sensitive in the inputs uncertainties while the latter attempts to balance the cost reduction with prescribed reliability requirements. It is an efficient and more realistic alternative to the use of *safety factors* as might be done in DDO.

In this paper, we are interested in RBDO such as described in (Tsompanakis et al., 2008). Its classical formulation reads:

$$\boldsymbol{d}^* = \arg\min_{\boldsymbol{d}\in\mathbb{D}} \mathfrak{c}(\boldsymbol{d}) \quad \text{s.t.} \begin{cases} \mathfrak{f}_j(\boldsymbol{d}) \leq 0, \quad \{j = 1,\dots,n_s\} \\ P[\mathfrak{g}_k(\boldsymbol{d},\boldsymbol{Z}) < 0] \leq P^t_{fk}, \quad \{k = 1,\dots,n_p\} \end{cases} \tag{6}$$

where $\mathfrak{c}$ is the cost function to minimize with respect to the so-called *design parameters* $\boldsymbol{d} = \{d_i, i = 1,\dots,s\} \in \mathbb{D} \subset \mathbb{R}^s$ subject to some *soft constraints* $\mathfrak{f} = \{\mathfrak{f}_j, j = 1,\dots,n_s\}$, which are simple deterministic functions bounding the *admissible design space*, and some probabilistic constraints associated to the collection of n_p so-called *limit-state surfaces* $\mathfrak{g}_k = 0$ which split the design space into the safe ($\mathfrak{g}_k > 0$) and the failure ($\mathfrak{g}_k < 0$) domains. In the context of reliability analysis, the uncertainties are modeled by random variables whose distributions are supposed to be known. They are represented in Eq. (6) by the environment variables $\boldsymbol{Z}$ whose realizations are denoted by $\boldsymbol{z}$. The probabilistic constraints consist then in finding a design for which a prescribed probability of failure P^t_{fk} is respected. This probability of failure is evaluated by integrating the joint probability density $f_z(\boldsymbol{d},\boldsymbol{z})$ of the random variables $\boldsymbol{Z}$ over the failure domain:

$$P_{fk}(\boldsymbol{d}) = P[\mathfrak{g}_k(\boldsymbol{d},\boldsymbol{Z}) < 0] = \int_{\mathfrak{g}_k(\boldsymbol{d},\boldsymbol{z})<0} f_z(\boldsymbol{d},\boldsymbol{z})\,\mathrm{d}\boldsymbol{z} \tag{7}$$

The solution of Eq. (6) in its classical form involves a nested optimization problem where the outer loop explores the design space and the inner loop solves the reliability analysis in the space of the random variables in order to evaluate the probability of failure for the current design. This requires intensive computational efforts. Tsompanakis et al. (2008) (Chapter 9) review the different techniques developed to solve the related numerical difficulties and identify three main groups: a) *two-level* approaches where the inner loop is solved through reliability techniques such as FORM/SORM or Monte Carlo simulations, b) *mono-level* approaches where Eq. (6) is reformulated so that the optimization and reliability analyses are solved simultaneously without nesting the problems and c) *decoupled* approaches where an equivalent deterministic or pseudo-deterministic problem is formulated and solved instead, avoiding the costly reliability analysis.

For practical purposes, we adopted in this paper a single loop approach based on quantiles computation and involving surrogate models.

3.2 *Surrogate-based RDBO*

The RBDO is applied here for lightweight design of automotive body structures in an industrial context. For this very application, the target probability of failure is rather high (say around 5%) and we find it not necessary to employ advanced reliability methods. Besides, the constraints result from multi-disciplinary time-consuming finite element simulations. They are hence replaced in the optimization procedure by easy-to-evaluate surrogate models. The optimization problem is then reformulated as follows:

$$\boldsymbol{d}^* = \arg\min_{\boldsymbol{d}\in\mathbb{D}} \mathfrak{c}(\boldsymbol{d}) \quad \text{s.t.} \begin{cases} \mathfrak{f}(\boldsymbol{d}) \leq \boldsymbol{0} \\ \widehat{\mathfrak{g}}_{\alpha\%}(\boldsymbol{d},\boldsymbol{z}) \leq \boldsymbol{0} \end{cases} \tag{8}$$

where the probabilistic constraint in Eq. (6) is replaced by a quantile estimation through the surrogate models of the performance functions. $\mathfrak{g}_{\alpha\%}$ is then the α-th percentile of the constraints computed by a Monte Carlo simulation.

The first step here is the construction of the surrogate models. For this, a design of experiments is primarily built in the joint space of the design and environment variables. More specifically, we sample the training input points in the unit hypercube. The physical design variables

are then obtained by a simple linear transformation: $\mathscr{L} : [\mathbf{0},\mathbf{1}] \mapsto [\boldsymbol{d}_{min},\boldsymbol{d}_{max}]$. Since the environment variables follow various distributions in the physical space, an iso-probabilistic transformation is made such that:

$$F_{Z_l}(z_l) = u_l, l = \{1,...,s_e\} \tag{9}$$

where Z_l is a random variable with cumulative distribution function F_{Z_l}, u_l is a realization of a uniformly distributed random variable and s_e is the number of environment parameters.

To compute the quantiles, N realizations of the random variables of the vector $\boldsymbol{Z}$ are drawn according to their respective distributions. To reduce numerical difficulties, the same realization of the underlying uniform random numbers is used throughout all the iterative steps of the optimization. Quantiles are then computed based on the surrogate models and then plugged in for performance checking.

This method is applied in the next section on two examples: an analytical problem involving the Euler critical force and the finite element model of the sidemember subsystem.

4 APPLICATIONS

4.1 *Critical Euler force*

This first example features a straight column of rectangular cross-section with dimensions $\{b,h\}$ in service under a compressive load F_{ser}. It is proposed to minimize its section while satisfying to the critical Euler force condition:

$$(b^*,h^*) = \arg\min_{b,h} bh \quad \text{s.t.} \begin{cases} h-b \leq 0 \\ F_{ser} - (kF_{cr})_{\alpha\%} > 0 \end{cases} \tag{10}$$

where F_{cr} is the critical Euler force and reads $F_{cr} = \frac{\pi^2 E b h^3}{12 l^2}$ with l and E being respectively the length of the column and the Young's modulus of its constitutive material. An additional noise parameter k is added here to mimic model error on the critical load, which is similar to the one in crash simulations.

The probabilistic model consists here of the three environment variables $\boldsymbol{Z} = \{k,E,l\}$ following lognormal distributions with parameters $(\lambda_\bullet,\zeta_\bullet)$, $\bullet = \{k,E,l\}$. With these distributions, it is possible to analytically compute the solution. Let us consider two cases:

- Deterministic solution (DDO): k, E and L are set to the mean values μ_k, μ_E and μ_L.

$$b^* = h^* = \left(\frac{12\mu_l^2 F_{ser}}{\mu_k \pi^2 \mu_E}\right)^{1/4} \tag{11}$$

- RDBO solution:

$$b^* = h^* = \left(\frac{12F_{ser}}{\pi^2 \exp\left(\lambda_k + \lambda_E - 2\lambda_l + u_\alpha \sqrt{\zeta_k^2 + \zeta_E^2 + 4\zeta_l^2}\right)}\right)^{1/4} \tag{12}$$

  where $u_\alpha = \Phi^{-1}(\alpha)$, Φ being the standard Gaussian cumulative distribution function. We set here α to 5%.

Table 1 gathers the parameters of the probabilistic model. The parameters of the lognormal distribution used in Eq. (12) are retrieved by the following transformation:

$$\zeta_\bullet = \sqrt{\log\left(1+\delta_\bullet^2\right)} \qquad \text{and} \qquad \lambda_\bullet = \log(\mu_\bullet) - \frac{1}{2}\zeta_\bullet^2 \tag{13}$$

The analytical solution gives $b^* = h^* = 227.25$ mm for DDO and $b^* = h^* = 238.45$ mm for RBDO.

Table 1: Parameters of the probabilistic model

Parameter	Distribution	Mean (μ)	Coef. of var. (δ)
k	Lognormal	0.6	0.1
E (MPa)	Lognormal	10000	0.05
l (mm)	Lognormal	3000	0.01

For the surrogate-based strategy, we adopted a 100 points 5-dimensional Sobol' design to construct the metamodels. Throughout this paper, Kriging models are built using DiceKriging in R (Roustant et al., 2012) while SVR is directly implemented in Matlab. Figure 1 below shows box-plots of the results for 100 realizations of the random variables used for the computation of the quantile. As the found result is sensitive to the size of the Monte Carlo population, the scatter of the solutions is investigated for $N = 10^3$ and $N = 10^4$. In Figure 1, solutions are shown to be extremely accurate. Kriging-based RBDO gives the best results. This is simply due to the fact that Kriging approximates more accurately the constraint. For $N = 10^4$, the scatter in the solution is sufficiently small and only this case will be treated later. Note that the use of *common random numbers* here greatly improves the stability of the optimizations.

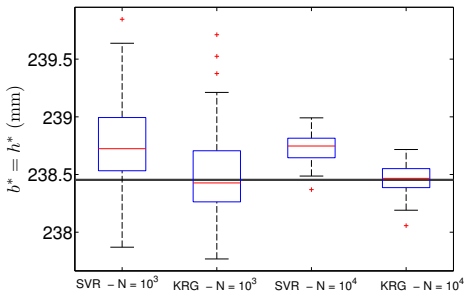
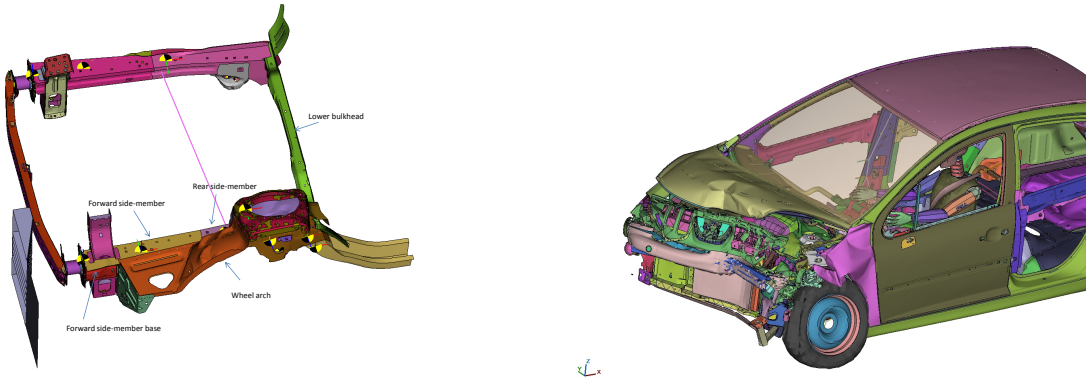


Figure 1: Solutions of the RBDO problem for 100 optimizations with SVR and Kriging (KRG) while considering $N = 10^3$ and $N = 10^4$ points in the Monte Carlo simulation for the quantile. The continuous thick line is the reference solution.

4.2 *Sidemember subsystem*



(a) Sidemember subsystem

(b) Entire model crash simulation

Figure 2: The sidemember subsystem.

This application involves the so-called *sidemember subsystem*. It is a subset of the entire car finite element model consisting of the sidemember itself and some parts around it, such as the bumper (Figure 2). They are made of metal sheets welded together. It is proposed

here to minimize their weight under some frontal impact related constraints. For this purpose, the thicknesses of the five parts, denoted in Figure 2(a), are selected as design parameters: $\boldsymbol{d} = \{d_1, d_2, d_3, d_4, d_5\}$. Their minimal, maximal and nominal values are gathered in table 2. This model exhibits the same level of noise as in the crash model of the entire car. Specifically, the crash behavior of the car is sensitive to the uncertainties in the initial conditions. The RBDO is applied here to account for these uncertainties. The probabilistic model is based on the EuroNCAP protocol information about the uncertain crash parameters. We consider here two important parameters, namely the initial velocity of the car and the variation of the barrier position:

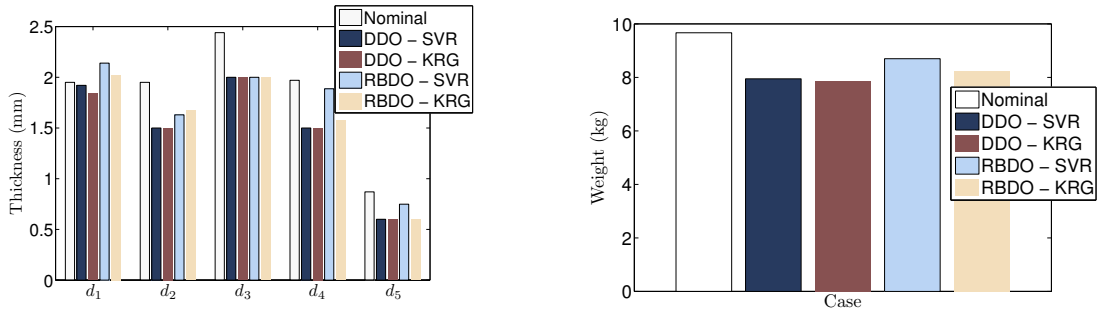
$$\boldsymbol{Z} = \{V, P\} : \qquad V \sim \mathscr{U}(34, 35) \quad \text{and} \quad P \sim \mathscr{N}(0, 2) \tag{14}$$

where V is the velocity in km/h, P is the variation of the barrier position in mm and $\mathscr{U}$ and $\mathscr{N}$ stand respectively for the uniform and normal distributions. The outputs selected as constrains

Table 2: Thicknesses of the five parts and corresponding weights.

Configuration	d_1 (mm)	d_2 (mm)	$d3$ (mm)	d_4 (mm)	d_5 (mm)	Weight (kg)
Minimum	1.50	1.50	2.00	1.50	0.60	7.51
Maximum	2.50	2.50	3.00	2.50	1.20	12.31
Nominal	1.95	1.95	2.44	1.97	0.87	9.67

for this analysis are the maximum sidemember compression (y_1) and wall force (y_2). Their limits are set respectively to 520 mm and 170 kN. Note that these values cannot be physically related to those encountered in a real car model. Both the DDO and RBDO are performed in order to assess how these uncertainties affect the weight savings. To construct the metamodels, the considered training set consists of a 285 points 7-dimensional Sobol' design. Figure 3(a)



(a) Thicknesses

(b) Weights

Figure 3: Optimal designs for different cases with the nominal design.

above shows the resulting thicknesses for DDO and RBDO with each surrogate model together with the initial design. The weight savings are illustrated in Figure 3(b). As expected, the weight savings are lesser when considering the model uncertainties. This makes sense as the optimal design is put further away from the boundaries of the admissible design space. The optimal designs are relatively close in DDO. It is not the case for RBDO. The reason for the difference in the latter case is certainly the quite poor fitting quality of the metamodels.

To validate these results, we simulate the actual constraints with the FE models at the optimal points. For RBDO, actual quantiles are computed. Since our computational budget is limited, the size of the Monte Carlo population for the quantiles estimation is reduced to 500. Results are shown in Table 3. While the designs are valid for SVR, they are not safe with Kriging because the output y_1 was underestimated. The accuracy of the metamodels, and hence the quantiles, can be improved here simply by enriching the design of experiments. Methods such as *Expected Global Reliability Analysis* should be investigated for the Kriging surrogate. It

would basically consist in adaptively enriching the design of experiments in regions of interest at each iteration of the optimization. This would result in gradually improving the accuracy of the quantiles.

Table 3: Actual values of the outputs for the different designs with mean values of $\boldsymbol{Z}$

Case	DDO		RBDO	
Design	Case #1 - SVR	Case #1 - KRG	Case #2 - SVR	Case #2 - KRG
$\widehat{y}_1$ (mm)	520.00	520.00	518.52	519.50
y_1 (mm)	513.73	530.63	515.11	528.67
$\widehat{y}_2$ (kN)	170.00	170.00	169.97	169.96
y_2 (kN)	157.22	137.24	153.80	155.34

5 CONCLUSION

In this paper, it was proposed to investigate a surrogate-based RBDO methodology for automotive body structures lightweight design. Two metamodels were selected for this study, namely Kriging and support vector regression. They were first briefly introduced, followed by the formulation of the RBDO problem. We considered here a quantile based formulation which is easier to apply in an industrial context. The methodology was first applied on the buckling of a straight column for which an analytical solution was available. The application of the methodology allowed us to validate the implementation of the methodology as the results converged to the reference solutions. Finally, the RDBO of a sidemember system was carried out. Results were consistent with what was expected. However, the insufficient accuracy of the metamodels remains an issue. Adaptive designs in an iterative scheme might help address this issue and will be further investigated in future works.

REFERENCES

Chapelle, O. (2002). *Support Vector Machines : principes d'induction, réglage automatique et connaissances a priori*. Ph. D. thesis, Université Pierre et Marie Curie.

Drucker, H., C. Burges, L. Kaufman, A. Smola, and V. Vapnik (1997). Support vector regression machines. In *Adv. Neu. In.*, Volume 9, pp. 155–161.

Koehler, J. R. and A. Owen (1996). Computer experiments. In S. Ghosh and C. Rao (Eds.), *Handbook of Statistics*, Volume 13, pp. 261–308. North Holland.

Matheron, G. (1971). La théorie des variables régionalisées et ses applications.

Rasmussen, C. E. and C. K. I. Williams (2005). *Gaussian processes for machine learning (adaptive computation and machine learning)*. The MIT Press.

Roustant, O., D. Ginsbourger, and Y. Deville (2012). DiceKriging, DiceOptim: two R packages for the analysis of computer experiments by Kriging-based metamodeling and optimization. *J. Stat. Softw. 51*(1), 1–55.

Rubinstein, R. and W. Davidson (1999). The cross-entropy method for combinatorial and continuous optimization. *Methodology and Computing in Applied Probability 1*(2), 127–190.

Santner, T., B. Williams, and W. Notz (2003). *The Design and Analysis of Computer Experiments*. Springer, New York.

Tsompanakis, Y., N. Lagaros, and M. Papadrakis (Eds.) (2008). *Structural design optimization considering uncertainties*. Taylor & Francis.

Vapnik, V. and O. Chapelle (1999). Bounds on error expectation for support vector machines.

Vapnik, V. N. (1995). *The nature of statistical learning theory*. New York, NY, USA: Springer-Verlag.