EEE 431: Telecommunications 1

Quiz 2

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Section: **1** (Thu. lectures) or **2** (Wed. lectures)

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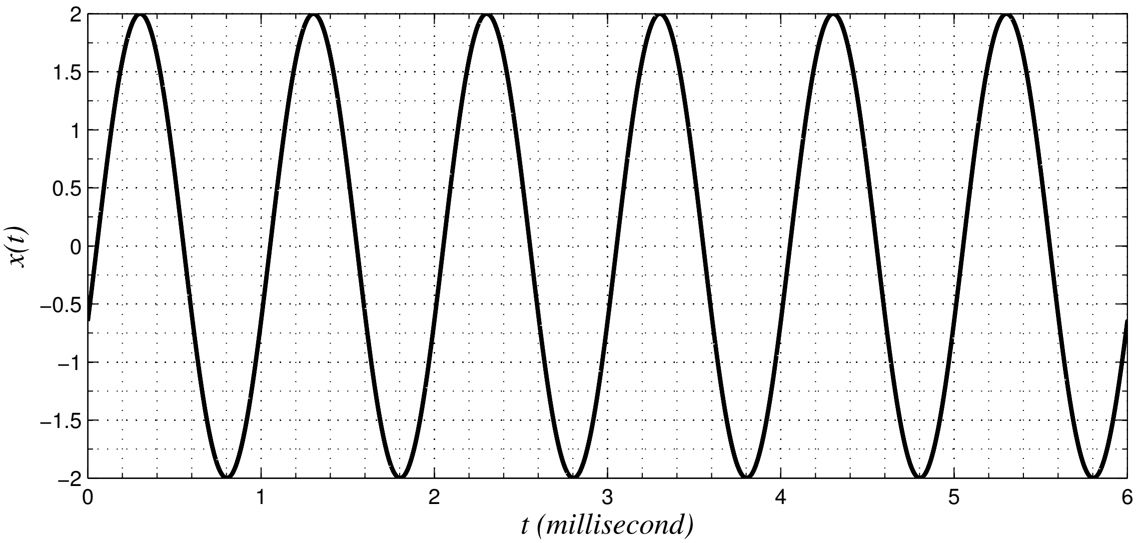
Prob. 3: ________ / 25

Prob. 4: ________ / 32

Total: ________ / 100

Some trigonometric identities: $\sin(2x) = 2\sin(x)\cos(x)$
$\cos(2x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
$\sin(x)\cos(y) = 0.5\sin(x + y) + 0.5\sin(x - y)$
$\cos(x)\cos(y) = 0.5\cos(x + y) + 0.5\cos(x - y)$
$\sin(x)\sin(y) = 0.5\cos(x - y) - 0.5\cos(x + y)$.

Problem 1 Consider analog signal $x(t)$ shown in the figure below.



Suppose that $x(t)$ is sampled with a sampling frequency of $(2500/3)$ samples/second for $t \in [0, 6]$ milliseconds. Then, each sample is passed through a 4-level uniform quantizer as defined below:

$$Q(x) = \begin{cases} \hat{x}_1, & \text{if } x \in [-2, -1) \\ \hat{x}_2, & \text{if } x \in [-1, 0) \\ \hat{x}_3, & \text{if } x \in [0, 1) \\ \hat{x}_4, & \text{if } x \in [1, 2] \end{cases}$$

where $\hat{x}_1 = -1.5$, $\hat{x}_2 = -0.5$, $\hat{x}_3 = 0.5$, and $\hat{x}_4 = 1.5$. After quantization, the encoder maps $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$, and $\hat{x}_4$ to 00, 01, 11, and 10, respectively.

a) List the outputs of the quantizer.

b) List the outputs of the encoder.

c) What is the required bandwidth for transmitting this signal?

Hint: For binary signaling, $R/2\,\text{Hz}$ of bandwidth is required for a data rate of R bits per second.

Problem 2 Suppose that a positive analog message $m(t)$ is inserted into a signal $x(t)$ as follows:

$x(t) = 2\,m(t)\sin(2\pi f_c t + \phi)$ where $m(t) > 0$ for all t.

Design a receiver to extract $m(t)$ by using local oscillator(s), multiplier(s), adders(s), square-root operator(s), and/or filter(s). Specify all the parameters at the receiver. Assume that the local oscillator(s) can generate any sinusoidal signal with frequency f_c and with any phase but the phase ϕ is not known at the receiver (i.e., parameter ϕ cannot be used in your receiver design). Show the final output of your receiver.

Problem 3 A source X generates outputs according to the following probability density function (PDF):

$$f_X(x) = \begin{cases} 0.5\, e^{-|x|}/(1 - e^{-4}), & \text{if } x \in [-4, 4] \\ 0, & \text{otherwise} \end{cases}.$$

This source is quantized by using the following 4-level quantizer:

$$Q(x) = \begin{cases} 2.5\,, & \text{if } x \in (1.5, 4] \\ 0.5\,, & \text{if } x \in (0, 1.5] \\ -0.5\,, & \text{if } x \in (-1.5, 0] \\ -2.5\,, & \text{if } x \in [-4, -1.5] \end{cases}$$

a) Determine the probability that $Q(X) > -1$. That is, calculate $P(Q(X) > -1)$.

b) Calculate the probability that the quantization error is larger than 1. That is, calculate $P(X - Q(X) > 1)$.

Problem 4 Consider random processes $X(t)$ and $Y(t)$ defined as $X(t) = 2A+3t$ and $Y(t) = 4B\cos(100\pi t+C)$, where A is a Gaussian random variable with mean 2 and variance 4, B is discrete random variable taking values of 1 and 2 with equal probabilities, and C is a continuous uniform random variable over $[0, 6\pi]$. Also, A, B, and C are independent.

a) Calculate the mean and autocorrelation function of $X(t)$. Is $X(t)$ wide-sense stationary (WSS)? Why or why not?

b) Calculate the mean and autocorrelation function of $Y(t)$. Is $Y(t)$ WSS? Why or why not?

c) Calculate the crosscorrelation function of $X(t)$ and $Y(t)$. Are $X(t)$ and $Y(t)$ jointly WSS? Why or why not?

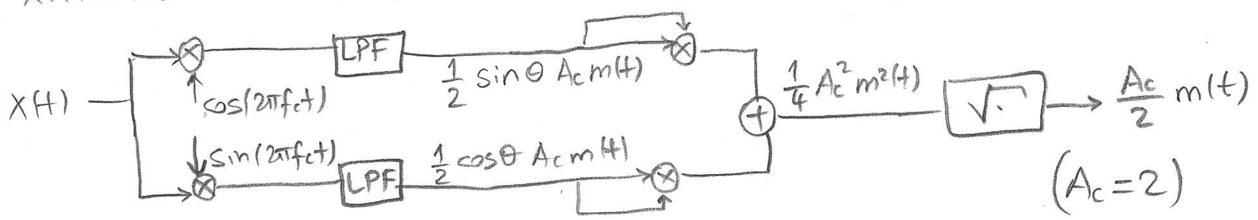
1) $T_s = \frac{3}{2500}$ sec $= \boxed{1.2 \text{ ms}}$

	t (ms)	0	1.2	2.4	3.6	4.8	6
a)	Quantizer	$\hat{x}_2$	$\hat{x}_4$	$\hat{x}_4$	$\hat{x}_2$	$\hat{x}_1$	$\hat{x}_2$
b)	Encoder	01	10	10	01	00	01

c) 2 bits/1.2 ms $= \frac{5000}{3}$ bits/sec $\longrightarrow \boxed{\frac{2500}{3} \text{ Hz}}$

2) $x(t) = A_c\, m(t) \sin(2\pi f_c t + \theta) \qquad m(t) > 0 \quad \forall t$

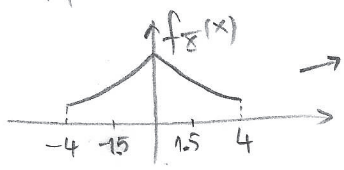
Similar to Q.2 of HW-4



$(A_c = 2)$

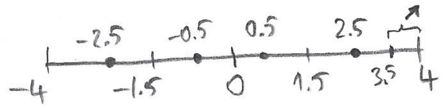
3) a) $P(Q(X) > -1) = P(X > -1.5) = 1 - P(X > 1.5) = 1 - \int_{1.5}^{4} \frac{0.5 e^{-x}}{1 - e^{-4}}\, dx$

by symmetry



$= \boxed{1 - \frac{0.5}{1 - e^{-4}} \left(e^{-1.5} - e^{-4}\right)}$

b) $P(X - Q(X) > 1) = P(X \in [3.5, 4]) = \int_{3.5}^{4} \frac{0.5 e^{-x}}{1 - e^{-4}}\, dx = \boxed{\frac{0.5}{1 - e^{-4}} \left(e^{-3.5} - e^{-4}\right)}$



4) a) $E[X(t)] = E[2A + 3t] = 2 \underbrace{E[A]}_{2} + 3t = \boxed{4 + 3t}$

$R_X(t_1, t_2) = E[X(t_1) X(t_2)] = E[(2A + 3t_1)(2A + 3t_2)] = 4 \underbrace{E[A^2]}_{8} + 6E[A] t_2 + 6E[A] t_1 + 9 t_1 t_2$

$= \boxed{32 + 12(t_1 + t_2) + 9 t_1 t_2}$

Not WSS

b) $E[Y(t)] = E[4B \cos(100\pi t + C)] \underset{B \,\&\, C \text{ indep.}}{=} 4E[B] \underbrace{E[\cos(100\pi t + C)]}_{0} = \boxed{0}$

$R_Y(t_1, t_2) = E[16 B^2 \cos(100\pi t_1 + C) \cos(100\pi t_2 + C)] = 8 \underbrace{E[B^2]}_{\frac{1}{2} 1^2 + \frac{1}{2} 2^2} \cos(100\pi(t_1 - t_2))$

WSS

$= \boxed{20 \cos(100\pi(t_1 - t_2))}$

c) $R_{XY}(t_1, t_2) = E[(2A + 3t_1)(4B \cos(100\pi t_2 + C))]$

$= \underbrace{E[2A + 3t_1]}_{4 + 3t} \underbrace{E[4B \cos(100\pi t_2 + C)]}_{0} = \boxed{0}$

Not jointly WSS since $X(t)$ is not WSS.