IE 411: Introduction to Nonlinear Optimization

Fall 2022 - Homework Assignment 3
Due: November 7, 2022

Question 1. For each of the following sets, determine whether they are convex or not (explain your choice)

a) $C_1 = \{x \in \mathbb{R}^n \mid \|x\|^2 = 1\}$.

b) $C_2 = \{x \in \mathbb{R}^n \mid \max_{i \in \{1,2,\ldots,n\}} x_i \leq 1\}$.

c) $C_3 = \{x \in \mathbb{R}^n \mid \min_{i \in \{1,2,\ldots,n\}} x_i \leq 1\}$.

d) $C_4 = \{x \in \mathbb{R}^2_{++} \mid x_1 x_2 \geq 1\}$.

Question 2. Let $C \subseteq \mathbb{R}^n$ be a convex set; f be a convex function over C and g be a strictly convex function over C. Show that the sum function $f+g$ is a strictly convex function over C.

Question 3. Let $g : \mathbb{R}_+ \to \mathbb{R}$ be defined on the non-negative real line as $g(x) := x^p$ for some $p > 1$. Is g a convex function? Prove or disprove.

Question 4. Show that the log-sum-exp function defined as $f : \mathbb{R}^n \to \mathbb{R}$, $f(x) := \log \sum_{i=1}^n e^{x_i}$ is not a strictly convex function.
(Hint: *Consider points from the line* $\{\mu 1 \mid \mu \in \mathbb{R}\}$, *where* $1 \in \mathbb{R}^n$ *is the vector of ones.*)

Question 5. Show that the following functions are convex over the specified domain C:

a) $f(x) = \|x\|_2^4$ over $\mathbb{R}^n$.

b) $f(x) = (2x_1^2 + 3x_2^2)(\frac{1}{2}x_1^2 + \frac{1}{3}x_2^2)$ over $\mathbb{R}^2$.

c) $f(x) = \max\{\sqrt{x_1^2 + x_2^2 + 20x_3^2 - x_1x_2 - 4x_2x_3 + 1}, (x_1^2+x_2^2+x_1+x_2+2)^2\}$ over $\mathbb{R}^3$.