EEE 431: Telecommunications I
Homework 8

1) Problem 13.2.

2) Problem 13.3.

3) Problem 13.4.

4) Problem 13.6.

5) Consider a (8,5) (binary) linear block code (i.e., $n = 8$, $k = 5$). Denote the message bits by $x_1, \ldots, x_5$, and the codeword bits by $c_1, c_2, \ldots, c_8$. Assume that the coded bits satisfy the following parity check equations.

$$\begin{aligned} c_1 + c_4 + c_5 + c_6 &= 0, \\ c_2 + c_3 + c_5 + c_7 &= 0, \\ c_1 + c_2 + c_3 + c_4 + c_8 &= 0. \end{aligned}$$

a) Determine the parity check and generator matrices of the code.

b) What is the codeword corresponding to the message $(x_1, x_2, x_3, x_4, x_5) = (1, 1, 0, 1, 0)$?

c) We obtain a new code from this one by adding one more parity bit such that all the codewords are of even weight. The new code is a $(9, 5)$ linear block code. What is its parity check matrix? What is its minimum distance?

6) Determine the rate, generator matrix and minimum distance of the smallest <u>linear block code</u> (i.e., the one with the lowest number of codewords) which contains the five codewords (of length $n = 10$) given below:

1 0 1 0 1 0 1 0 1 0,
0 1 0 1 0 1 0 1 0 1,
0 0 1 1 1 1 1 1 1 1,
1 1 1 1 1 1 1 1 1 1,
0 1 1 0 1 0 1 0 1 0.

7) Consider a linear block code with the generator matrix

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

a) How many codewords does the code have? List all of them.

b) What is the minimum distance of this code? What is its weight enumerator?

c) How many errors can this code detect? How many errors can it correct?

d) This code is used over a binary symmetric channel with cross-over probability 0.01. That is, the channel inputs and outputs are 0 and 1, and the channel is described by the transition probabilities: $P(\text{output is } 1 \mid \text{input is } 0) = P(\text{output is } 0 \mid \text{input is } 1) = 0.01$. Assuming that the code is used for error detection only, and that the errors in different channel uses are independent of each other, determine the probability that there will be undetected errors in the transmission.

8) Consider a cascade of K amplifier stages where the i-th amplifier has a gain of G_i and noise figure of F_i. Show that the overall noise figure of the cascade is given by

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_K - 1}{G_1 G_2 \ldots G_{K-1}}.$$

9) We want to use three amplifiers in cascade to improve the signal strength at a receiver. Amplifier A has a gain of 20 dB and a noise figure of 10 dB; amplifier B has a gain of 20 dB and a noise figure of 6 dB; amplifier C has a gain of 10 dB and a noise figure of 3 dB.

In which order would you connect these three amplifiers to ensure that the resulting signal quality is the highest at the output?

Assuming that the input SNR is 40 dB, what is the resulting SNR at the output?