Örnekler

Ör: $A = \begin{bmatrix} 0 & 0 & 1 & 3 & 5 \\ 0 & -1 & 1 & 1 & 2 \\ 0 & 2 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & 3 \end{bmatrix}$ veriliyor. A'ya denk eşelon matrisi bulunuz.

Çöz: $A \xrightarrow{r_1 \leftrightarrow r_4} \begin{bmatrix} 0 & 1 & 0 & 2 & 3 \\ 0 & -1 & 1 & 1 & 2 \\ 0 & 2 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3 & 5 \end{bmatrix} \xrightarrow[-2r_1+r_3 \to r_3]{r_1+r_2 \to r_2} \begin{bmatrix} 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & -1 & -3 & -5 \\ 0 & 0 & 1 & 3 & 5 \end{bmatrix}$

$\xrightarrow[-r_2+r_4 \to r_4]{r_2+r_3 \to r_3} \begin{bmatrix} 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ eşelon ve satır indirgenmiş eşelon matristir.

Ör: Aşağıdaki matrislerin varsa terslerini bulunuz.

1) $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, 2) $B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 2 \end{bmatrix}$ 3) $C = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

Çöz:

1) $[A:I] = \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & -1/2 & 1/2 \end{array}\right]$

$\rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & -1/2 & 1/2 \end{array}\right]$. $A^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$.

2) $[B:I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & -2 & 2 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 1 & 1 \end{array}\right]$ soldaki taraftaki matriste sıfır satır olduğundan A nın tersi yoktur.

3) Varsa C^{-1}'i de siz bulunuz.

Ör: $A = \begin{bmatrix} -1 & 1 & 2 & -1 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & 4 & 5 \end{bmatrix}$ matrisi veriliyor. $R = E_k \cdots E_2 E_1 A$ o.ş elemanter matrisler E_{k-1}, E_2, E_1 ve satır indirgenmiş eşelon matris R yi bulunuz.

Çöz: $A \xrightarrow{-1r_1} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & 4 & 5 \end{bmatrix} = E_1 A$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-1r_1} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_1$

$E_1 A \xrightarrow{-1r_1 + r_2 \to r_2} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 0 & 3 & 4 \\ 1 & -1 & 4 & 5 \end{bmatrix} = E_2 E_1 A$

$I_3 \xrightarrow{-1r_1 + r_2 \to r_2} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_2$

$E_2 E_1 A \xrightarrow{-1r_1 + r_3 \to r_3} \begin{bmatrix} +1 & -1 & -2 & -1 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 7 & 6 \end{bmatrix} = E_3 E_2 E_1 A$

$I_3 \xrightarrow{-1r_1 + r_3 \to r_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = E_3$

$E_3 E_2 E_1 A \xrightarrow{\frac{1}{3} r_2} \begin{bmatrix} 1 & -1 & -2 & -1 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 7 & 6 \end{bmatrix} = E_4 E_3 E_2 E_1 A$

$I_3 \xrightarrow{\frac{1}{3} r_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_4$

$E_4 E_3 E_2 E_1 A \xrightarrow{2r_2 + r_1 \to r_1} \begin{bmatrix} 1 & -1 & 0 & 5/3 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 7 & 6 \end{bmatrix} = E_5 E_4 E_3 E_2 E_1 A$

$I_3 \xrightarrow{2r_2 + r_1 \to r_1} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_5$

$E_5 E_4 E_3 E_2 E_1 A \xrightarrow{-7r_2 + r_3 \to r_3} \begin{bmatrix} 1 & -1 & 0 & 5/3 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 0 & -10/3 \end{bmatrix} = E_6 \cdots E_2 E_1 A$

$I_3 \xrightarrow{-7r_2 + r_3 \to r_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -7 & 1 \end{bmatrix} = E_6$

$I_3 \xrightarrow{-3/10 r_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3/10 \end{bmatrix} = E_7$

$$I_3 \xrightarrow{-3/4 r_3 + r_2 \to r_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3/4 \\ 0 & 0 & 1 \end{bmatrix} = E_8$$

$$I_3 \xrightarrow{-3/5 r_3 + r_1 \to r_1} \begin{bmatrix} 1 & 0 & -3/5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_9$$

olup $R = E_9 E_8 E_7 \cdots E_2 E_1 A$ elde edilir. R matrisi ise

$$R = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ dir.}$$

Ör: $A = \begin{bmatrix} 2 & 3 & -1 & 0 & 6 \\ 1 & -3 & 1 & 1 & 8 \\ 0 & 2 & 0 & 0 & 1 \end{bmatrix}$ veriliyor. R eşelon, P elemanter matrislerin çarpımı olmak üzere $R = PA$ şeklinde yazınız.

Çöz: $[A : I] \to$ esi $\to [B : P]$ şeklinde matris bulunduğunda $B = PA$ şeklindedir.

$$[A : I] = \left[\begin{array}{ccccc|ccc} 2 & 3 & -1 & 0 & 6 & 1 & 0 & 0 \\ 1 & -3 & 1 & 1 & 8 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \text{ matrisine elemanter}$$

satır işlemleri uygulayarak

$$\left[\begin{array}{ccccc|ccc} 1 & 0 & 1 & 1 & 19/2 & 0 & 1 & 3/2 \\ 0 & 1 & 0 & 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 2/3 & 11/2 & -1/3 & 2/3 & -3/2 \end{array}\right] \text{ matrisine ulaşalım}$$

$$R = \begin{bmatrix} 1 & 0 & 1 & 1 & 19/2 \\ 0 & 1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 2/3 & 11/2 \end{bmatrix} \text{ matrisi eşelon formdadır.}$$

$$P = \begin{bmatrix} 0 & 1 & 3/2 \\ 0 & 0 & 1/2 \\ -1/3 & 2/3 & -3/2 \end{bmatrix} \text{ de istenen elemanter matrislerin çarpımı olan matristir.}$$

$R = P \cdot A$ olduğunu görünüz.

Ör: $A = \begin{bmatrix} 1 & -1 & x & 2 \\ 2 & -1 & 2x-1 & 5 \\ -1 & 1 & 1 & -3 \\ 1 & -1 & x & 4 \end{bmatrix}$ matrisi veriliyor. A nın tersinin olması için x ne olmalıdır?

Çöz: $A \longrightarrow \begin{bmatrix} 1 & -1 & x & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & x+1 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & x & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & x+1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\longrightarrow \begin{bmatrix} 1 & -1 & x & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & x+1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Matrisin tersinin olması için sıfır satır bulundurmaması gerekir. O halde $x+1 \neq 0$ olmalıdır. Yani $x \neq -1$ dir.

Soru: $x=0$ için A^{-1} i bulunuz.

Ör: $A = \begin{bmatrix} 1 & -2 & 0 & 5 \\ -1 & 2 & 1 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ ve $B = \begin{bmatrix} -1 & 2 & 0 & -5 \\ -1 & 2 & 2 & 1 \\ 1 & -2 & -1 & 2 \end{bmatrix}$

matrisleri $C = \begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ satır indirgenmiş C ye satır denktir gösteriniz.

Çöz: $A \rightarrow \begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = C$

ve $B \rightarrow \begin{bmatrix} 1 & -2 & 0 & 5 \\ -1 & 2 & 2 & 1 \\ 1 & -2 & -1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & -1 & -3 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = C$

A ve B her ikiside aynı satır indirgenmiş eşelon C matrisine satır denk matrislerdir.