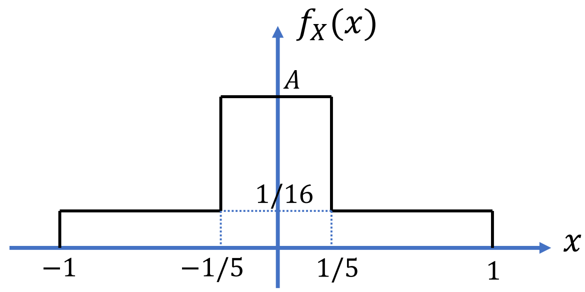
EEE 431: Telecommunications I
Homework 3

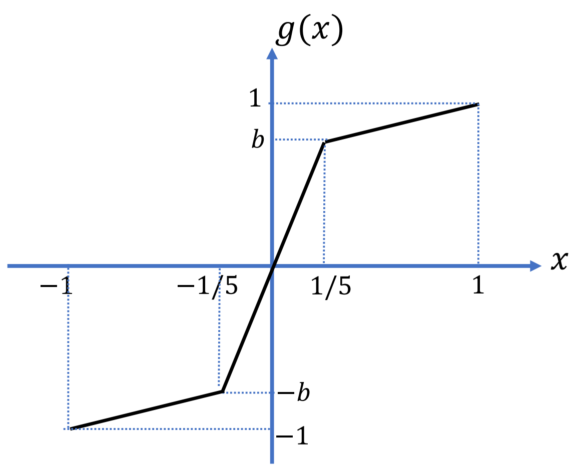
1) Assume that a discrete memoryless source X with probability density function (PDF) $f_X(x)$ given below is transmitted using pulse code modulation (PCM). The number of quantization levels used is 1024.



a) Determine the average power of the source $E[X^2]$. Note that you will also need to determine A.

b) Assuming that uniform PCM is used, determine the output signal to quantization noise ratio (SQNR) in dB.

c) We decide to transmit the same source with non-uniform PCM using a compander. For this purpose, we identify a compressor of the form given below.



Determine the parameter b that maximizes the average SQNR. Also find the resulting improvement in the SQNR in dBs compared to the uniform PCM used in the previous part.

2) A source output X modeled as a (continuous) uniform random variable on the interval $[0, 5]$ is input to a quantizer with the following description:

$$Q(x) = \begin{cases} 1\ , & if\ -\infty < x \leq 2, \\ 3\ , & if\ 2 < x \leq 4, \\ 4.5\ , & if\ 4 < x < \infty. \end{cases}$$

Determine the probability density function of the quantization error $X - Q(X)$.

3) Assume that a source with p.d.f.

$$f_X(x) = \begin{cases} 1/3 & \text{for} \quad -1 \le x < -1/2 \\ 2/3 & \text{for} \quad -1/2 \le x < 1/2 \\ 1/3 & \text{for} \quad 1/2 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

produces 1000 samples per second. We would like to transmit the source outputs using uniform PCM that uses 7 bits per sample.

a) Assuming that there are no bit errors in the transmission, what is the resulting SQNR (in dB) of the system?

b) What is the minimum bandwidth required to transmit the PCM signal?

c) Assuming that the bit error probability in the transmission is 10^{-6}, and *natural mapping* is used to map the quantization levels to bit sequences, what is the resulting SQNR (in dB)? How much loss is observed compared to part a?

4) The outputs of a stationary source is distributed according to the probability density function

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a) What is the resulting SQNR (in dB) if we use a four level quantizer given by

$$Q(x) = \begin{cases} -3/4 & \text{for} \quad -1 \le x < -1/2 \\ -1/4 & \text{for} \quad -1/2 \le x < 0 \\ 1/4 & \text{for} \quad 0 \le x < 1/2 \\ 3/4 & \text{for} \quad 1/2 \le x \le 1 \end{cases}$$

to quantize the outputs of this source.

b) Assume that we would like to use a compander that uses the four level quantizer of the previous part to improve the SQNR of the system. What kind of a compressor would you use? Give an approximate shape.

5) A continuous time source is being encoded using pulse code modulation (PCM). The sampling rate is $f_s = 20$k samples/second, and each sample is being quantized using $N = 1024$ levels. The samples have a probability density function given by $f_X(x) = \Lambda(x)$. Clearly, the range of source outputs is $[-1, 1]$.

a) Assuming that uniform PCM is used, determine the signal to quantization noise ratio (SQNR) and the minimum channel bandwidth needed for transmission (with binary modulation).

b) Consider part a, however, assume that while the quantization regions are of equal length, the reconstruction levels are not mid-points of the intervals. Instead, for each interval of the form $[a, b)$, the reconstruction level is at $(a + 3b)/4$, i.e., the reconstruction level is closer to the upper boundary of the quantization interval.

Determine the resulting SQNR.

c) Assume that we perform non-uniform PCM where the equivalent non-uniform quantizer is described as follows (let the input to the quantizer be x):
   - for $x \in [0, 1/4)$, we have 180 quantization regions of equal length;
   - for $x \in [1/4, 1/2)$, we have 140 quantization regions of equal length;
   - for $x \in [1/2, 3/4)$, we have 116 quantization regions of equal length;
   - for $x \in [3/4, 1)$, we have 76 quantization regions of equal length;
   - for negative values of x, the quantizer is symmetric; e.g., for $x \in [-3/4, -1/2)$, we have 116 quantization regions of equal length, and similar for other regions.
   - for all the quantization regions, the reconstruction levels are the mid-points of the intervals.

   Clearly, we have a large number of quantization levels, and we can approximate the quantization errors as uniform.
   Determine the resulting SQNR.

6) The message signal $m(t) = \frac{1}{2}\cos(4000\pi t) + \frac{1}{2}\sin(2000\pi t)$ is being transmitted using conventional AM. The modulated signal is given by $x(t) = 10(1 + m(t))\cos(400k\pi t)$.

a) Plot the Fourier Transforms of $m(t)$ and $x(t)$.
b) Determine the ratio of the average power in the sidebands to the overall average power in $x(t)$.
c) Determine the modulation index. Is the signal overmodulated?

7) Consider a message signal given by $m(t) = \sin(2\pi f_m t)$.

a) Plot the spectrum of $m(t)$.
b) Plot the spectrum of the modulated signal if DSB-SC modulation with $x(t) = m(t)sin(2\pi f_c t)$ is employed. Assume $f_c \gg f_m$.
c) Plot the spectrum of the modulated signal if conventional AM modulation with $x(t) = (1 + 0.5m(t))\cos(2\pi f_c t)$ is employed. Assume $f_c \gg f_m$.

8) A conventional AM modulated signal is given by

$$x(t) = 4\sin(2\pi 100kt) + \sin(2\pi 99kt) + \sin(2\pi 101kt) + \cos(2\pi 99kt) - \cos(2\pi 101kt).$$

The carrier frequency is $f_c = 100kHz$, and the amplitude sensitivity constant is $k_a = 1$.

a) Determine the power content in the sidebands, and the ratio of the sideband power to the total power of the modulated signal.
b) Determine the message signal and the modulation index.

9) Let a message signal be given by $m(t) = \sin(2\pi 10kt) + 2\cos(2\pi 20kt)$ is modulated using DSB-SC AM with a carrier signal $10\cos(2\pi 1000kt + \pi/3)$. The resulting signal is denoted by $x(t)$.

a) Determine and plot the Fourier transform of the modulated signal.
b) Determine the lower and upper sidebands of the modulated signal in time domain.
c) The signal $x(t)$ is demodulated using a carrier signal with an incorrect phase, i.e., using $\cos(2\pi 1000kt + \pi/6)$. Determine the ratio of the resulting output signal power to the one obtained by using the correct carrier phase.