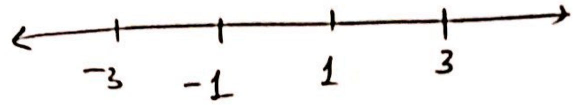
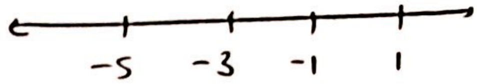
EEE 431 HW # 7

Question 1:

(I)



(II)



(III)

$d_{min,1} = d_{min,2} = 2, \quad d_{min,3} = 1$

$E_{av,1} = \frac{1}{4}(9+1+1+9) = 5$

$E_{av,2} = \frac{1}{4}(25+9+1+1) = 9$

$E_{av,3} = \frac{1}{4}(4+1+1+9) = 15/4$

$\Rightarrow \frac{d_{min,1}^2}{E_{av,1}} = \frac{4}{5}, \quad \frac{d_{min,2}^2}{E_{av,2}} = \frac{4}{9}, \quad \frac{d_{min,3}^2}{E_{av,3}} = \frac{4}{15}.$

Hence, in terms of power efficiency: $I > II > III$.

At the high SNR: I is better than II by $10\log_{10}\left(\frac{9}{5}\right) \cong 2.55\,dB$

II is better than III by $10\log_{10}\left(\frac{15}{9}\right) \cong 2.21\,dB$

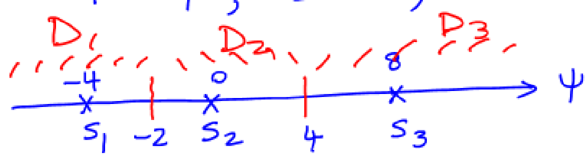
I is better than III by $10\log_{10}\left(\frac{15}{4}\right) \cong 4.77\,dB$.

Question 2

a)

Vector representation of the three signals (with the given basis):

$s_1 = -4\ ,\ s_2 = 0\ ,\ s_3 = 8.$



D_i: decision regions.

Equally likely symbols $\Rightarrow$ use the ML rule (over an AWGN channel), i.e., minimize $(r-s_i)^2$. i.e.,

$$\hat{m} = \begin{cases} 1 & \text{if } r < -2 \\ 2 & \text{if } -2 < r < 4 \\ 3 & \text{if } r > 4 \end{cases}$$

Prob of error: $P_e = \frac{1}{3} P_{e,1} + \frac{1}{3} P_{e,2} + \frac{1}{3} P_{e,3}$ with

$P_{e,i} = P(\text{error} \mid m_i \text{ sent})$. With $r = s_i + n$, $n \sim \mathcal{N}(0, \frac{N_0}{2})$

$$P_{e,1} = P(n > 2) = Q\left(\frac{2}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{8}{N_0}}\right)$$

$$P_{e,2} = P(n < -2 \text{ or } n > 4) = Q\left(\sqrt{\frac{8}{N_0}}\right) + Q\left(\sqrt{\frac{32}{N_0}}\right)$$

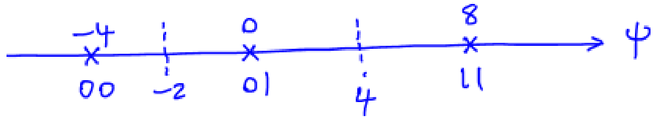
$$P_{e,3} = P(n < -4) = Q\left(\sqrt{\frac{32}{N_0}}\right).$$

With $E_s = \frac{(-4)^2 + 0^2 + 8^2}{3} = \frac{80}{3}$

we get:

$$\boxed{P_e = \frac{2}{3} \cdot \left(Q\left(\sqrt{\frac{3\gamma_s}{10}}\right) + Q\left(\sqrt{\frac{6\gamma_s}{5}}\right) \right)}$$

b)



let

$$P_{b,i} = P(\text{bit error} \mid m_i \text{ sent}).$$

$$P_{b,1} = \frac{1}{2} \cdot P\left(r \in (-2,4) \mid m_1 \text{ sent}\right) + P\left(r > 4 \mid m_1 \text{ sent}\right)$$

$$= \frac{1}{2} P\left(n \in (2,8)\right) + P(n > 8) = \frac{1}{2}\left(Q\left(\sqrt{\frac{8}{N_0}}\right) - Q\left(\sqrt{\frac{128}{N_0}}\right)\right) + Q\left(\sqrt{\frac{128}{N_0}}\right)$$

$$P_{b,2} = \frac{1}{2} P_{e,2}$$

(since $01 \rightarrow 00$ & $01 \rightarrow 11$ both incur 1 bit error out of 2 bits transmitted)

$$P_{b,3} = \frac{1}{2} P\left(r \in (-2,4) \mid m_3 \text{ sent}\right) + P\left(r < -2 \mid m_3 \text{ sent}\right)$$

$$= \frac{1}{2} P\left(n \in (-10,-4)\right) + P(n < -10)$$

$$= \frac{1}{2} \cdot \left(Q\left(\sqrt{\frac{32}{N_0}}\right) - Q\left(\sqrt{\frac{200}{N_0}}\right)\right) + Q\left(\sqrt{\frac{200}{N_0}}\right)$$

Hence, $P_b = \frac{1}{3} P_{b,1} + \frac{1}{3} P_{b,2} + \frac{1}{3} P_{b,3}$

$$= \frac{1}{3} Q\left(\sqrt{\frac{8}{N_0}}\right) + \frac{1}{3} Q\left(\sqrt{\frac{32}{N_0}}\right) + \frac{1}{6} Q\left(\sqrt{\frac{128}{N_0}}\right) + \frac{1}{6} Q\left(\sqrt{\frac{200}{N_0}}\right)$$

with $E_s = \frac{80}{3}$;

$$\boxed{P_b = \frac{1}{3} Q\left(\sqrt{\frac{3}{10}\gamma_s}\right) + \frac{1}{3} Q\left(\sqrt{\frac{6\gamma_s}{5}}\right) + \frac{1}{6} Q\left(\sqrt{\frac{24}{5}\gamma_s}\right) + \frac{1}{6} Q\left(\sqrt{\frac{15}{2}\gamma_s}\right)}$$

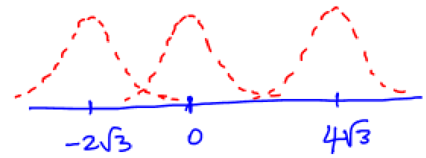
c)

$$\langle s_1(t), \psi'(t)\rangle = \int_0^2 -\sqrt{6}\, t \cdot \frac{1}{\sqrt{2}}\, dt = -2\sqrt{3}$$

$$\langle s_2(t), \psi'(t)\rangle = 0$$

$$\langle s_3(t), \psi'(t)\rangle = \int_0^2 2\sqrt{6}\, t \cdot \frac{1}{\sqrt{2}}\, dt = 4\sqrt{3}.$$

PDFs of $r' | m_i$ sent



Hence:

$$r' | m_1 \sim \mathcal{N}\left(-2\sqrt{3}, \frac{N_0}{2}\right)$$

$$r' | m_2 \sim \mathcal{N}\left(0, N_0/2\right)$$

$$r' | m_3 \sim \mathcal{N}\left(4\sqrt{3}, \frac{N_0}{2}\right)$$

ML receiver:

$$\hat{m} = \underset{i}{\operatorname{argmax}}\ P(r' | m_i \text{ sent})$$

hence the decision rule is again a threshold rule:

$$\Rightarrow \quad \hat{m} = \begin{cases} 1 & \text{if } r' < -\sqrt{3} \\ 2 & \text{if } -\sqrt{3} < r' < 2\sqrt{3} \\ 3 & \text{if } r' > 2\sqrt{3} \end{cases}$$

& Similar to part a:

$$P_e = \frac{1}{3}\Big(P(n > \sqrt{3}) + P(n < -\sqrt{3} \text{ or } n > 2\sqrt{3}) + P(n < -2\sqrt{3})\Big)$$

$$= \frac{2}{3}\Big(P(n > \sqrt{3}) + P(n > 2\sqrt{3})\Big) = \frac{2}{3}\left(Q\left(\sqrt{\frac{6}{N_0}}\right) + Q\left(\sqrt{\frac{24}{N_0}}\right)\right)$$

[Note $n \sim \mathcal{N}(0, \frac{N_0}{2})$]

with $E_s = \frac{80}{3}$:

$$\boxed{P_e = \frac{2}{3}\left(Q\left(\sqrt{\frac{9}{40}\gamma_s}\right) + \frac{2}{3}\, Q\left(\sqrt{\frac{9\gamma_s}{10}}\right)\right).}$$

Keeping only the dominant terms: this is $\approx \log_{10}\left(\frac{3/10}{9/40}\right) = 1.2\,\text{dB}$ worse than the result in part a.

d)

There would be no change with the noise part. I.e., $n \sim \mathcal{N}(0, N_0/2)$. However, the signal part will be different.

Considering $s_1(t)$:

$$s_1(t) * \psi(2-t)\Big|_{t=1.9} = -\sqrt{6}\cdot\sqrt{\tfrac{3}{8}}\cdot \int_0^2 \tau\,(2+\tau-t)\cdot\underbrace{\big(u(t-\tau)-u(t-\tau-2)\big)}_{=1 \text{ if } t-2<\tau<t} d\tau \Big|_{t=1.9}$$

$$= -\frac{3}{2}\cdot\int_0^{1.9} \tau\cdot(\tau+0.1)\,d\tau \cong -3.7$$

Similarly for $s_3(t) * \psi(2-t)\Big|_{t=1.9} \cong 7.4$. (For $s_2(t)$, we get "0")

Hence:

$r|m_1 \sim \mathcal{N}\left(-3.7, \frac{N_0}{2}\right)$ $\quad r|m_2 \sim \mathcal{N}(0, N_0/2)$

$r|m_3 \sim \mathcal{N}\left(7.4, \frac{N_0}{2}\right)$.

$$P_e = \frac{1}{3}\cdot\Big(P(r>-2\,|\,m_1) + P(r<-2 \text{ or } r>4\,|\,m_2) + P(r<4\,|\,m_3)\Big)$$

$$= \frac{1}{3}\cdot\Big(P(n>1.7) + P(n<-2 \text{ or } n>4) + P(n<-3.4)\Big)$$

$$= \frac{1}{3}\left(Q\left(\frac{1.7}{\sqrt{N_0/2}}\right) + Q\left(\frac{2}{\sqrt{N_0/2}}\right) + Q\left(\frac{4}{\sqrt{N_0/2}}\right) + Q\left(\frac{3.4}{\sqrt{N_0/2}}\right)\right)$$

with $E_s = \frac{80}{3}$:

$$P_e = \frac{1}{3}\left(Q\left(\sqrt{0.217\gamma_s}\right) + Q\left(\sqrt{0.3\gamma_s}\right) + Q\left(\sqrt{1.2\gamma_s}\right) + Q\left(\sqrt{0.8670\gamma_s}\right)\right)$$

which is approximately $10\log_{10}\left(\frac{0.3}{0.217}\right) \cong 1.4$ dB worse

Question 3

a)

We need to use MAP rule.

$$\hat{m} = \arg\max_{m=1,2,3} p_m \cdot P(r \mid s_m)$$

[where r is obtained from $r(t)$ as: $r(t) \to \boxed{\psi(T-t)} \xrightarrow{t=T} r$]

Basis: $\psi(t) = \frac{g(t)}{\sqrt{E_g}} \Rightarrow s_1 = -A\sqrt{E_g} \quad s_2 = 0 \quad s_3 = A\sqrt{E_g}$

$r \mid s_1 \sim \mathcal{N}\left(-A\sqrt{E_g}, \frac{N_0}{2}\right) \qquad r \mid s_2 \sim \mathcal{N}\left(0, \frac{N_0}{2}\right)$

$r \mid s_3 \sim \mathcal{N}\left(A\sqrt{E_g}, \frac{N_0}{2}\right)$

We need to compare
$$\begin{cases} p \cdot \frac{1}{\sqrt{\pi N_0}} e^{-(r+A\sqrt{E_g})^2/N_0} \\ (1-2p) \frac{1}{\sqrt{\pi N_0}} e^{-r^2/N_0} \\ p \frac{1}{\sqrt{\pi N_0}} e^{-(r-A\sqrt{E_g})^2/N_0} \end{cases}$$

Simplifying: (let $r_{th} = \frac{A\sqrt{E_g}}{2} + \frac{N_0}{2A\sqrt{E_g}} \ln\left(\frac{1-2p}{p}\right)$)

If $r_{th} > 0$;
$$\hat{m} = \begin{cases} 1 & \text{if } r < -r_{th} \\ 2 & \text{if } -r_{th} < r < r_{th} \\ 3 & \text{if } r > r_{th} \end{cases}$$

If $r_{th} < 0$;
$$\hat{m} = \begin{cases} 1 & \text{if } r < 0 \\ 3 & \text{if } r > 0 \end{cases}$$

b)

If $r_{th} < 0$ then "2" is not selected.

That is, if

$$-\frac{A\sqrt{E_g}}{2} + \frac{N_0}{2A\sqrt{E_g}} \ln\left(\frac{1-2p}{p}\right) < 0$$

$$\Rightarrow \quad \ln\left(\frac{1-2p}{p}\right) < \frac{A^2 E_g}{N_0}$$

$$\Rightarrow \quad \boxed{p > \frac{1}{2 + \exp\left(\frac{A^2 E_g}{N_0}\right)}}$$

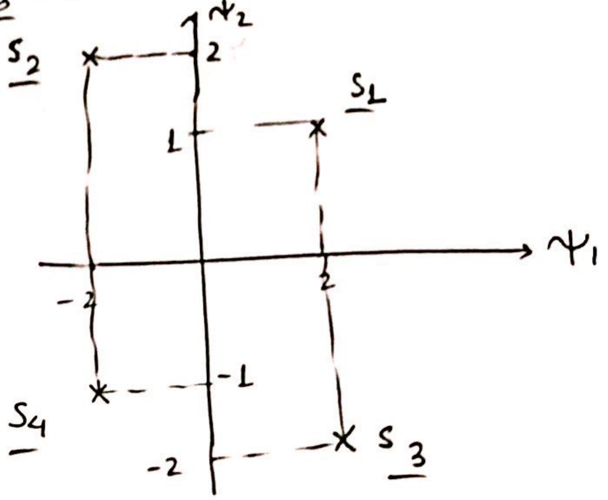
"2" is not selected. In this case the decision rule is simply;

$$\boxed{\hat{m} = \begin{cases} 1 & \text{if } r < 0 \\ 3 & \text{if } r > 0 \end{cases}}$$

Question 4:

a) $\underline{s}_1 = [2 \ \ 1]^T$, $\underline{s}_2 = [-2 \ \ 2]^T$, $\underline{s}_3 = [2 \ -2]^T$, $\underline{s}_4 = [-2 \ -1]^T$

$$\hat{m} = \begin{cases} 1, & r_1, r_2 > 0 \\ 2, & r_1 < 0, \ r_2 > 0 \\ 3, & r_1 > 0, \ r_2 < 0 \\ 4, & r_1, r_2 < 0 \end{cases}$$



Due to symmetry, $P_{e,1} = P_{e,4}$ & $P_{e,2} = P_{e,3}$

$$P_{e,1} = 1 - P(r_1, r_2 > 0 \mid \underline{s}_1 \text{ is sent})$$

$$= 1 - P(n_1 + 2 > 0, \ n_2 + 1 > 0)$$

$$= 1 - \left(1 - Q\left(\sqrt{\frac{8}{N_0}}\right)\right)\left(1 - Q\left(\sqrt{\frac{2}{N_0}}\right)\right)$$

$$P_{e,2} = 1 - P(r_1 < 0, \ r_2 > 0 \mid \underline{s}_2 \text{ is sent})$$

$$= 1 - P(n_1 - 2 < 0, \ n_2 + 2 > 0)$$

$$= 1 - \left(1 - Q\left(\sqrt{\frac{8}{N_0}}\right)\right)^2 = 2Q\left(\sqrt{\frac{8}{N_0}}\right) - Q^2\left(\sqrt{\frac{8}{N_0}}\right)$$

$$P_{e,1} = Q\left(\sqrt{\frac{8}{N_0}}\right) + Q\left(\sqrt{\frac{2}{N_0}}\right) - Q\left(\sqrt{\frac{8}{N_0}}\right)Q\left(\sqrt{\frac{2}{N_0}}\right).$$

$\overline{E_s} = \frac{1}{2}(5+8) = \frac{13}{2}$, i.e. $\frac{2}{N_0} = \frac{4}{13}\gamma$ and $\frac{8}{N_0} = \frac{16}{13}\gamma$

with $P_e = \frac{1}{4}\sum_{i=1}^{4} P_{e,i}$, we obtain

$$P_e = \frac{3}{2} Q\left(\sqrt{\frac{16\gamma}{13}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{4\gamma}{13}}\right) - \frac{1}{2}Q^2\left(\sqrt{\frac{16\gamma}{13}}\right) - \frac{1}{2}Q\left(\sqrt{\frac{4\gamma}{13}}\right)Q\left(\sqrt{\frac{16\gamma}{13}}\right)$$

b) we pick $\underline{s}_1$ if $\|\underline{r}-\underline{s}_1\|^2$ is the smallest among $\|\underline{r}-\underline{s}_i\|^2$

$\|\underline{r}-\underline{s}_1\|^2 \leq \|\underline{r}-\underline{s}_2\|^2 \Rightarrow (r_1-2)^2+(r_2-1)^2 \leq (r_1+2)^2+(r_2-2)^2$

$\Rightarrow -4r_1+5-2r_2 \leq 4r_1+8-4r_2$

$\Rightarrow \boxed{8r_1-2r_2+3 \geq 0.}$ --- (i)

$\|\underline{r}-\underline{s}_1\|^2 \leq \|\underline{r}-\underline{s}_3\|^2 \Rightarrow (r_1-2)^2+(r_2-1)^2 \leq (r_1-2)^2+(r_2+2)^2$

$\Rightarrow -2r_2+1 \leq 4r_2+4$

$\Rightarrow \boxed{r_2 \geq -1/2}$ --- (ii)

$\|\underline{r}-\underline{s}_1\|^2 \leq \|\underline{r}-\underline{s}_4\|^2 \Rightarrow (r_1-2)^2+(r_2-1)^2 \leq (r_1+2)^2+(r_2+1)^2$

$\Rightarrow \boxed{2r_1+r_2 \geq 0}$ --- (iii)

$Z_1 = \left\{ (r_1,r_2) \mid 8r_1-2r_2+3 \geq 0,\ r_2 \geq -1/2,\ 2r_1+r_2 \geq 0 \right\}.$

c) $\|\underline{s}_1 - \underline{s}_2\|^2 = 17, \quad \|\underline{s}_1 - \underline{s}_3\|^2 = 9, \quad \|\underline{s}_1 - \underline{s}_4\|^2 = 20$

$\|\underline{s}_2 - \underline{s}_3\|^2 = 32, \quad \|\underline{s}_2 - \underline{s}_4\|^2 = 9, \quad \|\underline{s}_3 - \underline{s}_4\|^2 = 17$

$$P_e \leq \frac{1}{4} \sum_{i=1}^{4} \sum_{\substack{j=1 \\ j \neq i}}^{4} Q\left(\frac{\|\underline{s}_i - \underline{s}_j\|}{\sqrt{2N_0}}\right)$$

$$= \frac{1}{4}\left[4Q\left(\sqrt{\frac{9}{2N_0}}\right) + 4Q\left(\sqrt{\frac{17}{2N_0}}\right) + 2Q\left(\sqrt{\frac{32}{2N_0}}\right) + 2Q\left(\sqrt{\frac{20}{2N_0}}\right)\right]$$

$$P_e \mathrel{\overset{=}{\leq}} Q\left(\sqrt{\frac{9\gamma}{13}}\right) + Q\left(\sqrt{\frac{17\gamma}{13}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{32\gamma}{13}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{20\gamma}{13}}\right).$$

At high SNR $\quad P_e \mathrel{\overset{\sim}{\leq}} Q\left(\sqrt{\frac{9\gamma}{13}}\right)$

[coefficients do not matter. Q(.) is an exponential decay.]

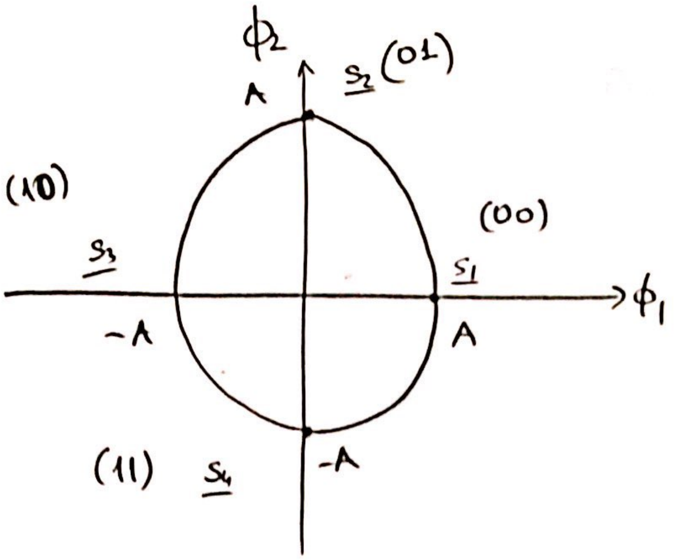
& $P_e \simeq \cancel{\frac{3}{2}} Q\left(\sqrt{\frac{4\gamma}{13}}\right)$ [from part a]

The dominant terms are those w/ the lowest arguments of the Q(.) function.

$\Rightarrow$ Optimal decision rule is superior by

$$\approx 10\log_{10}\left(\frac{9}{4}\right) \simeq \boxed{3.5\text{ dB}}$$

5)



$$\hat{m} = \begin{cases} 1, & r_1 \geq r_2,\ r_1 \geq -r_2 \\ 2, & r_1 < r_2 \ \& \ r_1 \geq -r_2 \\ 3, & r_1 < r_2 \ \& \ r_1 < -r_2 \\ 4, & r_1 \geq r_2 \ \& \ r_1 < -r_2 \end{cases}$$

$$P_{00 \to 01} = P(A + n_1 < n_2 \ \& \ A + n_1 \geq -n_2)$$

($n_1 - n_2$ and $n_1 + n_2$ are uncorrelated, hence independent)

$$= P(n_1 - n_2 < -A \ \& \ n_1 + n_2 \geq -A)$$

$$= P(n_1 - n_2 < -A)\, P(n_1 + n_2 > -A)$$

$$= Q\left(\frac{A}{\sqrt{N_0}}\right) Q\left(\frac{-A}{\sqrt{N_0}}\right)$$

$$P_{00 \to 10} = P(A + n_1 < n_2 \ ,\ A + n_1 < -n_2)$$

$$= P(n_1 - n_2 < -A)\, P(n_1 + n_2 < -A)$$

$$= Q\left(\frac{A}{\sqrt{N_0}}\right) Q\left(\frac{A}{\sqrt{N_0}}\right)$$

$$P_{00 \to 11} = P(A + n_1 \geq n_2 \ \& \ A + n_1 < -n_2)$$

$$= P(n_1 - n_2 \geq -A)\, P(n_1 + n_2 < -A)$$

$$= Q\left(\frac{-A}{\sqrt{N_0}}\right) Q\left(\frac{A}{\sqrt{N_0}}\right)$$

Due to symmetry;

$$P_b = (1 \cdot P_{00 \to 01} + 2\, P_{00 \to 11} + P_{00 \to 10})/2$$

$$= \frac{3Q\left(\frac{A}{\sqrt{N_0}}\right) Q\left(\frac{-A}{\sqrt{N_0}}\right) + Q\left(\frac{A}{\sqrt{N_0}}\right)^2}{2}$$

$$= \frac{3}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left(1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right) + Q\left(\sqrt{\frac{2E_b}{N_0}}\right)^2$$

$$= \frac{3}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right) - \frac{1}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right)^2$$

$$\approx \frac{3}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \text{ at the high SNR.}$$

when Gray mapping is used,

$$P_b = \frac{1}{2} P_e = \frac{1}{2}(1 - P_{c,1})$$

$$= \frac{1}{2}\left(1 - Q\left(\frac{-A}{\sqrt{N_0}}\right)^2\right)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) - Q^2\left(\sqrt{\frac{2E_b}{N_0}}\right)\frac{1}{2}$$

$$\approx Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{at the high SNR.}$$

P_b for gray mapping is definitely smaller than the natural binary coding.

Question 6

$$d_{min,16-PSK} = 2\sqrt{E_s}\sin\left(\frac{\pi}{16}\right)$$

For the 16-QAM, $E_s = \frac{2d^2 \cdot 15}{3} = 10d^2 \Rightarrow d = \sqrt{\frac{E_s}{10}}$

$$d_{min,16-QAM} = 2d = 2\sqrt{\frac{E_s}{10}}.$$

$$\Rightarrow \frac{d_{min,16-PSK}^2}{E_s} = 4\sin^2(\pi/16), \quad \frac{d_{min,16-QAM}^2}{E_s} = \frac{4}{10}.$$

Since $\sin\left(\frac{\pi}{16}\right)^2 < \frac{1}{10} \Rightarrow$ 16-QAM is more power efficient.

$\Rightarrow$ 16-QAM is better by $10\log_{10}\left(\frac{1/10}{\sin^2(\pi/16)}\right) \cong \boxed{4.19\,dB}$

Question 7

a)

We can use

$$\psi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_0 t)$$

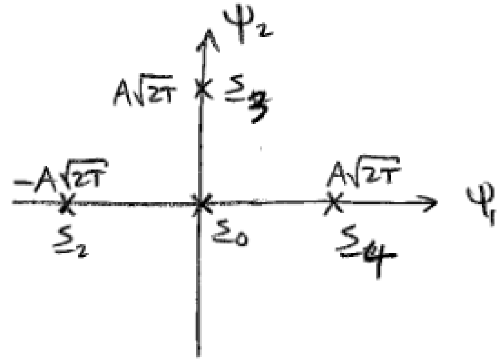
$$\psi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_0 t)$$

One can easily check that these are normalized and orthogonal!

Then:

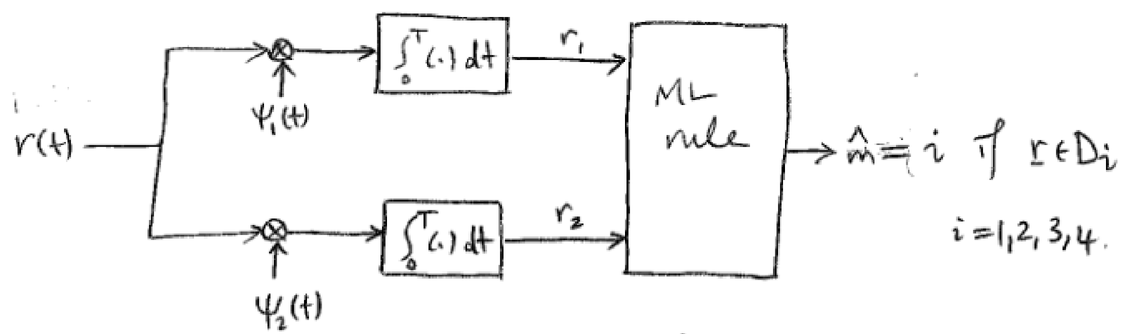
$$\underline{s}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \underline{s}_2 = \begin{bmatrix} -A\sqrt{2T} \\ 0 \end{bmatrix}$$

$$\underline{s}_3 = \begin{bmatrix} 0 \\ A\sqrt{2T} \end{bmatrix} \quad \underline{s}_4 = \begin{bmatrix} A\sqrt{2T} \\ 0 \end{bmatrix}$$

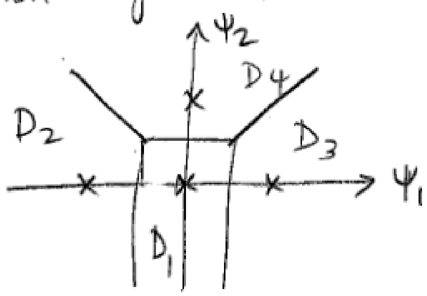


b)

ML receiver:



Decision regions (due to symmetry):



$$\Rightarrow \hat{m} = \begin{cases} 1 & \text{if } |r_1| < A\sqrt{\frac{T}{2}},\ r_2 < A\sqrt{\frac{T}{2}} \\ 2 & \text{if } r_1 < -A\sqrt{\frac{T}{2}},\ r_1 < -r_2 \\ 3 & \text{if } r_1 > A\sqrt{\frac{T}{2}},\ r_1 > r_2 \\ 4 & \text{if } r_2 > A\sqrt{\frac{T}{2}},\ r_2 > |r_1|. \end{cases}$$

c)

$$
\begin{aligned}
P_{e,1} &= P(\text{error} \mid \underline{s}_1 \text{ sent}) \\
&= 1 - P(\underline{r} \in D_1 \mid \underline{s}_1 \text{ sent}) \quad \text{if } \underline{s}_1 \text{ sent: } r_1 = n_1, r_2 = n_2 \\
&= 1 - P\left(-A\sqrt{\tfrac{T}{2}} < n_1 < A\sqrt{\tfrac{T}{2}},\ n_2 < A\sqrt{\tfrac{T}{2}}\right) \quad \text{independence of } n_1 \,\&\, n_2 \\
&= 1 - P\left(-A\sqrt{\tfrac{T}{2}} < n_1 < A\sqrt{\tfrac{T}{2}}\right) \cdot P\left(n_2 < A\sqrt{\tfrac{T}{2}}\right) \quad n_1, n_2 \sim \mathcal{N}\left(0, \tfrac{N_0}{2}\right) \\
&= 1 - \left(1 - 2Q\left(\frac{A\sqrt{T/2}}{\sqrt{N_0/2}}\right)\right)\left(1 - Q\left(\frac{A\sqrt{T/2}}{\sqrt{N_0/2}}\right)\right) \\
&= 3Q\left(\sqrt{\frac{A^2T}{N_0}}\right) - 2Q^2\left(\sqrt{\frac{A^2T}{N_0}}\right)
\end{aligned}
$$

Since: $\bar{E}_s = \frac{1}{4}\left(0 + A^2 2T + A^2 2T + A^2 2T\right) = \frac{3A^2T}{2}$

$$\Rightarrow \quad \bar{E}_b = \frac{3}{4}A^2T$$

$$\Rightarrow \boxed{P_{e,1} = 3Q\left(\sqrt{\frac{4}{3}\frac{\bar{E}_b}{N_0}}\right) - 2Q^2\left(\sqrt{\frac{4}{3}\cdot\frac{\bar{E}_b}{N_0}}\right)}$$