FEN FAKÜLTESİ
MAT 122 MATEMATİK II-Uygulama

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Kaynak: Thomas Calculus

1. Aşağıdaki integralleri değişken değiştirme yöntemini kullanarak hesaplayınız.

* $\int (2x+2)\, e^{x^2+2x+3}\, dx$

$$x^2+2x+3 = u \Rightarrow (2x+2)\,dx = du$$

$$\int (2x+2)\, e^{x^2+2x+3}\, dx = \int e^u\, du = e^u + C$$

$$= e^{x^2+2x+3} + C$$

* $\int (2x+5)\,(x^2+5x)^7\, dx$

$$x^2+5x = u \Rightarrow (2x+5)\,dx = du$$

$$I = \int u^7\, du = \frac{u^8}{8} + C$$

$$= \frac{1}{8}(x^2+5x)^8 + C$$

* $\int \dfrac{x^3}{(1+x^4)^{1/3}}\, dx = ?$

$$u = 1 + x^4 \Rightarrow du = 4x^3\,dx$$

$$\int \frac{x^3}{(1+x^4)^{1/3}}\,dx = \int \frac{1}{4}\,\frac{du}{u^{1/3}} = \frac{1}{4}\int u^{-1/3}\,du$$

$$= \frac{1}{4}\,u^{2/3}\cdot\frac{3}{2} + C$$

$$= \frac{3}{8}\,(1+x^4)^{2/3} + C$$

$$* \quad \int \frac{\sin(\ln x)}{x}\,dx = ?$$

$$u = \ln x \quad , \quad du = \frac{1}{x}\,dx$$

$$I = \int \frac{\sin(\ln x)}{x}\,dx = \int \sin u\,du = -\cos u + C$$

$$= -\cos(\ln x) + C$$

$$* \int_0^9 \sqrt{4-\sqrt{x}}\, dx = ?$$

$$u = 4-\sqrt{x} \Rightarrow \sqrt{x} = 4-u$$

$$x = (4-u)^2 = u^2 - 8u + 16$$

$$dx = (2u-8)\, du$$

$$x=0 \Rightarrow u = 4$$

$$x = 9 \Rightarrow u = 1$$

$$\int_0^9 \sqrt{4-\sqrt{x}}\, dx = \int_4^1 \sqrt{u}\, (2u-8)\, du$$

$$= \int_4^1 u^{1/2} (2u-8)\, du = \int_4^1 (2u^{3/2} - 8u^{1/2})\, du$$

$$= 2 \cdot \frac{2}{5} u^{5/2} - 8 \cdot \frac{2}{3} u^{3/2} \Big|_4^1$$

$$= \frac{188}{15}$$

$$* \quad \int \frac{(3+\ln x)^2(2-\ln x)}{4x}\,dx = ?$$

$$u = 3+\ln x \Rightarrow du = \frac{1}{x}\,dx \quad , \quad \ln x = u-3$$

$$I = \frac{1}{4}\int (3+\ln x)^2 (2-\ln x)\frac{1}{x}\,dx$$

$$= \frac{1}{4}\int u^2\,(2-(u-3))\,du$$

$$= \frac{1}{4}\int u^2(5-u)\,du$$

$$= \frac{1}{4}\left(5.\frac{u^3}{3} - \frac{u^4}{4}\right) + C$$

$$= \frac{5}{12}(3+\ln x)^3 - \frac{1}{16}(3+\ln x)^4 + C$$

* $\int x\sqrt{4-x}\,dx$

$u = 4-x \Rightarrow du = (-1)\,dx \quad , \quad -du = dx \quad , \quad x = 4-u$

$$I = \int (4-u)\sqrt{u}\,(-1)\,du$$

$$= -\int (4-u)\sqrt{u}\,du$$

$$= -\int (4-u)\,u^{1/2}\,du = -\int \left(4u^{1/2} - u^{3/2}\right)du$$

$$= -\left(4 \cdot \frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right) + C$$

$$= -\left(\frac{8}{3}(4-x)^{3/2} - \frac{2}{5}(4-x)^{5/2}\right) + C$$

$$= \frac{2}{5}(4-x)^{5/2} - \frac{8}{3}(4-x)^{3/2} + C$$

* $\int \frac{3}{x\ln x}\,dx = ?$

$u = \ln x \Rightarrow du = \frac{1}{x}\,dx \quad , \quad I = 3\int \frac{1}{u}\,du$

$$= 3\ln|u| + C$$

$$= 3\ln|\ln x| + C$$

$$* \int_1^2 \left(\frac{2+\sqrt{x}}{x} \right)^{1/2} dx = ?$$

$$2+\sqrt{x} = u \Rightarrow \frac{dx}{2\sqrt{x}} = du \quad , \quad \frac{dx}{\sqrt{x}} = 2\,du$$

$$\int_1^2 \frac{(2+\sqrt{x})^{1/2}}{\sqrt{x}} dx = \int_{x=1}^{x=2} u^{1/2}\, 2\, du = \int_3^{2+\sqrt{2}} 2u^{1/2}\, du$$

$$= 2u^{3/2} \frac{2}{3} \Big|_3^{2+\sqrt{2}}$$

$$= \frac{4}{3} \left((2+\sqrt{2})^{3/2} - 2^{3/2} \right)$$

$$* \int e^{2x} \sqrt{e^x+2}\, dx = ?$$

$$e^x + 2 = u \Rightarrow e^x\, dx = du$$

$$I = \int (u-2)\sqrt{u}\, du = \int (u^{3/2} - 2u^{1/2})\, du$$

$$= \frac{2}{5} u^{5/2} - 2 \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{5} (e^x+2)^{5/2} - \frac{4}{3} (e^x+2)^{3/2} + C$$

2. Aşağıdaki türevleri hesaplayınız:

* $y = \int_0^x \sqrt{1+t^2}\, dt \Rightarrow \frac{dy}{dx} = \sqrt{1+x^2}$

* $y = \int_{\sqrt{x}}^0 \sin t^2 \, dt$

$\sqrt{x} = u, \quad y = \int_u^0 \sin t^2 \, dt = -\int_0^u \sin t^2 \, dt$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{du} = -\frac{d}{du} \int_0^u \sin t^2 \, dt = -\sin u^2$$

$$\frac{du}{dx} = \frac{1}{2} x^{-1/2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -\sin(\sqrt{x})^2 \cdot \frac{1}{2} x^{-1/2}$$

$$= -\frac{1}{2\sqrt{x}} \sin x$$

$$* \quad y = x \int_{2}^{x^2} \sin(t^3)\, dt$$

$$\frac{dy}{dx} = 1 \cdot \int_{2}^{x^2} \sin(t^3)\, dt + x \cdot \frac{d}{dx} \int_{2}^{x^2} \sin t^3\, dt$$

$$\frac{d}{dx} \int_{2}^{x^2} \sin t^3\, dt = ? \qquad x^2 = u$$

$$\frac{d}{dx} \left[\int_{2}^{u} \sin t^3\, dt \right] = \frac{d}{du} \int_{2}^{u} \sin t^3\, dt \cdot \frac{du}{dx}$$

$$= \sin u^3 \cdot 2x$$

$$= (\sin x^6)\, 2x$$

$$\Rightarrow \quad y' = 2x^2 \sin x^6 + \int_{2}^{x^2} \sin t^3\, dt$$

3. f, $[-a, a]$ simetrik aralığı üzerinde sürekli olsun;

a. Eğer f çift ise $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

b. Eğer tek ise $\int_{-a}^{a} f(x)\,dx = 0$

olduğunu gösteriniz.

Çözüm:

a. $$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= -\int_{0}^{-a} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= -\int_{0}^{a} f(-u)\,(-du) + \int_{0}^{a} f(x)\,dx$$

$u = -x \Rightarrow du = -dx$

$x = 0 \Rightarrow u = 0$

$x = -a \Rightarrow u = a$

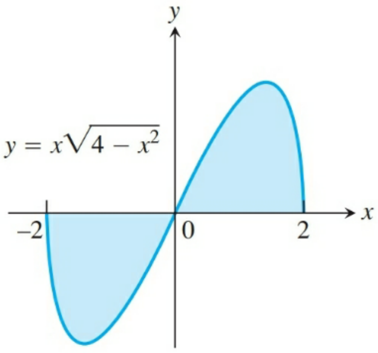
$$= \int_{0}^{a} f(u)\,du + \int_{0}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

$f(-u) = f(u)$

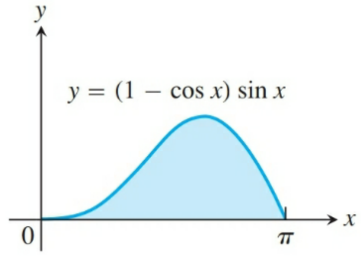
b. ÖDEV

4. Aşağıdaki taralı bölgelerin alanlarını bulunuz.

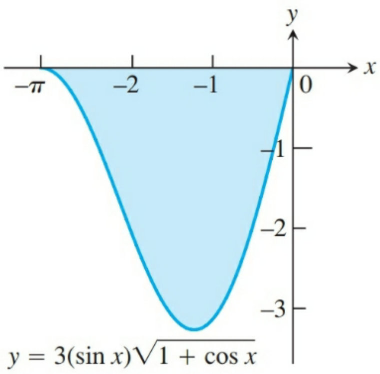
a.



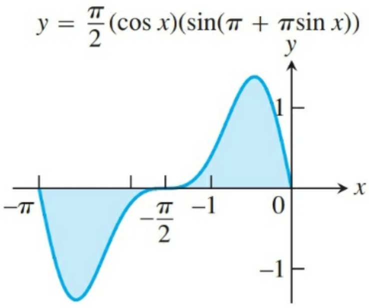
b.



c.



d.



Çözüm.

a. $$A = \left| \int_{-2}^{0} x\sqrt{4-x^2}\, dx \right| + \int_{0}^{2} x\sqrt{4-x^2}\, dx$$

$$= \underbrace{-\int_{-2}^{0} x\sqrt{4-x^2}\, dx}_{\substack{4-x^2=u \\ -2x\,dx=du}} + \underbrace{\int_{0}^{2} x\sqrt{4-x^2}\, dx}_{\substack{4-x^2=u \\ -2x\,dx=du}}$$

$$= \frac{1}{2}\int_{0}^{4} u^{1/2}\, du + \int_{0}^{4} \frac{1}{2} u^{1/2}\, du = \int_{0}^{4} u^{1/2}\, du = 16/3$$

b. $A = \int_0^{\pi} (1-\cos x)\sin x \, dx$

$1-\cos x = u \Rightarrow \sin x \, dx = du$

$x=0 \Rightarrow u=0$

$x=\pi \Rightarrow u=2$

$A = \int_0^2 u \, du = \frac{u^2}{2} \Big|_0^2 = 2$

c. $A = -\int_{-\pi}^{0} 3\sin x \sqrt{1+\cos x} \, dx$

$1+\cos x = u \Rightarrow -\sin x \, dx = du$

$x=0 \Rightarrow u=2$

$x=-\pi \Rightarrow u=0$

$A = -\int_0^2 3 \, u^{1/2} (-du) = 3\int_0^2 u^{1/2} du$

$= 2 \, u^{3/2} \Big|_0^2 = 2^{5/2}$

d. Simetriden dolayı

$$A = 2\int_{-\frac{\pi}{2}}^{0} \frac{\pi}{2}(\cos x)(\sin(\pi + \pi\sin x))\,dx$$

$$= 2\int_{0}^{\pi} \frac{\pi}{2}\sin u \;\frac{1}{\pi}\,du$$

$u = \pi + \pi\sin x$

$du = \pi\cos x\,dx \Rightarrow \frac{1}{\pi}du = \cos x\,dx$

$x = -\frac{\pi}{2} \Rightarrow u = 0$

$x = 0 \Rightarrow u = \pi$

4. f'nin grafiği ve x-ekseni arasında kalan bölgenin alanını bulunuz.

a. $f(x) = x^2 - 4x + 3$; $0 \leq x \leq 3$

b. $f(x) = 1 - \dfrac{x^2}{4}$; $-2 \leq x \leq 3$

c. $f(x) = 5 - 5x^{2/3}$; $-1 \leq x \leq 8$ (ÖDEV)

d. $f(x) = 1 - \sqrt{x}$; $0 \leq x \leq 4$ (ÖDEV)

Çözüm.

a. $f(x) = x^2 - 4x + 3 \Rightarrow (x-3)(x-1) = 0$, $x = 3$, $x = 1$

$f(x) = (x-3)(x-1)$

$0 \leq x \leq 1 \Rightarrow f(x) \geq 0$

$1 \leq x \leq 3 \Rightarrow f(x) \leq 0$

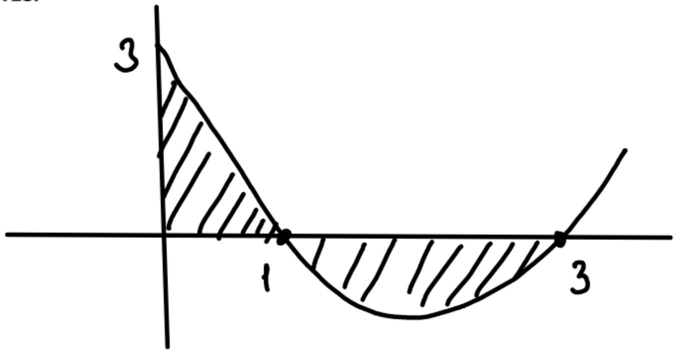
$$\text{Alan} = \int_0^1 f(x)\,dx - \int_1^3 f(x)\,dx$$

$$= \int_0^1 (x^2 - 4x + 3)\,dx - \int_1^3 (x^2 - 4x + 3)\,dx$$

$$= \left(\frac{x^3}{3} - 2x^2 + 3x\right)\Big|_0^1 - \left(\frac{x^3}{3} - 2x^2 + 3x\right)\Big|_1^3$$

$$= \left(\frac{1}{3} - 2 + 3\right) - \left[(9 - 18 + 9) - \left(\frac{1}{3} - 2 + 3\right)\right]$$

$$= \frac{8}{3}$$



$f(x) = x^2 - 4x + 3$

b. $f(x) = 1 - \frac{x^2}{4}$; $f(x) = 0 \Rightarrow 4 - x^2 = 0 \Rightarrow x = \mp 2$

$$f(x) = \frac{4 - x^2}{4} = \frac{1}{4}(2 - x)(2 + x)$$

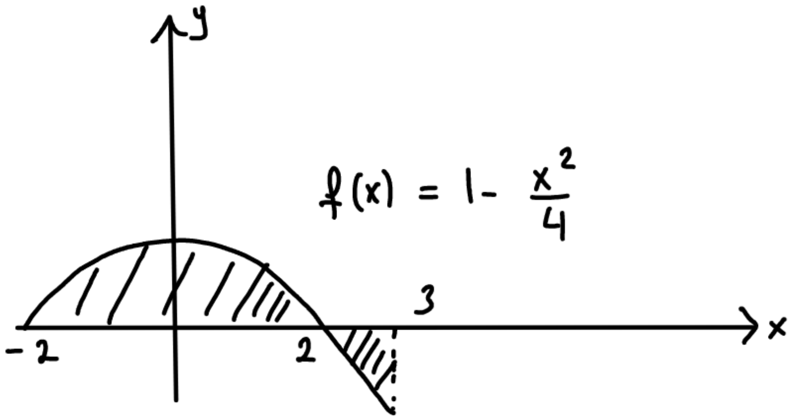
$-2 \leq x \leq 2 \Rightarrow f(x) \geq 0$

$2 \leq x \leq 3 \Rightarrow f(x) \leq 0$

$$\text{Alan} = \int_{-2}^{2} \left(1 - \frac{x^2}{4}\right) dx - \int_{2}^{3} \left(1 - \frac{x^2}{4}\right) dx$$

$$= \left(x - \frac{x^3}{12}\right)\Big|_{2}^{-2} - \left(x - \frac{x^3}{12}\right)\Big|_{2}^{3}$$

$$= \frac{13}{4}$$



c. $A = 62$

d. $A = 2$

– ÖDEV –

1. a. $\displaystyle\int_0^3 \sqrt{y+1}\, dy$ b. $\displaystyle\int_{-1}^0 \sqrt{y+1}\, dy$

2. a. $\displaystyle\int_0^1 r\sqrt{1-r^2}\, dr$ b. $\displaystyle\int_{-1}^1 r\sqrt{1-r^2}\, dr$

3. a. $\displaystyle\int_0^{\pi/4} \tan x \sec^2 x\, dx$ b. $\displaystyle\int_{-\pi/4}^0 \tan x \sec^2 x\, dx$

4. a. $\displaystyle\int_0^{\pi} 3\cos^2 x \sin x\, dx$ b. $\displaystyle\int_{2\pi}^{3\pi} 3\cos^2 x \sin x\, dx$

5. a. $\displaystyle\int_0^1 t^3(1+t^4)^3\, dt$ b. $\displaystyle\int_{-1}^1 t^3(1+t^4)^3\, dt$

6. a. $\displaystyle\int_0^{\sqrt{7}} t(t^2+1)^{1/3}\, dt$ b. $\displaystyle\int_{-\sqrt{7}}^0 t(t^2+1)^{1/3}\, dt$

7. a. $\displaystyle\int_{-1}^1 \frac{5r}{(4+r^2)^2}\, dr$ b. $\displaystyle\int_0^1 \frac{5r}{(4+r^2)^2}\, dr$

8. a. $\displaystyle\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\, dv$ b. $\displaystyle\int_1^4 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\, dv$

9. a. $\displaystyle\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx$ b. $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx$

10. a. $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\, dx$ b. $\displaystyle\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}}\, dx$

11. a. $\displaystyle\int_0^{\pi/6} (1-\cos 3t)\sin 3t\, dt$ b. $\displaystyle\int_{\pi/6}^{\pi/3} (1-\cos 3t)\sin 3t\, dt$

12. a. $\displaystyle\int_{-\pi/2}^0 \left(2+\tan\frac{t}{2}\right)\sec^2\frac{t}{2}\, dt$ b. $\displaystyle\int_{-\pi/2}^{\pi/2} \left(2+\tan\frac{t}{2}\right)\sec^2\frac{t}{2}\, dt$

13. a. $\displaystyle\int_0^{2\pi} \frac{\cos z}{\sqrt{4+3\sin z}}\, dz$ b. $\displaystyle\int_{-\pi}^{\pi} \frac{\cos z}{\sqrt{4+3\sin z}}\, dz$

14. a. $\displaystyle\int_{-\pi/2}^0 \frac{\sin w}{(3+2\cos w)^2}\, dw$ b. $\displaystyle\int_0^{\pi/2} \frac{\sin w}{(3+2\cos w)^2}\, dw$

15. $\displaystyle\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\, dt$

16. $\displaystyle\int_1^4 \frac{dy}{2\sqrt{y}\,(1+\sqrt{y})^2}$

17. $\displaystyle\int_0^{\pi/6} \cos^{-3} 2\theta \sin 2\theta\, d\theta$

18. $\displaystyle\int_{\pi}^{3\pi/2} \cot^5\left(\frac{\theta}{6}\right)\sec^2\left(\frac{\theta}{6}\right) d\theta$

19. $\displaystyle\int_0^{\pi} 5(5-4\cos t)^{1/4}\sin t\, dt$

20. $\displaystyle\int_0^{\pi/4} (1-\sin 2t)^{3/2}\cos 2t\, dt$

21. $\displaystyle\int_0^1 (4y-y^2+4y^3+1)^{-2/3}(12y^2-2y+4)\, dy$

22. $\displaystyle\int_0^1 (y^3+6y^2-12y+9)^{-1/2}(y^2+4y-4)\, dy$