1. $\int x(\ln x)^3\,dx$ belirsiz integralini bulunuz.

$u=(\ln x)^3$, $x\,dx=dv$

$du=3(\ln x)^2\cdot\frac{1}{x}\,dx$, $\frac{x^2}{2}=v$

$$\int x(\ln x)^3dx = \frac{x^2}{2}(\ln x)^3-\int \frac{x^2}{2}\,3(\ln x)^2\frac{1}{x}\,dx$$

$$= \frac{x^2}{2}(\ln x)^3-\frac{3}{2}\int x(\ln x)^2\,dx$$

$u=(\ln x)^2$, $x\,dx=dv$

$du=2(\ln x)\cdot\frac{1}{x}$, $\frac{x^2}{2}=v$

$$I=\frac{x^2}{2}(\ln x)^3-\frac{3}{2}\left(\frac{x^2}{2}(\ln x)^2-\int\frac{x^2}{2}\cdot 2\ln x\,\frac{1}{x}\,dx\right)$$

$$I=\frac{x^2}{2}(\ln x)^3-\frac{3}{4}x^2(\ln x)^2-\frac{3}{2}\int x\ln x\,dx$$

$u=\ln x$, $x\,dx=dv$

$du=\frac{1}{x}\,dx$, $\frac{x^2}{2}=v$

$$I=\frac{x^2}{2}(\ln x)^3-\frac{3}{4}x^2(\ln x)^2-\frac{3}{2}\left(\frac{x^2}{2}\ln x-\int\frac{x^2}{2}\,\frac{1}{x}\,dx\right)$$

$$I=\frac{1}{2}(\ln x)^3x^2-\frac{3}{4}(\ln x)^2x^2-\frac{3}{4}(\ln x)x^2-\frac{3}{8}x^2+C$$

2. $\int x^3 e^{x^2} dx$ belirsiz integralini hesaplayınız.

$$x^2 = y \Rightarrow 2x\,dx = dy \Rightarrow x\,dx = \frac{dy}{2}$$

$$\int x^3 e^{x^2} dx = \int x^2 e^{x^2} x\,dx = \int y e^y \frac{1}{2} dy = \frac{1}{2}\int y e^y dy$$

$$y = u, \quad e^y dy = dv$$

$$dy = du, \quad e^y = v$$

$$I = \frac{1}{2}\left(y e^y - \int e^y dy \right) = \frac{1}{2} y e^y - \frac{1}{2} e^y + C$$

$$I = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C$$

3. $\int \frac{\ln(\ln x)}{2x} dx$ belirsiz integralini hesaplayınız.

$$\ln x = y \Rightarrow \frac{1}{x} dx = dy$$

$$I = \int \frac{1}{2} \ln y\, dy = \frac{1}{2}\int \ln y\, dy$$

$$\ln y = u, \; dy = dv \Rightarrow \frac{1}{y} dy = du, \; y = v$$

$$I = \frac{1}{2}\left(y \ln y - \int y \frac{1}{y} dy \right) = \frac{1}{2} y \ln y - y + C$$

$$I = \frac{1}{2}\left(\ln x \cdot \ln(\ln x) - \ln x\right) + C$$

4. $\int_1^e \sin(\ln x)\,dx$ belirli integralini hesaplayınız.

$\ln x = y \Rightarrow \frac{1}{x}\,dx = dy \quad , \quad dx = x\,dy = e^y\,dy$

$$\int_1^e \sin(\ln x)\,dx = \int_0^1 \sin(y)\cdot e^y\,dy$$

$u = e^y \quad , \quad \sin y\,dy = dv$

$du = e^y\,dy \quad , \quad -\cos y = v$

$$I = -e^y \cos y + \int \cos y\, e^y\,dy$$

$u = e^y \quad , \quad \cos y\,dy = dv$

$du = e^y\,dy \quad , \quad \sin y = v$

$$I = -e^y \cos y + \left(e^y \sin y - \int \sin y\, e^y\,dy \right)$$

$$\int e^y \sin y\,dy = -e^y \cos y + e^y \sin y - \int e^y \sin y\,dy$$

$$\int_0^1 e^y \sin y\,dy = \frac{1}{2} e^y (\sin y - \cos y)\Big|_0^1$$

$$= \frac{1}{2} e\,(\sin 1 - \cos 1) + \frac{1}{2} e$$

5. $\int \dfrac{1+\cot x}{1+e^{-x}\csc x}\,dx$ belirsiz integralini bulunuz.

$$\frac{1+\cot x}{1+e^{-x}\csc x} = 1+\frac{\cos x}{\sin x} \Big/ 1+\frac{1}{e^x}\cdot\frac{1}{\sin x}$$

$$= \frac{\sin x+\cos x}{\sin x}\cdot\frac{e^x\sin x}{1+e^x\sin x}$$

$$= \frac{e^x\sin x + e^x\cos x}{1+e^x\sin x}$$

$$I = \int \frac{\overbrace{e^x\sin x + e^x\cos x}^{u'}}{\underbrace{1+e^x\sin x}_{u}} = \int \frac{u'}{u}\,du = \ln|u| + C$$

$$I = \ln|1+e^x\sin x| + C$$

6. $\int \sqrt{\frac{x^2+1}{x^8}}\, dx = ?$ $(x>0)$

$$I = \int \sqrt{\frac{x^2+1}{x^8}}\, dx = \int \frac{1}{x^3} \sqrt{1+\frac{1}{x^2}}\, dx$$

$$u = 1+\frac{1}{x^2} = 1+x^{-2}$$

$$du = -2x^{-3}\, dx$$

$$I = -\frac{1}{2}\int \sqrt{u}\, du = -\frac{1}{2}\cdot\frac{2}{3}\cdot u^{3/2} + C$$

$$= -\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2} + C$$

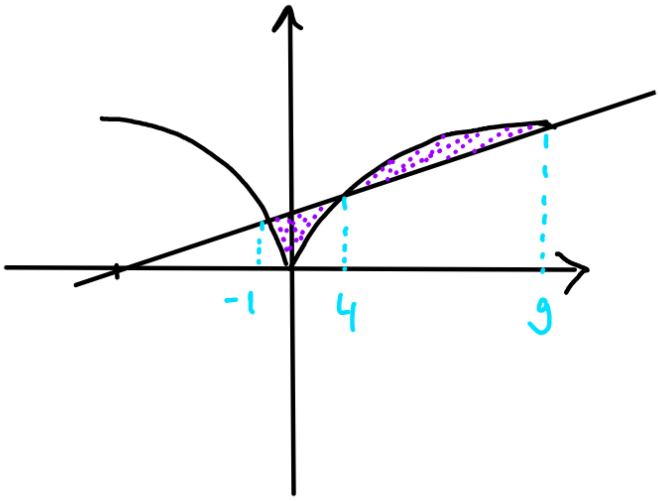
7. $\int x\, e^{x^3}(3x^3+2)\, dx$

$$u = x^2 e^{x^3} \Rightarrow du = \left(2x\, e^{x^3} + e^{x^3} 3x^2 . x^2\right) dx$$

$$du = x e^{x^3} . (3x^3+2)\, dx$$

$$I = \int du = u + C = x^2 e^{x^3} + C$$

8. $y=\sqrt{|x|}$ ve $5y = x+6$ ile sınırlı bölgenin alanını bulunuz.



$y=\sqrt{|x|}$, $5y = x + 6$

$x \geq 0 \Rightarrow 5\sqrt{x} = x+6 \Rightarrow 25x = x^2 + 12x + 36$

$x^2 - 13x + 36 = 0$

$x = 4$, $x = 9$

$x \leq 0 \Rightarrow 5\sqrt{-x} = x+6 \Rightarrow -25x = x^2 + 12x + 36$

$x^2 + 37x + 36 = 0$

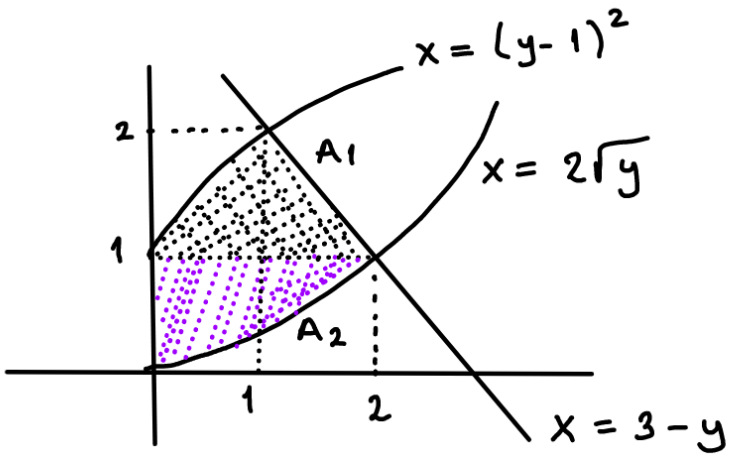
$x = -1$

$$A = \int_{-1}^{0} \left(\frac{x}{5} + \frac{6}{5} - \sqrt{-x} \right) dx + \int_{0}^{4} \left(\frac{x}{5} + \frac{6}{5} - \sqrt{x} \right) dx$$

$$+ \int_{4}^{9} \left(\sqrt{x} - \frac{x}{5} - \frac{6}{5} \right) dx$$

$A = 5|3$

9. Soldan y-ekseni, alttan $x = 2\sqrt{y}$, sol üstten $x = (y-1)^2$ ve sağ üstten $x = 3-y$ ile sınırlı bölgenin alanını bulunuz.



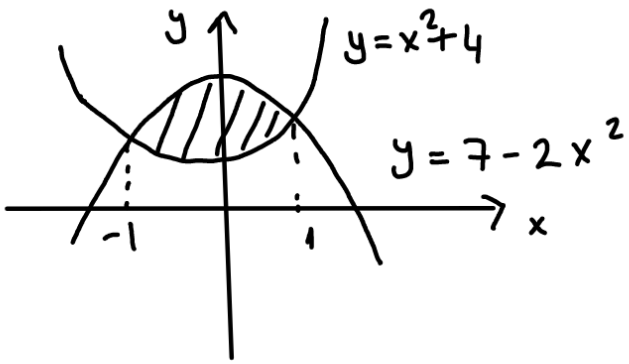
$$(y-1)^2 = 3-y \Rightarrow y^2 - y - 2 = 0 \Rightarrow y = 2,\ x = 1$$

$$2\sqrt{y} = 3-y \Rightarrow 4y = 9 - 6y + y^2 \Rightarrow y^2 - 10y + 9 = 0,\ y = 1,\ x = 2$$

$$A = A_1 + A_2 = \int_1^2 \left((3-y) - (y-1)^2\right) dy + \int_0^1 2\sqrt{y}\, dy = 5/2$$

$$A = \int_0^1 \left(\sqrt{x} + 1 - \frac{x^2}{4}\right) dx + \int_1^2 \left(3 - x - \frac{x^2}{4}\right) dx = \frac{5}{2}$$

10. $y = 7 - 2x^2$ ve $y = x^2 + 4$ eğrileri ile sınırlı bölgenin alanını bulunuz.



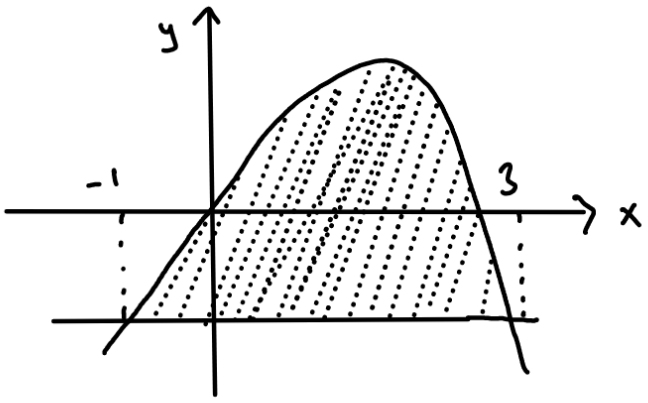
$x^2 + 4 = 7 - 2x^2$

$3x^2 = 3$, $x = \mp 1$

$$A = \int_{-1}^{1} (7 - 2x^2 - x^2 - 4)\, dx$$

$A = 4$

11. $y = 2x - x^2$ ve $y = -3$ ile sınırlı bölgenin alanını bulunuz.



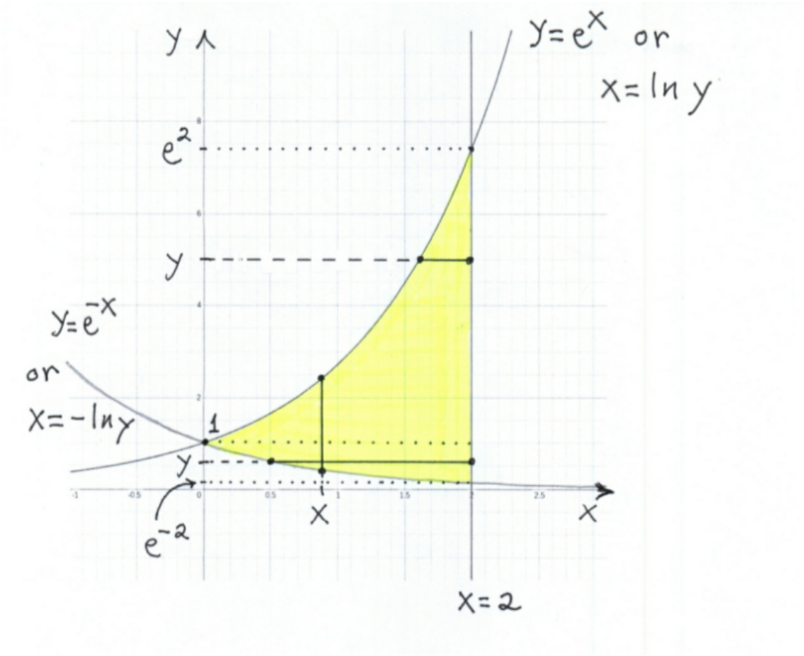
$-3 = 2x - x^2$

$x^2 - 2x - 3 = 0$

$x = 3$, $x = -1$

$$A = \int_{-1}^{3} (2x - x^2 - (-3))\, dx = 32/3$$

12. $y = e^x$, $y = e^{-x}$ ve $x = 2$ ile sınırlı bölgenin alanını bulunuz.

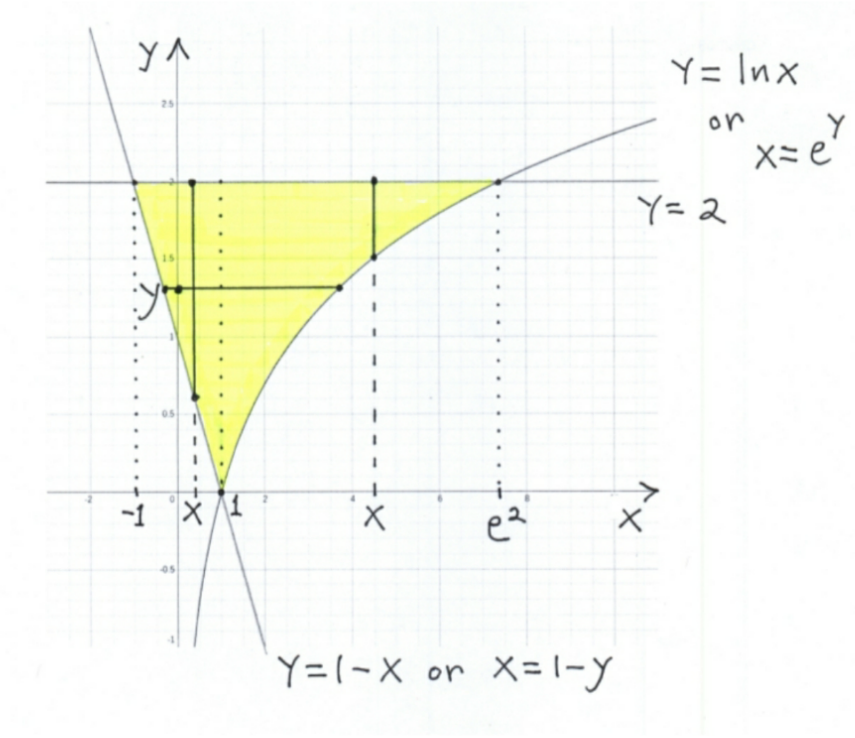


$e^x = e^{-x} \Rightarrow \quad x = -x, \quad x = 0 \quad (y = 1)$

$0 \le x \le 2$ ve $e^{-x} \le e^x$

$$A = \int_0^2 (e^x - e^{-x})\,dx = e^2 + e^{-2} - 2$$

13. $y = \ln x$, $y = 1 - x$ ve $y = 2$ ile sınırlı bölgenin alanını bulunuz. (x'e ve y'ye göre yazınız)



$$A = \int_{-1}^{1} (2 - (1 - x))\,dx + \int_{1}^{e^2} (2 - \ln x)\,dx = e^2 - 1$$

$$A = \int_{0}^{2} (e^y - (1 - y))\,dy = e^2 - 1$$

- ÖDEV -

Kısmi İntegrasyon

1–24 alıştırmalarındaki integralleri hesaplayın.

1. $\int x \sin \frac{x}{2}\, dx$
2. $\int \theta \cos \pi\theta\, d\theta$
3. $\int t^2 \cos t\, dt$
4. $\int x^2 \sin x\, dx$
5. $\int_1^2 x \ln x\, dx$
6. $\int_1^e x^3 \ln x\, dx$
7. $\int \tan^{-1} y\, dy$
8. $\int \sin^{-1} y\, dy$
9. $\int x \sec^2 x\, dx$
10. $\int 4x \sec^2 2x\, dx$
11. $\int x^3 e^x\, dx$
12. $\int p^4 e^{-p}\, dp$
13. $\int (x^2 - 5x)e^x\, dx$
14. $\int (r^2 + r + 1)e^r\, dr$
15. $\int x^5 e^x\, dx$
16. $\int t^2 e^{4t}\, dt$
17. $\int_0^{\pi/2} \theta^2 \sin 2\theta\, d\theta$
18. $\int_0^{\pi/2} x^3 \cos 2x\, dx$
19. $\int_{2/\sqrt{3}}^2 t \sec^{-1} t\, dt$
20. $\int_0^{1/\sqrt{2}} 2x \sin^{-1}(x^2)\, dx$
21. $\int e^\theta \sin\theta\, d\theta$
22. $\int e^{-y} \cos y\, dy$
23. $\int e^{2x} \cos 3x\, dx$
24. $\int e^{-2x} \sin 2x\, dx$

25-32 alıştırmalarında doğrular ve eğrilerle sınırlı bölgelerin alanlarını bulun.

25. $x = 2y^2, \quad x = 0, \quad$ ve $\quad y = 3$
26. $x = y^2 \quad$ ve $\quad x = y + 2$
27. $y^2 - 4x = 4 \quad$ ve $\quad 4x - y = 16$
28. $x - y^2 = 0 \quad$ ve $\quad x + 2y^2 = 3$
29. $x + y^2 = 0 \quad$ ve $\quad x + 3y^2 = 2$
30. $x - y^{2/3} = 0 \quad$ ve $\quad x + y^4 = 2$
31. $x = y^2 - 1 \quad$ ve $\quad x = |y|\sqrt{1 - y^2}$
32. $x = y^3 - y^2 \quad$ ve $\quad x = 2y$

33-40 alıştırmalarında doğrular ve eğrilerle sınırlı bölgelerin alanlarını bulun.

33. $y = 2 \sin x \quad$ ve $\quad y = \sin 2x, \quad 0 \le x \le \pi$
34. $y = 8 \cos x \quad$ ve $\quad y = \sec^2 x, \quad -\pi/3 \le x \le \pi/3$
35. $y = \cos(\pi x/2) \quad$ ve $\quad y = 1 - x^2$
36. $y = \sin(\pi x/2) \quad$ ve $\quad y = x$
37. $y = \sec^2 x, \quad y = \tan^2 x, \quad x = -\pi/4, \quad$ ve $\quad x = \pi/4$
38. $x = \tan^2 y \quad$ ve $\quad x = -\tan^2 y, \quad -\pi/4 \le y \le \pi/4$
39. $x = 3 \sin y \sqrt{\cos y} \quad$ ve $\quad x = 0, \quad 0 \le y \le \pi/2$
40. $y = \sec^2(\pi x/3) \quad$ ve $\quad y = x^{1/3}, \quad -1 \le x \le 1$