EEE 431: Telecommunications I
Homework 4

1) Problem 5.49.

2) Problem 5.51.

3) Problem 5.52.

4) Consider a wide sense stationary (WSS) Gaussian random process $X(t)$ with zero mean and autocorrelation function $R_X(\tau) = 2 - |\tau|$, for $|\tau| < 2$, and 0 else. This process is input to a linear system with input-output relationship

$$\mathcal{L}\{x(t)\} = tx(t) + x(t-2).$$

a) Determine the power spectral density of the input process $X(t)$.

b) Is the output process $Y(t)$ Gaussian? Is it WSS? Why or why not?

c) Determine the probability $P(Y(1) > 3Y(2) + 5)$.

5) A random process $X(t)$ is given by

$$X(t) = A\cos(2\pi f_0 t + \Theta),$$

where A and Θ are independent random variables. A is a uniform random variable on the interval $[1, 2]$, and Θ is a discrete random variable taking on three different values with $\mathbb{P}(\Theta = 0) = \mathbb{P}(\Theta = \frac{\pi}{2}) = \mathbb{P}(\Theta = \pi) = 1/3$.

Determine the mean and autocorrelation of the process $X(t)$, and show that the process is cyclo-stationary. Also determine the period and the average autocorrelation of the process.

6) A random process is given by

$$X(t) = A_1 \cos(2\pi 20k\ t + \Theta) + A_2 \cos(2\pi 30k\ t + \Theta)$$

where $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$ is a random vector with mean $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and covariance matrix $\mathbf{C} = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$, and Θ is a uniform random variable on the interval $[0, 2\pi]$. The random vector $[A_1 \quad A_2]^T$ and the random variable Θ are independent.

a) Determine the mean and autocorrelation functions of the random process $X(t)$. Is the process wide sense stationary? Is it cyclo-stationary?

b) The power spectral density of a cyclo-stationary process is defined as the Fourier transform of its average autocorrelation function, and it has the same interpretation as the one defined for wide sense stationary processes.
Determine the power spectral density of the random process $X(t)$ and its average power content.

7) A random process $X(t)$ is given by

$$X(t) = A_1 \cos(1000\pi t + \Theta_1) - A_2 \sin(1000\pi t + \Theta_2),$$

where A_1, A_2 are constants, and Θ_1 and Θ_2 are independent uniform random variables on the interval $[0, 2\pi)$.

a) Determine the mean and autocorrelation of the process $X(t)$. Is it wide sense stationary (WSS)?

b) This process is input to an LTI system with the impulse response $h(t) = 4000 \cdot sinc^2(2000t)$. Determine the power spectral density of the output process $Y(t)$.

c) What are the average powers of $X(t)$ and $Y(t)$? What are their average powers in the frequency band 900 Hz to 1100 Hz?

8) Consider a random process defined as

$$X(t) = Y \cos(2\pi f_0 t) + Z \sin(2\pi f_0 t)$$

where f_0 is a constant, and Y and Z are zero mean random variables. Prove that the random process $X(t)$ is wide sense stationary if and only if Y and Z are uncorrelated random variables with equal variance.