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Hacettepe Üniversitesi

İST 265-02 Matematiksel İstatistik

Ödev 5

X n, d hipergeometrik dağılıma sahip olsun. $M, N \to \infty$'da $p = M/(M+N)$ $0 \le p \le 1$ koşulu altında,

$$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} \to \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

, olur.

İspat:
$$p(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} = \frac{M!}{(M-x)!\,x!} \cdot \frac{(N-M)!}{(n-x)!\,(N-M-n+x)!} \cdot \frac{(N-n)!\,n!}{N!}$$

$$= \binom{n}{x} \frac{M!}{(M-x)!} \frac{(N-M)!}{(N-M-n+x)!} \frac{(N-n)!}{N!}$$

$$= \binom{n}{x} \frac{[M(M-1)(M-2)\dots(M-x+1)][(N-M)(N-M-1)\dots(N-M-(n-x)+1)]}{N(N-1)(N-2)\dots(N-n+1)}$$

$$= \binom{n}{x} \frac{M^x}{M^x} M(M-1)(M-2)\dots(M-x+1) \frac{(N-M)^{n-x}}{(N-M)^{n-x}} \cdot (N-M)(N-M-1)\dots(N-M-(n-x)+1)$$

$$\cdot \frac{N^n}{N^n} \frac{1}{N(N-1)\dots(N-n+1)}$$

$$= \binom{n}{x} M^x \left[1\left(1-\frac{1}{M}\right)\dots\left(1-\frac{(x-1)}{M}\right)\right] (N-M)^{n-x} \left[1\left(1-\frac{1}{N-M}\right)\dots\left(1-\frac{(n-x-1)}{N-M}\right)\right]$$

$$\cdot \frac{1}{N^n} \left[\frac{1}{1\left(1-\frac{1}{N}\right)\dots\left(1-\frac{n-1}{N}\right)}\right]$$

$$= \binom{n}{x} \frac{M^x (N-M)^{n-x}}{N^n} \left[1\left(1-\frac{1}{M}\right)\cdots\left(1-\frac{(x-1)}{M}\right)\right]\left[1\left(1-\frac{1}{N-M}\right)\cdots\left(1-\frac{(n-x-1)}{N-M}\right)\right]$$

$$\left[\frac{1}{1\left(1-\frac{1}{N}\right)\cdots\left(1-\frac{n-1}{N}\right)}\right]$$

Burada $M, N \to \infty$ ve $p = M/(M+N)$ olarak alındığında aşağıdaki binom o.f elde edilir.

$$p(x) = \binom{n}{x}\left(\frac{M}{N}\right)^x \left(\frac{N-M}{N}\right)^{n-x} = \binom{n}{x} p^x (1-p)^{n-x}$$

Kaynakça

- Matematiksel İstatistik, Doç. Dr. Yasemin KAYHAN ATILGAN Doç. Dr. Derya Ersel (pg. 142)