IE 411: Introduction to Nonlinear Optimization

Fall 2022 - Homework Assignment 5

Due: December 20 2022

Question 1. Consider the optimization problem:

$$\begin{aligned}
\text{minimize} \quad & x_1 - 4x_2 + x_3 \\
\text{subject to} \quad & x_1 + 2x_2 + 2x_3 = -2 \\
& x_1^2 + x_2^2 + x_3^2 \leq 1.
\end{aligned}$$

a) Given a KKT point of this problem, must it be an optimal solution? Explain/show your reasoning.

b) Solve the problem using KKT conditions.

Question 2. Consider the optimization problem:

$$\begin{aligned}
\text{minimize} \quad & x_1^2 - x_2^2 - x_3^2 \\
\text{subject to} \quad & x_1^4 + x_2^4 + x_3^4 \leq 1.
\end{aligned}$$

a) Is this a convex programming problem? Explain/show your reasoning.

b) Find all the KKT points of the problem.

c) Find the optimal solution of the problem.

Question 3. Use KKT conditions to solve the following problem. Explain/show your reasoning in detail.

$$\begin{aligned}
\text{minimize} \quad & x_1^4 - x_2^2 \\
\text{subject to} \quad & x_1^2 + x_2^2 \leq 1 \\
& 2x_2 + 1 \leq 0.
\end{aligned}$$

Question 4. Consider the optimization problem:

$$\text{minimize} \quad (x_1 - 3)^2 + (x_2 - 2)^2$$
$$\text{subject to } x_1 + x_2 = 1$$
$$x_1, x_2 \geq 0.$$

a) Solve the problem using KKT conditions. Explain each step clearly.

b) Derive the Lagrange dual problem. What can you say about strong duality without solnving the dual problem.

c) Solve the dual problem.

Question 5. Consider the optimization problem:

$$\text{minimize} \quad x_1^2 + 2x_2^2 + 2x_1x_2 + x_1 - x_2 - x_3$$
$$\text{subject to } x_1 + x_2 + x_3 \leq 1$$
$$x_3 \leq 3.$$

a) Is the problem convex?

b) Find an optimal solution to this problem. Explain each step clearly.

c) Derive the Lagrange dual problem. What can you say about strong duality without solnving the dual problem.

d) Solve the dual problem.