Q1 HW#3

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$f(x,y) \longrightarrow g(\ell,\theta)$

$f(x-x_0, y-y_0) \longrightarrow$?

$h(x,y) = f(x-x_0, y-y_0)$, $h(x,y) \longrightarrow k(\ell,\theta)$

$$R\{h(x,y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\,\delta(x\cos\theta + y\sin\theta - \ell)\,dx\,dy$$

$$k(\ell,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-x_0, y-y_0)\,\delta(x\cos\theta + y\sin\theta - \ell)\,dx\,dy$$

$x' = x - x_0$
$y' = y - y_0$

$$k(\ell,\theta) = \int_{-\infty}^{\infty} f(x',y')\,\delta(x'\cos\theta + x_0\cos\theta + y'\sin\theta + y_0\sin\theta - \ell)\,dx'dy'$$

$$k(\ell,\theta) = \int_{-\infty}^{\infty} f(x',y')\,\delta\Big(\underbrace{(\ell - x_0\cos\theta - y_0\sin\theta)}_{\text{new line argument}} - (x'\cos\theta + y'\sin\theta)\Big)\,dx'dy'$$

$$\Rightarrow k(\ell,\theta) = \boxed{R\{f(x-x_0, y-y_0)\} = g(\ell - x_0\cos\theta - y_0\sin\theta, \theta)}$$

Q2

$f(x,y) = e^{-\frac{x^2+y^2}{2}}$

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a) Let's find $F(u,v)$.

$$f(x,y) = e^{-\frac{x^2+y^2}{2}} = e^{-\pi\left(\left(\frac{x}{\sqrt{2\pi}}\right)^2 + \left(\frac{y}{\sqrt{2\pi}}\right)^2\right)} \xleftrightarrow{2D\ FT} F(u,v) = 2\pi e^{-2\pi^2(u^2+v^2)}$$

$$F(\rho) = 2\pi e^{-2\pi^2\rho^2}$$

No θ dependence

$$F(\rho\cos\theta, \rho\sin\theta) = F(u,v) \overset{\text{Projection Slice}}{=} G(\rho,\theta) = 2\pi e^{-2\pi^2\rho^2} = 2\pi e^{-\pi(\sqrt{2\pi}\rho)^2}$$

$\updownarrow$ 1D FT

$$g(\ell,\theta) = \sqrt{2\pi}\, e^{-\frac{\ell^2}{2}}$$

b) $W(\rho) = e^{-\frac{\rho^2}{4}} = e^{-\pi\left(\frac{\rho}{2\sqrt{\pi}}\right)^2} \xleftrightarrow{1D\ FT} 2\sqrt{\pi}\, e^{-4\pi^2\ell^2} = w(\ell)$

$$\hat{f}(x,y) = f(x,y) * \mathcal{R}^{-1}\{\tilde{h}(\ell)\}, \quad \tilde{h}(\ell) = w(\ell)$$

$$\Rightarrow h(r) = \mathcal{H}^{-1}(W(\rho))$$

$$h(r) = 4\pi e^{-4\pi^2 r^2}$$

$$\hat{f}(x,y) = e^{-\pi\left(\left(\frac{x}{\sqrt{2\pi}}\right)^2 + \left(\frac{y}{\sqrt{2\pi}}\right)^2\right)} * 4\pi e^{-\pi\left((2\sqrt{\pi}x)^2 + (2\sqrt{\pi}y)^2\right)}$$

$$\hat{F}(u,v) = 2\pi e^{-2\pi^2(u^2+v^2)} \cdot 4\pi \cdot \frac{1}{4\pi} e^{-\frac{1}{4}(u^2+v^2)} = 2\pi e^{-(u^2+v^2)\underbrace{\left(2\pi^2+\frac{1}{4}\right)}_{a}}$$

$$= 2\pi e^{-\pi\left(\left(\sqrt{\frac{a}{\pi}}u\right)^2 + \left(\sqrt{\frac{a}{\pi}}v\right)^2\right)}$$

$\updownarrow$ 2D FT

$$\hat{f}(x,y) = \frac{2\pi^2}{a} e^{-\frac{\pi^2}{a}(x^2+y^2)}$$

$$\hat{f}(x,y) = \frac{2\pi^2}{2\pi^2+\frac{1}{4}} e^{-\frac{\pi^2}{2\pi^2+\frac{1}{4}}(x^2+y^2)}$$

Q3

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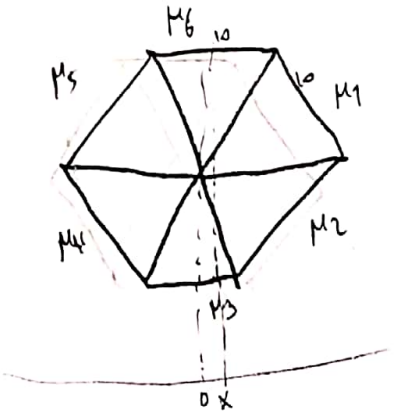
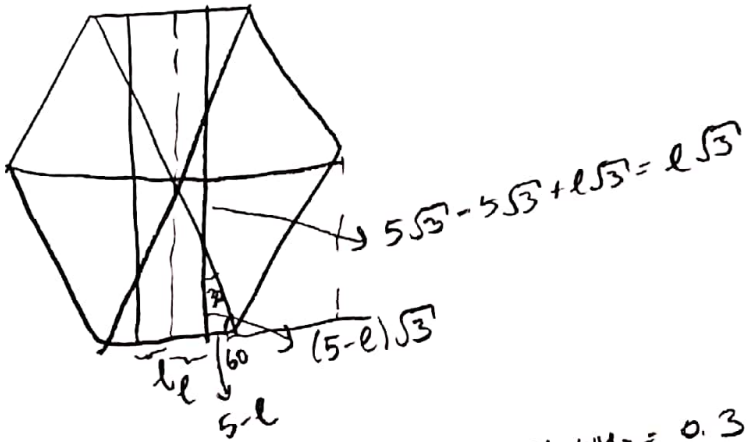
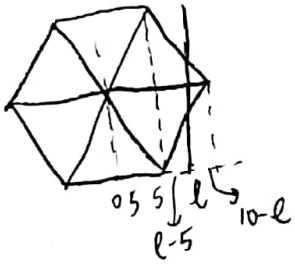
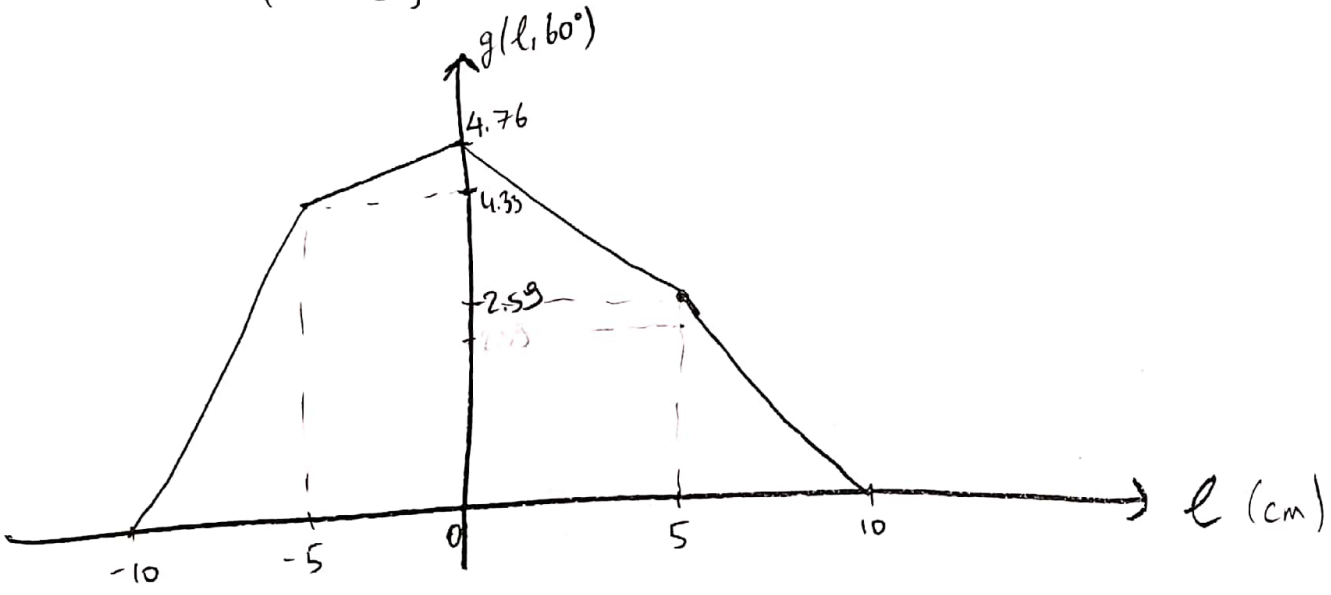






$\mu_1+\mu_2 = 0.3$
$\mu_3+\mu_6 = 0.55$
$\mu_4+\mu_5 = 0.5$

$$g(\ell, 60^\circ) = \begin{cases} (5-\ell)\sqrt{3}(\mu_3+\mu_6) + \ell\sqrt{3}(\mu_1+\mu_2), & 0 \le \ell < 5 \\ (10-\ell)\sqrt{3}(\mu_1+\mu_2), & 5 \le \ell \le 10 \\ (5+\ell)\sqrt{3}(\mu_3+\mu_6) - \ell\sqrt{3}(\mu_4+\mu_5), & -5 \le \ell < 0 \\ (10+\ell)\sqrt{3}(\mu_4+\mu_5), & -10 \le \ell < -5 \\ 0, & \text{o.w.} \end{cases}$$



Q4

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a) The smallest ~~cardiac~~ FOV's width/length ~~contour~~ is 10cm.

So, we must at least use a detector array which is 20 cms long.

b) $N_{proj} \geq \frac{\pi}{2} N_{points} \Rightarrow$ $\boxed{N_{proj_{min}} = 403}$

$$\Delta x = \frac{FOV}{256} = \frac{20cm}{256} = \boxed{0.078cm}$$

Small pixel width $\Rightarrow$ Better Resolution

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5) a)

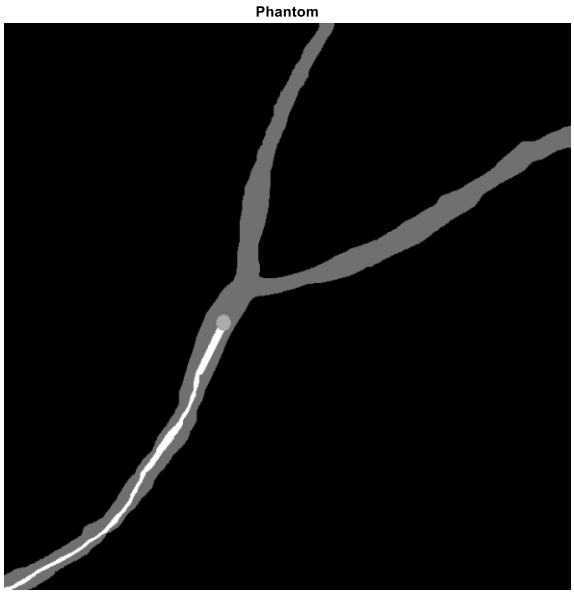

Figure 1.1 Original phantom, a human vessel, and a catheter in it.

b)

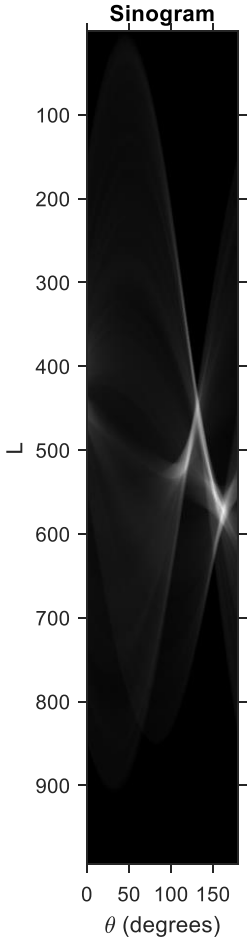

Figure 1.2 Sinogram image of the phantom (721 projections)

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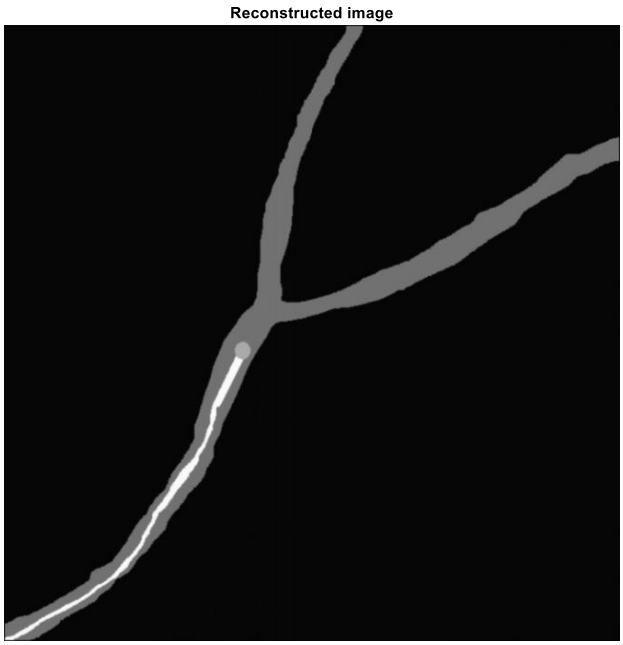

Figure 1.3 Reconstructed image

c)

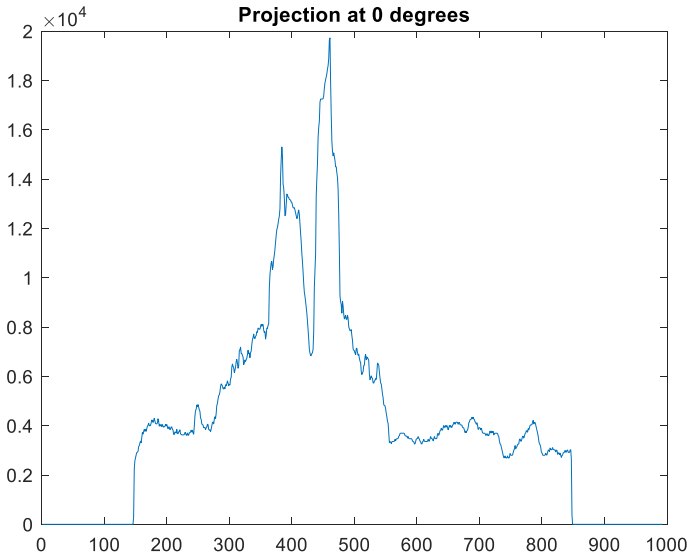

Figure 1.4 Projection of the image at 0 degrees.

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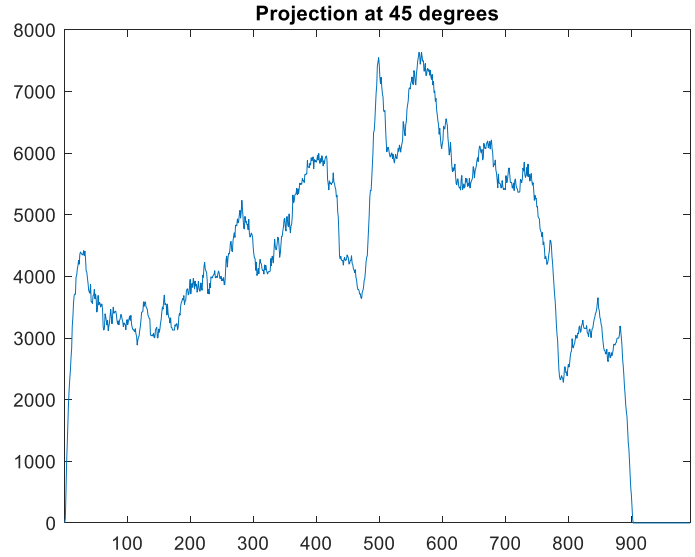

Figure 1.5 Projection of the image at 45 degrees.

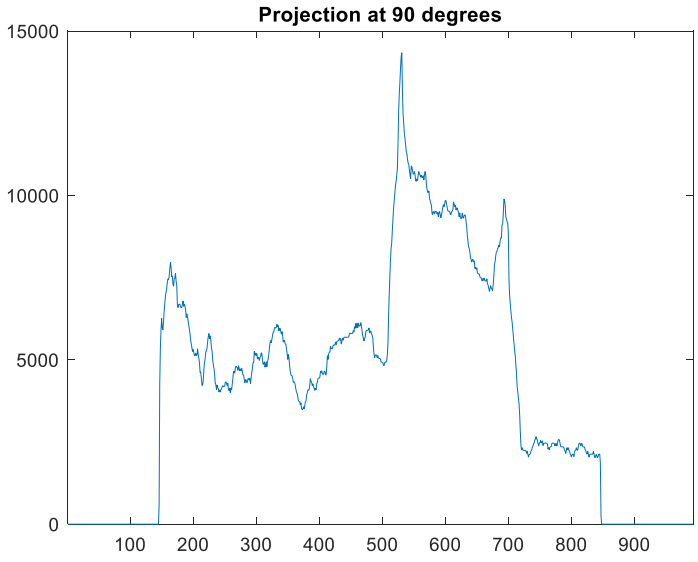

Figure 1.6 Projection of the image at 90 degrees.

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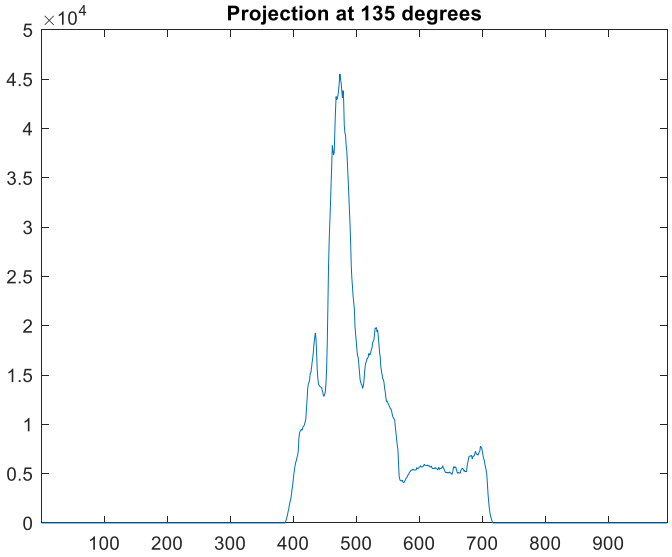

Figure 1.7 Projection of the image at 135 degrees.

d)

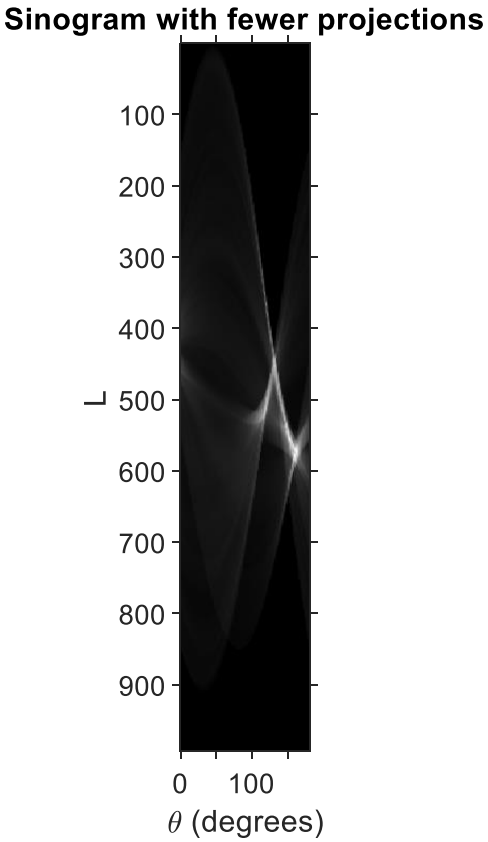

Figure 1.8 Sinogram image of the phantom with fewer projections (91 projections)

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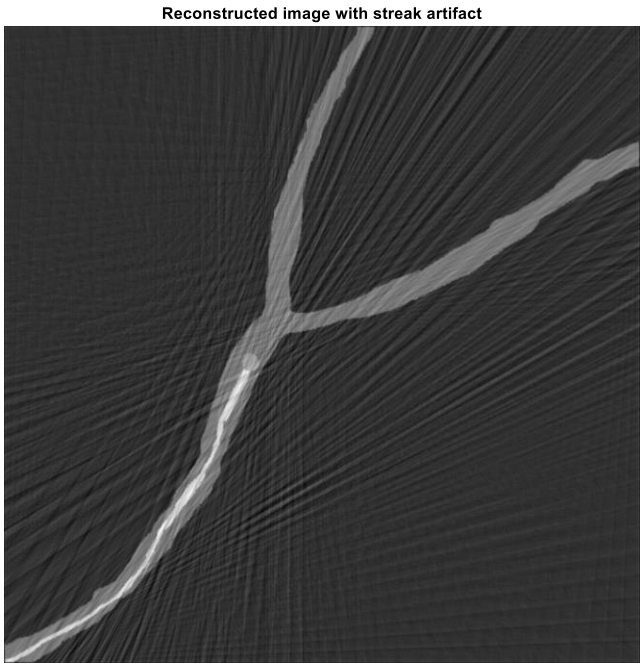

Figure 1.9 Reconstructed image with fewer projections. This artifact is a “streak” artifact.

e)

Figure 1.10 Reconstructed image, using Ram-Lak filter.

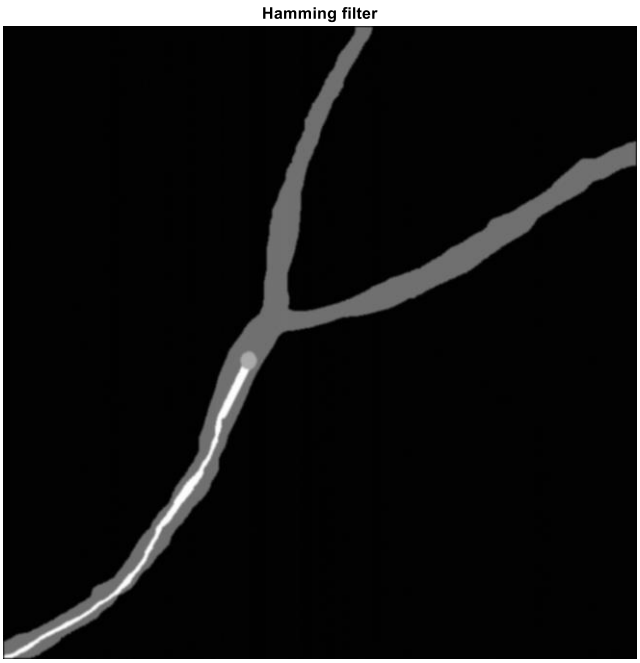

Figure 1.11 Reconstructed image, using Hamming filter.

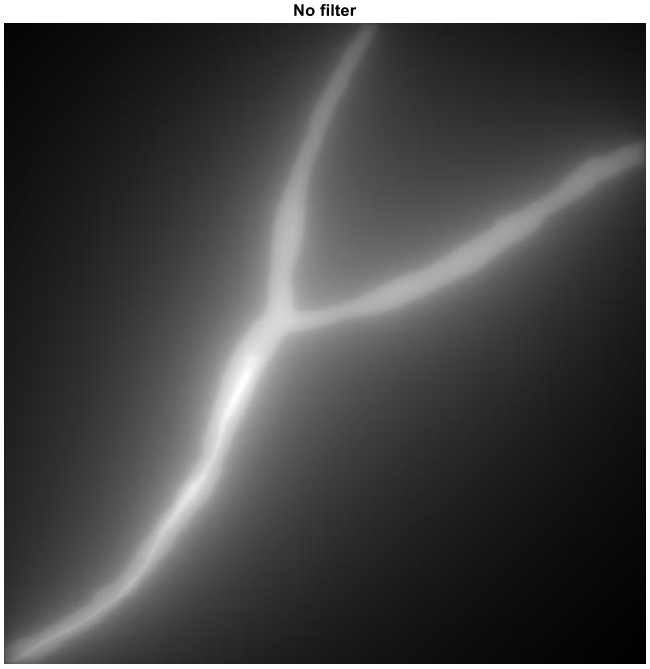

Figure 1.12 Reconstructed image, using no filter.

In the case of reconstruction with sufficient projections, Ram-Lak filter gives the best result. Even though the Hamming filter's result is good, it performs unnecessary low pass filtering in the Fourier domain. If we have enough projections, smoothing filter is unnecessary. The worst one is the one with no filter because it corresponds to the direct back-projection. In the case of back-projection, the reconstructed image has undesired haze in it.

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f)

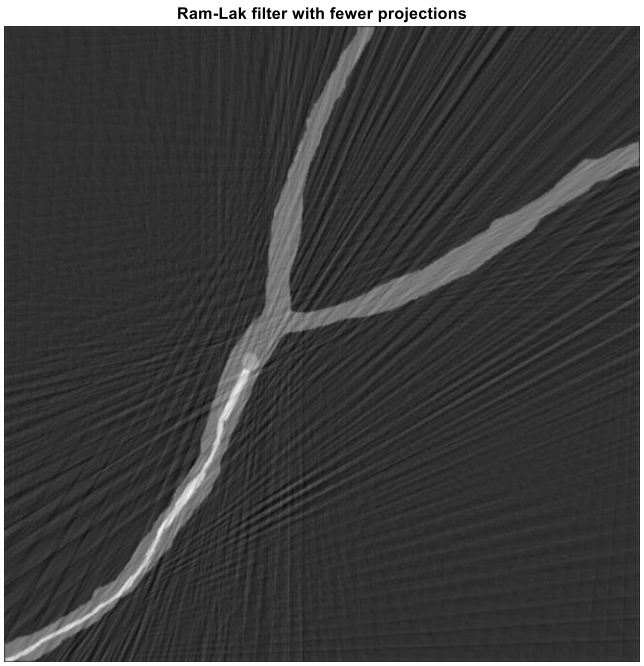

Figure 1.13 Reconstructed image, with fewer projections, using Ram-Lak filter

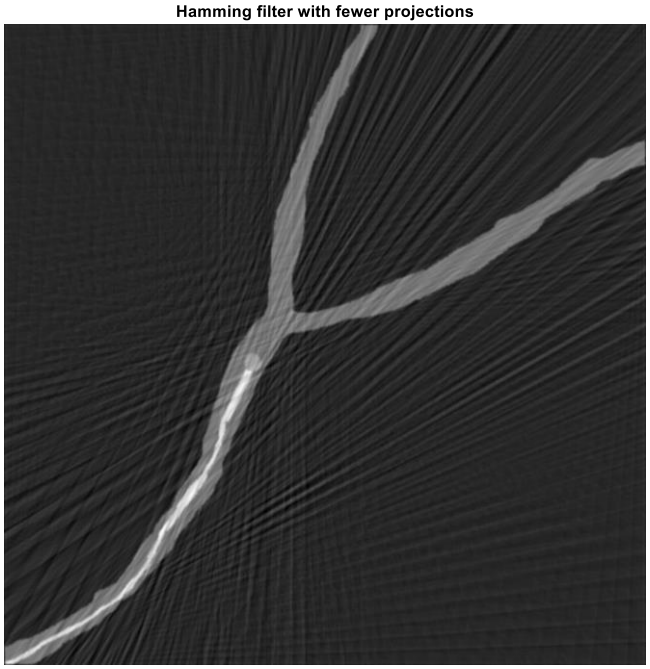

Figure 1.14 Reconstructed image, with fewer projections, using Hamming filter

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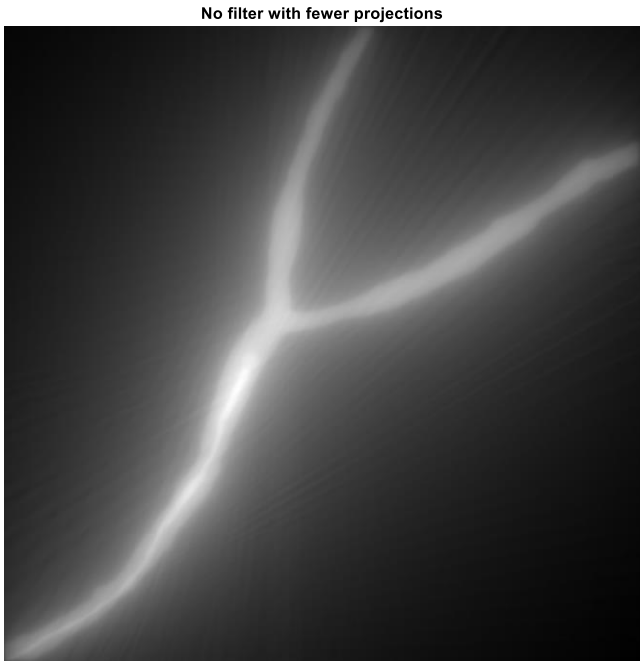

Figure 1.15 Reconstructed image, with fewer projections, using no filter

In the case of reconstruction with fewer projections, Hamming filter gives the best result. Even though the excessive low pass filtering of the Hamming filter was unnecessary in the case of reconstruction with sufficient projections, we need filtering to decrease the effect of the “streak” artifact in the case of reconstruction with fewer projections. Reconstruction with no filter still performs as the worst, due to the undesired haze in it.

Code

```
P = double(rgb2gray(imread('vessel_and_catheter.png')));
%a
imshow(P, []);
title("Phantom");

%b
theta = 0:0.25:180;
[R, xp] = radon(P, theta);

figure;
imshow(R,[], 'XData', theta);
title("Sinogram");
xlabel('\theta (degrees)')
ylabel('L')
axis on;

recon = iradon(R, theta);

figure;
imshow(recon, []);
```

```
title("Reconstructed image");

%c
proj0 = R(:,find(theta == 0));
proj45 = R(:,find(theta == 45));
proj90 = R(:,find(theta == 90));
proj135 = R(:,find(theta == 135));

figure;
plot(proj0);
title("Projection at 0 degrees");
xlim tight;

figure;
plot(proj45);
title("Projection at 45 degrees");
xlim tight;

figure;
plot(proj90);
title("Projection at 90 degrees");
xlim tight;

figure;
plot(proj135);
title("Projection at 135 degrees");
xlim tight;

%d
theta_less = 0:2:180;
R_less = radon(P, theta_less);
figure;
imshow(R_less, [], 'XData', theta_less);
title("Sinogram with fewer projections");
xlabel('\theta (degrees)')
ylabel('L')
axis on;

recon_alias = iradon(R_less, theta_less);

figure;
imshow(recon_alias, []);
title("Reconstructed image with streak artifact");
%this is called streak artifact

%e
recon_default = iradon(R, theta, 'linear');
```

```
figure;
imshow(recon_default, []);
title("Ram-Lak filter");

recon_hamming = iradon(R, theta, 'linear', 'Hamming');
figure;
imshow(recon_hamming, []);
title("Hamming filter");

recon_nothing = iradon(R, theta, 'linear', 'none');
figure;
imshow(recon_nothing, []);
title("No filter");

%f
recon_default_alias = iradon(R_less, theta_less,
'linear');
figure;
imshow(recon_default_alias, []);
title("Ram-Lak filter with fewer projections");

recon_hamming_alias = iradon(R_less, theta_less, 'linear',
'Hamming');
figure;
imshow(recon_hamming_alias, []);
title("Hamming filter with fewer projections");

recon_nothing_alias = iradon(R_less, theta_less, 'linear',
'none');
figure;
imshow(recon_nothing_alias, []);
title("No filter with fewer projections");
```