FALL 2021 - EEE 473/573 Medical Imaging
HW1 Solutions

1

There are endless possibilities of images.

2

a) **Linearity:** Let $g'(x,y)$ be the response of the system to the input $f'(x,y) = \sum_{k=1}^{K} w_k f_k(x,y)$:

$$\begin{aligned} g'(x,y) = f'(x,-1) + f'(0,y) &= \sum_{k=1}^{K} w_k f_k(x,-1) + \sum_{k=1}^{K} w_k f_k(0,y) \\ &= \sum_{k=1}^{K} w_k (f_k(x,-1) + f_k(0,y)) = \sum_{k=1}^{K} w_k g_k(x,y), \end{aligned}$$

where $g_k(x,y) = f_k(x,-1) + f_k(0,y)$. So, the system is LINEAR.

Shift Invariance: Let $g'(x,y)$ be the response of the system to the input $f'(x,y) = f(x - x_0, y - y_0)$:

$$\begin{aligned} g'(x,y) = f'(x,-1) + f'(0,y) &= f(x - x_0, -1 - y_0) + f(-x_0, y - y_0) \\ &\neq g(x - x_0, y - y_0), \end{aligned}$$

where $g(x - x_0, y - y_0) = f(x - x_0, -1) + f(0, y - y_0)$. So, the system is NOT SHIFT INVARIANT.

b) **Linearity:** Let $g_k(x,y) = \max\{f_k(x,y), 0\}$, and let $g'(x,y)$ be the response of the system to the input $f'(x,y) = \sum_{k=1}^{K} w_k f_k(x,y)$

$$\begin{aligned} g'(x,y) = \max\{f'(x,y), 0\} &= \max\{\sum_{k=1}^{K} w_k f_k(x,y), 0\} \\ &\neq \sum_{k=1}^{K} w_k \max\{f_k(x,y), 0\} = \sum_{k=1}^{K} w_k g_k(x,y). \end{aligned}$$

So, the system is NOT LINEAR. [1]

[1] At first sight, the system may look linear. In fact, if all $f_k(x,y) = h(x,y)$ (i.e., if all $f_k(x,y)$ are identical), then the system will be linear. Otherwise, the system will not be linear. For example, for $f_1(x,y) = x$, and $f_2(x,y) = sin(x)$, one can see that the system is not linear.

Shift Invariance: Let $g'(x,y)$, be the response of the system to the input $f'(x,y) = f(x - x_0, y - y_0)$,

$$g'(x,y) = max(f'(x,y), 0) = max(f(x-x_0, y-y_0), 0) = g(x-x_0, y-y_0).$$

So the system is SHIFT INVARIANT.

3

Given $f(x,y) = e^{j2\pi(x+y)}$, evaluate the following for given x_1 and y_1:

a) $f(x,y)\delta(x-x_1, y+y_1) = e^{j2\pi(x_1-y_1)}\delta(x-x_1, y+y_1)$

b) $f(x,y) * \delta(x-x_1, y+y_1) = f(x-x_1, y+y_1) = e^{j2\pi(x-x_1+y+y_1)}$

c) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(x-x_1, 3y+y_1)f(x,y)dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{3}\delta(x-x_1, y+\frac{y_1}{3})f(x,y)dxdy = \frac{e^{j2\pi(x_1-\frac{y_1}{3})}}{3}$

d) $f(x+1, -y_1) * \delta(x-x_1, y+1) = f(x-x_1+1, -y_1) = e^{j2\pi(x-x_1+1-y_1)}$

4

Find the 2D Fourier transforms of the following continuous signals:

a) Using the scaling and sifting property of the impulse:

$$\begin{aligned} G(u,v) &= \mathcal{F}\Big\{\delta(\frac{x}{x_1}, y_1 y - 1)\Big\} \\ &= |\frac{x_1}{y_1}|\mathcal{F}\Big\{\delta(x, y - \frac{1}{y_1})\Big\} \\ &= |\frac{x_1}{y_1}|e^{j2\pi\frac{v}{y_1}} \end{aligned}$$

b) Using the linearity property of FT and the transform table:

$$G(u,v) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\delta(u-n, v-m)$$

c) Using the scaling and shifting properties of FT and the transform table:

$$G(u,v) = |\frac{1}{3y_1}|e^{-j2\pi(\frac{-ux_1}{3}+\frac{v}{y_1})}\mathrm{rect}(\frac{u}{3}, \frac{v}{y_1})$$

d) Using the scaling and product properties of FT and the transform table:

$$\begin{aligned} G(u,v) &= \mathcal{F}\{\mathrm{rect}(x_1 x, \frac{y}{y_1})e^{j2\pi(u_0 x + 4v_0 y)}\} \\ &= \mathcal{F}\{\mathrm{rect}(x_1 x, \frac{y}{y_1})\} * \mathcal{F}\{e^{j2\pi(u_0 x + 4v_0 y)}\} \\ &= |\frac{y_1}{x_1}|\mathrm{sinc}(\frac{u}{x_1}, y_1 v) * \delta(u-u_0, v-4v_0) \\ &= |\frac{y_1}{x_1}|\mathrm{sinc}(\frac{u-u_0}{x_1}, y_1(v-4v_0)) \end{aligned}$$

e) Using the scaling and convolution properties of FT and the transform table:

$$\begin{aligned}
\mathcal{F}\big\{e^{-2\pi(4x^2+y^2)} * cos(2\pi x+\pi y)\big\} &= \mathcal{F}\big\{e^{-2\pi(4x^2+y^2)}\big\}\mathcal{F}\big\{cos(2\pi x+\pi y)\big\} \\
&= \mathcal{F}\big\{e^{-\pi((2\sqrt{2}x)^2+(\sqrt{2}y)^2)}\big\}\frac{1}{2}\Big(\delta\big(u-1, v-\frac{1}{2}\big)+\delta\big(u+1, v+\frac{1}{2}\big)\Big) \\
&= \frac{1}{4}e^{-\pi((\frac{u}{2\sqrt{2}})^2+(\frac{v}{\sqrt{2}})^2)}\frac{1}{2}\Big(\delta\big(u-1, v-\frac{1}{2}\big)+\delta\big(u+1, v+\frac{1}{2}\big)\Big) \\
&= \frac{1}{8}e^{-\pi(1/8+1/8)}\delta\big(u-1, v-\frac{1}{2}\big)+\frac{1}{8}e^{-\pi(1/8+1/8)}\delta\big(u+1, v+\frac{1}{2}\big) \\
&= \frac{e^{-\pi/4}}{8}\Big(\delta\big(u-1, v-\frac{1}{2}\big)+\delta\big(u+1, v+\frac{1}{2}\big)\Big)
\end{aligned}$$

f)

$$F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

and $f(x,y)$ is a real-valued signal,

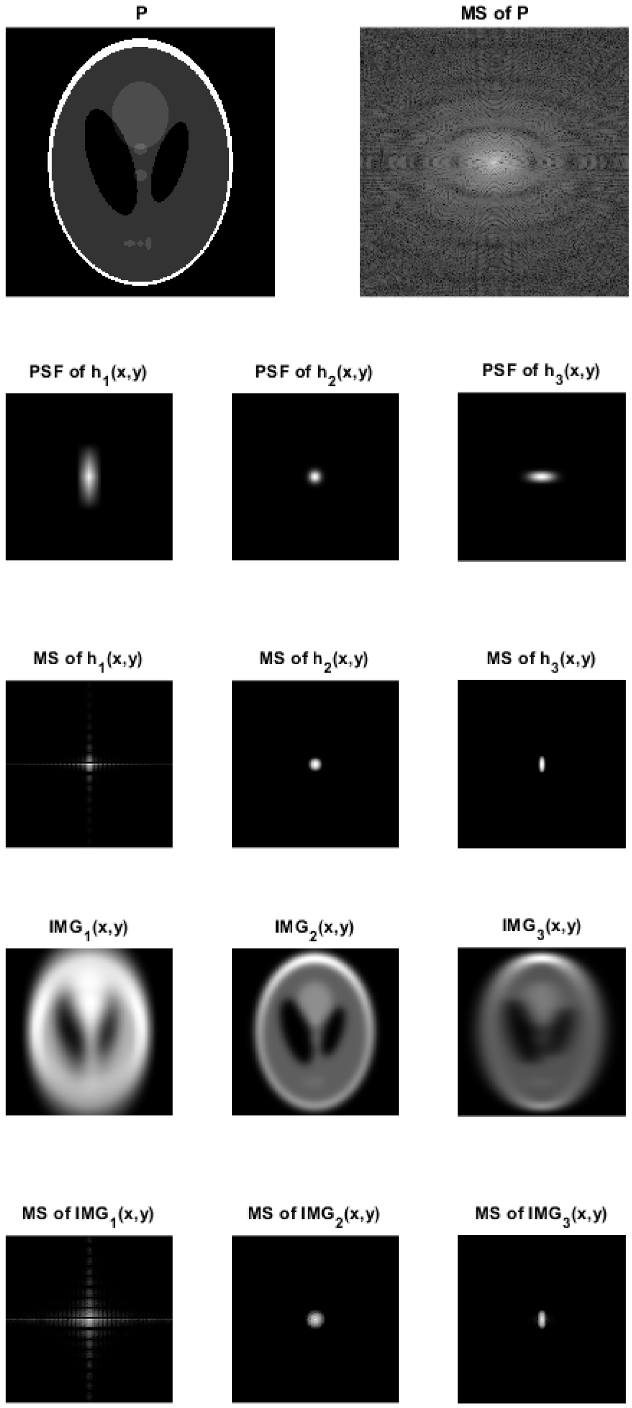
i.

$$\begin{aligned}
F^*(u,v) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[f(x,y)e^{-j2\pi(ux+vy)}]^* dxdy \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f^*(x,y)e^{j2\pi(ux+vy)}dxdy \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{j2\pi(ux+vy)}dxdy, \text{ since } f(x,y) \text{ is real valued,} \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(-\xi,-\zeta)e^{-j2\pi(u\xi+v\zeta)}d\xi d\zeta, \text{ with } \xi=-x \text{ and } \zeta=-y, \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi,\zeta)e^{-j2\pi(u\xi+v\zeta)}d\xi d\zeta, \text{ since } f(x,y)=f(-x,-y), \\
&= F(u,v).
\end{aligned}$$

ii. Very similar to (i).

5

Next page.


P
MS of P
PSF of h1(x,y)
PSF of h2(x,y)
PSF of h3(x,y)
MS of h1(x,y)
MS of h2(x,y)
MS of h3(x,y)
IMG1(x,y)
IMG2(x,y)
IMG3(x,y)
MS of IMG1(x,y)
MS of IMG2(x,y)
MS of IMG3(x,y)