EEE 431: Telecommunications 1

Quiz 2

March 5, 2016, 10:30

Instructor: Sinan Gezici

Name: ______________________________

Signature: ______________________________

Bilkent ID: ______________________________

Prob. 1: ________ / 20

Prob. 2: ________ / 25

Prob. 3: ________ / 30

Prob. 4: ________ / 25

Total: ________ / 100

Problem 1 Consider a signal $s(t)$ that is bandlimited to $100\,\text{kHz}$ (i.e., $S(f) = 0$ for $|f| \geq 100\,\text{kHz}$), and define $x(t) = s(t)\cos(200000\pi t)$. We would like to transmit $x(t)$ via uniform PCM (pulse code modulation). Suppose that $x(t)$ is first sampled with a sampling frequency which is equal to twice of the Nyquist rate. Then, a v-bit uniform quantizer is employed. If the generated PCM signal will be transmitted through a channel which has a bandwidth of $6.4\,\text{MHz}$, what is the maximum value of v for the quantizer? (assume binary signaling for transmission)

Hint: For binary signaling, $R/2\,\text{Hz}$ of bandwidth is required for a data rate of R bits per second.

Problem 2 Consider the following AM signal: $x(t) = A_c m(t)\sin(2\pi f_c t)$, where $m(t)$ is bandlimited to $200\,\text{kHz}$ and $f_c = 1\,\text{MHz}$. By using a local oscillator, a multiplier and a filter, design a receiver to extract $m(t)$ from $x(t)$. Specify the signal at the local oscillator and the frequency response of the filter. (You can assume that the local oscillator can generate any sinusoidal signal with the same phase as in $x(t)$. Also, scaling factors are not important.)

Hint: $\cos(2x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$ and $\sin(2x) = 2\cos(x)\sin(x)$.

Problem 3 A source X generates outputs according to the following probability density function (PDF):

$$f_X(x) = \begin{cases} (x+2)/4\,, & \text{if } x \in [-2, 0] \\ (-x+2)/4\,, & \text{if } x \in (0, 2] \\ 0\,, & \text{otherwise} \end{cases}$$

This source is quantized by using the following 4-level quantizer:

$$Q(x) = \begin{cases} 1.5\,, & \text{if } x \in (1, 2] \\ 0.5\,, & \text{if } x \in (0, 1] \\ -0.5\,, & \text{if } x \in (-1, 0] \\ -1.5\,, & \text{if } x \in [-2, -1] \end{cases}$$

Determine the cumulative distribution function (CDF) of $Q(X)$, and plot it.

Problem 4 Consider a 4-level quantizer specified by the following reconstruction levels $(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4)$ and the decision boundaries (a_1, a_2, a_3):

$$\hat{x}_1 = -3,\ \hat{x}_2 = -1,\ \hat{x}_3 = 1,\ \hat{x}_4 = 3,\ a_1 = -2,\ a_2 = 0,\ a_3 = 2\,.$$

(That is, if the input is lower than a_1, the reconstruction level is $\hat{x}_1$; if the input is between a_1 and a_2, the reconstruction level is $\hat{x}_2$; ... and so on.)

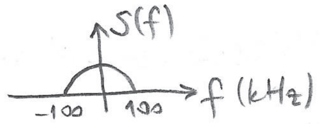
Suppose that an input X, which is uniformly distributed over the closed interval $[-5, 5]$, is processed by the quantizer specified above.

a) For the input X, calculate the expected value of X^2; i.e., $E\{X^2\}$.

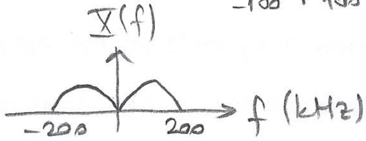
b) For the specified input and the quantizer, calculate the mean squared error distortion; i.e., $E\{(X - Q(X))^2\}$, where $Q(X)$ denotes the output of the quantizer.

EEE 431 Quiz 2 (Spring'16)

(1) $s(t) \rightarrow$ bandlimited to 100 kHz $\longrightarrow$



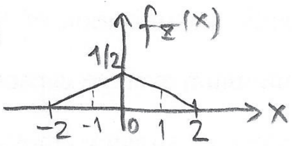
$x(t) = s(t)\cos(2\pi\,100000\,t) \rightarrow$

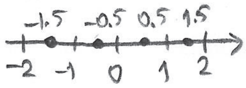


$f_s = 2\,(2 \times 200\,\text{kHz}) = 800\,\text{kHz}$

f_s samples/sec $\times$ v bits/sample $= f_s v$ bits/sec $\rightarrow \dfrac{\overset{800000}{f_s} v}{2}\,\text{Hz} = 6.4 \times 10^6 \Rightarrow \boxed{v = 16\text{ bits}}$

(3)





Uniform quantizer

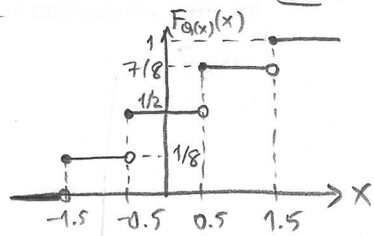
$$Q(x) = \begin{cases} 1.5, & x \in (1,2] \\ 0.5, & x \in (0,1] \\ -0.5, & x \in (-1,0] \\ -1.5, & x \in [-2,-1] \end{cases}$$

$$P(Q(X)=1.5) = P(X \in (1,2]) = \int_1^2 \frac{-x+2}{4}\,dx = \left(-\frac{x^2}{8} + \frac{x}{2}\right)\Big|_1^2 = -\frac{2^2}{8} + \frac{2}{2} - \left(-\frac{1^2}{8} + \frac{1}{2}\right) = \boxed{\frac{1}{8}}$$

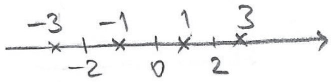
$P(Q(X)=0.5) = P(X \in (0,1]) = \int_0^1 \frac{-x+2}{4}\,dx = \boxed{\frac{3}{8}}$ Similarly, $P(Q(X)=-0.5) = \boxed{\frac{3}{8}}$ $P(Q(X)=-1.5) = \boxed{\frac{1}{8}}$

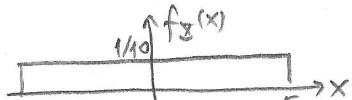
<u>CDF:</u>

$$F_{Q(x)}(x) = P(Q(X) \le x) = \begin{cases} 1, & \text{if } x \ge 1.5 \\ 7/8, & \text{if } 0.5 \le x < 1.5 \\ 1/2, & \text{if } -0.5 \le x < 0.5 \\ 1/8, & \text{if } -1.5 \le x < -0.5 \\ 0, & \text{if } x < -1.5 \end{cases}$$



(4)



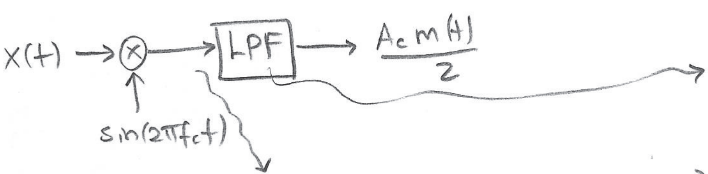


(a) $E[X^2] = \int_{-5}^{5} \frac{1}{10} x^2\,dx = \frac{x^3}{30}\Big|_{-5}^{5} = \boxed{\frac{25}{3}}$

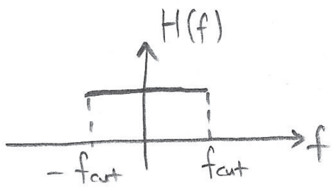
(b) $E[(X - Q(X))^2] = \int_{-5}^{-2} \frac{1}{10}(x+3)^2\,dx + \int_{-2}^{0} \frac{1}{10}(x+1)^2\,dx + \int_{0}^{2} \frac{1}{10}(x-1)^2\,dx + \int_{2}^{5} \frac{1}{10}(x-3)^2\,dx$

$= \boxed{11/15}$

(2) $x(t) = A_c m(t) \sin(2\pi f_c t)$



$A_c m(t) \sin^2(2\pi f_c t) = \frac{A_c m(t)}{2}(1 - \cos(4\pi f_c t))$



$f_{cut} \in (200\,\text{kHz}, 1800\,\text{kHz})$