EEE 473/573 Medical Imaging
Quiz 1 – Tuesday, 27 October 2020
Duration: 30 minutes

Write your Name and Student ID at the top of every page.
Write the following statement on the first page and sign below.

Honor Code: *"I have not given or received any aid during this quiz. I will do my share and take an active part in ensuring that others and I uphold the principles of honesty and integrity."*

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1) Given a continuous signal, $f(x,y) = y^2 + 2xy$, evaluate $g(x,y) = f(x,y)\delta\left(x-3, \frac{y}{4}+2\right)$.

Simplify your answer as much as possible.

$$\boldsymbol{g(x,y) = f(x,y)\delta\left(x-3, \frac{y+8}{4}\right) = 4\, f(3,-8)\delta(x-3, y+8) = 64\,\delta(x-3, y+8)}$$

2) Calculate the 2D convolution: $f(x,y) = exp\big(j2\pi(3x+4y)\big) * \exp\big(-(4x^2+9y^2)\big)$

Simplify your answer as much as possible.

$$\boldsymbol{f(x,y) = e^{j2\pi(3x+4y)} * e^{-(4x^2+9y^2)}}$$

$$\boldsymbol{\mathcal{F}_{2D}\{f(x,y)\} = F(u,v) = \mathcal{F}_{2D}\left\{e^{j2\pi(3x+4y)}\right\}\mathcal{F}_{2D}\left\{e^{-(4x^2+9y^2)}\right\}}$$

$$\boldsymbol{F(u,v) = \delta(x-3, y-4)\frac{\pi}{6}e^{-\pi^2\left(\frac{u^2}{4}+\frac{v^2}{9}\right)}}$$

$$\boldsymbol{F(u,v) = \delta(x-3, y-4)\frac{\pi}{6}e^{-\pi^2\left(\frac{3^2}{4}+\frac{4^2}{9}\right)} = \delta(x-3, y-4)\frac{\pi}{6}e^{-\frac{145\pi^2}{36}}}$$

$$\boldsymbol{f(x,y) = \mathcal{F}_{2D}\{F(u,v)\} = \frac{\pi}{6}e^{-\frac{145\pi^2}{36}}e^{j2\pi(3x+4y)}}$$

3) A 1D medical imaging system with PSF given by $h(x) = 3tri(x/2)$ is used for imaging an object given as $f(x) = 3 + \sin(\pi x)$. What is the modulation of this object? What is the modulation of the image generated by the system, $g(x)$?

$$\boldsymbol{f_{max} = 3+1 = 4, \qquad f_{min} = 3-1 = 2 \qquad \rightarrow m_f = \frac{4-2}{4+2} = \frac{1}{3} = 0.33}$$

$$H(u) = \mathcal{F}_{2D}\{3tri(x/2)\} = 3 \cdot 2sinc^2(2u) = 6sinc^2(2u)$$

$$G(u) = H(u)F(u) = H(u)\left\{3\delta(x) + \frac{1}{2j}\left(\delta(x - 1/2) - \delta(x + 1/2)\right)\right\}$$

$$G(u) = H(0)3\delta(x) + H\left(\frac{1}{2}\right)\frac{1}{2j}\left(\delta(x - 1/2) - \delta(x + 1/2)\right) \quad (since\ H\left(\frac{1}{2}\right) = H\left(-\frac{1}{2}\right))$$

$$g(x, y) = 3H(0) + H\left(\frac{1}{2}\right)sin(\pi x) = 3 \cdot 6 + 0 \cdot sin(\pi x) = 18$$

$$g_{max} = 18, \quad g_{min} = 18 \rightarrow m_g = 0$$

4) A 1D medical imaging system has two subsystems with the following PSFs:

$$h_1(x) = 3tri(x/2) \quad \text{and} \quad h_2(x) = 4e^{-\frac{x^2}{9}}$$

What is the approximate FWHM resolution for the overall imaging system?

$h_1(x) = 3tri\left(\frac{x}{2}\right)$ *is from -2 to 2 with maximum value of 3.*

Half max value 3/2 is reached at -1 to 1. Hence $FWHM_1 = 2$

$h_2(x) = 4e^{-\frac{x^2}{9}}$ *has its maximum at x = 0 with a value of 4.*

$$e^{-\frac{x_{half}^2}{9}} = \frac{1}{2} \xrightarrow{yields} \frac{x_{half}^2}{9} = ln2 \xrightarrow{yields} x_{half}^2 = 9ln2 \xrightarrow{yields} x_{half} = \pm 3\sqrt{ln2}$$

$$FWHM_2 = 2x_{half} = 6\sqrt{ln2} \approx 4.995$$

$$FWHM \approx \sqrt{FWHM_1^2 + FWHM_2^2} = \sqrt{4 + 24.95} \approx 5.38$$