Kuvvet serileri için uygulamalar (Taylor ve McLaurin serileri)

1) Verilen fonksiyonların ilgili noktalardaki Taylor serilerini bulunuz.

a) $f(x)=\sqrt{x}$, $a=1$, b) $f(x)=\ln x$, $a=1$.

Çözüm: $f(x)$ in $a=1$ noktadaki Taylor serisi

$$\sum_{k=0}^{\infty}\frac{f^{(k)}(a)}{k!}(x-a)^k=f(a)+\frac{f'(a)}{1!}(x-a)+\dots+\frac{f^{(k)}(a)}{k!}(x-a)^k+\dots$$

biçiminde olduğundan;

a) $f(x)=\sqrt{x}=(x)^{1/2}\rightarrow f(1)=1$

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$$f'(x)=\frac{1}{2}(x)^{-1/2}\rightarrow f'(1)=1/2$$

$$f''(x)=-\frac{1}{2^2}x^{-3/2}\rightarrow f''(1)=-1/2^2$$

$$f'''(x)=+\frac{1}{2^2}\cdot\frac{3}{2}x^{-5/2}\rightarrow f'''(1)=+\frac{1.3}{2^3}$$

$$f^{iv}(x)=-\frac{1.3.5}{2^4}\cdot x^{-7/2}\rightarrow f^{iv}(1)=-\frac{1.3.5}{2^4}$$

$\vdots$

$$f^{(k)}(x)=(-1)^{k+1}\cdot\frac{1.3.5\dots(2k-1)}{2^k}\rightarrow f^{(k)}(1)=(-1)^{k+1}\cdot\frac{1.3.5\dots(2k-1)}{2^k}$$

old. dan;

$$\sum_{k=0}^{\infty}\frac{f^{(k)}(1)}{k!}(x-1)^k=1+\frac{1/2}{1!}(x-1)-\frac{1/2^2}{2!}(x-1)^2+\frac{\frac{1.3}{2^3}}{3!}(x-1)^3+\dots$$
$$+(-1)^{k+1}\frac{\frac{1.3.5\dots(2k-1)}{2^k}}{k!}(x-1)^k$$

$$=1+\frac{1}{2}(x-1)-\frac{1}{2!\,2^2}(x-1)^2+\frac{1.3}{2^3.3!}(x-1)^3-\frac{1.3.5}{2^4.4!}(x-1)^4+\dots$$
$$+(-1)^{k+1}\frac{1.3.5\dots(2k-1)}{2^k.k!}(x-1)^k+\dots=1+\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1.3.5\dots(2k-1)}{2^k.k!}(x-1)^k$$ dır.

b) $y=f(x)=\ln x$, $a=1$ ise

$f(x)=\ln x\longrightarrow f(1)=\ln 1=0$

$f'(x)=\frac{1}{x}\rightarrow f'(1)=\frac{1}{1}=1.$

$f''(x)=-\frac{1}{x^2}\longrightarrow f''(1)=-1$

$f'''(x)=2.x^{-3}\rightarrow f'''(1)=2.$

$f^{iv}(x)=-3.2.x^{-4}\dots f^{iv}(1)=-3.2.1.$

$\vdots$

$f^{(k)}(x)=(-1)^{k}(k-1)!x^{-k}\rightarrow f^{(k)}(1)=(-1)^{k}(k-1)!$

old. dan $\ln x \longrightarrow \sum_{0} \frac{f^{(k)}(1)}{k!}(x-1)^k$

$= f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \dots + \frac{f^{(k)}(1)}{k!}(x-1)^k + \dots$

$= 0 + \frac{1}{1!}(x-1) - \frac{1}{2!}(x-1)^2 + \frac{2!}{3!}(x-1)^3 - \frac{3!}{4!}(x-1)^4 + \dots + \frac{(-1)^k (k-1)!}{k!}(x-1)^k + \dots$

$= (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots + (-1)^{k+1} \cdot \frac{(x-1)^k}{k} + \dots$

$= \sum_{1}^{\infty} (-1)^k \cdot \frac{(x-1)^k}{k}$ bulunur.

2) Aşağıdaki fonksiyonların McLaurin serilerini bulunuz.

a) $y = f(x) = \ln(x+6)$, b) $f(x) = \sin^2 x$ c) $f(x) = \sqrt{1+x^2}$

d) $f(x) = \cos\left(\frac{x}{3}\right)$, e) $f(x) = \sinh(2x)$

Çözüm: a) $y = \ln(6+x) = \ln\left(6\cdot\left(1+\frac{x}{6}\right)\right) = \ln(6) + \ln(1+x/6)$

dir. $\ln(1+x) = \sum_{1}^{\infty} (-1)^{k+1} \cdot \frac{x^k}{k}$ olduğundan $(-1 < x \leq 1$ için$)$

$$\ln\left(1+\frac{x}{6}\right) = \sum_{1}^{\infty} (-1)^{k+1} \cdot \frac{(x/6)^k}{k} = \sum_{1}^{\infty} (-1)^{k+1} \frac{x^k}{k \cdot 6^k} \quad \left(-1 < x/6 \leq 1 \iff -6 < x \leq 6\right)$$

olacaktır. Böylece;

$$y = \ln(6+x) = \ln(6) + \sum_{1}^{\infty} (-1)^{k+1} \frac{x^k}{k \cdot 6^k}, \quad (-6 < x \leq 6) \text{ olur.}$$

b) $f(x) = \sin^2 x = \frac{1-\cos 2x}{2} = \frac{1}{2} - \frac{1}{2}\cos 2x$ dir.

Ayrıca $\cos x = \sum_{0}^{\infty} (-1)^k \cdot \frac{x^{2k}}{(2k)!}$ $(x \in \mathbb{R})$ idi.

Böylece $\cos 2x = \sum_{0}^{\infty} (-1)^k \cdot \frac{(2x)^{2k}}{(2k)!} = \sum_{0}^{\infty} (-1)^k \cdot \frac{2^{2k} \cdot x^{2k}}{(2k)!}$

$= \sum_{0}^{\infty} (-1)^k \cdot \frac{x^{2k} \cdot 4^k}{(2k)!}$ olacaktır.

o halde $f(x) = \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x = \frac{1}{2} - \frac{1}{2}\sum_{0}^{\infty} (-1)^k \frac{x^{2k} \cdot 2^{2k}}{(2k)!}$

$= \frac{1}{2} + \sum_{0}^{\infty} (-1)^{k+1} \frac{x^{2k}\, 2^{2k-1}}{(2k)!}$ bulunur. $(\forall x \in \mathbb{R}$ için$)$.

c) $f(x)=\sqrt{1+x^2}$ olduğundan ve

$(1+x)^{\alpha}=\sum_{k=0}^{\infty}\binom{\alpha}{k}.x^k$, $(|x|<1)$ olduğu bilindiğinden;

Yani $\alpha=1/2$ için $(1+x)^{1/2}=\sqrt{1+x}=\sum_{0}^{\infty}\binom{1/2}{k}.x^k$ $(|x|<1)$

olacaktır. Böylece $f(x)=\sqrt{1+x^2}=(1+x^2)^{1/2}$

$=\sum_{0}^{\infty}\binom{1/2}{k}.(x^2)^k=\sum_{0}^{\infty}\binom{1/2}{k}.x^{2k}$, $\left(\begin{array}{c}|x^2|=|x|^2<1 \Leftrightarrow \\ |x|<1 \text{ için}\end{array}\right)$

$g(x)=(1+x)^{1/2}=\sqrt{1+x}$ için $\alpha=1/2$ ise $\binom{\alpha}{0}=1$, $\binom{\alpha}{1}=1/2$

$\binom{\alpha}{2}=\frac{\frac{1}{2}(1/2-1)}{2!}=\frac{-1/4}{2!}=-\frac{1}{2^2.2!}$, $\binom{\alpha}{3}=\frac{(\frac{1}{2})(1/2-1)(1/2-2)}{3!}=\frac{1.3}{2^3.3!}$

$\binom{\alpha}{4}=\frac{1/2(1/2-1)(\frac{1}{2}-2)(\frac{1}{2}-3)}{4!}=\frac{1/2.(-1/2)(-3/2)(-5/2)}{4!}=-\frac{1.3.5}{2^4.4!}$

$\dots\binom{\alpha}{k}=(-1)^{k+1}\frac{1.3.5\dots(2k-1)}{2^k.k!}$ olup;

$(1+x)^{1/2}=\binom{1/2}{0}+\binom{1/2}{1}x+\binom{1/2}{2}x^2+\dots+\binom{1/2}{k}x^k+\dots$

$=1+\frac{1}{2}x-\frac{1}{2^2.2!}x^2+\frac{1.3}{2^3.3!}x^3-\frac{1.3.5}{2^4.4!}x^4+-\frac{1.3.5.7}{2^5.5!}x^5+\dots+\frac{(-1)^{k+1}.1.3.5.(2k-1)}{2^k.k!}x^k+\dots$

$=1+\sum_{1}^{\infty}(-1)^{k+1}\frac{1.3.5\dots(2k-1)}{2^k.k!}x^k$ olduğundan;

$\sqrt{1+x^2}=(1+x^2)^{1/2}=1+\sum_{1}^{\infty}(-1)^{k+1}\frac{1.3.5\dots(2k-1)}{2^k.k!}(x^2)^k$ olur.

d) $f(x)=\cos(x/3)$ ve de $\cos x=\sum_{0}^{\infty}(-1)^k.\frac{x^{2k}}{(2k)!}$ old. dan

$f(x)=\cos(x/3)=\sum_{0}^{\infty}(-1)^k.\frac{(x/3)^{2k}}{(2k)!}=\sum_{0}^{\infty}(-1)^k.\frac{x^{2k}}{3^{2k}.(2k)!}$ olur.

e) $f(x)=\sinh(2x)$ ve de $\sinh x=\sum_{0}^{\infty}\frac{x^{2k+1}}{(2k+1)!}$ $(\forall x\in\mathbb{R})$

olduğundan;

$f(x)=\sinh 2x=\sum_{0}^{\infty}\frac{(2x)^{2k+1}}{(2k+1)!}=\sum_{0}^{\infty}\frac{2^{2k+1}.x^{2k+1}}{(2k+1)!}$ $\forall x\in\mathbb{R}$

bulunur.

3) İlgili McLaurin serilerini kullanarak aşağıdaki limitleri bulunuz.

a) $\lim\limits_{x\to 0}\dfrac{\cosh x-1}{\cos x-1}$, b) $\lim\limits_{x\to 0}\left(\dfrac{1}{\tan x}-\dfrac{1}{x}\right)$, c) $\lim\limits_{x\to 0}\dfrac{x-\tan^{-1}x}{x^3}$

d) $\lim\limits_{x\to 0}\dfrac{\ln(1-x)}{e^x-1}$

Çözüm: a) $\lim\limits_{x\to 0}\dfrac{\cosh x-1}{\cos x-1}=[0/0]=\lim\limits_{x\to 0}\dfrac{\cancel{1}+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\dots\cancel{-1}}{\cancel{1}-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots\cancel{-1}}$

$$\cosh x=\sum_0^\infty \frac{x^{2k}}{(2k)!}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\dots$$

$$\cos x=\sum_0^\infty (-1)^k\frac{x^{2k}}{(2k)!}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots$$

$$=\lim_{x\to 0}\frac{x^2\left(\frac{1}{2!}+\frac{x^2}{4!}+\frac{x^4}{6!}+\dots\right)}{x^2\left(-1/2!+\frac{x^2}{4!}-\frac{x^4}{6!}+\dots\right)}=\frac{1/2!}{-1/2!}=-1 \text{ bulunur.}$$

b) $\lim\limits_{x\to 0}\left(\dfrac{1}{\tan x}-\dfrac{1}{x}\right)=\lim\limits_{x\to 0}\left(\dfrac{\cos x}{\sin x}-\dfrac{1}{x}\right)=\lim\limits_{x\to 0}\dfrac{x.\cos x-\sin x}{x.\sin x}=[0/0]$

$$\cos x=\sum_0^\infty (-1)^k\frac{x^{2k}}{(2k)!}$$

$$\sin x=\sum_0^\infty (-1)^k\frac{x^{2k+1}}{(2k+1)!}$$

$$=\lim_{x\to 0}\frac{x\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots\right)-\left(\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots\right)}{x.\left(\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots\right)}$$

$$=\lim_{x\to 0}\frac{\cancel{x}-x^3/2!+x^5/4!-\frac{x^7}{6!}+\dots\cancel{-\frac{x}{1!}}+\frac{x^3}{3!}-\frac{x^5}{5!}+\frac{x^7}{7!}-\dots}{\frac{x^2}{1!}-\frac{x^4}{3!}+\frac{x^6}{5!}-\frac{x^8}{7!}+\dots}$$

$$=\lim_{x\to 0}\frac{\left(-\frac{1}{2!}+\frac{1}{3!}\right)x^3+\left(\frac{1}{4!}-1/5!\right)x^5+\left(-1/6!+\frac{1}{7!}\right)x^7+\dots}{x^2\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\dots\right)}$$

$$=\lim_{x\to 0}\frac{-\frac{2}{3!}x+\frac{4}{5!}x^3-\frac{6}{7!}x^5+\dots}{1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\dots}=\frac{0}{1}=0 \text{ bulunur.}$$

c) $\lim\limits_{x\to 0}\dfrac{x-\tan^{-1}x}{x^3}=[0/0]=\lim\limits_{x\to 0}\dfrac{x-\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\dots\right)}{x^3}$

$$\tan^{-1}x=\sum_0^\infty (-1)^k\frac{x^{2k+1}}{2k+1}\qquad =\lim_{x\to 0}\frac{\frac{x^3}{3}-\frac{x^5}{5}+\frac{x^7}{7}-\frac{x^9}{9}+\dots}{x^3}$$

$$=\lim_{x\to 0}\frac{\cancel{x^3}\left(\frac{1}{3}-\frac{x^2}{5}+\frac{x^4}{7}-\frac{x^6}{9}+\dots\right)}{\cancel{x^3}}=\lim_{x\to 0}\frac{\frac{1}{3}-\frac{x^2}{5}+\frac{x^4}{7}-\frac{x^6}{9}+\dots}{1}$$

$$=1/3-0=1/3 \text{ bulunur.}$$

d) $\lim\limits_{x\to 0} \dfrac{\ln(1-x)}{e^x-1} = [0/0] = \lim\limits_{x\to 0} \dfrac{-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-\frac{x^5}{5}-\cdots}{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right)-(1)}$

$\ln(1-x) = -x-\dfrac{x^2}{2}-\dfrac{x^3}{3}-\dfrac{x^4}{4}-\cdots-\dfrac{x^k}{k}-\cdots$

$e^x = \sum\limits_0^{\infty} \dfrac{x^k}{k!} = 1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\cdots$

$= \lim\limits_{x\to 0} \dfrac{\cancel{x}\left(-1-\frac{x}{2}-\frac{x^2}{3}-\frac{x^3}{4}-\frac{x^4}{5}-\cdots\right)}{\cancel{x}\left(1+\frac{x}{2!}+\frac{x^2}{3!}+\frac{x^3}{4!}+\cdots\right)}$

$= \lim\limits_{x\to 0} \dfrac{-1-x/2-\frac{x^2}{3}-\cdots}{1+x/2!+\frac{x^2}{3!}+\frac{x^3}{4!}+\cdots} = \dfrac{-1}{1} = -1$ bulunur.

4) $\dfrac{\pi}{2} = 1+\dfrac{1}{2}\cdot\dfrac{1}{3}+\dfrac{1\cdot 3}{2\cdot 4}\cdot 1/5+\cdots+\dfrac{1\cdot3\cdot5\cdots(2k-1)}{2\cdot4\cdot6\cdots(2k)}\cdot\dfrac{1}{2k+1}+\cdots$

eşitliğini gösteriniz.

Çözüm: $\dfrac{1}{\sqrt{1-x^2}} = \sum\limits_0^{\infty} \dfrac{1\cdot3\cdot5\cdots(2k-1)}{2\cdot4\cdot6\cdots(2k)}\cdot x^{2k}$, $(|x|<1)$ idi.

$\Rightarrow$ Taraf tarafa integral alınarak, $(|x|<1$ için$)$

$$\int_0^x \frac{dt}{\sqrt{1-t^2}} = \operatorname{Arcsin} x = \sin^{-1}x = \sum_0^{\infty} \frac{1\cdot3\cdot5\cdots(2k-1)}{2\cdot4\cdot6\cdots(2k)}\cdot\frac{x^{2k+1}}{2k+1}$$

dir.

Ayrıca bu seri $x=1$ için yakınsaktır! $\downarrow x=1$ de yakınsadığını görün lütfen

$$\pi/2 = \sin^{-1}(1) = \sum_{k=0}^{\infty} \frac{1\cdot3\cdot5\cdots(2k-1)}{2\cdot4\cdot6\cdots(2k)}\,\frac{1}{2k+1}$$ dir.

5) Aşağıdaki belirsiz integralleri bulunuz.

a) $\displaystyle\int \frac{\sin x}{x}\,dx$, b) $\displaystyle\int \frac{\tan^{-1}x}{x}\,dx$, c) $\displaystyle\int \sqrt{1-x^3}\,dx$

Çözüm (a)

$\sin x = \sum\limits_0^{\infty}(-1)^k \dfrac{x^{2k+1}}{(2k+1)!}$ $\longrightarrow$ $\displaystyle\int \frac{\sin x}{x}\,dx = \int \frac{\sum_0^{\infty}(-1)^k \frac{x^{2k+1}}{(2k+1)!}}{x}\,dx$

$$= \int \frac{x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}+\cdots}{x}\,dx$$

$$= \int\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\frac{x^8}{9!}+\cdots\right)dx = x-\frac{x^3}{3.3!}+\frac{x^5}{5.5!}-\frac{x^7}{7.7!}+\frac{x^9}{9.9!}+\cdots$$

$$= \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{(2k+1)(2k+1)!}$$ bulunur.

b) $$\int \frac{\tan^{-1}x}{x}\,dx = \int \frac{x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots}{x}\,dx$$

$$\tan^{-1}x = \sum_{0}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1}$$

$$= \int \left(1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \frac{x^8}{9} + \cdots\right)dx$$

$$= x - \frac{x^3}{3^2} + \frac{x^5}{5^2} - \frac{x^7}{7^2} + \frac{x^9}{9^2} - \cdots = \sum_{0}^{\infty}(-1)^k \frac{x^{2k+1}}{(2k+1)^2}$$ olur.

c) $$\int \sqrt{1-x^3}\,dx = \int \left(1 - \frac{1}{2}x^3 - \frac{1.3}{2.4}x^6 - \frac{1.3.5}{2.4.6}x^9 - \frac{1.3.5.7}{2.4.6.8}x^{12} - \cdots\right)dx$$

$$\sqrt{1+x} = 1 + \sum_{1}^{\infty}(-1)^{k+1}\frac{1.3.5\cdots(2k-1)}{2.4.6\cdots(2k)}x^k$$ idi.

$$\Rightarrow \sqrt{1-x} = 1 + \sum_{1}^{\infty}(-1)^{k+1}\frac{1.3.5\cdots(2k-1)}{2.4.6\cdots(2k)}\cdot(-x)^k$$

$$= 1 + \sum_{1}^{\infty}(-1)^{k+1}(-1)^k\frac{1.3.5\cdots(2k-1)}{2.4.6\cdots(2k)}\cdot x^k$$

$$= 1 - \sum_{1}^{\infty}\frac{1.3.5\cdots(2k-1)}{2.4.6\cdots(2k)}x^k$$ ve dk

$$= x - \frac{1}{2.4}x^4 - \frac{1.3}{2.4}\cdot\frac{x^7}{7} - \frac{1.3.5}{2.4.6}\cdot\frac{x^{10}}{10} - \frac{1.3.5.7}{2.4.6.8}\cdot\frac{x^{13}}{13} - \cdots$$

$$= x - \sum_{1}^{\infty}\frac{1.3.5\cdots(2k-1)}{2.4.6.8\cdots(2k)}\cdot\frac{x^{3k+1}}{(3k+1)}$$

$$\sqrt{1-x^3} = 1 - \sum_{1}^{\infty}\frac{1.3.5\cdots(2k-1)}{2.4.6\cdots(2k)}\cdot x^{3k}$$

6) $$\int_3^{\infty}\frac{dx}{x^{k+1}} = \frac{1}{k.3^k}$$ eşitliğini kullanarak

$$\frac{1}{1.3} + \frac{1}{2.3^2} + \cdots + \frac{1}{k.3^k} + \cdots = ?$$

$$\frac{1}{1.3} + \frac{1}{2.3^2} + \frac{1}{3.3^3} + \cdots + \frac{1}{k3^k} + \cdots = \sum_{1}^{\infty}\frac{1}{k.3^k}$$ dır.

Şimdi, $$f(x) = \frac{1}{3-x} = \frac{1}{3(1-\frac{x}{3})} = \frac{1}{3}\cdot\sum_{0}^{\infty}\left(\frac{x}{3}\right)^k = \sum_{0}^{\infty}\frac{x^k}{3^{k+1}}$$ dür.

Bu seri $|x|<3$ olan x ler için yakınsaktır. Taraf-tarafa integral alırsak, $$\int_0^x \frac{dt}{3-t} = -\ln(3-t)\Big|_0^x = \ln\left(\frac{3}{3-x}\right) = \sum_{0}^{\infty}\frac{x^{k+1}}{(k+1)\,3^{k+1}}$$

$$= \sum_{1}^{\infty}\frac{x^k}{k.3^k}$$ olup $x=1$ alınırsa,

$$\ln(3/2) = \sum_{1}^{\infty}\frac{1}{k.3^k}$$ bulunur.

$\int_3^\infty \frac{dx}{x^{k+1}} = \frac{1}{k.3^k}$ olduğu biliniyorsa; $\sum_1^\infty \frac{1}{k3^k}$

$= \sum_1^\infty \left(\int_3^\infty \frac{dx}{x^{k+1}} dx \right) = \int_3^\infty \left(\sum_{k=1}^\infty \frac{1}{x^{k+1}} \right) dx$ dir.

Ayrıca; $\frac{1}{1-1/x} = \sum_0^\infty \left(\frac{1}{x}\right)^k = \sum_0^\infty \frac{1}{x^k} = 1 + \frac{1}{x} + \sum_2^\infty \frac{1}{x^k}$

$= \frac{x+1}{x} + \sum_1^\infty \frac{1}{x^{k+1}}$ dir $\Rightarrow \sum_1^\infty \frac{1}{x^{k+1}} = \frac{x}{x-1} - \frac{x+1}{x} = \frac{1}{x(x-1)}$

dir. O halde $\sum_1^\infty \frac{1}{k.3^k} = \int_3^\infty \left(\sum_1^\infty \frac{1}{x^{k+1}} \right) dx = \int_3^\infty \frac{dx}{x(x-1)}$

$= \lim_{R\to\infty} \left(\ln\left(\frac{x-1}{x}\right) \right) \Big|_3^R = \lim_{R\to\infty} \ln\left(\frac{R-1}{R}\right) - \ln(2/3) = \ln(3/2)$ dir.