EEE 431: Telecommunications I
Homework 6

1) Problem 8.16.

2) Problem 8.33.

3) A zero mean white Gaussian noise process $X(t)$ with power spectral density $N_0/2$ is input to an LTI filter with impulse response

$$h(t) = \delta(t) - \frac{1}{2}\delta(t-1) - \frac{1}{2}\delta(t+1).$$

Denote the filter output by $Y(t)$.

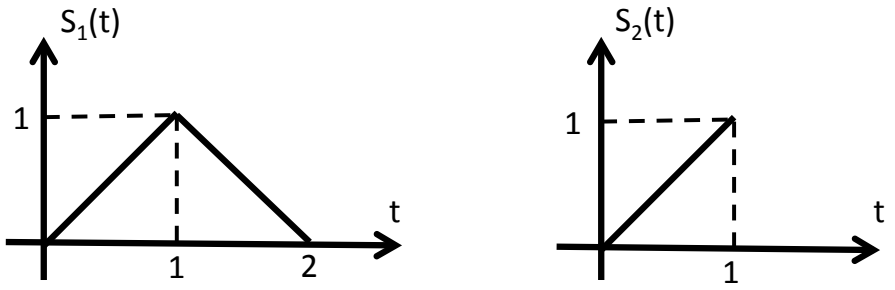
a) Is $Y(t)$ Gaussian random process? Why or why not?

b) Is $Y(t)$ a wide-sense stationary process? Is it strict sense stationary? Why or why not?

c) We sample the output process at a specific time t_0 and obtain the random variable $Y(t_0)$. Determine the probability that this sample exceeds a threshold value A, i.e., $\mathbb{P}(Yt_0 > A)$.

d) We sample the output process at two different time instances, t_1 and t_2 with $t_2 - t_1 = 2$ units. Determine the joint distribution of the random variables $Y(t_1)$ and $Y(t_2)$. Also determine the probability that the difference in the two samples is greater than a threshold A, i.e., $\mathbb{P}(|Y(t_2) - Y(t_1)| > A)$.

4) Consider a binary communication system using the two signals $s_1(t)$ and $s_2(t)$ shown in the figure below. The transmitted bits are equally likely, bit duration is $T = 2$, and the transmission is over an AWGN channel with power spectral density $N_0/2$.



a) Find an orthonormal basis for the signal space, and show the signal constellation.

b) Show the impulse responses matched to the basis functions, and describe the optimal matched filter type receiver. Is it possible to implement the optimal receiver using only one matched filter? If so, how? (Be specific, and provide a detailed block diagram as necessary.)

c) What is the average probability of error? Express your answer in terms of the average SNR per bit γ_b.

5) Consider an antipodal signaling scheme using $p(t)$ and $-p(t)$ to transmit "1" and "0" over an AWGN channel with power spectral density $N_0/2$. Assume that $p(t) = \cos(\pi t/T_b) + \sin(\pi t/T_b)$ where T_b is the bit duration.

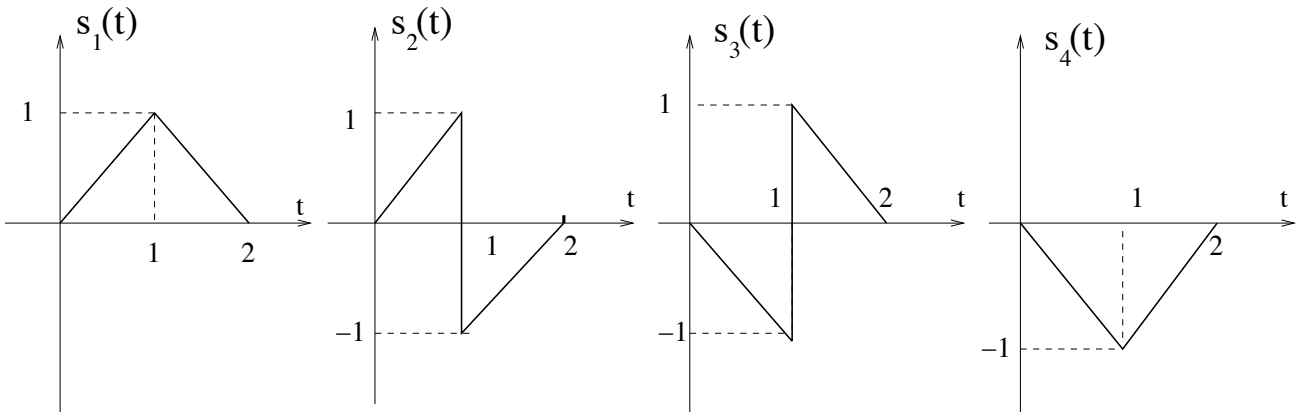
Assume that a receiver multiplies the received signal (AWGN channel output) with a rectangular pulse of a positive amplitude and integrates the result over the bit period to produce a decision variable r, and then it uses a threshold detector to make a decision on the transmitted bit. That is, the received declares "1" as a result if $r > r_{th}$, and "0", otherwise.

a) Determine the optimal value of the threshold r_{th} to minimize the average bit error probability?

b) Take $r_{th} = 0$ and compute the average error probability, and express the result in terms of E_b/N_0.

c) The above receiver structure is not optimal. Determine the optimal receiver structure, and compute the resulting average error probability. Compare the result with that of the previous part to identify how much inferior is the receiver in part a.

6) The four signals shown in the figure are used to transmit four different messages (where the symbol period is $T = 2$). Assume that the symbols are equally likely.



a) Find an orthonormal set of basis functions for this signal set.

b) Plot the signal constellation.

c) What are the impulse responses of the filters matched to the basis functions in part a?

d) This system is being used over an AWGN channel. Describe the structure of the optimal receiver.

e) Assume that mapping from two bits to the signals are as follows: 00 is transmitted by $s_1(t)$, 01 by $s_2(t)$, 10 by $s_3(t)$ and 11 by $s_4(t)$, and the system is used over an AWGN channel with power spectral density of $\frac{N_0}{2}$. What is the resulting (exact) bit error probability? Express your answer in the average signal to noise ratio per bit $\gamma_b = \frac{E_b}{N0}$.

7) Consider a ternary communication system using three signals

$$s_1(t) = 1 \text{ for } t \in [0, 1/2),\ 0 \text{ otherwise},$$
$$s_2(t) = 1 \text{ for } t \in [1/2, 1],\ 0 \text{ otherwise},$$
$$s_3(t) = 1 \text{ for } t \in [0, 1],$$

where the symbol period is $T = 1$. The three symbols transmitted are equally likely, and the channel is AWGN with power spectral density $N_0/2$.

a) Find an orthonormal set of basis functions for this set, find the corresponding vector representation for the transmitted signals, and plot the signal constellation. Also determine the optimal receiver structure to minimize the symbol error probability.

b) What is the (exact) conditional error probability assuming that $s_3(t)$ is transmitted?