

Name Lastname	
Student ID	
Signature	
Classroom #	EE-

Q1 (25 pts)	
Q2 (25 pts)	
Q3 (25 pts)	
Q4 (25 pts)	
TOTAL	

**EEE 473/573 – Spring 2014-2015**  
**FINAL EXAM**

24 May 2015, 9:00-11:00

- Open book, open notes.
  - Provide appropriate explanations in your solution and **show intermediate steps clearly.**  
**No credit will be given otherwise.**
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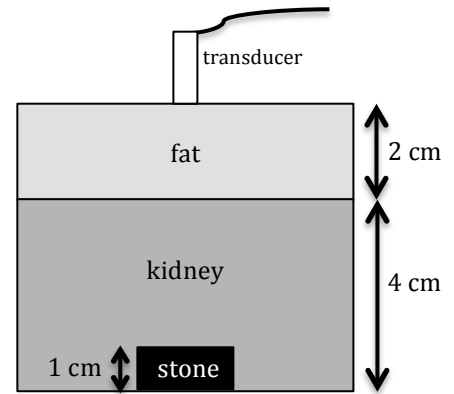
1) [25 points] The transfer function of a 2D LSI imaging system is given by  $H(u, v) = 1 + e^{-j2\pi x_0 u}$ .

- [5 points] What is the point spread function of this system?
- [10 points] The 2D Fourier Transform of the input to this system is given by  $F(u, v) = \exp\left(-\frac{u^2}{k_1^2}\right) \exp\left(-\frac{v^2}{k_2^2}\right)$ . What is the output image,  $g(x, y)$ ?
- [10 points] What is the condition on  $(x_0, k_1, k_2)$  to guarantee that the output image in part (b) has two spatially resolved peaks?

2) [25 points] Assume that we have a 6 MHz ultrasound transducer that can handle at most 70 dB pressure loss. As shown on the right, we would like to image a kidney stone with this system.

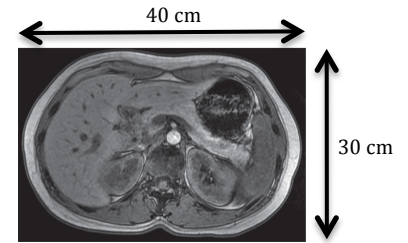
- $a_{fat} = 0.63 \text{ dB cm}^{-1}\text{MHz}^{-1}$ ,  $Z_{fat} = 1.35 \times 10^{-6} \text{ kg m}^{-2}\text{s}^{-1}$
- $a_{kidney} = 1 \text{ dB cm}^{-1}\text{MHz}^{-1}$ ,  $Z_{fat} = 1.62 \times 10^{-6} \text{ kg m}^{-2}\text{s}^{-1}$
- $a_{stone} = 6 \text{ dB cm}^{-1}\text{MHz}^{-1}$ ,  $Z_{stone} = 20 \times 10^{-6} \text{ kg m}^{-2}\text{s}^{-1}$

- [5 points] What is the depth of penetration in kidney for this ultrasound system?
- [15 points] What is the loss in dB for the ultrasound wave returning from the kidney/stone interface (received by the transducer)? Take into account both attenuation and reflection/transmission losses.
- [5 points] What is the optimum transducer size for resolving the kidney/stone interface? Assume  $c = 1500 \text{ m/s}$ , independent of the medium.



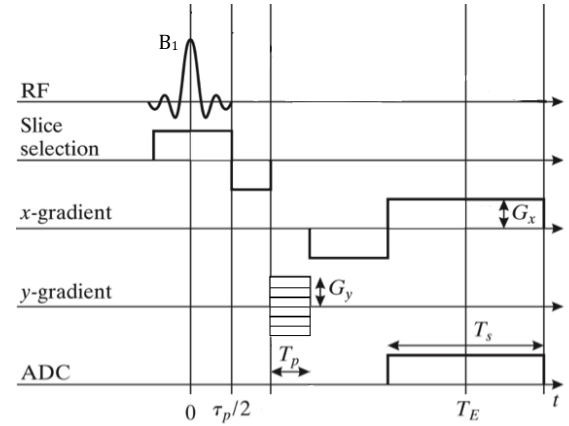
3) [25 points] We would like to image an axial cross-section of the abdomen as shown on the right. We want the field-of-view in the x-direction to be 40 cm and in the y-direction to be 30 cm. We want  $2\text{ mm} \times 2\text{ mm}$  resolution, with a 1 mm slice thickness.

- Our 1.5 T MRI scanner has a maximum gradient strength of 4 G/cm.
- During data acquisition, the samples are acquired  $16\text{ }\mu\text{s}$  apart.



We want to design a typical gradient echo sequence (i.e., line-by-line  $k$ -space acquisition), as shown on the right.

- [10 points] We want a  $90^\circ$  excitation and a 1 mm slice thickness. We are using a sinc RF pulse with main lobe and three side lobes on each side, as shown on the right. What is the minimum value for  $\tau_p$ , the duration of the RF pulse? What is the amplitude  $B_1$  for this  $\tau_p$ ?
- [10 points] What is  $T_s$ ? What is  $G_x$ ?
- [5 points] What is the minimum value for  $T_p$ ?



4) [25 points] The chemical shift of fat is 3.35 ppm lower relative to water. This will cause some problems in the reconstructed MRI images that contain both fat and water. For this question, assume the following:

- We have a 3 Tesla MRI scanner.
- We are using a typical gradient echo sequence with a constant readout gradient  $G_x$  as in Question 3.
- $TE \ll T_2$ , and there is negligible  $T_2$  relaxation during data acquisition.
- The receiver is tuned to the frequency of water (i.e.,  $\nu_0 = \nu_{water}$ ).
- Assume that if we image an impulse water at position  $(x_0, y_0)$ , the corresponding MRI image is an impulse at position  $(x_0, y_0)$  (i.e., there are no truncation artifacts due to covering a finite extent in  $k$ -space).

- a) [10 points] Show that if we image an impulse fat at position  $(x_0, y_0)$ , the position of the fat in the corresponding MRI image will be  $(x'_0, y_0)$  where

$$x'_0 = x_0 + \frac{\Delta\nu}{\gamma G_x}$$

Here,  $\Delta\nu$  is the Larmor frequency difference between fat and water.

- b) [10 points] Now, we image the object shown on the right, which is made up of one square (2 cm x 2 cm) filled with fat, and one circle (2 cm diameter) filled with water. We use  $G_x = 0.1$  G/cm. What is the corresponding MRI image? Sketch this image and clearly mark important positions on the image.

**Hint:** MRI is an LSI system if we are imaging water *only*, or fat *only*.

- c) [5 points] What would you do to make this problem less severe?

