Name Lastname	
Student ID	
Signature	
Classroom #	EE-

Q1 (25 pts)	
Q2 (10 pts)	
Q3 (25 pts)	
Q4 (15 pts)	
Q4 (25 pts)	
TOTAL	

EEE 473/573 – Spring 2014-2015
MIDTERM EXAM #2

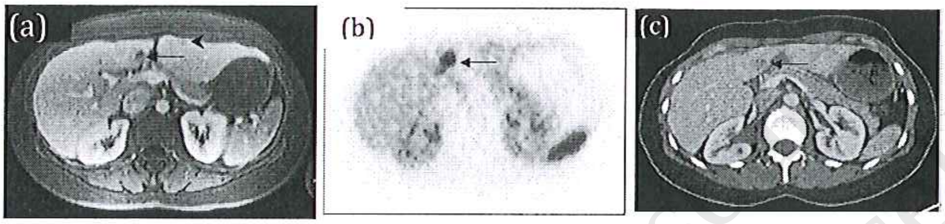
29 April 2015, 18:00-20:00

- Open book, open notes.
- Provide appropriate explanations in your solution and **show intermediate steps clearly. No credit will be given otherwise**.

---

1) [25 points] Name the imaging modality for each of the following images. Your choices are:
- X-ray, CT, MRI, or Nuclear Medicine (no need to specify if it is planar scintigraphy, SPECT, or PET).

Rules: Explain your reasoning with one or two (not more) sentences for each case. Name only one modality for each image. No points will be given if there is no explanation or if you list two options for one image.

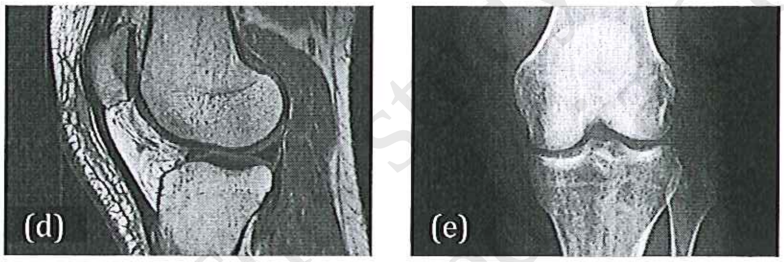


(a), (b), and (c) are showing the same slice in the abdomen.

a) MRI : good soft tissue contrast, good resolution, no bones.

b) Nuclear medicine : Bad resolution, shows function (arrow), no tissue contrast.

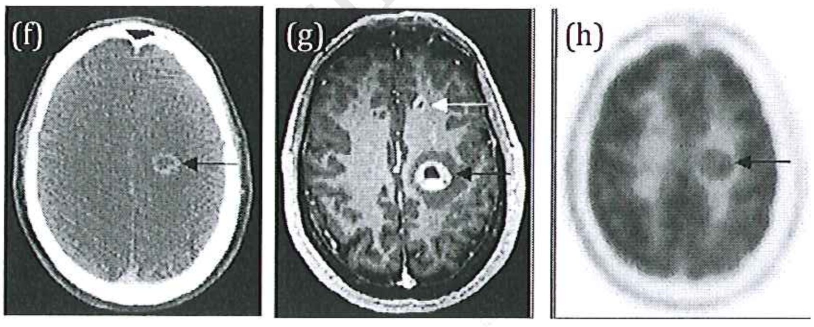
c) CT : good resolution, bones are clearly visible.



(d) and (e) are both showing images of the knee.

d) MRI : excellent soft tissue contrast good resolution, no signal from bone itself (we see bone marrow only).

e) X-ray : typical knee x-ray image, bones overlapping → projection format.



(f), (g), and (h) are showing the same slice in the brain.

f) CT : not good contrast from soft tissue of brain, but skull is clearly visible. bone

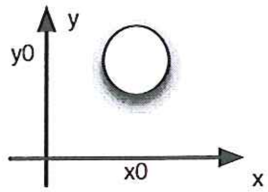
g) MRI : excellent soft tissue contrast, good resolution.

h) Nuclear medicine : bad resolution, shows function mostly (arrow)

2) [10 points] Consider a circle of radius r_0 centered at (x_0,y_0), as shown on the right.

a) [5 points] Express the circle as a 2D function $f(x,y)$ (or $f(r,\emptyset)$).

b) [5 points] Find the 2D Fourier Transform of this function.



a) $f(x,y) = \text{rect}\left(\frac{r}{2r_0}\right) * \delta(x-x_0, y-y_0)$ ← shifted circle

$$= \text{rect}\left(\frac{\sqrt{x^2+y^2}}{2r_0}\right) * \delta(x-x_0, y-y_0)$$

$$\boxed{f(x,y) = \text{rect}\left(\frac{\sqrt{(x-x_0)^2+(y-y_0)^2}}{2r_0}\right)}$$

b) $f(x,y) = \text{rect}\left(\frac{r}{2r_0}\right) * \delta(x-x_0, y-y_0)$

↓ Hankel T. ↓ Fourier T.

$F(u,v) = 4r_0^2 . \text{jinc}(2r_0 . q) \cdot e^{-j2\pi x_0 . u} e^{-j2\pi y_0 v}$, where $q = \sqrt{u^2+v^2}$

$$\boxed{F(u,v) = 4r_0^2 . \text{jinc}(2r_0\sqrt{u^2+v^2}) . e^{-j2\pi(x_0 u + y_0 v)}}$$

Some students thought that the question is asking for a "ring" of radius r_0. Note that a "ring" with finite amplitude would have zero area under it, hence its Fourier Transform would be zero. A more reasonable thing to ask would be a "delta ring" of radius r_0, which does have a finite area. The solution for that would be:

a) $f(x,y) = \delta(r-r_0) * \delta(x-x_0, y-y_0) = \delta\left(\sqrt{(x-x_0)^2+(y-y_0)^2} - r_0\right)$

↓ Hankel T. ↓ Fourier T.

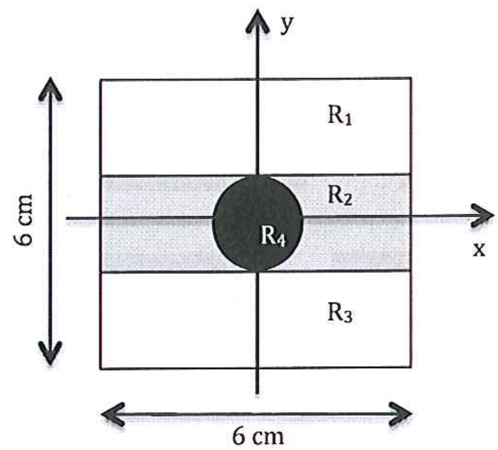
b) $F(u,v) = 2\pi r_0 . J_0(2\pi r_0 q) \cdot e^{-j2\pi x_0 u} e^{-j2\pi y_0 v}$, where $q = \sqrt{u^2+v^2}$

$$\boxed{F(u,v) = 2\pi r_0 . J_0\left(2\pi r_0\sqrt{u^2+v^2}\right) . e^{-j2\pi(x_0 u + y_0 v)}}$$

I gave full points to this solution, as well.

3) [25 points] Consider the 2D object shown, which consists of four regions R_1, R_2, R_3, R_4.

- R_1, R_2, and R_3 are 6 cm x 2 cm rectangles.
- R_4 is a circle with 2 cm diameter.
- The linear attenuation coefficient for each region is: $\mu_1 = 0.1\ \text{cm}^{-1}$, $\mu_2 = 0.3\ \text{cm}^{-1}$, $\mu_3 = 0.2\ \text{cm}^{-1}$, $\mu_4 = 0$.
- Only R_2 and R_4 contain radionuclides. Their relative concentrations are $f_2 = 1$ and $f_4 = 2$. (The absolute values/units are not important for this question).
- Assume perfect detection and ignore inverse square law.



a) [13 points] We image the radioactivity using a 2D SPECT scanner. What is the local contrast of the projection $g_{SPECT}(l, 0°)$? Let $g_{SPECT}(0,0°)$ be used as the intensity of the object of interest (i.e., the circle). When $\theta = 0°$, the camera is located on the +y-axis (above the object) looking down.

b) [12 points] Now assume the radionuclides in part (a) are replaced by positron emitting radionuclides with the same concentrations. We image the radioactivity using a 2D PET scanner. What is the local contrast of the projection $g_{PET}(l, 0°)$? Again, let $g_{PET}(0,0°)$ be used as the intensity of the object of interest.

Hint: For local contrast calculations, there is no need to fully calculate $g(l, 0°)$ for all l.

a) Calculate target intensity and background intensity only. This suffices to calculate the contrast.

$$g_t = g_{SPECT}(0,0°) = f_4 \int_{-1}^{1} \exp\{-(1-y)\cancel{\mu_4}^{0} - 2\mu_1\}\, dy = 2e^{-0.2}\int_{-1}^{1} dy = 4e^{-0.2} = 3.27$$

* The background signal is for $|l| > 1$:

$$g_b = g_{SPECT}(|l|>1, 0°) = f_2 \int_{-1}^{1} \exp\{-(1-y)\mu_2 - 2\mu_1\}\, dy = e^{-0.3}\cdot e^{-0.2}\int_{-1}^{1} \exp\{0.3y\}\, dy$$

$$= \frac{e^{-0.5}}{0.3}\cdot e^{0.3y}\Big|_{-1}^{1} = \frac{e^{-0.2} - e^{-0.8}}{0.3} = 1.23$$

* Local contrast is:

$$C = \frac{g_t - g_b}{g_b} = \frac{3.27 - 1.23}{1.23} = \boxed{1.66}$$

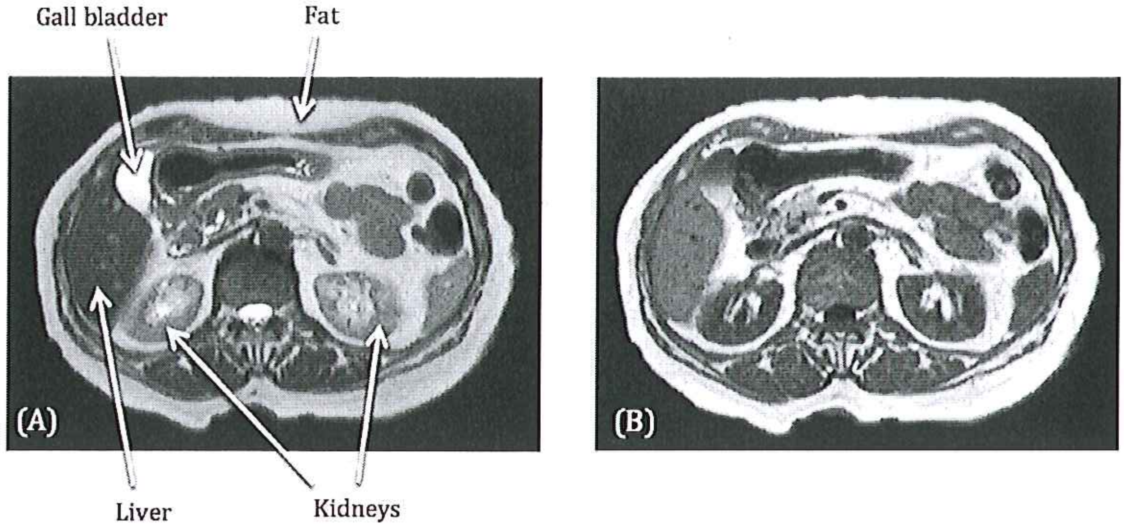
b) $$g_t = g_{PET}(0,0°) = f_4 \cdot \exp\{-2\mu_4 - 2\mu_1 - 2\mu_3\} \int_{-1}^{1} dy = 4 \cdot e^{-0.6} \simeq 2.20$$

$$g_b = g_{PET}(|l|>1, 0°) = f_2 \cdot \exp\{-2\mu_2 - 2\mu_1 - 2\mu_3\} \int_{-1}^{1} dy = 2 \cdot e^{-1.2} \approx 0.60$$

* The local contrast is:

$$C = \frac{g_t - g_b}{g_b} = \frac{2.20 - 0.60}{0.60} \approx \boxed{2.67}$$

4) [15 points] Two different MRI images of the same slice in the abdomen are shown below. The T_1, T_2 and relative proton density (P_D) values for some of the organs/tissues are also listed below.



	T_1 (ms)	T_2 (ms)	Relative P_D
Liver	450	43	0.60
Kidney	650	58	0.60
Fat	230	85	0.9
Gall Bladder	2000	300	1.00

a) [10 points] What type of MRI contrast do you see in (A) and (B)? Explain your reasoning in detail. <u>No points will be given without proper explanation or if you present two different choices for one image.</u>

b) [5 points] What could be resonable echo times (TE) and repetition times (TR) for these two images, assuming a 90° flip angle?

a)(A) Comparing signal values in the image; Gall bladder is the brightest, then fat, then kidneys, and least brightest is liver. This must be a <u>T_2-weighted MRI</u> image.

The signal intensity is proportional to e^{-TE/T_2}, so tissues with short T_2 (such as liver) has the least signal.

(B) Fat is the brightest, followed by liver, then kidneys. Gall bladder signal is mixed, but the above information is sufficient to deduce that this is <u>T_1-weighted contrast</u>.

The signal intensity is higher for shorter T_1.

b) * For T_2-weighted MRI: TE comparible to T_2 of tissues

$\longrightarrow$ TE around 100 ms or so.

Long TR (approximately $3 \cdot T_1^{max}$)

$\longrightarrow$ TR around 6000 ms or so.

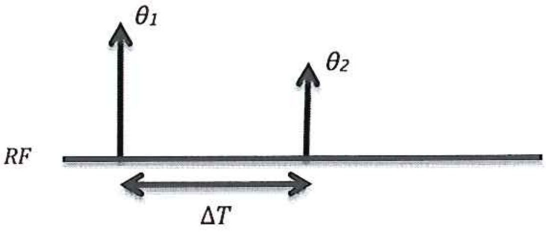
* For T_1-weighted MRI: Short TE $\longrightarrow$ 10-20 ms or so.

TR comparible to T_1 of tissues

$\longrightarrow$ TR around 500-600 ms.

5) [25 points] Consider the following MR sequence in which an RF excitation pulse of tip angle θ_1 is followed by a θ_2 pulse a fixed time ΔT later. Assume the following conditions are true:

- The object being imaged is homogeneous with a single value of T_1 and T_2.
- The magnetization is initially at equilibrium $M_0 = 1$.
- $\Delta T \gg T_2$ and $T_1 = 5\,\Delta T$.



a) [10 points] If $\theta_1 = \theta_2 = 60°$, what are the relative signal amplitudes for the two FIDs immediately after each excitation?

b) [10 points] Determine θ_1 such that the FID amplitude for the second excitation is always zero.

c) [5 points] Assuming <u>no T_1 relaxation</u> takes place during ΔT, find the $\theta_1 - \theta_2$ combination that produces the maximum possible signal amplitudes, while ensuring that the amplitudes for the two FIDs are the same.

a) $M_z(0^+) = M_0.\cos\theta_1 = 0.5$

FID 1 : $M_{xy}(0^+) = M_0.\sin\theta_1\, e^{-j2\pi\gamma_0 t}\, e^{j\phi}$

$|M_{xy}(0^+)| = M_0.\sin\theta_1 = \frac{\sqrt{3}}{2} = \boxed{0.866}$

* Then, M_z recovers during ΔT :

$M_z(\Delta T^-) = M_0(1-e^{-\Delta T/T_1}) + M_z(0^+)\,e^{-\Delta T/T_1}$

$= (1-e^{-0.2}) + 0.5.e^{-0.2} = 0.591$

FID 2 : $M_{xy}(\Delta T^+) = M_z(\Delta T^-).\sin\theta_2 .\, e^{-j2\pi\gamma_0 t}.e^{j\phi}$

$|M_{xy}(\Delta T^+)| = M_z(\Delta T^-).\sin\theta_2 = 0.591 \times \frac{\sqrt{3}}{2} = \boxed{0.511}$

b) To ensure zero signal for the second FID, we should have $M_z(\Delta T^-) = 0$.

$M_z(\Delta T^-) = M_0(1-e^{-\Delta T/T_1}) + M_0.\cos\theta_1\, e^{-\Delta T/T_1} = 0$

$1-e^{-0.2} + \cos\theta_1 . e^{-0.2} = 0$

$\cos\theta_1 = -\frac{(1-e^{-0.2})}{e^{-0.2}} = -0.22$

$\theta_1 = \cos^{-1}(-0.22) = \boxed{102.8°}$

c) $T_1 >> \Delta T$.

FID 1 : $|M_{xy}(0^+)| = M_0.\sin\theta_1$

Then, $M_z(\Delta T^-) = M_z(0^+) = M_0.\cos\theta_1$ (no T_1 relaxation)

FID 2 : $|M_{xy}(\Delta T^+)| = M_z(\Delta T^-).\sin\theta_2 = M_0.\cos\theta_1.\sin\theta_2$

* First, if we choose $\theta_1 > 45°$, we can not achieve FID1 = FID 2.

* Best case is $\boxed{\theta_1 = 45°}$, $\boxed{\theta_2 = 90°}$.

Then, FID 1 = FID 2 = $\frac{1}{\sqrt{2}}$