Name Lastname	
Student ID	
Signature	
Classroom #	EE-

Q1 (25 pts)	
Q2 (25 pts)	
Q3 (25 pts)	
Q4 (15 pts)	
Q5 (10 pts)	
TOTAL	

EEE 473/573 – Spring 2015-2016
FINAL EXAM

13 May 2016, 15:30-18:00

- Open book, open notes.
- Provide appropriate explanations in your solution and **show intermediate steps clearly. No credit will be given otherwise**.

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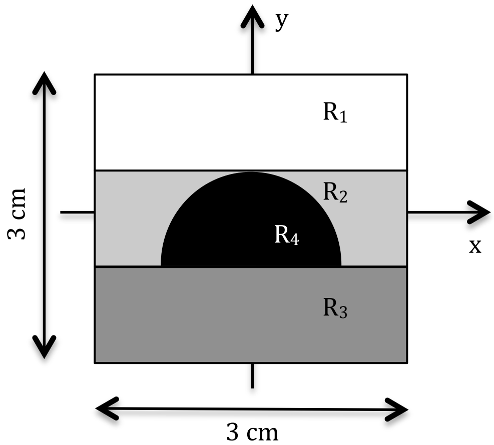
1) [25 points] Nuclear Medicine:
Consider the 2D object shown on the right.



- R_1, R_2, and R_3 are 3 cm x 1 cm rectangles.
- R_4 is a semi circle.
- The linear attenuation coefficient for each region is:
  $\mu_1 = 0.1\text{ cm}^{-1}$, $\mu_2 = 0.2\text{ cm}^{-1}$, $\mu_3 = 0.3\text{ cm}^{-1}$, $\mu_4 = 0.4\text{ cm}^{-1}$.
- Only R_2, R_3, and R_4 contain radionuclides. Their relative concentrations are $f_2 = 2$, $f_3 = 3$, $f_4 = 4$.
  (The absolute values/units are not important for this question).
- Assume perfect detection and ignore inverse square law.

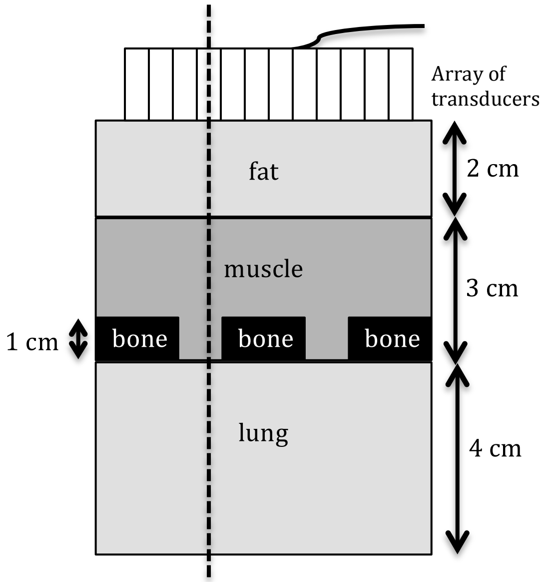
a) [13 points] We image the radioactivity using a 2D SPECT scanner. What is the local contrast of the projection $g_{SPECT}(l, 0°)$? Let $g_{SPECT}(0,0°)$ be used as the intensity of the object of interest (i.e., the semi circle). When $\theta = 0°$, the camera is located on the +y-axis (above the object) looking down.

b) [12 points] Now assume the radionuclides in part (a) are replaced by positron emitting radionuclides with the same concentrations. We image the radioactivity using a 2D PET scanner. What is the local contrast of the projection $g_{PET}(l, 0°)$? Again, let $g_{PET}(0,0°)$ be used as the intensity of the object of interest.

Hint: For local contrast calculations, there is no need to fully calculate $g(l, 0°)$ for all l.

2) [25 points] Ultrasound:

Assume that we have a 5 MHz ultrasound system as shown on the right, and we would like to image the lung interface with this system. Our system features an array of transducers, where each transducer transmits/receives to/from tissues directly along the line of its axis. The dashed line in the schematic demonstrates the line of axis for one of the transducers.



Tissue parameters are given as:

- $a_{fat} = 0.63$ dB cm^{-1}MHz^{-1}, $Z_{fat} = 1.35\times10^{-6}$ kg m^{-2}s^{-1}
- $a_{muscle} = 1$ dB cm^{-1}MHz^{-1}, $Z_{muscle} = 1.7\times10^{-6}$ kg m^{-2}s^{-1}
- $a_{bone} = 20$ dB cm^{-1}MHz^{-1}, $Z_{bone} = 5\times10^{-6}$ kg m^{-2}s^{-1}
- $a_{lung} = 40$ dB cm^{-1}MHz^{-1}, $Z_{lung} = 0.26\times10^{-6}$ kg m^{-2}s^{-1}

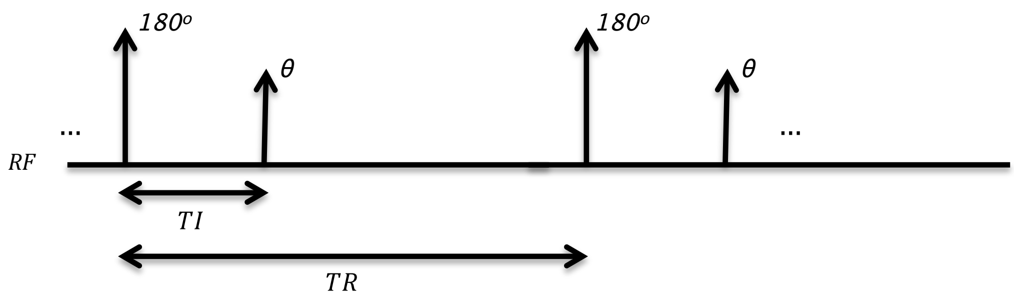
a) [10 points] For the transducers that do not "see" bone, what is the loss in dB for the ultrasound wave returning from the muscle/lung interface?

b) [10 points] For the transducers that "see" bone, what is the loss in dB for the ultrasound wave returning from the bone/lung interface?

c) [5 points] Based on these numbers, comment on how the presence of bone affects the image in ultrasound.

3) [25 points] A Fat Suppression Technique in MRI:
Consider the following MR sequence in which a 180° RF pulse is applied, followed by a θ degree RF excitation. After a total time of TR, the pulses are repeated. This is called an "inversion recovery sequence". Assume the following conditions:

- The magnetization has equilibrium value M_0.
- $TR \gg T_2$ for all tissues of interest.
- After numerous TRs, M_z magnetization right before the θ degree RF excitation will reach a steady-state value of M_z^{SS}.
- The MR signal is acquired right after θ RF pulse.
- For fat: T_1 = 290 ms and T_2 = 165 ms.
- For muscle: T_1 = 1130 ms and T_2 = 35 ms.



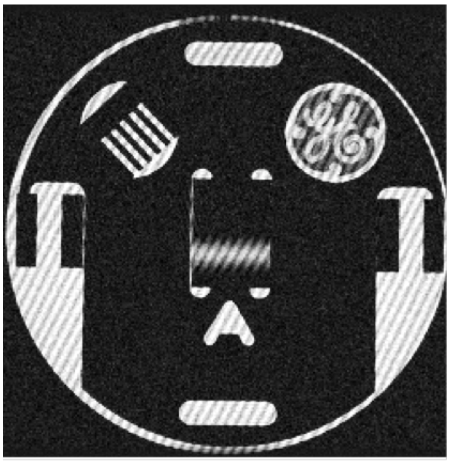
a) [10 points] Show that the "steady-state" magnetization M_z^{SS} is:

$$M_z^{SS} = M_0 \frac{1 - 2e^{-\frac{TI}{T_1}} + e^{-\frac{TR}{T_1}}}{1 + \cos\theta\, e^{-\frac{TR}{T_1}}}$$

b) [8 points] Normally, fat appears very bright in MRI images due to its relatively short T_1 and long T_2. We want to use the inversion recovery sequence to suppress the signal from fat while maximizing the signal from muscle. If TR = 1500 ms, find the optimum θ that maximizes the steady-state MR signal from muscle.

c) [7 points] For the TR and θ in part (b), what is the TI value that guarantees that the steady-state MR signal from fat is always zero?

4) [15 points] White Pixel Artifact in MRI:



Assume that we collect the k-space data in a typical line-by-line fashion. While collecting the data, a malfunction in the MRI scanner causes an additive "spike" signal to be recorded at k-space position (u_0,v_0). The amplitude of this spike is MUCH higher than the actual signal coming from the object that we are imaging, so we can model it as an additive delta at (u_0,v_0), i.e., $A\,\delta(u-u_0, v-v_0)$ where A is a constant.

The final displayed MRI image is the magnitude of the reconstructed image and it looks as shown on the right.

Derive an expression for the resulting image, $|g(x,y)|$. Express it in terms of the ideal image, $f(x,y)$. Assume that the ideal image $f(x,y)$ is real valued.

5) [10 points] Conjugate Symmetry for MRI?
Given fact: If $f(x,y)$ is a real-valued function, its 2D Fourier Transform has conjugate symmetry, i.e., $F(u,v) = F^*(-u,-v)$.

Porof. Zihni Sinir claims the following:
"MRI images are real valued, so they have conjugate symmetry in Fourier domain. In MRI, we are collecting the data in k-space (i.e., Fourier domain), typically line-by-line. So, we can take advantage of conjugate symmetry and acquire the data for only 50% of the k-space. This way, we can reduce the scan time by a factor of 2."

Explain his claim. Do you agree? If you think he is right, explain to what extent his claim is right. If you think he is wrong, explain to what extent his claim is wrong.