Elec 4700

Assignment 3 – Monte Carlo/ Finite Difference Method

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Introduction:

The purpose of this experiment involved combining the Monte Carlo simulation and Finite difference method from the past two assignments and seeing what happens when there's a field pushing the electrons.

Part 1:

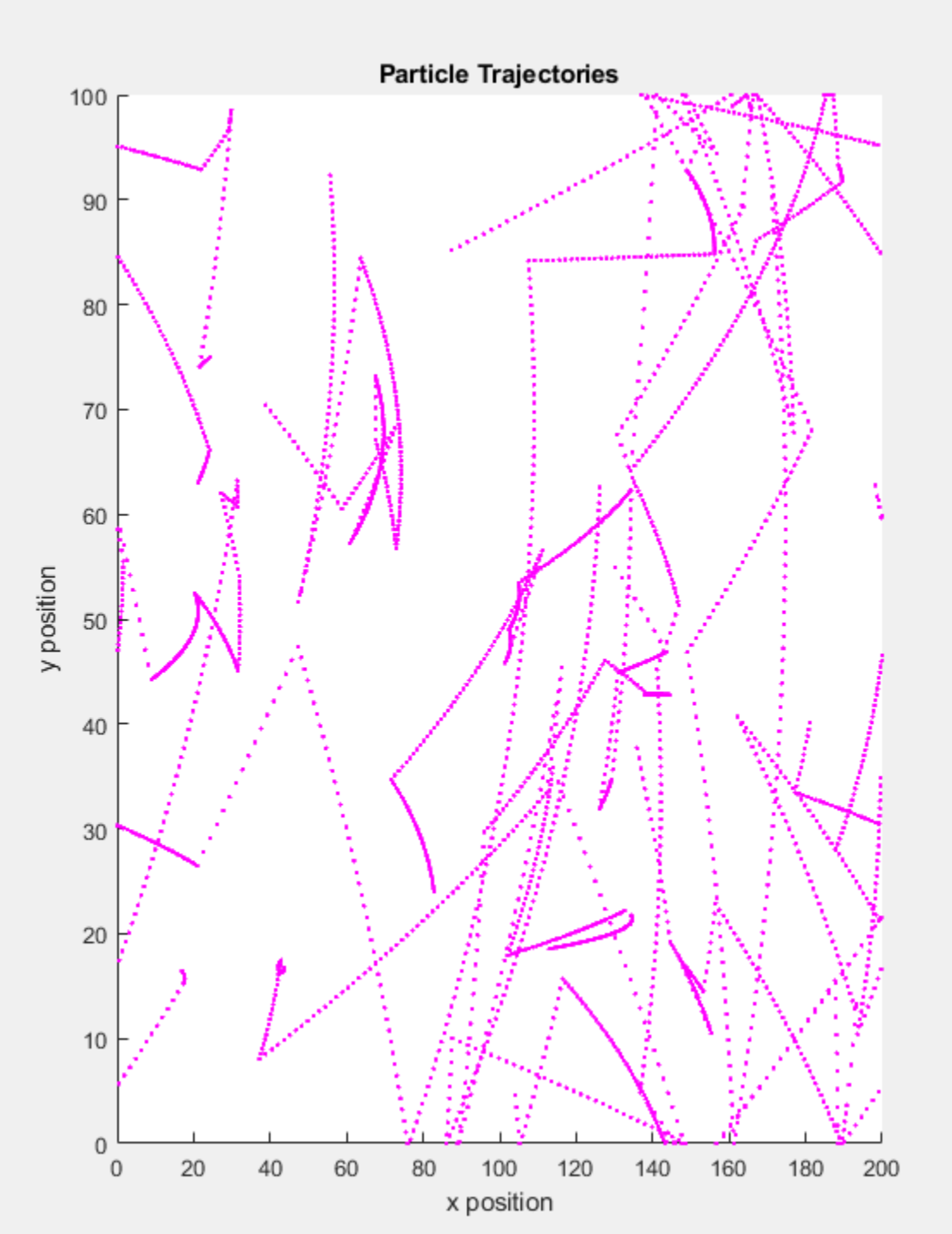

Figure 1: Particle trajectory without bottleneck

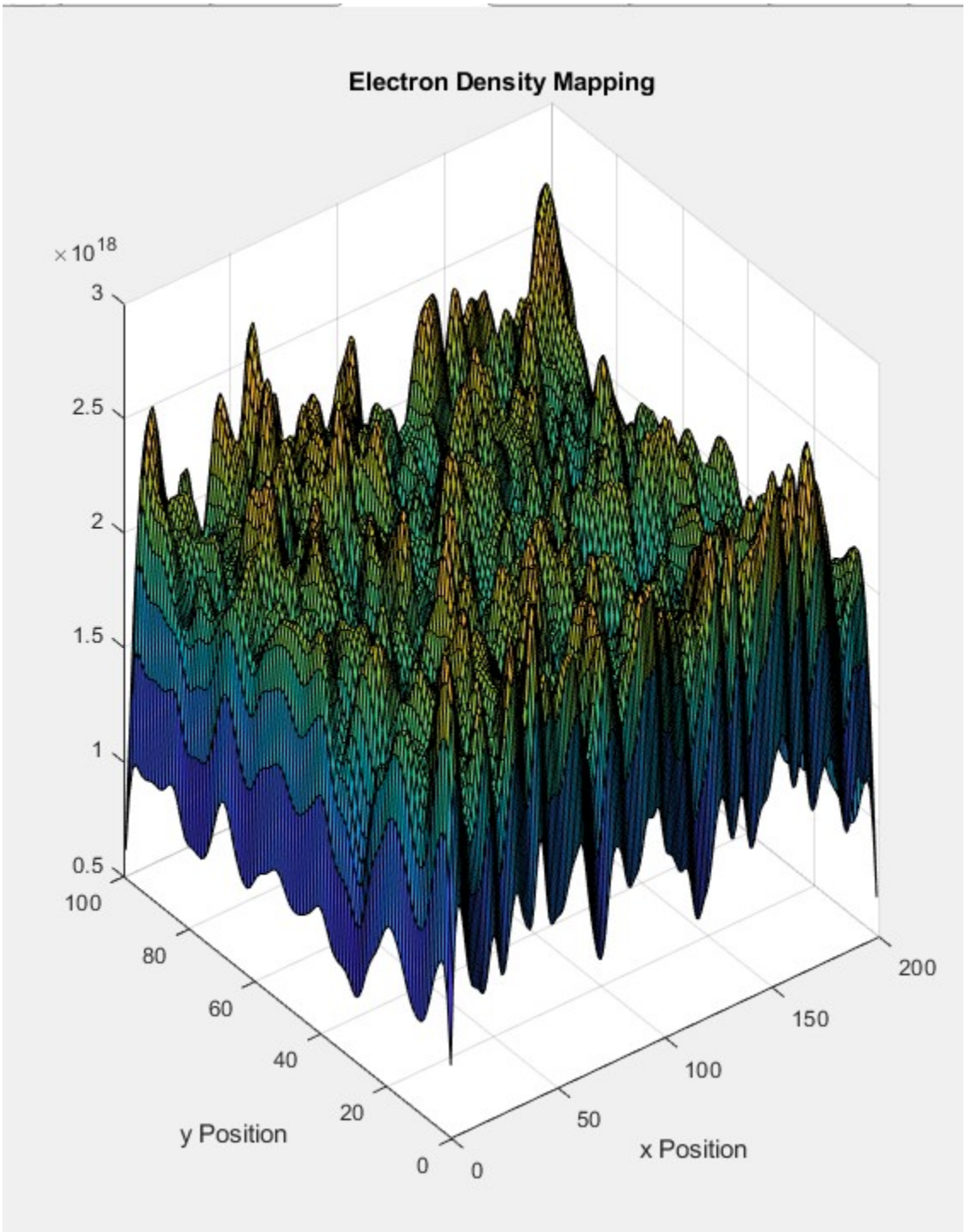

Figure 2: Electron Density Mapping.

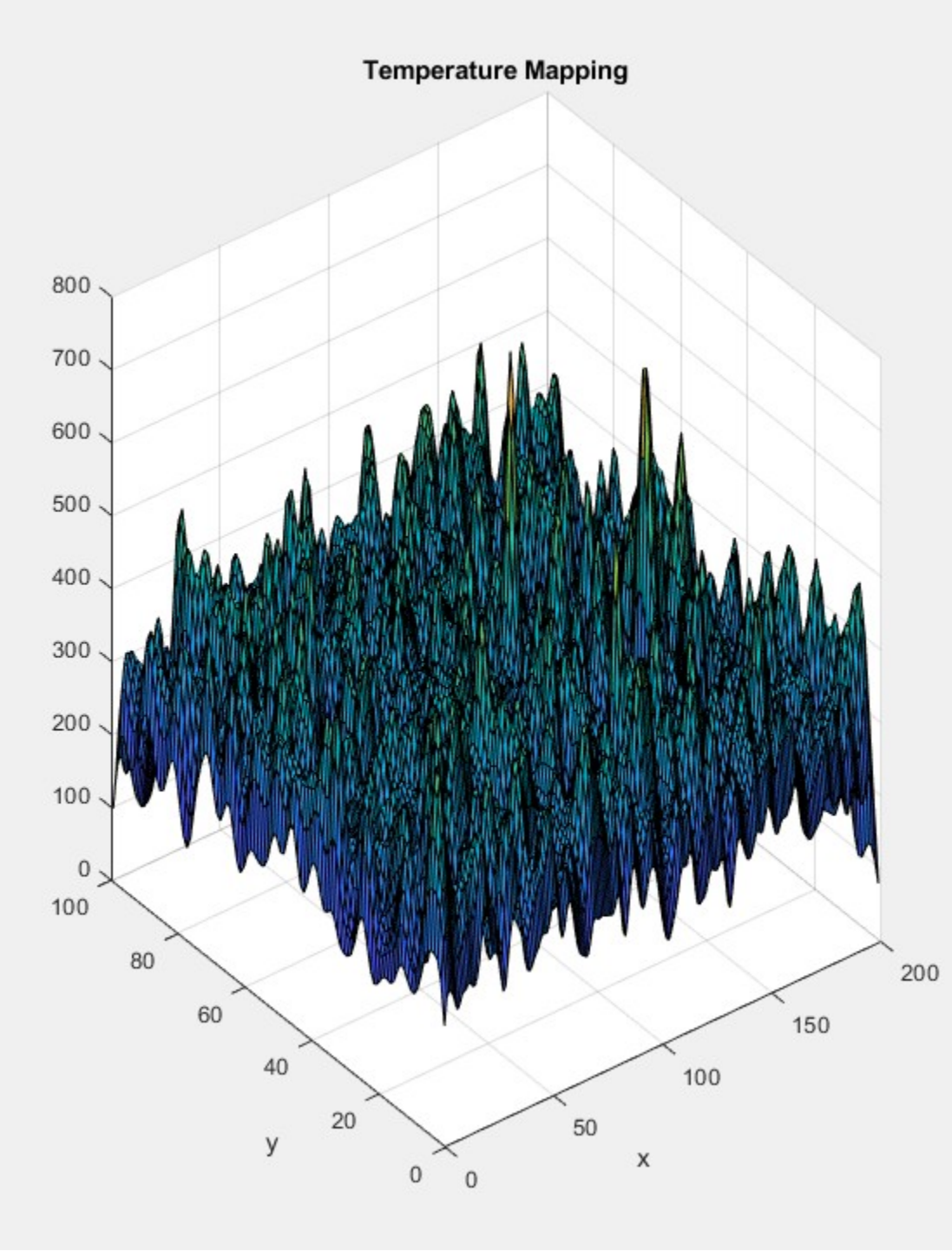

Figure 3: Temperature mapping

The relationship between the electron drift current density and the average carrier velocity is a linear relationship. This can be arrived at from the equation: J = V*n*q*Ny.

Where V is the average carrier velocity of the particles, n is the electron Charge Concentration which was provided to us in the assignment instruction as 10^{15} cm^{-2}, Q is the electron Charge, Ny is the length of the y boundary.

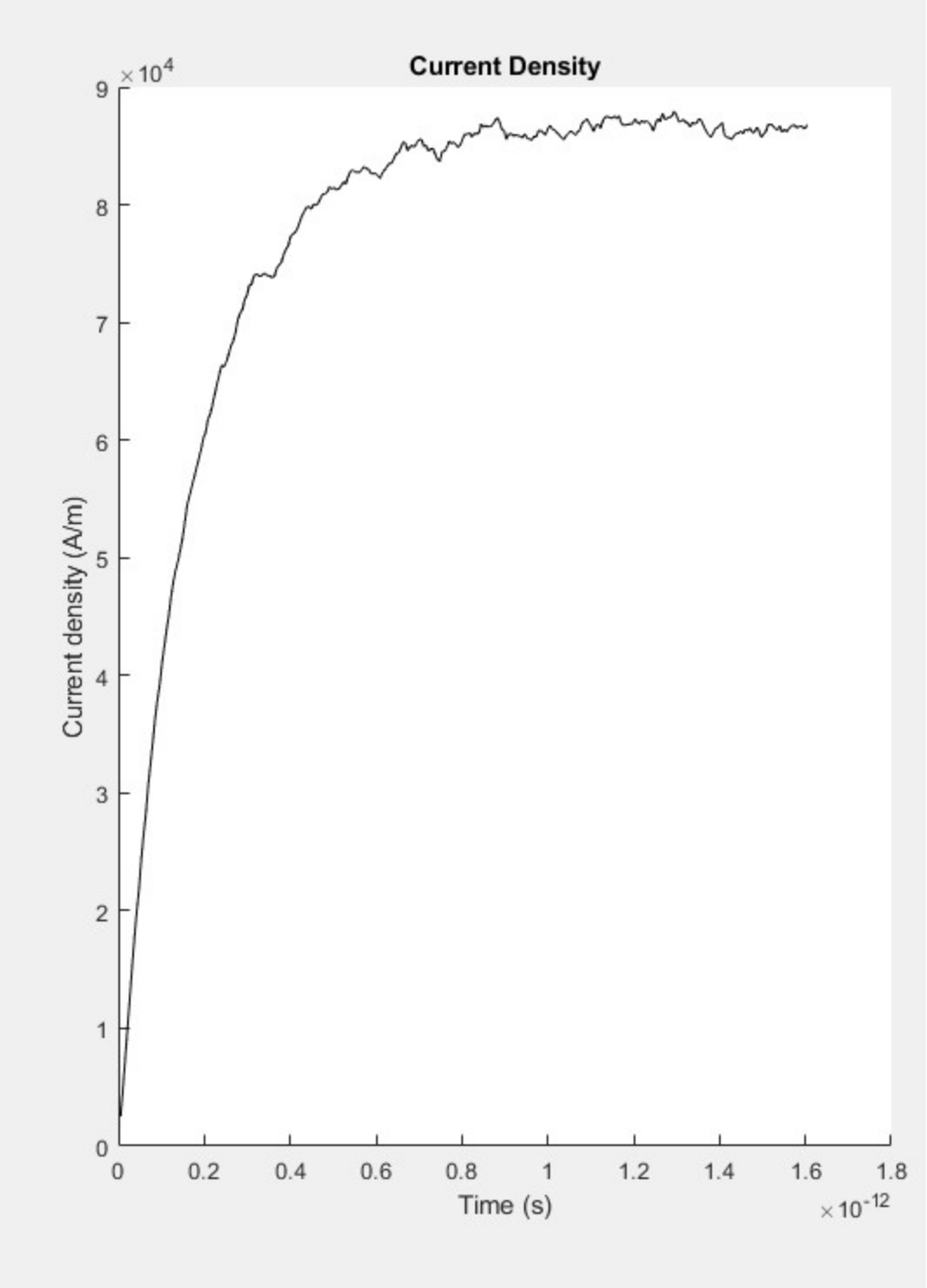

Figure 4: Current Density vs Time

As we can see from the plot above, the behavior of the current is as follows the current rises until it gets to a point where it saturates. What causes the saturation of the current is rethermalization which is resetting the velocities of the particles and this is caused by the scattering (rethermalization occurs as a result of the simulation getting to a point where the amount of scattering of the electrons begins to cancel out the acceleration of the electrons).

The electric field E, Force on the charge F and Acceleration A From the calculations found in the simulations are: E = $5x10^5$ V/m, F = 8.01 x 10^{-14} Newtons and A = 3.38 x 10^{17} m/s^2.

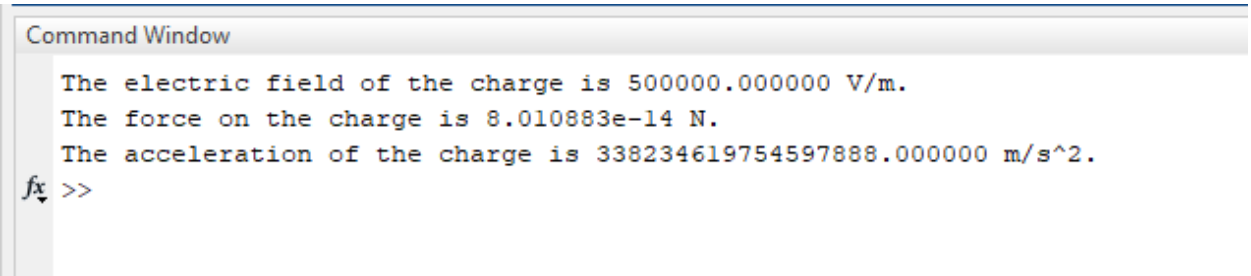

Figure 5: Electric Field, Acceleration and Force on charge from MATLAB command Line.

Doing the calculations manually to make sure the answers from the simulations are correct: From the relationship E = V/D. We know the voltage from the assignment instructions to be 0.1 and we also know the distance to be 0.2 micro-meters. Plugging that into the relationship E =V/D we get 500000 V/m which is the same as what was gotten from matlab.

The Force can be calculated by using the relationship F = QE where Q is the electron charge and E is the electric field of the particle. We get that Force = 500000*1.602e-19 = 8.01e-14. Also the same as the value gotten from matlab

The acceleration can then be calculated by the relationship of F = ma implying that a = F/ m where m is the effective mass. We get a = (8.01e-14)/((9.11e-31) *(0.26))= 3.38 x 10 17 m/s^2

Part 2:

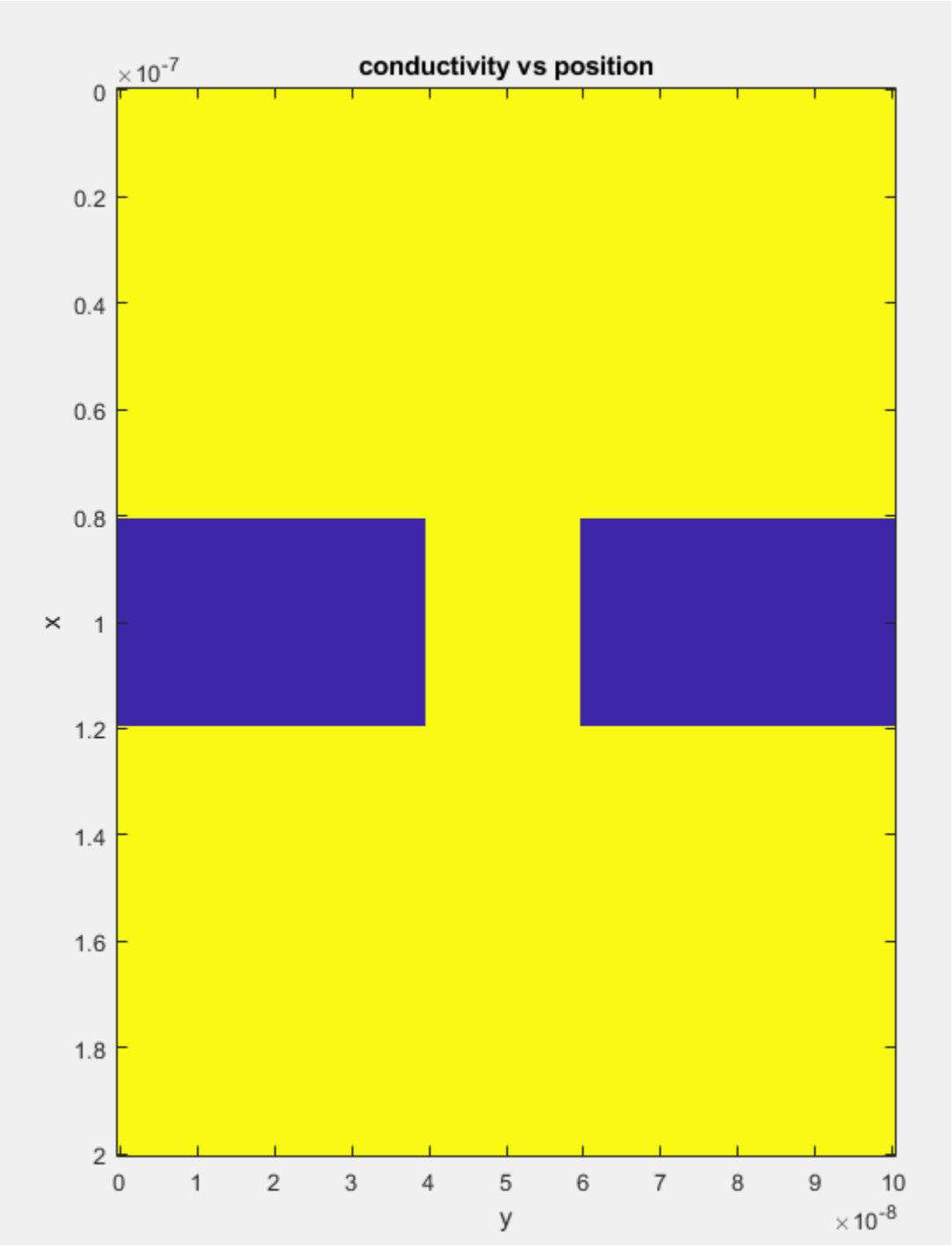

Figure 6: Conductivity vs Position

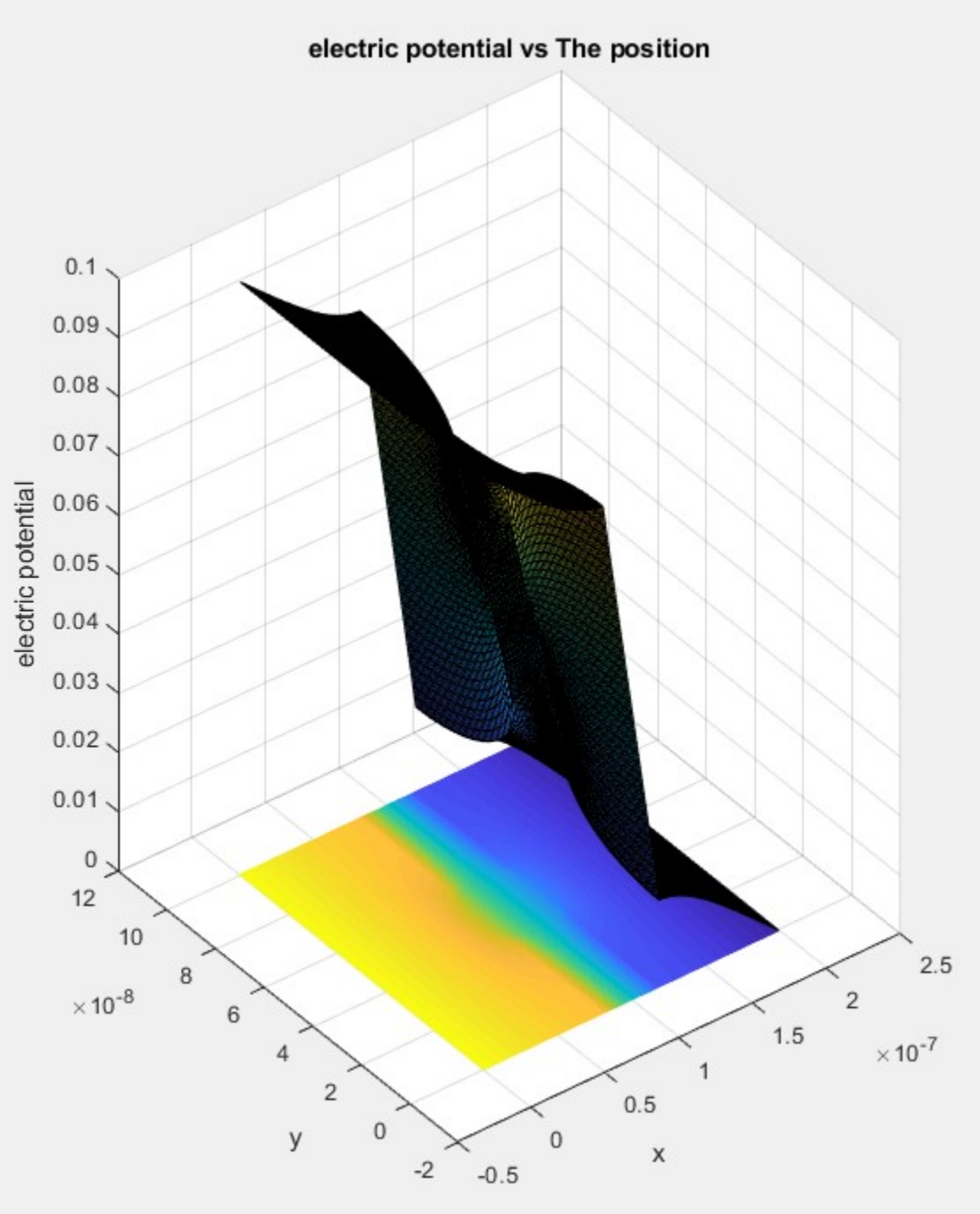

Figure 7: Electric potential vs Position

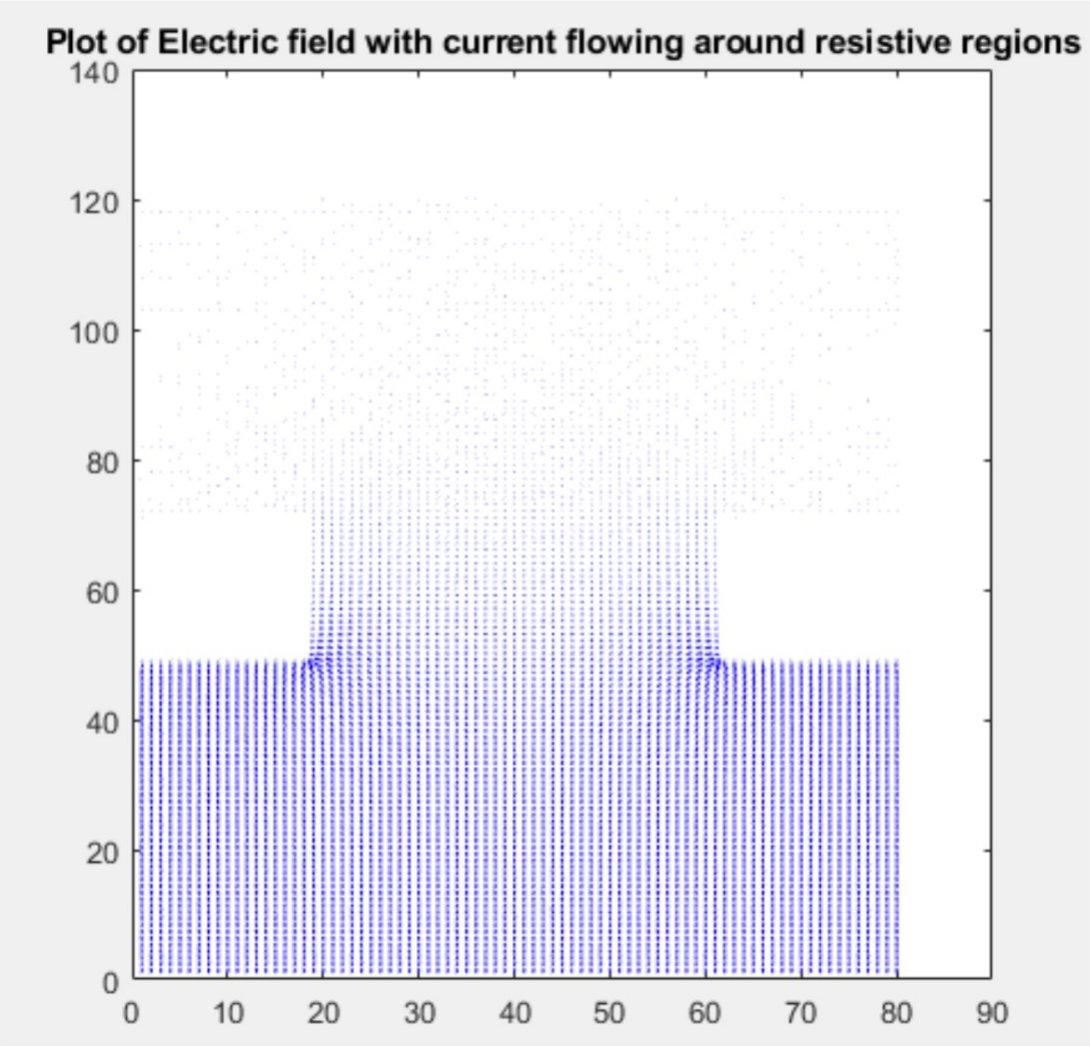

Figure 8: Electric field of the electrons

Part 3:

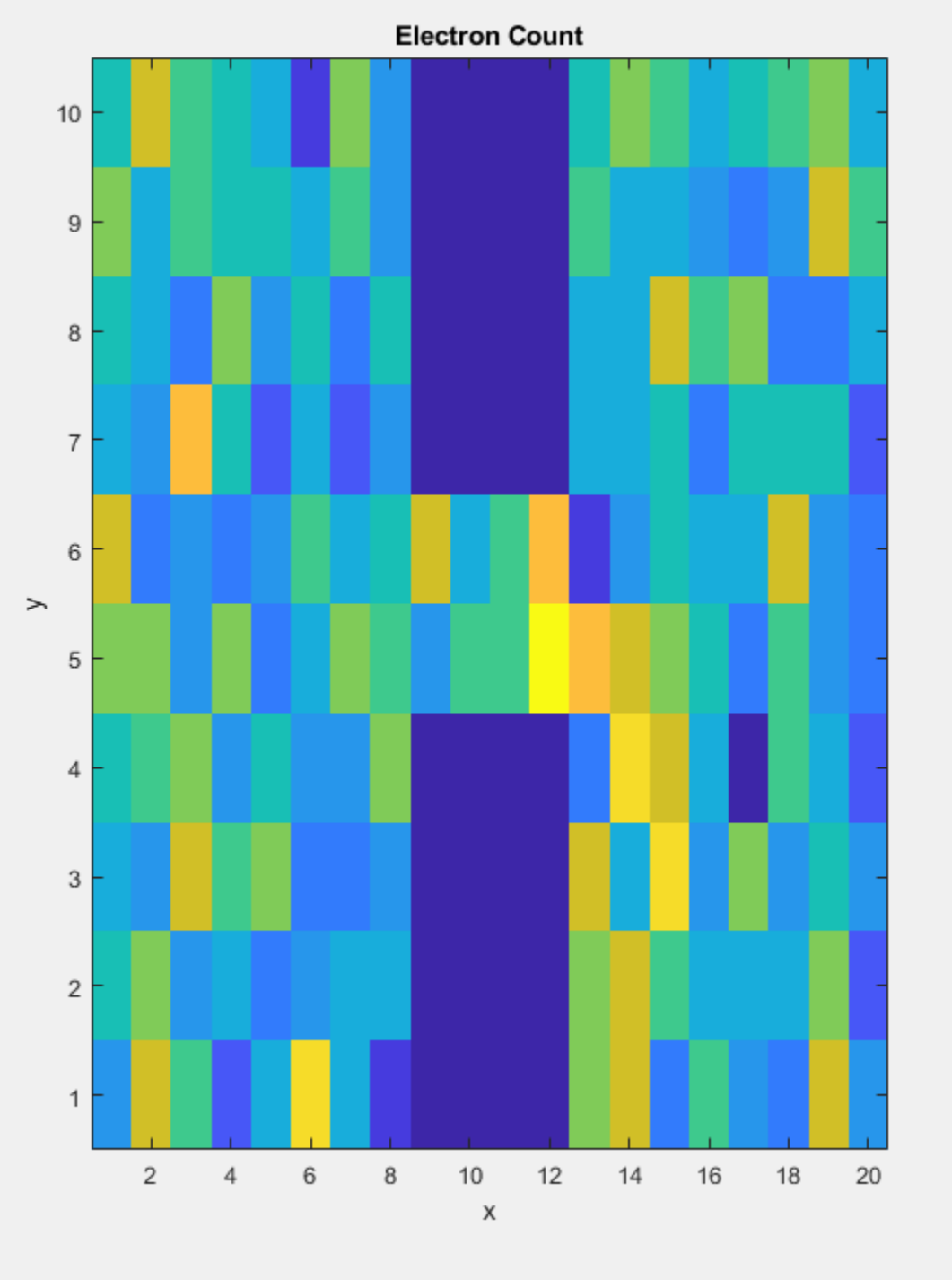

Figure 9: Density mapping

The density mapping was plot using imagesc function in MATLAB which is why it appears as seen as above. The density mapping from figure 9 above is basically telling us where the electrons are most concentrated as the simulation is taking place.

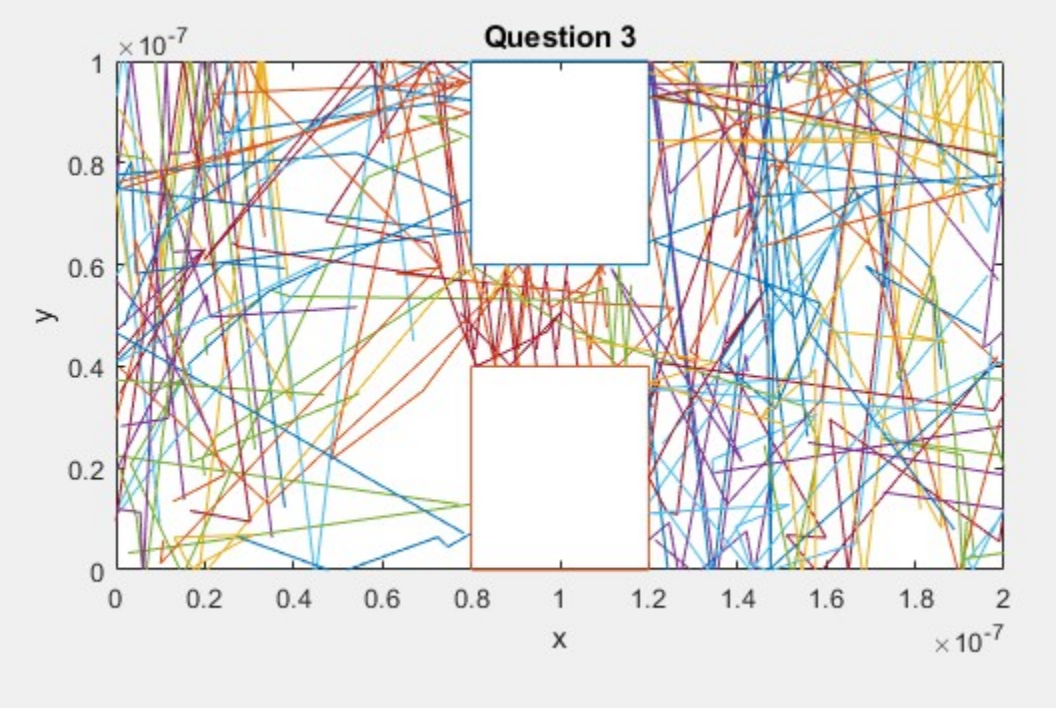

Figure 10: particle trajectory

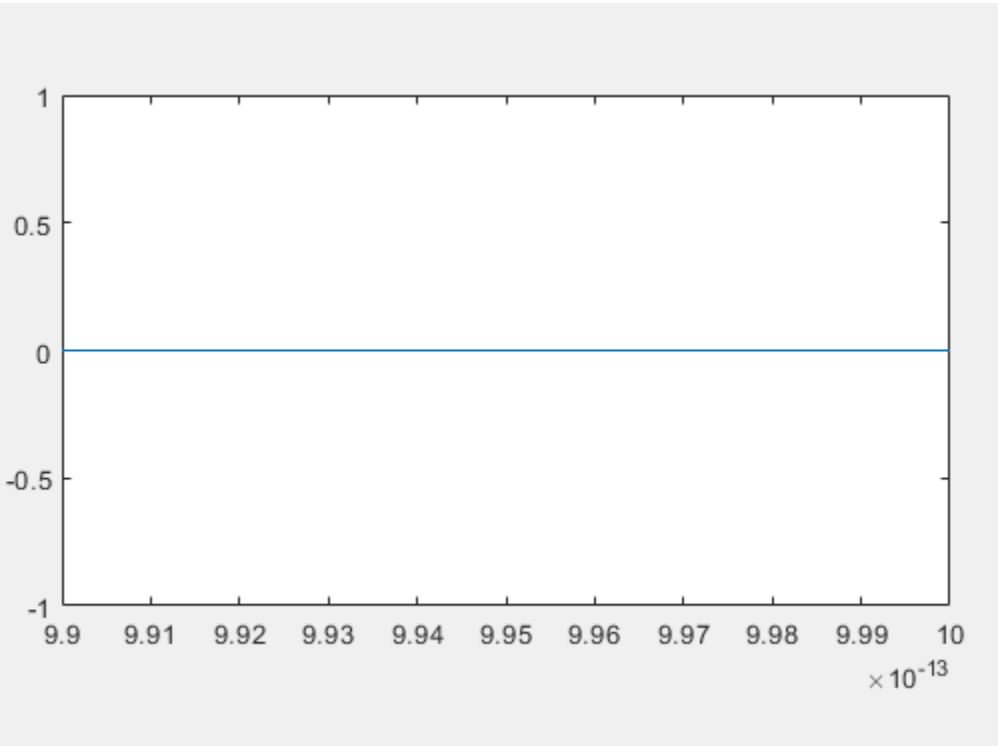

Figure 11: Temperature vs time step

This was what was gotten from our simulation at first but after slight modifications to our code we were able to get something similar to what the ideal plot should look like in theory. Ideally the temperature vs time step should look more like figure 12 below, that is it should fluctuate around a constant value.

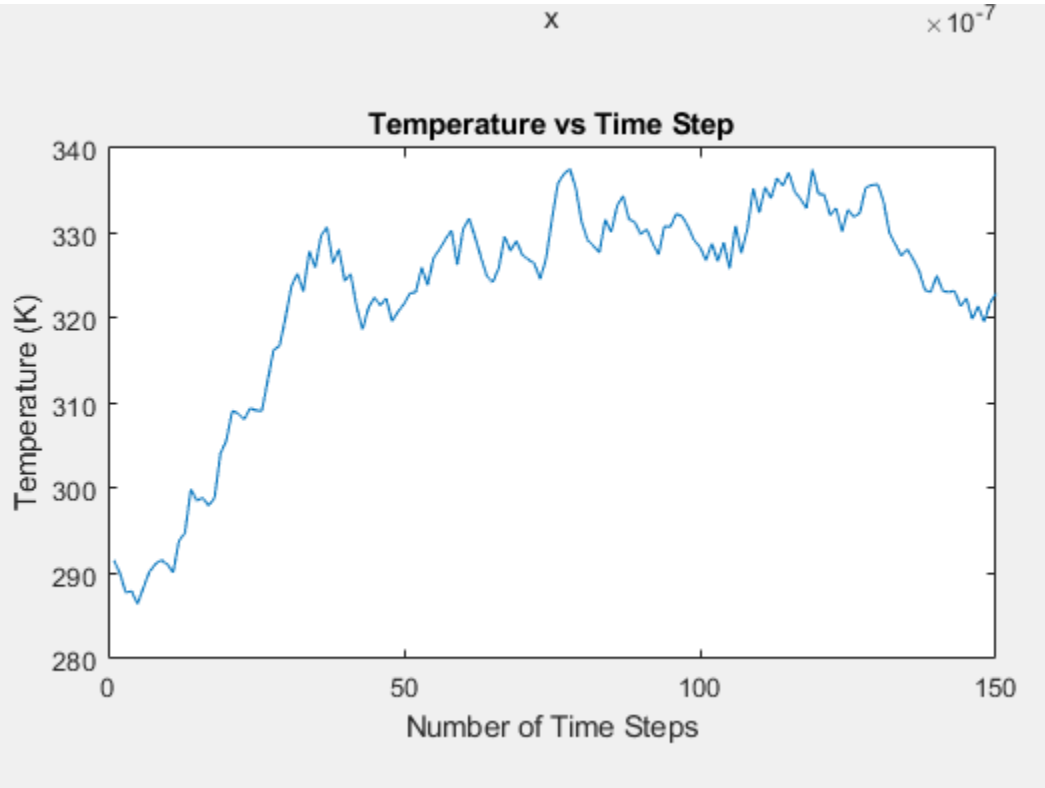

Figure 12: Temperature vs time step

The next step to make the simulation more accurate would be to increase the number of particles in the simulation, as by doing this we would be able to get a more accurate result. Another way would be to use a finer mesh in the simulation.

Conclusion:

In conclusion, the assignment was a success we were able to combine both assignment two and assignment 1 together and add a field into the setup in order to see what happens to the electrons. We were also able to see the relationship between the electron drift current density and carrier velocity of the electrons.

Appendix:

Question 1 and Question 2:

```
clc
clear
set(0,'DefaultFigureWindowStyle','docked')

% Jarikre Efe Jeffery
%101008461
% Part 1: Start with the Monte Carlo Simulator from Assignment 1 without
% the bottleneck from those assignments
wid_x = 200e-9; % width
len_y = 100e-9; % length
Voltage_x = 0.1; % Voltage across x dimension
Voltage_y = 0; % Voltage applied across y

Charge_electron = -1.60217662e-19; % Charge of electrons
electron_conc = 1e15*100^2; % Density of electrons
m0 = 9.10938356e-31; % electron rest mass
effective_m = 0.26*m0; % effective mass of electrons
Temperature = 300; % temperature in Kelvin
Boltz_const = 1.38064852e-23; % Boltzmann Constant
Thermal_v = sqrt(2*Boltz_const*Temperature/effective_m); % Thermal velocity of
the electrons
mean_free_path = Thermal_v*0.2e-12; % Mean free path
specular_top_bound = 0; % capable of being specular or diffusive
specular_bottom_bound = 0; % capable of being specular or diffusive
time_step = len_y/Thermal_v/100;
num_iterations = 300;
size_p = 40000;
pp = 10;
pscat = 1 - exp(-time_step/0.2e-12);
vel = makedist('Normal', 'mu', 0, 'sigma',
sqrt(Boltz_const*Temperature/effective_m));
Display_m = 0;
% Using the relationship E = V/D to calculate the electric field.
electricfield_x = Voltage_x/wid_x;
electricfield_y = Voltage_y/len_y;
electricfield_total = electricfield_x + electricfield_y;
fprintf('The electric field of the charge is %f V/m.\n',electricfield_total);
% The force on each electron is the sum of its individual components.
x_force = Charge_electron*electricfield_x;
y_force = Charge_electron*electricfield_y;
total_force = abs(x_force + y_force);
fprintf('The force on the charge is %d N.\n',total_force);
acceleration = total_force/effective_m; % Using the relationship of f=ma we
can calculate the accelration of the particle.
fprintf('The acceleration of the charge is %f m/s^2.\n',acceleration);
% The current formula is J = vnqNy. This formula shows the relationship
% between the electron drift current density and average carrier velocity
% Spoke to TA and he insisted on using this since matbe performing
% integration might complicate things.
change_vx = x_force*time_step/effective_m;
change_vy = y_force*time_step/effective_m;
change_vx = change_vx.*ones(size_p,1);
```

```
change_vy = change_vy.*ones(size_p,1);
positions = zeros(size_p, 4);
traj = zeros(num_iterations, pp*2);
temporary_a = zeros(num_iterations,1);
J = zeros(num_iterations,2);

% Initializing the positions of the particles
for i = 1:size_p
    theta = rand*2*pi;
    positions(i,:) = [wid_x*rand len_y*rand random(vel) random(vel)];
end
temperature_plot = animatedline;
figure(2);
current_plot = animatedline;
title('Current Density');
xlabel('Time (s)');
ylabel('Current density (A/m)');

% Iterate through the simulation
for i = 1:num_iterations
    positions(:,3) = positions(:,3) + change_vx;
    positions(:,4) = positions(:,4) + change_vy;
    positions(:,1:2) = positions(:,1:2) + time_step.*positions(:,3:4);
    j = positions(:,1) > wid_x;
    positions(j,1) = positions(j,1) - wid_x;
    j = positions(:,1) < 0;
    positions(j,1) = positions(j,1) + wid_x;
    j = positions(:,2) > len_y;

    if(specular_top_bound)
        positions(j,2) = 2*len_y - positions(j,2);
        positions(j,4) = -positions(j,4);
    else
        positions(j,2) = len_y;
        v = sqrt(positions(j,3).^2 + positions(j,4).^2);
        theta = rand([sum(j),1])*2*pi;
        positions(j,3) = v.*cos(theta);
        positions(j,4) = -abs(v.*sin(theta));
    end

    j = positions(:,2) < 0;

    if(specular_bottom_bound)
        positions(j,2) = -positions(j,2);
        positions(j,4) = -positions(j,4);
    else
        positions(j,2) = 0;
        v = sqrt(positions(j,3).^2 + positions(j,4).^2);
        theta = rand([sum(j),1])*2*pi;
        positions(j,3) = v.*cos(theta);
        positions(j,4) = abs(v.*sin(theta));
    end

    j = rand(size_p, 1) < pscat;
    positions(j,3:4) = random(vel, [sum(j),2]);
```

```
    temporary_a(i) = (sum(positions(:,3).^2) +
sum(positions(:,4).^2))*effective_m/Boltz_const/2/size_p;

    for j=1:pp
        traj(i, (2*j):(2*j+1)) = positions(j, 1:2);
    end

    J(i, 1) = Charge_electron.*electron_conc.*mean(positions(:,3));
    J(i, 2) = Charge_electron.*electron_conc.*mean(positions(:,4));
    addpoints(temperature_plot, time_step.*i, temporary_a(i));
    addpoints(current_plot, time_step.*i, J(i,1));
    if(Display_m && mod(i,5) == 0)
        figure(1);
        hold off;
        plot(positions(1:pp,1)./1e-9, positions(1:pp,2)./1e-9, 'o');
        axis([0 wid_x/1e-9 0 len_y/1e-9]);
        hold on;
        title('Particle Trajectories');
        % x and y positions are in nanometers
        xlabel('x position');
        ylabel('y position');
        pause(0.05);
    end
end

figure(1);
title('Particle Trajectories');
%x and y positions are in nanometers
xlabel('x position');
ylabel('y position');
axis([0 wid_x/1e-9 0 len_y/1e-9]);
hold on;

for i=1:pp
    plot(traj(:,i*2)./1e-9, traj(:,i*2+1)./1e-9, 'm.');
end

electron_conc = hist3(positions(:,1:2),[200 100])';
N = 20;
sigma = 1.5;
[x,y] = meshgrid(round(-N/2):round(N/2), round(-N/2):round(N/2)); % in order
to use the surf function later
f=exp(-x.^2/(2*sigma^2)-y.^2/(2*sigma^2));
f=f./sum(f(:));
figure(3);
electron_conc = conv2(electron_conc,f,'same');
electron_conc =
electron_conc/(len_y./size(electron_conc,1)*wid_x./size(electron_conc,2));
surf(conv2(electron_conc,f,'same'));
% contourmode(conv2(electron_conc,f)
title('Electron Density Mapping');
% x and y positions are in nanometers
xlabel('x Position');
ylabel('y Position');

% Temperature mapping
```

```
sum_x = zeros(ceil(wid_x/1e-9),ceil(len_y/1e-9));
sum_y = zeros(ceil(wid_x/1e-9),ceil(len_y/1e-9));
temp_num = zeros(ceil(wid_x/1e-9),ceil(len_y/1e-9));

% electron velocity
for i=1:size_p
    x = floor(positions(i,1)/1e-9);
    y = floor(positions(i,2)/1e-9);
    if(x==0)
        x = 1;
    end
    if(y==0)
        y= 1;
    end
    sum_y(x,y) = sum_y(x,y) + positions(i,3)^2;
    sum_x(x,y) = sum_x(x,y) + positions(i,4)^2;
    temp_num(x,y) = temp_num(x,y) + 1;
end

temporary_a = (sum_x + sum_y).*effective_m./Boltz_const./2./temp_num;
temporary_a(isnan(temporary_a)) = 0;
temporary_a = temporary_a';
N = 20;
sigma = 1.5;
[x,y] = meshgrid(round(-N/2):round(N/2), round(-N/2):round(N/2)); % useful to
use the surf function
f=exp(-x.^2/(2*sigma^2)-y.^2/(2*sigma^2));
f=f./sum(f(:));
figure(4);
surf(conv2(temporary_a,f,'same'));
% contourmode(conv2(temporary_a,f,'same'));
title('Temperature Mapping');
xlabel('x');
ylabel('y');


clc
clear
set(0,'DefaultFigureWindowStyle','docked')

% Jarikre Efe Jeffery
%101008461
% Part 2: Use The Finite Difference Method in Assignment 2 to calculate the
% electric field and then provide a field for the Monte Carlo bottleneck
% Simulation

length_y = 200e-9;
width_x = 100e-9;
Len_box = 40e-9;
Wid_box = 40e-9;
meshspace = 1e-9;
num_x = round(length_y/meshspace + 1);
num_y = round(width_x/meshspace + 1);

conductivity_outside = 1;
conductivity_inside = 1e-2;
```

```
% Conductivity mapping
conductivity_mapping = zeros(num_x,num_y);

for i = 1:num_x
   for j = 1:num_y
       if (i-1)>0.5*(length_y-Len_box)/meshspace&&(i-
1)<0.5*(length_y+Len_box)/meshspace&&((j-1)<Wid_box/meshspace||(j-1)>(width_x-
Wid_box)/meshspace)
           conductivity_mapping(i,j) = conductivity_inside;
       else
           conductivity_mapping(i,j) = conductivity_outside;
       end
   end
end

figure(5)
imagesc([0 width_x],[0 length_y],conductivity_mapping);
xlabel('y')
ylabel('x')
title('conductivity vs position')

G_matrix = sparse(num_x*num_y);
B_matrix = zeros(1,num_x*num_y);
for i = 1:num_x
    for j = 1:num_y
        n = j +(i-1)*num_y;
        n1 = j + (i - 1) * length_y;
        nxm1 = j + ((i-1) - 1) * length_y;
        nxp1 = j + ((i+1) - 1) * length_y;
        nym1 = (j-1) + (i - 1) * length_y;
        nyp1 = (j+1) + (i - 1) * length_y;

        if i == 1
        n1 = j + (i - 1) * length_y;
        nxm1 = j + ((i-1) - 1) * length_y;
        nxp1 = j + ((i+1) - 1) * length_y;
        nym1 = (j-1) + (i - 1) * length_y;
        nyp1 = (j+1) + (i - 1) * length_y;
            G_matrix(n,n) = 1;
            B_matrix(n) = 0.1;

        elseif i == num_x
        n1 = j + (i - 1) * length_y;
        nxm1 = j + ((i-1) - 1) * length_y;
        nxp1 = j + ((i+1) - 1) * length_y;
        nym1 = (j-1) + (i - 1) * length_y;
        nyp1 = (j+1) + (i - 1) * length_y;
        G_matrix(n,n) = 1;


        elseif j == 1
        n1 = j + (i - 1) * length_y;
        nxm1 = j + ((i-1) - 1) * length_y;
        nxp1 = j + ((i+1) - 1) * length_y;
        nym1 = (j-1) + (i - 1) * length_y;
```

```
    nyp1 = (j+1) + (i - 1) * length_y;
    nxm = j + (i-2)*num_y;
    nxp = j + i*num_y;
    nyp = j+1 + (i-1)*num_y;
    rxm = (conductivity_mapping(i,j) + conductivity_mapping(i-1,j))/2;
    rxp = (conductivity_mapping(i,j) + conductivity_mapping(i+1,j))/2;
    ryp = (conductivity_mapping(i,j) + conductivity_mapping(i,j+1))/2;

        G_matrix(n,n) = -(rxm + rxp + ryp);
        G_matrix(n,nxm) = rxm;
        G_matrix(n,nxp) = rxp;
        G_matrix(n,nyp) = ryp;


    elseif j == num_y
        nxm = j + (i-2)*num_y;
        nxp = j + i*num_y;
        nym = j-1 + (i-1)*num_y;
        n1 = j + (i - 1) * length_y;
        nxm1 = j + ((i-1) - 1) * length_y;
        nxp1 = j + ((i+1) - 1) * length_y;
        nym1 = (j-1) + (i - 1) * length_y;
        nyp1 = (j+1) + (i - 1) * length_y;

        rxm = (conductivity_mapping(i,j) + conductivity_mapping(i-1,j))/2;
        rxp = (conductivity_mapping(i,j) + conductivity_mapping(i+1,j))/2;
        rym = (conductivity_mapping(i,j) + conductivity_mapping(i,j-1))/2;

        G_matrix(n,n) = -(rxm + rxp + rym);
        G_matrix(n,nxm) = rxm;
        G_matrix(n,nxp) = rxp;
        G_matrix(n,nym) = rym;

    else
        nxm = j + (i-2)*num_y;
        nxp = j + i*num_y;
        nym = j-1 + (i-1)*num_y;
        nyp = j+1 + (i-1)*num_y;
        n1 = j + (i - 1) * length_y;
    nxm1 = j + ((i-1) - 1) * length_y;
    nxp1 = j + ((i+1) - 1) * length_y;
    nym1 = (j-1) + (i - 1) * length_y;
    nyp1 = (j+1) + (i - 1) * length_y;
        rxm = (conductivity_mapping(i,j) + conductivity_mapping(i-1,j))/2;
        rxp = (conductivity_mapping(i,j) + conductivity_mapping(i+1,j))/2;
        ryp = (conductivity_mapping(i,j) + conductivity_mapping(i,j+1))/2;
        rym = (conductivity_mapping(i,j) + conductivity_mapping(i,j-1))/2;

        G_matrix(n,n) = -(rxm + rxp + rym + ryp);
        G_matrix(n,nxm) = rxm;
        G_matrix(n,nxp) = rxp;
        G_matrix(n,nym) = rym;
        G_matrix(n,nyp) = ryp;

    end
end
```

```
end

% set up matrix and solve
V = G_matrix\B_matrix';
Voltage_map = zeros(num_x,num_y);
for i = 1:num_x
    for j = 1:num_y
        n = j +(i-1)*num_y;
        Voltage_map(i,j) = V(n);
    end
end

[X, Y] = meshgrid(0:meshspace:length_y,0:meshspace:width_x);
figure(6)
surf(X',Y',Voltage_map)
hold on
imagesc([0 length_y],[0 width_x],Voltage_map')
xlabel('x')
ylabel('y')
zlabel('electric potential')
title('electric potential vs The position')
hold off

[electricfield_y, electricfield_x] = gradient(Voltage_map,meshspace);
electricfield_x = -electricfield_x;
electricfield_y = -electricfield_y;

figure(7)
quiver(X',Y',electricfield_x,electricfield_y)
xlim([0 length_y])
ylim([0 width_x])
xlabel('x')
ylabel('y')
title('Electric Field of the electrons')
```

Question 3:

```
clc
clear
set(0,'DefaultFigureWindowStyle','docked')
% Jarikre Efe Jeffery
%101008461
% Part 3: Use the coupled simulations to investigate the "device" and
% extract simple trajectories
global C
global Ecount
global Vx Vy Vtotal x y
Ecount =1000;
C.mo = 9.10938215e-31;
C.k = 1.3806504e-23;
electron_charge = -1.60217662e-19;
Temperature =300;
effective_m = 0.26*C.mo;
Length = 200e-9;
Width = 100e-9;
Thermal_v = sqrt((2*C.k*Temperature)/effective_m);
```

```
time_step = 10e-15;
frame = 100*time_step;
x = zeros(Ecount, 2);
y = zeros(Ecount, 2);
Temperature = zeros(1,2);
Time = 0;
VisibleEcount = 50;
tmn = 0.2e-12;
PScat = 1 - exp(-time_step/tmn);
V_Histogram = zeros(Ecount, 1);
bottleneck_X = [80e-9 80e-9 120e-9 120e-9 80e-9];

bottleneck_Y1 = [100e-9 60e-9 60e-9 100e-9 100e-9];
bottleneck_Y2 = [40e-9 0 0 40e-9 40e-9];
Specular = true;
Inside_Box = true;
Mapping_S = 10e-9;
Density_Mapping = zeros(Width/Mapping_S, Length/Mapping_S);
Temperature_Mapping = zeros(Width/Mapping_S, Length/Mapping_S);

wid_x = 30;
len_y = 20;
change_x = Length/wid_x;
change_y = Width/len_y;
conduction_outside = 1;
conduction_inside = 01e-2;
conductivity = zeros(wid_x,len_y);
G_matrix = sparse (wid_x*len_y, wid_x*len_y);
V_matrix = zeros(1, wid_x*len_y);
Voltage_x = 0.1;

for i = 1:wid_x
    for j = 1:len_y
        n = j + (i - 1)*len_y;
        nxm = j + ((i-1) - 1)*len_y;
        nxp = j + ((i+1) - 1)*len_y;
        nym = (j-1) + (i - 1)*len_y;
        nyp = (j+1) + (i - 1)*len_y;
        if (i > (0.3*wid_x) || i < (0.6*wid_x)) && (j > (0.6*len_y) || j <
(0.3*len_y))
            conductivity(i,j) = conduction_inside;
        else
            conductivity(i,j) = conduction_outside;
        end

    end
end
for i = 1:wid_x
    for j = 1:len_y
        n = j + (i - 1)*len_y;
        nxm = j + ((i-1) - 1)*len_y;
        nxp = j + ((i+1) - 1)*len_y;
        nym = (j-1) + (i - 1)*len_y;
        nyp = (j+1) + (i - 1)*len_y;
        if (i == 1)
        n = j + (i - 1)*len_y;
```

```
        nxm = j + ((i-1) - 1)*len_y;
        nxp = j + ((i+1) - 1)*len_y;
        nym = (j-1) + (i - 1)*len_y;
        nyp = (j+1) + (i - 1)*len_y;
            V_matrix(n) = Voltage_x;
            G_matrix(n,n) = 1;
        elseif (i == wid_x)
        n = j + (i - 1)*len_y;
        nxm = j + ((i-1) - 1)*len_y;
        nxp = j + ((i+1) - 1)*len_y;
        nym = (j-1) + (i - 1)*len_y;
        nyp = (j+1) + (i - 1)*len_y;
            V_matrix(n) = 0;
            G_matrix(n,n) =1;
        elseif (j == 1)
        n = j + (i - 1)*len_y;
        nxm = j + ((i-1) - 1)*len_y;
        nxp = j + ((i+1) - 1)*len_y;
        nym = (j-1) + (i - 1)*len_y;
        nyp = (j+1) + (i - 1)*len_y;
            G_matrix(n,n) = -((conductivity(i,j) + conductivity(i-1,j))/2) -
((conductivity(i,j) + conductivity(i+1,j))/2) - ((conductivity(i,j) +
conductivity(i,j+1))/2);
            G_matrix(n, nxm) = ((conductivity(i,j) + conductivity(i-1,j))/2);
            G_matrix(n,nxp) = ((conductivity(i,j) + conductivity(i+1,j))/2);
            G_matrix(n, nyp) = ((conductivity(i,j) + conductivity(i,j+1))/2);
        elseif (j == len_y)
        n = j + (i - 1)*len_y;
        nxm = j + ((i-1) - 1)*len_y;
        nxp = j + ((i+1) - 1)*len_y;
        nym = (j-1) + (i - 1)*len_y;
        nyp = (j+1) + (i - 1)*len_y;
            G_matrix(n,n) = -((conductivity(i,j) + conductivity(i-1,j))/2) -
((conductivity(i,j) + conductivity(i+1,j))/2) - ((conductivity(i,j) +
conductivity(i,j-1))/2);
            G_matrix(n,nxm) = ((conductivity(i,j) + conductivity(i-1,j))/2);
            G_matrix(n,nxp) = ((conductivity(i,j) + conductivity(i+1,j))/2);
            G_matrix(n,nym) = ((conductivity(i,j) + conductivity(i,j-1))/2);
        else
        n = j + (i - 1)*len_y;
        nxm = j + ((i-1) - 1)*len_y;
        nxp = j + ((i+1) - 1)*len_y;
        nym = (j-1) + (i - 1)*len_y;
        nyp = (j+1) + (i - 1)*len_y;
            G_matrix(n,n) = -((conductivity(i,j) + conductivity(i-1,j))/2) -
((conductivity(i,j) + conductivity(i+1,j))/2) - ((conductivity(i,j) +
conductivity(i,j-1))/2) - ((conductivity(i,j) + conductivity(i,j+1))/2);
            G_matrix(n,nxm) = ((conductivity(i,j) + conductivity(i-1,j))/2);
            G_matrix(n,nxp) = ((conductivity(i,j) + conductivity(i+1,j))/2);
            G_matrix(n,nym) = ((conductivity(i,j) + conductivity(i,j-1))/2);
            G_matrix(n,nyp) = ((conductivity(i,j) + conductivity(i,j+1))/2);
        end
    end
end

Solution = G_matrix\V_matrix';
```

```
surface = zeros(wid_x,len_y);
for i = 1:wid_x
    for j = 1:len_y
        n = j + (i - 1)*len_y;
        nxm = j + ((i-1) - 1) * len_y;
        nxp = j + ((i+1) - 1) * len_y;
        nym = (j-1) + (i - 1) * len_y;
        nyp = (j+1) + (i - 1) * len_y;
        surface(i,j) = Solution(n);
    end
end
[Electricfield_x, Electricfield_y] = gradient(-surface);
Force_x = electron_charge*Electricfield_x;
Force_y = electron_charge*Electricfield_y;
Acceleration_x = Force_x /effective_m;
Acceleration_y = Force_y /effective_m;

for i = 1:Ecount
    x(i,1) = rand()*200e-9;
    y(i,1) = rand()*100e-9;
    Inside_Box = true;
    while Inside_Box == true
        if (x(i) >= 40e-9 && x(i) <= 120e-9) && (y(i) >= 60e-9 ||...
                y(i) <= 40e-9)
            x(i,1) = rand * 200e-9;
            y(i,1) = rand * 100e-9;
        else
            Inside_Box = false;
        end
    end

end

for i = 1:Ecount

Vx(1:Ecount) = Thermal_v * randn;
Vy(1:Ecount) = Thermal_v * randn;
end

figure(8)
subplot(2,1,1);
plot(bottleneck_X, bottleneck_Y1, bottleneck_X, bottleneck_Y2)
axis([0 Length 0 Width]);
title('Question 3');
xlabel('x');
ylabel('y');
hold on;

while Time < frame
    subplot(2,1,1)
    for j = 1:Ecount
        leaking = true;
        if PScat> rand
                Vx(j) = Thermal_v * randn;
                Vy(j) = Thermal_v * randn;
        end
```

```
x_index = round((x(j,2)/Length) * 30);
y_index = round((y(j,2)/Width)*20);
if x_index < 1
    x_index = 1;
elseif x_index > 30
        x_index = 30;
end
if y_index < 1
    y_index = 1;
elseif y_index > 20
    y_index = 20;
end

Vx(j) =  Vx(j) + Acceleration_x(x_index,y_index)*time_step;
Vy(j) =  Vy(j) + Acceleration_y(x_index,y_index)*time_step;
x(j,2) = x(j,1);
y(j,2) = y(j,1);
x(j,1) = x(j,1) + (time_step * Vx(j));
y(j,1) = y(j,1) + (time_step * Vy(j));

if (x(j,1) >= 80e-9 && x(j,1) <= 120e-9) && y(j,1) >= 60e-9

        if y(j,2) < 60e-9
            Vy(j) = -Vy(j);
            y(j,1) = 60e-9;
            y(j,2) = 60e-9;

        elseif x(j,2) < 80e-9
            Vx(j) = -Vx(j);
            x(j,1) = 80e-9;
            x(j,2) = 80e-9;

        elseif x(j,2) > 120e-9
            Vx(j) = -Vx(j);
            x(j,1) = 120e-9;
            x(j,2) = 120e-9;
        end

    if Specular == true

        x(j,1) = x(j,2) + Vx(j)*time_step;
        y(j,1) = y(j,2) + Vy(j)*time_step;
    else

     Vx(j) = Thermal_v * randn;
     Vy(j) = Thermal_v * randn;

     while leaking == true
         if(x(j,2) < 80e-9 && Vx(j) >= 0) || ...
                 (x(j,2) > 120e-9 && Vx(j) <= 0) || ...
                 (y(j,2) < 60e-9 && Vy(j) >= 0)
             Vx(j) = Thermal_v * randn;
             Vy(j) = Thermal_v * randn;
         else
             leaking = false;
```

```
            end
        end
        x(j,1) = x(j,2) + Vx(j)*time_step;
        y(j,1) = y(j,2) + Vy(j)*time_step;
    end
end
if (x(j,1) >= 80e-9 && x(j,1) <= 120e-9) && y(j,1) <= 40e-9
        if y(j,2) > 40e-9
            Vy(j) = -Vy(j);
            y(j,1) = 40e-9;
            y(j,2) = 40e-9;
        elseif x(j,2) < 80e-9
            Vx(j) = -Vx(j);
            x(j,1) = 80e-9;
            x(j,2) = 80e-9;
        elseif x(j,2) > 120e-9
            Vx(j) = -Vx(j);
            x(j,1) = 120e-9;
            x(j,2) = 120e-9;
        end
    if Specular == true
        x(j,1) = x(j,2) + Vx(j)*time_step;
        y(j,1) = y(j,2) + Vy(j)*time_step;
    else
        Vx(j) = Thermal_v * randn;
        Vy(j) = Thermal_v * randn;
        while leaking == true
            if(x(j,2) < 80e-9 && Vx(j) >= 0) || ...
                    (x(j,2) > 120e-9 && Vx(j) <= 0) || ...
                    (y(j,2) > 40e-9 && Vy(j) <= 0)
                Vx(j) = Thermal_v * randn;
                Vy(j) = Thermal_v * randn;
            else
                leaking = false;
            end
        end
        x(j,1) = x(j,2) + Vx(j)*time_step;
        y(j,1) = y(j,2) + Vy(j)*time_step;
    end
end
if x(j,1) > Length
    x(j,2) = 0;
    x(j,1) = time_step * Vx(j);
end
if x(j,1) < 0
    x(j,2) = Length;
    x(j,1) = x(j,2) + (time_step * Vx(j));
end
if y(j,1) > Width || y(j,1) < 0
    Vy(j) = -Vy(j);
end
XPlot = [x(j,2) x(j,1)];
YPlot = [y(j,2) y(j,1)];
if j < VisibleEcount
plot(XPlot,YPlot);
end
```

```
       VTotal = sqrt(Vx(j)^2 + Vy(j)^2);
    end

    AvgTemperature = Temperature(1,2)/Ecount;
    TemperaturePlot = [Temperature(1,1) AvgTemperature];
    TimePlot = [(Time - time_step) Time];
    subplot(2,1,2);
    plot(TimePlot, TemperaturePlot);
    Temperature(1,1) = AvgTemperature;
    AvgTemperature = 0;
    Temperature(1,2) = 0;
    pause(1e-19)
    Time = Time + time_step;
end
for i = 1:(Length/Mapping_S)
    for j = 1:(Width/Mapping_S)
        for m = 1:Ecount
            if(x(m,1) > Mapping_S*(i -1)) && ...
                    (x(m,1) < Mapping_S*(i)) && ...
                    (y(m,1) > Mapping_S*(j - 1)) && ...
                    (y(m,1) < Mapping_S*(j))

                Vtotal(m) = sqrt(Vx(m)^2 + Vy(m)^2);

                Density_Mapping(j, i) = Density_Mapping(j, i) + 1;
                Temperature_Mapping(j, i) = Temperature_Mapping(j,i) + ...
                    (effective_m*Vtotal(m)^2)/(2*C.k);
            end
            Temperature_Mapping(j,i) =
Temperature_Mapping(j,i)/Density_Mapping(j,i);
        end
    end
end
figure(9)
imagesc(Density_Mapping)
title('Density mapping of all electrons in the frame')
xlabel('x');
ylabel('y');
set(gca, 'Ydir', 'Normal')
title('Electron Count')
```