Middle East Technical University
Department of Mechanical Engineering
ME 310 Numerical Methods
Spring 2024 (Dr. Cuneyt Sert)
Study Set 5

For Homework 5 submit the answers of <u>questions 2, 3, 10 and 12. Their grade percentages are not known at this point. It will be decided later.</u>

Assigned: 08/05/2024 – Due: 20/05/2024, 10:30

<u>**Homework Rules and Suggestions**:</u>

- This assignment can be done individually or as a team of two students. Everything in your report should be the result of your own work or your team's work. You are allowed to discuss the questions with your classmates and teaching staff up to a certain detail on ODTUClass. You are not allowed to use an AI tool such as ChatGPT in writing codes or other parts of your report.
- Put the following honor pledge at the top of your homework report and behave accordingly.
  "I understand that this is an individual/team assignment. I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own/team's."
- If you do the homework as a team, put the percent contribution of each member at the beginning of your report.
- If you've exchanged ideas with other students outside ODTUClass, you need to put their names and the extent of your discussion at the beginning of your report.
- You need to submit a printed report. It is what we will be grading. You also need to upload the same report as a PDF document (not a Word document) together with all other files (such as codes) to ODTUClass. Name your MATLAB files properly. Follow MATLAB file naming rules such as "File names cannot start with a number", "They cannot contain special characters or spaces", etc.
- Late submission is not allowed unless you have a valid excuse. In such a case, you need let the whole teaching team know about it before the submission deadline, unless it is an emergency.
- Make sure that the codes in your report are formatted properly. Use a small sized, fixed width font and make sure that lines are not wrapped. If your code is very long, you can shorten it by getting rid of its noncritical parts and putting a note about this. Note that we grade what we see in your printed report. Do not expect us to run your codes for you to generate results, figures, etc. You should do that yourself and put all the results in your report.
- In writing your codes, follow good programming practices such as "use explanatory header lines", "explain inputs and outputs of functions", "use self-explanatory variable names", "use comments", "use empty lines and spaces for readability", "use indentation for code blocks", "divide long lines into multiple lines using MATLAB's '…' syntax", etc.
- Pay attention to the format of your report. It should look like a serious academic work, not like a high school student work. Font types and sizes, page margins, empty spaces on pages, equations, figures, tables, captions, colors, etc. are all important to give the desired "academic work feeling". Language used is also important. Reports with poor use of English will be penalized.
- Do not provide an unnecessarily long report. The shorter your report, the better it is, as long as it answers the questions properly. Avoid wasting paper. Print on both sides of sheets. Avoid using color unless it is really necessary. Format your report properly with small but readable fonts, small margins, no unnecessarily large figures, no useless spaces, etc. to reduce the number of sheets. Do not use a cover page.
- There are more than 100 students, and we can spend only about 10 minutes to grade each report. Your report should be easy to read and understand. We should be able to find the results and judge their correctness easily. We should not get lost in your report. The more we struggle to understand your report, the lower your grade will be. Use figures and tables cleverly for this purpose.
- Reports with only figures, tables and codes, but no text, comments or discussions will not get a good grade. Start answering each question with one paragraph of introduction. Even when a question does not specifically ask for a discussion or a comment, you need to write a few sentences on the key points and your key findings/learnings.
- Figures and tables should be numbered and should have captions (at the bottom for figures and at the top for tables). Their titles should be self-explanatory, i.e., we should be able understand everything about the table or figure just by reading its title. They should all be referred properly in the written text (such as "… as shown in Fig. 3" or "… (See Table 2)").
- Do not use any Appendices in your report.
- Do not forget to put a numbered reference list at the end of your report if you use references. In that case, you need to refer to the references in the text.
- If you are inexperienced in programming, converting an idea/algorithm into a code and writing it in a bug-free way can be time consuming and frustrating. This is not something that can be done at the last minute. You are advised to start working on the assignments as soon as they are assigned.

Reading Assignments:

Self-learning is an important skill. Not everything can be discussed in lectures. You need to learn certain things by yourself.

R1) Read page 516 of the 8th edition to learn the Newton-Gregory formula that is used to find interpolating polynomials for equi-spaced data points. Understand its similarity to the Taylor series expansion.

R2) Read pages 521-529 of the 8th edition to learn how quadratic and cubic splines can be formulated differently (although being equivalent mathematically) than what we did in class.

R3) Read pages 529-531 of the 8th edition to learn multi-dimensional interpolation.

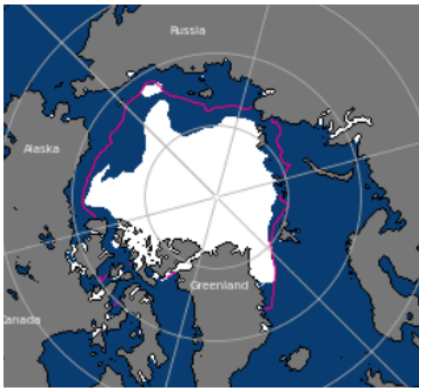
R4) Read the Epilogue section of Part 5 (pages 592-595 of 8th edition). Part 5 includes 4 chapters and its epilogue is at the end of Chapter 20. What we call "Chapter 5 Curve Fitting" in our lectures is this 5th part of the textbook. Note that we've skipped the Fourier Approximation chapter. You are not responsible for that chapter, but you are strongly advised to study it after the semester is over, because the material presented there is quite useful.

Questions:

Q1. The number of transistors on Intel central processing units since the early 1970s up to 2003 is given below. Fit the model $y = Ae^{Bt}$ to the data. Plot the data points together with the model curve. Comment on the quality of the fit. Use the model curve to estimate the number of transistors in year 2023. Compare this estimate to the actual value that you can find by a web search.

CPU	Year	Transistors
4004	1971	2,250
8008	1972	2,500
8080	1974	5,000
8086	1978	29,000
286	1982	120,000
386	1985	275,000
486	1989	1,180,000
Pentium	1993	3,100,000
Pentium II	1997	7,500,000
Pentium III	1999	24,000,000
Pentium 4	2000	42,000,000
Itanium	2002	220,000,000
Itanium 2	2003	410,000,000

Q2. The ice extent at the North Pole (in million km²) during the years of 2015 - 2016 is shown below. Fit the model $y = c_1 + c_2 t + c_3 \sin(2\pi t) + c_4 \cos(2\pi t)$ to the data, where y denotes the ice extent and t is time in years beginning January 2015 ($t = 0$ for January 2015, $t = 1/12$ for February 2015, $t = 11/12$ for December 2015, $t = 12/12$ for January 2016, $t = 23/12$ for December 2016). Plot the data points together with the fitted curve. Comment on the quality of the fit.



Month	2015	2016
Jan.	13.75	13.64
Feb.	14.51	14.32
Mar.	14.49	14.53
Apr.	13.98	13.83
May	12.69	12.08
Jun.	11.05	10.60
Jul.	8.83	8.13
Aug.	5.66	5.60
Sept.	4.68	4.72
Oct.	7.79	6.45
Nov.	10.11	9.08
Dec.	12.33	12.09

Q3. The measured level of a drug in a patient's blood is given below.

Time [h]	1	2	3	4	5	6	7	8
Concentration [ng/ml]	8.0	12.3	15.5	16.8	17.1	15.8	15.2	14.0

a) Fit the model $y = Ate^{Bt}$ to the data by applying linearization first. Plot the model curve together with the data points. Estimate the concentration at $t = 16\ \mathrm{h}$. Comment on the quality of the fit. Show calculation details.

b) Repeat part (a) without linearizing the data, i.e. by directly performing non-linear regression. Show calculation details.

Q4. An experiment provided a data set of y vs. x. According to the theory, the relation between these variables is of the form $y = [1/(Ax^2 + B)]^2$. Is this relation in the "general linear least squares regression form"? If not, can you suggest a way to linearize it?

Q5. The thermal conductivity of iron is found to vary with temperature as follows.

Temperature [K]	70	100	200	300	400	500	600
Conductivity [W/(mK)]	215	134	93	81	70	61	55

a) Plot the data and suggest a relation that you think will fit to it.

b) Perform least squares regression and calculate the constants of the suggested regression. Plot the regression curve and the data points. Comment on the quality of the fit.

Q6. The efficiency of a reaction type hydraulic turbine is found to vary with the output power and the available water head as follows. Develop a least-squares fit of the form $\eta = a_0 + a_1\mathcal{P} + a_2 h$. Comment on the quality of the fit.

Power ($\mathcal{P}$ [MW])	16.4	22.4	29.8	23.9	32.1	41.0	38.8	46.2	59.7
Water head (h [ft])	80	80	80	110	110	110	145	145	145
Efficiency (η [%])	75	80	85	75	80	85	75	80	85

Q7. Use Gauss-Newton method to fit a function of the form $y = c_1 e^{-c_2(x-c_3)^2}$ to the following data. Start with the initial guesses of $c_1 = c_2 = c_3 = 1$. Perform 5 iterations and tabulate the results. Show all calculation details. Plot the final equation together with the data points and comment.

x:	1.0	2.0	3.0	4.0
y:	3.0	6.0	5.0	1.0

Q8. An experiment produced the following data points

x:	-1	-0.96	-0.86	-0.79	0.22	0.5	0.93
y:	-1	-0.151	0.894	0.986	0.895	0.5	-0.306

a) Construct the finite divided difference table and determine the Newton's interpolating polynomial that passes through all the points. Plot the function together with the data points. Interpolate for $y(-0.2)$. Show all calculation details.

b) Repeat part (a), but this time determine the Lagrange interpolating polynomial. Show all calculation details.

c) Determine the natural cubic splines that interpolate the data set. Interpolate for $y(-0.2)$. Show all calculation details.

d) Compare the results and discuss.

Q9. The expected lifetime (in 1000 hours) of an industrial fan when operated at different temperatures is given below. Estimate the lifetime at 30 °C and 55 °C by using following interpolations. Show all calculation details.

a) Using the first three points. Use Newton's divided difference interpolating polynomials.

b) Using all four points. Use Lagrange interpolating polynomials.

c) Using natural cubic splines.

Temp [°C]:	25	40	50	60
Lifetime [×1000 h]:	95	75	63	54

Q10. A major use of polynomial interpolation is to replace evaluation of a complicated function by evaluation of a polynomial, which involves only elementary computer operations like addition, subtraction, and multiplication. Something complex is replaced with something simpler and computable, with perhaps some loss in accuracy that we will have to analyze. Our example here is from trigonometry.

a) Interpolate the function $f(x) = \sin(x)$ at 4 equally spaced points in the interval [0, $\pi/2$], i.e. using points 0, $\pi/6$, $\pi/3$, $\pi/2$. Determine the 3rd order interpolating polynomial $p(x)$. Plot $f(x)$ and $p(x)$ together to access the quality of the fit.

b) Plot $f(x) - p(x)$ and determine the maximum interpolation error in absolute sense, i.e. find $\max(|f(x) - p(x)|)$.

c) Study the reading assignment R1 and estimate the upper bound of the remainder of the Newton-Gregory formula for the same interpolation performed in part (a). Compare the answer with that of part (b).

d) Repeat parts (a), (b) and (c) using 8 equally spaced points in the same interval.

Q11. Runge's function is $f = 1/(1 + 25x^2)$. Plot this function in the interval [-1, 1] to see its behavior. This function is commonly used to demonstrate how high order interpolation with equally spaced points can give erroneous results, which is known as Runge's phenomenon.

a) Using values of Runge's function at 9 equally spaced points of x = -1, -0.75, -0.5, -0.25, . . ., 0.75, 1, determine the 8th order interpolating polynomial. Plot it and use it to estimate $f(0.8)$. Discuss the results.

c) Determine and plot natural cubic splines using the same 9 points and estimate $f(0.8)$. Compare the results with those of part (a).

Q12. (Work on the reading assignment R3 first) Temperatures are measured at 25 points on a heated square plate.

	x = 0	x = 2	x = 4	x = 6	x = 8
y = 0	100.00	90.00	80.00	70.00	60.00
y = 2	85.00	64.49	53.50	48.15	50.00
y = 4	70.00	48.90	38.43	35.03	40.00
y = 6	55.00	38.78	30.39	27.07	30.00
y = 8	40.00	35.00	30.00	25.00	20.00

Estimate the temperature at point $x = 3$, $y = 3.2$ by performing

a) bi-linear interpolation (linear in both directions) using 4 points shown in red color,

b) bi-quadratic interpolation (quadratic in both directions) using 9 points shown in blue color,

c) bi-quadratic interpolation (quadratic in both directions) using 9 points shown in green color,

Q13. Learn what the following MATLAB command do: polyfit, spline, interp1, interp2, interp3.

Q14. Read about Bezier curves, and NURBS curves and learn their relation with splines. Play the following online drawing game to practice with Bezier curves https://bezier.method.ac. If you want, you can share your efficient drawings that uses optimum number of points on ODTUClass.