EEE 473/573 Medical Imaging – Fall 2020-2021
Homework 4
Due 27 December 2021, Monday at 23:59

GUIDELINES FOR HOMEWORK SUBMISSION

1. NO submission via E-MAIL (all email submissions will be discarded).
2. Submit a PDF file. Other file types will not be accepted. If there are any handwritten parts, you can scan them (make sure they are legible) and insert into the PDF file. Unclear presentation of results will be penalized heavily. No partial credits to unjustified answers.
3. For the part labeled as "MATLAB Question", you can choose to use MATLAB or other softwares (e.g., Python). Make sure to include the relevant codes at the end of the PDF file to be submitted. If your codes are missing, that question will NOT be graded.
4. This is a Turnitin submission. The Turnitin system requires the submitted file to contain at least 20 words in it. If you are submitting a Word file with scanned pages only, the file will be rejected by the system. You can type your name multiple times at the beginning of the file to overcome this problem.
5. Submission system will remain open for 1 day after the deadline. No points will be lost if you submit your assignment within 12 hours of the deadline. There will be a 50% penalty if you submit after 12 hours but within 24 hours past the deadline. No submissions beyond 24 hours past the deadline.

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1) A sample is in the equilibrium condition under a static magnetic field $\boldsymbol{B_0}$. A short RF pulse is used to excite the sample:

$$B_1(t) = B_1^e(t)e^{-j2\pi\nu_0 t},$$

The envelope of the RF pulse with duration 2T is given as, in units of Gauss:

$$B_1^e(t) = \begin{cases} t\dfrac{2}{10T}, & 0 \le t \le \dfrac{T}{2} \\ \dfrac{1}{10}, & \dfrac{T}{2} \le t \le \dfrac{3T}{2} \\ \dfrac{4}{10} - \dfrac{2}{10T}t, & \dfrac{3T}{2} \le t \le 2T \\ 0, & otherwise \end{cases}$$

a) Find the tip angle α as a function of time for $0 < t < 2T$.

b) What is the value of T to make $B_1(t)$ a $\frac{\pi}{2}$ pulse.

2) If $\boldsymbol{M}$ is initially in equilibrium, the components of $\boldsymbol{M}$ after an RF pulse with tip angle α applied at t=0 are as follows:

$$M_z(t) = M_0\left(1 - e^{-\frac{t}{T_1}}\right) + M_0\cos(\alpha)\, e^{-\frac{t}{T_1}}$$

$$M_{xy}(t) = M_0\sin(\alpha)\, e^{j\theta} e^{-\frac{t}{T_2}}$$

Suppose that the sample is excited with repeated RF pulses of tip angle α, separated by a repetition time T_R. Show that after a sufficient number of repetitions, a steady-state will be reached as follows:

$$M_{xy}(t) = M_z^{ss}\sin(\alpha)\, e^{j\theta} e^{-\frac{t}{T_2}}$$

where,

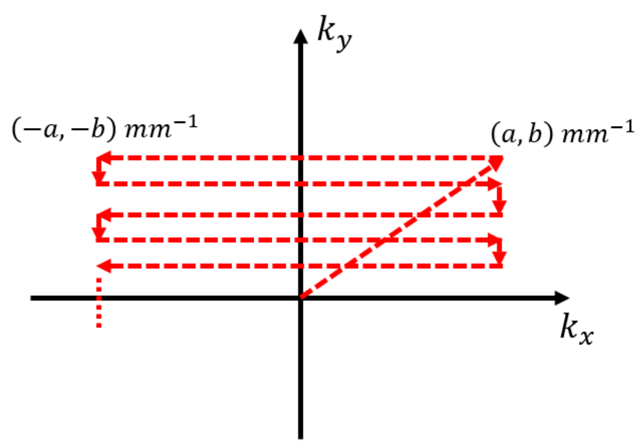
$$M_z^{ss} = M_0 \frac{1 - e^{-\frac{T_R}{T_1}}}{1 - \cos(\alpha)\, e^{-\frac{T_R}{T_1}}}$$

Here, M_z^{SS} is the steady-state value for $M_z(t)$ right before each RF pulse. Assume that the transverse magnetization has completely decayed before each RF pulse, i.e., $T_R \gg T_2$ and $M_{xy}(T_R) = 0$.

Hint: Relate M_z^n and M_z^{n+1}, i.e., the M_z values after n^{th} and $(n+1)^{th}$ pulses respectively.

3) Suppose we have a phantom containing water in an MRI system that uses $G_x = G_y = G_z = \pm 1$ G/cm . We want to image one slice of this phantom at $z = 10\ cm$ with slice thickness $\Delta z = 5$ mm.

- **a)** What should be the bandwidth of the RF waveform?
- **b)** Suppose that we want to image the slice at $z = 10\ cm$ with resolution $(\Delta x, \Delta y) = (1\ mm, 2\ mm)$. What should be the values for (a, b) for the first line of the multiple-line MRI trajectory given in the figure below.
- **c)** What should be the minimum number of lines to scan an image with FOV_y = 10 cm? Compute Δt_y for the y-gradient to move from one line to the next.
- **d)** What should be the sampling rate, f_s, of the ADC needed to scan an image with FOV_x = 10 cm?
- **e)** Considering the MRI trajectory given below, draw the timing diagram for the gradient waveforms and the ADC, assuming the starting point is the origin of k-space. Include the first 3 lines in your timing diagram.



4) MATLAB Question: Include your MATLAB codes in your solution.

T2 mapping: If you estimate the T_2 for every pixel in an MRI image and display this as an image, it is called a "T_2 map". So, every pixel in this "T_2 map" image corresponds to the estimated T_2 of the corresponding pixel in the MRI image.

Download the files *brainT2_mri.mat* and roiellipse.m posted on Moodle. The file *brainT2_mri.dat* contains two sets of simulated brain MRI images:

• *image1* and *image2*: Simulated T_2-weighted MRI images with TE = 60 ms and TE = 120 ms, respectively. For both images, = 90° and TR = 4000 ms. No noise added.

• *image1_noisy* and *image2_noisy*: Same as above, with a very small amount of Gaussian noise added to both images (noise is added in k-space, which is where the measurements are taken).

a) Derive an equation for how the T_2 map, $T_2(x, y)$ can be calculated from images at two different echo times: IMG_1 at TE_1 and IMG_2 at TE_2. You can assume the following for each image:

$$IMG_i(x, y) = A\, M_0(x, y) sin\alpha\; e^{-TE_i/T_2(x,y)}$$, where A is some constant.

b) Calculate and display the T_2 map for the noise-free dataset (i.e., image1 and image2), and the noisy dataset (i.e., image1_noisy and image2_noisy).
We recommend doing the following to restrict the range of T_2 values displayed to between 0 and 350 ms, and to make sure the estimated T_2 map is real-valued:

```
figure, imshow(abs(T2map),[0 350])
```

c) For the noise-free data set, estimate T_2 for white matter using roiellipse.m. Here is the MATLAB snippet that you can use to do that:

```
figure, imshow(T2map,[])

mask = roiellipse; % type "help roiellipse" to see how to use it

T2_est = mean(T2map(mask))
```

This code will create a figure displaying your T_2 map. It will then place a draggable and scalable ellipse on it. Drag this ellipse on a part of the image that contains ***white matter only*.** Select a reasonably large ellipse, so that the mean T_2 estimation is more accurate. We recommend maximizing the window containing the figure first, so that you can move/scale the ellipse more carefully. When you are ready, click the "Continue" button on the bottom left of the window. This will create a "mask" of the selected elliptical region, and then calculate the mean T_2 value in that region using the mask.
In your solution, include the image with ellipse showing the selected region, together with the estimated T_2 value.

d) Use the same "mask" in part (c) to estimate the T_2 for white matter for the noisy dataset. What is the estimated value? How much did it deviate (percentage-wise) from the noise-free dataset estimation?

e) Repeat parts (c-d) for gray matter. Include the image with ellipse in your solution, together with the estimated T_2 values.

f) Repeat parts (c-d) for CSF (cerebrospinal fluid). Include the image with ellipse in your solution, together with the estimated T_2 values.

g) Which noisy T_2 estimate showed the biggest deviation from its noise-free version? Why?

Hint: If you are not sure which part of the brain is gray matter, white matter, or CSF, here is a segmented version of the simulated MRI image: