Q1 HW#3

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$f(x,y) \longrightarrow g(l,\theta)$

$f(x-x_0, y-y_0) \longrightarrow ?$

$h(x,y) = f(x-x_0, y-y_0)$ , $h(x,y) \longrightarrow k(l,\theta)$

$$R\{h(x,y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,\delta(x\cos\theta + y\sin\theta - l)\,dx\,dy$$

$$k(l,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-x_0, y-y_0)\,\delta(x\cos\theta + y\sin\theta - l)\,dx\,dy$$

$x' = x - x_0$
$y' = y - y_0$

$$k(l,\theta) = \int_{-\infty}^{\infty} f(x',y')\,\delta(x'\cos\theta + x_0\cos\theta + y'\sin\theta + y_0\sin\theta - l)\,dx'dy'$$

$$k(l,\theta) = \int_{-\infty}^{\infty} f(x',y')\,\delta\big(\underbrace{(l - x_0\cos\theta - y_0\sin\theta)}_{\text{new line argument}} - (x'\cos\theta + y'\sin\theta)\big)\,dx'dy'$$

$$\Rightarrow k(l,\theta) = \boxed{R\{f(x-x_0, y-y_0)\} = g(l - x_0\cos\theta - y_0\sin\theta, \theta)}$$

## Q2

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$$f(x,y) = e^{-\frac{x^2+y^2}{2}}$$

a) Let's find $F(u,v)$.

$$f(x,y) = e^{-\frac{x^2+y^2}{2}} = e^{-\pi\left(\left(\frac{x}{\sqrt{2\pi}}\right)^2 + \left(\frac{y}{\sqrt{2\pi}}\right)^2\right)} \xleftrightarrow{\text{2D FT}} F(u,v) = 2\pi e^{-2\pi^2(u^2+v^2)}$$

$$F(\rho) = 2\pi e^{-2\pi^2\rho^2}$$

No $\theta$ dependence

$$F(\rho\cos\theta, \rho\sin\theta) = F(u,v) \overset{\text{Projection slice}}{=} G(\rho,\theta) = 2\pi e^{-2\pi^2\rho^2} = 2\pi e^{-\pi(\sqrt{2\pi}\rho)^2}$$

$$\Big\updownarrow \text{1D FT}$$

$$\boxed{g(l,\theta) = \sqrt{2\pi}\, e^{-\frac{l^2}{2}}}$$

b) $$W(\rho) = e^{-\frac{\rho^2}{4}} = e^{-\pi\left(\frac{\rho}{2\sqrt{\pi}}\right)^2} \xleftrightarrow{\text{1D FT}} 2\sqrt{\pi}\, e^{-4\pi^2 l^2} = w(l)$$

$$\hat{f}(x,y) = f(x,y) * R^{-1}\{\tilde{h}(l)\}, \quad \tilde{h}(l) = w(l)$$

$$\Rightarrow h(r) = \mathcal{H}^{-1}(W(\rho))$$

$$\boxed{h(r) = 4\pi e^{-4\pi^2 r^2}}$$

$$\hat{f}(x,y) = e^{-\pi\left(\left(\frac{x}{\sqrt{2\pi}}\right)^2 + \left(\frac{y}{\sqrt{2\pi}}\right)^2\right)} * 4\pi e^{-\pi\left((2\sqrt{\pi}x)^2 + (2\sqrt{\pi}y)^2\right)}$$

$$\hat{F}(u,v) = 2\pi e^{-2\pi^2(u^2+v^2)} \cdot \cancel{4\pi} \cdot \frac{1}{\cancel{4\pi}} e^{-\frac{1}{4}(u^2+v^2)} = 2\pi e^{-(u^2+v^2)\underbrace{\left(2\pi^2+\frac{1}{4}\right)}_{a}}$$

$$= 2\pi e^{-\pi\left(\left(\sqrt{\frac{a}{\pi}}u\right)^2 + \left(\sqrt{\frac{a}{\pi}}v\right)^2\right)}$$

$$\Big\updownarrow \text{2D FT}$$

$$\hat{f}(x,y) = \frac{2\pi^2}{a} e^{-\frac{\pi^2}{a}(x^2+y^2)}$$

$$\boxed{\hat{f}(x,y) = \frac{2\pi^2}{2\pi^2+\frac{1}{4}} e^{-\frac{\pi^2}{2\pi^2+\frac{1}{4}}(x^2+y^2)}}$$

# Q3

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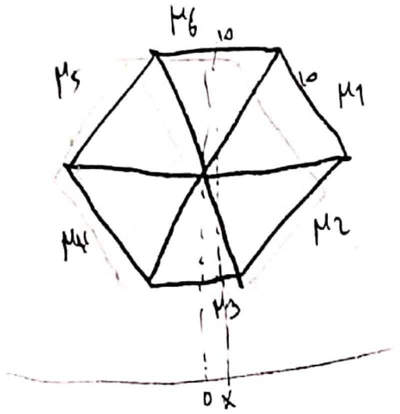
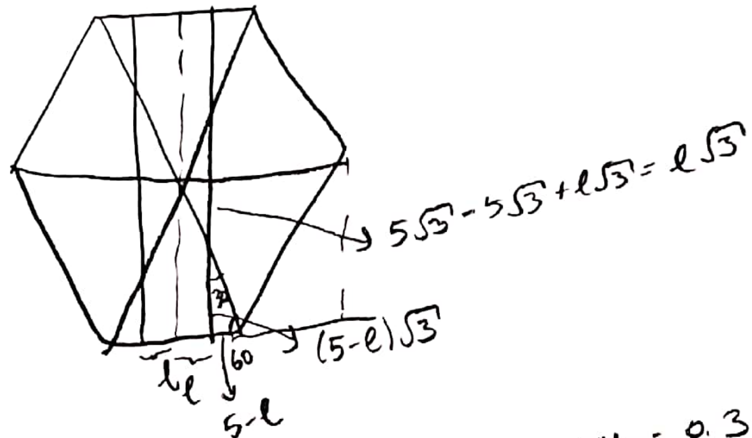
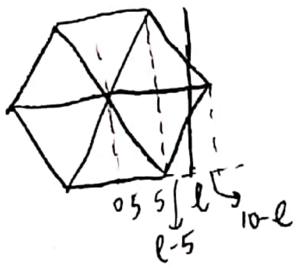


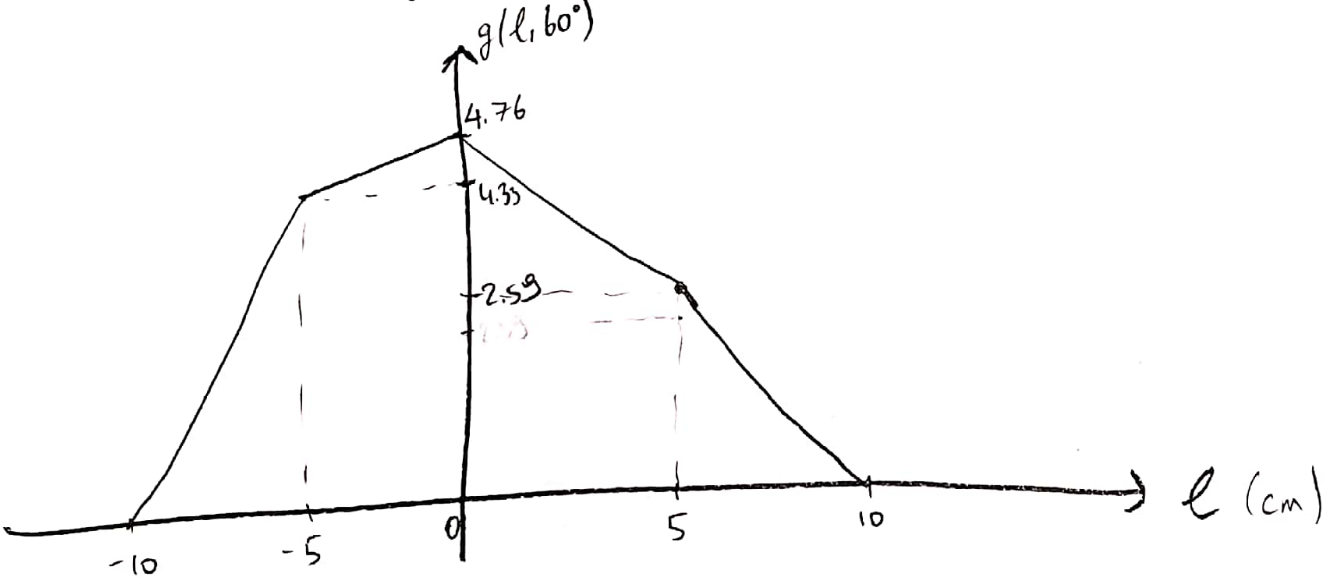




$\mu_1 + \mu_2 = 0.3$

$\mu_3 + \mu_6 = 0.55$

$\mu_4 + \mu_5 = 0.5$

$$g(\ell, 60^\circ) = \begin{cases} (5-\ell)\sqrt{3}(\mu_3+\mu_6) + \ell\sqrt{3}(\mu_1+\mu_2), & 0 \leq \ell < 5 \\ (10-\ell)\sqrt{3}(\mu_1+\mu_2), & 5 \leq \ell \leq 10 \\ (5+\ell)\sqrt{3}(\mu_3+\mu_6) - \ell\sqrt{3}(\mu_4+\mu_5), & -5 \leq \ell < 0 \\ (10+\ell)\sqrt{3}(\mu_4+\mu_5), & -10 \leq \ell < -5 \\ 0, & \text{o.w.} \end{cases}$$



Q4

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a) The smallest ~~...~~ FOV's ~~...~~ length width is 10cm.

So, we must at least use a detector array which is 20 cms long.

b) $N_{proj} \geq \frac{\pi}{2} N_{points} \Rightarrow \boxed{N_{proj\,min} = 403}$

$$\Delta X = \frac{FOV}{256} = \frac{20cm}{256} = \boxed{0.078cm}$$

Small pixel width ⇒ Better Resolution