Q1

HW #4

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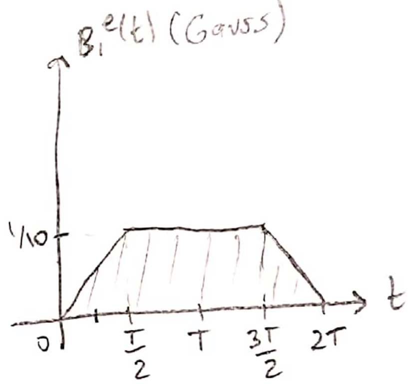
$$B_1^e(t) = \begin{cases} t\,\dfrac{2}{10T}, & 0 \le t \le \dfrac{T}{2} \\ \dfrac{1}{10}, & \dfrac{T}{2} \le t \le \dfrac{3T}{2} \\ \dfrac{4}{10} - \dfrac{2}{10T}t, & \dfrac{3T}{2} \le t \le 2T \\ 0, & \text{o.w.} \end{cases}$$

a)

$$\alpha = \gamma \int_0^{2T} B_1^e(t)\,dt$$



$$\alpha(t) = \begin{cases} \dfrac{\gamma t^2}{10T} \times 10^{-4}, & 0 \le t \le \dfrac{T}{2} \\ \gamma\left(\dfrac{T}{40} + \dfrac{\left(t - \frac{T}{2}\right)}{10}\right) \times 10^{-4}, & \dfrac{T}{2} < t \le \dfrac{3T}{2} \\ \gamma\left(\dfrac{T}{8} + \left(\dfrac{\left(\frac{1}{10} + \frac{4}{10} - \frac{2t}{10T}\right)\left(t - \frac{3T}{2}\right)}{2}\right)\right) \times 10^{-4}, & \dfrac{3T}{2} < t \le 2T \\ \gamma \dfrac{3T}{20} \times 10^{-4}, & 2T < t \\ 0, & \text{o.w.} \end{cases}$$

b)

$$\frac{\cancel{\pi}}{2} = 2\cancel{\pi}\,\gamma\,\frac{3T}{20} \times 10^{-4}$$

$$T = \frac{5}{3\gamma}10^4 = \frac{5}{3(42.58 \times 10^6)}10^4 \Rightarrow \boxed{T = 0.39142\text{ms}}$$

Q2

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$$M_z(t) = M_0(1-e^{-t/T_1}) + M_z(0^+)e^{-t/T_1}$$

$$M_{xy}(t) = M_0 \sin\alpha \, e^{j\theta} e^{-t/T_2} \rightarrow \text{Rotating frame}$$

$TR \gg T2$

M_z^{ss} is defined as the steady state value. So, if we try to calculate M_z after n repetitions, instead of M_0, we must use M_z^{ss} for the initial magnetization, s.t.

$$M_z^n(t) = M_0(1-e^{-t/T_1}) + M_z^{ss}\cos\alpha \, e^{-t/T_1} \quad (1)$$

Thus, $M_z^{n+1}(0) = M_z^n(TR) = M_z^{ss}$ (2) (Pulses are separated by TR)

So, if we plug (2) in (1):

$$M_z^n(TR) = M_0(1-e^{-\frac{TR}{T_1}}) + M_z^{ss}\cos\alpha \, e^{-\frac{TR}{T1}}$$

$$M_z^{ss}(1-\cos\alpha \, e^{-\frac{TR}{T_1}}) = M_0(1-e^{-\frac{TR}{T_1}})$$

$$\Rightarrow \boxed{M_z^{ss} = M_0 \frac{1-e^{-\frac{TR}{T_1}}}{1-\cos\alpha \, e^{-\frac{TR}{T_1}}}}$$

Since M_z^{ss} replaces M_0 for initial values,

$$M_{xy}(t) = M(0^+)\sin\alpha \, e^{j\theta} e^{-t/T_2}$$

$$\boxed{M_{xy}(t) = M_z^{ss}\sin\alpha \, e^{j\theta} e^{-t/T_2}}$$

Q3

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$G_x = G_y = G_z = \pm 1\ G/cm$

Desired: $z = 10cm$, $\Delta z = 5mm$

a) $\Delta v = \Delta z\, \gamma G_z = (5\times10^{-1}cm)\left(42.58\frac{MHz}{T}\right)\left(10^{-4}\frac{T}{cm}\right)$

$= \boxed{2.129\,kHz}$

b) $\Delta x = 1mm$, $\Delta y = 2mm$

$FWHM_x = \frac{1}{K_{x,extent}} = 1mm \Rightarrow K_{x,extent} = 1mm^{-1} \Rightarrow \boxed{a = 0.5mm^{-1}}$

$FWHM_y = \frac{1}{K_{y,extent}} = 2mm \Rightarrow K_{y,extent} = 0.5mm^{-1} \Rightarrow \boxed{b = 0.25mm^{-1}}$

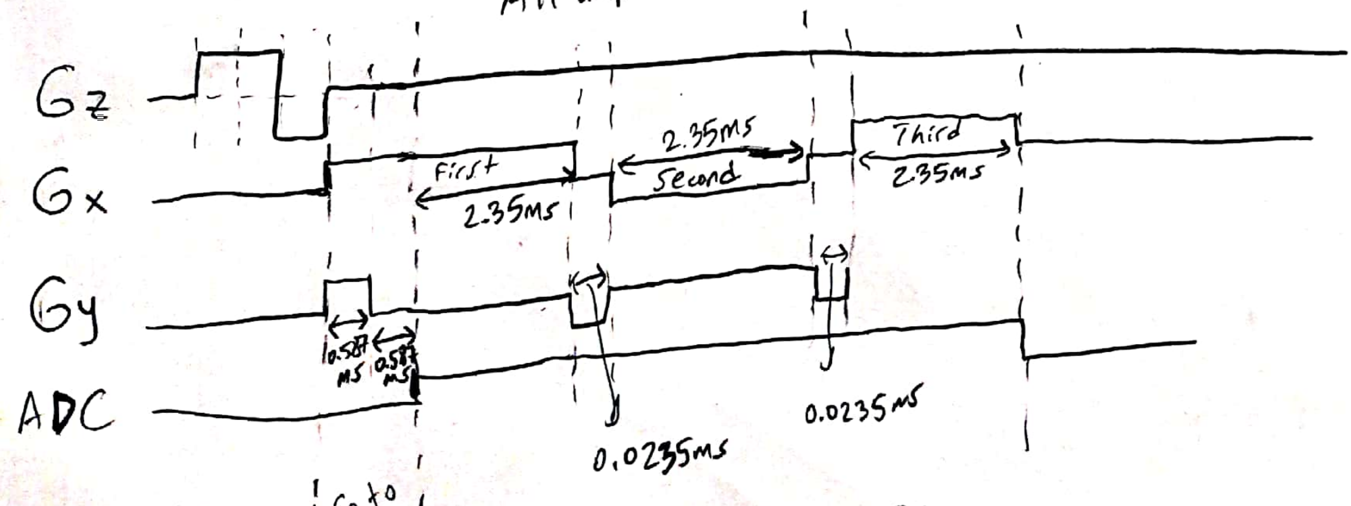
c) $FOV_y = 10cm \Rightarrow \boxed{\Delta k_y = 0.1cm^{-1} = 0.01mm^{-1}}$

$\frac{K_{y,extent}}{\Delta k_y} = \boxed{50 \text{ lines}}$

$t_y = \frac{\Delta k_y}{\gamma G_y} = \frac{10^{-1}}{(cm)\left(\frac{42.58\times10^6 Hz}{10^4 G}\right)\left(\frac{1G}{cm}\right)} = \boxed{0.0235ms}$

d) $f_s = \gamma G_x FOV_x = \frac{42.58\times10^6 Hz}{10^4 G}\cdot\frac{1G}{cm}\cdot 10cm = \boxed{42.58\,kHz}$

e) Assuming we do not read data during moving to (a,b).
All amplitudes are $\pm 1\ G/cm$.



Time to go to t_a: $\frac{5}{...} = 1.17ms$ ... $0.587ms$

Time to cover one line x: $\frac{\cancel{\ }}{} = 2\times1.17ms = 2.35ms$

Total ADC time: $(2.35ms)(3) + (0.0235ms)(2)$

$= \boxed{7.097ms}$

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Q4

a)
$$IMG_1(x,y) = AM_0(x,y)\sin\alpha \, e^{-\frac{TE_1}{T_2(x,y)}}$$
$$IMG_2(x,y) = AM_0(x,y)\sin\alpha \, e^{-\frac{TE_2}{T_2(x,y)}}$$

$$\frac{IMG_1(x,y)}{IMG_2(x,y)} = e^{-\left(\frac{TE_1 - TE_2}{T_2(x,y)}\right)}$$

$$\frac{-(TE_1 - TE_2)}{T_2(x,y)} = \ln(IMG_1(x,y)) - \ln(IMG_2(x,y))$$

$$\Rightarrow \boxed{T_2(x,y) = \frac{TE_1 - TE_2}{\ln(IMG_2(x,y)) - \ln(IMG_1(x,y))}}$$

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(b)

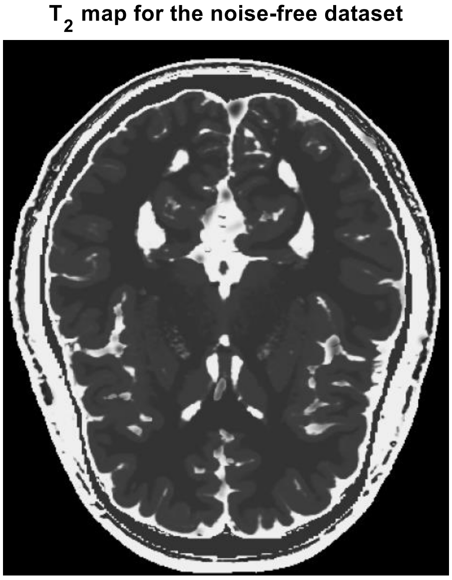

Figure 1.1.1 T_2 map for the noise-free dataset using the derived equation

(c)

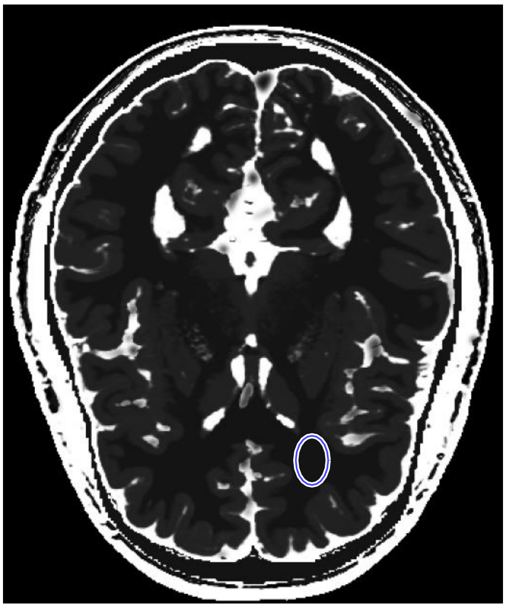

Figure 1.2.1 ROI ellipse for white matter, noise-free image. Estimated T_2 value for white matter is 70.05ms.

(d)

I used the same mask as in part (c), so there was no need to plot anything. Estimated T_2 value for white matter in the noisy T_2 map is 71.26ms. Percentage-wise deviation from the noise-free dataset is 1.74%.

(e)

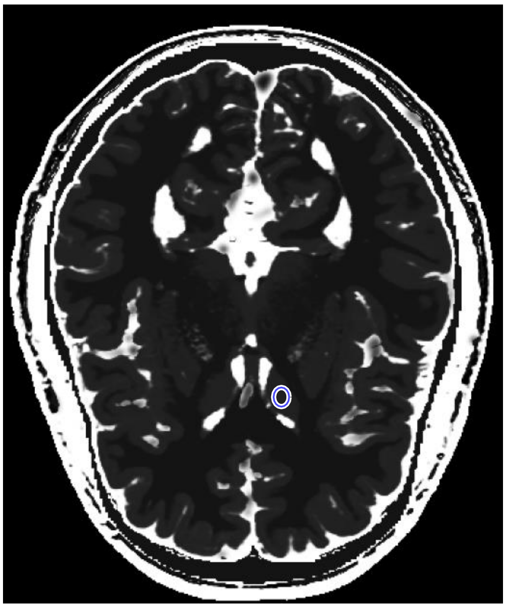

Figure 1.3.1 ROI ellipse for gray matter, noise-free image. Estimated T_2 value for gray matter is 83.97ms.

Estimated T_2 value for gray matter in the noisy T_2 map is 84.50ms. Percentage-wise deviation from the noise-free dataset is 0.63%.

(f)

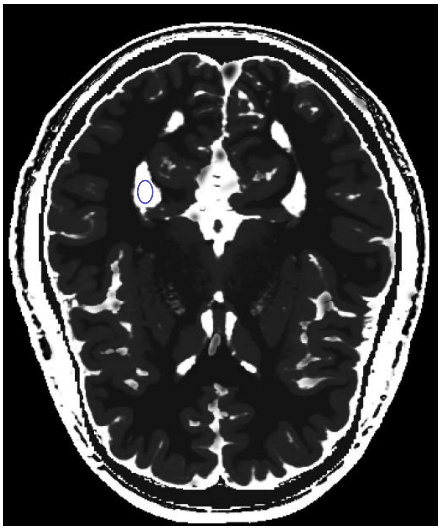

Figure 1.4.1 ROI ellipse for cerebrospinal fluid (CSF), noise-free image. Estimated T_2 value for CSF is 327.02ms.

Estimated T_2 value for CSF in the noisy T_2 map is 348.74ms. Percentage-wise deviation from the noise-free dataset is 6.64%.

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(g)

The largest deviation in T_2 estimation was seen in CSF. Between the three tissues, CSF has the T_2 value, so in T_2 weighted contrast images, CSF is the brightest between the tissues. Hence, even a small noise in the scan will be strengthened drastically.

Code

```
%Homework 4 - Q4
dataset = load("brainT2_mri.mat");
TE = dataset.TE;
image1 = dataset.image1;
image2 = dataset.image2;
TR = dataset.TR;
flip_degree = dataset.flip_degree;
image1_noisy = dataset.image1_noisy;
image2_noisy = dataset.image2_noisy;

%% b
T2map = (TE(1)-TE(2))./(log(image2)-log(image1));
figure;
imshow(abs(T2map),[0 350]);
title("T_2 map for the noise-free dataset");

%% c
figure;
imshow(T2map, []);
mask_wm = roiellipse;
T2_est_wm = mean(T2map(mask_wm));
%T2_est_wm = 70.0480 ms

%% d
T2map_noisy = (TE(1)-TE(2))./(log(image2_noisy)-
log(image1_noisy));
T2_est_wm_noisy = mean(T2map_noisy(mask_wm));
%T2_est_wm_noisy = 71.2636 ms

deviation_wm = (abs(T2_est_wm -
T2_est_wm_noisy)/T2_est_wm)*100;
%deviation_wm = 1.7355%

%% e
figure;
imshow(T2map, []);
mask_gm = roiellipse;
T2_est_gm = mean(T2map(mask_gm));
%T2_est_gm = 83.9685 ms
```

```
T2_est_gm_noisy = mean(T2map_noisy(mask_gm));
%T2_est_gm_noisy = 84.5014 ms

deviation_gm = (abs(T2_est_gm -
T2_est_gm_noisy)/T2_est_gm)*100;
%deviation_gm = 0.6347%

%% f
figure;
imshow(T2map, []);
mask_csf = roiellipse;
T2_est_csf = mean(T2map(mask_csf));
%T2_est_gm = 327.0154 ms

T2_est_csf_noisy = mean(T2map_noisy(mask_csf));
%T2_est_csf_noisy = 348.7403 ms

deviation_csf = (abs(T2_est_csf -
T2_est_csf_noisy)/T2_est_csf)*100;
%deviation_csf = 6.6434%
```