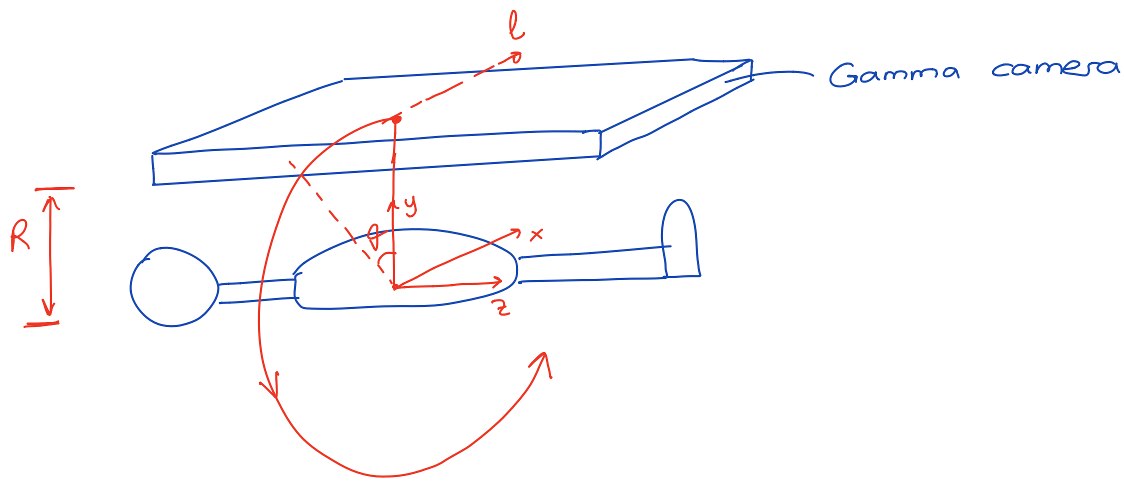
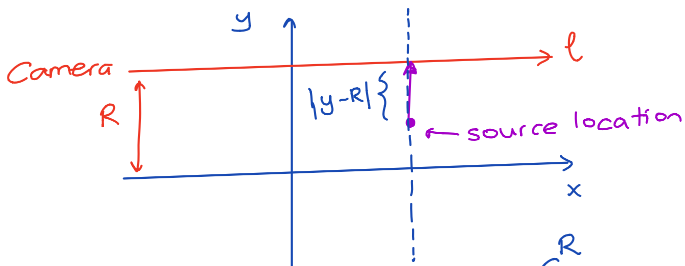
CHAPTER 9 - EMISSION COMPUTED TOMOGRAPHY

SPECT Image Formation: Change coordinate system

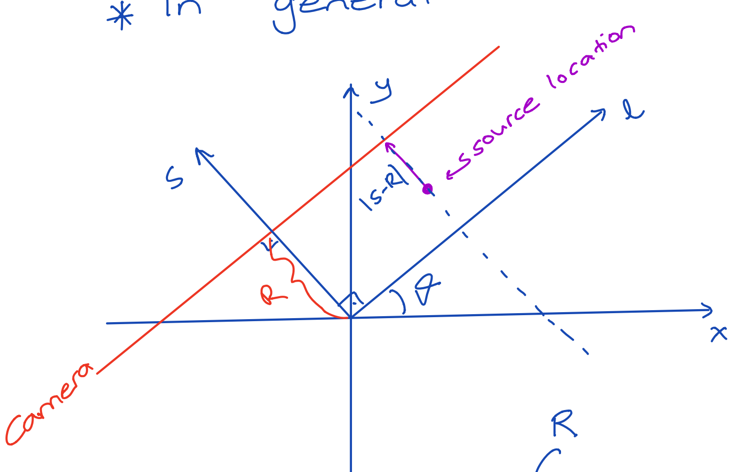


* when $\theta = 0$



$$\phi(z,\ell) = \int_{-\infty}^{R} \frac{A(x,y,z)}{4\pi(y-R)^2} \cdot \exp\left\{-\int_{y}^{R} \mu(x,y',z;E)\,dy'\right\} dy$$

* In general



$$L(\ell,\theta) = \{(x,y) \mid x\cos\theta + y\sin\theta = \ell\}$$

$$x(s) = \ell.\cos\theta - s.\sin\theta$$

$$y(s) = \ell.\sin\theta + s.\cos\theta$$

$$\phi(\ell,\theta) = \int_{-\infty}^{R} \frac{A(x(s),y(s))}{4\pi(s-R)^2} \exp\left\{-\int_{s}^{R} \mu(x(s'),y(s');E)\,ds'\right\} ds$$

Too complicated because of unknown μ in between the source and the camera, and also inverse square law.

* Ignore inverse square law and attenuation.
  → works resonably well in practice

Then,

$$\phi(\ell,\theta) = \int_{-\infty}^{\infty} A(x(s), y(s))\, ds$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(x,y)\, \delta(x\cos\theta + y\sin\theta - \ell)\, dx\, dy$$

⇒ same as CT imaging equation

* There is one difference: in CT, we first take the logarithm of the data to arrive at the necessary projections:

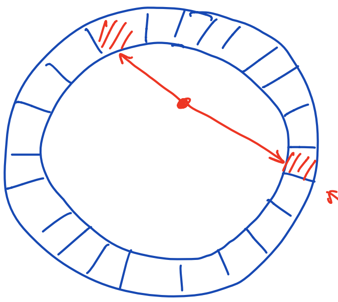
$$g(\ell,\theta) = -\ln\left(\frac{I_d}{I_o}\right)$$

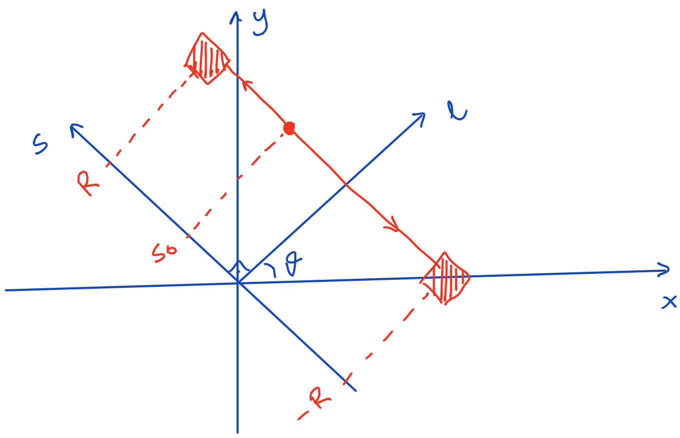
* In SPECT, recorded # of photons directly provides the projection data.

<u>Reconstruction</u>: same as in CT ⇒ filtered backprojection

Iterative reconstruction: First ignore inverse square law and attenuation and reconstruct. Then incorporate attenuation and depth dependency and reconstruct again, ...

PET Image Formation :





line of response : the line joining two opposing detectors that identified an event.

$\Longrightarrow$ event must have happened along that line.

$$N_c(s_0) = N_0 \cdot \underbrace{\exp\left\{-\int_{s_0}^{R} \mu(x(s'),y(s');E)\,ds'\right\}}_{\text{\% that survive and hit the upper detector}} \cdot \underbrace{\exp\left\{-\int_{-R}^{s_0} \mu(x(s'),y(s');E)\,ds'\right\}}_{\text{\% that survive and hit the lower detector}}$$

coincident photon count

# of annihilations

% of coincident photons

$$= N_0 \cdot \underbrace{\exp\left\{-\int_{-R}^{R} \mu(x(s'),y(s');E)\,ds'\right\}}_{\text{total attenuation along the line}}$$

total attenuation along the line <u>independent of</u> source location.

* Over the whole line :

$$\phi(\ell,\theta) = K \cdot \int_{-R}^{R} A(x(s),y(s)) \cdot \exp\left\{-\int_{-R}^{R} \mu(x(s'),y(s');E)\,ds'\right\} ds$$

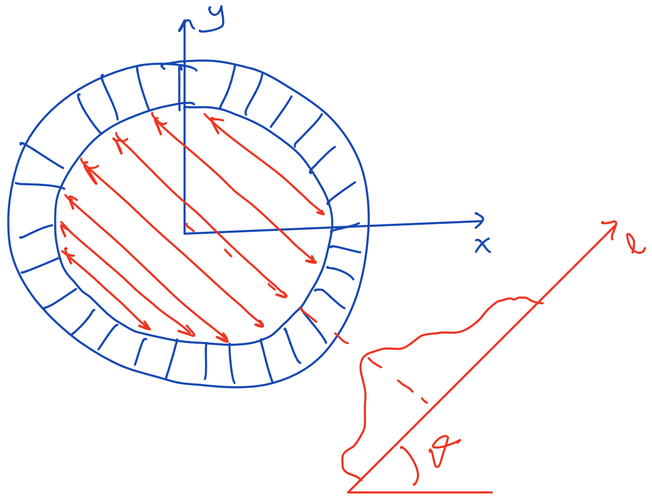
some constant

$$= K \cdot \underbrace{\int_{-R}^{R} A(x(s),y(s))\,ds}_{\text{total events along the line}} \cdot \underbrace{\exp\left\{-\int_{-R}^{R} \mu(x(s),y(s);E)\,ds\right\}}_{\text{attenuation along the line}}$$

* if we ignore attenuation:

$$\phi(\ell,\theta) = K \cdot \int_{-R}^{R} A(x(s),y(s))\,ds \quad \Big\} \text{ projection of } A(x,y) \text{ along angle } \theta$$

* organize the response into parallel-ray projection



* <u>Reconstruction</u> : filtered backprojection

* <u>Attenuation correction</u> : if $\mu(x,y)$ were known

$$\phi_c(\ell,\theta) = \frac{\phi(\ell,\theta)}{\underbrace{K \cdot \exp\left\{-\int_{-R}^{R} \mu(x(s),y(s);E)\,ds\right\}}_{\text{attenuation}}} = \int_{-R}^{R} A(x(s),y(s))\,ds$$

← actual measured activity

@511 keV in PET

$\mu(x,y)$ can be reconstructed from a CT scan.

@ 80-120 keV

$\rightarrow$ need to estimate μ @ 511 keV.

$\Rightarrow$ motivation for combined PET/CT scanner.