Name Lastname	
Student ID	
Signature	
Classroom #	EE-

Q1 (25 pts)	
Q2 (25 pts)	
Q3 (25 pts)	
Q4 (25 pts)	
TOTAL	

EEE 473/573 – Spring 2014-2015
FINAL EXAM

24 May 2015, 9:00-11:00

- Open book, open notes.
- Provide appropriate explanations in your solution and **show intermediate steps clearly. No credit will be given otherwise**.

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1) [25 points] The transfer function of a 2D LSI imaging system is given by $H(u,v) = 1 + e^{-j2\pi x_0 u}$.

a) [5 points] What is the point spread function of this system?

b) [10 points] The 2D Fourier Transform of the input to this system is given by $F(u,v) = exp\left(-\frac{u^2}{k_1^2}\right) exp\left(-\frac{v^2}{k_2^2}\right)$. What is the output image, $g(x,y)$?

c) [10 points] What is the condition on (x_0, k_1, k_2) to guarantee that the output image in part (b) has two spatially resolved peaks?

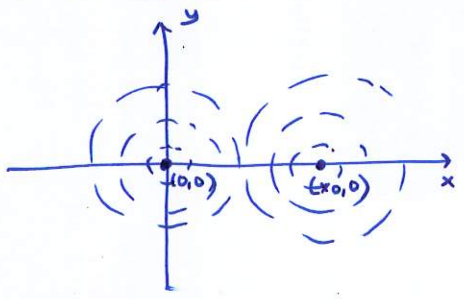
a) $h(x,y) = FT_{2D}^{-1}\left\{1 + e^{-j2\pi x_0 u}\right\} = \boxed{\delta(x,y) + \delta(x-x_0, y)}$

b) $f(x,y) = \pi k_1 k_2 \cdot exp\left(-\pi^2 k_1^2 x^2\right) exp\left(-\pi^2 k_2^2 y^2\right)$

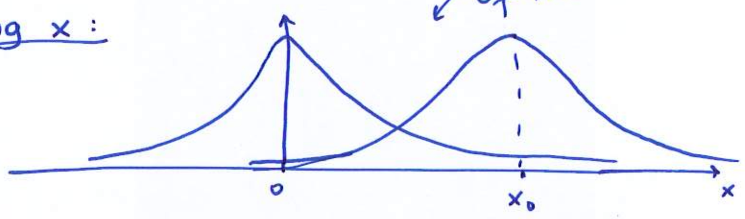
$g(x,y) = f(x,y) * h(x,y)$

$$\boxed{g(x,y) = \pi k_1 k_2 \cdot exp\left(-\pi^2 (k_1^2 x^2 + k_2^2 y^2)\right) + \pi k_1 k_2 \cdot exp\left(-\pi^2 \left(k_1^2 (x-x_0)^2 + k_2^2 y^2\right)\right)}$$

c) In $g(x,y)$, we have two 2D-Gaussian functions. These two functions are harder to resolve along the x-direction.



Along x :



To resolve the two peaks, the distance between peaks should be more than their FWHM along x-direction.

$x_0 > FWHM_x$

$exp\left(-\pi^2 k_1^2 x^2\right) = \frac{1}{2}$

$x = \sqrt{\frac{\ln 2}{\pi^2 k_1^2}} = \frac{\sqrt{\ln 2}}{\pi k_1}$

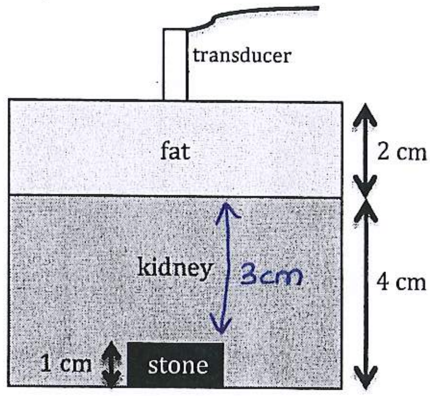
$FWHM = 2x = \frac{2\sqrt{\ln 2}}{\pi k_1}$

So, $\boxed{x_0 > \frac{2\sqrt{\ln 2}}{\pi k_1}}$

No condition on k_2.

2) [25 points] Assume that we have a 6 MHz ultrasound transducer that can handle at most 70 dB pressure loss. As shown on the right, we would like to image a kidney stone with this system.

- $a_{fat} = 0.63$ dB cm^{-1}MHz^{-1}, $Z_{fat} = 1.35\times10^{-6}$ kg m^{-2}s^{-1}
- $a_{kidney} = 1$ dB cm^{-1}MHz^{-1}, $Z_{fat} = 1.62\times10^{-6}$ kg m^{-2}s^{-1}
- $a_{stone} = 6$ dB cm^{-1}MHz^{-1}, $Z_{stone} = 20\times10^{-6}$ kg m^{-2}s^{-1}



a) [5 poinst] What is the depth of penetration in kidney for this ultrasound system?

b) [15 points] What is the loss in dB for the ultrasound wave returning from the kidney/stone interface (received by the transducer)? Take into account both attenuation and reflection/transmission losses.

c) [5 points] What is the optimum transducer size for resolving the kidney/stone interface? Assume c = 1500 m/s, independent of the medium.

a) $\alpha_{kidney} = a_{kidney} \cdot 6\text{ MHz} = 6\text{ dB/cm}$

$$d_p = \frac{1}{2\alpha} = \frac{70\text{ dB}}{2 \cdot 6\text{ dB/cm}} = \boxed{5.83\text{ cm}}$$

b) $\alpha_{fat} = 0.63 \times 6 = 3.78$ dB/cm

From attenuation: $2\times 2\text{cm} \times \alpha_{fat} + 2\times 3\text{cm} \times \alpha_{kidney}$

$$= 4\text{ cm} \times 3.78\text{ dB/cm} + 6\text{ cm} \times 6\text{ dB/cm} = \boxed{\begin{array}{l}51.12\text{ dB}\\ \text{attenuation}\end{array}}$$

Transmission from fat/kidney interface (normal incidence):

$$T_I = \frac{4 Z_1 Z_2}{(Z_1+Z_2)^2} = \frac{4 \times 1.35 \times 1.62}{(1.35+1.62)^2} = 0.9917$$

Reflection from kidney/stone interface (normal incidence):

$$R_I = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 = \left(\frac{20-1.62}{20+1.62}\right)^2 = 0.7227$$

* Ultrasound wave will be transmitted from fat/kidney interface, reflected from kidney/stone interface, then transmitted from kidney/fat interface.

Total transmission/reflection $= T_I \cdot R_I \cdot T_I = (0.9917)^2 \times 0.7227 = 0.7108$

↳ in dB $= 10 \cdot \log_{10}(0.7108) = \boxed{-1.48\text{ dB}}$ → i.e., 1.48 dB loss

because this is power (intensity), not amplitude

Total loss = loss from attenuation (in dB) + loss from transmission/reflection (in dB)

$= 51.12$ dB $+ 1.48$ dB

$= \boxed{52.6\text{ dB loss}}$

c) For optimality, we should be in "near field" and "far field" crossover region.

$$\frac{D_{opt}^2}{\lambda} = z_{max}$$

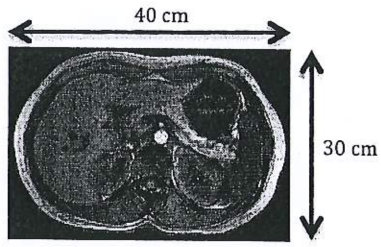
$$D_{opt} = \sqrt{z_{max} \cdot \lambda}$$

$z_{max} = 5$ cm (depth until the stone)

$$\lambda = \frac{c}{f} = \frac{1500 \text{ m/s}}{6.10^6 \text{ Hz}} = 0.25 \text{ mm}$$

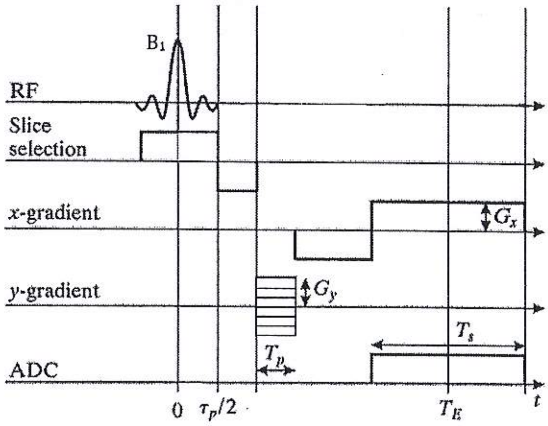
$$D_{opt} = \sqrt{50 \text{ mm} \cdot 0.25 \text{ mm}} = \boxed{3.54 \text{ mm}}$$

3) [25 points] We would like to image an axial cross-section of the abdomen as shown on the right. We want the field-of-view in the x-direction to be 40 cm and in the y-direction to be 30 cm. We want 2 mm × 2 mm resolution, with a 1 mm slice thickness.



- Our 1.5 T MRI scanner has a maximum gradient strength of 4 G/cm.
- During data acquisition, the samples are acquired 16 μs apart.

We want to design a typical gradient echo sequence (i.e., line-by-line *k*-space acquisition), as shown on the right.



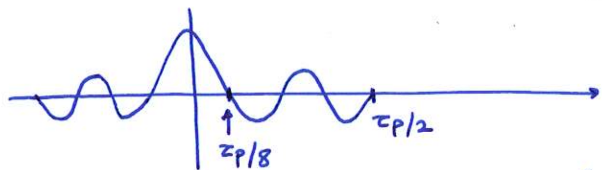
a) [10 points] We want a 90^0 excitation and a 1 mm slice thickness. We are using a sinc RF pulse with main lobe and three side lobes on each side, as shown on the right. What is the minimum value for τ_p, the duration of the RF pulse? What is the amplitude B_1 for this τ_p?

b) [10 points] What is T_s? What is G_x?

c) [5 points] What is the minimum value for T_p?

Note : $1\ G = 10^{-4}\ T = 0.1\ mT$

a) $\Delta z = \dfrac{\Delta \nu}{\gamma G_z}$



sinc pulse can be written as : $B_1^e(t) = B_1 . \text{sinc}\left(\frac{t}{\tau_p/8}\right) . \text{rect}\left(\frac{t}{\tau_p}\right)$

* Considering an infinitely long sinc :

$B_1^e(t) = B_1 . \text{sinc}\left(\frac{t}{\tau_p/8}\right)$ ← bandwidth for this pulse is $\Delta\nu = \dfrac{1}{\tau_p/8} = \dfrac{8}{\tau_p}$

so, $\Delta z = \dfrac{8/\tau_p}{\gamma G_z} \Rightarrow \tau_p = \dfrac{8}{\Delta z . \gamma . G_z}$

* For minimum τ_p, we should use maximum G_z . So, $\boxed{G_z = 4\ G/cm.}$

$$\tau_p = \frac{8}{1mm . 42.58 \frac{MHz}{Tesla} . \frac{0.4\,mT}{cm}} = \boxed{4.7\ ms}$$

* For $\frac{\pi}{2}$ flip angle :

$$\alpha(z=0) = \int \gamma B_1 . \text{sinc}\left(\frac{t}{\tau_p/8}\right) dt = \gamma . B_1 \frac{\tau_p}{8} = \frac{\pi}{2} \Rightarrow B_1 = \frac{4\pi}{2\pi \gamma \tau_p} \approx \boxed{10\ \mu T}$$

$= 0.1$ Gauss

b) $T_s = N_x . T$

we know that : $FWHM_x = \dfrac{1}{N_x . \Delta k_x}$, $FOV_x = \dfrac{1}{\Delta k_x}$

$$\underbrace{\qquad\qquad}_{N_x = \frac{FOV_x}{FWHM_x} = \frac{400mm}{2mm} = 200}$$

$T_s = 200 . 16\mu s = 3200 \mu s = \boxed{3.2\ ms}$

$$FWHM_x = \frac{1}{N_x \cdot \Delta k_x} = \frac{1}{N_x \cdot \gamma G_x \cdot T} = \frac{1}{T_s \cdot \gamma G_x}$$

$$G_x = \frac{1}{FWHM_x \cdot T_s \cdot \gamma} = \frac{1}{2 \cdot 10^{-3} \cdot 3.2 \times 10^{-3} \cdot 42.58 \cdot 10^{6}}$$

$$\boxed{G_x = 3.68 \ mT/m = 0.368 \ G/cm}$$

c) $$\underbrace{FWHM_y = \frac{1}{N_y \cdot \Delta k_y} \quad , \quad FOV_y = \frac{1}{\Delta k_y}}$$

$$N_y = \frac{FOV_y}{FWHM_y} = \frac{300 \, mm}{2 \, mm} = 150$$

$$FWHM_y = \frac{1}{N_y \cdot \Delta k_y} = \frac{1}{N_y \cdot \gamma \, \Delta G_y \cdot T_p} \Rightarrow T_p = \frac{1}{FWHM_y \cdot N_y \, \gamma \cdot \Delta G_y}$$

* Here, ΔG_y is the incremental change in y-gradient in each TR. y-gradient varies between $\pm \frac{N_y}{2} \Delta G_y$, which should be kept below 4 G/cm.

For minimum T_p,

$$G_y = \frac{N_y}{2} \Delta G_y = 4 \ G/cm$$

$$\Delta G_y = \frac{4}{75} \ G/cm$$

$$T_p = \frac{1}{2 \cdot 10^{-3} \cdot 150 \cdot 42.58 \times 10^{6} \cdot \frac{4}{75} \cdot \frac{10^{-4}}{10^{-2}}} \simeq \boxed{147 \ \mu s}$$

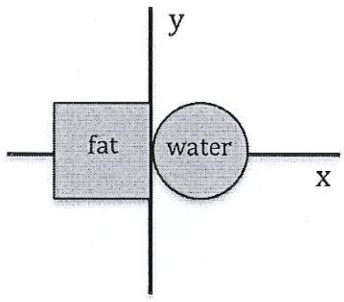
4) [25 points] The chemical shift of fat is 3.35 ppm lower relative to water. This will cause some problems in the reconstructed MRI images that contain both fat and water. For this question, assume the following:

- We have a 3 Tesla MRI scanner.
- We are using a typical gradient echo sequence with a constant readout gradient G_x as in Question 3.
- TE << T_2, and there is negligible T_2 relaxation during data acquisition.
- The receiver is tuned to the frequency of water (i.e., $\nu_0 = \nu_{water}$).
- Assume that if we image an impulse water at position (x_0, y_0), the corresponding MRI image is an impulse at position (x_0, y_0) (i.e., there are no truncation artifacts due to covering a finite extent in k-space).

a) [10 points] Show that if we image an impulse fat at position (x_0, y_0), the position of the fat in the corresponding MRI image will be (x_0', y_0) where

$$x_0' = x_0 + \frac{\Delta\nu}{\gamma G_x}$$

Here, $\Delta\nu$ is the Larmor frequency difference between fat and water.



b) [10 points] Now, we image the object shown on the right, which is made up of one square (2 cm x 2 cm) filled with fat, and one circle (2 cm diameter) filled with water. We use G_x = 0.1 G/cm. What is the corresponding MRI image? Sketch this image and clearly mark important positions on the image.

Hint: MRI is an LSI system if we are imaging water *only*, or fat *only*.

c) [5 points] What would you do to make this problem less severe?

a) Ignoring constants and T_2 relaxation, compare signals from impulse water and impulse fat.

For water:

$$s_w(t) = \iint_{-\infty}^{\infty} \delta(x-x_0, y-y_0)\, e^{-j2\pi\nu_0 t} \cdot e^{-j2\pi\gamma G_x . x . t}\, e^{-j2\pi\gamma n \Delta G_y . y . t_y}\, dx\,dy$$

$$= e^{-j2\pi\nu_0 t} \cdot e^{-j2\pi\gamma G_x . x_0 . t} \cdot e^{-j2\pi\gamma n \Delta G_y . y_0 t_y}$$

For fat:

$$s_f(t) = \underbrace{e^{-j2\pi(\nu_0+\Delta\nu)t} \cdot e^{-j2\pi\gamma G_x . x_0 t}}\, e^{-j2\pi\gamma . n \Delta G_y . y_0 . t_y}.$$

$$= e^{-j2\pi\nu_0 t} \cdot e^{-j2\pi[\Delta\nu + \gamma G_x . x_0]t} \cdot e^{-j2\pi\gamma . n \Delta G_y . y_0 t_y}$$

$$= e^{-j2\pi\nu_0 t} \cdot e^{-j2\pi\gamma G_x \underbrace{\left[x_0 + \frac{\Delta\nu}{\gamma G_x}\right]}_{x_0'} t}\, e^{-j2\pi\gamma n \Delta G_y . y_0 t_y}$$

$s_f(t)$ is as if it is from an impulse water at x_0'. So, since the receiver is tuned to water, impulse fat will be an impulse at

$$\boxed{x_0' = x_0 + \frac{\Delta\nu}{\gamma G_x}}$$

b) From given information, we see that the point spread functions for water and fat are:

$$h_{water} = \delta(x,y)$$

$$h_{fat} = \delta(x-\Delta x, y) \quad , \text{ where } \quad \Delta x = \frac{\Delta \nu}{\gamma G_x}$$

So,

$$ImG = ImG_{water} + ImG_{fat}$$

$$= M_{water}(x,y) * h_{water}(x,y) + M_{fat}(x,y) * h_{fat}(x,y)$$

$$\boxed{ImG(x,y) = M_{water}(x,y) + M_{fat}(x-\Delta x, y)}$$

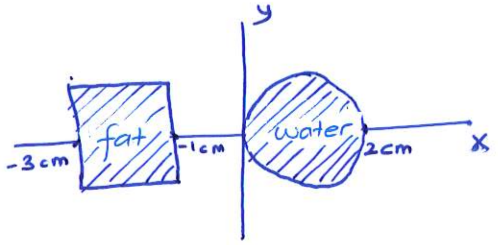
* calculate Δx,

3.35 ppm lower

$$\Delta x = \frac{\Delta \nu}{\gamma G_x} = \frac{-3.35 \cdot 10^{-6} \cdot 42.58 \ MHz/T \cdot 3T}{42.58 \frac{MHz}{T} \cdot 0.01 \frac{mT}{cm}}$$

$$\boxed{\Delta x = -1 \ cm}$$

fat will be shifted to the left by 1 cm.
water stays in place.



c) writing Δx more explicitly:

$$\Delta x = \frac{\Delta \nu}{\gamma G_x} = \frac{CS \cdot \gamma \cdot B_0}{\gamma G_x}$$

where CS is chemical shift (in ppm)

$$\boxed{\Delta x = \frac{CS \cdot B_0}{G_x}}$$

* We cannot change chemical shift (that is inherent).

* We can reduce B_0, e.g., use a 1.5 T MRI scanner, but this not such a practical solution (change scanner → expensive).

* We can increase G_x. Very practical solution. For example, for $G_x = 4 \ G/cm$, $\Delta x = -0.025$ cm.