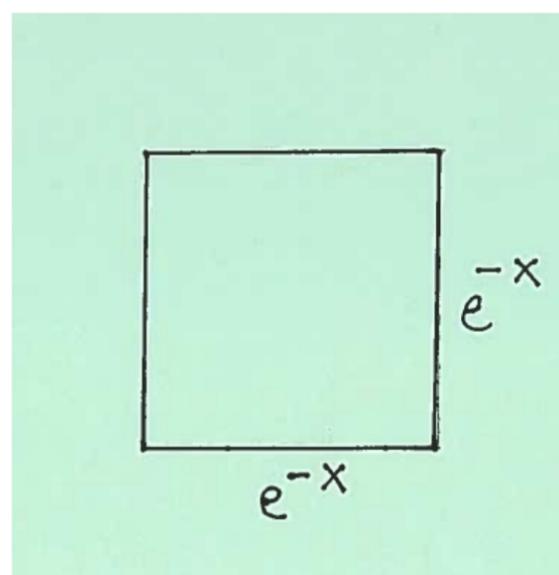
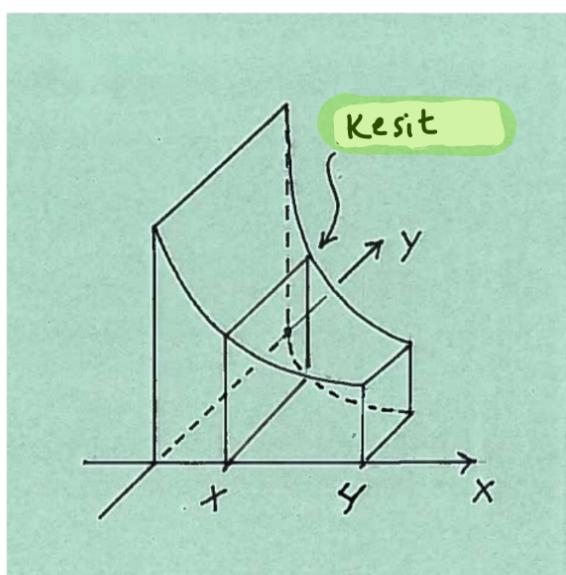
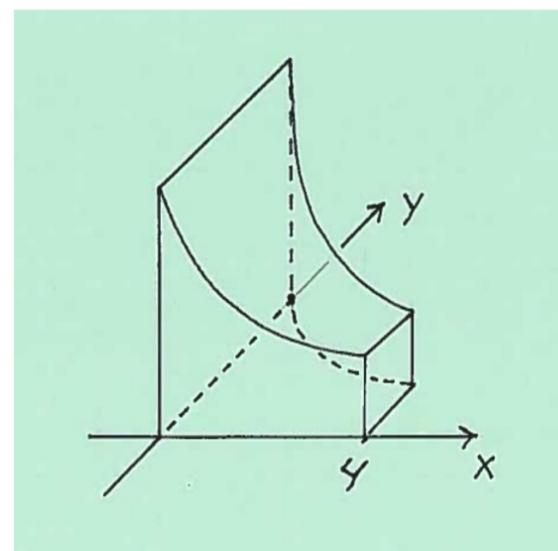
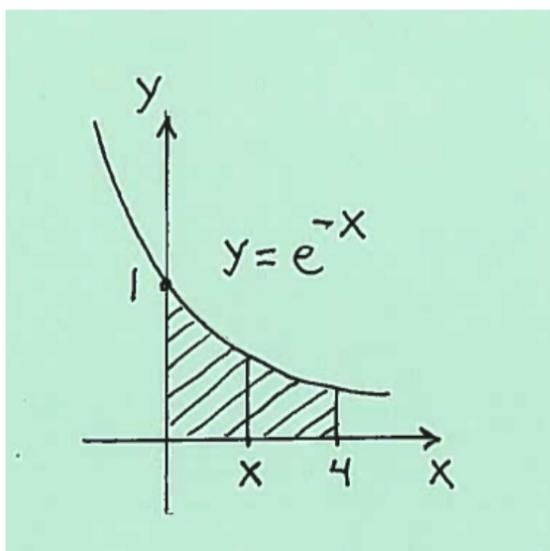


1. xy - düzleminde tabanı $y=0$, $y=e^{-x}$, $0 \leq x \leq 4$ ile sınırlı bölge olan katı cismin; x - eksene dik düzlemlerle kesiti kare olduğuna göre bu katı cismin hacmini bulunuz.

Gözüm.



$$A(x) = (e^{-x})^2 = e^{-2x}$$

$$\begin{aligned} V &= \int_0^4 A(x) dx = \int_0^4 e^{-2x} dx = -\frac{e^{-2x}}{2} \Big|_0^4 \\ &= -\frac{1}{2} (e^{-8} - 1) \end{aligned}$$

2. xy düzleminde tabanı x-ekseni, y-ekseni ve $x+2y=6$ doğrusu ile sınırlı bölge olan katı cismin x-eksenine dik düzlemlerle kesitleri

a. Kare

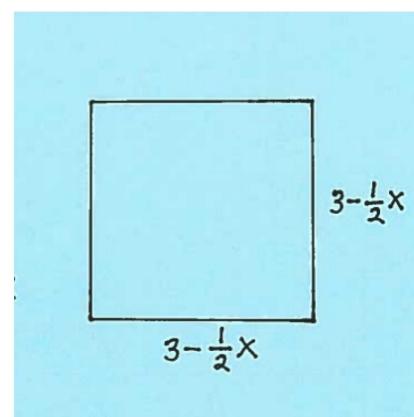
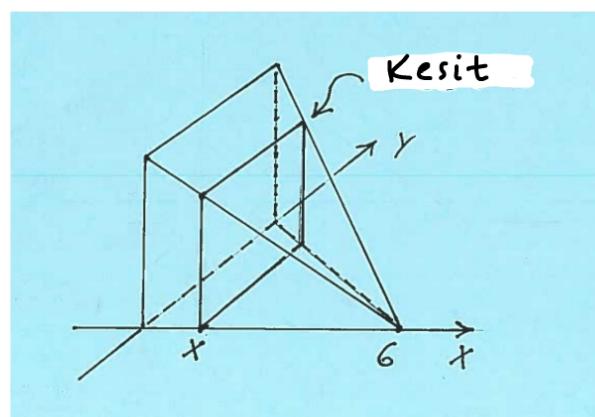
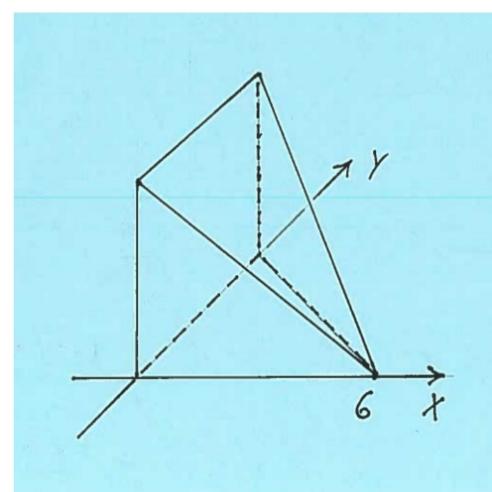
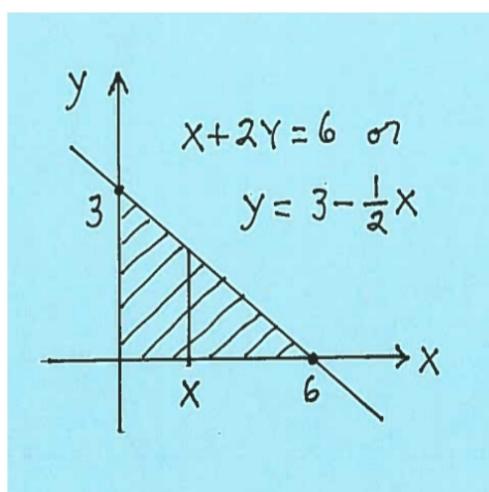
b. Yarım daire

c. Yüksekliği 5 olan dikdörtgen

ise bu katı cismin hacmini bulunuz.

Gözüm:

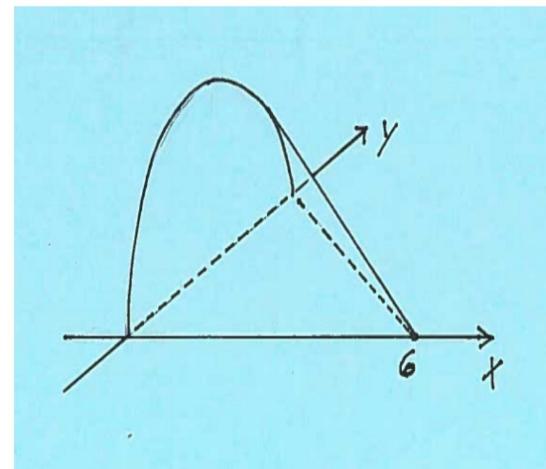
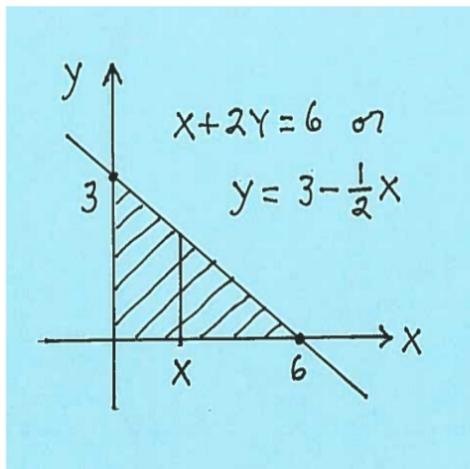
a.



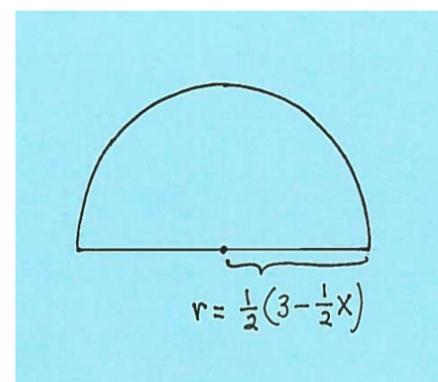
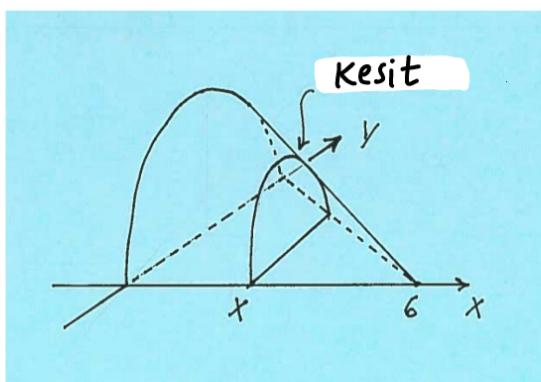
$$A(x) = \left(3 - \frac{1}{2}x\right)^2, \quad V = \int_0^6 A(x) dx$$

$$V = \int_0^6 \left(3 - \frac{1}{2}x\right)^2 dx$$

b.

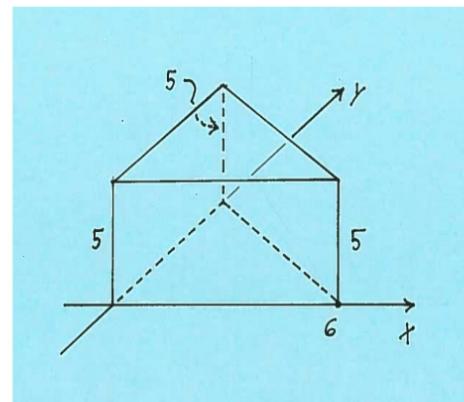
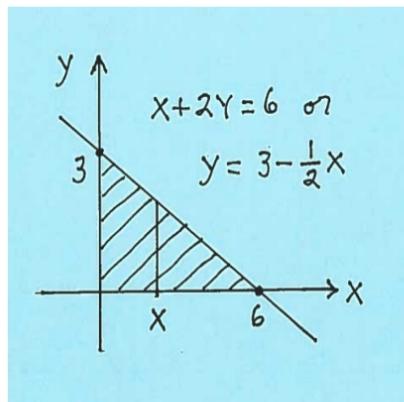


Kesit

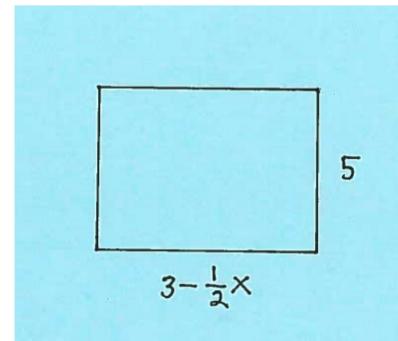
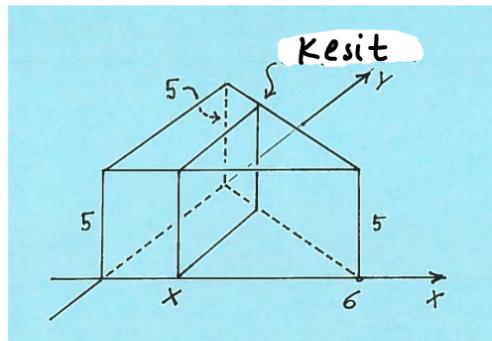


$$A(x) = \frac{1}{2}\pi \left(\frac{1}{2}(3 - \frac{1}{2}x)\right)^2, \quad V = \int_0^6 \frac{\pi}{2} \left[\frac{1}{2}(3 - \frac{x}{2})\right]^2 dx$$

c.



Kesit



$$A(x) = 5 \cdot (3 - \frac{1}{2}x), \quad V = \int_0^6 5 \cdot (3 - \frac{1}{2}x) dx$$

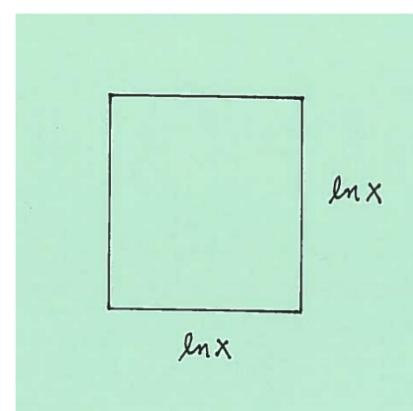
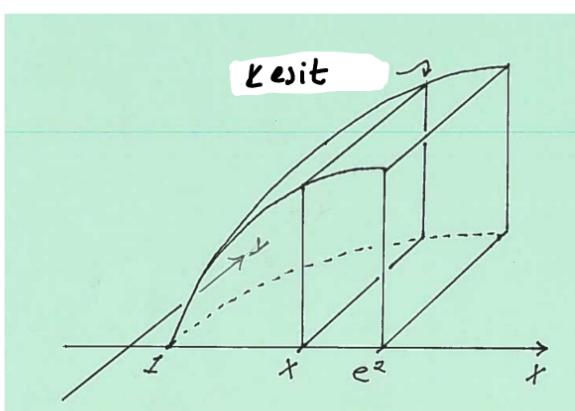
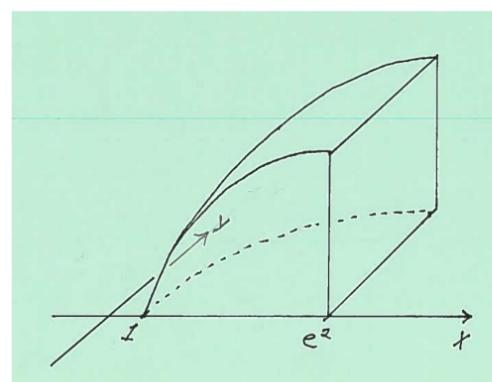
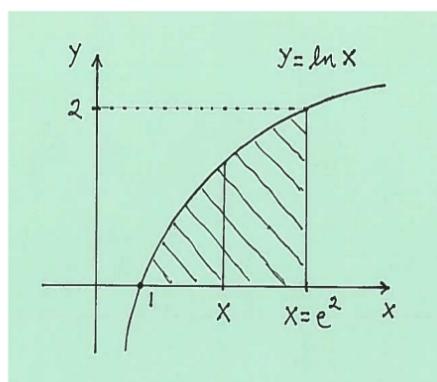
3. xy düzleminde tabanı x -ekseni, $x=e^2$ ve $y=\ln x$ eğrisi ile sınırlı bölge olan katı cisim, x -eksenine dik düzlemlerle kesitleri

- Kare
- Geyrek daire
- Yüksekliği 4 olan ikizkenar üçgen

ise bu katı cismin hacmini bulunuz.

Gözüm.

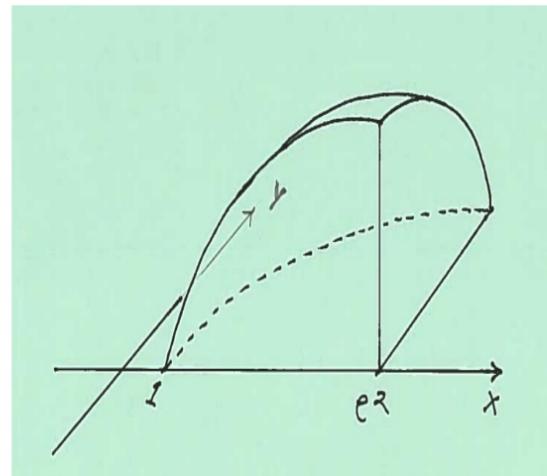
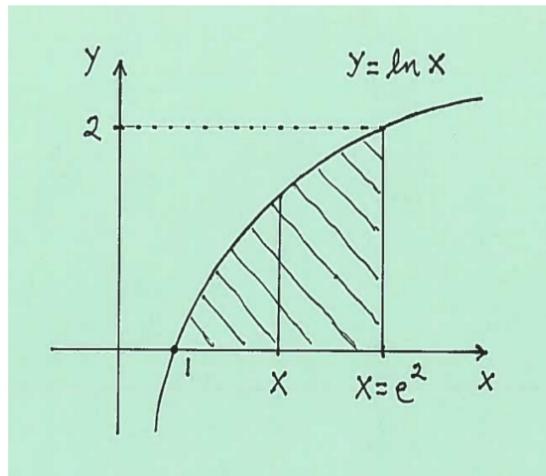
a.



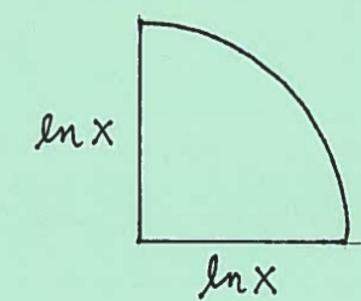
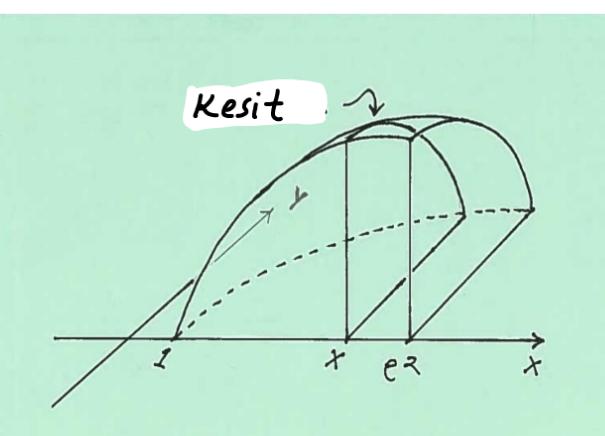
$$A(x) = (\ln x)^2$$

$$V = \int_1^{e^2} (\ln x)^2 dx$$

b.

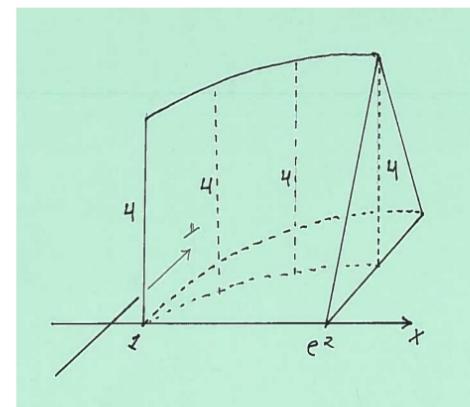
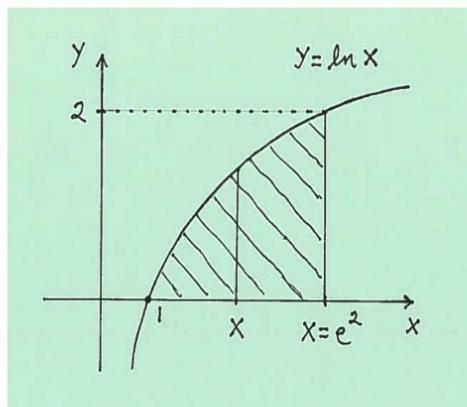


Kesit

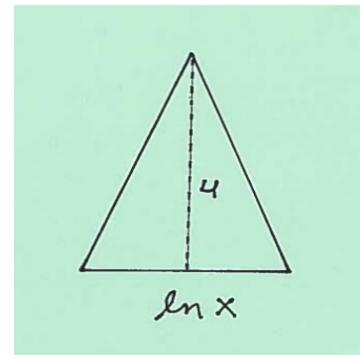
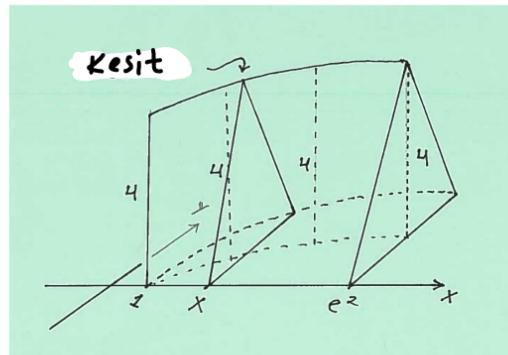


$$A(x) = \frac{1}{4} \pi r^2 = \frac{1}{4} \pi (\ln x)^2, \quad V = \int_1^{e^2} \frac{\pi}{4} (\ln x)^2 dx$$

c.



Kesit



$$A(x) = \frac{1}{2} (\ln x) \cdot 4 = 2 \ln x, \quad V = \int_1^{e^2} 2 \ln x dx$$

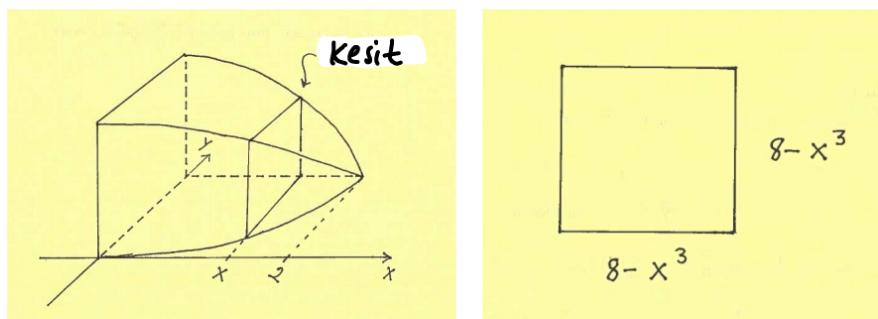
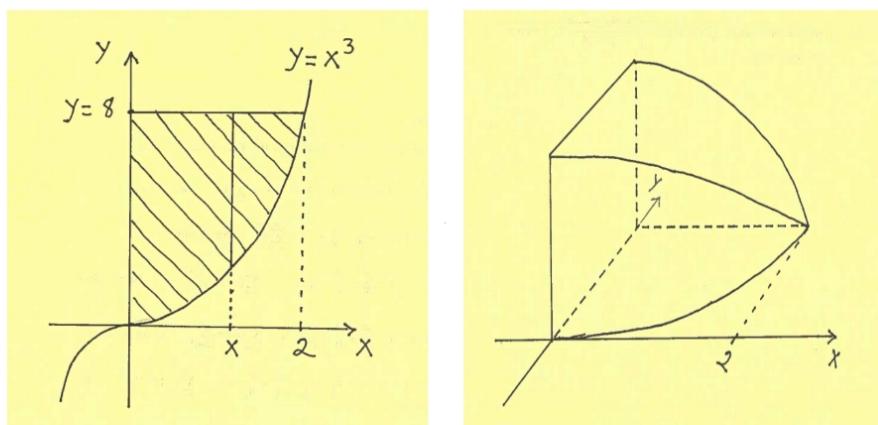
4. xy düzleminde tabanı x -ekseni, $y=8$ ve $y=x^3$ eğrisi ile sınırlı bölge olan katı cisim x -eksenine dik düzlemlerle kesitleri

- a. Kare
- b. Eşkenar üçgen
- c. Görüresi 16 olan dikdörtgen

ise bu katı cismin hacmini bulunuz.

Gözüm.

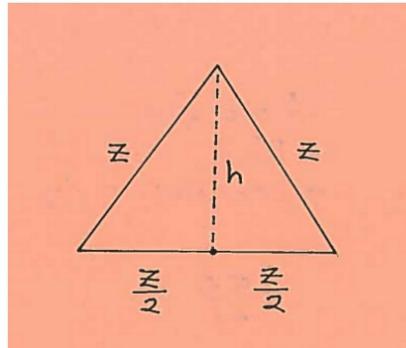
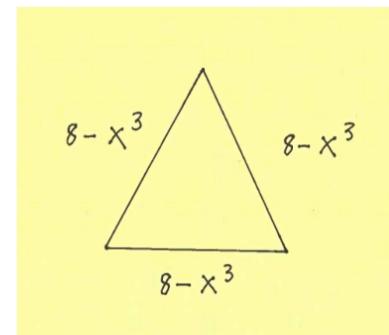
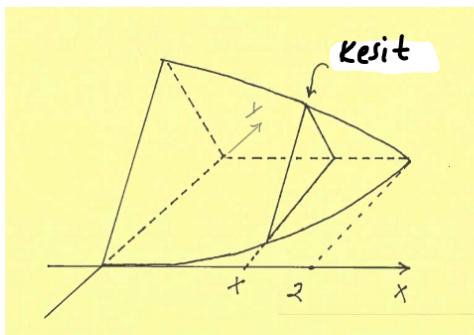
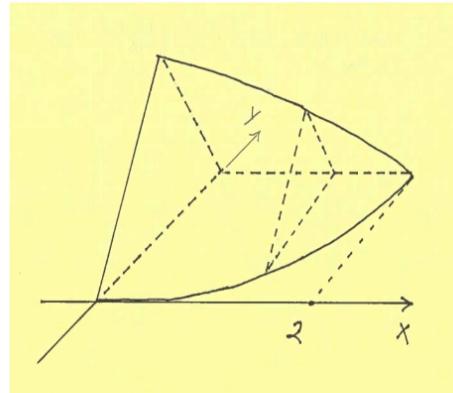
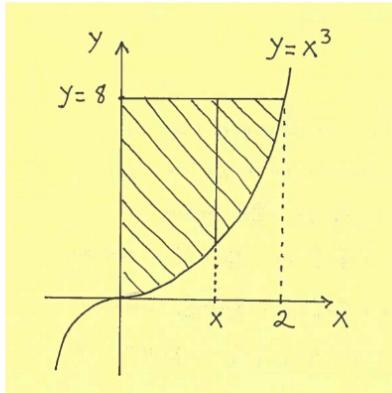
a.



$$A(x) = (8 - x^3)^2$$

$$V = \int_0^2 (8 - x^3)^2 dx$$

b.



$$h^2 + \left(\frac{z}{2}\right)^2 = z^2 \Rightarrow h^2 + \frac{z^2}{4} = z^2$$

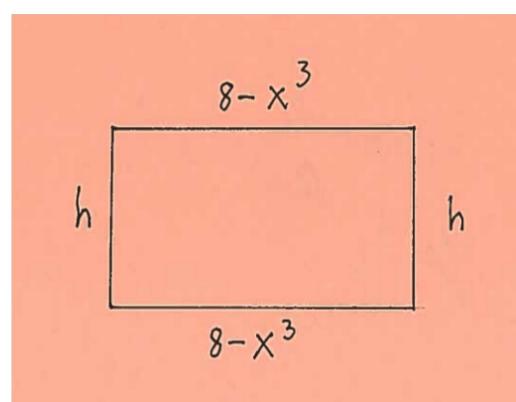
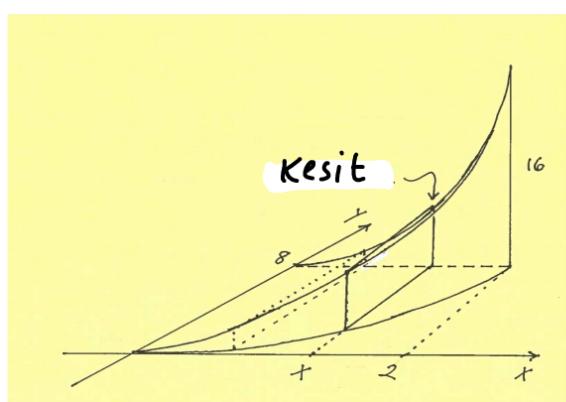
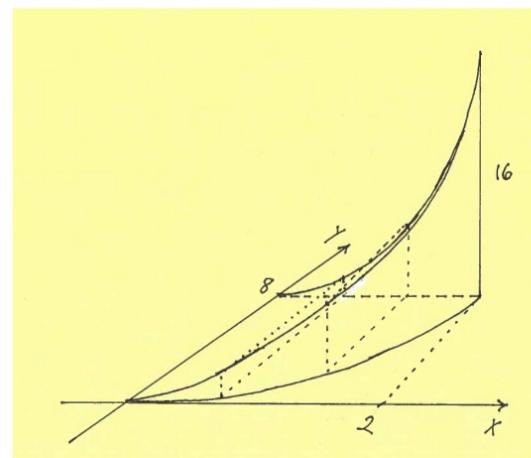
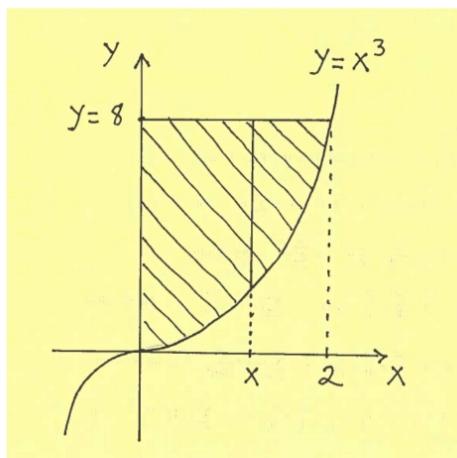
$$h^2 = \frac{3}{4} z^2$$

$$h = \frac{\sqrt{3}}{2} z$$

$$A(x) = \frac{1}{2} z \left(\frac{\sqrt{3}}{2} z \right) = \frac{\sqrt{3}}{4} z^2 = \frac{\sqrt{3}}{4} (8 - x^3)^2$$

$$V = \int_0^2 \frac{\sqrt{3}}{4} (8 - x^3)^2 dx$$

c.



$$2h + 2(8 - x^3) = 16$$

$$h + 8 - x^3 = 8 \Rightarrow h = x^3$$

$$A(x) = x^3 (8 - x^3) = 8x^3 - x^6$$

$$V = \int_0^2 (8x^3 - x^6) dx$$

5. $y = x^3$, $y = 0$ ve $x = 2$ ile sınırlı bölge

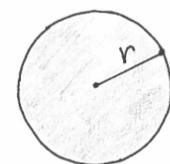
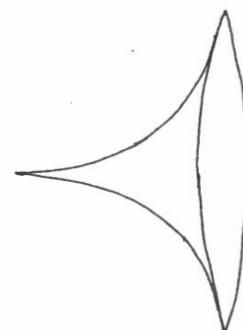
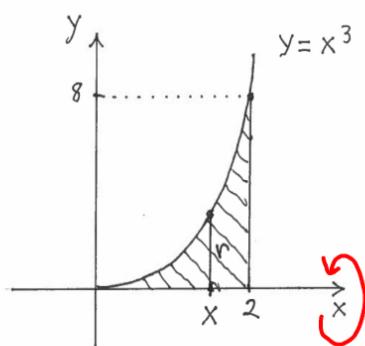
a. x - ekseni

b. y - ekseni

etrafında döndürülüğünde elde edilen dönel cismin hacmini bulunuz.

Gözüm

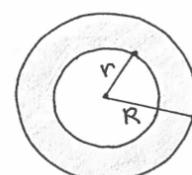
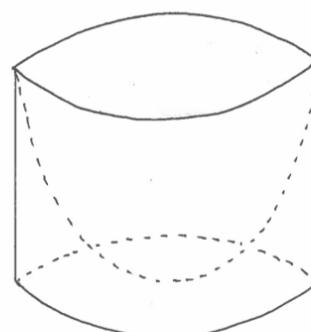
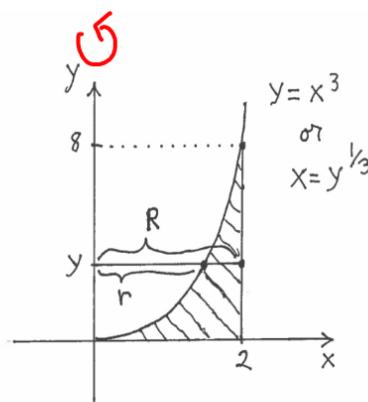
a.



$$A(x) = \pi r^2 = \pi (x^3)^2$$

$$V = \int_0^2 \pi (x^3)^2 dx = \int_0^2 \pi x^6 dx$$

b.



$$A(y) = \pi R^2 - \pi r^2 = \pi 2^2 - \pi (y^{1/3})^2$$

$$V = \int_0^8 (\pi 2^2 - \pi (y^{1/3})^2) dy$$

6. $y = \ln x$, $y = 0$ ve $x = e$ ile sınırlı bölge

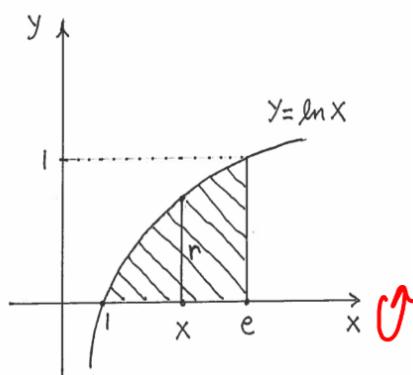
a. x - ekseni

b. $y = -1$

c. $y = 3$

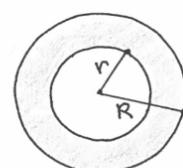
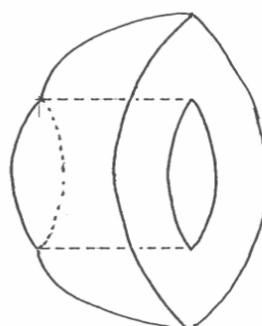
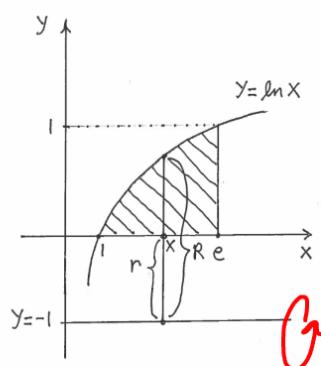
eşrafında döndürülüğünde elde edilen dönel cismin hacmini bulunuz.

a.



$$r = \ln x, \quad V = \int_1^e \pi (\ln x)^2 dx$$

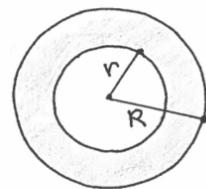
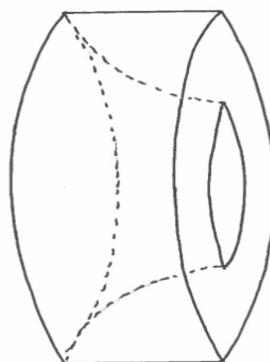
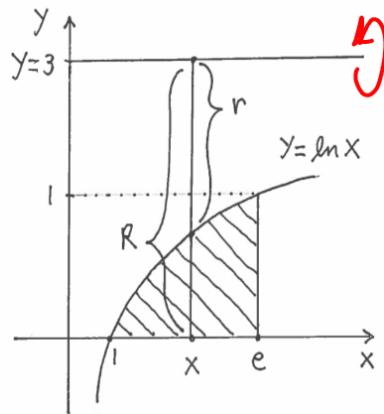
b.



$$R = 1 + \ln x, \quad r = 1$$

$$V = \int_1^e [\pi (1 + \ln x)^2 - \pi 1^2] dx$$

c.



$$R = 3, \quad r = 3 - \ln x$$

$$V = \pi \int_1^e [3^2 - (3 - \ln x)^2] dx$$

7. $2x+3y=6$, $y=0$ ve $x=0$ ile sınırlı bölge

a. y - ekseni

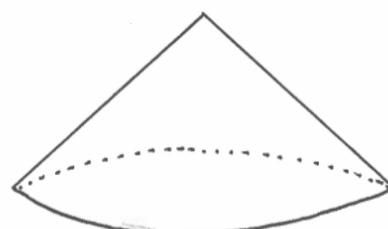
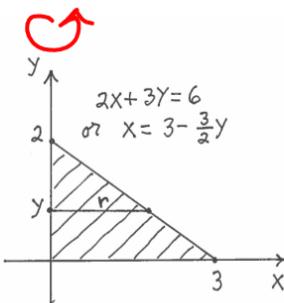
b. $x = -2$

c. $x = 4$

etrafında döndürülüğünde elde edilen dönel cismin hacmini bulunuz.

Cözüm

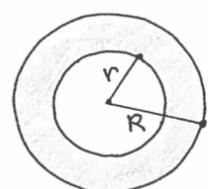
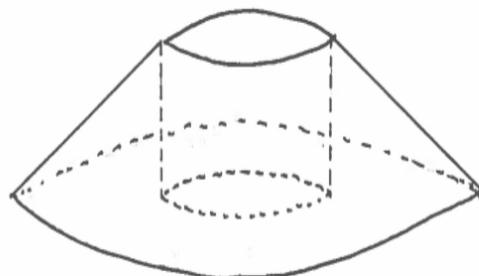
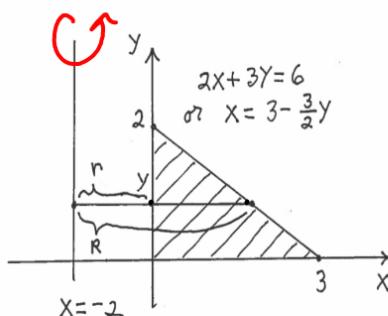
a.



$$\Gamma = x = 3 - \frac{3}{2}y$$

$$V = \pi \int_0^2 (3 - \frac{3}{2}y)^2 dy$$

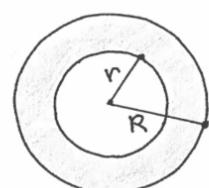
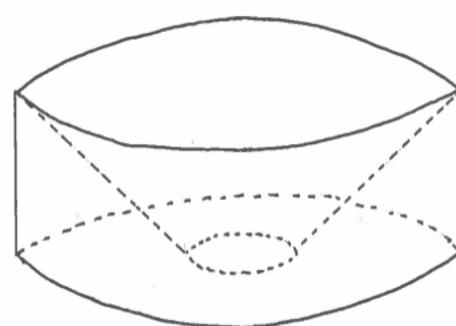
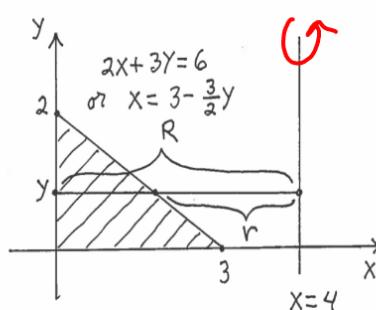
b.



$$R = 2 + \left(3 - \frac{3}{2}y\right), \quad r = 2$$

$$V = \pi \int_0^2 \left(\left(2 + 3 - \frac{3}{2}y\right)^2 - 2^2 \right) dy$$

c.



$$R = 4, \quad r = 4 - \left(3 - \frac{3}{2}y\right) = 1 + \frac{3}{2}y$$

$$V = \pi \int_0^2 \left[4^2 - \left(1 + \frac{3}{2}y\right)^2 \right] dy$$

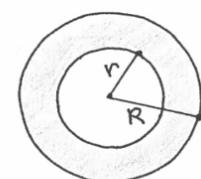
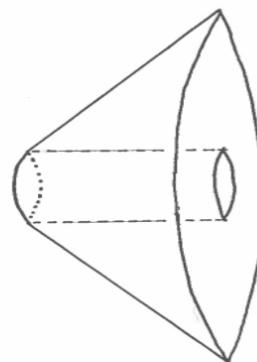
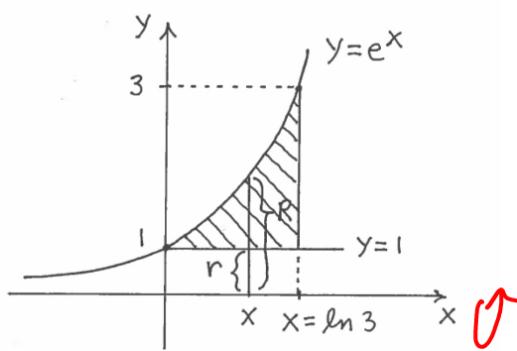
8. $y = e^x$, $y = 1$, $x = \ln 3$ ile sınırlı bölge

- a. x - ekseni
- b. y - ekseni
- c. $X = 4$
- d. $y = 1$
- e. $x = -3$
- f. $y = 3$

etrafında döndürülüğünde elde edilen dönel cismin hacmini bulunuz.

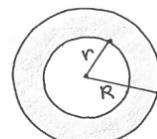
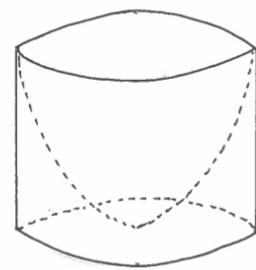
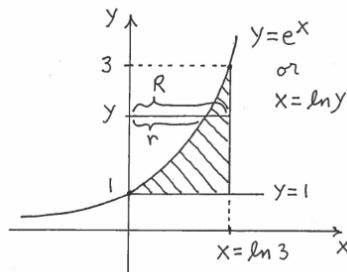
Cözüm

a.



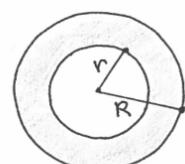
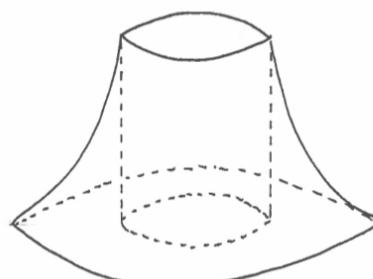
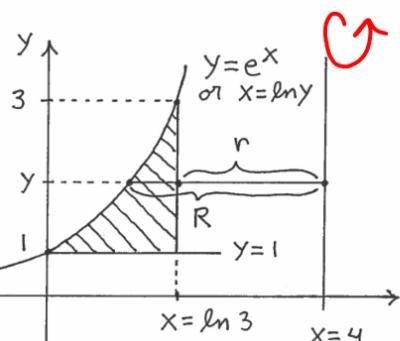
$$R = e^x, \quad r = 1 \quad \Rightarrow \quad V = \int_0^{\ln 3} \pi [(e^x)^2 - 1^2] dx$$

b.



$$R = \ln 3, \quad r = \ln y \quad V = \pi \int_1^3 ((\ln 3)^2 - (\ln y)^2) dy$$

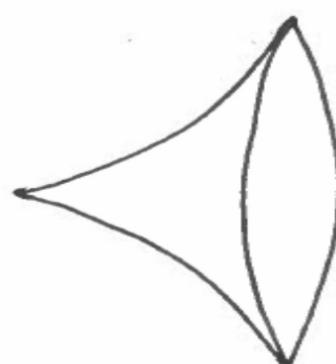
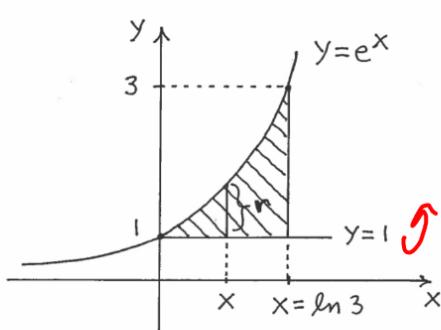
c.



$$R = 4 - \ln y, \quad r = 4 - \ln 3$$

$$V = \pi \int_1^3 [(4 - \ln y)^2 - (4 - \ln 3)^2] dy$$

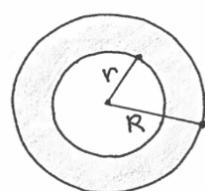
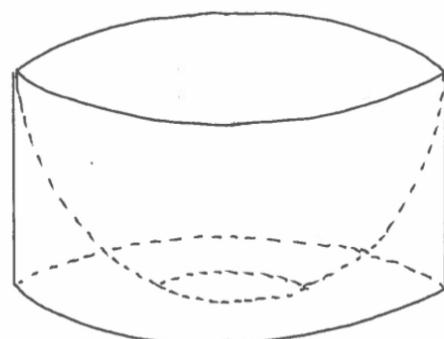
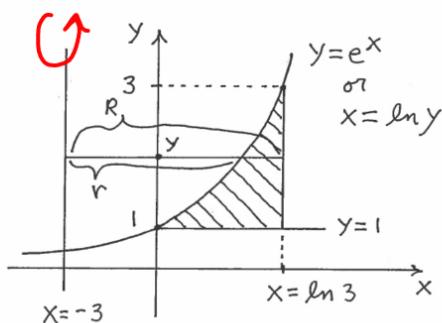
d.



$$r = e^x - 1$$

$$V = \pi \int_0^{\ln 3} (e^x - 1)^2 dx$$

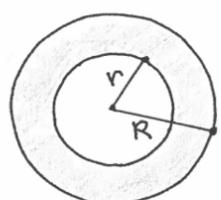
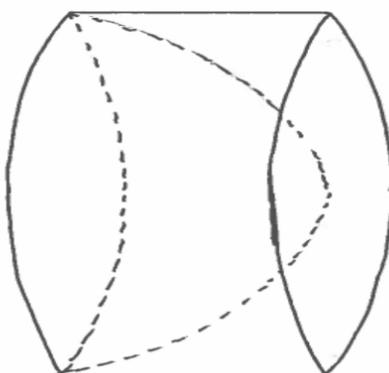
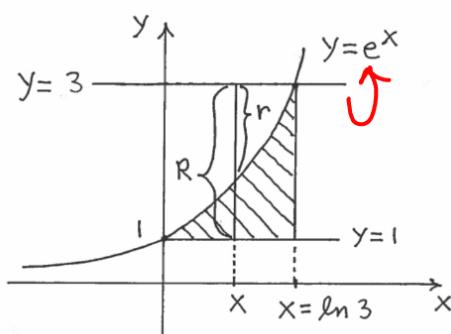
e.



$$R = 3 + \ln 3, \quad r = 3 + \ln y$$

$$V = \pi \int_1^3 [(3 + \ln 3)^2 - (3 + \ln y)^2] dy$$

f.

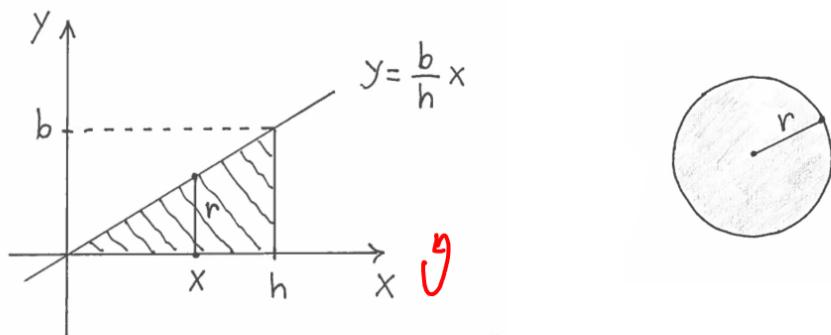


$$R = 3 - (e^x), \quad r = 3 - (e^x)$$

$$V = \pi \int_0^{\ln 3} [2^2 - (3 - e^x)^2] dx$$

9. Yüksekliği h ve taban yarıçapı b olan dik dairesel koninin hacmini disk yöntemi kullanarak bulunuz.

Gözüm.



$$r = \frac{b}{h} x$$

$$V = \int_0^h \pi \left(\frac{b}{h} x \right)^2 dx = \int_0^h \pi \frac{b^2}{h^2} x^2 dx$$

$$= \frac{\pi b^2}{h^2} \cdot \frac{x^3}{3} \Big|_0^h$$

$$= \frac{1}{3} \pi b^2 h$$

10. $y = \sqrt{x}$, $y=0$, $x=4$ ile sınırlı bölge

a. y - ekseni

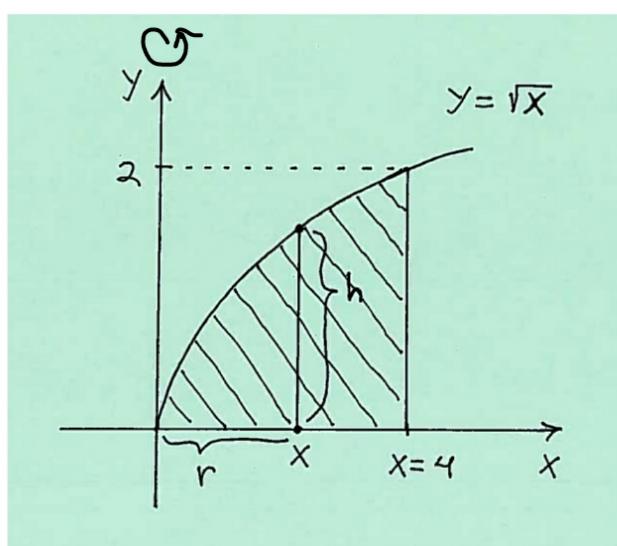
b. x - ekseni

etrafında döndürerek bir dönel cisim elde ediliyor.

Bu dönel cismin hacmini kabuk (silindir) yöntemi kullanarak bulunuz.

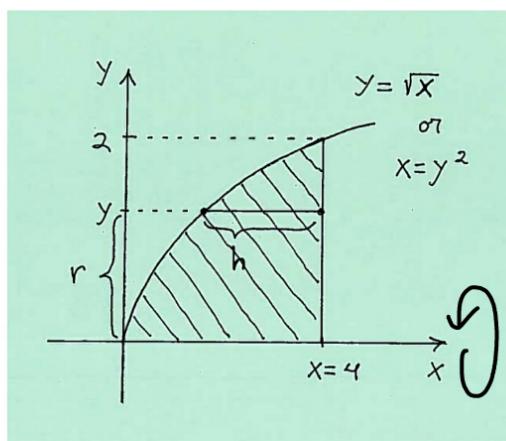
Cözüm.

a.



$$h = \sqrt{x}, \quad r = x \quad V = \int_0^4 2\pi \times \sqrt{x} \, dx$$

b.



$$r = y, \quad h = 4 - y^2, \quad V = \int_0^2 2\pi y (4 - y^2) \, dy$$

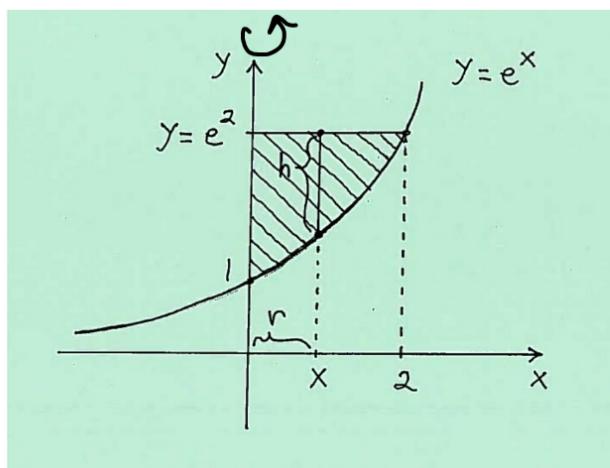
11. $y = e^x$, $y = e^2$, $x = 0$ ile sınırlı bölge

- y - ekseni
- x - ekseni

etrafında döndürerek bir dönel cisim elde ediliyor.
Bu dönel cismin hacmini kabuk (silindir) yöntemi
kullanarak bulunuz.

Cözüm

a.

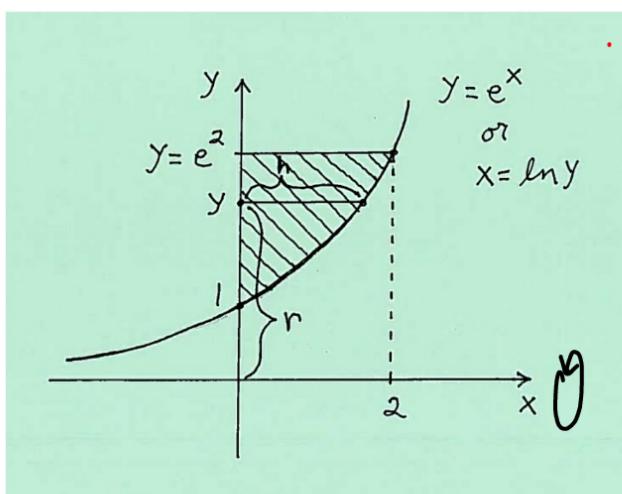


$$r = x$$

$$h = e^2 - e^x$$

$$V = \int_0^2 2\pi \times (e^2 - e^x) dx$$

b.



$$r = y$$

$$h = \ln y$$

$$V = \int_1^{e^2} 2\pi y \ln y dy$$

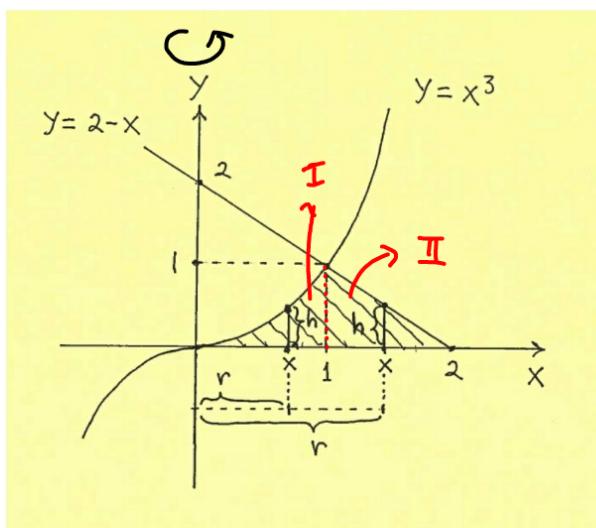
12. $y = x^3$, $y = 2 - x$, $y = 0$ ile sınırlı bölge

- y - ekseni
- x - ekseni

etrafında döndürerek bir dönel cisim elde ediliyor.
Bu dönel cismin hacmini kabuk (silindir) yöntemiyle
kullanarak bulunuz.

Gözüm

a.



I. Bölge için

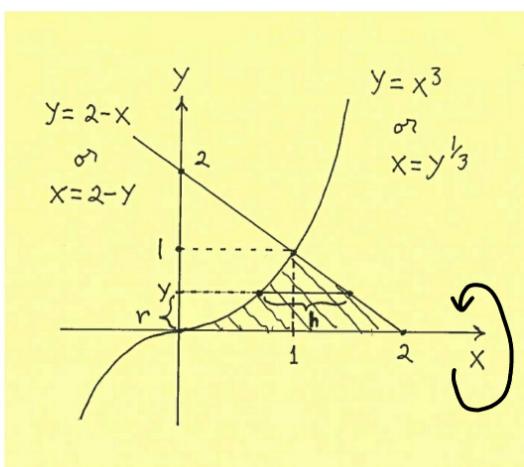
$$r = x, \quad h = x^3$$

II. bölge için

$$r = x, \quad h = 2 - x$$

$$V = \int_0^1 2\pi x \cdot x^3 dx + \int_1^2 2\pi x \cdot (2-x) dx$$

b.



$$r = y$$

$$h = (2-y) - (y^{1/3})$$

$$V = 2\pi \int_0^1 y (2-y - y^{1/3}) dy$$

13. $y = \tan x$, $y = \sqrt{3}$, $x = 0$ ile sınırlı bölge

a. y - ekseni

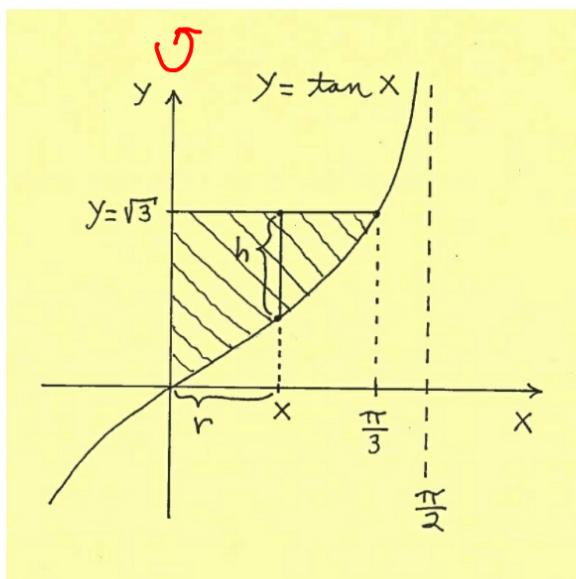
b. x - ekseni

etrafında döndürerek bir dönel cisim elde ediliyor.

Bu dönel cismin hacmini kabuk (silindir) yöntemi kullanarak bulunuz.

Gözüm

a.

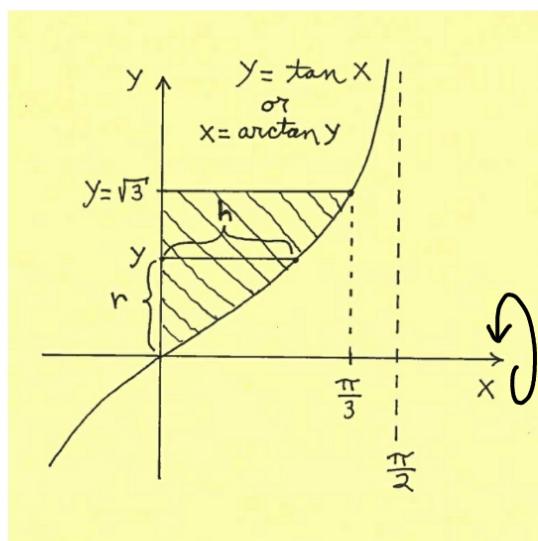


$$r = x$$

$$h = \sqrt{3} - \tan x$$

$$V = 2\pi \int_0^{\pi/3} x (\sqrt{3} - \tan x) dx$$

b.



$$r = y$$

$$h = \arctan y$$

$$V = \int_0^{\sqrt{3}} y \arctan y dy$$

14. $y = x^2$, $y = 8\sqrt{x}$ ile sınırlı bölge

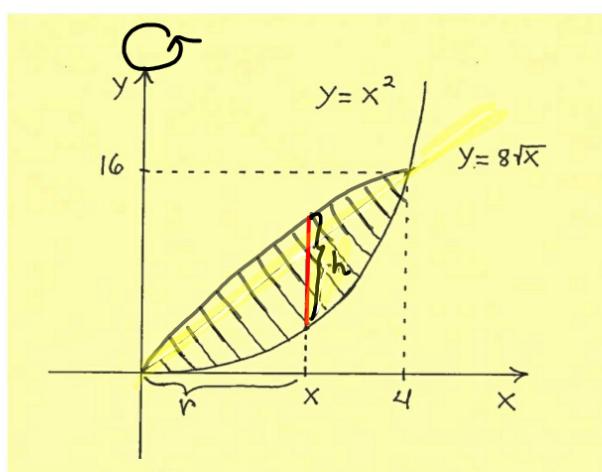
- a. y - ekseni
- b. x - ekseni
- c. $x = -2$
- d. $x = 6$
- e. $y = 20$
- f. $y = -1$

etrafında döndürerek bir dönel cisim elde ediliyor.

Bu dönel cismin hacmini kabuk (silindir) yöntemi kullanarak bulunuz.

Gözüm.

a.

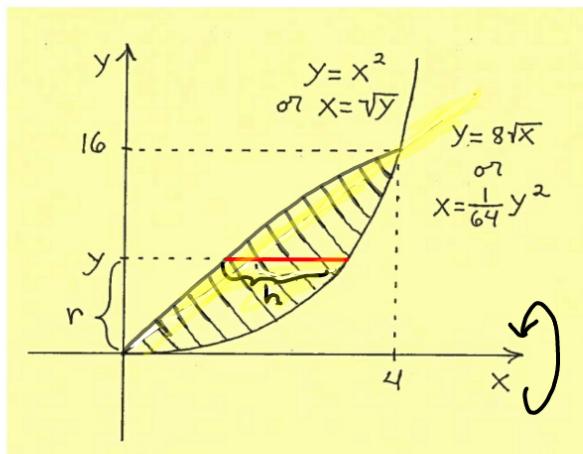


$$r = x$$

$$h = 8\sqrt{x} - x^2$$

$$V = \int_0^4 2\pi x (8\sqrt{x} - x^2) dx$$

b.

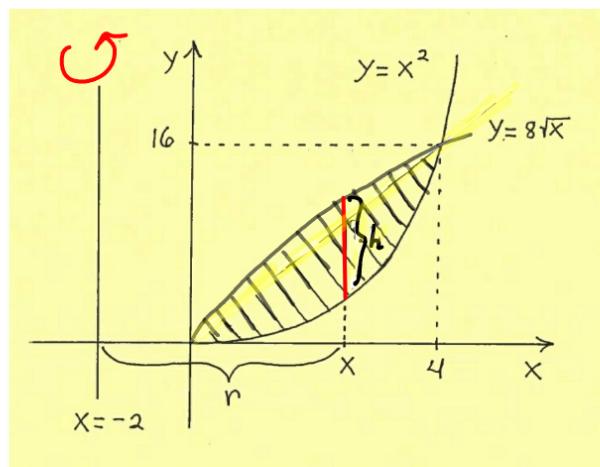


$$r = y$$

$$h = \frac{1}{64} y^2$$

$$V = \int_0^{16} 2\pi y (\sqrt{y} - \frac{1}{64} y^2)$$

c.

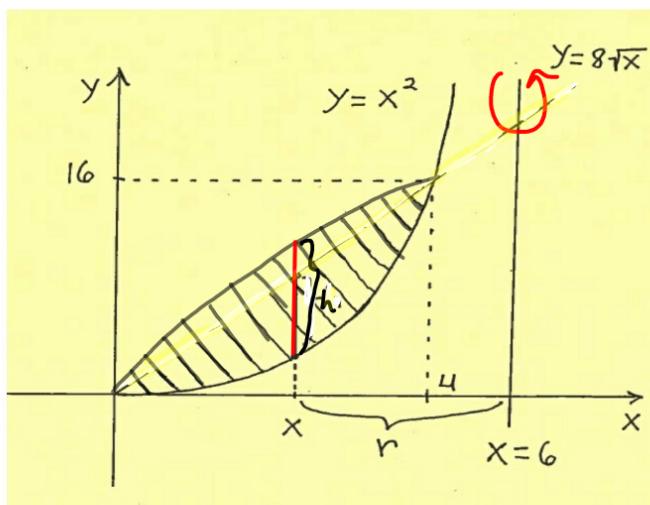


$$r = x + 2$$

$$h = 8\sqrt{x} - x^2$$

$$V = 2\pi \int_0^4 (x+2)(8\sqrt{x} - x^2) dx$$

d.

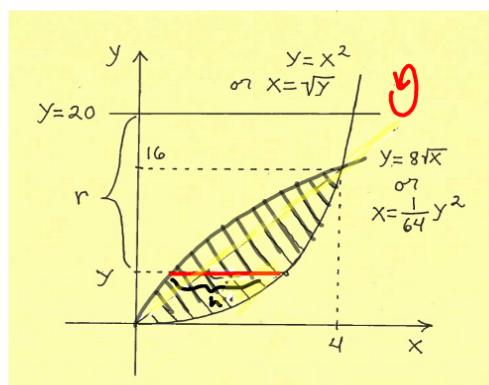


$$r = 6 - x$$

$$h = 8\sqrt{x} - x^2$$

$$V = 2\pi \int_0^4 (6-x)(8\sqrt{x} - x^2) dx$$

e.

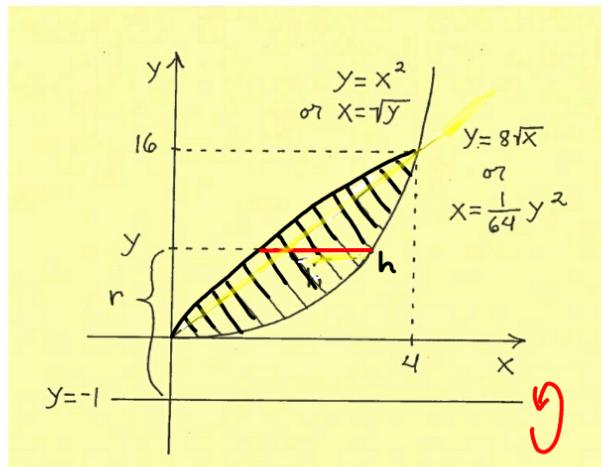


$$r = 20 - y$$

$$h = \sqrt{y} - \frac{1}{64}y^2$$

$$V = 2\pi \int_0^{16} (20-y) \cdot (\sqrt{y} - \frac{1}{64}y^2) dy$$

f.



$$r = 1 + y$$

$$h = \sqrt{y} - \frac{1}{64}y^2$$

$$V = 2\pi \int_0^{16} (1+y) (\sqrt{y} - \frac{1}{64}y^2) dy$$