

$$1-) \int \sec \theta \cdot d\theta = ?$$

$$\int \frac{1}{\cos \theta} \cdot d\theta = \int \frac{\cos \theta}{\cos^2 \theta} d\theta = \int \frac{\cos \theta}{1 - \sin^2 \theta} d\theta, \quad \begin{array}{l} u = \sin \theta \text{ olsun.} \\ du = \cos \theta \cdot d\theta \text{ olur.} \end{array}$$

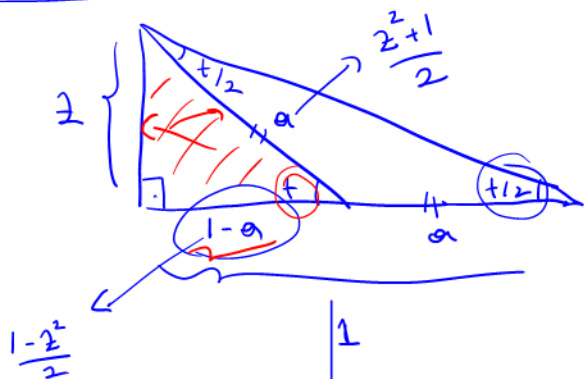
$$= \int \frac{du}{1-u^2} \rightarrow \frac{A}{(1-u)} + \frac{B}{(1+u)}$$

$$= \int \frac{1}{2} \frac{du}{1+u} + \int \frac{1}{2} \frac{du}{1-u}$$

$$= \ln|1+u| - \ln|1-u| = \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + C \text{ olur.}$$

$$2-) \int \frac{dt}{\sin t - \cos t} = ?$$

$$\tan \frac{t}{2} = z \quad \text{dönüşüm: } m\bar{o} \quad \text{ya da } \text{padé} \quad i$$



$$\Rightarrow a^2 = z^2 + (1-a)^2 = z^2 + a^2 + 1 - 2a$$

$$\Rightarrow 2a = z^2 + 1$$

$$\Rightarrow \frac{z^2 + 1}{2} = a$$

$$1-a = 1 - \frac{z^2 + 1}{2}$$

$$= \frac{1-z^2}{2}$$

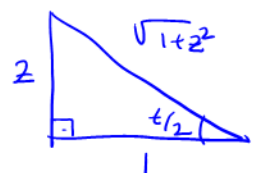
$$\cos t = \frac{\frac{1-z^2}{2}}{\frac{z^2+1}{2}} = \frac{1-z^2}{1+z^2}$$

$$\sin t = \frac{z}{\frac{z^2+1}{2}} = \frac{2z}{z^2+1} \quad \text{bulunur.}$$

$$\left( \sec^2 \frac{t}{2} \right) \cdot \frac{1}{2} \cdot dt = dz \Rightarrow dt = \underbrace{2 \cdot \cos^2 \frac{t}{2}}_{\frac{2}{1+z^2}} \cdot dz$$

$$= \frac{2}{1+z^2} dz$$

$$\int \frac{2}{1+z^2} dz$$



$$\cos \frac{t}{2} = \frac{1}{\sqrt{1+z^2}}$$

$$\cos^2 \frac{t}{2} = \frac{1}{1+z^2}$$

$$\Rightarrow \int \frac{2}{\underbrace{2z-1+z^2}_{(z+1)^2-2}} dz = \int \frac{2 dz}{(z+1-\sqrt{2})(z+1+\sqrt{2})} \Rightarrow \frac{A}{z+1-\sqrt{2}} + \frac{B}{z+1+\sqrt{2}} = \frac{2}{2z-1+z^2}$$

Özellik ↓

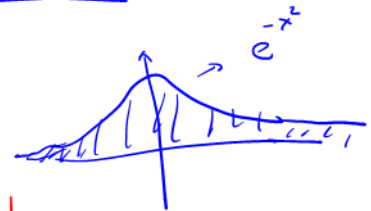
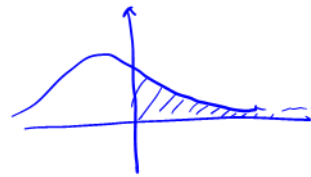
$$\Rightarrow A = \frac{1}{\sqrt{2}} \quad , \quad B = -\frac{1}{\sqrt{2}}$$

$$= \int \frac{1}{\sqrt{2}} \cdot \frac{dz}{z+1-\sqrt{2}} - \int \frac{1}{\sqrt{2}} \cdot \frac{dz}{z+1+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \left( \ln |z+1-\sqrt{2}| - \ln |z+1+\sqrt{2}| \right)$$

3-)  $\int_0^{\infty} \frac{16 \tan^{-1} x}{1+x^2} dx = ?$

I



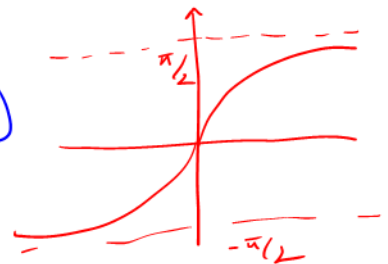
$$\tan^{-1} x = u \Rightarrow \frac{1}{1+x^2} dx = du$$

$$I = \lim_{\alpha \rightarrow \infty} \int_0^{\alpha} \frac{16 \tan^{-1} x}{1+x^2} dx$$

$$= \lim_{\alpha \rightarrow \infty} \int_0^{\tan^{-1} \alpha} 16 u \cdot du = \lim_{\alpha \rightarrow \infty} 8 u^2 \Big|_0^{\tan^{-1} \alpha}$$

$$= \lim_{\alpha \rightarrow \infty} 8 \left\{ (\tan^{-1} \alpha)^2 - 0 \right\}$$

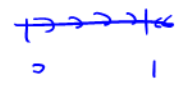
$$= 8 \cdot \frac{\pi^2}{4} = 2\pi^2 \quad \square$$



4-)  $\int_0^2 \frac{dx}{\sqrt{|x-1|}} = ?$

$f(x) = \frac{1}{\sqrt{|x-1|}}$

$x \rightarrow 1 \quad f(x) \rightarrow \infty$



$$I = \int_0^1 \frac{dx}{\sqrt{1-x}} + \int_1^2 \frac{dx}{\sqrt{x-1}}$$

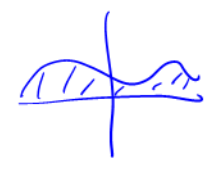
$$= \lim_{a \rightarrow 1^-} \int_0^a \frac{dx}{\sqrt{1-x}} + \lim_{b \rightarrow 1^+} \int_b^2 \frac{dx}{\sqrt{x-1}}$$

$$= \lim_{a \rightarrow 1^-} \left. -2\sqrt{1-x} \right|_0^a + \lim_{b \rightarrow 1^+} \left. 2\sqrt{x-1} \right|_b^2$$

$$= \lim_{a \rightarrow 1^-} \left( \underbrace{-2\sqrt{1-a}}_0 + 2 \right) + \lim_{b \rightarrow 1^+} \left( 2 - \underbrace{2\sqrt{b-1}}_0 \right)$$

$$= 2 + 2 = 4$$

5-)  $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$  işleminin yakınsak olup olmadığını bulunuz.



Çözüm

$f(x) = \frac{1}{e^x + e^{-x}}$  Fonksiyon çifttir. Aşağıda;

$$f(-x) = \frac{1}{e^{-x} + e^{-(-x)}} = \frac{1}{e^x + e^{-x}} = f(x) \rightarrow$$



$$\int_{-a}^a \frac{dx}{e^x + e^{-x}} = 2 \cdot \int_0^a \frac{dx}{e^x + e^{-x}}$$

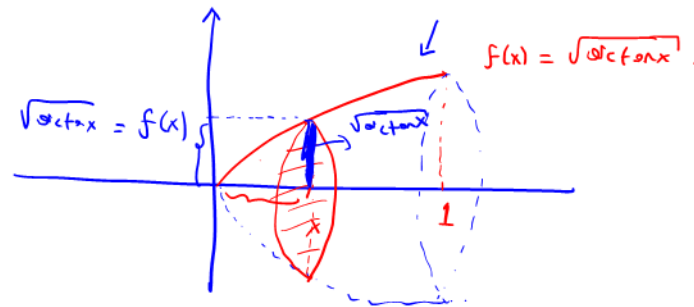
$$I = \lim_{a \rightarrow \infty} \int_{-a}^a \frac{dx}{e^x + e^{-x}} = \lim_{a \rightarrow \infty} 2 \int_0^a \frac{dx}{e^x + e^{-x}}$$

$\forall x \in \mathbb{R}$  için  $e^x > 0$  old. not edelim.

$$0 < \frac{1}{e^x + e^{-x}} < \frac{1}{e^{-x}} \quad \text{ve} \quad \text{ayrıca} \quad 2 \int_0^{\infty} \frac{1}{e^x} dx = 2 \quad \text{ve} \quad \text{gelmektedir.}$$

Konvergenste bir fonksiyon  $\int \frac{1}{e^x + e^{-x}} dx$  gelmektedir olur.

6-)  $f(x) = \sqrt{\arctan x}$  fonksiyonu  $[0,1]$  aralığında tanımlanmış. Bu fonk'nun  $x$  eksenini etrafında döndürülmesiyle elde edilen cismin hacmi nedir?



$$\begin{aligned} P_x &= \pi r^2 \\ &= \pi \cdot (\sqrt{\arctan x})^2 \\ &= \pi \cdot \arctan x \quad \text{olur.} \end{aligned}$$

$\forall x \in [0,1]$  için  $P_x$  alanlarını toplayarak hacmi elde ederiz.

$$V = \int_0^1 \pi \cdot \arctan x \, dx \quad \arctan x = u \quad \text{ve} \quad dx = du$$

$$\frac{1}{1+x^2} \cdot dx = du \quad (\leftrightarrow) \quad x = u$$

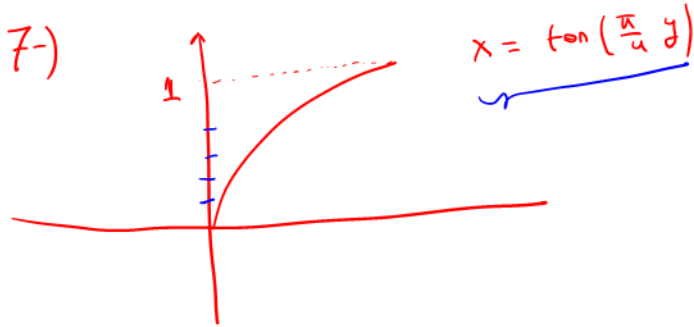
$$= \pi \left[ \arctan x \cdot x \Big|_0^1 - \frac{\ln(1+x^2)}{2} \Big|_0^1 \right] \quad \int \arctan x \, dx = \arctan x \cdot x - \int \frac{x}{1+x^2} dx$$

$$\downarrow \quad \begin{matrix} 1+x^2 = p \\ 2x \, dx = dp \end{matrix} \quad \int \frac{dp}{2p}$$

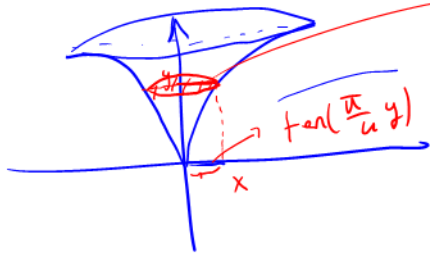
$$= \pi \left\{ \frac{\pi}{u} \cdot 1 - \left[ 0 \cdot 0 \right] - \left[ \frac{\ln 2}{2} - \frac{\ln 1}{2} \right] \right\}$$

$$= \pi \left( \frac{\pi}{u} - \frac{\ln 2}{2} \right) \underline{\underline{\pi}}$$

$$\hookrightarrow \frac{1}{2} h v = \frac{\ln(1+x^2)}{2}$$



$x = \tan\left(\frac{\pi}{u} y\right)$  Fonksiyonu  $y$  ekseninde döndürerek elde edilen şeklin hacmi ne olur?



$$Dy = \pi \cdot r^2$$

$$Dy = \pi \cdot \tan^2\left(\frac{\pi}{u} y\right) \text{ bulunur.}$$

$\forall y \in [0,1]$  için tüm  $Dy$ 'lerin toplamı istenilen hacmi verecektir.

$$V = \int_0^1 \pi \cdot \tan^2\left(\frac{\pi}{u} y\right) \cdot dy, \quad \frac{\pi}{u} y = u$$

$$\Rightarrow \frac{\pi}{u} dy = du$$

$$\Rightarrow dy = \frac{u}{\pi} du$$

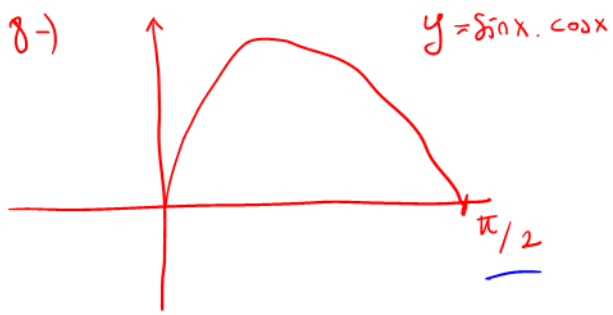
$$\frac{s^2}{c^2} + \frac{c^2}{c^2} = \frac{s^2 + c^2}{c^2} = \frac{1}{c^2}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$V = u \int_0^{\pi/u} \tan^2(u) \cdot du = u \int_0^{\pi/u} (\sec^2 u - 1) du = u \left[ \tan u - u \right]_0^{\pi/u}$$

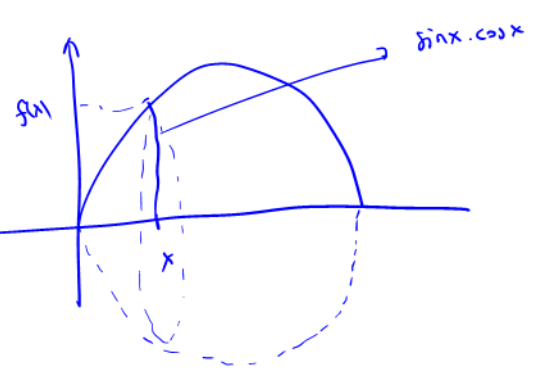
$$= u \left[ 1 - \frac{\pi}{u} - 0 \right] = \underline{\underline{u - \pi}}$$

8-)



Yandaki f'unk.'nın x-ekseni etrafında döndürülmesi ile oluşan şeklin hacmini bulun.

Çözüm:



$$dV_x = \pi \cdot \sin^2 x \cdot \cos^2 x$$

$$V = \int_0^{\pi/2} \pi \cdot \sin^2 x \cdot \cos^2 x \cdot dx$$

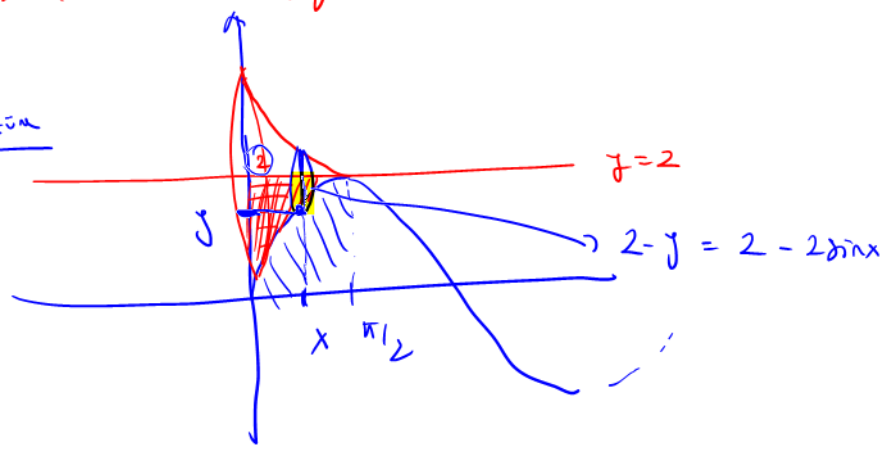
$$= \int_0^{\pi/2} \pi \cdot (\underbrace{\sin x \cdot \cos x}_{\frac{1}{2} \cdot \sin 2x})^2 dx$$

$$= \frac{\pi}{4} \int_0^{\pi/2} (\sin 2x)^2 dx, \quad 2x=0 \Rightarrow 2dx=du$$

$$= \frac{\pi}{8} \cdot \int_0^{\pi} \sin^2 u \cdot du = \frac{\pi^2}{16}$$

9-) Birinci bölgede üstten  $y=2$  doğrusu, alttan  $y=2 \sin x$ ,  $x \in [0, \frac{\pi}{2}]$  eğri ve soldan y-ekseni ile sınırlı bölgenin y=2 etrafında döndürülmesiyle elde edilen şeklin hacmini bulunuz.

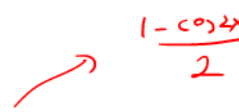
Çözüm



$$dV_x = \pi \cdot [2 - 2 \sin x]^2$$

$$\int_0^{\pi/2} \pi \cdot [2 - 2\sin x]^2 dx = 4\pi \int_0^{\pi/2} (1 - \sin x)^2 dx$$

$$= 4\pi \int_0^{\pi/2} 1 + \sin^2 x - 2\sin x dx$$


 $\frac{1 - \cos 2x}{2}$

$$= \pi [3\pi - 8]$$