Q1

HW #4

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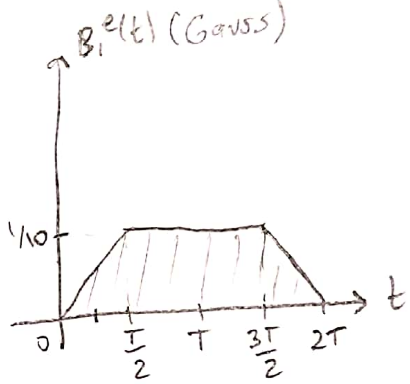
$$B_1^e(t) = \begin{cases} t\,\dfrac{2}{10T}, & 0 \le t \le \dfrac{T}{2} \\ \dfrac{1}{10}, & \dfrac{T}{2} \le t \le \dfrac{3T}{2} \\ \dfrac{4}{10} - \dfrac{2}{10T}t, & \dfrac{3T}{2} \le t \le 2T \\ 0, & \text{o.w.} \end{cases}$$

a)

$$\alpha = \gamma \int_0^{2T} B_1^e(t)\,dt$$



$$\alpha(t) = \begin{cases} \dfrac{\gamma t^2}{10T} \times 10^{-4}, & 0 \le t \le \dfrac{T}{2} \\ \gamma\left(\dfrac{T}{40} + \dfrac{\left(t - \frac{T}{2}\right)}{10}\right) \times 10^{-4}, & \dfrac{T}{2} < t \le \dfrac{3T}{2} \\ \gamma\left(\dfrac{T}{8} + \left(\dfrac{\left(\frac{1}{10} + \frac{4}{10} - \frac{2t}{10T}\right)\left(t - \frac{3T}{2}\right)}{2}\right)\right) \times 10^{-4}, & \dfrac{3T}{2} < t \le 2T \\ \gamma \dfrac{3T}{20} \times 10^{-4}, & 2T < t \\ 0, & \text{o.w.} \end{cases}$$

b)

$$\frac{\cancel{\pi}}{2} = 2\cancel{\pi}\cancel{\gamma}\,\frac{3T}{20} \times 10^{-4}$$

$$T = \frac{5}{3\cancel{\gamma}} 10^4 = \frac{5}{3(42.58 \times 10^6)} 10^4 \Rightarrow \boxed{T = 0.39142\,ms}$$

Q2

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$$M_z(t) = M_0(1-e^{-t/T_1}) + M_z(0^+)e^{-t/T_1}$$

$$M_{xy}(t) = M_0 \sin\alpha \, e^{j\theta} e^{-t/T_2} \longrightarrow \text{Rotating frame}$$

$$TR >> T2$$

M_z^{ss} is defined as the steady state value. So, if we try to calculate M_z after n repetitions, instead of M_0, we must use M_z^{ss} for the initial magnetization, s.t.

$$M_z^n(t) = M_0(1-e^{-t/T_1}) + M_z^{ss}\cos\alpha \, e^{-t/T_1} \quad (1)$$

Thus, $M_z^{n+1}(0) = M_z^n(TR) = M_z^{ss}$ (2) (Pulses are separated by TR)

So, if we plug (2) in (1):

$$M_z^n(TR) = M_0(1-e^{-\frac{TR}{T_1}}) + M_z^{ss}\cos\alpha \, e^{-\frac{TR}{T_1}}$$

$$M_z^{ss}(1-\cos\alpha \, e^{-\frac{TR}{T_1}}) = M_0(1-e^{-\frac{TR}{T_1}})$$

$$\Rightarrow \boxed{M_z^{ss} = M_0 \frac{1-e^{-\frac{TR}{T_1}}}{1-\cos\alpha \, e^{-\frac{TR}{T_1}}}}$$

Since M_z^{ss} replaces M_0 for initial values,

$$M_{xy}(t) = M(0^+)\sin\alpha \, e^{j\theta} e^{-t/T_2}$$

$$\boxed{M_{xy}(t) = M_z^{ss}\sin\alpha \, e^{j\theta} e^{-t/T_2}}$$

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Q3

$G_x = G_y = G_z = \pm 1\ G/cm$

Desired: $z = 10cm$, $\Delta z = 5mm$

a) $\Delta \nu = \Delta z\, \gamma G_z = (5\times10^{-1}cm)\left(42.58\ \frac{MHz}{T}\right)\left(10^{-4}\ \frac{T}{cm}\right)$

$= \boxed{2.129\,kHz}$

b) $\Delta x = 1mm$, $\Delta y = 2mm$

$FWHM_x = \frac{1}{K_{x,extent}} = 1mm \Rightarrow K_{x,extent} = 1mm^{-1} \Rightarrow \boxed{a = 0.5mm^{-1}}$

$FWHM_y = \frac{1}{K_{y,extent}} = 2mm \Rightarrow K_{y,extent} = 0.5mm^{-1} \Rightarrow \boxed{b = 0.25mm^{-1}}$

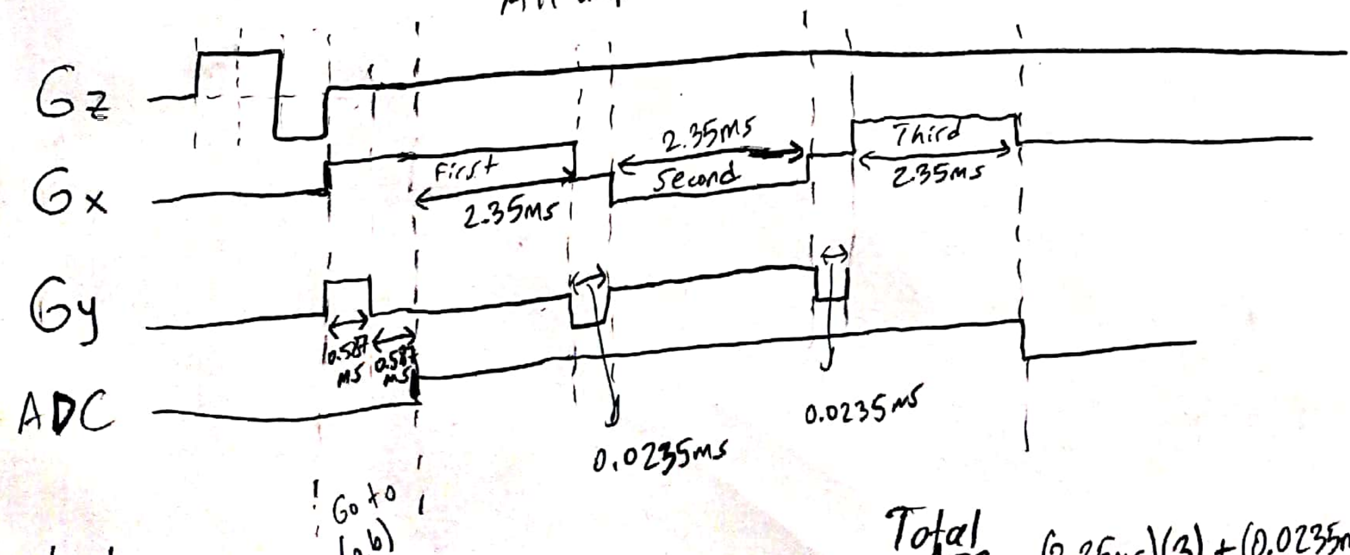
c) $FOV_y = 10cm \Rightarrow \boxed{\Delta k_y = 0.1cm^{-1} = 0.01mm^{-1}}$

$\frac{K_{y,extent}}{\Delta k_y} = \boxed{50\ lines}$

$t_y = \frac{\Delta k_y}{\gamma G_y} = \frac{10^{-1}}{(cm)\left(\frac{42.58\times10^6 Hz}{10^4 G}\right)\left(\frac{1G}{cm}\right)} = \boxed{0.0235ms}$

d) $f_s = \gamma G_x FOV_x = \frac{42.58\times10^6 Hz}{10^4 G} \cdot \frac{1G}{cm} \cdot 10cm = \boxed{42.58\,kHz}$

e) Assuming we do not read data during moving to (a,b).
All amplitudes are $\pm 1\ G/cm$.



Time to go to t_a: $\frac{5}{\ldots} = 1.17ms$ ... t_b: 0.587ms

Time to cover one line x: $= 2\times1.17ms = 2.35ms$

Total ADC time: $(2.35ms)(3) + (0.0235ms)(2)$
$= \boxed{7.097ms}$

Q4

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a)

$$IMG_1(x,y) = AM_0(x,y)\sin\alpha \, e^{-\frac{TE_1}{T_2(x,y)}}$$

$$IMG_2(x,y) = AM_0(x,y)\sin\alpha \, e^{-\frac{TE_2}{T_2(x,y)}}$$

$$\frac{IMG_1(x,y)}{IMG_2(x,y)} = e^{-\left(\frac{TE_1 - TE_2}{T_2(x,y)}\right)}$$

$$\frac{-(TE_1 - TE_2)}{T_2(x,y)} = \ln(IMG_1(x,y)) - \ln(IMG_2(x,y))$$

$$\Rightarrow \boxed{T_2(x,y) = \frac{TE_1 - TE_2}{\ln(IMG_2(x,y)) - \ln(IMG_1(x,y))}}$$