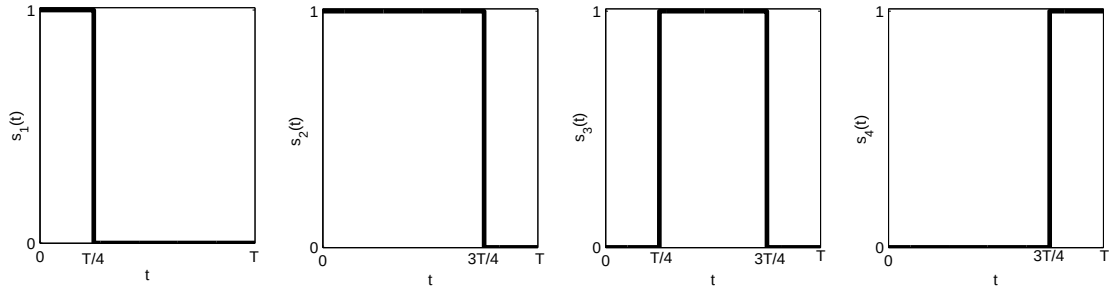
EEE-431 TELECOMMUNICATIONS I

Homework 5

1) Find a set of orthonormal basis functions for the following signals, and express each signal as a vector in the signal space defined by those orthonormal basis functions.



2) Consider the following signals that are defined for $t \in [0, 4]$.

$$\psi_1(t) = \begin{cases} 0.5\ , & t \in [0, 2] \\ -0.5\ , & t \in [2, 4] \end{cases}$$

$$\psi_2(t) = 0.5\ \ , \quad t \in [0, 4]$$

$$\psi_3(t) = \begin{cases} 0.5\ , & t \in [0, 1],\ \text{or, } t \in [2, 3] \\ -0.5\ , & t \in [1, 2],\ \text{or, } t \in [3, 4] \end{cases}$$

a) Show that $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$ are orthonormal.

b) Does the following signal reside in the three-dimensional space spanned by $\psi_1(t)$, $\psi_2(t)$ and $\psi_3(t)$? Prove your answer.

$$x(t) = \begin{cases} -1\ , & 0 \le t < 1 \\ 1\ , & 1 \le t < 3 \\ -1\ , & 3 \le t \le 4 \end{cases} .$$

3) Consider a 4-ary communications system that transmits the following signals for different messages:

$$s_1(t) = \begin{cases} 1\ , & t \in [0, T/2] \\ -1\ , & t \in (T/2, T] \end{cases} , \quad s_2(t) = \begin{cases} 1\ , & t \in [0, T/2] \\ 3\ , & t \in (T/2, T] \end{cases} ,$$

$$s_3(t) = \begin{cases} -3\ , & t \in [0, T/2] \\ 3\ , & t \in (T/2, T] \end{cases} , \quad s_4(t) = \begin{cases} -3\ , & t \in [0, T/2] \\ -1\ , & t \in (T/2, T] \end{cases} ,$$

where the signals are defined for $t \in [0, T]$ and are zero otherwise, with T denoting the symbol interval.

a) Find a set of orthonormal basis functions for the signals, and express each signal as a vector in the signal space defined by those orthonormal basis functions.

b) Assuming an additive white Gaussian noise (AWGN) channel and equally likely symbols, obtain the optimal receiver for this system. Please provide the mathematical expressions for the operations at the receiver.