

$$1) \int \sec \theta \cdot d\theta = ?$$

$$\int \frac{1}{\cos \theta} \cdot d\theta = \int \frac{\cos \theta}{\cos^2 \theta} \cdot d\theta = \int \frac{\cos \theta}{1 - \sin^2 \theta} \cdot d\theta, \quad u = \sin \theta \text{ obw.}$$

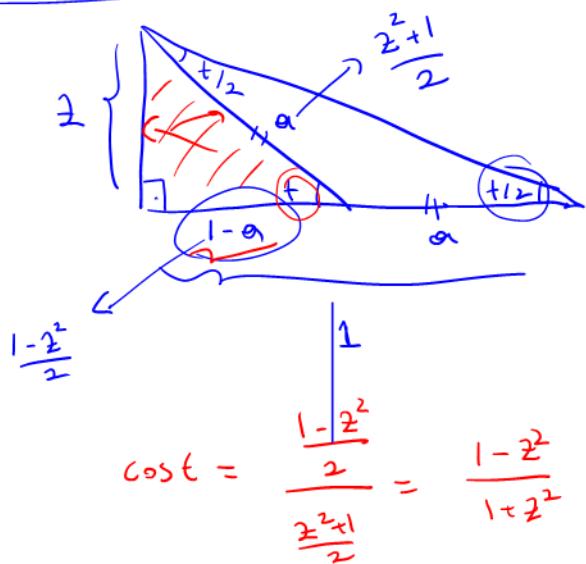
$$du = \cos \theta \cdot d\theta \text{ obw.}$$

$$= \int \frac{du}{1-u^2} \rightarrow \frac{A}{1-u} + \frac{B}{1+u}$$

$$= \int \frac{1}{2} \frac{du}{1+u} + \int \frac{1}{2} \frac{du}{1-u}$$

$$= \ln|1+u| - \ln|1-u| = \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + C \text{ obw.}$$

$$2) \int \frac{dt}{\sin t - \cos t} = ? \quad \tan \frac{t}{2} = z \quad \text{dönmez mi ya palam?}$$



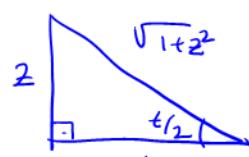
$$\Rightarrow \alpha^2 = z^2 + (1-\alpha)^2 = z^2 + \alpha^2 + 1 - 2\alpha$$

$$\Rightarrow 2\alpha = z^2 + 1$$

$$\Rightarrow \frac{z^2+1}{2} = \alpha \quad 1-\alpha = 1 - \frac{z^2+1}{2} \\ = \frac{1-z^2}{2}$$

$$\left(\sec^2 \frac{t}{2}\right) \cdot \frac{1}{2} \cdot dt = dz \Rightarrow dt = \underbrace{2 \cdot \cos^2 \frac{t}{2} \cdot dz}_{\frac{2}{1+z^2} dz}$$

$$\int \frac{\frac{2}{1+z^2} dz}{\frac{2z}{1+z^2} - \frac{1+z^2}{1+z^2}}$$



$$\cos \frac{t}{2} = \frac{1}{\sqrt{1+z^2}}$$

$$\cos^2 \frac{t}{2} = \frac{1}{1+z^2}$$

$$\Rightarrow \int \frac{2}{\underbrace{(2z-1+z^2)}_{(2+1)^2-2}} dz = \int \frac{2dz}{(2+1-\sqrt{2})(2+1+\sqrt{2})} \Rightarrow \frac{A}{2+1-\sqrt{2}} + \frac{B}{2+1+\sqrt{2}} = \frac{2}{2z-1+z^2}$$

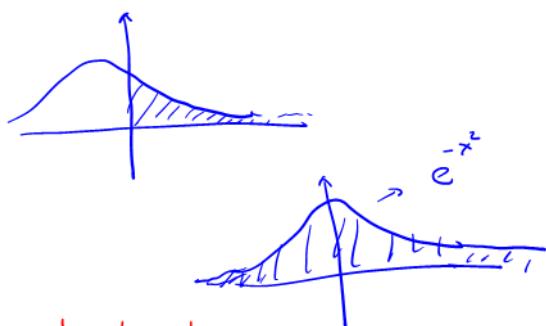
$\downarrow$   
 $\partial z / \partial t$

$$\Rightarrow A = \frac{1}{\sqrt{8}}, \quad B = -\frac{1}{\sqrt{8}}$$

$$= \int \frac{1}{\sqrt{8}} \cdot \frac{dz}{2+1-\sqrt{2}} - \int \frac{1}{\sqrt{8}} \cdot \frac{dz}{2+1+\sqrt{2}}$$

$$= \frac{1}{\sqrt{8}} \left( \ln |2+1-\sqrt{2}| - \ln |2+1+\sqrt{2}| \right).$$

3-)  $\int_0^\infty \frac{16 \tan^{-1} x}{1+x^2} dx = ?$



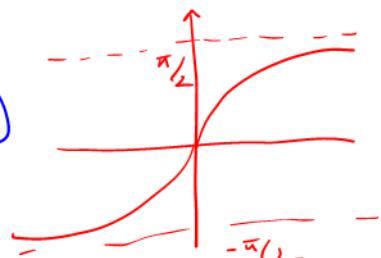
$$I = \lim_{\alpha \rightarrow \infty} \int_0^\alpha \frac{16 \tan^{-1} x}{1+x^2} dx$$

$\tan^{-1} x = 0 \Rightarrow \frac{1}{1+x^2} dx = du$

$$= \lim_{\alpha \rightarrow \infty} \int_0^{\tan^{-1} \alpha} 16u \cdot du = \lim_{\alpha \rightarrow \infty} 8u^2 \Big|_0^{\tan^{-1} \alpha}$$

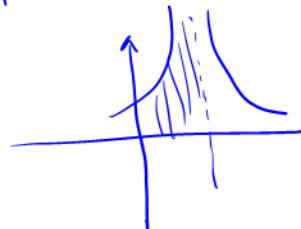
$$= \lim_{\alpha \rightarrow \infty} 8 \left[ (\tan^{-1} \alpha)^2 - 0 \right]$$

$$= 8 \cdot \frac{\pi^2}{4} = 2\pi^2 \quad \square$$



$$4-) \int_0^2 \frac{dx}{\sqrt{|x-1|}} = ?$$

$$f(x) = \frac{1}{\sqrt{|x-1|}} \quad x \rightarrow 1 \quad f(x) \rightarrow \infty$$



$$I = \int_0^1 \frac{dx}{\sqrt{1-x}} + \int_1^2 \frac{dx}{\sqrt{x-1}}$$

$\rightarrow$   $\rightarrow$   $\rightarrow$   $\rightarrow$

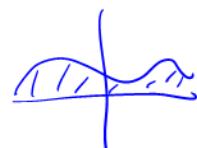
$$= \lim_{a \rightarrow 1^-} \int_0^a \frac{dx}{\sqrt{1-x}} + \lim_{b \rightarrow 1^+} \int_b^2 \frac{dx}{\sqrt{x-1}}$$

$$= \lim_{a \rightarrow 1^-} -2\sqrt{1-a} \Big|_0^a + \lim_{b \rightarrow 1^+} 2\sqrt{x-1} \Big|_b^2$$

$$= \lim_{a \rightarrow 1^-} \frac{(-2\sqrt{1-a} + 2)}{0} + \lim_{b \rightarrow 1^+} \frac{2 - 2\sqrt{b-1}}{\infty}$$

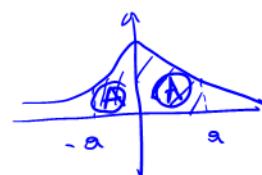
$$= 2 + 2 = 4.$$

5-)  $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$  ifadesinin yakınsak olup olmadığını bulunuz.



$$f(x) = \frac{1}{e^x + e^{-x}} \quad \text{Fonksiyon çiftti. asimetriktir;}$$

$$f(-x) = \frac{1}{e^{-x} + e^{-(x)}} = \frac{1}{e^x + e^{-x}} = f(x) \rightarrow$$



$$\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}} = 2 \cdot \int_0^{\infty} \frac{dx}{e^x + e^{-x}}$$

$$I = \lim_{a \rightarrow \infty} \int_{-\infty}^a \frac{dx}{e^x + e^{-x}} = \lim_{a \rightarrow \infty} 2 \int_0^a \frac{dx}{e^x + e^{-x}}$$

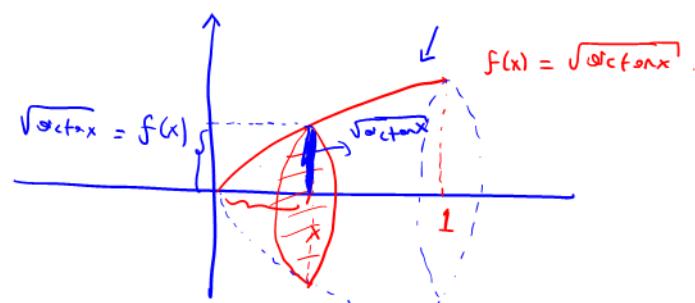
$\forall x \in \mathbb{R}$  ikin  $e^x > 0$  old. not edelin.

$$0 < \frac{1}{e^x + e^{-x}} < \frac{1}{e^{-x}}$$

ve eylem  $\int_0^{\infty} \frac{1}{e^x} dx = 2$  ve galans.

Konsantina tayfinden  $\int \frac{1}{e^x + e^{-x}} dx$  galans olur.

6-)  $f(x) = \sqrt{\operatorname{arctan} x}$  fonksiyonu  $\{0, 1\}$  aralığında tanımlanır. Bu fonksiyon  $x$  ekseni etrafında döndürmekle elde edilen seklin hacmi nedir?



$$\begin{aligned} D_x &= \pi r^2 \\ &= \pi \cdot (\sqrt{\operatorname{arctan} x})^2 \\ &= \pi \cdot \operatorname{arctan} x \text{ olur.} \end{aligned}$$

$\forall x \in \{0, 1\}$  ikin  $D_x$  alanını toplayarak hacmi elde ederiz.

$$\begin{aligned} V &= \int_0^1 \pi \cdot \operatorname{arctan} x dx \\ \operatorname{arctan} x &= u \quad \text{ve} \quad dx = du \\ \frac{1}{1+x^2} \cdot dx &= du \Leftrightarrow x = u \end{aligned}$$

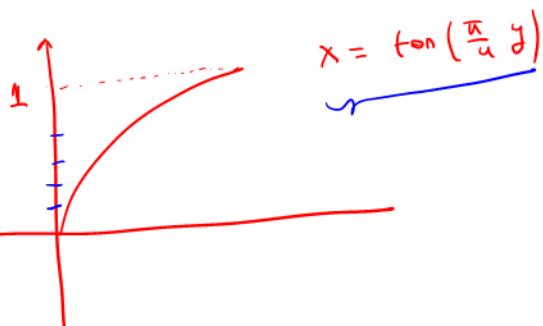
$$\begin{aligned} &= \pi \left[ \operatorname{arctan} x \cdot x \right]_0^1 - \left[ \frac{1}{2} \ln(1+x^2) \right]_0^1 \\ &\quad \int \operatorname{arctan} x dx = \operatorname{arctan} x \cdot x - \int \frac{x}{1+x^2} dx \\ &\quad \int \frac{dx}{1+x^2} = \frac{d\theta}{2} \quad 2x dx = d\theta \end{aligned}$$

$$= \pi \left\{ \frac{\pi}{u} \cdot 1 - \left[ 0 \cdot 0 \right] - \underbrace{\left[ \frac{\ln 2}{2} - \frac{\ln 1}{2} \right]}_{\text{}} \right\}$$

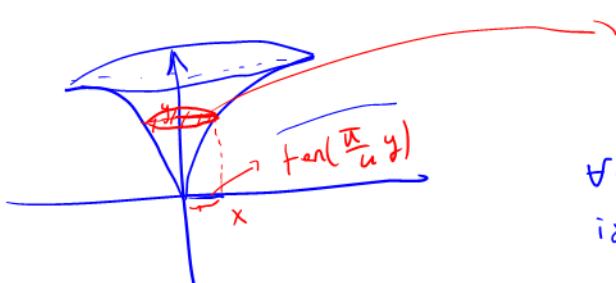
$$= \pi \left( \frac{\pi}{u} - \frac{\ln 2}{2} \right) \quad .$$

$$\frac{1}{2} \ln u = \frac{\ln(1+u^2)}{2}$$

7)



$x = \tan(\frac{\pi}{u}y)$  Punktum  $y$  elgeni etrafında döndürür elde edilen şekilde hachı ne olur?



$\forall y \in \mathbb{R}$  iken  $t = D_y$ 'nin toplam istenilen hachı verecektir.

$$D_y = \pi \cdot r^2$$

$$D_y = \pi \cdot \tan^2(\frac{\pi}{u}y) \quad \text{bulunur.}$$

$$\vartheta = \int_0^{\frac{\pi}{u}} \pi \cdot \tan^2(\frac{\pi}{u}y) \cdot dy, \quad \frac{\pi}{u}y = u$$

$$\Rightarrow \frac{\pi}{u} dy = du$$

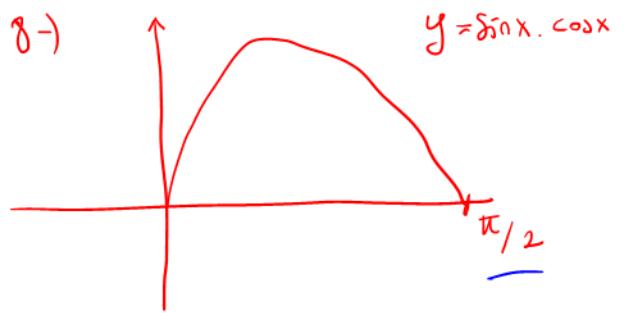
$$\Rightarrow dy = \frac{u}{\pi} du$$

$$\boxed{\frac{s^2}{c^2} + \frac{c^2}{c^2} = \frac{s^2 + c^2}{c^2} = \frac{1}{c^2}}$$

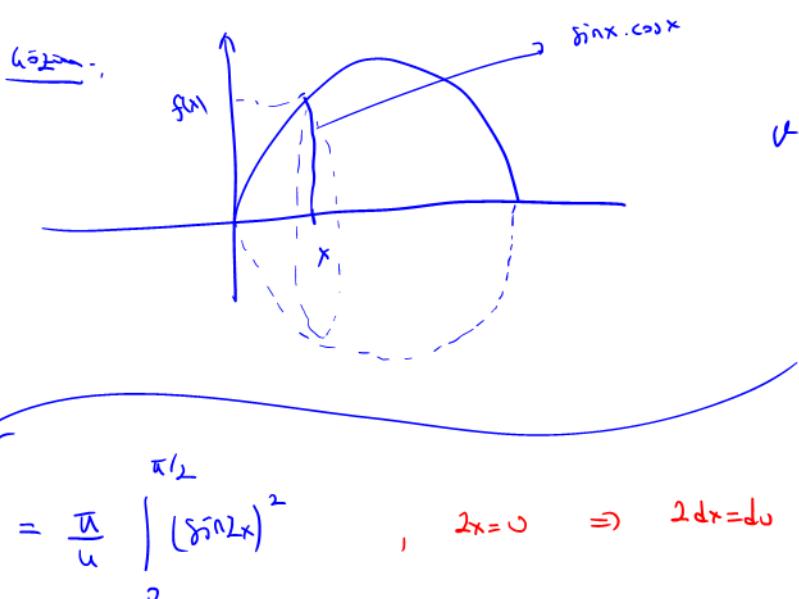
$$\tan^2 x + 1 = \sec^2 x$$

$$\vartheta = u \int_0^{\frac{\pi}{u}} \tan^2(u) \cdot du = u \int_0^{\frac{\pi}{u}} (\sec^2 u - 1) du = u \left\{ \tan u - u \Big|_0^{\frac{\pi}{u}} \right\}$$

$$= u \left\{ 1 - \frac{\pi}{u} - 0 \right\} = \boxed{u - \bar{u}}$$



Yondaiki funk. 'nın  $x$ - ekseni  
etra fonda döndürmeye ile  
olsun selen hacmini bulun.



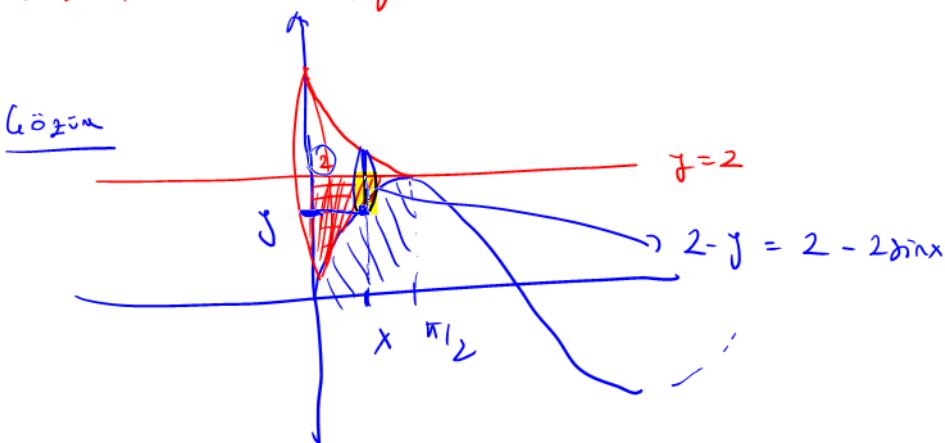
$$D_x = \pi \cdot \sin^2 x \cdot \cos^2 x$$

$$V = \int_0^{\pi/2} \pi \cdot \sin^2 x \cdot \cos^2 x \cdot dx$$

$$= \int_0^{\pi/2} \pi \cdot (\underbrace{\sin x \cdot \cos x}_{\frac{1}{2} \cdot \sin 2x})^2 dx$$

$$= \frac{\pi}{8} \cdot \int_0^{\pi} \sin^2 u du = \frac{\pi^2}{16}$$

g-) Birinci bölgeleri öften  $y=2$  doğrusu, alttan  $y=2 \sin x$ ,  
 $x \in [0, \frac{\pi}{2}]$  egrisi ve soldan  $y$ - ekseni ile sınırlı bölgelerin  $y=2$   
etra fonda döndürmeye elde edilen selen hacmini bulunuz.



$$D_x = \pi \cdot [2 - 2 \sin x]^2$$

$$\int_0^{\pi/2} \bar{u} \cdot \{2 - 2\sin x\}^2 dx = u\bar{u} \int_0^{\pi/2} (1 - \sin x)^2$$

$$= u\bar{u} \int_0^{\pi/2} 1 + \sin^2 x - 2\sin x$$

$$= \bar{u} \{3\bar{u} - 8\}$$