EEE 473/573 - Medical Imaging
Quiz 2 – Friday, 18 December 2020
Duration: 30 minutes

Write your Name and Student ID at the top of every page.
Write the following statement on the cover page and sign below.

***Honor Code**: "I have not given or received any aid during this quiz. I will do my share and take an active part in ensuring that others and I uphold the principles of honesty and integrity."*

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1) Find the 2D radon transform $g(l,\theta)$ of $f(x,y) = e^{-\left(\frac{x^2}{4}+\frac{y^2}{9}\right)}$. Simplify your answer as much as possible. **Hint:** Use Projection-Slice Theorem.

$$\boldsymbol{F(u,v) = \mathcal{F}_{2D}\{f(x,y)\} = 6\pi e^{-\pi^2(4u^2+9v^2)}}$$

$$\boldsymbol{G(\rho,\theta) = F(\rho\, cos\theta, \rho\, sin\theta) = 6\pi e^{-\pi^2(4\rho^2\, cos^2\theta+9\rho^2\, sin^2\theta)} = 6\pi e^{-\pi^2\rho^2(4cos^2\theta+9sin^2\theta)}}$$

$$\boldsymbol{g(l,\theta) = \mathcal{F}_{1D}^{-1}\{G(\rho,\theta)\} = \frac{6\pi}{\sqrt{\pi(4cos^2\theta+9sin^2\theta)}} e^{-\frac{l^2}{4cos^2\theta+9sin^2\theta}}}$$

2) Given the projection $g(l,\theta) = \delta(l - a\cdot cos\theta)$, find the associated object, $f(x,y)$. Simplify your answer as much as possible. **Hint:** Use Projection-Slice Theorem.

$$\boldsymbol{g(l,\theta) = \delta(l - a\cdot cos\theta)}$$

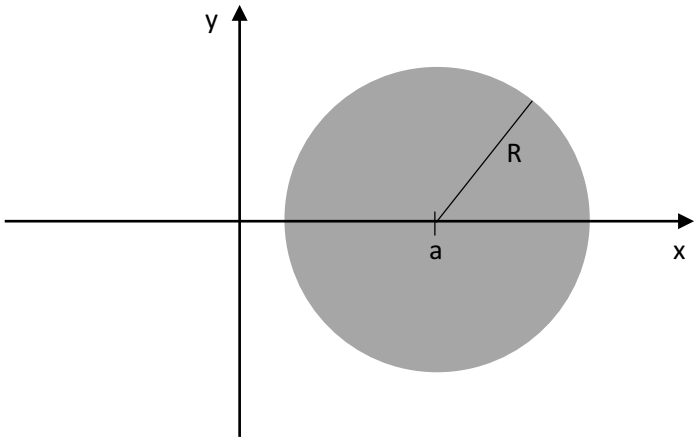
$$\boldsymbol{G(\rho,\theta) = \mathcal{F}_{1D}\{\delta(l - a\cdot cos\theta)\} = e^{-j2\pi\, a\cdot cos\theta\, \rho}}$$

$$\boldsymbol{F(u,v)|_{\substack{u=\rho cos\theta \\ v=\rho sin\theta}} = F(\rho\, cos\theta, \rho\, sin\theta) = G(\rho,\theta) = e^{-j2\pi\, a\cdot(\rho cos\theta)}}$$

$$\boldsymbol{F(u,v) = e^{-j2\pi\, a\, u}}$$

$$\boldsymbol{f(x,y) = \delta(x - a, y)}$$

3) What is $g(l, 0°)$ for the following object (i.e., the projection onto the x-axis)? Assume that $f(x, y) = 1$ in the shaded region and is zero otherwise.



$$g(l,\theta) = \begin{cases} \sqrt{R^2 - (a-l)^2}, & if\ a - R < l < a + R \\ 0, & otherwise \end{cases}$$

4) What is the Fourier transform of $g(l, 0°)$ from Question 3? **Hint:** Use Projection-Slice Theorem.

$$f(x,y) = rect\left(\frac{\sqrt{(x-a)^2 + y^2}}{2R}\right) = rect\left(\frac{\sqrt{x^2 + y^2}}{2R}\right) * \delta(x-a, y)$$

$$f(x,y) = rect\left(\frac{r}{2R}\right) * \delta(x-a, y)$$

$$F(u,v) = 4R^2 jinc\left(2R\sqrt{u^2 + v^2}\right) \cdot e^{-j2\pi au}$$

$$G(\rho,\theta) = F(\rho\, cos\theta, \rho\, sin\theta) = 4R^2 jinc\left(2R\sqrt{(\rho\, cos\theta)^2 + (\rho\, sin\theta)^2}\right) \cdot e^{-j2\pi a\rho cos\theta}$$

$$G(\rho,\theta) = 4R^2 jinc(2R\rho) \cdot e^{-j2\pi a\, \rho\, cos\theta}$$

$$G(\rho, 0°) = 4R^2 jinc(2R\rho) \cdot e^{-j2\pi a\, \rho}$$