Name Lastname	
Student ID	
Signature	
Classroom #	EE-

Q1 (25 pts)	
Q2 (25 pts)	
Q3 (25 pts)	
Q4 (25 pts)	
TOTAL	

EEE 473/573 – Spring 2014-2015
MIDTERM EXAM #1

5 April 2015, 14:00-16:00

- Open book, open notes.
- Provide appropriate explanations in your solution and **show intermediate steps clearly. No credit will be given otherwise**.

1) [25 points] Answer the following questions. Simplify your answers as much as possible.

a) [5 points] Calculate the 2D convolution $\text{rect}\left(\frac{x}{4}, y\right) * \delta(x-1, y-2)$.

b) [5 points] Calculate the 2D convolution $\text{rect}\left(\frac{x}{4}, y\right) * \delta(y-2)$.

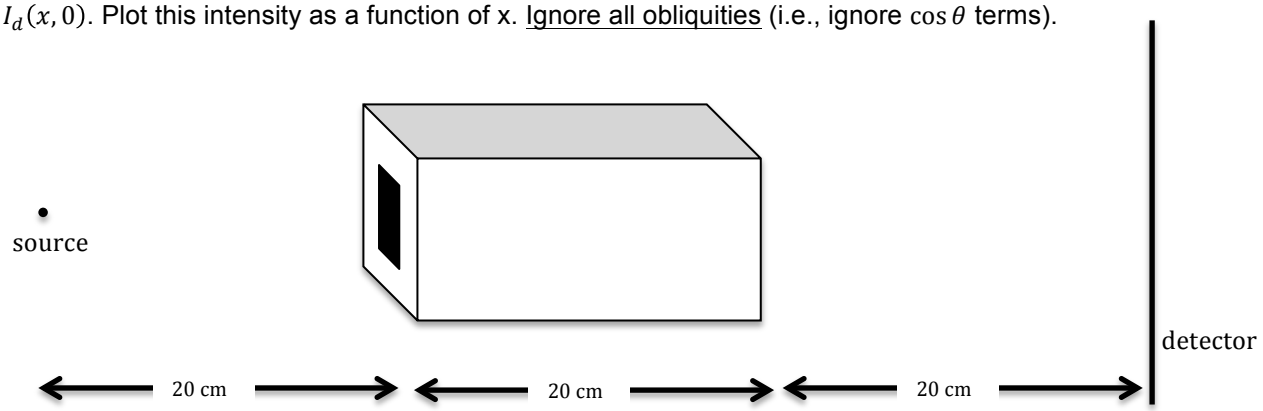
c) [5 points] Calculate the 2D convolution $\cos(2\pi x + 4\pi y) * \text{sinc}(2x, 3y)$.

d) [5 points] Calculate the 2D convolution $\cos(2\pi u_0 x + 2\pi v_0 y) * \exp(-x^2 - y^2)$.

e) [5 points] What is the 2D Fourier transform of the following function?

$f(x,y) = f(r) = \text{rect}\left(\frac{r-a}{b}\right)$, where a $>$ b .

2) [25 points] A hollow prism (i.e., with a hole at its center) has length L = 20 cm, outer width 8cm X 8cm, inner width 2cm X 2cm, and a constant linear attenuation coefficient of $\mu_0 = 0.05\ \text{cm}^{-1}$. This prism is imaged with a point source x-ray imaging system, as shown below. Formulate the intensity on the detector along the x-axis, $I_d(x, 0)$. Plot this intensity as a function of x. Ignore all obliquities (i.e., ignore $\cos\theta$ terms).



3) [25 points] A 2D function $f(x,y)$ (or $f(r,\emptyset)$) produces 1D projections given by

$$g(l,\theta) = 4\,sinc(2l)\cos(4\pi l)$$

a) [13 points] Determine the 2D function $f(x,y)$ (or $f(r,\emptyset)$).

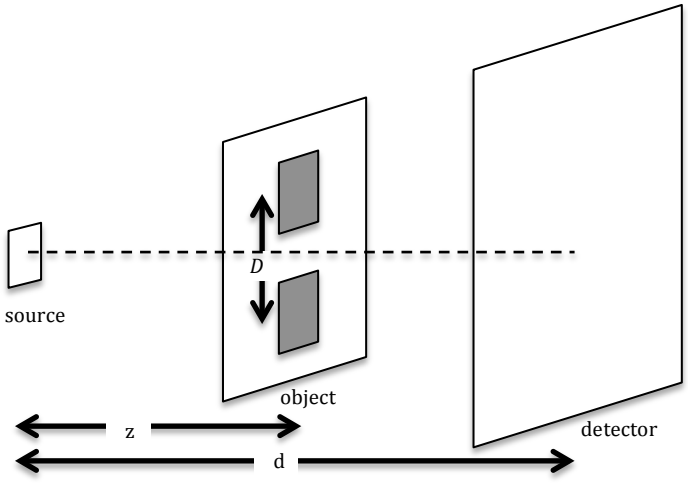
b) [12 points] If the CT image reconstruction is performed with a filtered backprojection system using a modified filter $|\rho|\text{rect}\left(\frac{\rho}{2\rho_0}\right)$, determine the resultant reconstructed image as a function of ρ_0. Simplify your answer as much as possible.

4) [25 points] A square source of size *L* by *L* is used to image a planar object that contains two square holes (lesions), each size *W* by *W*. The rest of the planar object has zero transmittivity. The centers of the two holes are separated by a distance *D* along the x-direction. The exact depth of the planar object is not known, except that it is between $z = d/2$ and $z = 2d/3$.

a) [10 points] Find the largest source size, *L*, that ensures that the two lesions remain fully resolved, i.e., they remain not touching in the image, for all z within the range specified.

b) [5 points] What is the largest value of *L* if $D = 9W/4$?

c) [10 points] Using the value from part (b), find the value(s) of z (within the range specified) that maximize the image intensity at the center of the lesions.



Ignore all obliquities (i.e., ignore $\cos\theta$ terms).
Hint: There is no need to fully calculate $I_d(x, y)$ for this question.