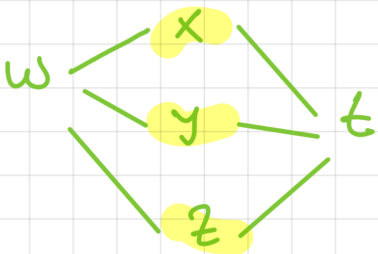
# 3. Hafta

1. $w=\ln(x+2y-z^2)$, $x=2t-1$, $y=\frac{1}{t}$, $z=\sqrt{t}$

ise $\frac{dw}{dt}=?$

Çöz:




$$\frac{dw}{dt}=\frac{\partial w}{\partial x}\frac{dx}{dt}+\frac{\partial w}{\partial y}\frac{dy}{dt}+\frac{\partial w}{\partial z}\frac{dz}{dt}$$

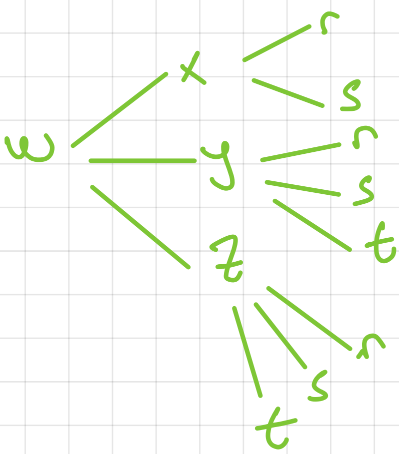
$$=\frac{1}{x+2y-z^2}\cdot 2+\frac{1}{x+2y-z^2}\cdot 2\cdot\left(-\frac{1}{t^2}\right)+$$

$$\frac{1}{x+2y-z^2}\cdot(-2z)\cdot\frac{1}{2}t^{-1/2}$$

2. $w=4x+y^2+z^3$, $x=e^{rs^2}$, $y=\ln\left(\frac{r+s}{t}\right)$, $z=rst^2$

ise $\frac{\partial w}{\partial s}$, $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial t}=?$

Çöz:




$$\frac{\partial w}{\partial s}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial s}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$=4\cdot 2rs\,e^{rs^2}+2y\cdot\frac{t}{r+s}\cdot\frac{1}{t}+3z^2\cdot rt^2$$

Digerleri ödev.

3. x ve y bağımsız değişkenler olmak üzere
$e^{xy}+2z-e^{z}-x^2y+3=0$ ise

$$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}=?$$

Çöz: $z(x,y)$

$$e^{xy}.y+2\frac{\partial z}{\partial x}-e^{z}.\frac{\partial z}{\partial x}-2xy=0$$

$$(2-e^{z}).\frac{\partial z}{\partial x}=2xy-ye^{xy}$$

$$\frac{\partial z}{\partial x}=\frac{2xy-ye^{xy}}{2-e^{z}}$$

2. Yol $F(x,y,z)=e^{xy}+2z-e^{z}-x^2y+3$

$$\frac{\partial z}{\partial x}=\frac{-F_x}{F_z}=-\frac{e^{xy}.y-2xy}{2-e^{z}}$$

$$e^{xy}.x+2.\frac{\partial z}{\partial y}-e^{z}.\frac{\partial z}{\partial y}-x^2=0$$

$$\frac{\partial z}{\partial y}=\frac{-F_y}{F_z}=\frac{x^2-e^{xy}.x}{2-e^{z}}$$

4. $z=x.f\left(\frac{z}{y}\right)$, f türevlenebilir ise $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}=?$

Çöz: $\frac{\partial z}{\partial x}=1.f\left(\frac{z}{y}\right)+x.f'\left(\frac{z}{y}\right).\frac{1}{y}\frac{\partial z}{\partial x}$

$$\frac{\partial z}{\partial x}\left(1-\frac{x}{y}f'\left(\frac{z}{y}\right)\right)=f\left(\frac{z}{y}\right) \Rightarrow$$

$$\frac{\partial z}{\partial x} = \frac{f(z/y)}{1 - \frac{x f'(z/y)}{y}}$$

$$z = x f\left(\frac{z}{y}\right)$$

$$\frac{\partial z}{\partial y} = x \cdot f'\left(\frac{z}{y}\right) \frac{\frac{\partial z}{\partial y} \cdot y - z \cdot 1}{y^2}$$

$$\frac{\partial z}{\partial y} = \frac{x \cdot f'\left(\frac{z}{y}\right) \cdot \frac{z}{y^2}}{\frac{x \cdot f'(z/y)}{y} - 1}$$

II. YOL:

$$F(x,y,z) = z - x f\left(\frac{z}{y}\right)$$

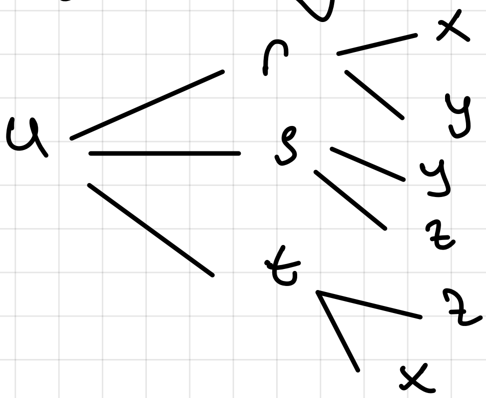
$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}, \quad \frac{\partial z}{\partial y} = \frac{-F_y}{F_z}.$$

ÖDEV.

7. $u = u(r,s,t)$ , $r = x-y$, $s = y-z$ , $t = z-x$
ve u türevlenebilir ise

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$ olduğunu gösteriniz.

Çöz:




$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial x}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial x}$$

$$=\frac{\partial u}{\partial r}\cdot 1+\frac{\partial u}{\partial s}\cdot 0+\frac{\partial u}{\partial t}\cdot(-1)$$

$$=\frac{\partial u}{\partial r}-\frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial r}\frac{\partial r}{\partial y}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial y}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$$

$$=\frac{\partial u}{\partial r}(-1)+\frac{\partial u}{\partial s}\cdot 1+\frac{\partial u}{\partial t}\cdot 0$$

$$=\frac{\partial u}{\partial s}-\frac{\partial u}{\partial r}$$

$$\frac{\partial u}{\partial z}=\frac{\partial u}{\partial r}\frac{\partial r}{\partial z}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial t}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial z}$$

$$=\frac{\partial u}{\partial r}\cdot 0+\frac{\partial u}{\partial s}\cdot(-1)+\frac{\partial u}{\partial t}\cdot 1$$

$$=\frac{\partial u}{\partial t}-\frac{\partial u}{\partial s}$$

$-2 \quad +8 \quad +4$

9. $x^2y + y^2z + z^2x = 10$ yüzeyinin $P_0 = (1, -2, 2)$ noktasındaki teğet düzlemi ve normal doğrusunu bulunuz.

Çöz: $F(x,y,z) = x^2y + y^2z + z^2x - 10 = 0$

$\nabla F = \langle F_x, F_y, F_z \rangle$ → normal doğrusu, geç. nok. $(x_0, y_0, z_0)$

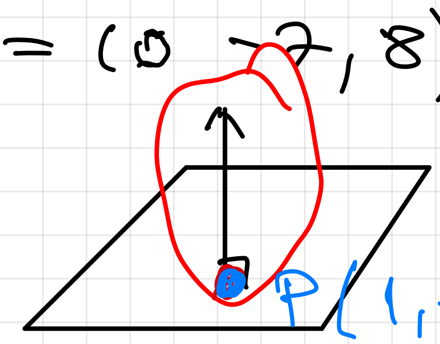
T. D: $F_x(x-x_0) + F_y(y-y_0) + F_z(z-z_0) = 0$

normal doğrusu: $(F_x, F_y, F_z)\big|_{(x_0, y_0, z_0)}$

$$(2xy + z^2,\ x^2 + 2yz,\ y^2 + 2xz)\big|_{(1,-2,2)}$$

$$= (2.1.-2 + 4,\ 1 + 2.(-2).2,\ 4 + 2.1.2)$$

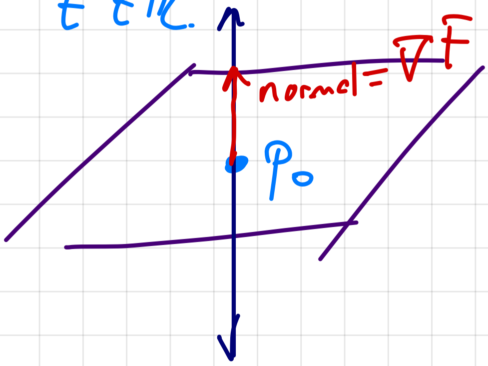
$$= (0, -7, 8)$$ ⇒ normal doğrusu




$$0.(x-1) - 7.(y+2) + 8.(z-2) = 0$$

normal doğrunun paralel olduğu vektör
$n = \langle 0, -7, 8 \rangle$
geçtiği nokta $P_0 = (1, -2, 2)$

parametrik denk: $\begin{cases} x = 1 + 0t \\ y = -2 + -7t \\ z = 2 + 8t \end{cases}$ $t \in \mathbb{R}$.




ye da

simetrik denklem: $x = 1, \quad \dfrac{y+2}{-7} = \dfrac{z-2}{8}$

normal doğrusu denklemleridir.

10. $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{a}$ $(a>0)$ yüzeyinin herhangi bir teğet düzleminin eksenleri kestiği noktaların toplamının $a$'ya eşit olduğunu gösteriniz.

Çöz: $(x_0, y_0, z_0)$ yüzeyin üzerinde bir nokta olsun. Yani

$$\sqrt{x_0} + \sqrt{y_0} + \sqrt{z_0} = \sqrt{a}$$

$(x_0, y_0, z_0)$'daki teğet düzlemi:

$$F(x,y,z) = \sqrt{x} + \sqrt{y} + \sqrt{z} - \sqrt{a}$$

$$F_x = \frac{1}{2\sqrt{x}}, \quad F_y = \frac{1}{2\sqrt{y}}, \quad F_z = \frac{1}{2\sqrt{z}}$$

T. D. $\dfrac{1}{2\sqrt{x_0}}(x - x_0) + \dfrac{1}{2\sqrt{y_0}}(y - y_0) + \dfrac{1}{2\sqrt{z_0}}(z - z_0) = 0$

Eksenleri kestiği noktalar!

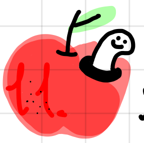
$$x=0,\ y=0 \Rightarrow z = \left( \overset{=\sqrt{x_0}}{\frac{x_0}{2\sqrt{x_0}}} + \overset{=\sqrt{y_0}}{\frac{y_0}{2\sqrt{y_0}}} + \overset{=\sqrt{z_0}}{\frac{z_0}{2\sqrt{z_0}}} \right) 2\sqrt{z_0}$$

$$y=0,\ z=0 \Rightarrow x = \left( \frac{y_0}{2\sqrt{y_0}} + \frac{z_0}{2\sqrt{z_0}} + \frac{x_0}{2\sqrt{x_0}} \right) \cdot 2\sqrt{x_0}$$

$$x=0,\ z=0 \Rightarrow y = \left( \frac{x_0}{2\sqrt{x_0}} + \frac{z_0}{2\sqrt{z_0}} + \frac{y_0}{2\sqrt{y_0}} \right) 2\sqrt{y_0}$$

$$x+y+z = \frac{1}{2}\left( \underbrace{\sqrt{x_0}+\sqrt{y_0}+\sqrt{z_0}}_{\sqrt{a}} \right) 2 \cdot \left( \underbrace{\sqrt{x_0}+\sqrt{y_0}+\sqrt{z_0}}_{\sqrt{a}} \right)$$

$$= \sqrt{a}\cdot\sqrt{a} = a \quad //$$



11. $x^2+y^2+z^2=2$ küresi ile $xy=1$ hiperbolik silindirinin $(1,1,0)$ noktasında birbirine teğet olduğunu gösteriniz.

Çöz: $F(x,y,z) = x^2+y^2+z^2-2$

$x_0=1$

$y_0=1$

$z_0=0$

$G(x,y,z) = xy-1$

T.D: $F_x(x-x_0) + F_y(y-y_0) + F_z(z-z_0) = 0$

$$2x_0\Big|_{x_0=1}(x-1) + 2y_0\Big|_{y_0=1}(y-1) + 2z_0\Big|_{z_0=0}(z-0) = 0$$

$$2(x-1) + 2(y-1) = 0$$

$$x+y=2$$

$G(x,y,z) = xy - 1$

T.D: $G_x(x-x_0) + G_y(y-y_0) + G_z(z-z_0) = 0$

$y_0|(x-1) + x_0|(y-1) + 0(z-0) = 0$

$y_0 = 1$ $\qquad$ $x_0 = 1$

$= (x-1) + (y-1) = 0$

$x + y = 2$

$\Rightarrow$ Bu iki yuzey $(1,1,0)$'da tegettir.