EEE 473/573 - Medical Imaging
Quiz 3 – Friday, 25 December 2020
Duration: 30 minutes

Write your Name and Student ID at the top of every page.
Write the following statement on the cover page and sign below.

Honor Code: *"I have not given or received any aid during this quiz. I will do my share and take an active part in ensuring that others and I uphold the principles of honesty and integrity."*

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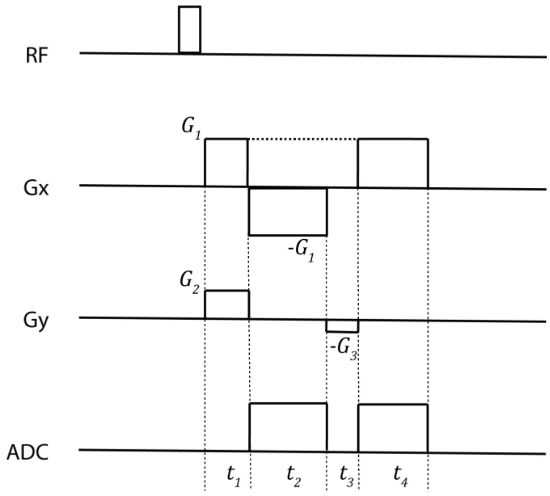
1) Consider the following spin echo MR sequence, with a repetition time TR. The RF pulses are applied along the x-axis. Assume that $M_z(0^-) = M_0$ and $M_{xy}(0^-) = 0$, where M_0 is the equilibrium magnetization. Assume that $\mathrm{TR} \gg \mathrm{T}_2$. Do NOT assume that $\mathrm{TR} \gg \mathrm{T}_1$.

- **a)** Find $M_z(0^+)$ and $M_{xy}(0^+)$.
- **b)** Find $M_z\left(\frac{TE^-}{2}\right)$ and $M_{xy}\left(\frac{TE^-}{2}\right)$.
- **c)** Find $M_z\left(\frac{TE^+}{2}\right)$ and $M_{xy}\left(\frac{TE^+}{2}\right)$.
- **d)** Find $M_{xy}(TE)$.
- **e)** Find $M_z(TR^-)$ and $M_{xy}(TR^-)$.

$\boldsymbol{M_z(0^-) = M_0}$	$\boldsymbol{M_{xy}(0^-) = 0}$
$\boldsymbol{M_z(0^+) = M_0 cos90^\circ = 0}$	$\boldsymbol{M_{xy}(0^+) = M_0 e^{j\frac{\pi}{2}}}$
$\boldsymbol{M_z\left(\frac{TE^-}{2}\right) = M_0\left(1 - e^{-TE/2T_1}\right)}$	$\boldsymbol{M_{xy}\left(\frac{TE^-}{2}\right) = M_0 e^{j\frac{\pi}{2}} e^{-TE/2T_2^*}}$
$\boldsymbol{M_z\left(\frac{TE^+}{2}\right) = -M_0\left(1 - e^{-TE/2T_1}\right)}$	$\boldsymbol{M_{xy}\left(\frac{TE^+}{2}\right) = M_0 e^{-j\frac{\pi}{2}} e^{-TE/2T_2^*}}$
	$\boldsymbol{M_{xy}(TE) = M_0 e^{-j\frac{\pi}{2}} e^{-TE/T_2}}$
$\boldsymbol{M_z(TR^-) = M_z\left(\frac{TE^+}{2}\right) e^{-\left(TR - \frac{TE}{2}\right)/T_1} + M_0\left(1 - e^{-\left(TR - \frac{TE}{2}\right)/T_1}\right)}$ $\boldsymbol{= M_0 - M_0 e^{-TR/T_1}\left(2e^{TE/2T_1} - 1\right)}$	$\boldsymbol{M_{xy}(TR^-) = M_{xy}\left(\frac{TE^+}{2}\right) e^{-(TR - TE/2)/T_2^*}}$ ≈ 0

2) Draw the k-space trajectory for the pulse sequence shown on the right. Assume the durations are: $t_1 = 2\text{ ms}$, $t_2 = 4\text{ ms}$, $t_3 = 1\text{ ms}$, and $t_4 = 4\text{ ms}$. The gradient amplitudes are: $G_1 = 10\text{ mT/m}$, $G_2 = 4\text{ mT/m}$, and $G_3 = 2\text{ mT/m}$.

Mark the data acquisition part of the trajectory (i.e., t_2 and t_4 intervals) with solid lines, and the other parts with dashed lines. Put arrows to mark the direction of the trajectory.



$$k_x = \bar{\gamma} \int G_x dt \quad \text{and} \quad k_y = \bar{\gamma} \int G_y dt$$

During t_1:

$k_{x,start} = 0, \quad k_{x,end} = 42.58x10^6\ G_1 t_1 \cong 0.85\ mm^{-1}$

$k_{y,start} = 0, \quad k_{y,end} = 42.58x10^6\ G_2 t_1 \cong 0.34\ mm^{-1}$

During t_2:

$k_{x,start} = 0.85\ mm^{-1}, \quad k_{x,end} = 0.85\ mm^{-1} - 42.58x10^6\ G_1 t_2 \cong -0.85\ mm^{-1}$

No change in k_y.

During t_3:

No change in k_x.

$k_{y,start} = 0.34\ mm^{-1}, \quad k_{y,end} = 0.34\ mm^{-1} - 42.58x10^6\ G_3 t_3 \cong 0.26\ mm^{-1}$

During t_4:

$k_{x,start} = -0.85\ mm^{-1}, \quad k_{x,end} = -0.85\ mm^{-1} + 42.58x10^6\ G_1 t_4 \cong 0.85\ mm^{-1}$

No change in k_y.