MAT 122 ARASINAV ÇÖZÜMLERİ

1) → (a) $\lim\limits_{x\to 2}\dfrac{\int_4^{x^2}\sin^2(t-4)\,dt}{(x^2-4)^3} = [0/0] \overset{\text{L'Hospital}}{=} \lim\limits_{x\to 2}\dfrac{\frac{d}{dx}\left(\int_4^{x^2}\sin^2(t-4)\,dt\right)}{\frac{d}{dx}\left((x^2-4)^3\right)}$

12

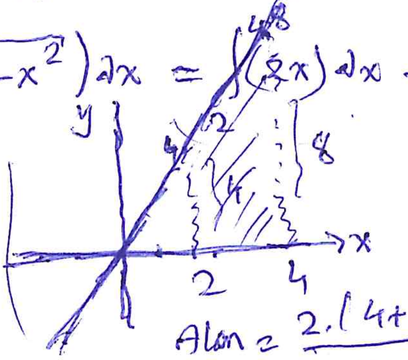
$y = x^2-4$ ve $x\to 2 \Leftrightarrow y\to 0$

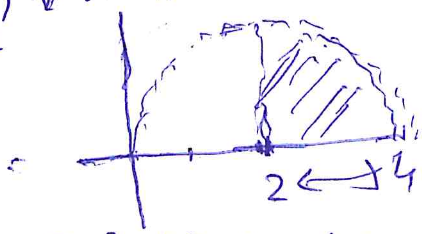
$= \lim\limits_{x\to 2}\dfrac{\sin^2(x^2-4)\cdot \cancel{2x}}{3\cdot(x^2-4)^2\,\cancel{2x}} = \dfrac{1}{3}\lim\limits_{y\to 0}\dfrac{\sin^2 y}{y^2} = \dfrac{1}{3}\left(\lim\limits_{y\to 0}\left(\dfrac{\sin y}{y}\right)\right)^2 = \dfrac{1}{3}\cdot 1^2 = 1/3$

b) → $\int_2^4 \left(2x+\sqrt{4x-x^2}\right)dx = \int_2^4 (2x)\,dx + \int_2^4 \sqrt{4x-x^2}\,dx$

13

$4x-x^2 = -(x^2-4x)$
$= -(x-2)^2+4$
$= 4-(x-2)^2$





Alan $= \dfrac{2.(4+8)}{2} = 12\ br^2$. $\dfrac{\text{Dairenin alanı}}{4} = \dfrac{\pi\cdot 2^2}{4} = \pi$

$= 12+\pi\ br^2$ olarak elde edilir.

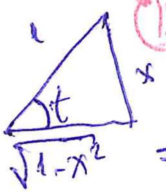
2) → (a) 13

$\int x\cdot \arcsin x\,dx = \dfrac{x^2}{2}\arcsin x - \dfrac{1}{2}\int x^2\dfrac{1}{\sqrt{1-x^2}}\,dx$

$u = \arcsin x \Rightarrow du = \dfrac{dx}{\sqrt{1-x^2}}$

$dv = x\,dx \Rightarrow v = \dfrac{x^2}{2}$

$x = \sin t \Rightarrow dx = \cos t\,dt$



$= \dfrac{x^2}{2}\text{Arcsin}x - \dfrac{1}{2}\int\dfrac{\sin^2 t\cdot \cancel{\cos t}\,dt}{\cancel{\cos t}} = \dfrac{x^2}{2}\text{Arcsin}x - \dfrac{1}{2}\int\dfrac{1-\cos 2t}{2}\,dt$

$\sin^2 t = \dfrac{1-\cos 2t}{2}$; $2\sin t\cos t$

$= x^2\text{Arcsin}x - \dfrac{1}{2}\left(\dfrac{t}{2}-\dfrac{\sin 2t}{4}\right)+c = \dfrac{x^2}{2}\text{Arcsin}x - \dfrac{\text{Arcsin}x}{4} + \dfrac{x\cdot\sqrt{1-x^2}}{2}+C$

b) → $\int\dfrac{x^4+1}{x^3-1}\,dx = \int\left(x+\dfrac{x+1}{x^3-1}\right)dx = \dfrac{x^2}{2}+\int\dfrac{x+1}{(x-1)(x^2+x+1)}\,dx = \dfrac{x^2}{2}+\int\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+x+1}$

$\begin{array}{r|l} x^4+1 & x^3-1 \\ \hline -x^4+x & x \\ \hline x+1 & \end{array}$

12

$x+1 = A(x^2+x+1)+(x-1)(Bx+C)$
$Bx^2+Cx-Bx-C$

$\Leftrightarrow A+B=0$
$A+C-B=1$
$A-C=1$

$2A+C=1$
$+A-C=1$
$3A=2 \Rightarrow A=2/3$

$B=-2/3$

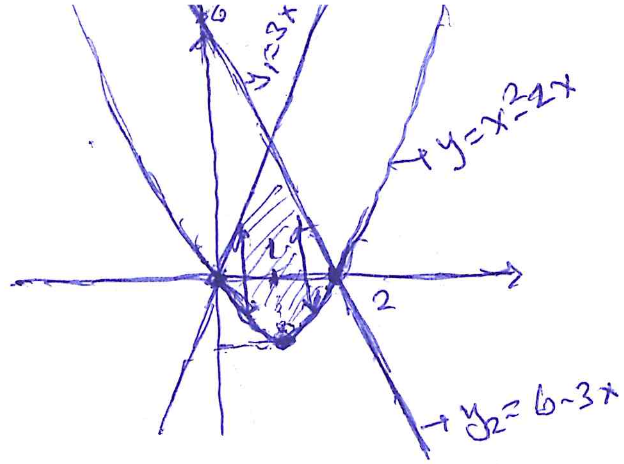
$C=\dfrac{2}{3}-1=-1/3$

$= \dfrac{x^2}{2}+\int\dfrac{2/3}{x-1}\,dx-\dfrac{1}{3}\int\dfrac{2x+1}{x^2+x+1}\,dx$

$= \dfrac{x^2}{2}+\dfrac{2}{3}\ln|x-1|-\dfrac{1}{3}\ln|x^2+x+1|+C = \dfrac{x^2}{2}+\dfrac{1}{3}\ln\left(\dfrac{(x-1)^2}{x^2+x+1}\right)+C$

3) → (a)

13



Taralı Alan $= \int_0^1 (y_1-y)\,dx + \int_1^2 (y_2-y)$

$= \int_0^1 (3x-x^2+2x)\,dx + \int_1^2 (6-3x-x^2+2x)$

$= \int_0^1 (5x-x^2)\,dx + \int_1^2 (6-x-x^2)\,dx$

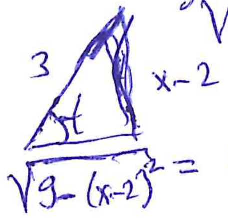
$= \left(\frac{5}{2}x^2-\frac{x^3}{3}\right)\Big|_0^1 + \left(6x-\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_1^2$

12

$= \left(\frac{5}{2}-\frac{1}{3}\right) + \left[\overbrace{12-2}^{10}-\frac{8}{3}-\overbrace{\left(6-\frac{1}{2}-\frac{1}{3}\right)}^{-6+1/2+1/3}\right] = \frac{13}{6}+\frac{4}{1}-\frac{8}{3}+\frac{1}{2} = \frac{13+\overbrace{24-16}^{10}+3}{6} = \frac{13}{3}$

b) → $\int \frac{(x-2)^2}{\sqrt{5+4x-x^2}}\,dx = \int \frac{(x-2)^2\,dx}{\sqrt{9-(x-2)^2}} = \int \frac{(3\sin t)^2\cdot \cancel{3\cos t}\,dt}{\cancel{3\cos t}} = \int 9\sin^2 t\,dt$

$\cos^2$



$x-2 = 3\sin t \Rightarrow dx = 3\cos t\,dt$

$= 9\int \frac{1-\cos 2t}{2}\,dt = \frac{9}{2}\left(t-\frac{\sin 2t}{2}\right)+C = \frac{9}{2}\left(\operatorname{Arcsin}\left(\frac{x-2}{3}\right)-\frac{9}{2}\sin t\cos t\right)$

$= \frac{9}{2}\operatorname{Arcsin}\left(\frac{x-2}{3}\right) - \frac{\cancel{9}}{2}\frac{(x-2)}{\cancel{3}}\cdot\frac{\sqrt{9-(x-2)^2}}{\cancel{3}} + C = \frac{9}{2}\operatorname{Arcsin}\left(\frac{x-2}{3}\right) - \frac{(x-2)\cdot\sqrt{5+4x-x^2}}{2}$

4) → (a)

15

$\int_0^\infty \frac{dx}{x^2+x} = \underbrace{\int_0^1 \frac{dx}{x^2+x}}_{2.\text{tip}} + \underbrace{\int_1^\infty \frac{dx}{x^2+x}}_{1.\text{tip}} = \lim_{a\to 0^+}\int_a^1 \frac{dx}{x^2+x} + \lim_{R\to\infty}\int_1^R \frac{dx}{x^2+x}$

$\int \frac{dx}{x(x+1)} = \int \left(\frac{A}{x}+\frac{B}{x+1}\right)dx$

$= \int \frac{dx}{x} - \int \frac{dx}{x+1} = \ln\left|\frac{x}{x+1}\right|$

$= \lim_{a\to 0^+} \ln\left|\frac{x}{x+1}\right|\Big|_a^1 + \lim_{R\to\infty} \ln\left(\frac{x}{x+1}\right)\Big|_1^R$

$= \lim_{a\to 0^+}\left(\ln(1/2) - \ln\frac{a}{a+1}\right) + \lim_{R\to\infty}\left(\ln\left(\frac{R}{R+1}\right) - \ln 1/2\right)$

$= \ln 1/2 + \infty + \ln 1 - \cancel{\ln 1/2} = +\infty \rightarrow$ integral ıraksaktır.

b) $\int \tan^3 \cdot \sec^2 x\,dx \underset{u=\tan x \Rightarrow du=\sec^2 x\,dx}{=} \int u^3\,du = \frac{u^4}{4}+C = \frac{\tan^4 x}{4}+C$ //

10