

Problemler 1

$$1) \begin{array}{l} x + 2y = 8 \\ 3x - 4y = 4 \end{array}$$

denklemlerini çözünүү.

$$\text{Çözü}: \begin{array}{l} x + 2y = 8 \\ 3x - 4y = 4 \end{array} \Rightarrow \begin{array}{l} 2x + 4y = 16 \\ 3x - 4y = 4 \end{array} \Rightarrow \begin{array}{l} 2x + 4y = 16 \\ 5x = 20 \end{array} \Rightarrow \begin{array}{l} x = 4 \\ x + 2y = 8 \end{array} \Rightarrow \begin{array}{l} y = 2 \\ x = 4 \end{array}$$

$$2) \begin{array}{l} 2x - 3y + 4z = -12 \\ x - 2y + z = -5 \\ 3x + y + 2z = 1 \end{array} \quad \begin{array}{l} x - 2y + z = -5 \\ 2x - 3y + 4z = -12 \\ 3x + y + 2z = 1 \end{array} \Rightarrow \begin{array}{l} x - 2y + z = -5 \\ y + 2z = -2 \\ 7y - 2 = 11 \end{array}$$

$$\Rightarrow \begin{array}{l} x - 2y + z = -5 \\ y + 2z = -2 \\ -15z = 30 \end{array} \quad \begin{array}{l} x - 2y + z = -5 \\ y + 2z = -2 \\ z = -2 \end{array} \quad \begin{array}{l} x - 2y + z = -5 \\ y = 2 \\ z = -2 \end{array} \Rightarrow \begin{array}{l} x = 4 \\ y = 2 \\ z = -2 \end{array}$$

$$3) \begin{array}{l} 3x + 2y + z = 2 \\ 4x + 2y + 2z = 8 \\ x - y + z = 4 \end{array} \quad 4) \begin{array}{l} x + y = 5 \\ 3x + 3y = 10 \end{array} \quad 5) \begin{array}{l} 2x + 4y + 6z = -12 \\ 2x - 3y - 4z = 15 \\ 3x + 4y + 5z = -8 \end{array}$$

$$6) \begin{array}{l} x + y - 2z = 5 \\ 2x + 3y + 4z = 2 \end{array}$$

denklemlerini çözünүү

$$7) \begin{array}{l} 2x - y = 5 \\ 4x - 2y = t \end{array}$$

denklemi veriliyor

- a) t nin hangi degeri icin denklemin çözümü varsa?
- " " " " " yoksa?

- b) t " " " " " forki degerini bulay.
- c) Denklemin çözümü olmasi icin t nin forki degerini bulay.

$$\text{Çözü}: \begin{array}{l} 2x - y = 5 \\ 4x - 2y = t \end{array} \Rightarrow \begin{array}{l} 2x - y = 5 \\ 0x + 0y = t - 10 \end{array}$$

- a) $t - 10 = 0 \Rightarrow t = 10$ icin denklemin sonsuz adet çözüm var.

- b) $t = 0$ alruk $0 = -10$ olur ki anlaý. çözüm yok

- c) $t \neq 10$ icin çözüm yoxdur.

8 - Homojen denklemler sisteminin her zaman çözümü var mıdır?

Sözlük: Evet $x_1=0, \dots, x_n=0$ daima bir çözümüdür yani homojen sistem çözümüne deðiktir.

9)

$$2x + 3y - z = 11$$

$$x - y + 2z = -7$$

$$4x + y - 2z = 12 \quad \text{veriliyor } x=1, y=2, z=r \text{ çözüm}$$

olması için r ne olmalıdır.

Sözlük: $2+6-r=11 \Rightarrow r=-3$. 3. denkleminde şöplor (görün)

o halde $x=1, y=2, z=-3$ çözümüdür

$$\begin{array}{l} 10) \quad \begin{array}{rcl} 3x & -2z & = 4 \\ x & -4y + z & = -5 \\ -2x & +3y + 2z & = 9 \end{array} \quad , \quad x=r, y=2, z=t \quad \text{aynı form} \end{array}$$

$$\underline{\text{Sözlük}}: 3r - 2t = 4 \Rightarrow r = \frac{4+t}{3} = 2$$

$$\begin{array}{l} 2-8+1=-5 \\ -4+6+2=4 \end{array} \quad \begin{array}{l} 3. \text{ denklemi şöplaşıy} \\ \text{o halde çözüm 2 buluy} \end{array}$$

$$\text{II}) \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ -2 & 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix} \quad \text{matrisleri veriliyor.}$$

Aşağıdakiler (tanımlı olmasa da) heraplayınız.

$$C+E, \quad AB, \quad BA, \quad 2C-3E, \quad CB+D, \quad AB+D^2, \quad DA+B, \quad EC,$$

$$C\bar{E}, \quad EB+\bar{F}, \quad F^T, \quad FD-3B, \quad AB-2D, \quad 3.(2A), \quad 6A$$

$$2F-3(A-E), \quad B^T+A\bar{E}, \quad A(BD), \quad (AB)D, \quad A(C+E)$$

$$AC+A\bar{E}, \quad A(C-3E), \quad A^T, \quad (AB)^T, \quad D^T A^T, \quad (C+E)^T, \quad A(2B)$$

$$\underline{\text{Sözlük}}: C+E = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}, \quad F^T C_{3 \times 3} \text{ tanımlı değil.}$$

Digerlerini sjy yapın.

12) $A = [r \ 1 \ 2]$, $B = [1 \ 3 \ -1]$ vérilecektir. $AB^T = 0$ olsun
 r ye belieğim

$$\text{Sö} : AB = [r \ 1 \ 2] \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = [0] \Rightarrow r + 3 - 2 = 0 \Rightarrow r = -1$$

$$\begin{aligned} 13) \quad & 2x + 3y - 3z + w + t = 7 \\ & 3x \quad + 2z \quad + 3t = -2 \\ & 2x + 3y \quad - 4w \quad = 3 \\ & z + w + t = 5 \quad \text{vérilecektir} \end{aligned}$$

- a) Katsayılar matrisini yapın.
- b) Lineer sistemi matris formunda yapın.
- c) Genişletilmiş katsayı matrisini yapın.

$$\text{Sö a)} \quad A = \begin{bmatrix} 2 & 3 & -3 & 1 & 1 \\ 3 & 0 & 2 & 0 & 3 \\ 2 & 3 & 0 & -4 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\text{b)} \quad \left\{ \begin{array}{l} \left[\begin{array}{ccccc|c} 2 & 3 & -3 & 1 & 1 & 7 \\ 3 & 0 & 2 & 0 & 3 & -2 \\ 2 & 3 & 0 & -4 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 & 5 \end{array} \right] \\ \left[\begin{array}{c} x \\ y \\ z \\ w \\ t \end{array} \right] = \left[\begin{array}{c} 7 \\ -2 \\ 3 \\ 5 \end{array} \right] \end{array} \right. , \quad A \cdot X = B$$

$$\text{c)} \quad \left[\begin{array}{ccccc|c} 2 & 3 & -3 & 1 & 1 & 7 \\ 3 & 0 & 2 & 0 & 3 & -2 \\ 2 & 3 & 0 & -4 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 & 5 \end{array} \right]$$

$$\text{d)} \quad \left[\begin{array}{cc|c} a+b & c+d & 4 \\ c-d & a-b & 6 \end{array} \right] = \left[\begin{array}{cc} 4 & 6 \\ 10 & 2 \end{array} \right] \quad \text{ne} \quad a=1, b=1, c=1, d=1$$

$$\text{Sö:} \quad \begin{aligned} a+b &= 4 & 2a &= 6 \Rightarrow a=3 \\ a-b &= 2 \Rightarrow & 2c &= 16 \quad c=8 \\ c+d &= 6 & b &= 1 \\ c-d &= 10 & d &= -2 \end{aligned}$$

$$15) \text{ a)} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + y \begin{pmatrix} 2 \\ 5 \end{pmatrix} + z \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{b)} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + y \begin{pmatrix} 2 \\ 5 \end{pmatrix} + z \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (\text{lineer sistemi})$$

matrix formunda yazın.

$$\text{a)} \begin{pmatrix} x \\ 2x \end{pmatrix} + \begin{pmatrix} 2y \\ 5y \end{pmatrix} + \begin{pmatrix} 0 \\ 3z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x+2y \\ 2x+5y+3z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{b)} \begin{pmatrix} x \\ x+2y \\ x+2z \\ 2x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$
$$\begin{pmatrix} x \\ x+2y \\ x+2z \\ 2x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

16) $A\mathbf{x} = 0$ homogen denklemler sistemi için \mathbf{x}, \mathbf{Y}

ile çözümleri olun.

a) $\mathbf{x} + \mathbf{y}$, b) $\mathbf{x} - \mathbf{y}$, c) $r \in \mathbb{R}, r\mathbf{x}$, d) $r, s \in \mathbb{R} r\mathbf{x} + s\mathbf{y}$

de çözümleri gösterin.

SöL: d) $A\mathbf{x} = 0, A\mathbf{y} = 0$ olsun. $A(r\mathbf{x} + s\mathbf{y})$

$$= rA\mathbf{x} + sA\mathbf{y} = r \cdot 0 + s \cdot 0 = 0. \quad \text{o halde } r\mathbf{x} + s\mathbf{y}$$

cözümlür.

a) $r = s = 1$ al, b) $r = 1, s = -1$ al, c) $r = s = 0$ al.

17) $A\mathbf{x} = \mathbf{B}$ denklemler sisteminin birden fazla çözüm varsa

sönsüz çözüm vardır gösterin.

SöL: \mathbf{x} ve \mathbf{y} çözüm olun. $A\mathbf{x} = \mathbf{B}, A\mathbf{y} = \mathbf{B}$ olsun.

SöL: \mathbf{x} ve \mathbf{y} çözüm olun. $A\mathbf{x} = \mathbf{B}, A\mathbf{y} = \mathbf{B}$ olsun.

$r \in \mathbb{R}$ alalım. $\mathbf{z} = r\mathbf{x} + (1-r)\mathbf{y}$ in çözümüne bakalım.

$$A\mathbf{z} = A(r\mathbf{x} + (1-r)\mathbf{y}) = rA\mathbf{x} + (1-r)A\mathbf{y} = r\mathbf{B} + \mathbf{B} - r\mathbf{B} = \mathbf{B}$$

$A\mathbf{z} = \mathbf{B}$ olsup her $r \in \mathbb{R}$ için $r\mathbf{x} + (1-r)\mathbf{y}$ çözümü varır yani sönsüz çözüm varılır.