Q5

$$f(x,y) = 5x^4 + 4x^2y - xy^3 + 4y^4 - x$$

b) Newton's Method $(x_0 = 5,\ y_0 = 4)$

$\partial f/\partial x = 20x^3 + 8xy - y^3 - 1$ $\qquad$ $\partial f/\partial y = 4x^2 - 3xy^2 + 16y^3$

$\dfrac{\partial^2 f}{\partial x^2} = 60x^2 + 8y$ $\qquad$ $\partial^2 f/\partial y^2 = -6xy + 48y^2$

$\dfrac{\partial^2 f}{\partial y \partial x} = 8x - 3y^2 = \dfrac{\partial^2 f}{\partial x \partial y}$

Iteration 1: $H_1 = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$ and $\nabla f_1 = \begin{Bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{Bmatrix}$ at $\begin{matrix} x = x_0 = 5 \\ y = y_0 = 4 \end{matrix}$

$$H_1 = \begin{bmatrix} 60 \cdot 5^2 + 8 \cdot 4 & 8 \cdot 5 - 3 \cdot 4^2 \\ 8 \cdot 5 - 3 \cdot 4^2 & -6 \cdot 5 \cdot 4 + 48 \cdot 4^2 \end{bmatrix} = \begin{bmatrix} 1532 & -8 \\ -8 & 738 \end{bmatrix}$$

$$\nabla f_1 = \begin{Bmatrix} 20 \cdot 5^3 + 8 \cdot 5 \cdot 4 - 4^3 - 1 \\ 4 \cdot 5^2 - 3 \cdot 5 \cdot 4^2 + 16 \cdot 4^3 \end{Bmatrix} = \begin{Bmatrix} 2595 \\ 884 \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix}_1 = -\left[H_1^{-1}\right] \left\{\nabla f_1\right\} = -\frac{\begin{bmatrix} 738 & 8 \\ 8 & 1532 \end{bmatrix}}{(1532 \cdot 738 - 8^2)} \begin{Bmatrix} 2595 \\ 884 \end{Bmatrix}$$

$$= \frac{-1}{1.131 \times 10^6} \begin{Bmatrix} 738 \cdot 2595 + 8 \cdot 884 \\ 8 \cdot 2595 + 1532 \cdot 884 \end{Bmatrix} = \begin{Bmatrix} -1.7 \\ -1.216 \end{Bmatrix} = \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix}$$

$$\left.\begin{matrix} x_1 = x_0 + \Delta x = 5 - 1.7 \\ y_1 = y_0 + \Delta y = 4 - 1.216 \end{matrix}\right\} \Rightarrow \boxed{x_1 = 3.3 \text{ and } y_1 = 2.784}$$

Iteration 2: H_2 and ∇f_2 are calculated @ $x = x_1 = 3.3$, $y = y_1 = 2.784$

$$H_2 = \begin{bmatrix} 60 \cdot 3.3^2 + 8 \cdot 2.784 & 8 \cdot 3.3 - 3 \cdot 2.784 \\ 8 \cdot 3.3 - 3 \cdot 2.784 & -6 \cdot 3.3 \cdot 2.784 + 48 \cdot 2.748^2 \end{bmatrix}$$

$$= \begin{bmatrix} 675.67 & 18.05 \\ 18.05 & 327.75 \end{bmatrix}$$

$$\nabla f_2 = \begin{Bmatrix} 20 \cdot 3.3^3 + 8 \cdot 3.3 \cdot 2.784 - 2.784^3 - 1 \\ 4 \cdot 3.3^2 - 3 \cdot 3.3 \cdot 2.784^2 + 16 \cdot 2.784^3 \end{Bmatrix} = \begin{Bmatrix} 1670.92 \\ 312.07 \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix}_2 = \frac{-1}{675.67 \cdot 327.75 - 18.05^2} \begin{bmatrix} 327.75 & -18.05 \\ -18.05 & 675.67 \end{bmatrix} \begin{Bmatrix} 1677.92 \\ 321.07 \end{Bmatrix}$$

$$= \begin{Bmatrix} -2.45 \\ -2.69 \end{Bmatrix} \Longrightarrow \begin{matrix} x_2 = 3.3 - 2.45 \\ y_2 = 2.784 - 2.69 \end{matrix} \Rightarrow \boxed{\begin{matrix} x_2 = 0.85 \\ y_2 = 0.09 \end{matrix}}$$

Iteration 3: H_3 and ∇f_3 are calculated @ $x = x_2 = 0.85$, $y = y_2 = 0.09$

$$H_3 = \begin{bmatrix} 60 \cdot (0.85)^2 + 8 \cdot 0.09 & 8 \cdot 0.85 - 3 \cdot 0.09 \\ 8 \cdot 0.85 - 3 \cdot 0.09 & -6 \cdot 0.85 \cdot 0.09 + 48 \cdot 0.09^2 \end{bmatrix} = \begin{bmatrix} 44.07 & 6.53 \\ 6.53 & -0.07 \end{bmatrix}$$

$$\nabla f_3 = \begin{Bmatrix} 20 \cdot 0.85^3 + 8 \cdot 0.85 \cdot 0.09 - 0.09^3 - 1 \\ 4 \cdot 0.85^2 - 3 \cdot 0.85 \cdot 0.09^2 + 16 \cdot 0.09^3 \end{Bmatrix} = \begin{Bmatrix} 14.06 \\ 2.88 \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix}_3 = \frac{-1}{44.07 \cdot (-0.07) - 6.53^2} \begin{bmatrix} -0.07 & -6.53 \\ -6.53 & 44.07 \end{bmatrix} \begin{Bmatrix} 14.06 \\ 2.88 \end{Bmatrix}$$

$$= \begin{Bmatrix} -0.4 \\ -0.77 \end{Bmatrix} \Longrightarrow \begin{matrix} x_3 = 0.85 - 0.4 \\ y_3 = 0.09 - 0.77 \end{matrix} \Rightarrow \boxed{\begin{matrix} x_3 = 0.45 \\ y_3 = -0.68 \end{matrix}}$$

c) optimum steepest descent ($x_0 = 5$. $y_0 = 4$)

$$\frac{\partial f}{\partial x} = 20x^3 + 8xy - y^3 - 1 \qquad \frac{\partial f}{\partial y} = 4x^2 - 3xy^2 + 16y^3$$

$$\frac{\partial^2 f}{\partial x^2} = 60x^2 + 8y \qquad \frac{\partial^2 f}{\partial y^2} = -6xy + 48y^2$$

$$\frac{\partial^2 f}{\partial y \partial x} = 8x - 3y^2 = \frac{\partial^2 f}{\partial x \partial y}$$

Iteration 1

$$\nabla f_1 = \begin{Bmatrix} 20 \cdot 5^3 + 8 \cdot 5 \cdot 4 - 4^3 - 1 \\ 4 \cdot 5^2 - 3 \cdot 5 \cdot 4^2 + 16 \cdot 4^3 \end{Bmatrix} = \begin{Bmatrix} 2595 \\ 884 \end{Bmatrix}$$

$$f\left(x_0 + h \cdot \partial f/\partial x \, , \, y_0 + h \frac{\partial f}{\partial y}\right) = g(h) =$$

$\rightarrow g'(h) =$

$g'(h^*) = 0 \rightarrow h^* = -0.00232 \rightarrow x_1 = x_0 + h^* \cdot \partial f/\partial x = 5 + (-0.00232) \cdot 2595$

$\rightarrow \boxed{x_1 = -1.02} \rightarrow y_1 = y_0 + h^* \frac{\partial f}{\partial y} = 4 + (-0.00232) \cdot 884$

$\rightarrow \boxed{y_1 = 1.95}$

Iteration 2

$$\nabla f_2 = \begin{Bmatrix} 20 \cdot (-1.02)^3 + 8 \times (-1.02) \times 1.95 - 1.95^3 - 1 \\ 4 \times 1.02^2 - 3 \times (-1.02) \times 1.95^2 + 16 \times 1.95^3 \end{Bmatrix} = \begin{Bmatrix} -45.5 \\ 134.47 \end{Bmatrix}$$

$$f\left(x_1 + h \, \partial f/\partial x \, , \, y_1 + h \frac{\partial f}{\partial y}\right) = g(h) =$$

$\rightarrow g'(h) =$

$g'(h^*) = 0 \rightarrow h^* = -0.0169 \rightarrow \boxed{\begin{matrix} x_2 = -0.25 \\ y_2 = -0.32 \end{matrix}}$

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Iteration 3

$$\nabla f_2 = \begin{Bmatrix} 20 \times 0.25^3 + 8 \times (-0.25)(-0.32) - (-0.32)^3 - 1 \\ 4 \times 0.25^2 - 3 \times (-0.25)(-0.32)^2 + 16 \times (-0.32)^3 \end{Bmatrix}$$

$$= \begin{Bmatrix} -0.6313 \\ -0.2179 \end{Bmatrix}$$

$$f\left(x_2 + h\frac{\partial f}{\partial x},\ y_2 + h\frac{\partial f}{\partial y}\right) = g(h) =$$

$$\rightarrow g'(h) =$$

$$g'(h^*) = 0 \longrightarrow h^* = -1.005 \longrightarrow x_3 = x_2 + h^* \partial f/\partial x = \boxed{0.38 = x_3}$$

$$y_3 = y_2 + h^* \frac{\partial f}{\partial y} = \boxed{-0.10 = y_3}$$