HACETTEPE ÜNİVERSİTESİ

SORULAR:

1) $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ matrisinin satır denk olduğu satır indirgenmiş eşelon formunu bulunuz.

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \xrightarrow[-2R_1+R_2 \to R_2]{R_1+R_3 \to R_3} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2+R_1 \to R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3+R_1 \to R_1]{R_3+R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

2) $A = \begin{bmatrix} -2 & 6 & -2 & 2 \\ 1 & -1 & 0 & 1 \end{bmatrix}$ matrisinin satırdenk olduğu R satır indirgenmiş eşelon matrisi ve $R = P.A$ olacak şekilde P tersinir matrisi bulunuz.

$$A \xrightarrow[R_1 \leftrightarrow R_2]{\varepsilon_1} \begin{bmatrix} 1 & -1 & 0 & 1 \\ -2 & 6 & -2 & 2 \end{bmatrix} \xrightarrow[\varepsilon_2]{2R_1+R_2 \to R_2} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 4 & -2 & 4 \end{bmatrix}$$

$$\xrightarrow[\varepsilon_3]{\frac{R_2}{4} \to R_2} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & -\frac{1}{2} & 1 \end{bmatrix} \xrightarrow[\varepsilon_4]{R_2+R_1 \to R_1} \begin{bmatrix} 1 & 0 & -1/2 & 2 \\ 0 & 1 & -1/2 & 1 \end{bmatrix} = R$$

P tersinir matrisini bulmak için 2 yol var.

1.yol: Elementer matrisler yardımıyla.

$\varepsilon_1(I) = E_1$, $\varepsilon_2(I) = E_2$, $\varepsilon_3(I) = E_3$, $\varepsilon_4(I) = E_4$

$$\boxed{P = E_4 . E_3 . E_2 . E_1}$$

$$\begin{bmatrix}1&0\\0&1\end{bmatrix} \xrightarrow[E_1]{R_1 \leftrightarrow R_2} \begin{bmatrix}0&1\\1&0\end{bmatrix} = E_1 \qquad \begin{bmatrix}1&0\\0&1\end{bmatrix} \xrightarrow{\frac{R_2}{4} \to R_2} \begin{bmatrix}1&0\\0&1/4\end{bmatrix} = E_3$$

$$\begin{bmatrix}1&0\\0&1\end{bmatrix} \xrightarrow[E_2]{2R_1+R_2 \to R_2} \begin{bmatrix}1&0\\2&1\end{bmatrix} = E_2 \qquad \begin{bmatrix}1&0\\0&1\end{bmatrix} \xrightarrow{R_2+R_1 \to R_1} \begin{bmatrix}1&1\\0&1\end{bmatrix} = E_4$$

$$P = E_4 . E_3 . E_2 . E_1 = \begin{bmatrix}1&1\\0&1\end{bmatrix} \cdot \begin{bmatrix}1&0\\0&1/4\end{bmatrix} \cdot \begin{bmatrix}1&0\\2&1\end{bmatrix} \cdot \begin{bmatrix}0&1\\1&0\end{bmatrix} = \begin{bmatrix}1/4&3/2\\1/4&1/2\end{bmatrix} //$$

2. yol: $[A|I] \longrightarrow [R|P]$

$$\left[\begin{array}{cccc|cc}-2&6&2&-2&1&0\\1&-1&0&1&0&1\end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|cc}1&-1&0&1&0&1\\-2&6&2&-2&1&0\end{array}\right] \xrightarrow{2R_1+R_2 \to R_2}$$

$$\left[\begin{array}{cccc|cc}1&-1&0&1&0&1\\0&4&2&0&1&2\end{array}\right] \xrightarrow{\frac{R_2}{4} \to R_2} \left[\begin{array}{cccc|cc}1&-1&0&1&0&1\\0&1&\frac{1}{2}&0&\frac{1}{4}&\frac{1}{2}\end{array}\right] \xrightarrow{R_2+R_1 \to R_1}$$

$$\left[\begin{array}{cccc|cc}1&0&1/2&1&1/4&3/2\\0&1&1/2&0&1/4&1/2\end{array}\right] //$$

$$\underbrace{\qquad\qquad}_{\shortparallel \atop R} \qquad \underbrace{\qquad}_{\shortparallel \atop P}$$

3) $A = \begin{bmatrix}1&2&0&-1&1\\-1&2&1&4&3\\2&4&0&-2&2\end{bmatrix}$ ve $B = \begin{bmatrix}1&-2&0&1&-1\\1&2&1&2&5\\0&0&1&3&4\end{bmatrix}$

matrislerinin satır denk olduklarını gösteriniz.

$A \longrightarrow \cdots \longrightarrow R_1$ → A'nın satır ind eşelon matrisi

$B \longrightarrow \cdots \longrightarrow R_2$ → B'nin satır ind. eşelon matrisi

Eğer iki matrisin satır indirgenmiş eşelon matrisler eşit ise bu iki matrisler satır denktir. Eşit değilse satır denk olamazlar.

$$A \xrightarrow[-2R_1+R_3\to R_3]{R_1+R_2\to R_2} \begin{bmatrix} 1 & 2 & 0 & -1 & 1 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = R_1$$

$$B \xrightarrow{R_1+R_2\to R_2} \begin{bmatrix} -1 & -2 & 0 & 1 & -1 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 3 & 4 \end{bmatrix} \xrightarrow[-R_2+R_3\to R_3]{-R_1\to R_1} \begin{bmatrix} 1 & 2 & 0 & -1 & 1 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = R_2$$

$R_1 = R_2$ olduğundan $A \approx B$'dir.