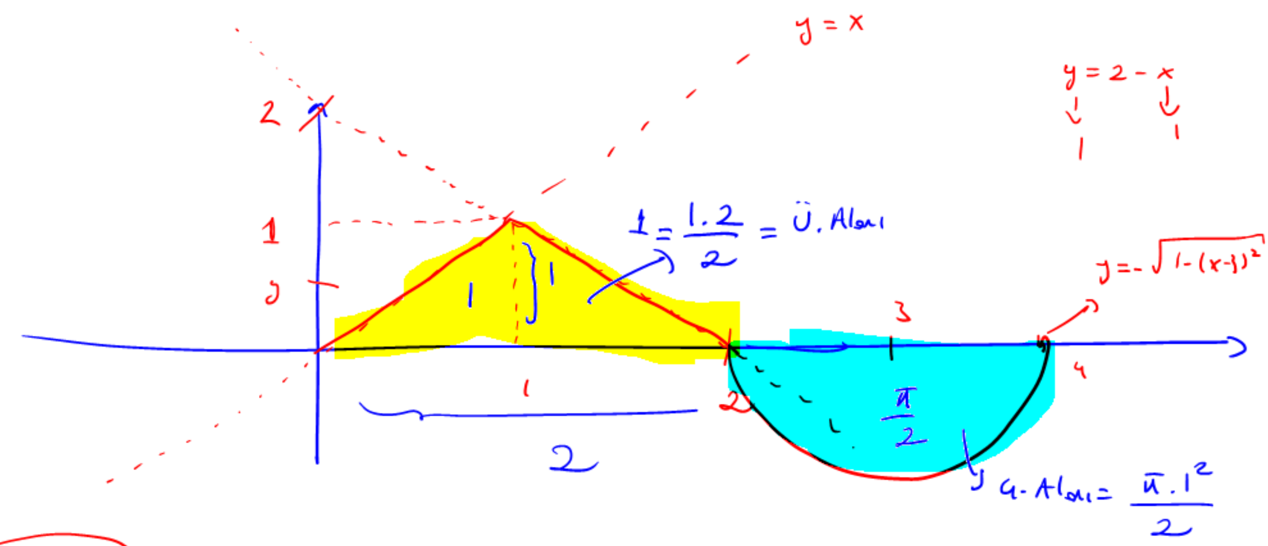
Genel Tekrar

1-) $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2 \\ -\underbrace{\sqrt{1-(x-3)^2}}, & 2 \leq x \leq 4 \end{cases}$ tanımlansın.

a-) $\displaystyle\int_0^4 f(x)\,dx = ?$

b-) f fonk. ile x ekseni arasında kalan sınırlı bölgenin alanı nedir?

Çözüm:



$\underbrace{y = -\sqrt{1-(x-3)^2}} \iff \underbrace{y^2 = 1 - (x-3)^2} \iff (x-3)^2 + y^2 = 1$

$\iff$ (3,0) merkezli 1 yarıçaplı çember.

$\underbrace{y = \sqrt{1-(x-3)^2}}$

b-) $1 + \frac{\pi}{2}$

a-) $\displaystyle\int_0^4 f(x)\,dx = 1 - \frac{\pi}{2}$

2-) $\underbrace{\displaystyle\int_0^x f(t)\,dt} = e^{2x} \cdot \cos x + c \;\cdots (*)$ koşulunu sağlayan $c \in \mathbb{R}$ sabitini ve f(t) fonk. bulunuz.

Çözüm: $\int_0^x f(t)\,dt = F(x)$ öyle ki $F'(x) = f(x)$ (Kalkülüsün temel teoremi)

$$\frac{d}{dx}\int_0^x f(t)\,dt = \frac{d}{dx}F(x) = F'(x) = f(x)$$

(*)'ın x'e göre türevini alalım.

$$\boxed{f(x) = 2 \cdot e^{2x} \cdot \cos x - e^{2x} \cdot \sin x}$$

(*)'da $x=0$ kabul edelim.

$$\underbrace{\int_0^0 f(t)\,dt}_{} = e^{2\cdot 0} \cdot \cos 0 + c \iff 0 = 1 \cdot 1 + c$$

$$\iff \boxed{c = -1}$$

3-) $F(x) = \int_0^1 e^{-x^2}\,dx$ olsun. $\int_0^1 x^2 \cdot e^{-x^2}\,dx$ integralini $F(x)$ cinsinden ifade ediniz.

$$F(x) = \int_0^1 e^{-x^2}\,dx$$

$e^{-x^2} = u \qquad dx = dv$

$-2x \cdot e^{-x^2} \cdot dx = du \qquad x = v$

$$= x \cdot e^{-x^2}\Big|_0^1 - \int_0^1 x \cdot (-2x) \cdot e^{-x^2} \cdot dx$$

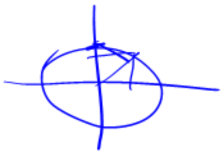
$$= e^{-1} + 2\int_0^1 x^2 \cdot e^{-x^2} \cdot dx$$

$$\Rightarrow \boxed{\frac{F(x) - e^{-1}}{2} = \int_0^1 x^2 \cdot e^{-x^2}\,dx}$$

4-) g fonk.nu $[0, \frac{\pi}{2}]$ aralığında pozitif bir fonk. olmak üzere

$$\underbrace{\int_0^{\pi/2} x\cdot g(\cos x)\,dx}_{I} = \underbrace{\frac{\pi}{2}\int_0^{\pi/2} g(\sin x)\,dx - \int_0^{\pi/2} x\cdot g(\sin x)\,dx}$$

koşulunu sağladığını gösteriniz.



Çözüm: $x = \frac{\pi}{2} - u$ olsun.

$dx = -du$, $\underline{\cos x = \cos\left(\frac{\pi}{2} - u\right) = \sin u}$

$$I = -\int_{\pi/2}^{0}\left(\frac{\pi}{2} - u\right)\cdot g(\sin u)\,du = -\int_{\pi/2}^{0}\frac{\pi}{2}\cdot g(\sin u)\,du + \int_{\pi/2}^{0} u\cdot g(\sin u)\,du$$

$$= \frac{\pi}{2}\int_0^{\pi/2} g(\sin u)\,du - \int_0^{\pi/2} u\cdot g(\sin u)\,du$$

$$= \frac{\pi}{2}\int_0^{\pi/2} g(\sin x)\,dx - \int_0^{\pi/2} x\cdot g(\sin x)\,dx$$

5-) $\underbrace{\int_0^{\pi/2} x\cdot\left(\frac{\sin x}{\cos 2x} - \frac{\cos x}{\cos 2x}\right)dx}_{I}$ int. hesaplayınız.

Çözüm: $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$ biliyoruz.

$$I = \int_0^{\pi/2} x\left(\frac{\sin x}{1 - 2\sin^2 x} \downarrow - \frac{\cos x}{2\cos^2 x - 1}\right)dx = \int_0^{\pi/2} x\left(\frac{\sin x}{\underbrace{1 - 2\sin^2 x}} + \frac{\cos x}{1 - 2\cos^2 x}\right)$$

$\uparrow$ $\uparrow$

$g(x) := \dfrac{x}{1-2x^2}$ tanımlayalım.

$$= \int_0^{\pi/2} x \cdot \left(g(\sin x) + g(\cos x) \right) dx$$

$$= \int_0^{\pi/2} x \cdot g(\sin x)\, dx + \int_0^{\pi/2} x\, g(\cos x)\, dx .$$

(4. soru)

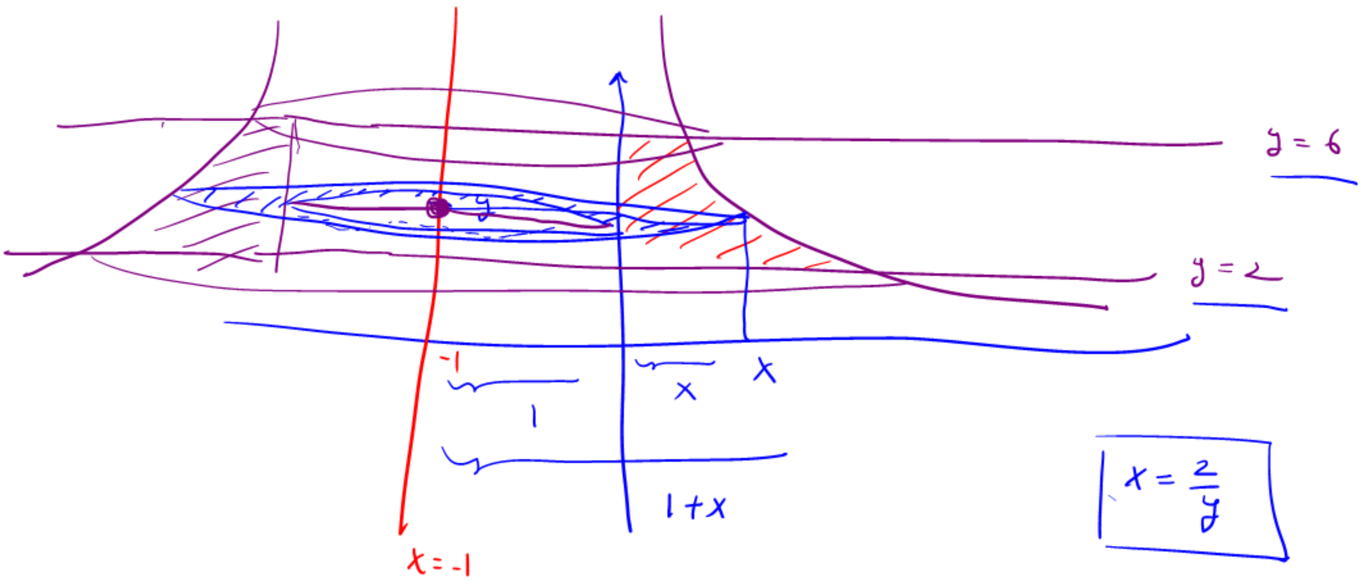
$$= \frac{\pi}{2} \int_0^{\pi/2} g(\sin x)$$

$$= \frac{\pi}{2} \int_0^{1} \frac{\sin x}{1 - 2\sin^2 x}$$

$$= \frac{\pi}{2} \int_0^{1} \frac{\sin x}{2\cos^2 x - 1}\, dx$$

, $\cos x = u$ dönüşümü ile kalanı çözülür.

6-) $x = \dfrac{2}{y}$ eğrisi, $y = 2$, $y = 6$ ve $x = 0$ doğruları arasında kalan bölgenin $x = -1$ doğrusu etrafında döndürülmesi sonucunda elde edilen cismin hacmi nedir?



$$\boxed{x = \frac{2}{y}}$$

Disk metodu ile çözelim.

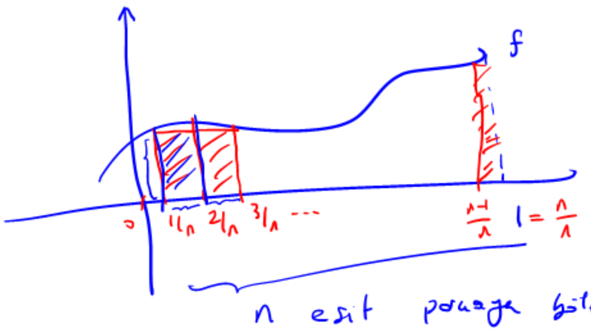
$$D_y = \pi \cdot (1+x)^2 - \pi \cdot 1^2$$

$$= \pi \left(1 + \frac{2}{y}\right)^2 - \pi$$

$$V = \int_2^6 \left(\pi \cdot \left(1+\frac{2}{y}\right)^2 - \pi\right) dy = \left[4(\ln 6 - \ln 2) - 4\cdot\left(\frac{1}{6}-\frac{1}{2}\right)\right]\pi$$

7-) $$\lim_{n\to\infty} \frac{1}{n^3}\sum_{k=1}^{n} k^2 \cdot \sin\left(\frac{k^3}{n^3}\right) \quad \text{---} \quad (*)$$ limiti hesaplayın.

Çözüm: Bu bir dizi-seri sorusu değil!



$$f\left(\frac{1}{n}\right)\cdot\frac{1}{n} + f\left(\frac{2}{n}\right)\cdot\frac{1}{n} + \dots + f\left(\frac{n-1}{n}\right)\cdot\frac{1}{n}$$

$$\frac{1}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \dots + f\left(\frac{n-1}{n}\right)\right]$$

$$\frac{1}{n}\sum_{k=1}^{n-1} f\left(\frac{k}{n}\right) \approx A$$

$$\Rightarrow \boxed{A = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n-1} f\left(\frac{k}{n}\right) = \int_0^1 f(x)\,dx} \quad \text{---} \quad (1)$$

$$(*) = \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2 \cdot \sin\left(\left(\frac{k}{n}\right)^3\right)\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\cdot\frac{1}{n}$$ öyle ki $f(x) := x^2 \cdot \sin(x^3)$.

$$\overset{(1)'\text{den}}{=} \int_0^1 f(x)\,dx$$

$$= \int_0^1 x^2 \cdot \sin x^3 \, dx$$, $x^3 = u \Rightarrow 3x^2 \cdot dx = du$

$$= \frac{1}{3}\int_0^1 \sin u \cdot du = \frac{1}{3}\left[-\cos u \Big|_0^1\right] = \frac{1}{3}\left(-\cos 1 - \underbrace{-\cos 0}\right)$$

$$= \frac{1}{3} - \frac{1}{3}\cos 1$$

8-) $\displaystyle \lim_{x\to 0} \frac{\int_0^{\sin 2x} \cos(5t)\cdot dt}{\sin x} = I$ limitini hesaplayın.

Çözüm: $\left(\frac{0}{0}\right)$ belirsizliği var dolayısıyla L'hopital teoremini uygulayalım.

$$I = \lim_{x\to 0} \frac{\overbrace{\cos(5\cdot\underbrace{\sin 2x}_{0})}^{1} \cdot \underbrace{\cos 2x}_{1} \cdot 2}{\underbrace{\cos x}_{1}}$$

$$= 2$$

$\int_0^{u(x)} f(t)\,dt = F(u(x))$,

$F'(u(x)) \cdot u'(x) = f(x)$

9-) $\displaystyle \int_0^{2\sqrt{2}} \frac{x^3}{\sqrt{16-x^2}} \cdot dx = I$

Çözüm: $x = 4\sin\theta \Rightarrow dx = 4\cdot\cos\theta\cdot d\theta$, $\sqrt{16-16\sin^2\theta} = \sqrt{16\cos^2\theta} = 4\cos\theta$

$0 = 4\sin\theta$, $2\sqrt{2} = 4\sin\theta$

$$I = \int_0^{\pi/4} \frac{4^3\cdot\sin^3\theta}{\cancel{4\cos\theta}} \cdot \cancel{4\cos\theta}\cdot d\theta = \int_0^{\pi/3} 4^3\cdot\sin^3\theta\cdot d\theta = \int_0^{\pi/3} 4^3\cdot \underbrace{\sin^2 x}_{(1-\cos^2 x)}\cdot\sin x\cdot dx$$