Determinantın Özellikleri ve Örnekler

1- A kare matris olsun. $\det(A) = \det(A^T)$ dir.

Ör $\begin{vmatrix}1&2\\3&4\end{vmatrix} = \begin{vmatrix}1&3\\2&4\end{vmatrix}$ Ör $\begin{vmatrix}1&2&3\\4&5&6\\7&8&9\end{vmatrix} = \begin{vmatrix}1&4&7\\2&5&8\\3&6&9\end{vmatrix}$

2- A kare matris olsun. A nın herhangi iki satırı değişirse determinant -1 ile çarpılmış olur.

Ör: $\begin{vmatrix}1&2\\3&4\end{vmatrix} = -\begin{vmatrix}3&4\\1&2\end{vmatrix}$ Ör $\begin{vmatrix}1&2&3\\4&5&6\\7&8&9\end{vmatrix} = -\begin{vmatrix}4&5&6\\1&2&3\\7&8&9\end{vmatrix}$

3- Eğer herhangi iki satır ya da sütun aynı ise determinant sıfırdır.

Ör $\begin{vmatrix}1&2&3\\4&5&6\\1&2&3\end{vmatrix} = 0$, Ör $\begin{vmatrix}1&1&4\\2&2&5\\3&3&6\end{vmatrix} = 0$

4- Eğer herhangi bir satır (sütun) sıfır ise determinant sıfıra eşittir.

Ör $\begin{vmatrix}1&2&3\\0&0&0\\4&5&6\end{vmatrix} = 0$, Ör $\begin{vmatrix}1&2&0\\3&4&0\\5&6&0\end{vmatrix} = 0$

5- Eğer herhangi bir satır bir c sayısı ile çarpılırsa matrisin determinantı c ile çarpılır.

Ör $\begin{vmatrix}1&2\\2&4\end{vmatrix} = 2\begin{vmatrix}1&2\\1&2\end{vmatrix} = 0$

Ör: $\begin{vmatrix}1&2&3\\4&5&6\\3&6&9\end{vmatrix} = 3\begin{vmatrix}1&2&3\\4&5&6\\1&2&3\end{vmatrix} = 3.0 = 0$

Ör :
$$\begin{vmatrix} x+1 & 2x+3 & 3x+5 \\ 3x+3 & 5x+7 & 7x+11 \\ 5x+6 & 8x+12 & 14x+24 \end{vmatrix} = 0 \text{ denklemini çözünüz.}$$

Çöz :
$$\begin{vmatrix} x+1 & 2x+3 & 3x+5 \\ 0 & -x-2 & -2x-4 \\ 1 & -2x-3 & -x-1 \end{vmatrix} = \begin{vmatrix} x+1 & 2x+3 & 3x+5 \\ 0 & -x-2 & -2x-4 \\ 1 & -3x-5 & -3x-5 \end{vmatrix}$$

$$= (-x-2)(-3x-5) \begin{vmatrix} x+1 & 2x+3 & 3x+5 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= (x+2)(3x+5) \begin{vmatrix} x+1 & 2x+3 & 3x+5 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{vmatrix} = (x+2)(3x+5)(-x-1)$$

$$= 0 \Rightarrow x = -2,\ x = -\frac{5}{3},\ x = -1.$$

Ör : $A = \begin{bmatrix} 0 & \cos x & \sin x \\ \cos y & -\sin x \sin y & \cos x \sin y \\ \sin y & \sin x \cos y & -\cos x \cos y \end{bmatrix}$ veriliyor, $\det(A) = ?$

Çöz : $|A| = -\cos x \begin{vmatrix} \cos y & \cos x \sin y \\ \sin y & -\cos x \cos y \end{vmatrix} + \sin x \begin{vmatrix} \cos y & -\sin x \sin y \\ \sin y & \sin x \cos y \end{vmatrix}$

$$= -\cos x\left(-\cos^2 y \cos x - \cos x \sin^2 y\right) + \sin x\left(\sin x \cos^2 y + \sin x \sin^2 y\right)$$

$$= \cos^2 x\left(\cos^2 y + \sin^2 y\right) + \sin^2 x\left(\cos^2 y + \sin^2 y\right) = \cos^2 x + \sin^2 x = 1.$$

SORU : $\begin{vmatrix} 2x-3 & 3x-5 & 4x-8 \\ 3x-5 & 5x-9 & 6x-12 \\ 4x-6 & 6x-10 & 9x-19 \end{vmatrix} = 0$ denklemini çözünüz. Ç: $x=1, x=2, x=3$

SORU : $\begin{vmatrix} 6x+1 & 3x+1 & 2x+1 \\ 9x+1 & 5x+1 & 3x+1 \\ 14x+4 & 7x+3 & 5x+3 \end{vmatrix} = 0$ " " Ç: $x=1, x=-1, x=0$

Ör: $\begin{vmatrix} x-2 & 3 \\ 4 & x+2 \end{vmatrix} = 0$ denklemini çözünüz.

Çöz: $\begin{vmatrix} x-2 & 3 \\ 4 & x-2 \end{vmatrix} = (x-2)(x+2) - 12 = x^2 - 4 - 12 = x^2 - 16 = 0 \Rightarrow x = \mp 4$

Ör $\begin{vmatrix} x+1 & x & x-4 \\ 2 & 1 & -4 \\ 3 & 5 & 1 \end{vmatrix} = 0$ denklemini çözünüz.

Çöz: Determinantı 1. satıra göre açalım.

$$\begin{vmatrix} x+1 & x & x-4 \\ 2 & 1 & -4 \\ 3 & 5 & 1 \end{vmatrix} = (x+1)\begin{vmatrix} 1 & -4 \\ 5 & 1 \end{vmatrix} - x\begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix} + (x-4)\begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix}$$

$$= (x+1)(1+20) - x(2+12) + (x-4)(10-3)$$

$$= 21(x+1) - 24x + 7(x-4) = 4x - 7 = 0 \Rightarrow x = \frac{7}{4}$$

Ör: Determinantın özelliklerini kullanarak

$$\begin{vmatrix} 1 & yz & yz(y+z) \\ 1 & xz & xz(x+z) \\ 1 & xy & xy(x+y) \end{vmatrix}$$ determinantını bulunuz.

Çöz: $$\begin{vmatrix} 1 & yz & yz(y+z) \\ 1 & xz & xz(x+z) \\ 1 & xy & xy(x+y) \end{vmatrix} = \begin{vmatrix} 1 & yz & y^2z+yz^2 \\ 0 & xz-yz & x^2z+xz^2-y^2z-yz^2 \\ 0 & xy-yz & x^2y+xy^2-y^2z-yz^2 \end{vmatrix}$$

$$= zy\begin{vmatrix} 1 & yz & y^2z+yz^2 \\ 0 & x-y & x^2+xz-y^2-yz \\ 0 & x-z & xy+x^2-yz-z^2 \end{vmatrix} = yz\begin{vmatrix} 1 & yz & y^2z+yz^2 \\ 0 & x-y & x^2+xz-y^2-yz \\ 0 & y-z & xy-z^2+y^2-xz \end{vmatrix}$$

$$= yz(y-z)\begin{vmatrix} 1 & yz & y^2z+yz^2 \\ 0 & x-y & x^2-y^2+z(x-y) \\ 0 & 1 & x+y+z \end{vmatrix} \; yz(y-z)(x-y)\begin{vmatrix} 1 & yz & y^2z+yz^2 \\ 0 & 1 & x+y+z \\ 0 & 1 & x+y+z \end{vmatrix}$$

$$= yz(y-z)(x-y)\cdot 0 = 0$$

Ör: $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 8 & 9 \end{bmatrix}$ veriliyor

a) $|A| = ?$, b) $ek(A) = ?$, c) $A^{-1} = ?$

Çöz: a) $|A| = 1 \cdot \begin{vmatrix} 3 & 4 \\ 8 & 9 \end{vmatrix} - 1 \begin{vmatrix} 2 & 4 \\ 5 & 9 \end{vmatrix} + 1 \begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix}$

$= (27-32) - (18-20) + (16-15) = -5+2+1 = -2$

b) $ek(A) = \begin{bmatrix} \begin{vmatrix} 3 & 4 \\ 8 & 9 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 5 & 9 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 8 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 5 & 9 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 5 & 8 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} -5 & 2 & 1 \\ -1 & 4 & -3 \\ 1 & -2 & 1 \end{bmatrix}^T$

$= \begin{bmatrix} -5 & -1 & 1 \\ 2 & 4 & -2 \\ 1 & -3 & 1 \end{bmatrix}$

c) $A^{-1} = \dfrac{ek(A)}{|A|} = \begin{bmatrix} \frac{5}{2} & \frac{1}{2} & -\frac{1}{2} \\ -1 & -2 & 1 \\ -1/2 & 3/2 & -1/2 \end{bmatrix}$

Ör
$2x + 3y - z = 1$
$3x + 5y + 2z = 8$
$x - 2y - 3z = -1$
denklemini çözün

Çöz: $|A| = \begin{vmatrix} 2 & 3 & -1 \\ 3 & 5 & 2 \\ 1 & -2 & -3 \end{vmatrix} = 22$

$|A_1| = \begin{vmatrix} 1 & 3 & -1 \\ 8 & 5 & 2 \\ -1 & -2 & -3 \end{vmatrix} = 66$, $|A_2| = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 8 & 2 \\ 1 & -1 & 3 \end{vmatrix} = -22$

$|A_3| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 5 & 8 \\ 1 & -2 & -1 \end{vmatrix} = 44$, $x = \frac{66}{22} = 3$, $y = \frac{-22}{22} = -1$, $z = \frac{44}{22} = 2$

Ör $\begin{vmatrix} 12 & 4 & 8 \\ 3 & 4 & 5 \\ 6 & 10 & 11 \end{vmatrix} = 4 \begin{vmatrix} 3 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 10 & 11 \end{vmatrix} = 3 \cdot 4 \begin{vmatrix} 1 & 1 & 2 \\ 1 & 4 & 5 \\ 2 & 10 & 11 \end{vmatrix}$

6 - A kare matris olsun. A nın herhangi satırını (sütun) c ile çarpıp başka satıra (sütun) eklenirse determinant değişmez.

Ör: $\begin{vmatrix} 1 & 2 & 3 \\ 2 & -2 & 4 \\ 3 & 1 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -6 & -2 \\ 0 & -5 & -3 \end{vmatrix} = -2 \begin{vmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & -5 & -3 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 5 & 3 \end{vmatrix}$

$= 2 \cdot 5 \cdot \begin{vmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 1 & 3/5 \end{vmatrix} = 10 \begin{vmatrix} 3 & 1 \\ 1 & 3/5 \end{vmatrix} = 10 \cdot (\frac{9}{5} - 1) = 10 \cdot \frac{4}{5} = 8$

Ör $\begin{vmatrix} 4 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & 4 & 6 \end{vmatrix} \underset{\frac{1}{2}r_3 \to r_3}{=} 2 \begin{vmatrix} 4 & 3 & 2 \\ 3 & -2 & 5 \\ 1 & 2 & 3 \end{vmatrix} \underset{r_1 \leftrightarrow r_3}{=} -2 \begin{vmatrix} 1 & 2 & 3 \\ 3 & -2 & 5 \\ 4 & 3 & 2 \end{vmatrix}$

$\underset{\substack{-3r_1+r_2 \\ \to r_2}}{=} -2 \begin{vmatrix} 1 & 2 & 3 \\ 0 & -8 & -4 \\ 4 & 3 & 2 \end{vmatrix} \underset{-4r_1+r_3 \to r_3}{=} -2 \begin{vmatrix} 1 & 2 & 3 \\ 0 & -8 & -4 \\ 0 & -5 & -10 \end{vmatrix} \underset{\frac{1}{4}r_2 \to r_2}{=} (-2) \cdot 4 \begin{vmatrix} 1 & 2 & 3 \\ 0 & -2 & -1 \\ 0 & -5 & -10 \end{vmatrix}$

$\underset{\frac{1}{5}r_3 \to r_3}{=} (-2)\,4 \cdot 5 \begin{vmatrix} 1 & 2 & 3 \\ 0 & -2 & -1 \\ 0 & -1 & -2 \end{vmatrix} \underset{-\frac{1}{2}r_2+r_3 \to r_3}{=} (-2)\,4 \cdot 5 \begin{vmatrix} 1 & 2 & 3 \\ 0 & -2 & -1 \\ 0 & 0 & -\frac{3}{2} \end{vmatrix}$

$= (-2) \cdot 4 \cdot 5 \cdot 1 \cdot (-2) \cdot (-\frac{3}{2}) = -120$

7 - iki matrisin çarpımının determinantı determinantları çarpımıdır

$\det(AB) = \det(A) \cdot \det(B)$

Ör $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, $|A| = -2$, $|B| = 5$

$AB = \begin{bmatrix} 4 & 3 \\ 10 & 5 \end{bmatrix}$, $|AB| = -10 = (-2) \cdot 5 = |A| \cdot |B|$