EEE 431 (Fall 2021)
MATLAB Assignment 2 (Due: Dec. 24, 17:00)

Part I: Signal Spaces (20 points)

In this part, you will use the signal data provided in Moodle. If the remainder of your ID when divided by 3 is equal to i, then use "signaldata$i$.mat". For instance, if your ID number is a multiple of 3, the use "signaldata0.mat".
The signal data in each file is the sampled version of the following signal with the sampling frequency $F_s = 1200$Hz:

$$x(t) = \sum_{k=1}^{10} a_k \sin(2\pi f_k t) p(t) \tag{1}$$

where $p(t)$ is the rectangular pulse with an amplitude of 1 for $t \in [0, 1)$, $f_k \in \{30k-20,\ 30k-10,\ 30k\}$ Hz and $a_k \in \mathbb{N}$ for any k. The exact values of a_k and f_k are unknown. Our aim is to identify the values of a_k and f_k from the signal data.

(a) (No MATLAB in this part) Find an orthonormal signal space that spans all possible signals in the form of (1). What would be the dimension of the corresponding signal space?

(b) (No MATLAB in this part) Based on correlations of $x(t)$ with basis functions, develop a technique to estimate a_k and f_k values based on $x(t)$. (Do not use Fourier domain approach.)

(c) Based on the technique developed in Part, extract a_k and f_k values from the signal data in MATLAB.

(d) Add zero mean Gaussian noise signals with variances $\sigma^2 = 1$, $\sigma^2 = 25$ and $\sigma^2 = 100$ to the signal data. While generating the noise signal, assume that each element of the noise vector is indepedently and identically generated from the normal distribution with mean 0 and variance σ^2.

In each case, plot the original signal together with the corrupted signal in the same figure for $t \in [0, 1)$.

(e) Apply the procedure in Part (b) to find f_k and a_k values in all three cases for $\sigma^2 = 1, 25,$ and 100. Comment on your observations.

Part II: Binary Modulation (45 points)

In this part, we will perform binary modulation and demodulation. Let the remainder of your ID when divided by 2 be equal to i. We will generate random bits uniformly (0's and 1's) and map them to $\{\Lambda_i(t)\}_{i=0}^{1}$ as follows:

$$0 \to -\Lambda_i(t) \tag{2}$$
$$1 \to \Lambda_i(t) \tag{3}$$

where $\{\Lambda_i(t)\}_{i=0}^{1}$ is defined as follows

$$\Lambda_0(t) = \begin{cases} t, & \text{if } 0 \le t \le T/4, \\ T/2 - t, & \text{if } T/4 < t \le T/2, \\ t - T/2, & \text{if } T/2 < t \le 3T/4, \\ T - t, & \text{if } 3T/4 < t \le T, \\ 0, & \text{otherwise.} \end{cases} \text{ and } \Lambda_1(t) = \begin{cases} t, & \text{if } 0 \le t \le T/4, \\ T/2 - t, & \text{if } T/4 < t \le 3T/4, \\ t - T, & \text{if } 3T/4 < t \le T, \\ 0, & \text{otherwise.} \end{cases}$$

where $T = 0.1$ second. For instance, if 010 is generated, the modulated signal, $x(t)$ is given by

$$x(t) = -\Lambda_i(t) + \Lambda_i(t - T) - \Lambda_i(t - 2T). \tag{4}$$

(a) Generate 5 random bits and construct the modulated signal $x(t)$. Use sampling frequency $F_s = 1$kHz. Plot $x(t)$.

(b) Obtain an orthonormal signal space for this type of modulation scheme (you may consider the time interval of $[0, T)$). What would be the dimension of this signal space? Plot the basis function(s). Represent $-\Lambda_i(t)$ and $\Lambda_i(t)$ as vectors in the signal space.

(c) At the receiver, it is assumed that there is an additional noise term. For that purpose, add zero-mean white Gaussian noise with variances 10^{-4}, 10^{-2} and 10^0 to the generated signal in Part (a). Then plot the received signal together with the original signal in the same figure for each case. Discuss about the signal-to-noise ratio (SNR) definition. Does the number of samples per each symbol affect the definition of SNR?

(d) (No MATLAB in this part) Based on the orthonormal signal space, provide the optimal receiver (both show the block diagram and specify the ML rule). Moreover, calculate the probability of error.

(e) Perform the optimal receiver for different values of σ^2 such that probability of error versus the SNR ranges from 0.5 to about 10^{-4}. (For each σ^2, generate 10^5 random bits and use $F_s = 1$ kHz.) Plot also the theoretical probability of error versus SNR. Use "semilogy" command of MATLAB. Comment on the results.

(f) (No MATLAB in this part) From now on, assume that symbols are not equally likely. Assume that $\Pr\{1 \text{ is generated}\} = 0.1$ and $\Pr\{0 \text{ is generated}\} = 0.9$. Discuss about the optimal receiver structure. Is the receiver in Part (d) optimal in this case? You do not need to derive the theoretical probability of error expression.

(g) Perform the optimal receiver for different values of σ^2 such that probability of error versus the SNR ranges from 0.5 to about 10^{-4}. (For each σ^2, generate 10^5 random bits and use sampling frequency $F_s = 1$ kHz.) Also use the receiver structure in (e), and compare the results. Use "semilogy" command of MATLAB. Comment on the results.

(h) Now, consider $\Pr\{1 \text{ is generated}\} = \alpha$ as a variable. Add zero-mean white Gaussian noise with variance 10^{-2} to the original signal. Perform the optimal receiver for various values of $\alpha \in (0, 0.5)$. (Generate 10^5 random bits and use sampling frequency $F_s = 1$ kHz for each value of α.) Plot the probability of error versus α for both the optimal receiver together with the receiver structure in Part (e) in the same figure. Use "semilogy" command of MATLAB. Comment on the results.

Part III: Frequency Shift Keying (FSK) (35 points)

Let the remainder of your ID when divided by 2 is equal to i. We will generate random bits uniformly (0's and 1's) and map them to $\{s_i^{(k)}\}_{k=0}^3$ as follows:

$$00 \to s_i^{(0)}(t),\ 01 \to s_i^{(1)}(t), 10 \to s_i^{(2)}(t),\ 11 \to s_i^{(3)}(t), \tag{5}$$

where $s_i^{(0)}(t) = \cos(2\pi f_i t)g(t)$, $s_i^{(1)}(t) = -s_i^{(0)}(t)$, $s_i^{(2)}(t) = \cos(2\pi 2 f_i t)g(t)$, $s_i^{(3)}(t) = -s_i^{(2)}(t)$, $f_0 = 125$Hz, $f_1 = 250$Hz, and $g(t)$ is the rectangular pulse with an amplitude of 1 for $t \in [0, T)$, where $T = 0.1$. Set the sampling frequency $F_s = 5$kHz. For instance, if 010011 is generated, the modulated signal $x(t)$ is given by

$$x(t) = s_i^{(1)}(t) + s_i^{(0)}(t - T) + s_i^{(3)}(t - 2T). \tag{6}$$

(a) Generate 5 random symbols and construct the modulated signal $x(t)$. Plot $x(t)$.

(b) Obtain an orthonormal signal space for this type of modulation scheme. What would be the dimension of this signal space? Plot the basis function(s). Represent $\{s_i^{(k)}\}_{k=0}^3$'s in the signal space.

(c) At the receiver, it is assumed that there is an additional noise term. For that purpose, add zero-mean white Gaussian noise with variances 10^{-2}, 10^0 and 10^2 to the generated signal in Part (a). Then plot the received signal together with the original signal in the same figure for each case. Discuss about the signal-to-noise ratio (SNR) definition. Does the number of samples per each symbol affect the definition of SNR?

(d) (No MATLAB in this part) Based on the orthonormal signal space, provide the optimal receiver (both show the block diagram and specify the ML rule). Moreover, calculate the probability of symbol error.

(e) Perform the optimal receiver for different values of σ^2 such that probability of symbol error versus the SNR ranges from 0.5 to about 10^{-4}. (For each σ^2, generate 10^5 random symbols and use $F_s = 5$ kHz.) Plot also the theoretical probability of symbol error versus SNR. Comment on the results.

Technical/Reporting Requirements:

Note that you are not allowed to use the built-in functions from Matlab (or, other resources) to complete the project. You must write your own code, and conduct your simulations using that code. Both the Matlab files and the project reports will be processed by turnitin. In addition, the m files will be checked via the Moss software.

Your report should contain all the relevant information about the set-up used (the specific schemes implemented, parameters selected, etc), results obtained and your comments on the results. The specific format is up to you, but please make sure to properly label each figure, include relevant captions, point to the right results in your explanations, etc. It should include a title page, brief introduction and outline as well as any references used. The references used should be cited within the report wherever they are used.

The report must be typed using an advanced wordprocessor (e.g. latex, word, etc), and should be submitted as a pdf file on the course Moodle site. Please also submit your Matlab codes as a separate (single) file. *The submission links for the pdf report and the Matlab m file will be separate.*

The file name format is LastName-FirstName.pdf or LastName-FirstName.m.