EEE 473/573 - Medical Imaging   Quiz 1   19/10/2021

Name	Student ID	Signature

1) Evaluate $f(x,y) = (3x^3 + 4y)\delta(x+2,y)$. Simplify your answer as much as possible.

$$f(x,y) = \left(3(-2)^3 + 4\cdot 0\right)\cdot \delta(x+2,y)$$
$$= -24 \cdot \delta(x+2,y)$$

2) Consider the input-output equation $g(x,y) = f(x-1,-y)$. Determine whether the system is linear and/or shift-invariant. Justify your answers.

* Linearity : $f'(x,y) = \sum_k w_k f_k(x,y)$, where $g_k(x,y) = f_k(x-1,-y)$

$$g'(x,y) = f'(x-1,-y) = \sum_k w_k f_k(x-1,-y)$$
$$= \sum_k w_k g_k(x,y)$$

The system is LINEAR.

* Shift Invariance : $f'(x,y) = f(x-x_0, y-y_0)$

$$g'(x,y) = f'(x-1,-y) = f(x-1-x_0, -y-y_0)$$
$$\neq g(x-x_0, y-y_0)$$

Because $g(x-x_0, y-y_0) = f(x-x_0-1, -y+y_0)$

The system is NOT Shift-invariant.

3) Calculate the 2D convolution: $f(x,y) = e^{j6\pi y} * e^{-\pi(9x^2+4y^2)}$.
Simplify your answer as much as possible.

$$F(u,v) = \mathcal{F}_{2D}\{e^{j6\pi y}\} \cdot \mathcal{F}_{2D}\{e^{-\pi(9x^2+4y^2)}\}$$

$$\mathcal{F}_{2D}\{e^{j6\pi y}\} = \mathcal{F}_{2D}\{e^{j2\pi(0.x+3y)}\} = \delta(u, v-3)$$

$$\mathcal{F}_{2D}\{e^{-\pi(9x^2+4y^2)}\} = \mathcal{F}_{2D}\{e^{-\pi((3x)^2+(2y)^2)}\} = \frac{1}{3.2} e^{-\pi\left(\left(\frac{u}{3}\right)^2+\left(\frac{v}{2}\right)^2\right)}$$

$$= \frac{1}{6} e^{-\pi\left(\frac{u^2}{9}+\frac{v^2}{4}\right)}$$

So, $F(u,v) = \delta(u,v-3) \cdot \frac{1}{6} e^{-\pi\left(\frac{u^2}{9}+\frac{v^2}{4}\right)}$

$$= \delta(u,v-3)\, \frac{1}{6} e^{-\pi\left(\frac{0^2}{9}+\frac{3^2}{4}\right)} = \delta(u,v-3) \cdot \frac{1}{6} e^{-\frac{9\pi}{4}}$$

Then,

$$f(x,y) = \frac{1}{6} e^{-\frac{9\pi}{4}} e^{j6\pi y}$$

4) Calculate the 2D Fourier Transform of $f(x,y) = rect\left(3x-1, \frac{y}{2}+1\right)$.
Simplify your answer as much as possible.

$$f(x,y) = rect\left(3\left(x-\frac{1}{3}\right), \frac{1}{2}(y+2)\right)$$

$$F(u,v) = \frac{1}{3 \cdot \frac{1}{2}} \cdot sinc\left(\frac{u}{3}, \frac{v}{1/2}\right) \cdot e^{-j2\pi\left(\frac{1}{3}u - 2v\right)}$$

$$= \frac{2}{3} sinc\left(\frac{u}{3}, 2v\right) e^{-j2\pi\left(\frac{u}{3} - 2v\right)}$$