

1) Aşağıda verilen kümelerin geometrik tanımını veriniz.

- a) $x = 1, y = 0$
- b) $y^2 + z^2 = 1, x = 0$
- c) $x^2 + (y - 1)^2 + z^2 = 4, y = 0$
- d) $x^2 + y^2 \leq 1, z = 3$
- e) $x = y$ z 'de kısıtlama yok.

2) Verilen vektörler için aşağıdakileri elde ediniz:

$$\mathbf{v} \cdot \mathbf{u}, |\mathbf{v}|, |\mathbf{u}|$$

\mathbf{v} ile \mathbf{u} arasındaki açının kosinüsü,

(i) $\mathbf{v} = 2\mathbf{i} + 10\mathbf{j} - 11\mathbf{k}, \mathbf{u} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (ii) $\mathbf{v} = 5\mathbf{i} + \mathbf{j}, \mathbf{u} = 2\mathbf{i} + \sqrt{17}\mathbf{j}$

(i) $\mathbf{v} = 2\mathbf{i} + 10\mathbf{j} - 11\mathbf{k} \quad \mathbf{u} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

$$\mathbf{v} \cdot \mathbf{u} = \langle 2, 10, -11 \rangle \cdot \langle 2, 2, 1 \rangle = 4 + 20 - 11 = 13$$

$$\mathbf{v} \cdot \mathbf{u} = |\mathbf{v}| \cdot |\mathbf{u}| \cdot \cos \alpha \quad \cos \alpha = \frac{|\mathbf{v}| \cdot |\mathbf{u}|}{\mathbf{v} \cdot \mathbf{u}}$$

$$|\mathbf{v}| = \sqrt{4 + 100 + 121} = 15 \quad |\mathbf{u}| = \sqrt{4 + 4 + 1} = 3$$

(ii) $\mathbf{u} \cdot \mathbf{v} = \langle -2, \sqrt{17} \rangle \cdot \langle 5, 1 \rangle = 10 + \sqrt{17}$

$$|\mathbf{v}| = \sqrt{25 + 1} = \sqrt{26} \quad |\mathbf{u}| = \sqrt{4 + 17} = \sqrt{21}$$

$$\cos \alpha = \frac{10 + \sqrt{17}}{\sqrt{26} \sqrt{21}}$$

- 3) Paralel ve dik vektörler $u = 5\mathbf{i} - \mathbf{j} + \mathbf{k}$, $v = \mathbf{j} - 5\mathbf{k}$, $w = -15\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ olsun. Varsa, hangi vektörler (a) dik, (b) paraleldir? Yanıtlarınızı açıklayın.

$$u = \langle 5, -1, 1 \rangle \quad v = \langle 0, 1, -5 \rangle \quad w = \langle -15, 3, -3 \rangle$$

$$u \cdot v = -6 \quad u \perp w \quad u \cdot w = -81 \quad u \perp w$$

$$v \cdot w = 18 \quad v \perp w$$

$$\begin{aligned} u \times v &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -1 & 1 \\ 0 & 1 & -5 \end{vmatrix} = \mathbf{i} \begin{vmatrix} -1 & 1 \\ 1 & -5 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 5 & 1 \\ 0 & -5 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 5 & -1 \\ 0 & 1 \end{vmatrix} \\ &= \mathbf{i}(5 - 1) - \mathbf{j}(-25) + \mathbf{k}(5) \\ &= 4\mathbf{i} + 25\mathbf{k} + 5\mathbf{k} \neq 0 \end{aligned}$$

$u \not\parallel v$

$$\begin{aligned} u \times w &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -1 & 1 \\ -15 & 3 & -3 \end{vmatrix} = \mathbf{i} \begin{vmatrix} -1 & 1 \\ 3 & -3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 5 & 1 \\ -15 & -3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 5 & -1 \\ -15 & 3 \end{vmatrix} \\ &= 0 \quad u \parallel w \end{aligned}$$

$$v \times w \neq 0 \quad v \not\parallel w$$

- 4) a) Hangi a değeri veya değerleri için $\mathbf{u} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ ve $\mathbf{v} = -4\mathbf{i} - 8\mathbf{j} + a\mathbf{k}$ vektörleri paralel olur?
 b) $|\mathbf{v}| = 2$, $|\mathbf{w}| = 3$ ve \mathbf{v} ile \mathbf{w} arasındaki açı $\pi/3$ ise, $|\mathbf{v} - 2\mathbf{w}|$ 'yu bulun.

a) $\mathbf{u} = \langle 2, 4, -5 \rangle \quad \mathbf{v} = \langle -4, -8, a \rangle$

$$\mathbf{u} \parallel \mathbf{v} \Leftrightarrow \mathbf{u} \times \mathbf{v} = 0$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & -5 \\ -4 & -8 & a \end{vmatrix} = (4a - 40)\mathbf{i} + (20 - 2a)\mathbf{j} + 0\mathbf{k} = 0$$

$$a = 10$$

b) $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k} \quad \mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k}$. olsun.

$$\begin{aligned} |\mathbf{v} - 2\mathbf{w}|^2 &= |(v_1 - 2w_1)\mathbf{i} + (v_2 - 2w_2)\mathbf{j} + (v_3 - 2w_3)\mathbf{k}|^2 \\ &= (v_1 - 2w_1)^2 + (v_2 - 2w_2)^2 + (v_3 - 2w_3)^2 \\ &= v_1^2 + v_2^2 + v_3^2 - 4(v_1w_1 + v_2w_2 + v_3w_3) + 4(w_1^2 + w_2^2 + w_3^2) \\ &= |\mathbf{v}|^2 - 4\mathbf{v} \cdot \mathbf{w} + 4|\mathbf{w}|^2 \\ &= |\mathbf{v}|^2 - 4|\mathbf{v}| |\mathbf{w}| \cos \alpha + 4|\mathbf{w}|^2 \\ &= 4 - 4 \cdot (2) \cdot (3) \cdot \cos \frac{\pi}{3} + 36 = 2\sqrt{7} \end{aligned}$$

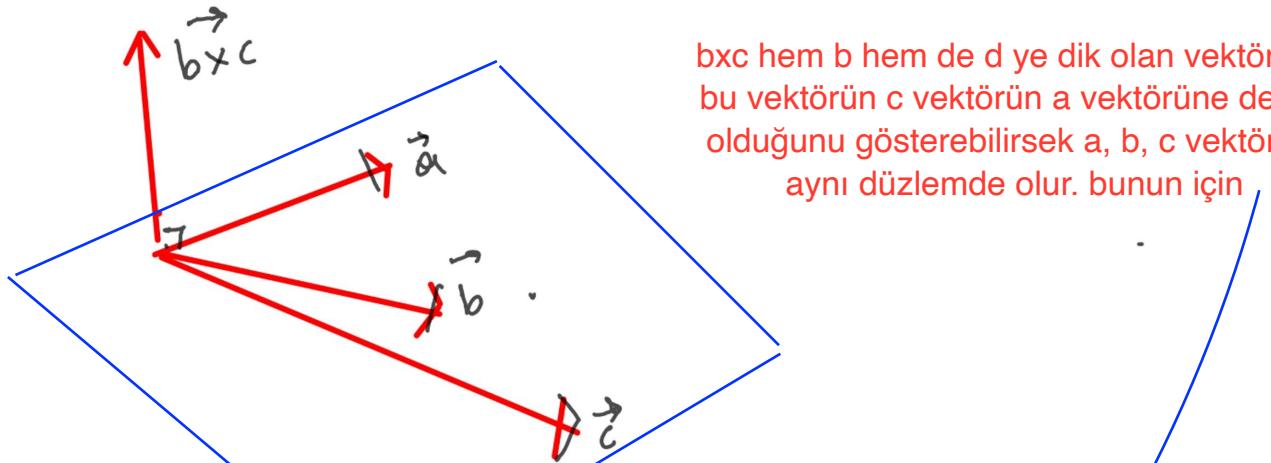
5)

Aşağıdaki vektörler düzlemsel midir, yani aynı düzlem üzerinde midir?

$$\vec{a} = i + 4j - 7k, \vec{b} = 2i - j + 4k, \vec{c} = -9j + 18k$$

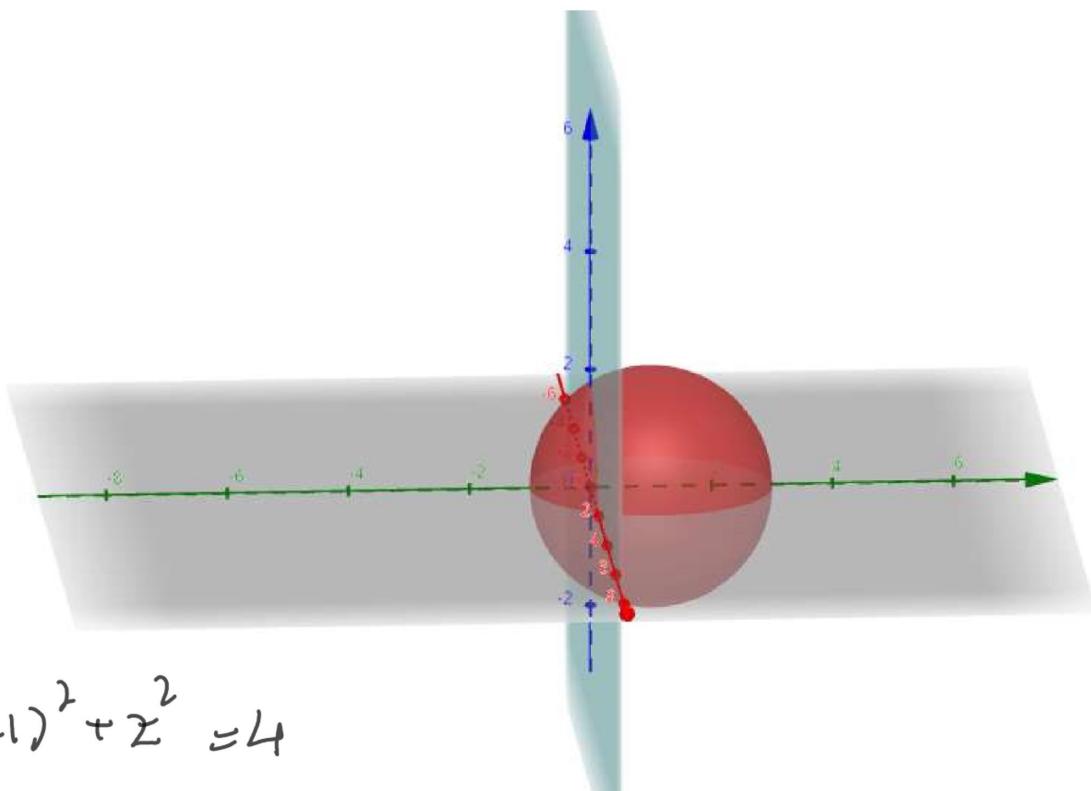
Verilen vektörlerin düzlemsel olabilmeleri için tümünün aynı düzlem içinde olmaları gereklidir.

$$\vec{a} = \langle 1, 4, -7 \rangle \quad \vec{b} = \langle 2, -1, 4 \rangle \quad \vec{c} = \langle 0, 9, -18 \rangle$$



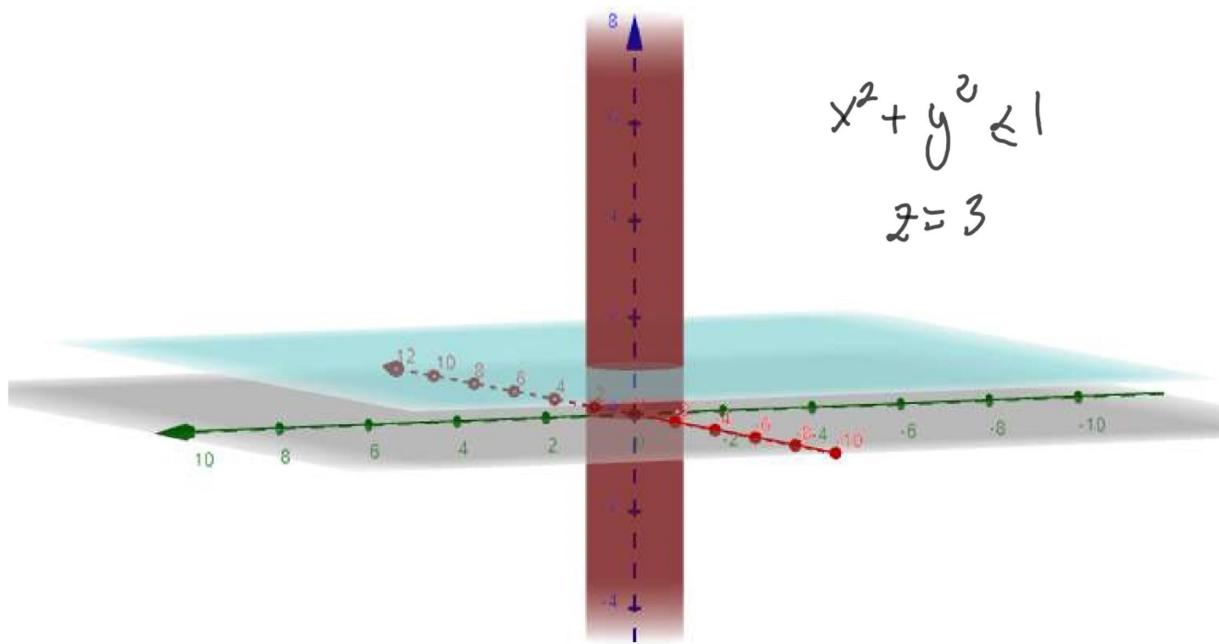
$b \times c$ hem b hem de c ye dik olan vektördür. bu vektörün c vektörünün a vektörüne de dik olduğunu gösterebilirsek a, b, c vektörleri aynı düzlemede olur. bunun için

$$\vec{a}, \vec{b} \text{ ve } \vec{c} \text{ düzlemseldir} \Leftrightarrow a \cdot (b \times c) = 0$$



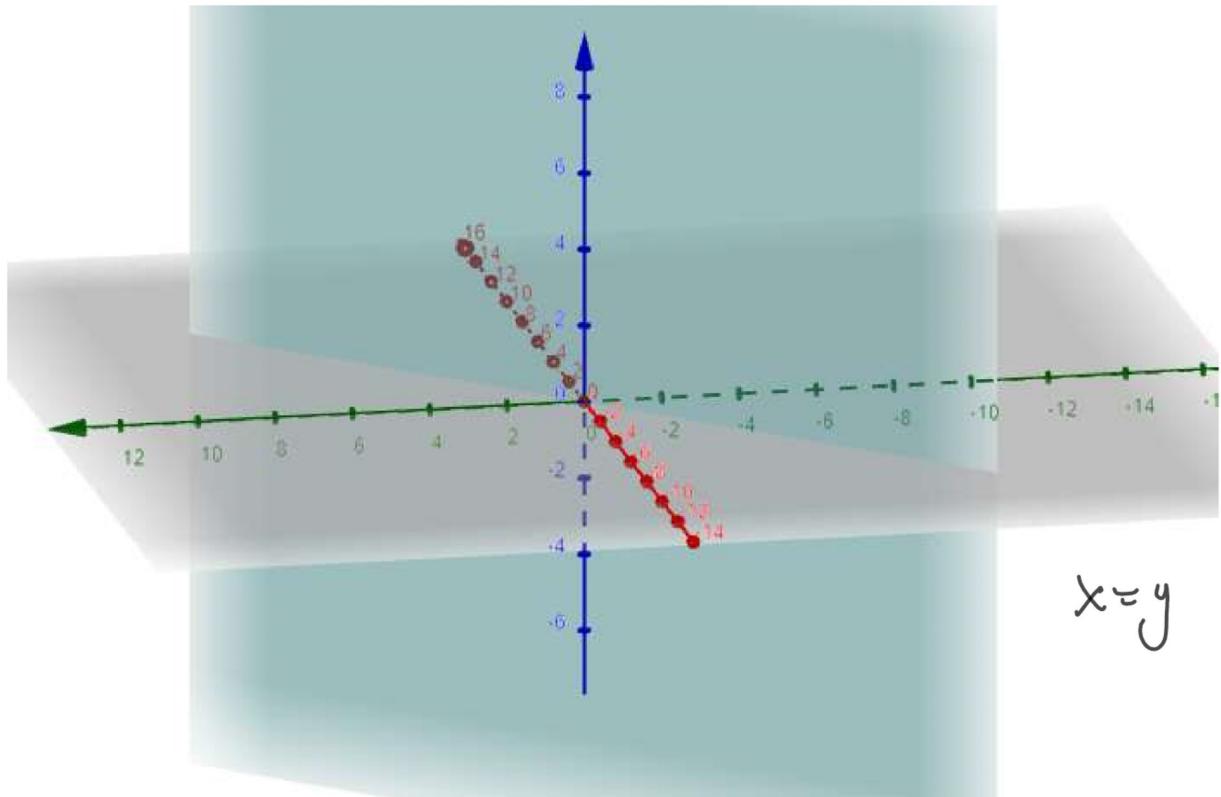
$$x^2 + (y+1)^2 + z^2 = 4$$

$$y=0$$

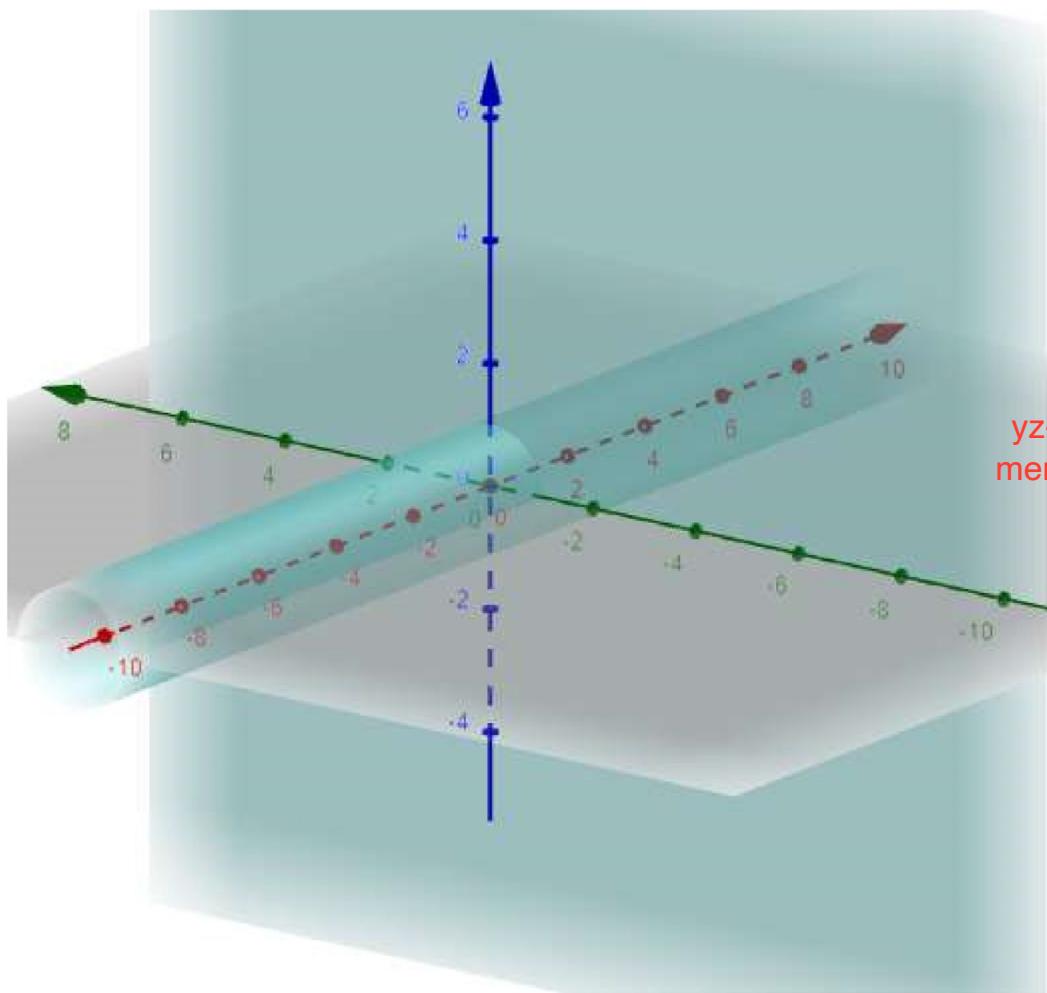


$$x^2 + y^2 \leq 1$$

$$z=3$$

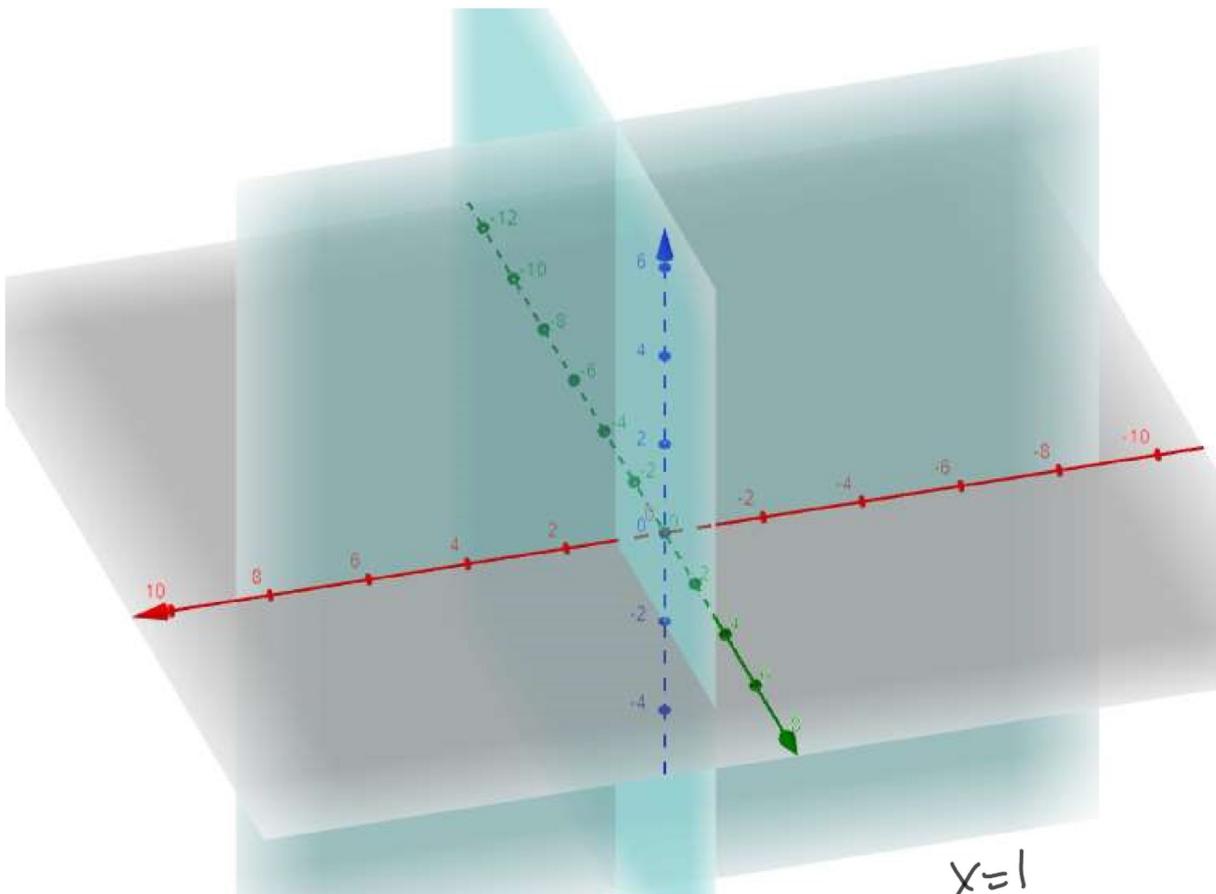


$$x = y \quad \exists t \in \mathbb{R}$$



$$y^2 + z^2 = 1$$

$x = 0$



xz-düzleminde x eksenini 1 de dik kesen z eksenine parallel doğru