1-) $\ln(1-x)$'in $x=0$'daki kuvvet serisini bulunuz.

<u>Çözüm:</u> $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

$$\Rightarrow \int \frac{1}{1-x}\,dx = \int \sum_{n=0}^{\infty} x^n . dx$$

$$\Rightarrow -\ln(1-x) = \sum_{n=0}^{\infty} \int x^n . dx$$

$$\Rightarrow \ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} + C$$

$x=0$ için $\ln(1-0) = -\sum_{n=0}^{\infty} \frac{0^{n+1}}{n+1} + c$

$$0 = c$$

$$\ln(1-x) = -\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$$

2-) $\sinh x$'in kuvvet seri açılımını bulunuz.

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$= \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^n}{n!} - \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}\right)$$

$(-1)^n . x^n$

$$= \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{\overbrace{(1-\overbrace{(-1)^n}^{\pm})}^{2} . x^n}{n!}\right)$$

$$= \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

3-) $\ln(2+2x+x^2)$'nin $x=1$'deki kuvvet seri açılımını bulunuz

$$\ln(1+(1+2x+x^2)) = \ln(1+(1+x)^2)\, dx$$

$$\frac{1}{1+(\underbrace{1+x}_{1+x=u})^2} = \frac{1}{1+u^2} = \sum_{n=0}^{\infty} (-1)^n . u^n$$

$$= \sum_{n=0}^{\infty} (-1)^n . (1+x)^n$$

$$\boxed{\begin{aligned} \frac{1}{1-x} &= \sum_{0}^{\infty} x^n \\ \Rightarrow \frac{1}{1+x} &= \sum_{0}^{\infty} (-1)^n . x^n \end{aligned}}$$

$$\Rightarrow \quad \ln(1+(1+x)^2) = \int \sum_{n=0}^{\infty} (-1)^n . (1+x)^n \, dx$$

$$= \sum_{n=0}^{\infty} \int (-1)^n . (1+x)^n dx$$

$$= \underbrace{\sum_{n=0}^{\infty} (-1)^n . \frac{(1+x)^{n+1}}{n+1}}_{} + C$$

$x=-1$ için $\quad \ln(1) = C = 0 .$

4-) $\int_0^x \frac{e^{t^2}-1}{t^2}\,dt = F(x)$ int. ile tanımlı Fonk.nun kuvvet serisi açılımını bulunuz.

$t^2 = u$ olsun. $e^{t^2} = e^u = \sum \frac{u^n}{n!} = \sum \frac{(t^2)^n}{n!}$

$$e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!} \Rightarrow e^{t^2} = \sum_{n=0}^{\infty} \frac{t^{2n}}{n!}$$

$e^t \cdot e^t$

$(\sum \ldots) \cdot (\sum \ldots)$

$$\Rightarrow e^{t^2} - 1 = \sum_{n=0}^{\infty} \frac{t^{2n}}{n!} - 1 =$$

$$= \sum_{n=1}^{\infty} \frac{t^{2n}}{n!}$$

$$\Rightarrow \frac{e^{t^2}-1}{t^2} = \frac{1}{t^2} \sum_{n=1}^{\infty} \frac{t^{2n}}{n!} = \sum_{n=1}^{\infty} \frac{t^{2n-2}}{n!}$$

$$\Rightarrow F(x) = \int_0^x \frac{e^{t^2}-1}{t^2}\,dt = \int_0^x \sum_{n=1}^{\infty} \frac{t^{2n-2}}{n!}\,dt$$

$$= \sum_{n=1}^{\infty} \int_0^x \frac{t^{2n-2}}{n!}\,dt$$

$$= \sum_{n=1}^{\infty} \frac{t^{2n-1}}{(2n-1).n!} \Big|_0^x$$

$$= \sum_{1}^{\infty} \frac{x^{2n-1}}{(2n-1)\,n!}$$

6-) $f(x) = \sin x$ fonk.nun $\frac{\pi}{4}$'de 4. dereceden Taylor Polinomunu bulun.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a) \cdot (x-a)^n}{n!}$$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$\Rightarrow$

$f(\pi/4) = \frac{1}{\sqrt{2}}$

$f'(\pi/4) = \frac{1}{\sqrt{2}}$

$f''(\pi/4) = -\frac{1}{\sqrt{2}}$

$= -\frac{1}{\sqrt{2}}$

$= 1/\sqrt{2}$

$$P_4(x) = \frac{1}{\sqrt{2}} \cdot \frac{(x-\frac{\pi}{4})^0}{0!} + \frac{1}{\sqrt{2}} \cdot (x-\pi/4)^1 + \frac{(x-\pi/4)^2}{-2\sqrt{2}}$$

$$+ \frac{(x-\pi/4)^3}{-6\sqrt{2}} + \frac{(x-\pi/4)^4}{24\sqrt{2}}.$$

7-) $\ln(0{,}98)$'in yaklaşık değerini bulmak için 3. dereceden Taylor polinomunu kullanınız.

$f(x) = \ln x$ olsun.

$f'(x) = \frac{1}{x} \to 1$

$f'' = -\frac{1}{x^2} \to -1$

$f''' = \frac{2}{x^3} \to 2$

$a=1$ için Taylor açılımı:

$$\ln x = 0 \cdot \frac{(x-1)^0}{0!} + 1 \cdot \frac{(x-1)^1}{1!} - \frac{(x-1)^2}{2!} + \cancel{2} \cdot \frac{(x-1)^3}{\cancel{3!}}$$

$$= (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$$

$$\ln(0{,}98) = (0{,}98-1) - \frac{(0{,}98-1)^2}{2} + \frac{(0{,}98-1)^3}{3}$$

$$= -(0{,}02) - \frac{(0{,}02)^2}{2} + \frac{(-0{,}02)^3}{3}$$

$$= -0{,}0202027$$

8-) $\lim\limits_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$ limitini Taylor formülü ile bulalım.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad , \quad e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^n}{n!}$$

$$\lim_{x \to 0} \frac{\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots\right) + \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots\right) - 2}{x^2}$$

$$\Rightarrow \lim_{x \to 0} \frac{2\left(\frac{\cancel{x^2}}{2} + \frac{\cancel{x^4}}{4!} + \frac{\cancel{x^6}}{6!} + \dots\right)}{\cancel{x^2}}$$

$$\Rightarrow \lim_{x \to 0} 2\left(\frac{1}{2} + \cancel{\frac{x^2}{4!}} + \cancel{\frac{x^4}{6!}} + \cancel{\dots}\right) = 1$$