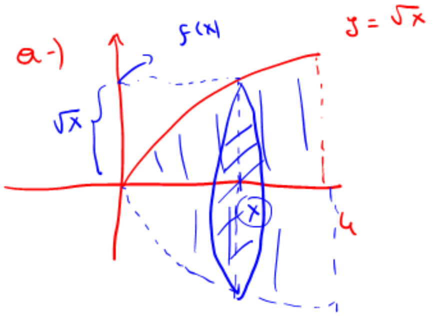
1-) Verilen bölgelerin x ve y ekseni etrafında döndürülmeleri sonucu elde edilen hacmi hem disk hem de kabuk metodu ile hesaplayınız.

a-)



Çözüm: Disk metodu:

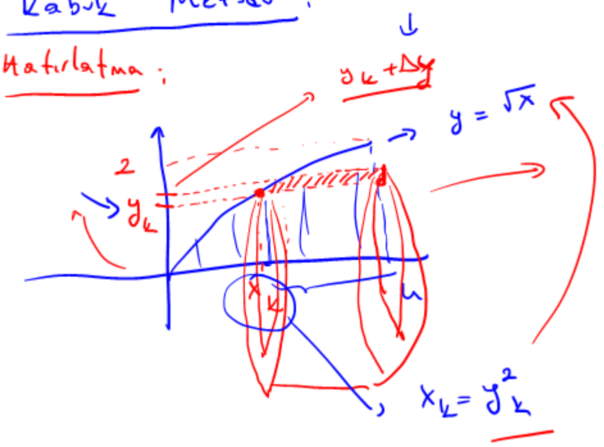
$$D_x = \pi (\sqrt{x})^2 = \pi . x$$

$$\int_0^4 \pi . x = \pi . \frac{x^2}{2} \Big|_0^4 = 8\pi$$

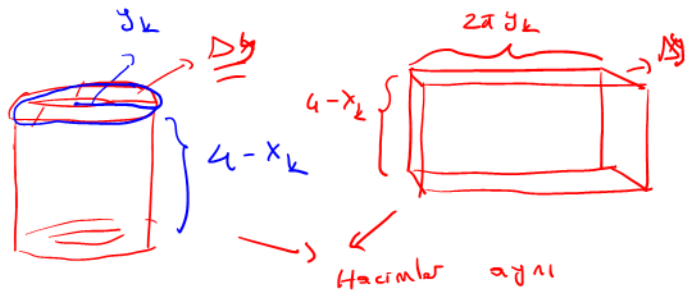
Formül: $V = \int_a^b \pi . [f(x)]^2 . dx$

Kabuk Metodu:

Hatırlatma:



$$\sqrt{y_k^2} = y_k$$



Hacimler aynı

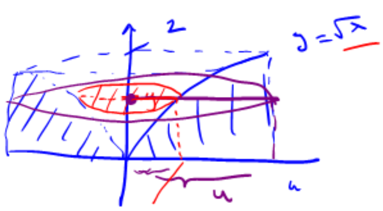
$$V_k = (4 - x_k) . 2\pi y_k . \Delta y$$

$$= (4 - y_k^2) . 2\pi y_k . \Delta y$$

$$\Rightarrow V = \int_0^2 2\pi . y . (4 - y^2) . dy$$

$$= 8\pi$$

y eksenine göre

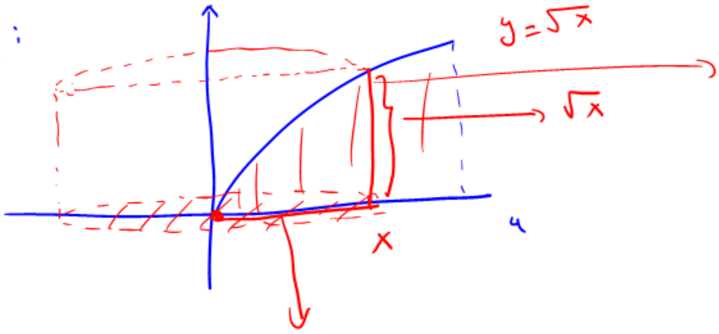


Disk Metodu:

$$V = \int_0^2 \pi . [4^2 - (y^2)^2] . dy = \frac{128\pi}{5}$$

$x = y^2$

Kabuk Metodu:



Kabuk yüksekliği

Kabuk yarıçapı = x

Kabuk metodu

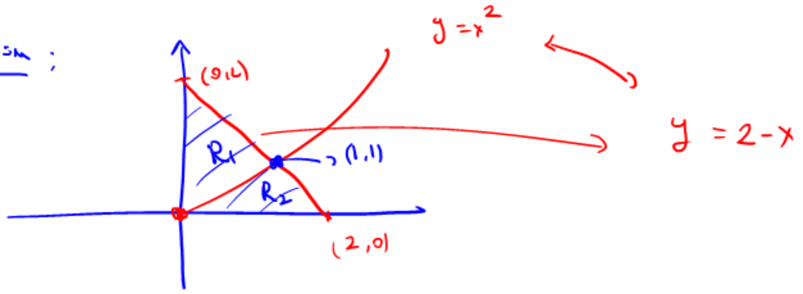
$$V = (2\pi . x).\sqrt{x}$$

$$V = \int_0^4 2\pi . x . \sqrt{x} \, dx = 8\pi$$

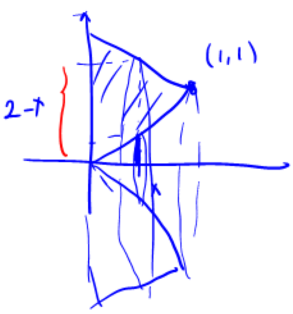
Formül: $V = 2\pi \int_a^b$ (kabuk yarıçapı).(kabuk yüksekliği)

2-) $y=x^2$ parabolü, köşeleri $(0,0)$, $(2,0)$ ve $(0,2)$ olan üçgeni iki bölgeye ayırmaktadır. Oluşan her bir bölgenin x-ekseni etrafında döndürülmesiyle elde edilen cisimlerin hacmi nedir?

Çözüm:

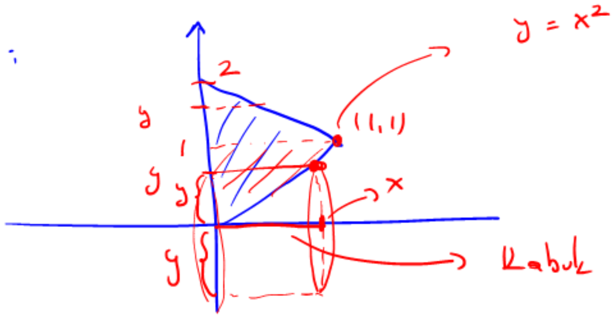


R_1 için:



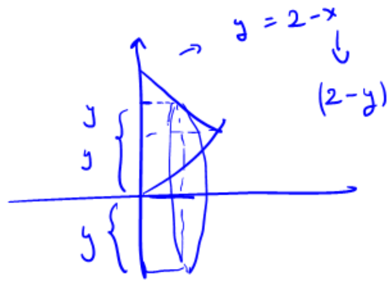
$$V = \pi \int_0^1 \left[(2-x)^2 - (x^2)^2\right] dx = \frac{32\pi}{15} \quad \text{II}$$

Kabuk Metodu :



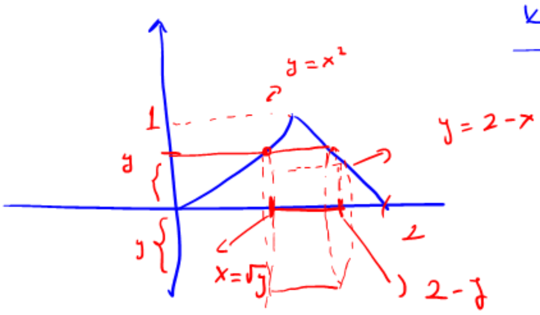
Kabuk yüksekliği $= x = \sqrt{y}$

$$V = 2\pi\left[\int_0^1 y\cdot\sqrt{y}\,dy + \int_1^2 y\cdot(2-y)\cdot dy\right] = \frac{32\pi}{15}$$



R₂ bölgesi

Kabuk Metodu.



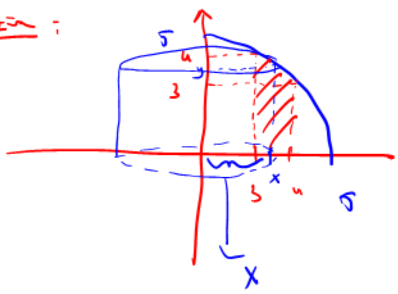
Kabuk yüksekliği $(2-y) - \sqrt{y}$

Kabuk yarıçapı $= y$

$$V = 2\pi\int_0^1 y\left(2-y-\sqrt{y}\right)dy = \frac{8\pi}{15}$$

3-) $y = \sqrt{25-x^2}$, $3 \leq x \leq 4$, y-ekseni etrafında döndür. Hacim ?

Çizim :



$y^2 = 25 - x^2$

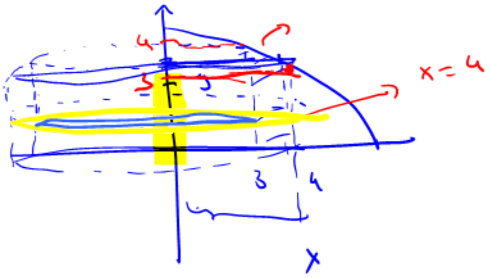
Kabuk yöntemi

Kabuk yüksekliği : $y = \sqrt{25-x^2}$

Kabuk yarıçapı : x

$$V = 2\pi\int_3^4 x\cdot\sqrt{25-x^2}\cdot dx = \frac{74\pi}{3}$$

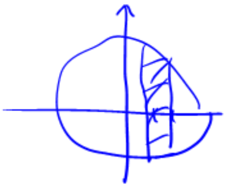
Disk yöntemi :



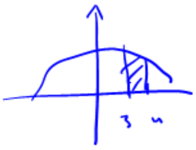
$$V = \pi\left[\int_3^4 \left[\left(\sqrt{25-y^2}\right)^2 - 3^2\right]\cdot dy + \int_0^3 \left(4^2 - 3^2\right)\cdot dy\right]$$

$$= \frac{74\pi}{3}$$

$y^2 = 25 - x^2 \rightarrow$ Tüm çember



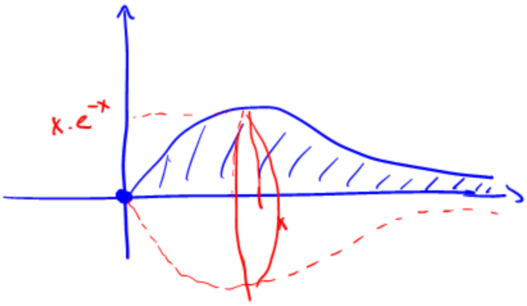
$y = \sqrt{25 - x^2}$

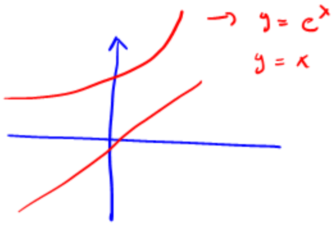


4-) $y = x.e^{-x}$, $0 \leq x < \infty$, x -ekseni etrafında döndürelim.

$$\lim_{x\to\infty} x.e^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = 0 \quad \rightarrow \text{l'hopital.}$$

$x = 0 \Rightarrow y = 0$



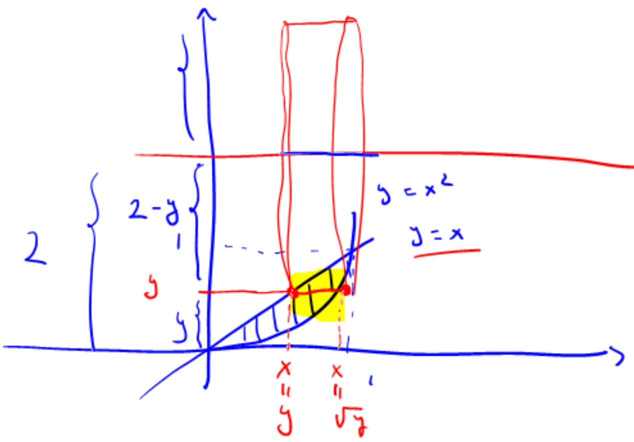


$$V = \pi \int_0^\infty (x.e^{-x})^2.dx = \pi \lim_{\alpha\to\infty} \int_0^\alpha x^2.e^{-2x}.dx = \pi/4$$

iki defa kısmi int. yapılmalı.

5-) $y = x$ ve $y = x^2$ eğrileri tarafından sınırlandırılan R bölgesi $y = 2$ doğrusu etrafında döndürülüyor. Oluşan şekil hacmi?

Çözüm: Kabuk M.:



K. yüksekliği: $\sqrt{y} - y$

K. yarıçapı: $2 - y$

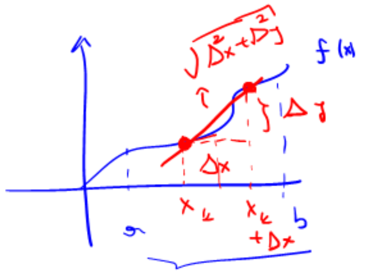
$$V = 2\pi \int_0^1 (2-y)\cdot(\sqrt{y} - y)\, dy$$

$$= \frac{8\pi}{15}$$

Disk Metodu: Ödev.

6-) $y = \ln(\sec x)$ eğrisinin $0 \le x \le \frac{\pi}{4}$ aralığındaki uzunluğunu bulun.

Hatırlatma:



$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{\Delta x^2 \left[1 + \frac{\Delta y^2}{\Delta x^2}\right]}$$

$$= \Delta x \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}$$

Çözüm

$$\frac{dy}{dx} = \frac{\cancel{\sec x}\cdot \tan x}{\cancel{\sec x}}$$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/4} \sec x\, dx$$

ödev

$$= \ln(\sqrt{2} + 1)$$