**EEE 473/573 - Medical Imaging** **Quiz 2** **09/11/2021**

| Name | Student ID | Signature |
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**1) [40 pts]** Consider an LSI medical imaging system with PSF given by $h(x,y) = e^{-\pi(3x^2+2y^2)}$.

- **a.** Calculate the MTF associated with this system, $MTF(u,v)$.
- **b.** An object $f(x,y) = 4 + 3cos(4\pi x)$ is imaged with the system. What is the modulation of this object? What is the modulation of the image generated by the system?

a) $h(x,y) = e^{-\pi\left[(\sqrt{3}x)^2 + (\sqrt{2}y)^2\right]}$

$H(u,v) = \frac{1}{\sqrt{3}\cdot\sqrt{2}} \cdot e^{-\pi\left[\left(\frac{u}{\sqrt{3}}\right)^2 + \left(\frac{v}{\sqrt{2}}\right)^2\right]} = \frac{1}{\sqrt{6}} e^{-\pi\left(\frac{u^2}{3}+\frac{v^2}{2}\right)}$

$MTF(u,v) = \frac{|H(u,v)|}{H(0,0)} = e^{-\pi\left(\frac{u^2}{3}+\frac{v^2}{2}\right)}$

b) $m_f = \frac{3}{4}$ (modulation of the object)

$g(x,y) = f(x,y) * h(x,y) = (4 + 3\cos(4\pi x)) * h(x,y)$

$G(u,v) = \left[4\,\delta(u,v) + \frac{3}{2}\left(\delta(u-2,v) + \delta(u+2,v)\right)\right] \cdot H(u,v)$

$= 4 \cdot H(0,0) \cdot \delta(u,v) + \frac{3}{2}\left[H(2,0)\cdot\delta(u-2,v) + H(-2,0)\,\delta(u+2,v)\right]$

$= 4\,H(0,0)\cdot\delta(u,v) + H(2,0)\cdot\frac{3}{2}\left(\delta(u-2,v) + \delta(u+2,v)\right)$

$g(x,y) = 4H(0,0) + 3\,H(2,0)\cdot\cos(4\pi x)$

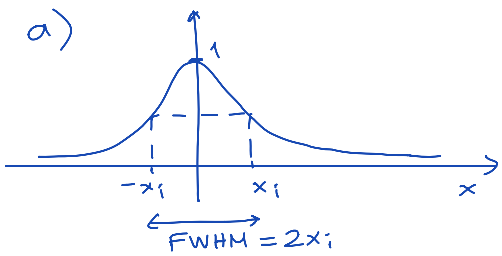
$m_g = \frac{3|H(2,0)|}{4\,H(0,0)} = \frac{3}{4}\,MTF(2,0) = \frac{3}{4}\,e^{-\pi\frac{4}{3}}$ (modulation of the image)

**2) [30 pts]** A 1D medical imaging system has two subsystems with the following PSFs:

$$h_1(x) = e^{-\pi x^2} \quad , \quad h_2(x) = e^{-3x^2}$$

- **a.** What is the FWHM associated with each subsystem?
- **b.** What is the FWHM associated with the cascaded system of these two subsystems. Is your result exact or approximate? Briefly explain why.

a)




Both $h_1(x)$ and $h_2(x)$ has max. val = 1.

$h_1(x_1) = \frac{1}{2} = e^{-\pi x_1^2}$

$x_1 = \sqrt{\frac{\ln 2}{\pi}}$

$FWHM_1 = 2x_1 = 2\sqrt{\frac{\ln 2}{\pi}}$

$h_2(x_i) = \frac{1}{2} = e^{-3x_i^2}$

$x_i = \sqrt{\frac{\ln 2}{3}}$

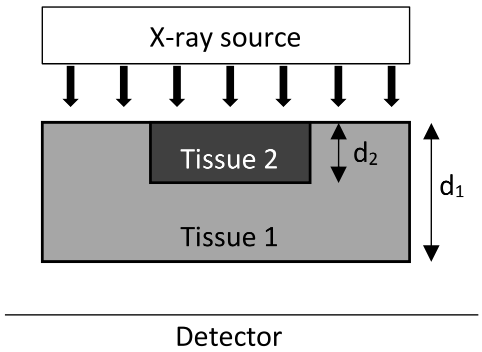
$FWHM_2 = 2x_2 = 2\sqrt{\frac{\ln 2}{3}}$

b) $FWHM = \sqrt{FWHM_1 + FWHM_2}$

$= \sqrt{4\cdot\frac{\ln 2}{\pi} + 4\,\frac{\ln 2}{3}} = 2\sqrt{\frac{(3+\pi)\ln 2}{3\pi}}$

This result is exact, since both $h_1(x)$ and $h_2(x)$ are Gaussian PSFs.

**3) [30 pts]** Suppose that we are imaging a body with a homogeneous tissue (Tissue 1), which contains a diseased tissue region (Tissue 2), as shown on the right. Assume narrow beam, parallel-ray geometry with monoenergetic x-ray photons.




Thicknesses and linear coefficients are:

- $d_1$=5 cm, $d_2$=2.5 cm
- $\mu_1$=0.2 cm$^{-1}$ for Tissue 1, $\mu_2$=0.4 cm$^{-1}$ for Tissue 2

Consider Tissue 1 as the background tissue. Calculate the local contrast for the diseased tissue in the x-ray image.

$$I_{background} = e^{-\mu_1 d_1} = e^{-0.2\times5} = e^{-1}$$

$$I_{target} = e^{-\mu_2 d_2} \cdot e^{-\mu_1(d_1-d_2)}$$

$$= e^{-0.4\times2.5} \cdot e^{-0.2\times2.5} = e^{-1.5}$$

* Then,

$$C = \frac{I_t - I_b}{I_b} = \frac{e^{-1.5} - e^{-1}}{e^{-1}} = \boxed{e^{-0.5} - 1}$$

negative contrast, since the target intensity is lower than the background.