

# CSE 211 Digital Design

Akdeniz University

Week01: Introduction to Digital Design

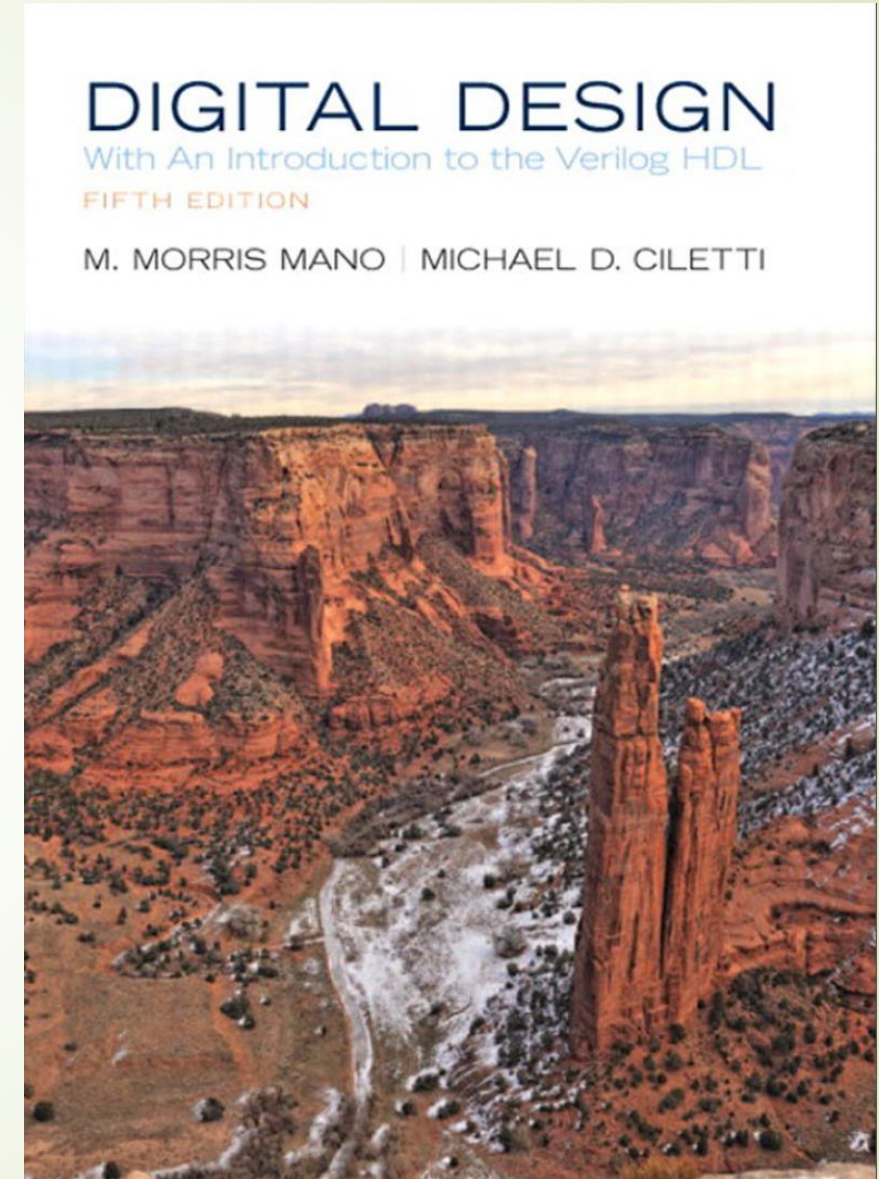
Assoc.Prof.Dr. Taner Danişman

[tdanisman@akdeniz.edu.tr](mailto:tdanisman@akdeniz.edu.tr)

Güz Yarıyılı		Bahar Yarıyılı
04 Ağustos 2023	Ders Görevlendirmelerinin Rektörlüğe Bildirilmesinin Son Günü	12 Ocak 2024
18 Ağustos 2023	Özel Öğrenci Başvurusu İçin Son Gün	26 Ocak 2024
25-29 Eylül 2023	Katkı Payı/Öğrenim Ücretleri Yatırma ve Kayıt Yenileme Süresi	12-16 Şubat 2024
29 Eylül 2023	Öğrenime Ara İzni Başvurusu İçin Son Gün	16 Şubat 2024
02 Ekim 2023	Derslerin Başlaması	19 Şubat 2024
02-06 Ekim 2023	Ders Bırakma ve Ders Ekleme Süresi (Ekle-Çıkar)	19-23 Şubat 2024
09 Ekim 2023	Şubelere Ayrılan Derslerin Rektörlüğe Bildirilmesinin Son Günü	26 Şubat 2024
20 Ekim 2023	Dersten Çekilmenin Son Günü	08 Mart 2024
05 Ocak 2024	Ara Sınav Sonuçlarının ve Diğer Yıl/Yarıyıl İçi Ölçme Araçları Sonuçlarının Otomasyon Sistemine Girilmesinin Son Tarihi	31 Mayıs 2024
05 Ocak 2024	Derslerin Sona Ermesi	31 Mayıs 2024
08-19 Ocak 2024	Yarıyıl Sonu Sınavları	03-14 Haziran 2024
22 Ocak 2024	Yarıyıl Sonu Sınav Sonuçlarının Otomasyon Sistemine Girilmesinin Son Günü	20 Haziran 2024
20-27 Ocak 2024	Yıl/Yarıyıl Sonu İkinci Sınavı(Bütünleme) Başvuru Tarihleri	15-22 Haziran 2024
29 Ocak-02 Şubat 2024	Yıl/Yarıyıl Sonu İkinci Sınavı(Bütünleme) Tarihleri	24-28 Haziran 2024
05 Şubat 2024	Yıl/Yarıyıl Sonu İkinci Sınavı(Bütünleme) Sonuçlarının Otomasyon Sistemine Girilmesinin Son Günü	01 Temmuz 2024

# Evaluation (absolute)

- Assignments %20
- Midterm %30
- Final %50
- Teaching Assistant: Erdiñç TÜRK
- Academic misconduct: **do not let this happen**
- **Monday 13:30-15:20 (Location : Amphi 1)**
- **Thursday 13:30:15:20 ( Location: Ylab 1 )**
- **Textbook :** Digital Design, With an Introduction to the Verilog HDL 5th Edition, Morris Mano, Michael Ciletti, Pearson.



## Can I use the previous year's Lab grades?

- ➡ **YES**, you can, but you should inform the Teaching Asistant in the beginning of the semestre (first week).

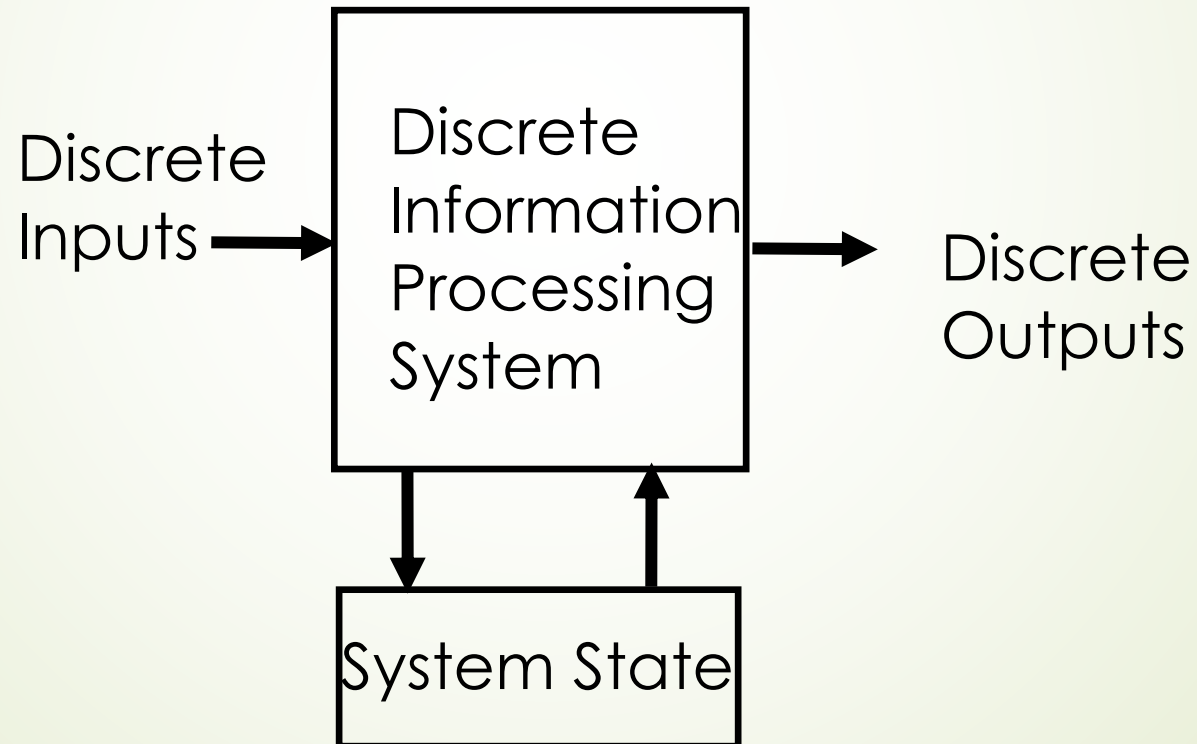
# Course program

Week 01	2-Oct-23	Introduction
Week 02	9-Oct-23	Digital Systems and Binary Numbers I
Week 03	16-Oct-23	Digital Systems and Binary Numbers II
Week 04	23-Oct-23	Boolean Algebra and Logic Gates I
Week 05	30-Oct-23	Boolean Algebra and Logic Gates II
Week 06	6-Nov-23	Gate Level Minimization
Week 07	13-Nov-23	Karnaugh Maps
Week 08	20-Nov-23	Midterm
Week 09	27-Nov-23	Karnaugh Maps
Week 10	4-Dec-23	Combinational Logic
Week 11	11-Dec-23	Combinational Logic
Week 12	18-Dec-23	Timing, delays and hazards
Week 13	25-Dec-23	Synchronous Sequential Logic
Week 14	1-Jan-24	Synchronous Sequential Logic



# Digital System

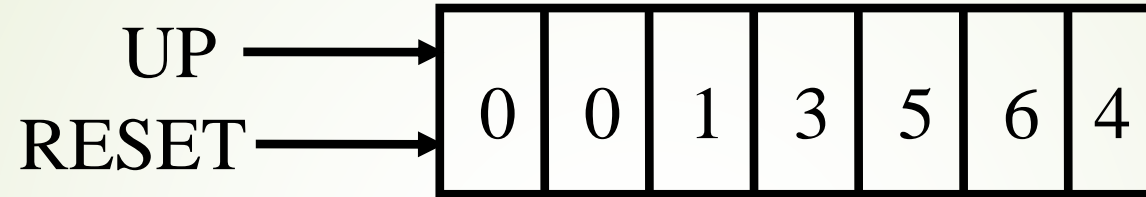
- Takes a set of discrete information inputs and discrete internal information (system state) and generates a set of discrete information outputs.



# Types of Systems

- ▶ With no state present
  - ▶ Combinational logic system
  - ▶  $\text{Output} = \text{Function}(\text{Input})$
- ▶ With state present
  - ▶ State updated at discrete times (e.g., once per clock tick)
    - Synchronous sequential system
  - ▶ State updated at any time
    - Asynchronous sequential system

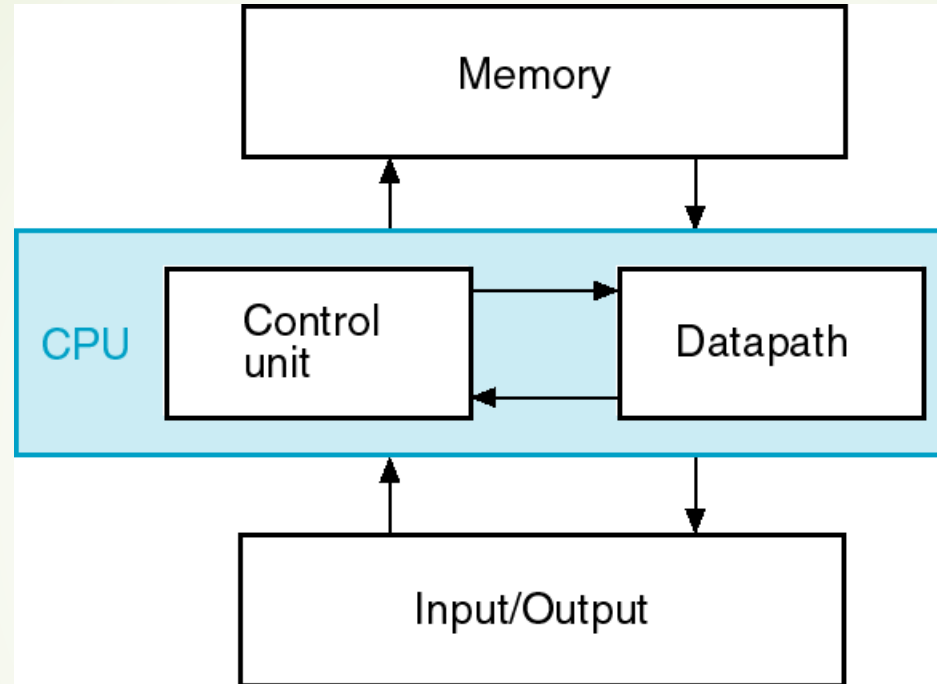
# Example: Digital Counter (e.g., Odometer)



- Inputs: Count Up, Reset
- Outputs: Visual Display
- State: “Value” of stored digits



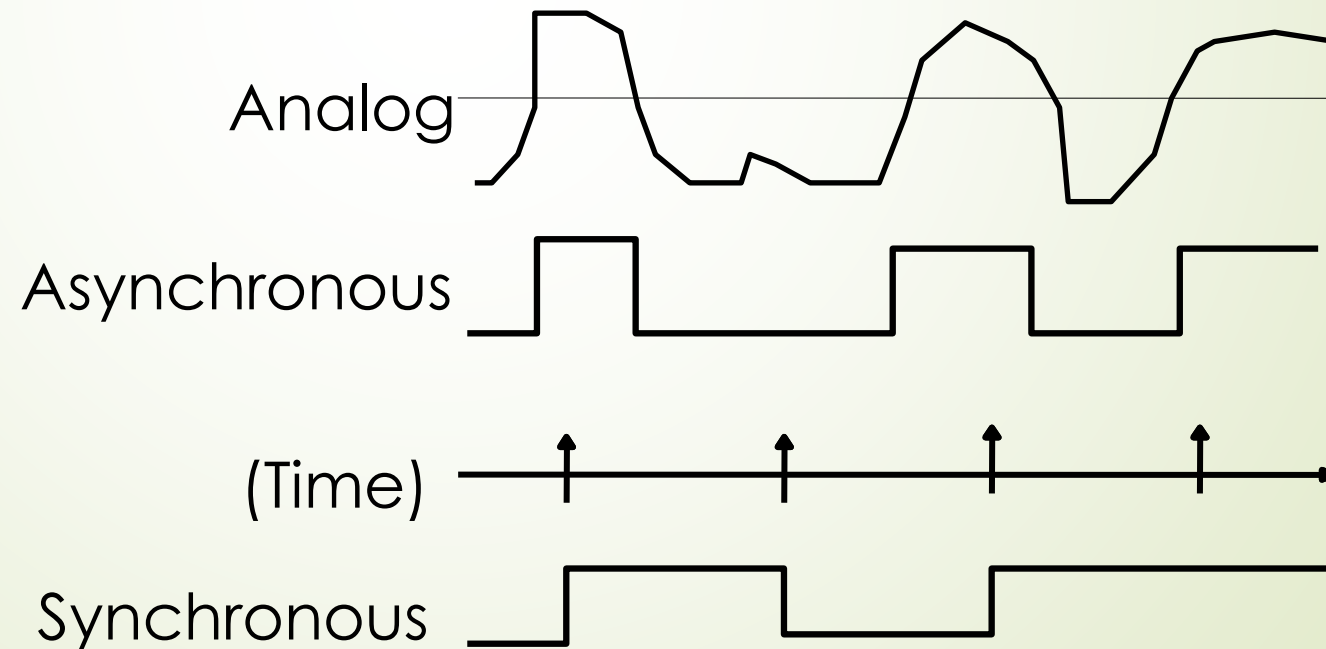
# Example: Digital Computer



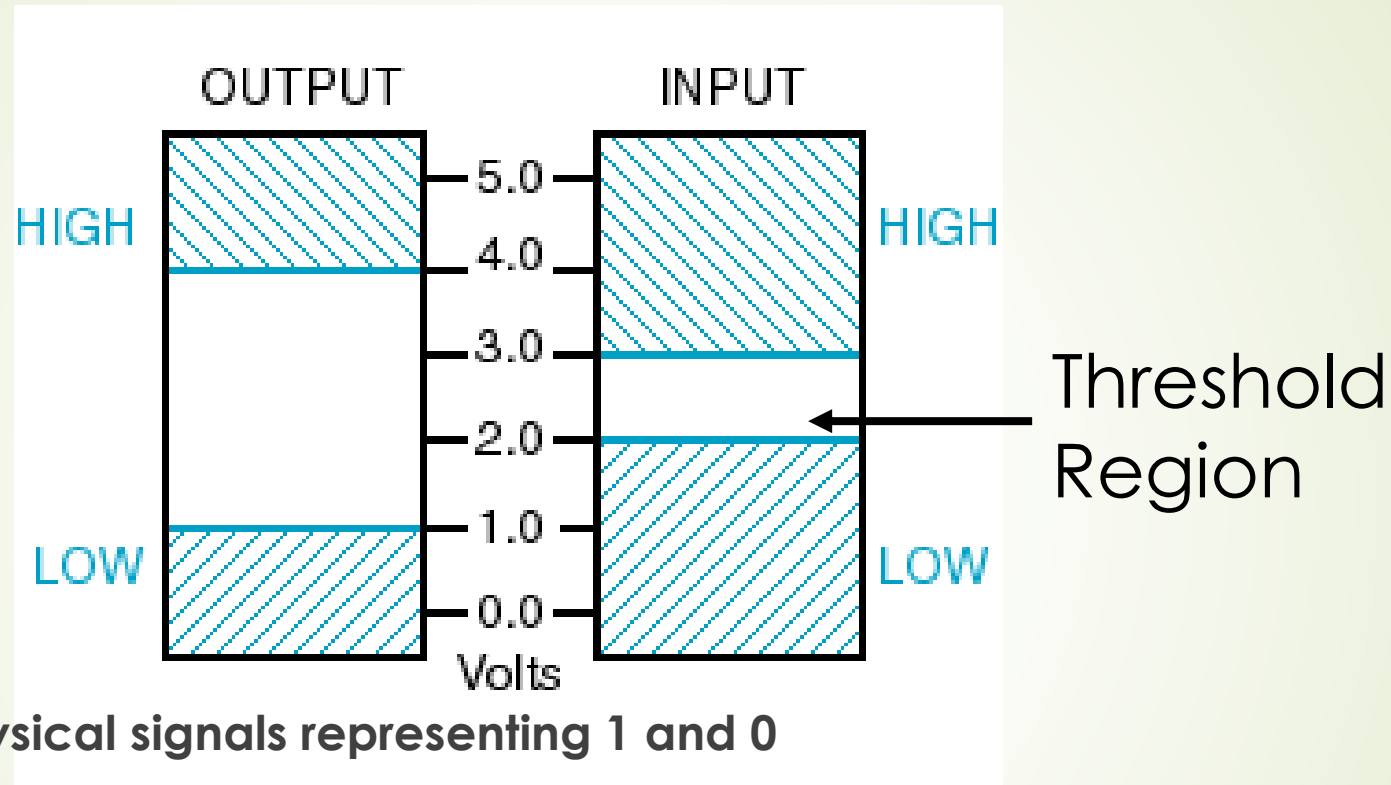
- Inputs: keyboard, mouse, modem, microphone
- Outputs: CRT, LCD, modem, speakers
- Is this system synchronous or asynchronous?

# Signals

- Information variables mapped to physical quantities
- In digital systems, the quantities take on *discrete* values
  - Two-level, or *binary*, values are the most prevalent values in digital systems
  - Binary values are represented abstractly by digits 0 and 1
- Signal examples over time:



# Physical Signal Example - Voltage



## ➤ Other physical signals representing 1 and 0

- CPU Voltage
- Disk Magnetic field direction
- CD Surface pits / light
- Dynamic RAM Charge

# Number Systems

## ➤ Decimal Numbers

➤ What does 5,634 represent?

➤ Expanding 5,634:

$$5 \times 10^3 = 5,000$$

$$+ 6 \times 10^2 = 600$$

$$+ 3 \times 10^1 = 30$$

$$+ 4 \times 10^0 = 4 \rightarrow 5,634$$

➤ What is “10” called in the above expansion?

➤ The radix.

➤ What is this type of number system called?

➤ Decimal.

➤ What are the digits for decimal numbers?

➤ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

➤ What are the digits for radix-r numbers?

➤ 0, 1, ..., r-1.

# Powers of 2

- Noteworthy powers of 2:
  - $2^{10}$  = kilo- = K;
  - $2^{20}$  = mega- = M;
  - $2^{30}$  = giga- = G;
  - ... tera-, peta-, ...

# General Base Conversion

## ■ Number Representation

Given a number of radix  $r$  of

“ $n$ ” integer digits  $a_{n-1}, \dots, a_0$

and

$n$

“ $m$ ” fractional digits  $a_{-1}, \dots, a_{-m}$

written as:

$a_{n-1} a_{n-2} a_{n-3} \dots a_2 a_1 a_0 . a_{-1} a_{-2} \dots a_{-m}$

has value:

$$\begin{aligned} \text{(Number)}_r &= \left( \sum_{i=0}^{i=n-1} a_i \cdot r^i \right) + \left( \sum_{j=-1}^{j=-m} a_j \cdot r^j \right) \\ &\quad \text{(Integer Portion)} + \text{(Fraction Portion)} \end{aligned}$$



# Commonly Occurring Bases

Name	Radix	Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F (= 0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)

# Converting Binary to Decimal

- To convert to decimal, use decimal arithmetic to sum the weighted powers of two:
- Converting  $11010_2$  to  $N_{10}$ :

$$\begin{aligned} N_{10} &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 26 \end{aligned}$$

# Converting Decimal to Binary

- **Method 1 (Method 2 – repeated division – next slide)**
  - Subtract the largest power of 2 that gives a positive result and record the power.
  - Repeat, subtracting from the prior result, until the remainder is zero.
  - Place 1's in the positions in the binary result corresponding to the powers recorded; in all other positions place 0's.
- **Example:  $625_{10} \rightarrow 1001110001_2$** 
  - $625 - 512 = 113 \rightarrow 9$
  - $113 - 64 = 49 \rightarrow 6$
  - $49 - 32 = 17 \rightarrow 5$
  - $17 - 16 = 1 \rightarrow 4$
  - $1 - 1 = 0 \rightarrow 0$
  - Place 1's in the the positions recorded and 0's elsewhere
- **Converting binary to decimal:** sum weighted powers of 2 using decimal arithmetic, e.g.,  $512 + 64 + 32 + 16 + 1 = 625$

# Conversion Between Bases

## ➤ Convert the Integral Part

- Repeatedly divide the number by the radix you want to convert to and save the remainders. The new radix digits are the remainders in reverse order of computation.

*Why does this work?*

*This works because, the remainder left in the division is always is the coefficient of the radix's exponent.*

- If the new radix is  $> 10$ , then convert all remainders  $> 10$  to digits A, B, ...

## ➤ Convert the Fractional Part

- Repeatedly multiply the fraction **by the radix** and save the integer digits that result. The new radix fraction digits are the integer numbers in **computed order**.

*Why does this work?*

*To convert fractional part, it should be divided by reciprocal of radix, which is same as multiplying with radix.*

- If the new radix is  $> 10$ , then convert all integer numbers  $> 10$  to digits A, B, ...

## ➤ Join together with the radix point

# Example: Convert $46.6875_{10}$ To Base 2

## ► Convert 46 to Base 2

$$46/2 = 23 \text{ remainder } = 0$$

$$23/2 = 11 \text{ remainder } = 1$$

$$11/2 = 5 \text{ remainder } = 1$$

$$5/2 = 2 \text{ remainder } = 1$$

$$2/2 = 1 \text{ remainder } = 0$$

$$1/2 = 0 \text{ remainder } = 1$$

Read off in reverse order:  $101110_2$

## ► Convert 0.6875 to Base 2:

$$0.6875 * 2 = 1.3750 \text{ int} = 1$$

$$0.3750 * 2 = 0.7500 \text{ int} = 0$$

$$0.7500 * 2 = 1.5000 \text{ int} = 1$$

$$0.5000 * 2 = 1.0000 \text{ int} = 1$$

$$0.0000$$

Read off in forward order:  $0.1011_2$

## ► Join together with the radix point: $1011110.1011_2$

# Converting Among Octal, Hexadecimal, Binary

## ➤ Octal (Hexadecimal) to Binary:

- Restate the octal (hexadecimal) as three (four) binary digits, starting at radix point and going both ways

## ➤ Binary to Octal (Hexadecimal):

- Group the binary digits into three (four) bit groups starting at the radix point and going both ways, padding with zeros as needed in the fractional part
- Convert each group of three (four) bits to an octal (hexadecimal) digit

## ➤ Example: Octal to Binary to Hexadecimal

6   3   5   .   1   7   7   <sub>8</sub>

= 110 | 011 | 101 . 001 | 111 | 111 <sub>2</sub>

= 1 | 1001 | 1101 . 0011 | 1111 | 1(000) <sub>2</sub> (regrouping)

= 1   9   D   .   3   F   <sub>16</sub> (converting)



# Non-numeric Binary Codes

- Given  $n$  binary digits (called bits), a binary code is a mapping from a subset of the  $2^n$  binary numbers to some set of represented elements.

- **Example: A binary code for the seven colors of the rainbow**

Binary Number	Color
000	Red
001	Orange
010	Yellow
011	Green
100	(Not mapped)
101	Blue
110	Indigo
111	Violet

- Flexibility of representation
  - can assign binary code word to any numerical or non-numerical data as long as data uniquely encoded.

# Number of Bits Required

- Given  $M$  elements to be represented by a binary code, the minimum number of bits,  $n$ , needed satisfies the following relationships:
  - $2^n \geq M > 2^{n-1}$
  - $n = \text{ceil}(\log_2 M)$  where  $\text{ceil}(x)$  is the smallest integer greater than or equal to  $x$
- Example: How many bits are required to represent decimal digits with a binary code?
  - $M = 10 \rightarrow n = 4$

# Number of Elements Represented

- Given  $n$  digits in radix  $r$ , there are  $r^n$  distinct elements that can be represented.
- But, can represent  $m$  elements,  $m < r^n$
- Examples:
  - Can represent 4 elements in radix  $r = 2$  with  $n = 2$  digits: (00, 01, 10, 11)
  - Can represent 4 elements in radix  $r = 2$  with  $n = 4$  digits: (0001, 0010, 0100, 1000)
    - This code is called a "one hot" code

# Binary Coded Decimal (BCD)

- The BCD code is the 8,4,2,1 code.
- This code is the simplest, most intuitive binary code for decimal digits and uses the same weights as a binary number, but only encodes the first ten values from 0 to 9.
- Example:  $1001 (9) = 1000 (8) + 0001 (1)$
- *How many “invalid” code words are there?*
- *What are the “invalid” code words?*

# Gray Code

- What property does this Gray code have?
  - Counting up or down changes only one bit at a time (including counting between 9 and 0)

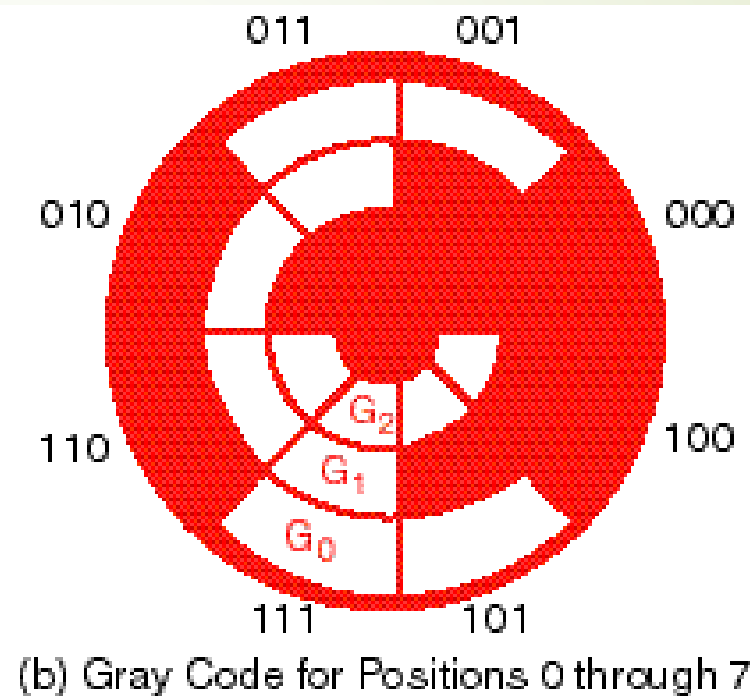
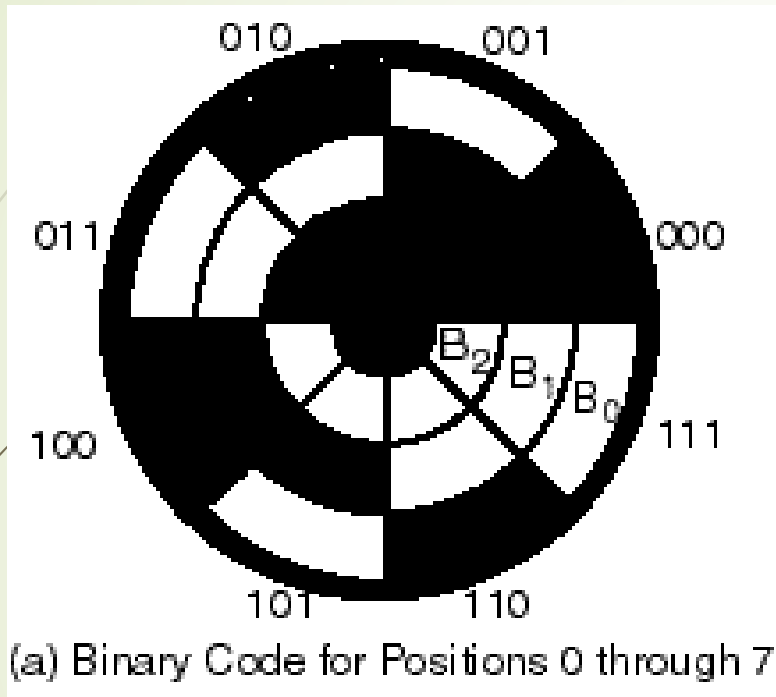
Decimali 3 alalım mesela en soldaki biti yazdık başa ne oldu 0xxx sonrasında xor denklemine sokalım hep

$0 \text{ xor } 0 = 0$   $0 \text{ xor } 1 = 1$   $1 \text{ xor } 1 = 0$  ne oldu tek tek yerlestirelim 0010 sağdaki tabloyla aynı bakınız.

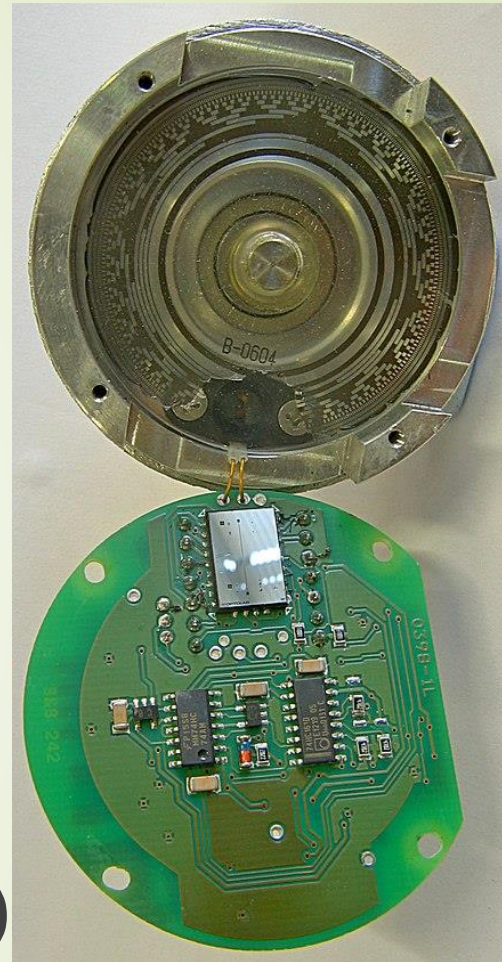
Decimal	Binary	Gray
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000



# Gray Code: Optical Shaft Encoder



- Shaft encoder: Capture angular position (e.g., compass)
- For binary code, what values can be read if the shaft position is at boundary of “3” and “4” (011 and 100) ?
- For Gray code, what values can be read ?





# Warning: Conversion or Coding?

- ▶ Do NOT mix up conversion of a decimal number to a binary number with coding a decimal number with a BINARY CODE.
- ▶  $13_{10} = 1101_2$  (This is conversion)
- ▶  $13 \Leftrightarrow 0001 \mid 0011$  (This is coding)