D-Chain Take-Home Exam

(Deadline: 17th of June 2022, Friday, 23:59)

1 Problem

In this THE, you will look at a variation of the shortest addition chain problem. A *distinct addition chain* or *d-chain* generating a positive integer n is a sequence of integers $\langle s_1, \ldots, s_m \rangle$ such that

- $s_1 = 1$
- $s_2 = 2$
- For $i \geq 3$, $s_i = s_j + s_k$ for some pair j, k where $j, k < i$ and $j \neq k$.
- s_m = n

In other words, a d-chain for n is an addition chain for n starting with "$1, 2$" and its further elements are obtained by summing two previous *distinct* elements in the chain.

Write a program that prints out a shortest d-chain for a given n.

2 Input Format

```
N
```

`N` = The integer whose shortest d-chain is to be found.

3 Output Format

```
S1 ... SM
```

$\texttt{S}_i$ = The ith element of the found shortest d-chain for `N`.
`M` = Shortest chain length.

4 Limits

$1 \leq \texttt{N} \leq 50000$
$1 \leq \texttt{M} \leq 25$
Time Limit: 1 second
Memory Limit: 256 MB

5 Clarifications

- Your solution is expected as a C++ program source named `dchain.cpp` that reads from the standard input and writes to the standard output.
- It is OK to copy code from the sample codes we shared in our course website in ODTÜClass for this THE.
- You are supposed to submit your code via ODTÜClass, via the auto-grader that we provided.
- The grade from the auto-grader is not final. We can later do further evaluations of your code and adjust your grade. Solutions that do not attempt a "reasonable solution" to the given task may lose points.
- We will compile your code on g++ with options: `-std=c++17 -O2 -lm -Wall -Wextra -Wpedantic`
- Late submissions are not allowed.

6 Hints

- We suggest using DFID (depth-first iterative deepening) coupled with clever improvement and a reasonably accurate admissible heuristic to solve this problem.
- We studied the similar shortest addition chain problem in the class. We suggest that you try to adapt the improvements and the heuristics that we discussed to this particular problem.

7 Examples

Sample Input 1

```
12
```

Sample Output 1

```
1 2 3 5 7 12 # Length: 6.
```

Sample Input 2

```
64
```

Sample Output 2

```
1 2 3 5 8 13 21 22 43 64 # Length: 10
```

Sample Input 3

```
748
```

Sample Output 3

```
1 2 3 5 8 10 18 28 46 74 120 194 314 434 748 # Length: 15.
```

Sample Input 4

```
38604
```

Sample Output 4

```
1 2 3 5 8 13 21 34 55 89 144 233 377 521 898
953 1851 2749 4600 7349 11949 19298 19306 38604 # Length: 24
```