Quiz: a-) $sgn(x) = \begin{cases} 1 & , x \geq 0 \\ -1 & , x < 0 \end{cases}$

$$\int_1^3 \frac{sgn(x-2)}{x^2}\,dx = I$$

Not: $x \in [1,2]$ için $-1 \leq x-2 \leq 0$

$\Rightarrow sgn(x-2) = -1$

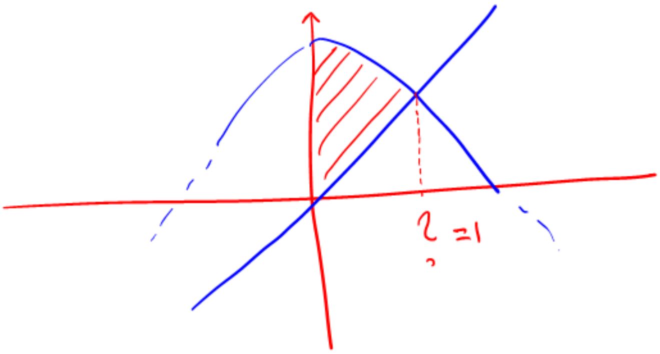
$x \in [2,3] \Rightarrow sgn(x-2) = 1$

$$I = \int_1^2 \frac{-1}{x^2}dx + \int_2^3 \frac{1}{x^2}dx = +\frac{1}{x}\Big|_1^2 - \frac{1}{x}\Big|_2^3 \longrightarrow \int \frac{1}{x^2} = \int x^{-2} = \frac{x^{-1}}{-1}$$

$$= \frac{1}{2} - 1 - \left\{\frac{1}{3} - \frac{1}{2}\right\}$$

$$= -\frac{1}{3}$$

b-) $f(x) = \sqrt{2}.\cos\left(\frac{\pi x}{4}\right)$, $g(x) = x$ ve y ekseni.



$\cancel{\sqrt{2}}.\cos\left(\frac{\pi}{4}x\right) = x$

$x = 1 \Rightarrow \cos\left(\frac{\pi}{4}\right) = \frac{1}{\cancel{\sqrt{2}}}$

$$\int_0^1 \left[\sqrt{2}.\cos\left(\frac{\pi}{4}x\right) - x\right]dx = \frac{\sqrt{2}.\sin\left(\frac{\pi}{4}x\right)}{\pi/4} - \frac{x^2}{2}\Big|_0^1$$

$$= \frac{4\sqrt{2}}{\pi}.\overbrace{\sin\left(\frac{\pi}{4}\right)}^{1/\sqrt{2}} - \frac{1}{2} - [0 - 0]$$

$$= \frac{4}{\pi} - \frac{1}{2}$$

1-) $\int_1^\infty \frac{\arctan x}{x^2}\,dx = I$ hesaplayınız.

$$I = \lim_{\alpha\to\infty}\int_1^\alpha \frac{\arctan x}{x^2}\,dx$$ kısmi integrasyon yapalım;

$\arctan x = u$ ve $\frac{dx}{x^2} = dv$ olsun.

$\Rightarrow \frac{1}{1+x^2}\,dx = du$ ve $-\frac{1}{x} = v$ olur.

$$I = \lim_{\alpha\to\infty}\left[-\frac{\arctan x}{x}\Big|_1^\alpha \cancel{-} + \int_1^\alpha \cancel{-}\frac{1}{x\,.\,(1+x^2)}\,dx\right]$$

Basit kesirler metodu ile;

$$\boxed{1} \over x(1+x^2)$$ $= \frac{A}{x}_{(1+x^2)} + \frac{Bx+C}{1+x^2}_{(x)}$

$$= \frac{\boxed{x^2(A+B) + Cx + A}}{x(x^2+1)}$$

$\Rightarrow$ $A+B = 0$
$C = 0$ $\Rightarrow$ $B = -1$
$A = 1$

$$I = \lim_{\alpha\to\infty}\left[-\frac{\arctan\alpha}{\alpha} - \left(-\frac{\arctan 1}{1}\right) + \int_1^\alpha \frac{1}{x}\,dx - \int\frac{x}{1+x^2}\,dx\right]$$

$$= \frac{\pi}{4} + \lim_{\alpha\to\infty} \ln x\Big|_1^\alpha - \frac{1}{2}\ln|x^2+1|\Big|_1^\alpha$$

$$= \frac{\pi}{4} + \lim_{\alpha\to\infty} \ln\alpha - \cancel{\ln 1}^{\,0} \;\left(-\frac{1}{2}\right)\left\{\ln(\alpha^2+1) + \ln 2\right\}$$

$\infty - \infty$

$$= \frac{\pi}{4} + \lim_{\alpha\to\infty} \cancel{\ln\left(\frac{\alpha}{\sqrt{\alpha^2+1}}\right)} + \ln 2$$

$\frac{\alpha}{\alpha\sqrt{1+\frac{1}{\alpha^2}}} = 1$

$= \frac{\pi}{4} + \ln 2$ II

2-) $\int_0^1 \frac{dx}{\sqrt{x-x^2}} = I$ hesaplayınız.

$f(x) = \frac{1}{\sqrt{x-x^2}}$, $\sqrt{x-x^2} \to 0$

$x \to 0^+$ iken $f(x) \to \infty$

$x \to 1^-$ iken $f(x) \to \infty$

Herhangi bir $c \in (0,1)$ seçelim.

$$I = \int_0^c \frac{dx}{\sqrt{x-x^2}} + \int_c^1 \frac{dx}{\sqrt{x-x^2}}$$

$$I = \lim_{u \to 0^+} \int_u^c \frac{dx}{\sqrt{x-x^2}} + \lim_{v \to 1^-} \int_c^v \frac{dx}{\sqrt{x-x^2}}$$

$\int \frac{dx}{\sqrt{x-x^2}}$,

$$x - x^2 = -(x^2 - x)$$
$$= -\left(\left(x-\frac{1}{2}\right)^2 - \frac{1}{4}\right)$$
$$= \frac{1}{4} - \left(x-\frac{1}{2}\right)^2$$

$$= \int \frac{dx}{\sqrt{\frac{1}{4} - \left(x-\frac{1}{2}\right)^2}}$$, $x - \frac{1}{2} = \frac{1}{2}u \Rightarrow dx = \frac{1}{2}du$

$$= \frac{1}{2}\int \frac{du}{\sqrt{\frac{1}{4} - \frac{1}{4}u^2}} = \cancel{\frac{1}{2}} \int \frac{du}{\cancel{\frac{1}{2}}\sqrt{1-u^2}} = \arcsin u = \arcsin 2x-1$$

$$I = \lim_{u \to 0^+} \arcsin(2x-1) \Big|_u^c + \lim_{v \to 1^-} \arcsin(2x-1) \Big|_c^v$$

$$= \lim_{u \to 0^+} \left| \arcsin(2c-1) - \arcsin(2u-1) \right| + \lim_{v \to 1} \arcsin(2v-1) - \arcsin(2c-1)$$

$= \cancel{\arcsin(2c-1)} - \arcsin(-1) + \arcsin 1 - \cancel{\arcsin(2c-1)}$

$= \arcsin 1 + \arcsin 1$

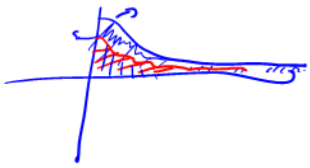
$= \frac{\pi}{2} + \pi/2 = \pi$.

3-) Aşagidaki integrallerin yakınsaklığını inceleyin.

a-) $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x^3+1}} = I$

$0 < f(x) < g(x) \to \displaystyle\int_0^\infty g(x)\,dx = L \ , \ L \in \mathbb{R}$

$\displaystyle\int_0^\infty f(x)\,dx$ yakınsaktır.



$0 < \dfrac{1}{\sqrt{x^3+1}} < \dfrac{1}{\sqrt{x^3}} = x^{-3/2}$

Eğer $\displaystyle\int_0^\infty x^{-3/2}dx$ yakınsak ise I'nın yakınsaklığını söyleriz.

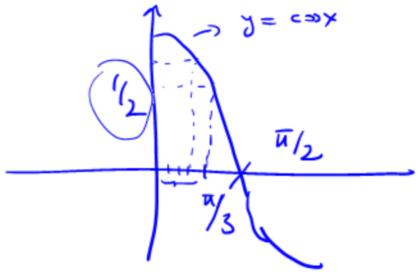
$f(x) = \dfrac{1}{\sqrt{x^3+1}}$ Fonk. $[0,1]$ aralığında süreklidir.

$I = \underbrace{\displaystyle\int_0^1 \frac{dx}{\sqrt{x^3+1}}}_{\text{Sonlu bir sayıya eşit!}} + \underbrace{\displaystyle\int_1^\infty \frac{dx}{\sqrt{x^3+1}}}_{\text{Has olmayan int.}}$ ---(1)

$\Rightarrow \displaystyle\int_1^\infty x^{-3/2}dx = 2$ bulunur ve dolayısıyla $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$ yakınsaktır. ---(2)

$\Rightarrow$ (1) ve (2)'den I yakınsaktır.

b-) $\int_0^{\pi/2} \frac{\cos x}{x}\,dx = ? = I$



$\forall x \in [0, \pi/3]$ için $\cos x > \frac{1}{2}$ old. elde ederiz.

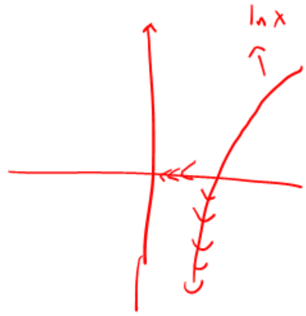
$\Rightarrow \frac{\cos x}{x} > \frac{1}{2x} \Rightarrow \frac{1}{2x} < \frac{\cos x}{x}$ bulunur.

$$I = \underbrace{\int_0^{\pi/3}\frac{\cos x}{x}\,dx}_{\text{Has olmayan kısım}} + \underbrace{\int_{\pi/3}^{\pi/2}\frac{\cos x}{x}\,dx}_{\text{Sonlu bir değer!}}$$

, $f(x)=\frac{\cos x}{x}$, $[\frac{\pi}{3},\frac{\pi}{2}]$ aralığında sürekli.

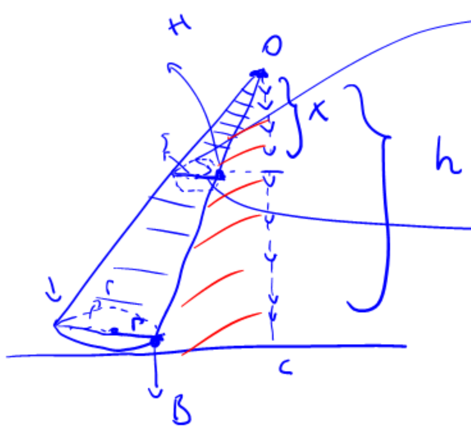
$$\int_0^{\pi/3}\frac{1}{2x}\,dx = \lim_{u\to 0^+} \frac{1}{2}\ln x \Big|_u^{\pi/3} = \lim_{u\to 0^+} \frac{1}{2}\ln\frac{\pi}{3} - \frac{1}{2}\ln u$$

$= \infty \Rightarrow$ ıraksak.



$\Rightarrow$ ıraksaktır! $\Rightarrow I$ ıraksaktır.

4-) Yüksekliği h ve yarıçapı r olan eğik dairesel koninin hacmini bulunuz.



D_x diskin hacmini versin.

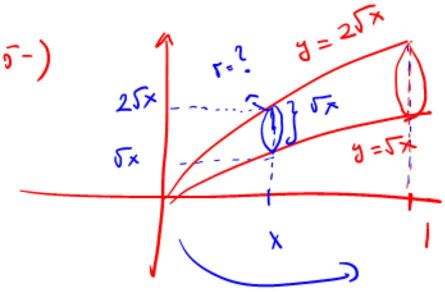
s diskin yarıçapı.

$$\begin{cases} \frac{2s}{2r} = \frac{OH}{OB} \\ \frac{OH}{OB} = \frac{x}{h} \end{cases} \Rightarrow \frac{s}{r} = \frac{x}{h} \Rightarrow \boxed{s = \frac{r \cdot x}{h}}$$

$$D_x = \pi S^2 = \pi \frac{r^2 \cdot x^2}{h^2} \qquad V = \int_0^h \pi \frac{r^2 \cdot x^2}{h^2} \cdot dx = \frac{\pi r^2}{h^2}\left(\frac{x^3}{3} \Big|_0^h \right)$$

$$= \frac{\pi r^2 h}{3}$$

5-)



x - eksenine dik kesitler ile boyunuz şekillerdeki cismin hacmini bulunuz.

Herhangi bir $x \in [0,1]$ alalım ve buna bağlı olarak oluşacak diskin alanını bulalım.

$r = \frac{\sqrt{x}}{2}$ olur ve dolayısıyla $D_x = \pi \frac{x}{4}$

$$\int_0^1 \frac{\pi x}{4} dx = \text{ödev.}$$

Hatırlatma Sorusu : $(\cosh x + \sinh x)^a = \cosh(ax) + \sinh(ax)$ old. ispatlayın.

$$\cosh x = \frac{e^x + e^{-x}}{2}, \qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$(\cosh x + \sinh x)^a = (e^x)^a = e^{ax}$$

$$\frac{e^{ax} + e^{-ax}}{2} + \frac{e^{ax} - e^{-ax}}{2} = e^{ax}.$$

7-) $\int \operatorname{sech}(\ln x)\, dx = ? = I$

$$I = \int \frac{1}{\cosh(\ln x)} dx = \int \frac{1}{\frac{e^{\ln x} + e^{-\ln x}}{2}} dx = \int \frac{2\, dx}{x + \frac{1}{x}} = \int \frac{2x}{x^2+1} dx$$

$$= \ln(x^2+1) + C \quad \#$$