Name Lastname	
Student ID	
Signature	
Classroom #	EE-

Q1 (25 pts)	
Q2 (25 pts)	
Q3 (25 pts)	
Q4 (15 pts)	
Q5 (10 pts)	
TOTAL	

EEE 473/573 – Spring 2015-2016
FINAL EXAM
13 May 2016, 15:30-18:00

- Open book, open notes.
- Provide appropriate explanations in your solution and **show intermediate steps clearly. No credit will be given otherwise**.

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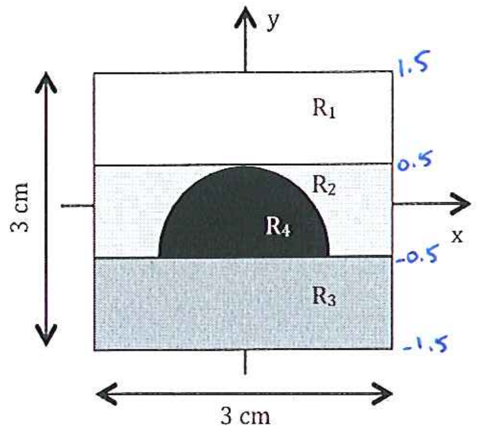
1) [25 points] Nuclear Medicine:
Consider the 2D object shown on the right.



- R_1, R_2, and R_3 are 3 cm x 1 cm rectangles.
- R_4 is a semi circle.
- The linear attenuation coefficient for each region is: $\mu_1 = 0.1\ \text{cm}^{-1}$, $\mu_2 = 0.2\ \text{cm}^{-1}$, $\mu_3 = 0.3\ \text{cm}^{-1}$, $\mu_4 = 0.4\ \text{cm}^{-1}$.
- Only R_2, R_3, and R_4 contain radionuclides. Their relative concentrations are $f_2 = 2$, $f_3 = 3$, $f_4 = 4$. (The absolute values/units are not important for this question).
- Assume perfect detection and ignore inverse square law.

a) [13 points] We image the radioactivity using a 2D SPECT scanner. What is the local contrast of the projection $g_{SPECT}(l, 0°)$? Let $g_{SPECT}(0,0°)$ be used as the intensity of the object of interest (i.e., the semi circle). When $\theta = 0°$, the camera is located on the +y-axis (above the object) looking down.

b) [12 points] Now assume the radionuclides in part (a) are replaced by positron emitting radionuclides with the same concentrations. We image the radioactivity using a 2D PET scanner. What is the local contrast of the projection $g_{PET}(l, 0°)$? Again, let $g_{PET}(0,0°)$ be used as the intensity of the object of interest.

Hint: For local contrast calculations, there is no need to fully calculate $g(l, 0°)$ for all l.

a) $$g_t = g_{SPECT}(0,0°) = \int_{-1.5}^{-0.5} f_3 \cdot \exp\left\{-\int_y^{-0.5}\mu_3\,dy' - 1\cdot\mu_4 - 1\cdot\mu_1\right\}dy + \int_{-0.5}^{0.5} f_4 \cdot \exp\left\{-\int_y^{0.5}\mu_4\,dy' - 1\cdot\mu_1\right\}dy$$

$$= 3\cdot\int_{-1.5}^{-0.5}\exp\{0.15 + 0.3y - 0.5\}\,dy + 4\cdot\int_{-0.5}^{0.5}\exp\{-0.2 + 0.4y - 0.1\}\,dy$$

$$= 3\cdot e^{-0.35}\cdot\frac{1}{0.3}e^{0.3y}\Big|_{-1.5}^{-0.5} + 4\cdot e^{-0.3}\cdot\frac{1}{0.4}e^{0.4y}\Big|_{-0.5}^{0.5}$$

$$= 10\cdot e^{-0.35}\cdot\left(e^{-0.15} - e^{-0.45}\right) + 10\cdot e^{-0.3}\cdot\left(e^{0.2} - e^{-0.2}\right)$$

$$= \boxed{4.555}$$

Background is where $|\ell| > 1$ (no contribution from the semicircle).

$$g_b = g_{SPECT}(|\ell|>1, 0^\circ) = \int_{-1.5}^{-0.5} f_3 . \exp\left\{-\int_y^{-0.5} \mu_3 dy' - 1.\mu_2 - 1.\mu_1\right\} dy + \int_{-0.5}^{0.5} f_2 . \exp\left\{-\int_y^{0.5} \mu_2 dy' - 1.\mu_1\right\} dy$$

$$= 3. \int_{-1.5}^{-0.5} \exp\{0.15 + 0.3y - 0.3\} dy + 2. \int_{-0.5}^{0.5} \exp(-0.1 + 0.2y - 0.1) dy$$

$$= 3. e^{-0.15} . \frac{1}{0.3} e^{0.3y} \Big|_{-1.5}^{-0.5} + 2. e^{-0.2} . \frac{1}{0.2} e^{0.2y} \Big|_{-0.5}^{0.5}$$

$$= 10. e^{-0.15} . (e^{-0.15} - e^{-0.45}) + 10. e^{-0.2} . (e^{0.1} - e^{-0.1}) = \boxed{3.56}$$

Contrast is: $C = \dfrac{g_t - g_b}{g_b} = \dfrac{4.555 - 3.56}{3.56} \approx \boxed{0.28}$

b) In PET, attenuation is the same for every point source along the line of integration.

$$g_t = g_{PET}(0, 0^\circ) = \int_{-1.5}^{-0.5} f_3 . \exp(-1.\mu_3 - 1.\mu_4 - 1.\mu_1) dy + \int_{-0.5}^{0.5} f_4 . \exp(-1.\mu_3 - 1.\mu_4 - 1.\mu_1) dy$$

$$= 3.1. e^{-0.8} + 4.1. e^{-0.8} = 7. e^{-0.8} = \boxed{3.145}$$

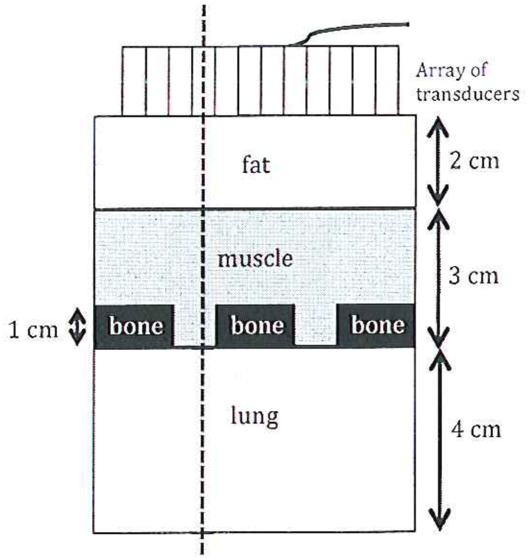
Again, background is where $|\ell| > 1$.

$$g_b = g_{PET}(|\ell|>1, 0^\circ) = \int_{-1.5}^{-0.5} f_3 . \exp(-1.\mu_3 - 1.\mu_2 - 1.\mu_1) dy + \int_{-0.5}^{0.5} f_2 . \exp(-1.\mu_3 - 1.\mu_2 - 1.\mu_1) dy$$

$$= 3. e^{-0.6} + 2. e^{-0.6} = 5. e^{-0.6} = \boxed{2.744}$$

Contrast is: $C = \dfrac{g_t - g_b}{g_b} = \dfrac{3.145 - 2.744}{2.744} \approx \boxed{0.15}$

2) [25 points] Ultrasound:
Assume that we have a 5 MHz ultrasound system as shown on the right, and we would like to image the lung interface with this system. Our system features an array of transducers, where each transducer transmits/receives to/from tissues directly along the line of its axis. The dashed line in the schematic demonstrates the line of axis for one of the transducers.



Tissue parameters are given as:

- $\alpha_{fat} = 0.63$ dB cm^{-1}MHz^{-1}, $Z_{fat} = 1.35\times10^{-6}$ kg m^{-2}s^{-1}
- $\alpha_{muscle} = 1$ dB cm^{-1}MHz^{-1}, $Z_{muscle} = 1.7\times10^{-6}$ kg m^{-2}s^{-1}
- $\alpha_{bone} = 20$ dB cm^{-1}MHz^{-1}, $Z_{bone} = 5\times10^{-6}$ kg m^{-2}s^{-1}
- $\alpha_{lung} = 40$ dB cm^{-1}MHz^{-1}, $Z_{lung} = 0.26\times10^{-6}$ kg m^{-2}s^{-1}

a) [10 points] For the transducers that do not "see" bone, what is the loss in dB for the ultrasound wave returning from the muscle/lung interface?

b) [10 points] For the transducers that "see" bone, what is the loss in dB for the ultrasound wave returning from the bone/lung interface?

c) [5 points] Based on these numbers, comment on how the presence of bone affects the image in ultrasound.

$\alpha_{fat} = 0.63 \times 5 = 3.15$ dB/cm , $\alpha_{muscle} = 5$ dB/cm , $\alpha_{bone} = 20\times5 = 100$ dB/cm

a) Attenuation loss : $2 \times (2cm \times \alpha_{fat} + 3cm \times \alpha_{muscle}) = 2\times(2\times3.15 + 3\times5)$
round-trip
$= \boxed{42.6 \text{ dB loss}}$

Transmission from fat/muskle interface : $T_{fm} = \dfrac{4Z_f \cdot Z_m}{(Z_f+Z_m)^2} = \dfrac{4\times1.35\times1.7}{(1.35+1.7)^2} = 0.9868$

Reflection from muscle/lung interface : $R_{mL} = \dfrac{(Z_m - Z_l)^2}{(Z_m+Z_l)^2} = \left(\dfrac{1.7-0.26}{1.7+0.26}\right)^2 = 0.5398$

* Ultrasound wave will be transmitted from fat/muscle interface, reflected from muscle/lung interface, then transmitted from muscle/fat interface.

Total loss due to transmission/reflection : $10 \cdot \log_{10}(T_{fm} \cdot R_{ml} \cdot T_{fm}) = -2.79$ dB
because we calculated power
$\boxed{2.79 \text{ dB loss}}$

So, total loss = loss due to attenuation (in dB) + loss due to transmission/reflection (in dB)

$= 42.6 + 2.79 = \boxed{45.39 \text{ dB loss}}$

b) Attenuation loss : $2\times(2\times\alpha_{fat} + 2\times\alpha_{muscle} + 1\times\alpha_{bone})$

$= 2\times(2\times3.15 + 2\times5 + 1\times100) = \boxed{232.6 \text{ dB loss}}$

Transmission from muscle/bone interface : $T_{mb} = \frac{4Z_m Z_b}{(Z_m+Z_b)^2} = \frac{4\times1.7\times5}{(1.7+5)^2} = 0.7574$

Reflection from bone/lung interface : $R_{bl} = \left(\frac{Z_b - Z_l}{Z_b + Z_l}\right)^2 = \left(\frac{5-0.26}{5+0.26}\right)^2 = 0.8121$

* ultrasound wave will be transmitted from fat to muscle, then from muscle to bone, then reflected from bone/lung interface, then transmitted from bone to muscle, and then from muscle to fat.

Total loss due to transmissions/reflections : $10.\log_{10}(T_{fm}.T_{mb}.R_{bl}.T_{mb}.T_{fm}) = -3.43 \text{ dB}$

$\boxed{3.43 \text{ dB loss}}$

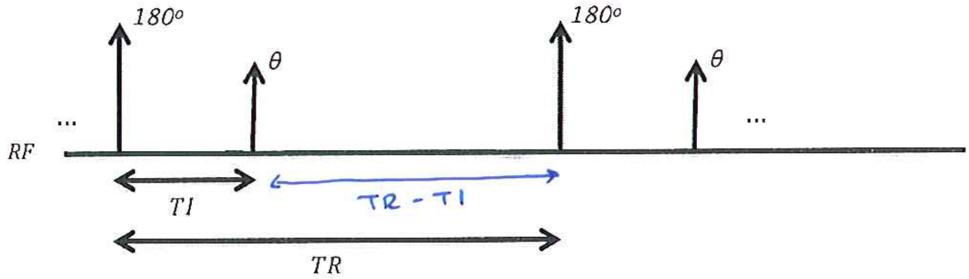
So, total loss $= 232.6 \text{ dB} + 3.43 \text{ dB} = \boxed{236.03 \text{ dB loss}}$

c) The presence of bone causes an extreme attenuation of the ultrasound wave. If we are trying to image any tissues that are located behind bone, the signal from those tissues will be VERY low (most probably below the detection limit). So, we will not be able to image behind the bone.

3) [25 points] A Fat Suppression Technique in MRI:

Consider the following MR sequence in which a 180° RF pulse is applied, followed by a θ degree RF excitation. After a total time of TR, the pulses are repeated. This is called an "inversion recovery sequence". Assume the following conditions:

- The magnetization has equilibrium value M_0.
- $TR \gg T_2$ for all tissues of interest.
- After numerous TRs, M_z magnetization right before the θ degree RF excitation will reach a steady-state value of M_z^{SS}.
- The MR signal is acquired right after θ RF pulse.
- For fat: T_1 = 290 ms and T_2 = 165 ms.
- For muscle: T_1 = 1130 ms and T_2 = 35 ms.



a) [10 points] Show that the "steady-state" magnetization M_z^{SS} is:

$$M_z^{SS} = M_0 \frac{1 - 2e^{-\frac{TI}{T_1}} + e^{-\frac{TR}{T_1}}}{1 + \cos\theta \, e^{-\frac{TR}{T_1}}}$$

b) [8 points] Normally, fat appears very bright in MRI images due to its relatively short T_1 and long T_2. We want to use the inversion recovery sequence to suppress the signal from fat while maximizing the signal from muscle. If TR = 1500 ms, find the optimum θ that maximizes the steady-state MR signal from muscle.

c) [7 points] For the TR and θ in part (b), what is the TI value that guarantees that the steady-state MR signal from fat is always zero?

a) Start from $t = TI^-$, and go one TR. we should get the same magnetization value in steady-state.

$$M_z(TI^-) = M_z^{ss}$$

$$M_z(TI^+) = M_z(TI^-).\cos\theta = M_z^{ss}.\cos\theta$$

$$M_z(TR^-) = M_0(1 - e^{-(TR-TI)/T_1}) + M_z(TI^+).e^{-(TR-TI)/T_1}$$

$$= M_0(1 - e^{-(TR-TI)/T_1}) + M_z^{ss}.\cos\theta.e^{-(TR-TI)/T_1}$$

$$M_z(TR^+) = M_z(TR^-).\cos(180°) = -M_z(TR^-)$$

$$M_z(TR+TI^-) = M_0(1 - e^{-TI/T_1}) + M_z(TR^+).e^{-TI/T_1}$$

$$= M_0(1 - e^{-TI/T_1}) - M_z(TR^-).e^{-TI/T_1}$$

in steady state, $M_z(TR + TI^-) = M_z(TI^-) = M_z^{ss}$. So,

$$M_z^{ss} = M_0(1 - e^{-TI/T_1}) - \left[M_0(1 - e^{-(TR-TI)/T_1}) + M_z^{ss}.\cos\theta.e^{-(TR-TI)/T_1}\right].e^{-TI/T_1}$$

$$M_z^{ss} = M_0(1 - e^{-TI/T_1}) - M_0(e^{-TI/T_1} - e^{-TR/T_1}) - M_z^{ss}.\cos\theta.e^{-TR/T_1}$$

$$\boxed{M_z^{ss} = M_0 . \frac{1 - 2e^{-TI/T_1} + e^{-TR/T_1}}{1 + \cos\theta.e^{-TR/T_1}}}.$$

b) MRI signal is proportional to M_{xy}. during data acquisition.

Right after θ RF pulse:

$$M_{xy} = M_z^{ss} \cdot \sin\theta = \underbrace{M_0 \cdot \left(1 - 2e^{-TI/T_1} + e^{-TR/T_1}\right)}_{\text{does not depend on } \theta} \cdot \frac{\sin\theta}{1 + \cos\theta \cdot e^{-TR/T_1}}$$

calling this A

To maximize MR signal from muscle; maximize M_{xy}

$$\frac{d M_{xy}}{d\theta} = A \cdot \frac{d}{d\theta}\left(\frac{\sin\theta}{1 + \cos\theta \cdot e^{-TR/T_1}}\right) = 0$$

$$= A \cdot \left(\frac{\cos\theta\,(1 + \cos\theta\, e^{-TR/T_1}) + \sin\theta \cdot \sin\theta\, e^{-TR/T_1}}{(1 + \cos\theta \cdot e^{-TR/T_1})^2}\right) = 0$$

Numerator should be zero:

$$\cos\theta + (\cos^2\theta + \sin^2\theta)\, e^{-TR/T_1} = 0$$

$$\cos\theta = -e^{-TR/T_1} \Rightarrow \boxed{\theta = \cos^{-1}\left(-e^{-TR/T_1}\right)}$$

for $TR = 1500$ ms, $T_1 = 1130$ ms, $\theta = \cos^{-1}\left(e^{-1500/1130}\right) \approx \boxed{105.38^\circ}$

c) We want M_{xy} for fat to be equal to zero.

$$M_{xy} = M_0 \underbrace{\left(1 - 2e^{-TI/T_1} + e^{-TR/T_1}\right)}_{\text{should be zero}} \cdot \frac{\sin\theta}{1 + \cos\theta \cdot e^{-TR/T_1}}$$

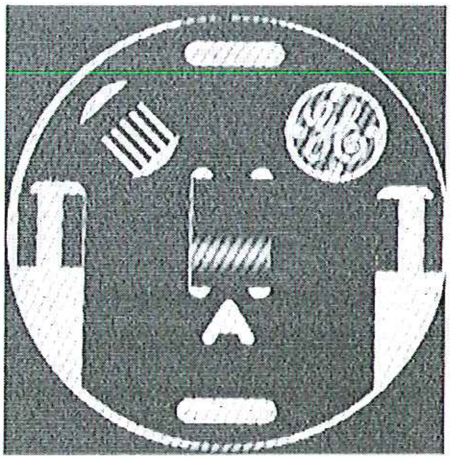
$$2e^{-TI/T_1} = 1 + e^{-TR/T_1}$$

$$\boxed{TI = T_1 \cdot \log\left(\frac{2}{1 + e^{-TR/T_1}}\right)}$$

for $TR = 1500$ ms, $T_1 = 290$ ms, $\Rightarrow \boxed{TI \simeq 199.4 \text{ ms}}$

4) [15 points] White Pixel Artifact in MRI:

Assume that we collect the k-space data in a typical line-by-line fashion. While collecting the data, a malfunction in the MRI scanner causes an additive "spike" signal to be recorded at k-space position (u_0,v_0). The amplitude of this spike is <u>MUCH higher</u> than the actual signal coming from the object that we are imaging, so we can model it as an additive delta at (u_0,v_0), i.e., $A\,\delta(u-u_0, v-v_0)$ where A is a constant.



The final displayed MRI image is the <u>magnitude</u> of the reconstructed image and it looks as shown on the right.

Derive an expression for the resulting image, $|g(x,y)|$. Express it in terms of the ideal image, $f(x,y)$. Assume that the ideal image $f(x,y)$ is real valued.

With the added spike, k-space data can be expressed as:

$$G(u,v) = \underbrace{F(u,v)}_{\text{ideal k-space data}} + A \cdot \delta(u-u_0, v-v_0)$$

then,

$$g(x,y) = f(x,y) + A \cdot e^{j2\pi(u_0x+v_0y)}$$

$$|g(x,y)| = |\underbrace{f(x,y)}_{\text{given as real-valued}} + A \cdot \cos(2\pi(u_0x+v_0y)) + jA \cdot \sin(2\pi(u_0x+v_0y))|$$

$$= \sqrt{\underbrace{(f(x,y) + A\cdot\cos(2\pi(u_0x+v_0y)))^2}_{\text{real part}} + \underbrace{(A\cdot\sin(2\pi(u_0x+v_0y)))^2}_{\text{imaginary part}}}$$

$$\boxed{|g(x,y)| = \sqrt{f^2(x,y) + 2Af(x,y)\cdot\cos(2\pi(u_0x+v_0y)) + A^2}}$$

Perfectly fits the artifact in the displayed image. It is as if we have a cosine added to the image. But, note that the background (black regions) do not have any cosine component. This is because $f(x,y) \approx 0$ there, so cosine component diminishes there.

5) [10 points] Conjugate Symmetry for MRI?

Given fact: If $f(x,y)$ is a real-valued function, its 2D Fourier Transform has conjugate symmetry, i.e., $F(u,v) = F^*(-u,-v)$.

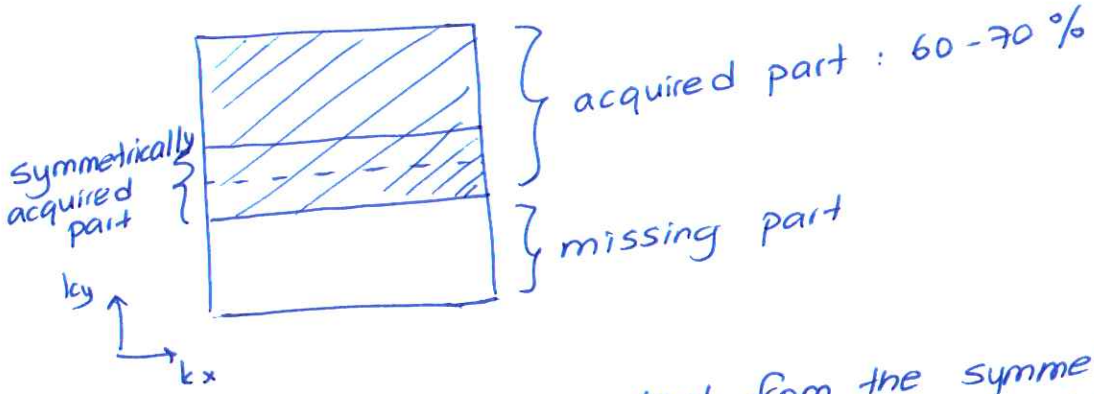
Porof. Zihni Sinir claims the following:

"MRI images are real valued, so they have conjugate symmetry in Fourier domain. In MRI, we are collecting the data in k-space (i.e., Fourier domain), typically line-by-line. So, we can take advantage of conjugate symmetry and acquire the data for only 50% of the k-space. This way, we can reduce the scan time by a factor of 2."

Explain his claim. Do you agree? If you think he is right, explain to what extent his claim is right. If you think he is wrong, explain to what extent his claim is wrong.

He is mostly correct. This is a technique called "partial Fourier" or "half Fourier" acquisition. The only problem is that the MRI images are NOT real-valued in reality. Field inhomogeneities, susceptibility, chemical shift all causes additional (position dependent) phase in the MRI image. Luckily, this phase is slowly varying.

So, 50% of k-space is NOT sufficient, but 60-70% is typically sufficient



Phase is estimated from the symmetrically sampled part and the missing part is filled in (typically iteratively) using conjugate symmetry.

Drawback : SNR is proportional to $\sqrt{N}$ in MRI.
↑ # of lines

so, this technique will result in a lower SNR image when compared to a fully sampled image.

Note : You were not expected to provide all this information to get full points.