Q1

HW#2

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Prove: $\mathcal{H}\{rect(\frac{r}{a})\} = a^2 jinc(qa)$

$$f(r) = rect(\frac{r}{a})$$

Then, $$F(q) = 2\pi \int_0^{\infty} r\, rect(\frac{r}{a})\, J_0(2\pi q r)\, dr$$

$$F(q) = 2\pi \int_0^{a/2} r\, J_0(2\pi q r)\, dr$$

$2\pi q r = u$

$dr = \frac{du}{2\pi q}$

$$= \frac{1}{q} \cdot \frac{1}{2\pi q} \int_0^{a\pi q} u\, J_0(u)\, du$$

Hint $$= \frac{1}{2\cancel{\pi} q^{\cancel{2}}} \cdot a\cancel{\pi}\cancel{q} J_1(a\pi q) = \frac{a}{2q} J_1(a\pi q)$$

$$= \frac{a^2}{2aq} J_1(a\pi q)$$

$$= \boxed{a^2 jinc(aq)}$$

Q2

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$$h_1(x,y) = e^{-\pi\left(\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2\right)} \quad , \quad h_2(x,y) = \text{sinc}(4x,y)$$

a) $MTF_1(u,v) = \dfrac{|H_1(u,v)|}{H_1(0,0)}$

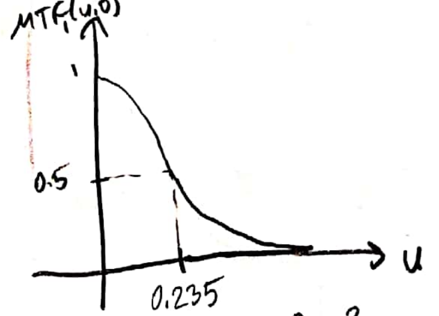
$$h_1(x,y) \xleftrightarrow{2D\ FT} H_1(u,v) = \frac{1}{\frac{1}{2}\cdot\frac{1}{2}} e^{-\pi((2u)^2+(3v)^2)} = 4e^{-\pi(4u^2+9v^2)}$$

$$H_1(0,0) = 4 \Rightarrow MTF_1(u,v) = \boxed{e^{-\pi(4u^2+9v^2)}}$$

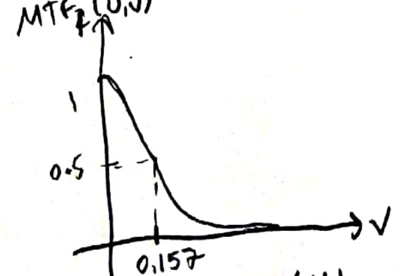
$$MTF_2(u,v) = \frac{|H_2(u,v)|}{H_2(0,0)}$$

$$h_2(x,y) \xleftrightarrow{2D\,FT} H_2(u,v) = \frac{1}{4}\,\text{rect}\left(\frac{u}{4}, v\right), \quad H_2(0,0) = \frac{1}{4} \Rightarrow MTF_2(u,v) = \boxed{\text{rect}\left(\frac{u}{4}, v\right)}$$

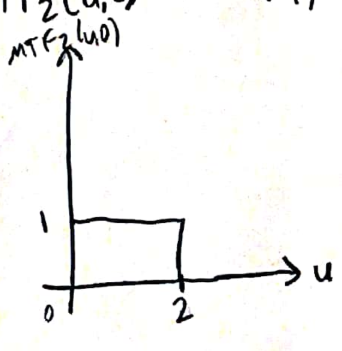
b) $MTF_1(u,0) = e^{-4\pi u^2} = \frac{1}{2} \Rightarrow u = \frac{1}{2}\sqrt{\frac{\ln 2}{\pi}} \approx 0.235$



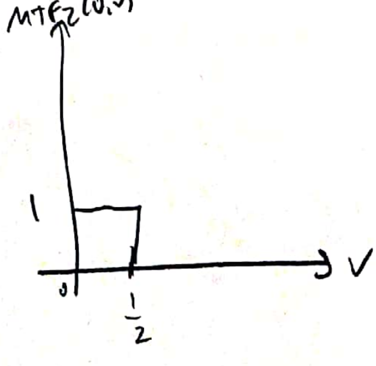
$MTF_2(0,v) = e^{-9\pi v^2} = \frac{1}{2} \Rightarrow v = \frac{1}{3}\sqrt{\frac{\ln 2}{\pi}} \approx 0.157$



$MTF_2(u,0) = \text{rect}\left(\frac{u}{4}\right)$



$MTF_2(0,v) = \text{rect}(v)$



c) $f(x,y) = 3 + 2\sin(2\pi(x+y))$

$u=1,\ v=1$

$\boxed{m_f = \frac{2}{3}}$

$m_{g_1} = MTF_1(1,1)\, m_f = \boxed{\frac{2e^{-13\pi}}{3}} \approx \boxed{1.222 \times 10^{-18}}$

$m_{g_2} = MTF(1,1)\, m_f = \boxed{\frac{2\,\mathrm{rect}(\frac{1}{4}, 1)}{3}} = \boxed{0}$

Q3

a) $e^{-\frac{\pi x^2}{4}} = \frac{1}{2} \Rightarrow x = 2\sqrt{\frac{\ln 2}{\pi}} \approx 0.939 \Rightarrow \boxed{FWHM_{1x} = 1.879}$

$e^{-\frac{\pi y^2}{9}} = \frac{1}{2} \Rightarrow y = 3\sqrt{\frac{\ln 2}{\pi}} \approx 1.409 \Rightarrow \boxed{FWHM_{1y} = 2.818}$

$\mathrm{sinc}(4x) = \frac{1}{2} \Rightarrow \frac{\sin(4\pi x)}{4\pi x} = \frac{1}{2} \Rightarrow x \approx 0.151 \Rightarrow \boxed{FWHM_{2x} = 0.302}$

$\mathrm{sinc}(y) = \frac{1}{2} \Rightarrow \frac{\sin(\pi y)}{\pi y} = \frac{1}{2} \Rightarrow y \approx 0.603 \Rightarrow \boxed{FWHM_{2y} = 1.206}$

b) Approximation:

$FWHM_x = \sqrt{FWHM_{1x}^2 + FWHM_{2x}^2} = \boxed{1.903}$

$FWHM_y = \sqrt{FWHM_{1y}^2 + FWHM_{2y}^2} = \boxed{3.065}$

Q4

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$\frac{\text{Dark}}{\text{Light}} > \text{Threshold} \Rightarrow \text{Diagnosis}$

a) Threshold = 1/10

$a+b+c+d = 8000$

		Disease +	Disease −
Test	+	183 (a)	72 (b)
	−	320 (c)	7425 (d)

Accuracy = 0.951

$$\text{Prevalence} = \frac{a+c}{a+b+c+d} = \frac{503}{8000} = 0.0629$$

$$\text{Sensitivity} = \frac{a}{a+c} = \frac{183}{503} = 0.3638$$

$$\text{Specificity} = \frac{d}{b+d} = \frac{7425}{7497} = 0.9904$$

$$\text{PPV} = \frac{a}{a+b} = \frac{183}{255} = 0.7176$$

$$\text{NPV} = \frac{d}{c+d} = \frac{7425}{7745} = 0.9587$$

b) Threshold = 1/8

		Disease +	Disease −
Test	+	143 (a)	39 (b)
	−	360 (c)	7458 (d)

Accuracy = 0.950

$$\text{Prevalence} = \frac{a+c}{a+b+c+d} = \frac{503}{8000} = 0.0629$$

$$\text{Sensitivity} = \frac{a}{a+c} = \frac{143}{503} = 0.2843$$

$$\text{Specificity} = \frac{d}{b+d} = \frac{7458}{7497} = 0.9948$$

$$\text{PPV} = \frac{a}{a+b} = \frac{143}{182} = 0.7857$$

$$\text{NPV} = \frac{d}{c+d} = \frac{7458}{7818} = 0.9539$$

Threshold is increased. Naturally, # of positives assigned by the ANN also dropped because $\frac{1}{10} < \frac{\text{Dark}}{\text{Light}} < \frac{1}{8}$ samples are now considered negative.

c) Second threshold is better in terms of PPV, so it is more selective when it determines (+). However, I believe that choosing a threshold depends on the application. If we were to use a test with high NPV, then we a test with high PPV, second threshold serves a better purpose.

Q5

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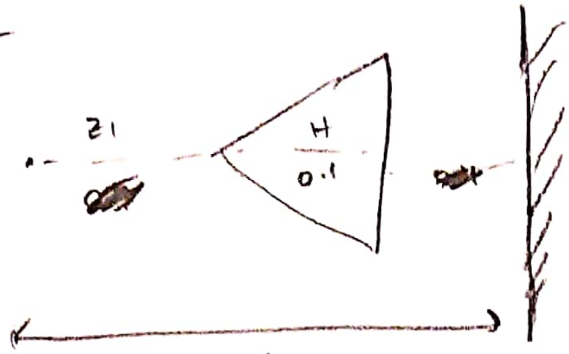

$\mu_0 = 0.1\,cm^{-1} = 10\,m^{-1}$

$H = 0.1\,m$

$\cos\theta = \frac{1}{\sqrt{1+x^2}}$

P.S. This comes from: $\text{rect}\left(\frac{x}{\frac{1}{5\sqrt{3}} + (z-0.6)\frac{2}{\sqrt{3}}}\right)$

a) $z_1 = 0.5m$

$$\mu(x,z) = \text{rect}\left(\frac{z-0.55}{0.1}\right)\underbrace{\text{rect}\left(\frac{x}{\frac{2z}{\sqrt{3}} - \frac{1}{\sqrt{3}}}\right)}_{\text{Triangular region}}$$

$$I_d(x,0) = I_0 \cos^3\theta \, e^{-\frac{1}{\cos\theta}\int_0^d \mu\left(\frac{x}{M}, z\right)dz}$$

$$= I_0\left(\frac{1}{1+x^2}\right)^{3/2} e^{-\frac{10m^{-1}}{\left(\frac{1}{1+x^2}\right)^{1/2}}\int_0^1 \text{rect}\left(\frac{z-0.55}{0.1}\right)\text{rect}\left(\frac{zx}{\frac{2z}{\sqrt{3}} - \frac{1}{\sqrt{3}}}\right)dz}$$

This holds for example one can find that $x > \frac{1}{6\sqrt{3}}$, there is zero attenuation due to μ_0, and for $x = \frac{1}{6\sqrt{3}}$, integral is equal to zero.

b) $z_1 = 0.8m$

$$\mu(x,z) = \text{rect}\left(\frac{z-0.85}{0.1}\right)\text{rect}\left(\frac{x}{\frac{2z}{\sqrt{3}} - \frac{16}{10\sqrt{3}}}\right)$$

$$I_d(x,0) = I_0 \cos^3\theta \, e^{-\frac{1}{\cos\theta}\int_0^d \mu\left(\frac{x}{M}, z\right)dz}$$

$$= I_0\left(\frac{1}{1+x^2}\right)^{3/2} e^{-\frac{10m^{-1}}{\left(\frac{1}{1+x^2}\right)^{1/2}}\int_0^1 \text{rect}\left(\frac{z-0.85}{0.1}\right)\text{rect}\left(\frac{zx}{\frac{2z}{\sqrt{3}} - \frac{16}{10\sqrt{3}}}\right)dz}$$

Similar to the previous one, $x > \frac{1}{3\sqrt{3}} \Rightarrow$ integral results in zero.