HACETTEPE ÜNİVERSİTESİ

huzem

FEN FAKÜLTESİ

MAT 122 MATEMATİK II-Uygulama

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Kaynak: Thomas Calculus

- 1.) $\int \cos x \ dx = \sin x + C$
- 2.) $\int \sin x \ dx = -\cos x + C$
- 3.) $\int \sec^2 x \ dx = \tan x + C$
- 4.) $\int \csc^2 x \ dx = -\cot x + C$
- 5.) $\int \sec x \tan x \ dx = \sec x + C$
- 6.) $\int \csc x \cot x \ dx = -\csc x + C$
- 7.) $\int \tan x \ dx = \ln|\sec x| + C$
- 8.) $\int \cot x \ dx = \ln|\sin x| + C$
- 9.) $\int \sec x \ dx = \ln|\sec x + \tan x| + C$
- 10.) $\int \csc x \ dx = \ln|\csc x - \cot x| + C$

- A.) $\sin 2x = 2 \sin x \cos x$
- B.) $\cos 2x = 2\cos^2 x - 1$ so that $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- C.) $\cos 2x = 1 - 2\sin^2 x$ so that $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- D.) $\cos 2x = \cos^2 x - \sin^2 x$
- E.) $1 + \cot^2 x = \csc^2 x$ so that $\cot^2 x = \csc^2 x - 1$

1. Aşağıdaki integralleri hesaplayınız.

1. $\int \sqrt{1-x^2}\,dx$ 2. $\int \frac{(x^2-1)^{3/2}}{x}\,dx$. 3. $\int \frac{1}{(1-x^2)^{3/2}}\,dx$.

4. $\int \frac{\sqrt{x^2+1}}{x}\,dx$. 5. $\int x^3\sqrt{4-9x^2}\,dx$.

Çözüm.

1. $\int \sqrt{1-x^2}\,dx$

$x = \sin\theta, \quad dx = \cos\theta\,d\theta$

$$I = \int \sqrt{1-x^2}\,dx = \int \sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$$

$$= \int \sqrt{\cos^2\theta}\,\cos\theta\,d\theta$$

$$= \int \cos^2\theta\,d\theta$$

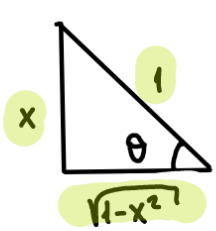
$$= \int \frac{1}{2}(1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\int (1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{1}{2}\left(\theta + \sin\theta\cos\theta\right) + C$$

$$= \frac{1}{2}\left(\arcsin x + x\sqrt{1-x^2}\right) + C$$



$\sin\theta = x$

2. $\int \frac{(x^2-1)^{3/2}}{x}\,dx$

$x = \sec\theta \Rightarrow dx = \sec\theta\tan\theta\,d\theta$

$$I = \int \frac{(x^2-1)^{3/2}}{x}\,dx = \int \frac{(\sec^2\theta-1)^{3/2}}{\sec\theta}\cdot\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{(\tan^2\theta)^{3/2}}{\sec\theta}\sec\theta\tan\theta\,d\theta$$

$$= \int \tan^4\theta\,d\theta$$

$$= \int \tan^2\theta\tan^2\theta\,d\theta$$

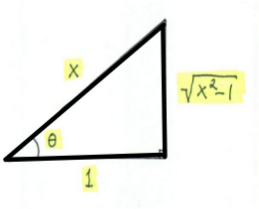
$$= \int \tan^2\theta(\sec^2\theta-1)\,d\theta$$

$$= \int \tan^2\theta\sec^2\theta\,d\theta - \int(\sec^2\theta-1)\,d\theta$$

$$= \int \tan^2\theta\sec^2\theta\,d\theta - \int\sec^2\theta\,d\theta + \int d\theta$$

$\tan\theta = u \Rightarrow \sec^2\theta\,d\theta = du$

$$= \frac{u^3}{3} - \tan\theta + \theta + C = \frac{(\tan\theta)^3}{3} - \tan\theta + \theta + C$$



$\sec\theta = x$, $\tan\theta = \sqrt{x^2-1}$

$$I = \frac{1}{3}\left(\sqrt{x^2-1}\right)^3 - \sqrt{x^2-1} + \operatorname{arcsec}x + C$$

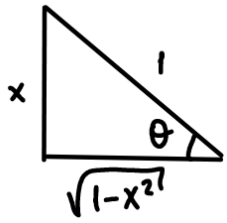
3. $\int \frac{1}{(1-x^2)^{3/2}}\, dx$

$x = \sin\theta, \qquad dx = \cos\theta\, d\theta$

$$I = \int \frac{1}{(1-\sin^2\theta)^{3/2}} \cos\theta\, d\theta = \int \frac{1}{(\cos^2\theta)^{3/2}} \cdot \cos\theta\, d\theta$$

$$= \int \frac{1}{\cos^3\theta} \cos\theta\, d\theta = \int \sec^2\theta\, d\theta = \tan\theta + C$$

$$I = \frac{x}{\sqrt{1-x^2}} + C$$



4. $\int \frac{\sqrt{x^2+1}}{x}\, dx$

$x = \tan\theta \quad , \quad dx = \sec^2\theta\, d\theta$

$$I = \int \frac{\sqrt{\sec^2\theta}}{\tan\theta}\, \sec^2\theta\, d\theta$$

$$I = \int \frac{\sec^3\theta}{\tan\theta}\, d\theta = \int \frac{\sec\theta \cdot \sec^2\theta}{\tan\theta}\, d\theta$$

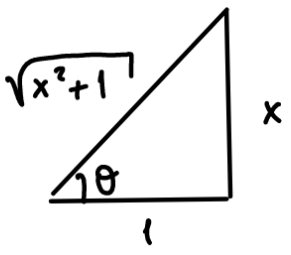
$$= \int \frac{\sec\theta\,(1+\tan^2\theta)}{\tan\theta}\, d\theta$$

$$= \int \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta}\,(1+\tan^2\theta)\, d\theta$$

$$= \int \left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\,\frac{\sin\theta}{\cos\theta}\right) d\theta$$

$$= \int \csc\theta\, d\theta + \int (\sec\theta \cdot \tan\theta)\, d\theta$$

$$= \ln|\csc\theta - \cot\theta| + \sec\theta + C$$



$$= \ln\left|\frac{\sqrt{x^2+1}}{x} - \frac{1}{x}\right| + \sqrt{x^2+1} + C$$

5. $\int x^3 \sqrt{4-9x^2}\, dx$

$= \int x^3\, 3.\sqrt{\frac{4}{9}-x^2}\, dx = 3\int x^3 \sqrt{\left(\frac{2}{3}\right)^2-x^2}\, dx$

$x = \frac{2}{3}\sin\theta \Rightarrow dx = \frac{2}{3}\cos\theta\, d\theta$

$= 3.\int \frac{8}{27}\sin^3\theta \sqrt{\frac{4}{9}-\frac{4}{9}\sin^2\theta}\cdot\frac{2}{3}\cos\theta\, d\theta$

$= \int \frac{8}{9}\sin^3\theta \frac{2}{3}.\cos\theta \frac{2}{3}.\cos\theta\, d\theta$

$= \frac{32}{81}\int \sin^3\theta\cos^2\theta\, d\theta$

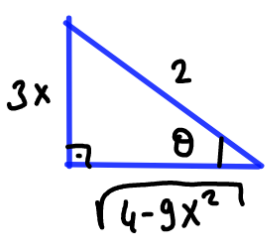
$\cos\theta = u \Rightarrow -\sin\theta\, d\theta = du$

$= \frac{32}{81}\int (1-\cos^2\theta)\cos^2\theta.\sin\theta\, d\theta$

$= \frac{32}{81}\int (1-u^2).u^2\,(-du)$

$= \frac{32}{81}\int (u^4-u^2)\, du$

$= \frac{32}{81}\left[\frac{u^5}{5}-\frac{u^3}{3}\right]+C = \frac{32}{81}\left(\frac{1}{5}\cos^5\theta-\frac{1}{3}\cos^3\theta\right)+C$



$\sin\theta = \frac{3x}{2}$

$\cos\theta = \frac{\sqrt{4-9x^2}}{2}$

$= \frac{4}{243}(4-9x^2)^{3/2}+\frac{1}{405}(4-9x^2)^{5/2}+C$

6. $\int \frac{dx}{x^3+9x}$ belirsiz integralini hesaplayınız.

$$\frac{1}{x^3+9x} = \frac{1}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$$

$$\Rightarrow \quad 1 = A(x^2+9) + (Bx+C)x$$

$$1 = (A+B)x^2 + Cx + 9A$$

$$9A = 1 \Rightarrow A = 1/9, \quad B = -1/9, \quad C = 0$$

$$\int \frac{dx}{x^3+9x} = \frac{1}{9}\int \frac{dx}{x} - \frac{1}{9}\int \frac{x}{x^2+9}$$

$$= \frac{1}{9}\ln|x| - \frac{2}{9}\ln(x^2+9) + C$$

7. $\int \frac{dx}{x^4 - 3x^3}$

$$\frac{1}{x^4 - 3x^3} = \frac{1}{x^3(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-3}$$

$$A(x^3 - 3x^2) + B(x^2 - 3x) + C(x-3) + Dx^3 = 1$$

$$\left.\begin{aligned} A + D &= 0 \\ -3A + B &= 0 \\ -3B + C &= 0 \\ 1 &= -3C \end{aligned}\right\} \quad \begin{aligned} A &= -1/27 \qquad D = 1/27 \\ B &= -1/9 \\ C &= -1/3 \end{aligned}$$

$$I = -\frac{1}{27}\int \frac{dx}{x} - \frac{1}{9}\int \frac{dx}{x^2} - \frac{1}{3}\int \frac{dx}{x^3} + \frac{1}{27}\int \frac{1}{x-3}\,dx$$

$$= -\frac{1}{27}\ln|x| + \frac{1}{9x} + \frac{1}{6x^2} + \frac{1}{27}\ln|x-3| + C$$

8. $\int \frac{x^2+1}{6x-9x^2}\,dx$

$$\int \frac{x^2+1}{6x-9x^2}\,dx = \frac{1}{9}\int \frac{9x^2-6x+6x+9}{6x-9x^2}$$

$$= \int \left(-\frac{1}{9} + \frac{1}{9}\,\frac{2x+3}{2x-3x^2}\right)dx$$

$$= -\frac{x}{9} + \frac{1}{9}\int \frac{2x+3}{x(2-3x)}\,dx$$

$$\frac{2x+3}{x(2-3x)} = \frac{A}{x} + \frac{B}{2-3x} \Rightarrow A(2-3x)+Bx = 2x+3$$

$$A = 3/2,\quad B = 13/2$$

$$\int \frac{x^2+1}{6x-9x^2}\,dx = -\frac{x}{9} + \frac{1}{6}\int \frac{dx}{x} + \frac{13}{18}\int \frac{dx}{2-3x}$$

$$= -\frac{x}{9} + \frac{1}{6}\ln|x| - \frac{13}{54}\ln|2-3x| + C$$

9. $\int \frac{x\,dx}{(x^2-x+1)^2}$

$$x^2-x+1 = \left(x-\frac{1}{2}\right)^2+\frac{3}{4}$$

$$I = \int \frac{x\,dx}{\left[\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\right]^2} \quad ; \quad x-\frac{1}{2}=u \; ; \quad dx=du$$

$$x = u+\frac{1}{2}$$

$$I = \int \frac{u\,du}{\left(u^2+\frac{3}{4}\right)^2} + \frac{1}{2}\int \frac{du}{\left(u^2+\frac{3}{4}\right)^2}$$

$u^2+\frac{3}{4}=y$

$2u\,du = dy$

$u = \frac{\sqrt{3}}{2}\tan\theta$

$du = \frac{\sqrt{3}}{2}\sec^2\theta\,d\theta$

$$= \frac{1}{2}\int \frac{dy}{y^2} + \frac{1}{2}\int \frac{\sqrt{3}}{2}\sec^2\theta\,d\theta \Big/ \frac{9}{16}\cdot\sec^4\theta$$

$$= -\frac{1}{2}\cdot\frac{1}{(x^2-x+1)} + \frac{4}{3\sqrt{3}}\int \cos^2\theta\,d\theta$$

$$= -\frac{1}{2(x^2-x+1)} + \frac{2}{3\sqrt{3}}(\theta+\sin\theta\cos\theta) + C$$

$$= \frac{2}{3\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}} + \frac{x-2}{3(x^2-x+1)} + C$$

10. $\int \frac{dx}{\sqrt{2x+3} - \sqrt[4]{2x+3}}$

$u^4 = 2x+3 \Rightarrow 4u^3\,du = 2dx$

$2u^3\,du = dx$

$$I = \int \frac{2u^3}{u^2 - u}\,du = \int \frac{2u^2}{u-1}\,du = \int \left(2u + \frac{2u}{u-1}\right)du$$

$$= \int \left(2u + 2\left(1 + \frac{1}{u-1}\right)\right)du$$

$$= 2\int u\,du + 2\int du + 2\int \frac{1}{u-1}\,du$$

$$= u^2 + 2u + 2\ln|u-1| + C$$

$$I = \sqrt{2x+3} + 2\sqrt[4]{2x+3} + 2\ln\left|\sqrt[4]{2x+3} - 1\right| + C$$

11. $$\int \frac{dx}{(4x^2+4x+5)^2}$$

$$(4x^2+4x+5)^2 = ((2x+1)^2+4)^2 = (y^2+4)^2$$

$$\downarrow$$

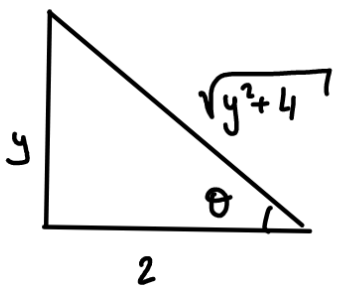
$$y = 2x+1 \quad , \quad dy = 2\,dx$$

$$I = \frac{1}{2}\int \frac{dy}{(y^2+4)^2} \quad ; \quad y = 2\tan\theta \; ; \quad dy = 2\sec^2\theta\, d\theta$$

$$I = \frac{1}{2}\int \frac{2\sec^2\theta\, d\theta}{16(1+\tan^2\theta)^2} = \frac{1}{16}\int \frac{\sec^2\theta}{\sec^4\theta}\, d\theta$$

$$= \frac{1}{16}\int \cos^2\theta\, d\theta \quad ; \quad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$I = \frac{1}{32}\int (\cos 2\theta + 1)\, d\theta = \frac{1}{64}\sin 2\theta + \frac{\theta}{32} + C$$



$$I = \frac{1}{32}\,\frac{y}{\sqrt{y^2+4}}\cdot\frac{2}{\sqrt{y^2+4}} + \frac{1}{32}\arctan\frac{y}{2} + C$$

$$= \frac{1}{16}\,\frac{2x+1}{4x^2+4x+5} + \frac{1}{32}\arctan\left(\frac{2x+1}{2}\right) + C$$

12. $\int \frac{dx}{(4x-x^2)^{3/2}}$

$$4x - x^2 = -(x^2 - 4x) = -(x^2 - 4x + 4) + 4$$
$$= -(x-2)^2 + 4$$

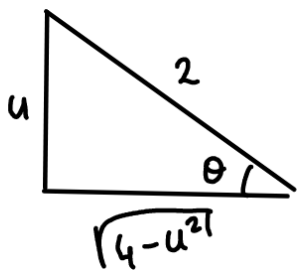
$$\int \frac{dx}{(4x-x^2)^{3/2}} = \int \frac{dx}{(4-(x-2)^2)^{3/2}} \quad ; \quad x-2 = u, \quad dx = du$$

$$= \int \frac{du}{(4-u^2)^{3/2}} \quad ; \quad u = 2\sin\theta, \quad du = 2\cos\theta\, d\theta$$

$$= \int \frac{2\cos\theta\, d\theta}{(4-4\sin^2\theta)^{3/2}} = \int \frac{2\cos\theta\, d\theta}{8\cos^3\theta} = \frac{1}{4}\int \sec^2\theta\, d\theta$$

$$= \frac{1}{4}\tan\theta + C = \frac{1}{4}\frac{u}{\sqrt{4-u^2}} + C$$

$$= \frac{1}{4}\frac{(x-2)}{\sqrt{4x-x^2}} + C$$



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1. $\int_0^1 \frac{dx}{(x+1)(x^2+1)}$

2. $\int_1^{\sqrt{3}} \frac{3t^2+t+4}{t^3+t}\,dt$

3. $\int \frac{y^2+2y+1}{(y^2+1)^2}\,dy$

4. $\int \frac{8x^2+8x+2}{(4x^2+1)^2}\,dx$

5. $\int \frac{2s+2}{(s^2+1)(s-1)^3}\,ds$

6. $\int \frac{s^4+81}{s(s^2+9)^2}\,ds$

7. $\int \frac{2\theta^3+5\theta^2+8\theta+4}{(\theta^2+2\theta+2)^2}\,d\theta$

8. $\int \frac{\theta^4-4\theta^3+2\theta^2-3\theta+1}{(\theta^2+1)^3}\,d\theta$

9. $\int_0^1 \frac{x^3\,dx}{x^2+2x+1}$

10. $\int_{-1}^0 \frac{x^3\,dx}{x^2-2x+1}$

11. $\int \frac{2x^3-2x^2+1}{x^2-x}\,dx$

12. $\int \frac{x^4}{x^2-1}\,dx$

13. $\int \frac{9x^3-3x+1}{x^3-x^2}\,dx$

14. $\int \frac{16x^3}{4x^2-4x+1}\,dx$

15. $\int \frac{y^4+y^2-1}{y^3+y}\,dy$

16. $\int \frac{2y^4}{y^3-y^2+y-1}\,dy$

1. $\int \frac{dy}{\sqrt{9+y^2}}$

2. $\int \frac{3\,dy}{\sqrt{1+9y^2}}$

3. $\int_{-2}^2 \frac{dx}{4+x^2}$

4. $\int_0^2 \frac{dx}{8+2x^2}$

5. $\int_0^{3/2} \frac{dx}{\sqrt{9-x^2}}$

6. $\int_0^{1/2\sqrt{2}} \frac{2\,dx}{\sqrt{1-4x^2}}$

7. $\int \sqrt{25-t^2}\,dt$

8. $\int \sqrt{1-9t^2}\,dt$

9. $\int \frac{dx}{\sqrt{4x^2-49}}, \quad x > \frac{7}{2}$

10. $\int \frac{5\,dx}{\sqrt{25x^2-9}}, \quad x > \frac{3}{5}$

11. $\int \frac{\sqrt{y^2-49}}{y}\,dy, \quad y > 7$

12. $\int \frac{\sqrt{y^2-25}}{y^3}\,dy, \quad y > 5$

13. $\int \frac{dx}{x^2\sqrt{x^2-1}}, \quad x > 1$

14. $\int \frac{2\,dx}{x^3\sqrt{x^2-1}}, \quad x > 1$

15. $\int \frac{x^3\,dx}{\sqrt{x^2+4}}$

16. $\int \frac{dx}{x^2\sqrt{x^2+1}}$

17. $\int \frac{8\,dw}{w^2\sqrt{4-w^2}}$

18. $\int \frac{\sqrt{9-w^2}}{w^2}\,dw$

19. $\int_0^{\sqrt{3}/2} \frac{4x^2\,dx}{(1-x^2)^{3/2}}$

20. $\int_0^1 \frac{dx}{(4-x^2)^{3/2}}$

21. $\int \frac{dx}{(x^2-1)^{3/2}}, \quad x > 1$

22. $\int \frac{x^2\,dx}{(x^2-1)^{5/2}}, \quad x > 1$