1-) $\int 4x \cdot \sec^2 2x \, dx = ?$

$2x = u$ olsun.

$\Rightarrow 2dx = du$ olur.

$\int u \cdot \sec^2 u \cdot du$ elde edilir.

$u = p$ ve $\sec^2 u \cdot du = dv$ olsun.

$\Rightarrow du = dp$ ve $\int \sec^2 u \cdot du = \int dv$

$\Rightarrow \tan u = v$ olur.

$\frac{\sin u}{\cos u}$

$\Rightarrow \int u \cdot \sec^2 u \, du = u \cdot \tan u - \int \tan u \cdot du$

$= u \cdot \tan u + \ln|\cos u|$

$= 2x \cdot \tan 2x + \ln|\cos 2x|$

kısmi integrasyon formülünden: **Hatırlatma**

$\int f \cdot g' \cdot dx = f \cdot g - \int f' \cdot g \cdot dx$

$f(x) = u$ ve $g'(x) \cdot dx = dv$

$\Rightarrow f'(x) \cdot dx = du$ ve $g(x) = v$

$\Rightarrow \int u \cdot dv = u \cdot v - \int v \cdot du$

2-) $\int \sin(\ln x) \, dx = ? = I$

$u = \ln x$ olsun. Böylece $du = \frac{1}{x} \cdot dx$

$x = e^u$

$\Rightarrow x \cdot du = dx$ olur.

$I = \int \sin(u) \cdot x \cdot du = \int \sin(u) \cdot e^u \cdot du$ olur.

$\sin(u) = p$ ve $e^u \cdot du = dv$

$\Rightarrow \cos(u) \cdot du = dp$ ve $e^u = v$

$\Rightarrow I = \sin(u) \cdot e^u - \int \cos(u) \cdot e^u \cdot du$

$\cos(u) = q$ ve $e^u \cdot du = w$

$\Rightarrow -\sin(u) \cdot du = dq$ $\longleftrightarrow$ $e^u = w$

$$\Rightarrow \cos u \cdot e^u - \int -\sin(u) \cdot e^u \cdot du$$

$$\Rightarrow \quad I = \sin(u)\, e^u - \cos u \cdot e^u - \underbrace{\int \sin u \cdot e^u \, du}_{I}$$

$$\Rightarrow \boxed{\cancel{2}I = \frac{\sin u \cdot e^u - \cos u \cdot e^u}{2}} = \frac{\sin(\ln x) \cdot e^{\ln x} - \cos(\ln x) \cdot e^{\ln x}}{2}$$

3.) $\int \ln(x+x^2)\,dx = ? = I$

$\ln(x+x^2) = u$ ve $dx = dv$ olsun.

$\Rightarrow \dfrac{1+2x}{x+x^2} \cdot dx = du$ ve $x = v$ elde edilir.

$$I = \ln(x+x^2) \cdot x - \int x \cdot \frac{1+2x}{x+x^2}\, dx$$

$$\frac{x+2x^2}{x+x^2} = \frac{x+x^2}{x+x^2} + \frac{x^2}{x+x^2}$$

$$= 1 + \frac{x^2}{\underbrace{x+x^2}_{x(1+x)}}$$

$$= 1 + \frac{x}{\underbrace{1+x}} - 1 + 1$$

$$= 2 - \frac{1}{1+x}$$

$$= \ln(x+x^2) \cdot x - \int 2 - \frac{1}{1+x}\, dx$$

$$= \ln(x+x^2) \cdot x - 2x + \ln|1+x| + C.$$

4-) $\forall n \in \mathbb{N}$ için

$$\int_0^1 (1-x^2)^n \cdot dx = \frac{2^{(2n)} \cdot (n!)^2}{(2n+1)!}$$ old. ispatlayınız.

ispat: $I_n = \int_0^1 (1-x^2)^n \cdot dx$ olsun.

$(1-x^2)^n = u$ ve $dx = dv$ olsun.

$\Rightarrow n \cdot (1-x^2)^{n-1} \cdot (-2x) \cdot dx = du$ ve $x = v$ olur.

$$\Rightarrow I_n = \underbrace{(1-x^2)^n \cdot x \Big|_0^1}_{0} - \int x \cdot n \cdot (1-x^2)^{n-1} \cdot (-2x) \cdot dx \quad \text{olur.}$$

$$= 2n \underbrace{\int x^2 \cdot (1-x^2)^{n-1} dx}_{} \quad \text{elde edilir.} \quad \ldots \quad (*)$$

$\Rightarrow Q$ diyelim.

(*) eşitliğinde şimdilik duralım.

$$I_n = \int_0^1 (1-x^2)^n \cdot dx = \int_0^1 (1-x^2) \cdot (1-x^2)^{n-1} \cdot dx$$

$$= \int_0^1 (1-x^2)^{n-1} dx - \underbrace{\int x^2 \cdot (1-x^2)^{n-1} \cdot dx}_{Q} \quad \text{olur.}$$

$$= I_{n-1} - Q$$

$\Rightarrow Q = I_{n-1} - I_n$ elde edilir. Bu eşitliği (*)'da kullanalım.

$$\Rightarrow I_n = 2n \cdot Q = 2n \cdot (I_{n-1} - I_n)$$

$$\Rightarrow I_n = 2n \cdot I_{n-1} - 2n I_n$$

$$\Rightarrow \underline{I_n} = \frac{2n}{\underline{1+2n}} \cdot \underbrace{I_{n-1}}$$ olur.

$$I_{n-1} = \frac{2(n-1)}{2(n-1)+1} \cdot I_{n-2} = \frac{2n-2}{2n-1} \cdot I_{n-2}$$

$$I_{n-2} = \frac{2(n-2)}{2(n-2)+1} \cdot I_{n-3} = \frac{2n-4}{2n-3} \cdot I_{n-3}$$

$\vdots$

I_1

$$\Rightarrow I_n = \frac{2n}{2n+1} \cdot \frac{2n-2}{2n-1} \cdot \frac{2n-4}{2n-3} \cdots I_1$$

$2/3$

$$\boxed{I_1 = \int_0^1 (1-x^2)^1 dx = 2/3}$$

$$= \frac{2 \cdot n \cdot 2 \cdot (n-1) \cdot 2 \cdot (n-2) \cdots 2 \cdot 1}{\underbrace{(2n+1)(2n-1) \cdot (2n-3) \cdots 3}_{P}}$$

$$= \frac{2^n \cdot n!}{(2n+1)(2n-1)\cdots 3} = \frac{2^n \cdot n!}{\frac{(2n+1)!}{2^n \cdot n!}} = \frac{2^{2n} \cdot [n!]^2}{(2n+1)!}$$

$P = (2n+1)(2n-1) \cdots 3$

$$P \cdot [\underbrace{2n \cdot (2n-2) \cdot (2n-4) \cdots}] = (2n+1) \cdot 2n \cdot (2n-1) \cdots 3 \cdot 2$$

↓ yukarda bulduk.

$$P \cdot 2^n \cdot n! = (2n+1)!$$

$$\Rightarrow P = \frac{(2n+1)!}{2^n \cdot n!}$$

5-) $\displaystyle\int_{1/2}^{1} \frac{y+4}{y^2+y}\,dy = ?$

$$\frac{y+4}{\underbrace{y^2+y}_{y\cdot(y+1)}} = \frac{A}{\underset{(y+1)}{y}} + \frac{B}{\underset{y}{y+1}}$$

$$\Rightarrow \frac{y+4}{y^2+y} = \frac{Ay+A+By}{y\cdot(y+1)} = \frac{y(A+B)+A}{y^2+y}$$

$$\Rightarrow \begin{cases} A+B=1 \\ A=4 \end{cases} \Rightarrow B=-3$$

$$\Rightarrow \int_{1/2}^{1} \frac{y+4}{y^2+y}\,dy = \int_{1/2}^{1} \frac{4}{y} - \frac{3}{y+1}$$

$$= 4\ln|y| - 3.\ln|y+1| \Big|_{1/2}^{1} = \text{ödev.}$$

6-) $\displaystyle\int \frac{x^2}{(x-1).\underbrace{(x^2+2x+1)}_{(x+1)^2}}\,dx = ? = I$

$$\frac{x^2}{(x-1)(x+1)^2} = \frac{A}{\underset{(x+1)^2 \to x^2+2x+1}{x-1}} + \frac{B}{\underset{(x+1)(x-1) \to x^2-1}{(x+1)}} + \frac{C}{\underset{(x-1)}{(x+1)^2}} = \frac{Ax^2+2Ax+A+Bx^2-B+Cx-C}{(x-1).(x+1)^2}$$

$$= \frac{x^2(A+B)+x(2A+C)+A-B-C}{(x-1)(x+1)^2}$$

$$\Rightarrow \begin{cases} A+B=1 & \ldots (1) \\ 2A+C=0 & \ldots (2) \\ A-B-C=0 & \ldots (3) \end{cases}$$

(1) ve (3)'ü toplayalım.

$\Rightarrow\ 2A-C=1$ olur. Bunu

(2) ile toplayalım.

$\Rightarrow\ 4A=1 \Rightarrow A=1/4$

$\Rightarrow\ B=3/4\ ,\ C=-1/2$

$$I = \int \underbrace{\frac{1}{4} \cdot \frac{1}{x-1}} + \underbrace{\frac{3}{4} \cdot \frac{1}{x+1}} - \frac{1}{2} \cdot \frac{1}{(x+1)^2} \, dx \qquad \frac{1}{(x+1)^2} \rightarrow (x+1)^{-2}$$

$$= \frac{1}{4} \ln|x-1| + \frac{3}{4} \cdot \ln|x+1| - \frac{1}{2} \frac{(x+1)^{-1}}{-1} \qquad \square$$

7-) $\displaystyle\int \frac{2s+2}{(s^2+1)\cdot(s-1)^3} \, ds = ? = I$

$$\frac{2s+2}{(s^2+1)\cdot(s-1)^3} = \frac{Ax+B}{s^2+1} + \frac{C}{(s-1)} + \frac{D}{(s-1)^2} + \frac{E}{(s-1)^3}$$

$$(s-1)^3 \qquad (s-1)^2\cdot(s^2+1) \qquad (s-1)(s^2+1) \qquad (s^2+1)$$

işlemler ödev

$$\begin{cases} A + C = 0 \\ -3A + B - 2C + D = 0 \\ 3A - 3B + 2C - D + E = 0 \\ -A + 3B - 2C + D = 2 \\ -B + C - D + E = 2 \end{cases}$$

sistemi elde edilir.

ipucu

Tüm denklemler toplanırsa $E = 2$ bulunur.

(2) ve (3) denklemler toplanırsa B bulunur.

(3) ve (4) toplanırsa A ve C bulunur.

Çözüm ödev.

$$\left[\begin{matrix} A = 0, & C = 0 \\ D = -1, & B = 1 \end{matrix} \right]$$

8-) $\displaystyle\int_0^{\pi/2} 35\cdot \underline{\sin^4 x}\cdot \cos^3 x\, dx = ?$

$\cos^3 x = \cos^2 x \cdot \cos x$

$= (1-\sin^2 x)\cdot \cos x$

$$= 35\int_0^{\pi/2} \sin^4 x\cdot\cos x - \sin^6 x\cdot\cos x \quad dx = 35\left\{\int_0^{\pi/2} \sin^4 x\cdot \underbrace{\cos x\, dx}_{du} - \int_0^{\pi/2} \sin^6 x\cdot \underbrace{\cos x\, dx}_{du}\right\}$$

$u = \sin x$

$\Rightarrow \quad \Rightarrow \quad du = \cos x\cdot dx$

$$\Rightarrow = 35\left[\int_0^1 u^4\cdot du - \int_0^1 u^6\cdot du\right] = 35\left\{\frac{u^5}{5}\Big|_0^1 - \frac{u^7}{7}\Big|_0^1\right\}$$

$$= 35\left(\frac{1}{5} - 0 - \left\{\frac{1}{7} - 0\right\}\right)$$

$$= 35\left\{\frac{1}{5} - \frac{1}{7}\right\}$$

$$= 2$$

9-) $\displaystyle\int \frac{dy}{y\cdot\sqrt{1+(\ln y)^2}} = I$

$\sqrt{1+x^2}$, $x = \tan\theta$

$\ln y \to \tan\theta\ (?)$ $\qquad y = e^{\tan\theta}$ dön. yapalım

$dy = \sec^2\theta\cdot e^{\tan\theta}\cdot d\theta$

$\sqrt{1+(\ln y)^2}$

$= \sqrt{1+(\ln e^{\tan\theta})^2}$

$= \sqrt{1+\tan^2\theta}$

$= \sec\theta$

$$I = \int \frac{\sec^2\theta\cdot e^{\tan\theta}\cdot d\theta}{e^{\tan\theta}\cdot \sec\theta}$$

$$= \int \sec\theta\cdot d\theta$$