EEE 473/573 Medical Imaging – Fall 2021-2022
Homework 3
Due 5 December 2021, Sunday at 23:59

GUIDELINES FOR HOMEWORK SUBMISSION

1. NO submission via E-MAIL (all email submissions will be discarded).
2. Submit a PDF file. Other file types will not be accepted. If there are any handwritten parts, you can scan them (make sure they are legible) and insert into the PDF file. Unclear presentation of results will be penalized heavily. No partial credits to unjustified answers.
3. For the part labeled as "MATLAB Question", you can choose to use MATLAB or other softwares (e.g., Python). Make sure to include the relevant codes at the end of the PDF file to be submitted. If your codes are missing, that question will NOT be graded.
4. This is a Turnitin submission. The Turnitin system requires the submitted file to contain at least 20 words in it. If you are submitting a Word file with scanned pages only, the file will be rejected by the system. You can type your name multiple times at the beginning of the file to overcome this problem.
5. Submission system will remain open for 1 day after the deadline. No points will be lost if you submit your assignment within 12 hours of the deadline. There will be a 50% penalty if you submit after 12 hours but within 24 hours past the deadline. No submissions beyond 24 hours past the deadline.

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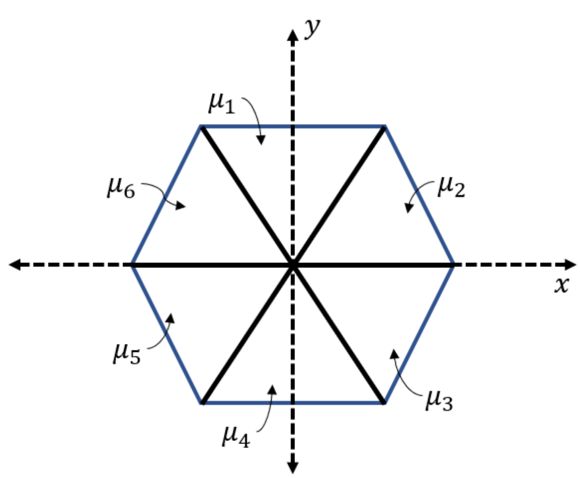
1) Assume that the 2D Radon transform of $f(x,y)$ is $g(l,\theta)$. Show that the 2D Radon transform of $f(x-x_0, y-y_0)$ is $g(l-x_0\cos\theta - y_0\sin\theta,\ \theta)$.

2) A 2D Gaussian object is given as $f(x,y) = e^{-\frac{x^2+y^2}{2}}$.

a) Find the 2D Radon transform, $g(l,\theta)$.

b) Assume that a Gaussian window function, $W(\rho) = e^{-\frac{\rho^2}{4}}$, is used during filtered backprojection. Find the reconstructed CT image $\hat{f}(x,y)$.

3) A 2D equilateral hexagon object is shown to the right, where each side of the hexagon is, $a = 10\ cm$. The linear attenuation coefficients are $\mu_1 = 0.25\ cm^{-1}$, $\mu_2 = 0.05\ cm^{-1}$, $\mu_3 = 0.35\ cm^{-1}$, $\mu_4 = 0.15\ cm^{-1}$, $\mu_5 = 0.35\ cm^{-1}$, and $\mu_6 = 0.20\ cm^{-1}$. This object is imaged with a CT scanner using parallel-ray geometry. Find and sketch $g(l, 60°)$.



4) For Question 3, assuming parallel-ray geometry,

a) What is the shortest length of the detector array that will cover the FOV?

b) Suppose the detector array has 256 elements. What is the minimum number of projections that need to be acquired to avoid aliasing? What is the resolution of the reconstructed image assuming the image covers the entire FOV?

5) **MATLAB Question:** As we covered in Chapter 6, the process of taking the projections of a 2D function is also called "Radon Transform", and the inverse process of reconstructing the images from projections is called the "Inverse Radon Transform". These transforms are available in MATLAB as built-in functions "*radon*" and "*iradon*".

Load the provided vessel image in to MATLAB using the following command:

```
P = double(rgb2gray(imread('vessel_and_catheter.png')));
```

This digital phantom shows a human vessel and a catheter placed in it (e.g., during an angiographic intervention). "P" is our "ideal" image.

a) Display the phantom P.

b) Using Radon transform, take projections of P. Reconstruct the image using inverse Radon transform function. Display the sinogram and the reconstructed image. Make sure that the number of angles you chose reconstructs the image with no visible artifacts.

c) Using the computed sinogram, plot the projections of P for the following angles: 0°, 45°, 90°, and 135°.

d) Repeat part (b), but for fewer number of projections. Make sure that there are some artifacts visible in the reconstructed image. Display the sinogram and the reconstructed image. What kind of an artifact are you seeing?

e) For the projection set in (b), reconstruct the image using three different filters:

i) Default filter in "*iradon*" (cropped ramp filter, i.e., ramp filter multiplied with a rect function. This is called "Ram-Lak" filter in Matlab),

ii) Hamming windowed filter,

iii) No filter (this would be direct backprojection reconstruction, without any filtering). These are options available in "*iradon*" function, so type "*help iradon*" to see how you can use these filters. Display the resulting images. Comment on the differences that you see in the reconstructed images. Which filter provides the best image? Why?

f) Repeat part (e) for part (d). Display the resulting images. Comment on the differences. Which filter provides the best image? Why?