Middle East Technical University
Department of Mechanical Engineering
ME 310 Numerical Methods
Spring 2024 (Dr. C. Sert)
Study Set 1

For Homework 1 submit the answers of questions 1, 2, 5, 9 and 12. Their grade percentages may not be equal.

Assigned: 02/03/2024 – Due: 11/03/2024, 10:30 (online and in class)

Homework Rules and Suggestions:

- This assignment can be done individually or as a team of two students. Everything in your report should be the result of your own work or your team's work. You are allowed to discuss the questions with your classmates and teaching staff up to a certain detail on ODTUClass. You are not allowed to use an AI tool such as ChatGPT in writing codes or other parts of your report.
- Put the following honor pledge at the top of your homework report and behave accordingly.
  "I understand that this is an individual/team assignment. I affirm that I have not given or received any unauthorized help on this assignment, and that this work is my own/team's."
- If you do the homework as a team, put the percent contribution of each member at the beginning of your report.
- If you've exchanged ideas with other students outside ODTUClass, you need to put their names and the extent of your discussion at the beginning of your report.
- You need to submit a printed report. It is what we will be grading. You also need to upload the same report as a PDF document (not a Word document) together with all other files (such as codes) to ODTUClass. Name your MATLAB files properly. Follow MATLAB file naming rules such as "File names cannot start with a number", "They cannot contain special characters or spaces", etc.
- Late submission is not allowed unless you have a valid excuse. In such a case, you need let the whole teaching team know about it before the submission deadline, unless it is an emergency.
- Make sure that the codes in your report are formatted properly. Use a small sized, fixed width font and make sure that lines are not wrapped. If your code is very long, you can shorten it by getting rid of its noncritical parts and putting a note about this. Note that we grade what we see in your printed report. Do not expect us to run your codes for you to generate results, figures, etc. You should do that yourself and put all the results in your report.
- In writing your codes, follow good programming practices such as "use explanatory header lines", "explain inputs and outputs of functions", "use self-explanatory variable names", "use comments", "use empty lines and spaces for readability", "use indentation for code blocks", "divide long lines into multiple lines using MATLAB's '…' syntax", etc.
- Pay attention to the format of your report. It should look like a serious academic work, not like a high school student work. Font types and sizes, page margins, empty spaces on pages, equations, figures, tables, captions, colors, etc. are all important to give the desired "academic work feeling". Language used is also important. Reports with poor use of English will be penalized.
- Do not provide an unnecessarily long report. The shorter your report, the better it is, as long as it answers the questions properly. Avoid wasting paper. Print on both sides of sheets. Avoid using color unless it is really necessary. Format your report properly with small but readable fonts, small margins, no unnecessarily large figures, no useless spaces, etc. to reduce the number of sheets. Do not use a cover page.
- There are more than 100 students, and we can spend only about 10 minutes to grade each report. Your report should be easy to read and understand. We should be able to find the results and judge their correctness easily. We should not get lost in your report. The more we struggle to understand your report, the lower your grade will be. Use figures and tables cleverly for this purpose.
- Reports with only figures, tables and codes, but no text, comments or discussions will not get a good grade. Start answering each question with one paragraph of introduction. Even when a question does not specifically ask for a discussion, you need write some comments on the key points and your key findings/learnings.
- Figures and tables should be numbered and should have captions (at the bottom for figures and at the top for tables). Their titles should be self-explanatory, i.e., we should be able understand everything about the table or figure just by reading its title. They should all be referred properly in the written text (such as "… as shown in Fig. 3" or "… (See Table 2)").
- Do not use any Appendices in your report.
- Do not forget to put a numbered reference list at the end of your report if you use references. In that case, you need to refer to the references in the text.
- If you are inexperienced in programming, converting an idea/algorithm into a code and writing it in a bug-free way can be time consuming and frustrating. This is not something that can be done at the last minute. You are advised to start working on the assignments as soon as they are assigned.

Reading Assignments:

Self-learning is an important part of education. Not everything can be discussed in lectures. You need to learn certain things by yourself.

R1) Read section PT1.1.2 Numerical Methods and Engineering Practice (page 4 of 8^{th} edition).

R2) Read section 3.1 Significant Figures (page 58 of 8^{th} edition).

R3) Read about the Scarborough criteria and learn why and how it is used in Section 3.3 (page 63 of 8^{th} edition).

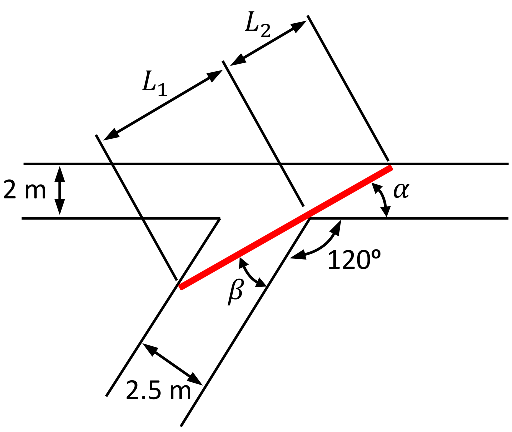
R4) Read Section 4.2 Error Propagation (page 99 of 8^{th} edition).

R5) Read Section 4.3 Total Numerical Error (page 108 of 8^{th} edition).

R6) Read the Epilogue section of Part 1 (page 112 of 8^{th} edition). Part 1 includes the first 4 chapters and its epilogue is at the end of Chapter 4. What we call "Chapter 1 Introduction" in our lectures is the first part (first 4 chapters) of the textbook.

Questions:

Q1. Two mine shafts meet at an angle of 120° as shown. The top and bottom shafts are 2 m and 2.5 m wide, respectively. Miners want to move a ladder in between the shafts and need to know the longest ladder that can turn at the intersection.



Length of the ladder can be formulated as

$$L = L_1 + L_2 = \frac{2.5}{\sin(\beta)} + \frac{2}{\sin(\alpha)}$$

using $\beta = \pi - 120\frac{\pi}{180} - \alpha$ (b and c are in radians)

$$L(\alpha) = \frac{2.5}{\sin\left(\pi - 120\frac{\pi}{180} - \alpha\right)} + \frac{2}{\sin(\alpha)}$$

Minimum value of this function is the length of the longest ladder that can make the turn.

Use the following approaches to determine the minimum of this function and the value of α that gives this minimum.

a) Plot the given $L(\alpha)$ function (with any software you want) and determine the minimum value of L and the corresponding α value visually. If there are multiple minimums, determine the physically meaningful one. This is a graphical solution.

b) Determine the α value that gives $dL/d\alpha = 0$. This is an analytical solution.

c) Write a code that determines the minimum L numerically. Starting from a low α value (in radians), evaluate L with small increments. For example, start from 0.01 and evaluate L at 0.01, 0.02, 0.03, 0.03, … . L values will drop initially, but then start to increase at some point. Your code should detect where this change occurs and print out the α interval where L has a minimum. Does your finding match with the results of parts (a) and (b)?

d) MATLAB has `fminbnd` function to find the minimum of a function in a given interval. Read its documentation to learn how it works. Use it to determine the minimum value of L. Show how you used the function and provide the result you get.

Q2. Watch the uploaded MATLAB tutorials about numerical integration codes shared by students. Close to the end of Tutorial 4, it is mentioned that MATLAB's built-in function `polyfit` can be used to simplify the developed codes.

a) Modify the uploaded code General_polynomial_itegration.m to use the `polyfit` function. Explain the changes you've made.

b) Use the developed code to integrate $\sin(x)\,/(1+x)$ from 0 to 1 using n=4 as the polynomial order and N=2, 4, 8, 16, 32, 64, 128 as the number of segments. Tabulate the obtained true errors for each N. Plot true error vs N. What is the slope of the plotted curve and what does this mean?

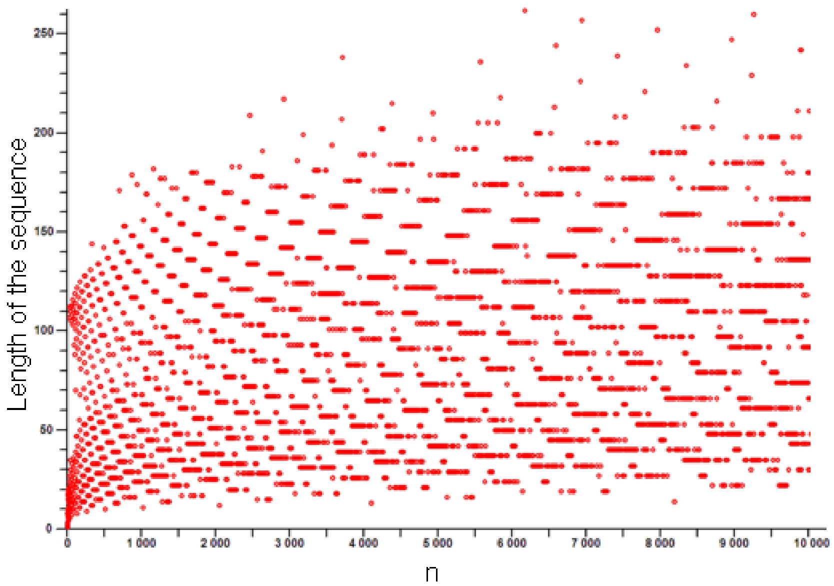
c) Repeat part (b) for the function $\sin(x^3)\cos(x)$ with limits of 0 to 2.2. Can you get the same convergence behavior as part (b)? Discuss.

Q3. There is a famous unsolved problem in number theory, known as the 3n+1 problem (also called the Collatz conjecture). Introduced in 1937 by German mathematician Lothar Collatz, the Collatz conjecture is a seemingly straightforward question with a surprisingly elusive answer. Start with any positive integer n, and repeat the following steps to generate a sequence:

- If n=1, stop.
- If n is even, replace it with n/2 and continue.
- If n is odd, replace it with 3n + 1 and continue.

For example, starting with n=7 produces the sequence 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. The sequence terminates after 17 steps. Note that whenever n reaches a power of 2, the sequence terminates in $\log_2 n$ more steps. The unanswered question is, does the process always terminate? Or is there some starting value that causes the process to go on forever, either because the numbers get larger and larger, or because some periodic cycle is generated?

Write a code that generates the following plot, which shows the length of the sequences for n values up to 10000.



Q4. The numerator of the following mathematical expression simplifies to x^2 and the division results in 1.

$$\frac{(x+y)^2 - (2xy + y^2)}{x^2}$$

Calculate this expression using a code for the following numbers using both single and double precision. Provide your results as a table. Comment on the results and discuss the cause of the behavior you see. Which operation of the above equation is mainly responsible for the accuracy loss? Read your textbook's related section to get a hint about subtractive cancellation.

x	y
0.01	1000
0.001	1000
0.0001	1000
0.00001	1000
0.000001	1000
0.0000001	1000

Q5. Write a code to evaluate e^{-10} using the following two equations. Print out the result and the relative percent true errors after adding each term. Do this using both double and single precision. Discuss the results and the behaviors you see in terms truncation and round-off errors. Which equation is better in terms of accuracy? Why? Discuss.

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \cdots \qquad \text{and} \qquad e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots}$$

Q6. a) What is machine epsilon? What does MATLAB's `eps` function return and what does it mean? What about `eps(1)`, `eps(100)` and `eps(1e6)`?

b) True or false: Floating-point numbers are distributed uniformly throughout their range.

Q7. a) Explain why a divergent infinite series, such as

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

can have a finite sum in floating-point arithmetic.

b) At what point will the partial sums stop to change?

Q8. Consider the function $f(x) = (e^x - 1)/x$. It is possible to show that

$$\lim_{x \to 0} f(x) = 1$$

a) Check this result by writing a program to compute $f(x)$ for $x = 10^{-k}, k = 1, 2, 3, \ldots, 15$. Provide the result of each calculation. Do your results agree with the expectations? Explain and discuss.

b) Repeat part (a) using the following mathematically equivalent formulation

$$f(x) = (e^x - 1)/\log{(e^x)}$$

Evaluate this function as given, with no simplification. If this works any better, can you explain why? Note that $\log()$ is the natural logarithm function.

Q9. The polynomial $(x-1)^6$ is zero at $x = 1$ and is positive elsewhere. The expanded form of the polynomial, $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$, is mathematically equivalent but may not give the same results numerically. Compute and plot the values of this polynomial, using each of the two forms, for 101 equally spaced points in the interval $[0.995;\ 1.005]$, i.e., with a spacing of 0.0001. Explain this behavior.

Q10. Write a program that sums N random, single-precision floating-point numbers x_i, uniformly distributed on the interval $[0, 1]$. Sum the numbers in each of the following ways (use only single-precision floating-point variables unless specifically indicated otherwise).

a) Sum the numbers in the order in which they were generated, using a double-precision variable in which to accumulate the sum.

b) Sum the numbers in the order in which they were generated, this time using a single-precision accumulator.

c) Sum the numbers in order of increasing magnitude. This will require that the numbers be sorted before summing.

d) Sum the numbers in order of decreasing magnitude (i.e., reverse the order of summation from part (c).
Run your program for various values of N and compare the results for methods (a) through (d). You may need to use a fairly large value for n to see a substantial difference. How do the methods rank in terms of accuracy, and why? How do the methods compare in cost?

Q11. We want to calculate the cosine of a number (in radians) using the 5th order Maclaurin series expansion. Using the remainder term of the series and the fact that the cosine function and its derivatives stay bounded in [-1,1], determine the upper bound of the error in calculating $\cos(0.25)$.

Q12. Derive the 0th, 1st and 2nd order Taylor series expansions of the function $f = 2\sin(x) + \cos(y)$ around point (0.6, 0.5). Evaluate these series at point (0.62, 0.48). Calculate the true error for each of these. Tabulate your results. Hint: See Eqn. (4.26) of your textbook for the Taylor series expansion of a multivariable function.

Q13. (Work on the "R4 reading assignment" before solving this question) Deflection at the tip of a cantilever beam with a point load acting at its tip is given by

$$d_{tip} = \frac{4PL^3}{Ebh^3}$$

where P is the point load, L is the length of the beam, E is the elastic modulus of the beam material, b is the width of the beam, and h is the height of the beam. Determine the deflection at the tip and its uncertainty for the following measured values: $P = 1000 \pm 20 \text{ N}$, $L = 1 \pm 0.01 \text{ m}$, $E = 30 \pm 0.3 \text{ GPa}$, $b = 10 \pm 0.1 \text{ cm}$ and $h = 15 \pm 0.15 \text{ cm}$.