1-) $\int \sqrt{\underbrace{1+\sin^2(x-1)}} \cdot \underbrace{\sin(x-1) \cdot \cos(x-1)\, dx} = ?$

$u = 1+\sin^2(x-1) \Rightarrow du = 2 \cdot \sin(x-1) \cdot \cos(x-1) \cdot dx$

$\Rightarrow \; = \int \sqrt{u} \cdot \frac{du}{2}$ $\quad \left(\sqrt{u} = u^{1/2}\right)$

$= \dfrac{u^{3/2}}{3/2 \cdot 2} = \dfrac{u^{3/2}}{3} = \dfrac{\sqrt{\left(1+\sin^2(x-1)\right)^3}}{3} + C.$

2-) $\int \dfrac{\sin\sqrt{\theta}}{\underbrace{\sqrt{\theta \cdot \cos^3\sqrt{\theta}}}}\, d\theta = ?$

$= \int \dfrac{\sin\sqrt{\theta}}{\sqrt{\theta} \cdot \underbrace{\sqrt{\cos^3\sqrt{\theta}}}} \cdot d\theta$

$\begin{cases} \cos\sqrt{\theta} = u \quad \text{olsun.} \\ \Rightarrow -\sin\sqrt{\theta} \cdot \dfrac{1}{2\sqrt{\theta}} \cdot d\theta = du \end{cases}$

$\Rightarrow \dfrac{\sin\sqrt{\theta}}{\sqrt{\theta}} \cdot d\theta = -2\,du$

$= \int \dfrac{-2\,du}{\sqrt{u^3}}$ $\quad \left(\sqrt{u^3} = u^{-3/2}\right)$

$= \dfrac{-2u^{-1/2}}{-1/2} = 4 \cdot u^{-1/2} = \dfrac{4}{\sqrt{u}} = \dfrac{4}{\sqrt{\cos\sqrt{\theta}}} + C.$

3-) $\int_0^1 \dfrac{10\sqrt{u}}{\underbrace{(1+u^{3/2})^2}}\, du = ?$

$1+u^{3/2} = p \quad$ olsun.

$\Rightarrow \dfrac{3}{2} \cdot u^{1/2}\, du = dp$

$\Rightarrow \sqrt{u} \cdot du = \dfrac{2}{3}\, dp$

$$\Rightarrow \int_{p(0)=1}^{p(1)=2} \frac{10}{p^2} \cdot \frac{2}{3}\, dp = \frac{20}{3} \int_1^2 p^{-2}\, dp = \frac{20}{3} \frac{p^{-1}}{-1} \Big|_1^2$$

$$= \frac{20}{3} \left\{ -\frac{1}{2} - (-1) \right\}$$

$$= \frac{10}{3}$$

4-) $\displaystyle\int_{\pi}^{3\pi/2} \cot^5\left(\frac{\theta}{6}\right) \cdot \sec^2\left(\frac{\theta}{6}\right) d\theta = ?$ $\qquad \cot\left(\frac{\theta}{6}\right) = \dfrac{1}{\tan\frac{\theta}{6}}$

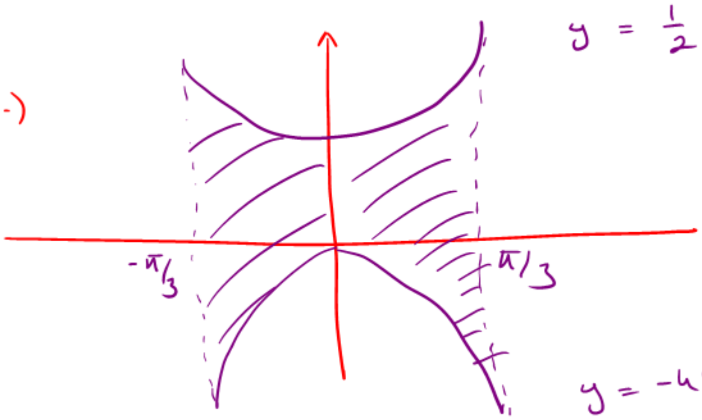
$\tan\left(\frac{\theta}{6}\right) = u \quad \Rightarrow \quad \sec^2\left(\frac{\theta}{6}\right) \cdot \frac{1}{6} \cdot d\theta = du$

$$\int_{u(\pi)=\frac{\sqrt{3}}{3}}^{u\left(\frac{3\pi}{2}\right)=1} \frac{1}{u^5} \cdot 6\, du = \frac{6 \cdot u^{-4}}{-4} \Big|_{\sqrt{3}/3}^{1} = \text{ödev.}$$

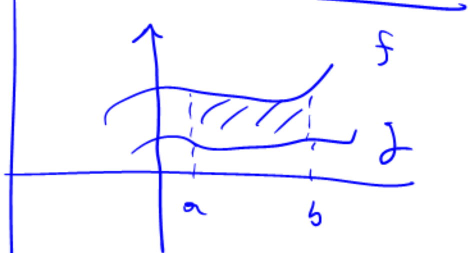
5-) Aşağıdaki bölgelerin alanlarını hesaplayınız.

a-)

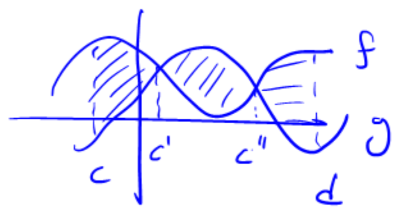
$y = \frac{1}{2}\sec^2 t$



$y = -4\sin^2 t$



$\displaystyle\int_a^b f - g$

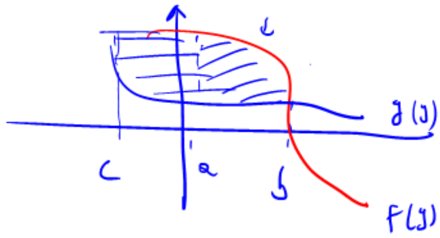


$$\text{Alan} = \int_{-\pi/3}^{\pi/3} \left(\frac{1}{2}\sec^2 t - \left(-4\sin^2 t\right) \right) dt$$

$$= \int_{-\pi/3}^{\pi/3} \frac{1}{2}\sec^2 t + \int_{-\pi/3}^{\pi/3} 4 \sin^2 t \, dt$$

$$\cos 2t = 1 - 2\sin^2 t$$

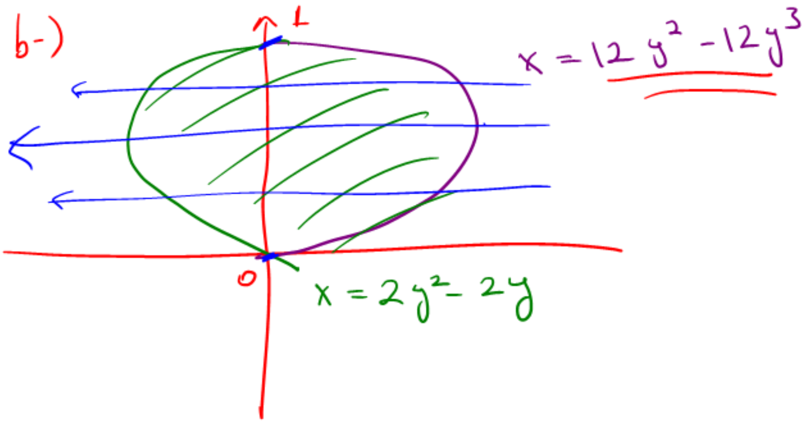
$$\sin^2 t = \frac{1}{2} - \frac{\cos 2t}{2}$$



$$\int_c^b f(y) - g(y)$$

$$= \frac{1}{2}\tan \Big|_{-\pi/3}^{\pi/3} + 4 \cdot \left\{ \frac{t}{2} - \frac{\sin 2t}{4} \Big|_{-\pi/3}^{\pi/3} \right\} = \frac{4\pi}{3}$$

b-)

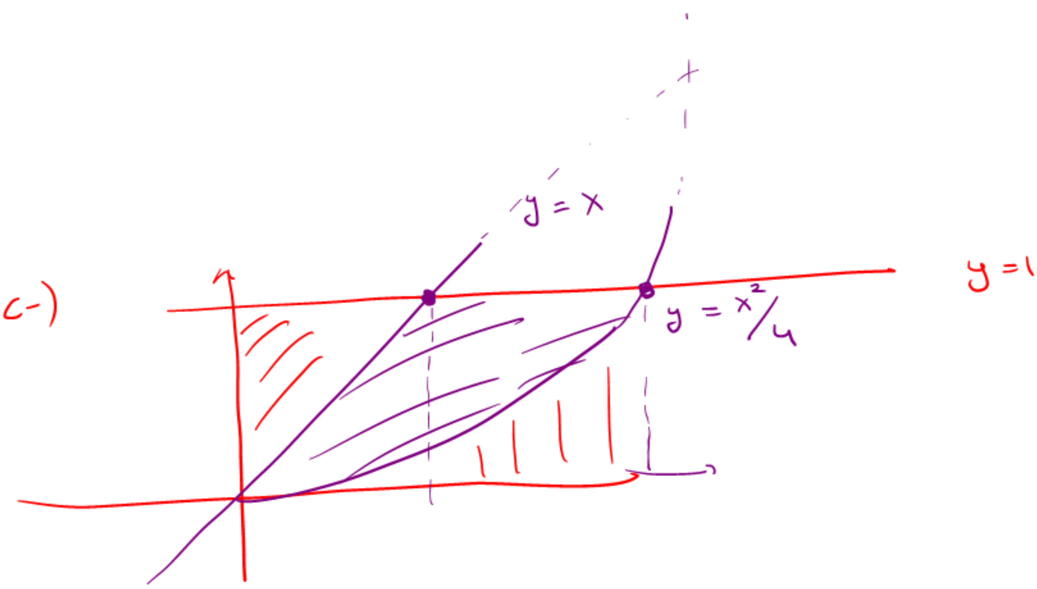


$$12y^2 - 12y^3 = 2y^2 - 2y$$

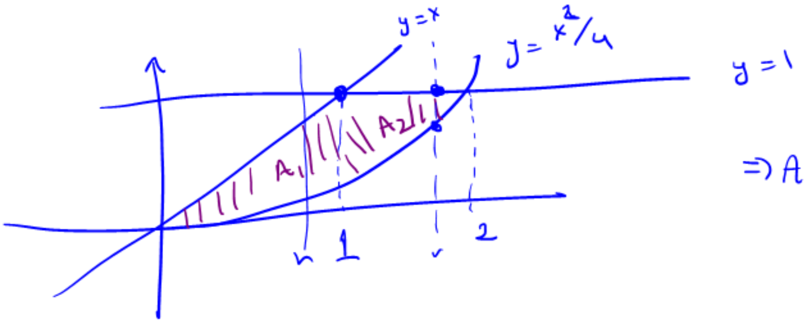
$$\text{Alan} = \int_0^1 \left(12y^2 - 12y^3 - \left(2y^2 - 2y\right) \right) dy = 4/3$$

çözüm ödev.

c-)



Çözüm: 1. yol



$A = A_1 + A_2$

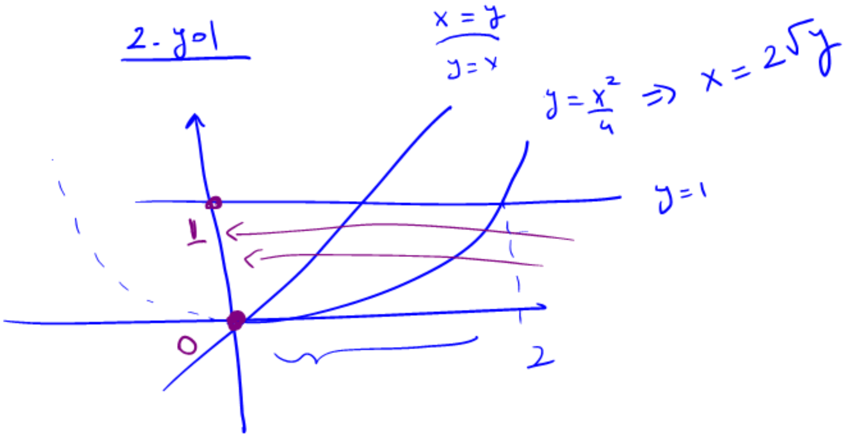
$$\Rightarrow A = \int_0^1 \left(x - \frac{x^2}{4}\right) dx + \int_1^2 \left[1 - \frac{x^2}{4}\right] dx$$

$$= \frac{x^2}{2} - \frac{x^3}{12} \Big|_0^1 + x - \frac{x^3}{12} \Big|_1^2$$

$$= \frac{1}{2} - \frac{1}{12} - 0 + \left(2 - \frac{8}{12} - \left[1 - \frac{1}{12}\right]\right)$$

$$= 5/6$$

2. yol



$y = x^2/4$

$\Rightarrow 4y = x^2$

$\Rightarrow \mp 2\sqrt{y} = x$

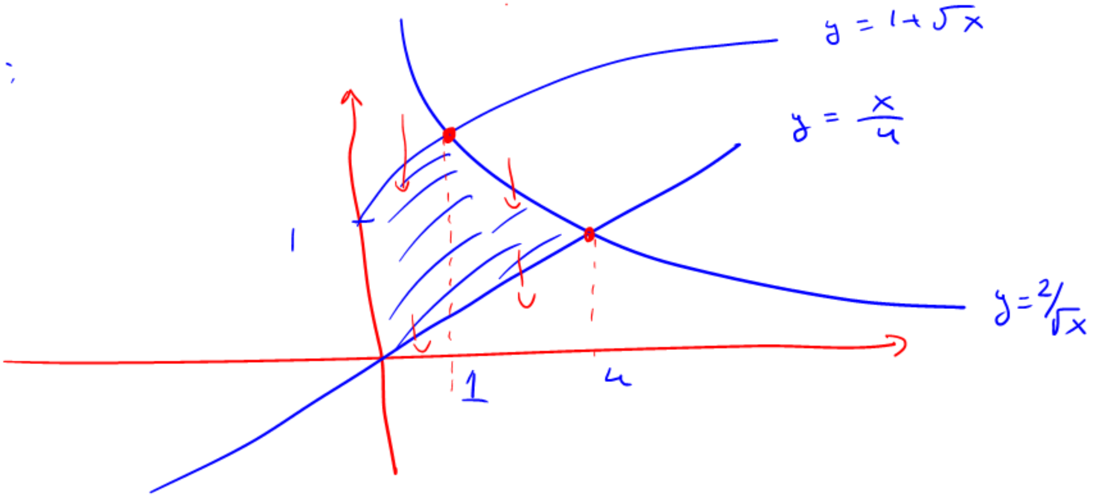
$x > 0 \quad \Rightarrow \quad x = 2\sqrt{y}$

$$A = \int_0^1 (2\sqrt{y} - y)\, dy = 2 \cdot \frac{y^{3/2}}{3/2} - \frac{y^2}{2} \Big|_0^1 = 2 \cdot \frac{1}{3/2} - \frac{1}{2} - [0]$$

$$= \frac{4}{3} - \frac{1}{2} = \frac{8-3}{6} = 5/6.$$

6-) Birinci bölgede soldan y-ekseni, alttan $y = \frac{x}{4}$ doğrusu, sol üstten $y = 1 + \sqrt{x}$ eğrisi ve sağ üstten $y = \frac{2}{\sqrt{x}}$ ile sınırlı bölgenin alanını bulun.

Çözüm:



$$1+\sqrt{x} = \frac{2}{\sqrt{x}} \Rightarrow \sqrt{x} + x = 2$$
$$\Rightarrow \sqrt{x} = 2 - x$$
$$\Rightarrow x = 4 + x^2 - 4x$$
$$\Rightarrow 0 = x^2 - 5x + 4$$
$$\Rightarrow \boxed{x=1}, \ \boxed{\cancel{x=4}}$$

Denklemi sağlamaz.

$$\frac{2}{\sqrt{x}} = \frac{x}{4} \Leftrightarrow \frac{4}{x} = \frac{x^2}{16}$$
$$\Rightarrow x^3 = 64 \Rightarrow \boxed{x = 4}$$

$$A = \int_0^1 \left(1+\sqrt{x} - \frac{x}{4}\right) + \int_1^4 \left(\frac{2}{\sqrt{x}} - \frac{x}{4}\right) = \frac{11}{3} \quad \#$$

7-) $f(x) = \frac{\sin x}{x}$, $x > 0$ fonksiyonunun bir ters türevi olduğunu varsayalım. Bu ters türev $F(x)$ olsun.

Buna göre $\int_1^3 \frac{\sin 2x}{x} dx$ 'in F cinsinden değeri nedir?

$$\int \frac{\sin x}{x} = F(x)$$

Çözüm:

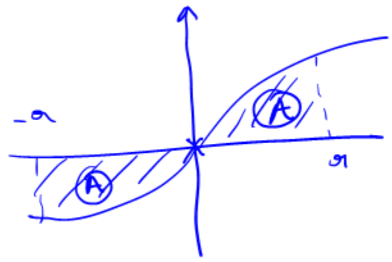
$$I = \int_1^3 \frac{\sin 2x}{x} dx, \quad 2x = u$$
$$\Rightarrow 2dx = du$$
$$\Rightarrow dx = \frac{du}{2}$$

$$I = \frac{1}{2}\int_2^6 \frac{\sin u \cdot du}{u/2} = \int_2^6 \frac{\sin u}{u}\, du = \int_2^6 \frac{\sin x}{x}\, dx = F(x)\Big|_2^6 = F(6) - F(2)$$

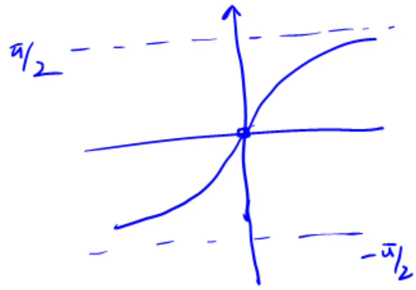
$\boxed{f(-x) = -f(x)}$

8-) f fonk. $[-\pi/8, \pi/8]$ aralığında tek ve integrallenebilir bir fonk olsun. Buna göre

$$\int_{-\pi/8}^{\pi/8} \arctan\left(\frac{x^2}{f(x)}\right) = 0$$ old. ispatlayın.



Çözüm: Eğer $g(x) = \arctan\left(\frac{x^2}{f(x)}\right)$ tek ise ispat biter.



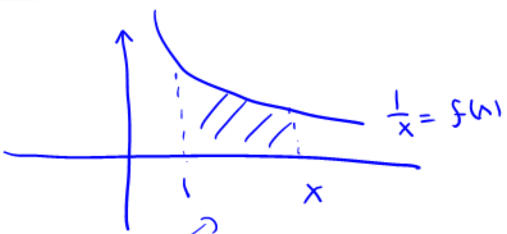
$$g(-x) = \arctan\left(\frac{(-x)^2}{f(-x)}\right) = \arctan\left(\frac{x^2}{-f(x)}\right)$$

$$= -\arctan\left(\frac{x^2}{f(x)}\right)$$

$$= -g(x)$$

$\Rightarrow g(-x) = -g(x) \Rightarrow$ g tektir ve dolayısıyla ispat biter.

$\int \frac{1}{x} = \ln|x| \rightarrow$



$\int_1^x \frac{1}{t}\, dt = \ln x$ (Tanım).

$f(x) = e^x \Rightarrow f^{-1}(x) = \ln x$

$\Rightarrow (\ln x)' = \frac{1}{x} \Rightarrow \int \frac{1}{x} = \ln x$