SORULAR:

(1)
$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 7 & -5 & 0 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 & 0 \\ 5 & 1 & -1 & 3 & 0 & 0 \\ 2 & 1 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 1.(-5).1.3.3.1 = -45$$

(2)
$$\begin{vmatrix} 1 & a & 2 \\ -1 & 1 & b \\ a & 2 & 3b \end{vmatrix} = 4$$ olduğu biliniyor.

a)
$$\begin{vmatrix} x^2 & ax & 2x \\ -x & 1 & b \\ ax & 2 & 3b \end{vmatrix} = x \begin{vmatrix} x & a & 2 \\ -x & 1 & b \\ ax & 2 & 3b \end{vmatrix} = x^2 . \underbrace{\begin{vmatrix} 1 & a & 2 \\ -1 & 1 & b \\ a & 2 & 3b \end{vmatrix}}_{4} = 4x^2$$

b)
$$\begin{vmatrix} -1 & -2a & -2 \\ -2 & 4 & 2b \\ a & 4 & 3b \end{vmatrix} = 2 \begin{vmatrix} -1 & -a & -2 \\ -2 & 2 & 2b \\ a & 2 & 3b \end{vmatrix} = 2.2.(-1). \underbrace{\begin{vmatrix} 1 & a & 2 \\ -1 & 1 & b \\ a & 2 & 3b \end{vmatrix}}_{4} = -16$$

c)
$$\begin{vmatrix} a+1 & a+2 & 2+3b \\ -1 & 1 & b \\ a & 2 & 3b \end{vmatrix} = \underbrace{\begin{vmatrix} \boxed{a \quad 2 \quad 3b} \\ -1 \quad 1 \quad b \\ \boxed{a \quad 2 \quad 3b} \end{vmatrix}}_{\substack{\text{aynı satır} \\ 0}} + \underbrace{\begin{vmatrix} 1 & a & 2 \\ -1 & 1 & b \\ a & 2 & 3b \end{vmatrix}}_{4} = 4$$

d) $$\begin{vmatrix} x & a+bx & 2 \\ -x & 1-bx & b \\ ax & 2+abx & 3b \end{vmatrix} = \begin{vmatrix} x & a & 2 \\ -x & 1 & b \\ ax & 2 & 3b \end{vmatrix} + \begin{vmatrix} x & bx & 2 \\ -x & -bx & b \\ ax & abx & 3b \end{vmatrix}$$

$$= x \underbrace{\begin{vmatrix} 1 & a & 2 \\ -1 & 1 & b \\ a & 2 & 3b \end{vmatrix}}_{4} + bx^2 \cdot \underbrace{\begin{vmatrix} 1 & 1 & 2 \\ -1 & -1 & b \\ a & a & 3b \end{vmatrix}}_{\text{aynı sütun} = 0} = 4x$$

3) $$\begin{vmatrix} 1 & 4 & 2 & x+4x^2+2x^3 \\ -1 & 2 & 3 & -x+2x^2+3x^3 \\ 3 & 1 & 2 & 3x+x^2+2x^3 \\ 0 & 1 & 1 & 3+x^2+x^3 \end{vmatrix} = \underbrace{\begin{vmatrix} 1 & 4 & 2 & x \\ -1 & 2 & 3 & -x \\ 3 & 1 & 2 & 3x \\ 0 & 1 & 1 & 0 \end{vmatrix}}_{\substack{x\text{'i çıkarınca} \\ \text{aynı sütun} = 0}} + \underbrace{\begin{vmatrix} 1 & 4 & 2 & 4x^2 \\ -1 & 2 & 3 & 2x^2 \\ 3 & 1 & 2 & x^2 \\ 0 & 1 & 1 & x^2 \end{vmatrix}}_{\substack{x^2\text{'yi çıkarınca} \\ \text{aynı sütun} = 0}} + \underbrace{\begin{vmatrix} 1 & 4 & 2 & 2x^3 \\ -1 & 2 & 3 & 3x^3 \\ 3 & 1 & 2 & 2x^3 \\ 0 & 1 & 1 & x^3 \end{vmatrix}}_{\substack{x^3\text{'ü çıkarınca} \\ \text{aynı sütun} = 0}}$$

$$+ \underbrace{\begin{vmatrix} 1 & 4 & 2 & 0 \\ -1 & 2 & 3 & 0 \\ 3 & 1 & 2 & 0 \\ 0 & 1 & 1 & 3 \end{vmatrix}}_{?}$$

$$\begin{vmatrix} 1 & 4 & 2 & 0 \\ -1 & 2 & 3 & 0 \\ 3 & 1 & 2 & 0 \\ 0 & 1 & 1 & 3 \end{vmatrix} = 3 \cdot \begin{vmatrix} 1 & 4 & 2 \\ -1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix} \xrightarrow[-3R_1+R_3 \to R_3]{R_1+R_2 \to R_2} 3 \cdot \begin{vmatrix} 1 & 4 & 2 \\ 0 & 6 & 5 \\ 0 & -11 & -4 \end{vmatrix} = 3 \cdot 1 \cdot \underbrace{\begin{vmatrix} 6 & 5 \\ -11 & -4 \end{vmatrix}}_{-24+55 = 31} = 93$$

4) E bir idempotent matris ise ($E^2 = E$) ve $E \neq I$ ise $\det(E) = 0$'dır.

$$E^2 = E \Rightarrow \det(E^2) = \det(E).\det(E) = \det(E)$$

$$\Rightarrow \underbrace{\det(E)}_{\text{sayı}}.(\underbrace{\det(E) - 1}_{\text{sayı}}) = 0$$

$$\Rightarrow \det(E) = 0 \quad \text{yada} \quad \underbrace{\det(E) = 1}_{\text{E tersinirdir.}} \text{'dır.}$$

$$E^{-1}E^2 = E\,E^{-1} \Rightarrow E = I \ \#$$

$$\Rightarrow \det(E) = 0 \text{ 'dır.}$$

5) c skaler $A_{n\times n}$ matrisi ise $\boxed{\det(c.A) = c^n \det(A)}$ 'dır.

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & \cdots & \cdots & A_{2n} \\ \vdots & & & \\ A_{n1} & \cdots & \cdots & A_{nn} \end{bmatrix} \Rightarrow c.A = \begin{bmatrix} cA_{11} & cA_{12} & \cdots & cA_{1n} \\ \vdots & & & \\ cA_{n1} & \cdots & \cdots & cA_{nn} \end{bmatrix}$$

$$\begin{vmatrix} cA_{11} & cA_{12} & \cdots & cA_{1n} \\ cA_{12} & cA_{22} & \cdots & cA_{2n} \\ \vdots & & & \\ cA_{1n} & \cdots & \cdots & cA_{nn} \end{vmatrix} \begin{matrix} \to c \\ \to c \\ \\ \to c \end{matrix} = \underbrace{c.c \ldots c}_{\text{n satırdan n tane c gelir.}} . \begin{vmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \\ A_{n1} & \cdots & A_{nn} \end{vmatrix} = c^n.\det(A)$$

6)

$$A=\begin{vmatrix} 2 & -5 & 4 & -1 & 1 \\ -7 & 3 & -2 & 1 & 0 \\ 1^{+} & 0^{-} & 0^{+} & 0^{-} & 0^{+} \\ 0 & 4 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & -2 \end{vmatrix} = \underbrace{1.(-1)^{3+1}}_{1} \cdot \begin{vmatrix} -5 & 4 & -1 & 1 \\ 3 & -2 & 1 & 0 \\ 4 & 1 & 0 & 0 \\ 1 & 0 & 0 & -2 \end{vmatrix}$$

$$\begin{array}{cccc} - & + & - & + \end{array}$$

$$= \underbrace{(-1).1}_{-1} . \begin{vmatrix} 4 & -1 & 1 \\ -2 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} - 2 . \begin{vmatrix} -5 & 4 & -1 \\ 3 & -2 & 1 \\ 4 & 1 & 0 \end{vmatrix} = - \underbrace{\begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}}_{-1} - 2 . \left(-1 . \underbrace{\begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix}}_{\substack{3+8 \\ =13}} - 1 . \underbrace{\begin{vmatrix} -5 & 4 \\ 4 & 1 \end{vmatrix}}_{\substack{-5-16 \\ = \\ =21}} \right)$$

$$= +(+1) - 2 (\underbrace{-13-21}_{-34}) = 1 + 68 = \underline{\underline{69}}$$

7)

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix} \xrightarrow[=]{R_2+R_3 \to R_3} \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a+b+c & a+b+c & a+b+c \end{vmatrix} = (a+b+c) . \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

aynı satır
$=$
0

Sonuç 0'dır.

8) $A_{n\times n}$ ortogonal $(A^{-1}=A^{T})$ ise $\det(A) = \mp 1$ 'dir.

$$A^{-1} = A^{T} \Rightarrow \det(A^{-1}) = \det(A^{T})$$

$$\Rightarrow \frac{1}{\det(A)} = \det(A) \Longrightarrow \det(A)^2 = 1$$

sayı

$$\Rightarrow \det(A) = \mp 1 \text{ 'dir.}$$

9) $A_{n\times n}$ matrisi ters-simetrik ve n tek ise $\det(A)=0$ 'dır.

A ters simetrik $\Rightarrow A^T=-A$

$$\Rightarrow \det(A^T)=\det(-A)$$

$$\Rightarrow \det(A)=(-1)^n.\det(A) \overset{n \text{ tek}}{=} -\det(A)$$

$$\Rightarrow 2\det(A)=0 \Rightarrow \det(A)=0.$$

10) $\begin{vmatrix} 2 & 4 & 5 \\ 0 & -6 & 2 \\ 0 & 0 & 3 \end{vmatrix} = 2.(-6).3=-36$

11) $AB=AC$ ve $\det(A)\neq 0$ olsun. O zaman $B=C$'dir.

$\det(A)\neq 0 \Rightarrow A$ tersinirdir.

$$\Rightarrow \underbrace{A^{-1}.A}_{I}.B=\underbrace{A^{-1}.A}_{I}.C \Rightarrow B=C$$

12) $A=\begin{bmatrix} t-1 & 0 & 1 \\ -2 & t+2 & -1 \\ 0 & 0 & t+1 \end{bmatrix}$ matrisi hangi t değeri için tersinir değildir?

1. yol $A \rightarrow \cdots\cdots \rightarrow$ indirgeme yoluyla. (4. hafta ders notlarında var.)

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2. yol: $\det(A) \neq 0 \Leftrightarrow$ tersinirdir. (tersinir değildir $\Leftrightarrow \det A = 0$)

$$|A| = (t+1) \cdot \begin{vmatrix} t-1 & 0 \\ -2 & t+2 \end{vmatrix} = (t+1) \cdot (t-1) \cdot (t+2) = 0$$

$\Rightarrow$ $t=-1$, $t=1$ ve $t=-2$ için $\det A = 0$ olur yani A tersinir olamaz.