EEE 431: Telecommunications 1

Quiz 1

February 27, 2015, 18:00-19:00.

Instructors: Sinan Gezici & Ali Taha Koç

Name: _____

Section: _____

Signature: _____

Bilkent ID: _____

Last Name in Morse code (2 points): _____

Prob. 1: _____ / 20

Prob. 2: _____ / 15

Prob. 3: _____ / 30

Prob. 4: _____ / 35

Total: _____ / 102

Problem 1 Suppose that there exist 5 different cell phone companies, named as A, B, C, D, and E, which have 30%, 30%, 20%, 10%, and 10% market shares, respectively. The probabilities of a cell phone failure during the first two years are given by 0.1, 0.1, 0.3, 0.6, and 0.5 for companies A, B, C, D, and E, respectively.

(a) What is the probability that a cell phone does not fail during the first two years?

(b) If a cell phone fails during the first two years, what is the probability that it is made by company C ?

Problem 2 Consider a scalar random variable X, which is Gaussian distributed with mean -3 and variance 4. Express the probability that X is smaller than 2, that is, $P(X < 2)$, in terms of the Q function.
Hint: Q function is defined as $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-0.5t^2} dt$, which is equal to the probability that a zero-mean, unit-variance Gaussian random variable is larger than or equal to y).

Problem 3 Consider a discrete memoryless source (DMS) consisting of 5 symbols, $\{X, Y, Z, W, R\}$, in its alphabet with probabilities 0.125, 0.35, 0.2, 0.125, 0.2, respectively. Perform Huffman coding for this DMS and list the codewords. Calculate the average codeword length.

Problem 4 Consider two random variables X and Y, which are distributed according to the following joint probability density function (PDF):

$$f_{X,Y}(x,y) = \begin{cases} (2x+3y)/48 \ , & \text{if } 2 \le x \le 4 \text{ and } 1 \le y \le 3 \\ 0 \ , & \text{otherwise} \end{cases}$$

(a) Find the marginal PDF of X, and the marginal PDF of Y.

(b) Are X and Y independent? Why/why not?

(c) Find the conditional expectation of X given Y; that is, $\mathrm{E}\{X|Y\}$.