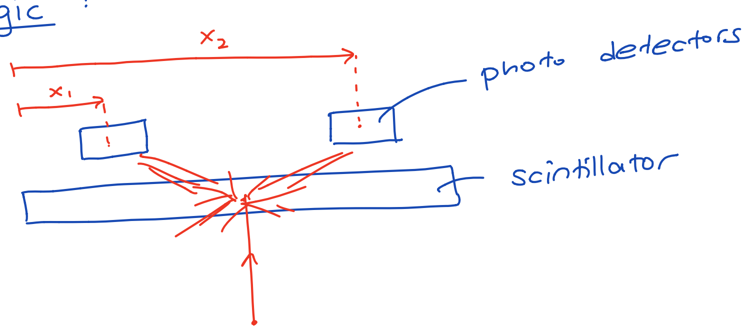
CHAPTER 8 - PLANAR SCINTIGRAPHY

Positioning Logic :

in 1D :



$$\bar{x} = \frac{I_1 \cdot x_1 + I_2 \cdot x_2}{I_1 + I_2}$$

Image Formation :

Event position Estimation :

* the height of the response from each photomultiplier tube is related to its distance to the scintillation event.

a_k : amplitude response from photomultiplier tube k

(x_k, y_k) : positions of " " "

$$z = \sum_{k=1}^{K} a_k$$: total amplitude response ("mass" of the light distribution)

Then,

$$X = \frac{1}{z} \sum_{k=1}^{K} x_k a_k$$

$$y = \frac{1}{z} \sum_{k=1}^{K} y_k a_k$$

"Center of mass" equations

So,

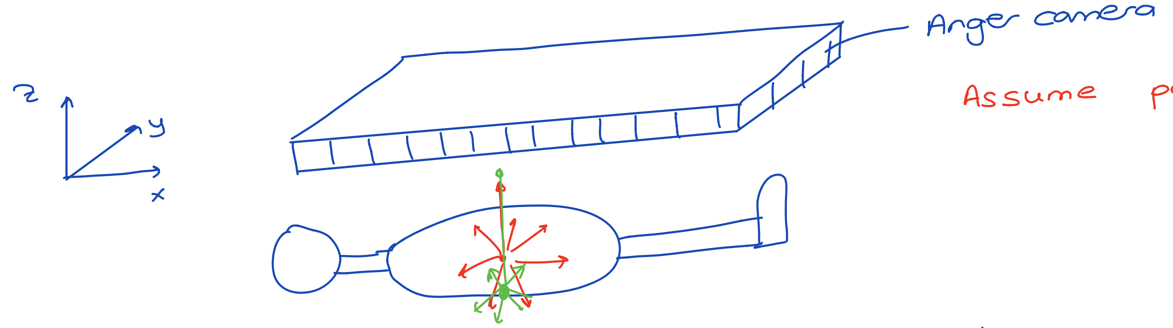
$$(\underbrace{x, y}_{\text{position}}, \underbrace{z}_{\text{amplitude of signal}})$$

Anger Camera Imaging Equation

Radioactivity in the body : $A(x,y,z)$

Energy of each photon (γ-ray photon) : E

We want to image : $\underbrace{\text{photon fluence rate}}_{\#\text{ of photons per unit area per unit time.}}$ coming from the patient



Assume parallel-hole collimator

* For radioactivity at point (x,y,z) only :

$$\phi_d = \underbrace{\frac{A}{4\pi z^2}}_{\text{photons are emitted in all directions}} \cdot \underbrace{\exp\left\{ - \int_z^0 \mu(x,y,z';E)\, dz' \right\}}_{\gamma\text{-photons have to pass through the body} \Rightarrow \text{attenuation along the line}}$$

* Combine all sources in line with the collimation :

$$\phi(x,y) = \int_{-\infty}^{0} \frac{A(x,y,z)}{4\pi z^2} \exp\left\{ - \int_z^0 \mu(x,y,z';E)\, dz' \right\} dz$$

* This is like projection
  - simpler because energy range is restricted
  - more complex because two sources of depth-dependent signal loss :
    1) inverse square law
    2) object dependent attenuation

$\Rightarrow$ Activity close to the camera contributes more to the image.

Planar source : a simplified case

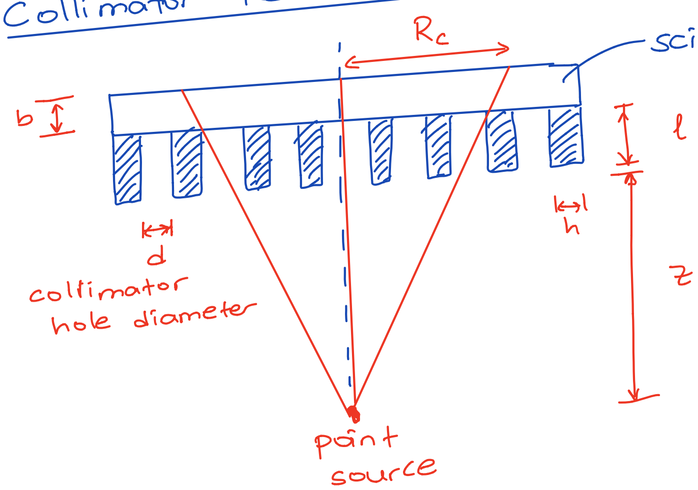
$$A(x,y,z) = A_{z_0}(x,y) \cdot \delta(z - z_0)$$

$$\phi(x,y) = \frac{A_{z_0}(x,y)}{4\pi z_0^2} \exp\left\{ - \int_{z_0}^{0} \mu(x,y,z';E)\, dz' \right\}$$

Even for planar source, intensity depends on μ along the path, which is not uniform.

Image Quality : resolution vs. sensitivity trade-off

Collimator Resolution :



Point source spread out in the image

from geometry : $\dfrac{d}{\ell} = \dfrac{R_c}{\ell + b + |z|}$

$$R_c = \frac{d}{\ell}(\ell + b + |z|)$$

resolution depends on depth $|z|$

So, targets farther away are blurred more.

Solution ? make collimator hole longer : $\ell \rightarrow 2\ell$

$$R_c = \frac{d}{2\ell}(2\ell + b + |z|) = d + \frac{b + |z|}{2\ell}$$

$\Rightarrow$ reduces sensitivity . fewer activity is detected.

* limit to resolution R_c : $\ell \rightarrow \infty$, $R_c \rightarrow d$
* making d smaller reduces sensitivity : less events detected.

Trade-off between resolution and sensitivity.

* Collimator PSF $\sim$ Gaussian with FWHM $= R_c$

Remember : for a Gaussian function

$$\text{FWHM} = 2\sigma\sqrt{2\ln 2}$$

So,

$$h_c(x, y; |z|) = \exp\left\{-4(x^2+y^2)\ln 2 / R_c^2(|z|)\right\}$$

depends on source depth

* Additional blurring in scintillator itself
  → called the intrinsic resolution of the Anger camera

Also modeled as a Gaussian:

$$h_I(x,y) = \exp\left\{-4(x^2+y^2)\ln 2 / R_I^2\right\}$$

does not depend on source depth

* So, for a planar source : $A(x,y,z) = A_{z_0}(x,y)\,\delta(z-z_0)$

$$\phi(x,y) = \frac{A_{z_0}(x,y)}{4\pi z_0^2}\exp\left\{-\int_{z_0}^{0}\mu(x,y,z';E)\,dz'\right\} * h_c(x,y;|z_0|) * h_I(x,y)$$

Typically, $R_I \ll R_c$. So, collimator's response dominates.

<u>SNR</u> : SNR $\propto \sqrt{\bar{N}}$ mean total number of acquired photons