FALL 2021 – EEE 473/573 Medical Imaging HW2 Solutions

1)

$$u = x - x_0 \implies du = dx$$
$$v = y - y_0 \implies dv = dy$$
$$f(x - x_0, y - y_0) = f(u, v)$$

Then,

$$\begin{aligned} g(\ell,\theta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\delta([u + x_0]\cos\theta + [v + y_0]\sin\theta - \ell)dudv \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\delta(u\cos\theta + v\sin\theta - [\ell - x_0\cos\theta - y_0\sin\theta])dudv \\ &= g(\ell - x_0\cos\theta - y_0\sin\theta\,,\theta) \end{aligned}$$

2)

a) $f(x,y) = e^{-\frac{(x^2+y^2)}{2}}$ is circularly symmetric, so, for all angles, $g(l,\theta)$ are the same. We can use the projection slice theorem here,

$$F(u,v) = \mathcal{F}_{2D}\{f(x,y)\} = 2\pi e^{-\pi^2 2(u^2+v^2)}$$

Then,

$$G(\rho,\theta) = F(\rho\cos\theta\,,\rho\sin\theta) = 2\pi e^{-\pi^2 2\rho^2}$$
$$g(\ell,\theta) = \mathcal{F}_{1D}^{-1}\{2\pi e^{-\pi^2 2\rho^2}\} = \sqrt{2\pi}e^{-\frac{\ell^2}{2}}$$

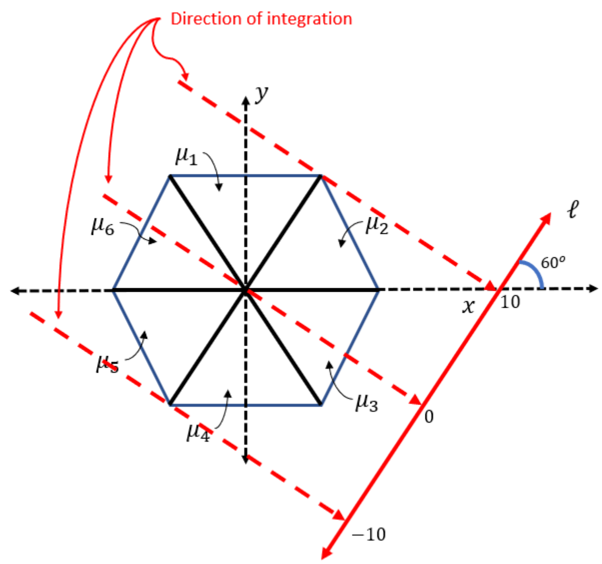
b)

$$\hat{f}(x,y) = \mathcal{F}_{2D}^{-1}\left\{G(\rho,\theta)W(\rho)|_{\rho=\sqrt{u^2+v^2},\theta=\operatorname{atan}\left(\frac{u}{v}\right)}\right\}$$

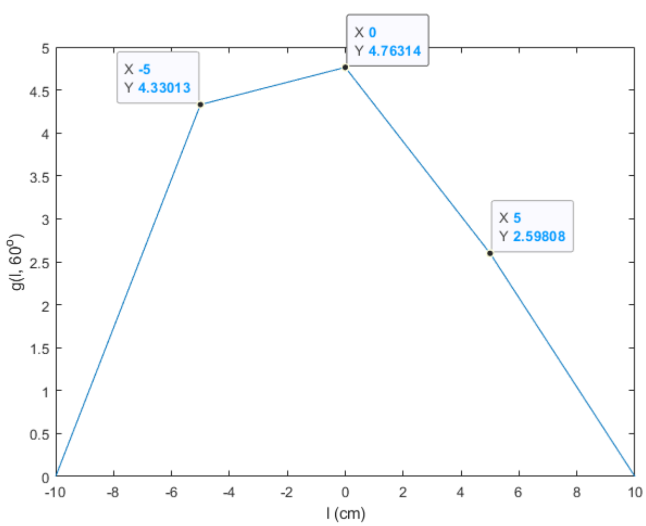
Note that $G(\rho,\theta)$ is independent of the angle θ.

$$\begin{aligned} \hat{f}(x,y) &= \mathcal{F}_{2D}^{-1}\left\{2\pi e^{-\pi^2 2\rho^2}e^{-\frac{\rho^2}{4}}|_{\rho=\sqrt{u^2+v^2}}\right\} \\ &= 2\pi\mathcal{F}_{2D}^{-1}\left\{e^{-\rho^2\left(\pi^2 2+\frac{1}{4}\right)}|_{\rho=\sqrt{u^2+v^2}}\right\} = 2\pi\mathcal{F}_{2D}^{-1}\left\{e^{-\frac{\pi(8\pi^2+1)}{\pi\quad 4}\rho^2}|_{\rho=\sqrt{u^2+v^2}}\right\} \\ &= 2\pi\mathcal{F}_{2D}^{-1}\left\{e^{-\pi\frac{(8\pi^2+1)}{4\pi}(u^2+v^2)}\right\} = 2\pi\frac{4\pi}{8\pi^2+1}e^{-\pi\frac{4\pi}{8\pi^2+1}(x^2+y^2)} \\ &= \frac{8\pi^2}{8\pi^2+1}e^{-\frac{4\pi^2}{8\pi^2+1}(x^2+y^2)} \\ &= 0.9875e^{-0.4937(x^2+y^2)} \approx f(x,y) \end{aligned}$$

3)



$$g(\ell, 60^o) = \begin{cases} (\mu_5 + \mu_4)|-10 - \ell|\sqrt{3}, & -10 \le \ell \le -5 \\ (\mu_5 + \mu_4)|\ell|\sqrt{3} + (\mu_6 + \mu_3)|-5 - \ell|\sqrt{3}, & -5 \le \ell \le 0 \\ (\mu_6 + \mu_3)|5 - \ell|\sqrt{3} + (\mu_1 + \mu_2)|\ell|\sqrt{3}, & 0 \le \ell \le 5 \\ (\mu_1 + \mu_2)|10 - \ell|\sqrt{3}, & 5 \le \ell \le 10 \\ 0, & \text{otherwise} \end{cases}$$



4)

a) Considering parallel ray geometry, the shortest length of the detector has to be 20 cm.

b) If detector has 256 elements, resolution of the image is:

$$\frac{20\ cm}{256} = 0.0781\ cm = 0.781\ mm$$

The minimum number of projections needed:

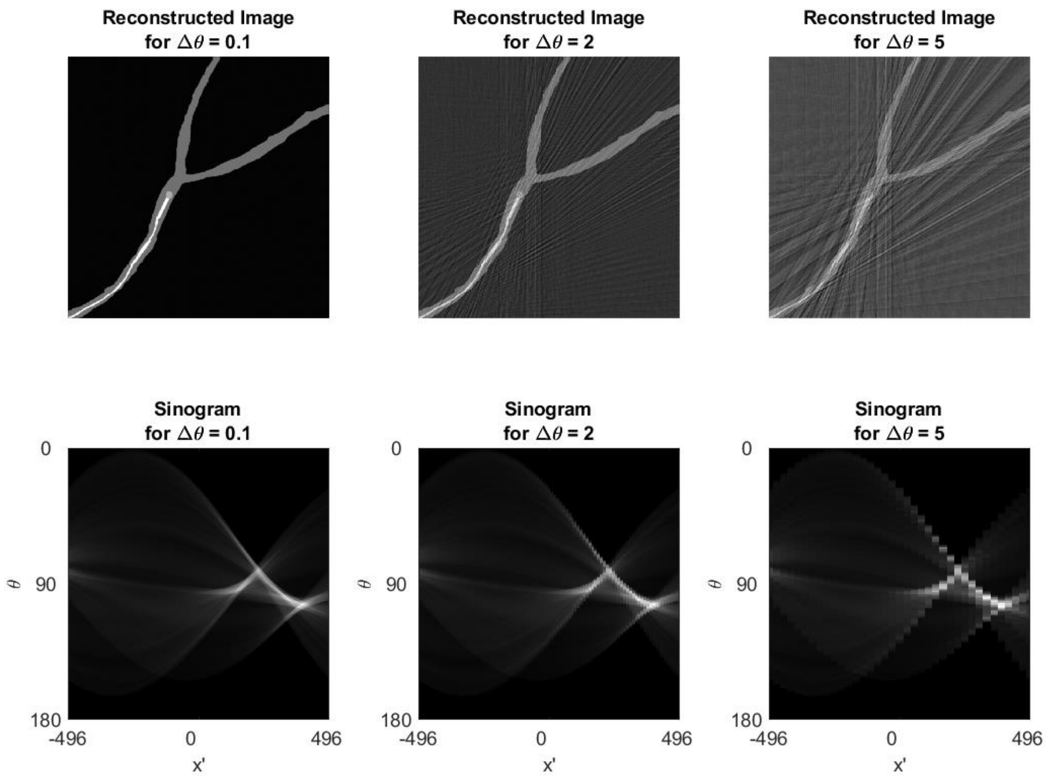
$$N_{proj} \geq \frac{\pi}{2} \times 256 \cong 402.1$$

Then, $N_{proj} = 403$.

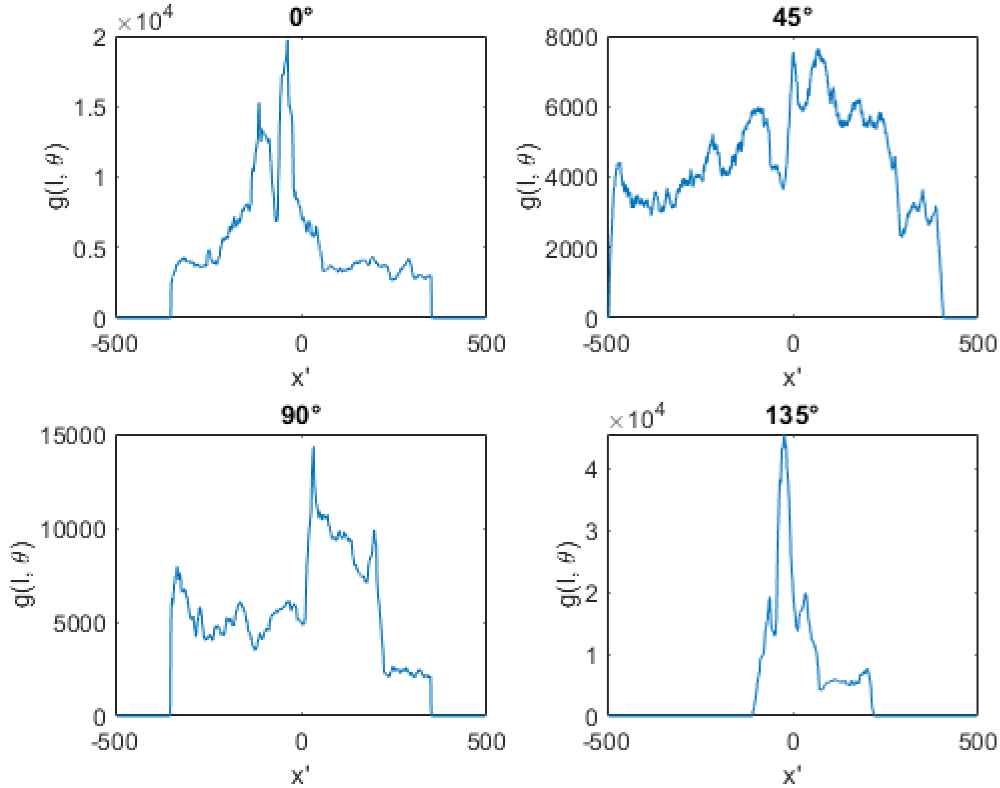
5)

a)

b) Reconstructions and sinograms for different $\Delta\theta$ (includes part **d**)

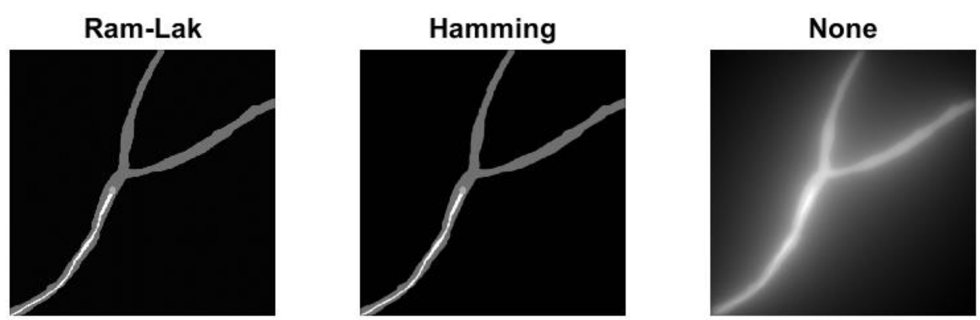


c)

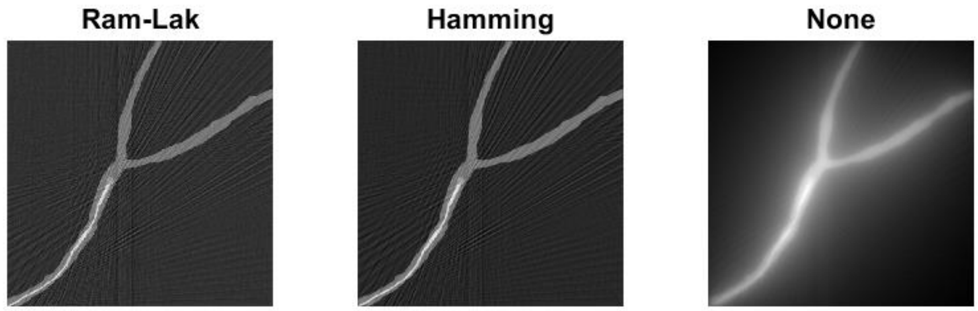


d)

e) Backprojection for $\Delta\theta = 0.1$



f) Backprojection for $\Delta\theta = 2$



Backprojection for $\Delta\theta = 5$