

CSE 211 Digital Design

Akdeniz University

Week01: Introduction to Digital Design

1

Assoc.Prof.Dr. Taner Danişman

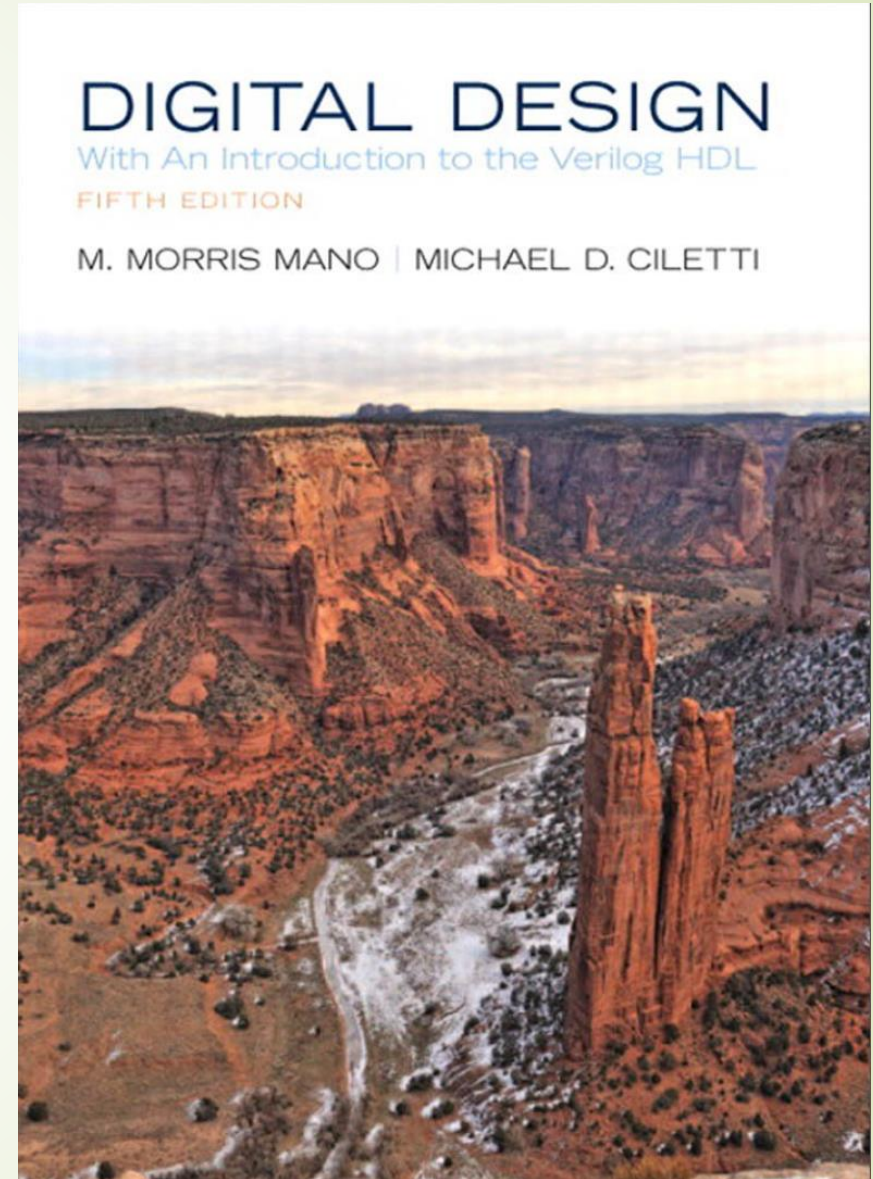
tdanisman@akdeniz.edu.tr

Güz Yarıyılı		Bahar Yarıyılı
05 Ağustos 2022	Ders Görevlendirmelerinin Rektörlüğe Bildirilmesinin Son Günü	30 Aralık 2022
12 Ağustos 2022	Özel Öğrenci Başvurusu İçin Son Gün	13 Ocak 2023
12-16 Eylül 2022	Katkı Payı/Öğrenim Ücretleri Yatırma ve Kayıt Yenileme Süresi	06-10 Şubat 2023
16 Eylül 2022	Öğrenime Ara İzni Başvurusu İçin Son Gün	10 Şubat 2023
19 Eylül 2022	Derslerin Başlaması	13 Şubat 2023
19-23 Eylül 2022	Ders Bırakma ve Ders Ekleme Süresi (Ekle-Çıkar)	13-17 Şubat 2023
26 Eylül 2022	Şubelere Ayrılan Derslerin Rektörlüğe Bildirilmesinin Son Günü	20 Şubat 2023
07 Ekim 2022	Dersten Çekilmenin Son Günü	03 Mart 2023
30 Aralık 2022	Ara Sınav Sonuçlarının ve Diğer Yıl/Yarıyıl İçi Ölçme Araçları Sonuçlarının Otomasyon Sistemine Girilmesinin Son Tarihi	26 Mayıs 2023
30 Aralık 2022	Derslerin Sona Ermesi	26 Mayıs 2023
02-13 Ocak 2023	Yarıyıl Sonu Sınavları	29 Mayıs-09 Haziran 2023
17 Ocak 2023	Yarıyıl Sonu Sınav Sonuçlarının Otomasyon Sistemine Girilmesinin Son Günü	13 Haziran 2023
14-21 Ocak 2023	Yıl/Yarıyıl Sonu İkinci Sınavı(Bütünleme) Son Başvuru Tarihi	10-17 Haziran 2023
23-29 Ocak 2023	Yıl/Yarıyıl Sonu İkinci Sınavı(Bütünleme)	19-25 Haziran 2023
30 Ocak 2023	Yıl/Yarıyıl Sonu İkinci Sınavı(Bütünleme) Sonuçlarının Otomasyon Sistemine Girilmesinin Son Günü	27 Haziran 2023
Ara sınavlar, yıl/yarıyıl içi ölçme araçları ve mazeret sınavları, ilgili Birimin belirlediği tarih ve saatte yapılır.		

Evaluation (absolute)

- Assignments %20
- Midterm %30
- Final %50
- Teaching Assistant: Erdiñç TÜRK
- Academic misconduct: **do not let this happen**
- **Monday 13:30-15:20 (Location : Yazılım Lab 1)**
- **Thursday 13:30:15:20 (Location: BB01)**

- **Textbook :** Digital Design, With an Introduction to the Verilog HDL 5th Edition, Morris Mano, Michael Ciletti, Pearson.



Can I use the previous year's Lab grades?

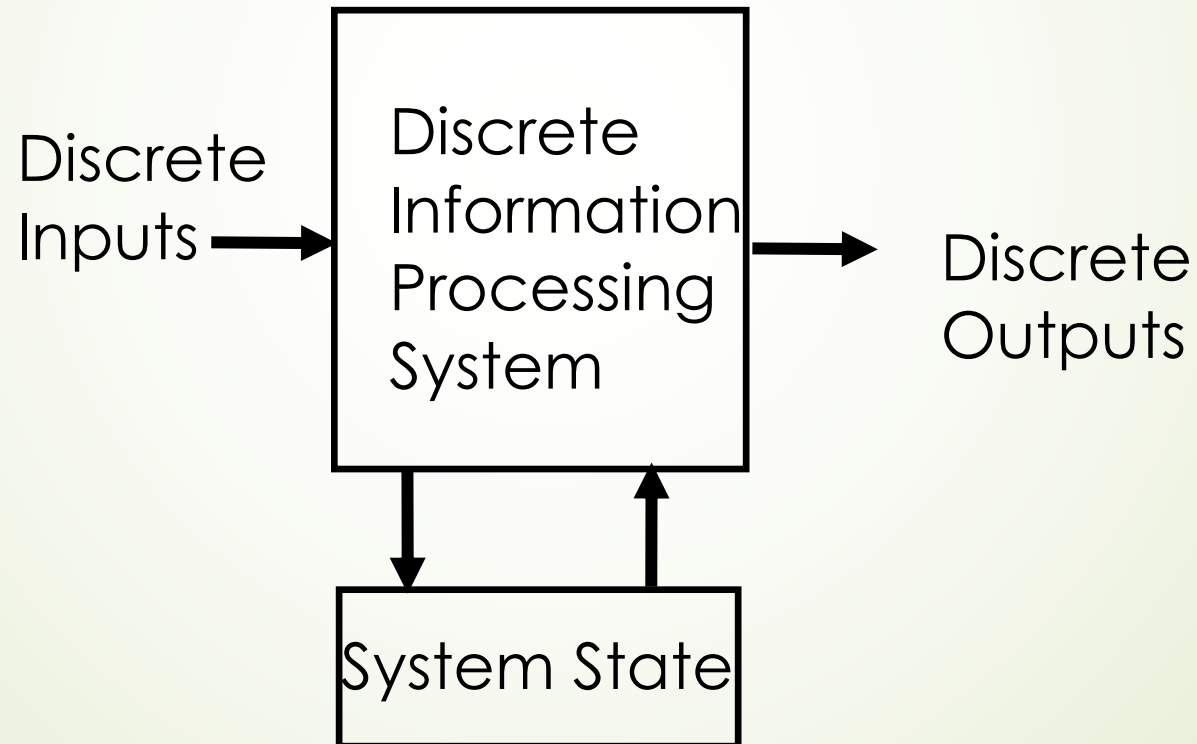
- ➔ **YES**, you can, but you should inform the Teaching Asistant in the beginning of the semestre.

Course program

Week 01	19-Sep-22	Introduction
Week 02	26-Sep-22	Digital Systems and Binary Numbers I
Week 03	03-Oct-22	Digital Systems and Binary Numbers II
Week 04	10-Oct-22	Boolean Algebra and Logic Gates I
Week 05	17-Oct-22	Boolean Algebra and Logic Gates II
Week 06	24-Oct-22	Boolean Algebra and Logic Gates III
Week 07	31-Oct-22	Gate Level Minimization
Week 08	07-Nov-22	Midterm
Week 09	14-Nov-22	Karnaugh Maps
Week 10	21-Nov-22	Karnaugh Maps
Week 11	28-Nov-22	Combinational Logic
Week 12	05-Dec-22	Combinational Logic
Week 13	12-Dec-22	Timing, delays and hazards
Week 14	19-Dec-22	Synchronous Sequential Logic
Week 15	26-Dec-22	Arduino Programming

Digital System

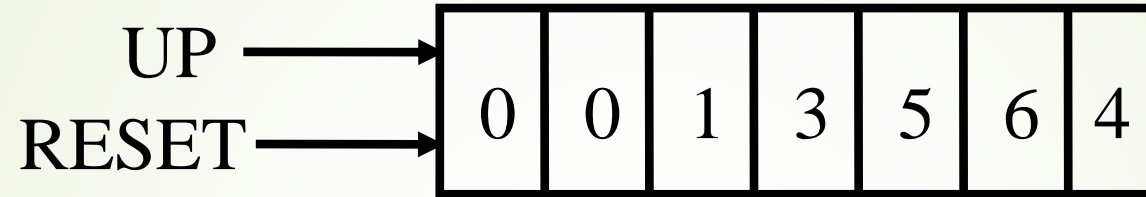
- Takes a set of discrete information inputs and discrete internal information (system state) and generates a set of discrete information outputs.



Types of Systems

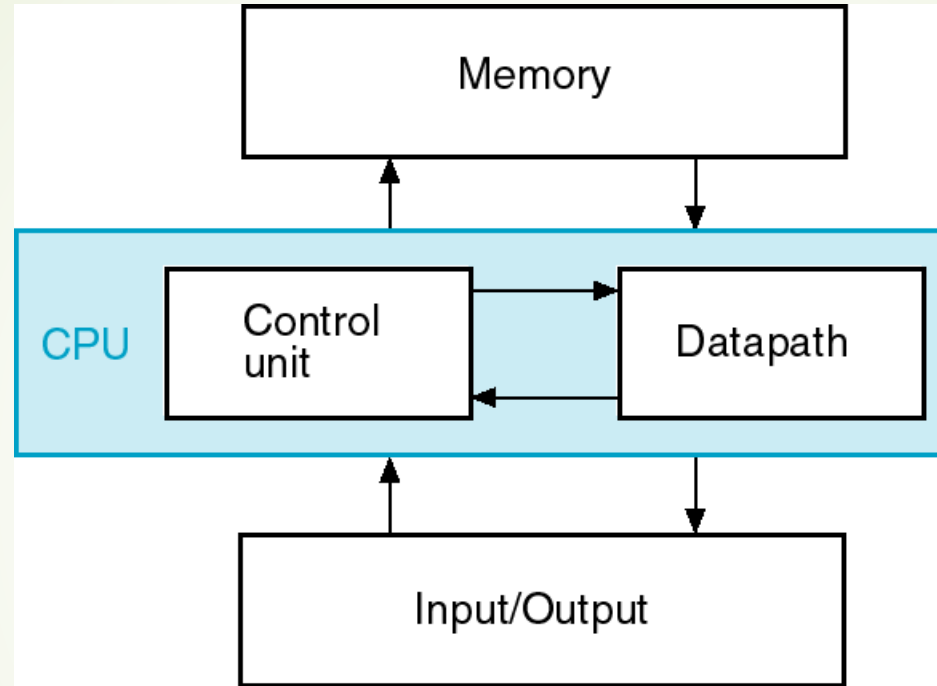
- ▶ With no state present
 - ▶ Combinational logic system
 - ▶ $\text{Output} = \text{Function}(\text{Input})$
- ▶ With state present
 - ▶ State updated at discrete times (e.g., once per clock tick)
 - Synchronous sequential system
 - ▶ State updated at any time
 - Asynchronous sequential system

Example: Digital Counter (e.g., Odometer)



- Inputs: Count Up, Reset
- Outputs: Visual Display
- State: “Value” of stored digits

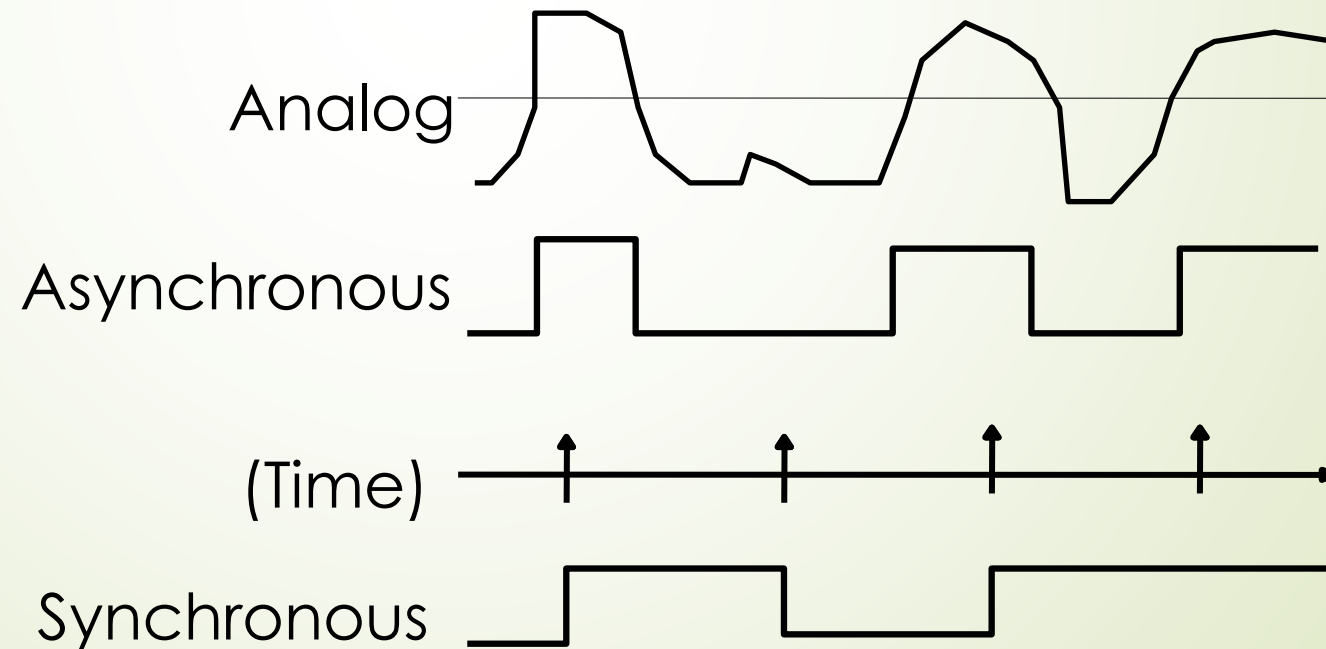
Example: Digital Computer



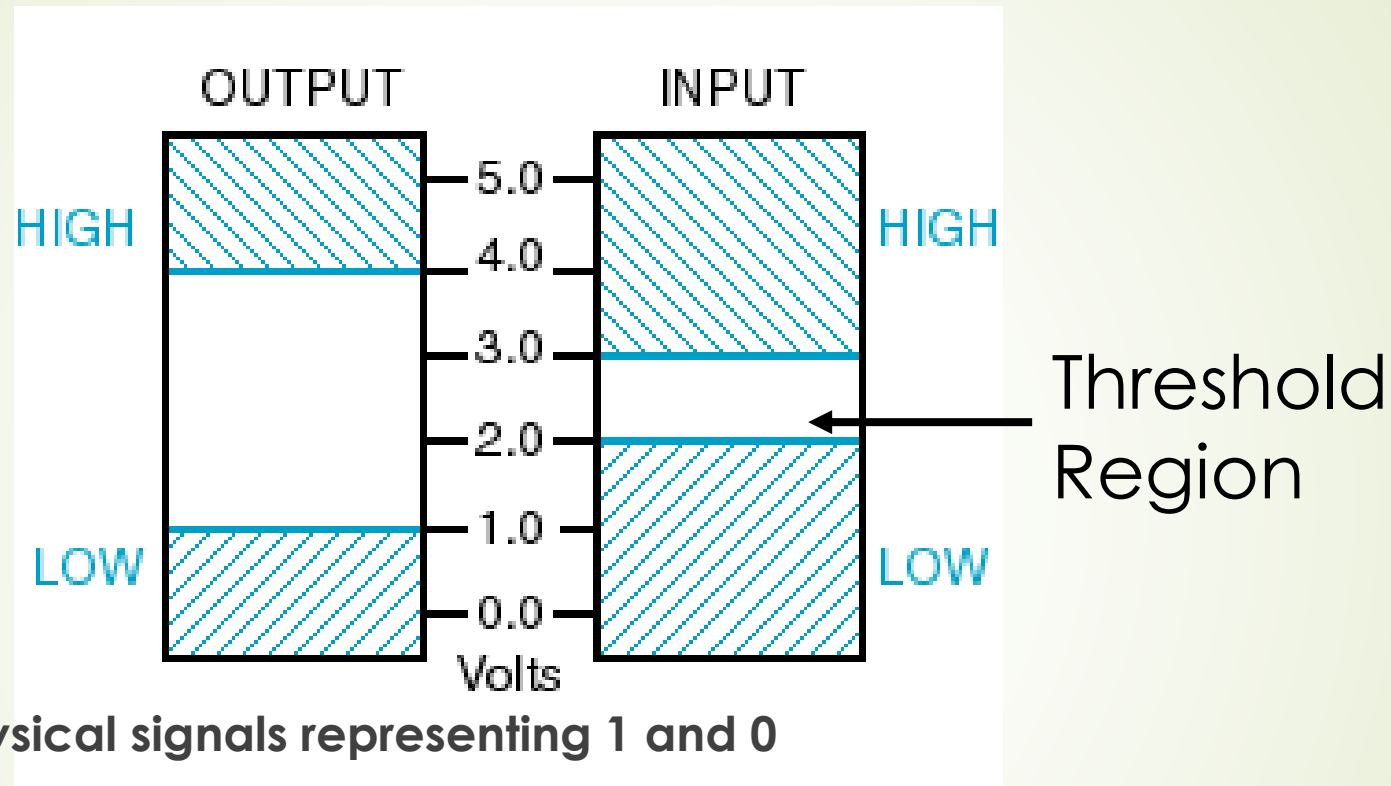
- Inputs: keyboard, mouse, modem, microphone
- Outputs: CRT, LCD, modem, speakers
- Is this system synchronous or asynchronous?

Signals

- Information variables mapped to physical quantities
- In digital systems, the quantities take on *discrete* values
 - Two-level, or *binary*, values are the most prevalent values in digital systems
 - Binary values are represented abstractly by digits 0 and 1
- Signal examples over time:



Physical Signal Example - Voltage



➤ Other physical signals representing 1 and 0

- CPU Voltage
- Disk Magnetic field direction
- CD Surface pits / light
- Dynamic RAM Charge

Number Systems

➤ Decimal Numbers

➤ What does 5,634 represent?

➤ Expanding 5,634:

$$5 \times 10^3 = 5,000$$

$$+ 6 \times 10^2 = 600$$

$$+ 3 \times 10^1 = 30$$

$$+ 4 \times 10^0 = 4 \rightarrow 5,634$$

➤ What is “10” called in the above expansion?

➤ The radix.

➤ What is this type of number system called?

➤ Decimal.

➤ What are the digits for decimal numbers?

➤ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

➤ What are the digits for radix-r numbers?

➤ 0, 1, ..., r-1.

Powers of 2

- Noteworthy powers of 2:
 - 2^{10} = kilo- = K;
 - 2^{20} = mega- = M;
 - 2^{30} = giga- = G;
 - ... tera-, peta-, ...

General Base Conversion

■ Number Representation

Given a number of radix r of

“ n ” integer digits a_{n-1}, \dots, a_0

and

n

“ m ” fractional digits a_{-1}, \dots, a_{-m}

written as:

$a_{n-1} a_{n-2} a_{n-3} \dots a_2 a_1 a_0 . a_{-1} a_{-2} \dots a_{-m}$

has value:

$$\begin{aligned} \text{(Number)}_r &= \left(\sum_{i=0}^{i=n-1} a_i \cdot r^i \right) + \left(\sum_{j=-1}^{j=-m} a_j \cdot r^j \right) \\ &\quad \text{(Integer Portion)} + \text{(Fraction Portion)} \end{aligned}$$

Commonly Occurring Bases

Name	Radix	Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F (= 0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)

Converting Binary to Decimal

- To convert to decimal, use decimal arithmetic to sum the weighted powers of two:
- Converting 11010_2 to N_{10} :

$$\begin{aligned} N_{10} &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 26 \end{aligned}$$

Converting Decimal to Binary

- **Method 1 (Method 2 – repeated division – next slide)**
 - Subtract the largest power of 2 that gives a positive result and record the power.
 - Repeat, subtracting from the prior result, until the remainder is zero.
 - Place 1's in the positions in the binary result corresponding to the powers recorded; in all other positions place 0's.
- **Example: $625_{10} \rightarrow 1001110001_2$**
 - $625 - 512 = 113 \rightarrow 9$
 - $113 - 64 = 49 \rightarrow 6$
 - $49 - 32 = 17 \rightarrow 5$
 - $17 - 16 = 1 \rightarrow 4$
 - $1 - 1 = 0 \rightarrow 0$
 - Place 1's in the the positions recorded and 0's elsewhere
- **Converting binary to decimal:** sum weighted powers of 2 using decimal arithmetic, e.g., $512 + 64 + 32 + 16 + 1 = 625$

Conversion Between Bases

➤ Convert the Integral Part

- Repeatedly divide the number by the radix you want to convert to and save the remainders. The new radix digits are the remainders in reverse order of computation.

Why does this work?

This works because, the remainder left in the division is always is the coefficient of the radix's exponent.

- If the new radix is > 10 , then convert all remainders > 10 to digits A, B, ...

➤ Convert the Fractional Part

- Repeatedly multiply the fraction by the radix and save the integer digits that result. The new radix fraction digits are the integer numbers in computed order.

Why does this work?

To convert fractional part, it should be divided by reciprocal of radix, which is same as multiplying with radix.

- If the new radix is > 10 , then convert all integer numbers > 10 to digits A, B, ...

➤ Join together with the radix point

Example: Convert 46.6875_{10} To Base 2

► Convert 46 to Base 2

$$46/2 = 23 \text{ remainder} = 0$$

$$23/2 = 11 \text{ remainder} = 1$$

$$11/2 = 5 \text{ remainder} = 1$$

$$5/2 = 2 \text{ remainder} = 1$$

$$2/2 = 1 \text{ remainder} = 0$$

$$1/2 = 0 \text{ remainder} = 1$$

Read off in reverse order: 101110_2

► Convert 0.6875 to Base 2:

$$0.6875 * 2 = 1.3750 \text{ int} = 1$$

$$0.3750 * 2 = 0.7500 \text{ int} = 0$$

$$0.7500 * 2 = 1.5000 \text{ int} = 1$$

$$0.5000 * 2 = 1.0000 \text{ int} = 1$$

$$0.0000$$

Read off in forward order: 0.1011_2

► Join together with the radix point: 1011110.1011_2

Converting Among Octal, Hexadecimal, Binary

➤ Octal (Hexadecimal) to Binary:

- Restate the octal (hexadecimal) as three (four) binary digits, starting at radix point and going both ways

➤ Binary to Octal (Hexadecimal):

- Group the binary digits into three (four) bit groups starting at the radix point and going both ways, padding with zeros as needed in the fractional part
- Convert each group of three (four) bits to an octal (hexadecimal) digit

➤ Example: Octal to Binary to Hexadecimal

6 3 5 . 1 7 7 ₈

= 110 | 011 | 101 . 001 | 111 | 111 ₂

= 1 | 1001 | 1101 . 0011 | 1111 | 1(000) ₂ (regrouping)

= 1 9 D . 3 F 8 ₁₆ (converting)

Non-numeric Binary Codes

- Given n binary digits (called bits), a binary code is a mapping from a subset of the 2^n binary numbers to some set of represented elements.

- **Example: A binary code for the seven colors of the rainbow**

Binary Number	Color
000	Red
001	Orange
010	Yellow
011	Green
100	(Not mapped)
101	Blue
110	Indigo
111	Violet

- Flexibility of representation
 - can assign binary code word to any numerical or non-numerical data as long as data uniquely encoded.

Number of Bits Required

- Given M elements to be represented by a binary code, the minimum number of bits, n , needed satisfies the following relationships:
 - $2^n \geq M > 2^{n-1}$
 - $n = \text{ceil}(\log_2 M)$ where $\text{ceil}(x)$ is the smallest integer greater than or equal to x
- Example: How many bits are required to represent decimal digits with a binary code?
 - $M = 10 \rightarrow n = 4$

Number of Elements Represented

- Given n digits in radix r , there are r^n distinct elements that can be represented.
- But, can represent m elements, $m < r^n$
- Examples:
 - Can represent 4 elements in radix $r = 2$ with $n = 2$ digits: (00, 01, 10, 11)
 - Can represent 4 elements in radix $r = 2$ with $n = 4$ digits: (0001, 0010, 0100, 1000)
 - This code is called a "one hot" code

Binary Coded Decimal (BCD)

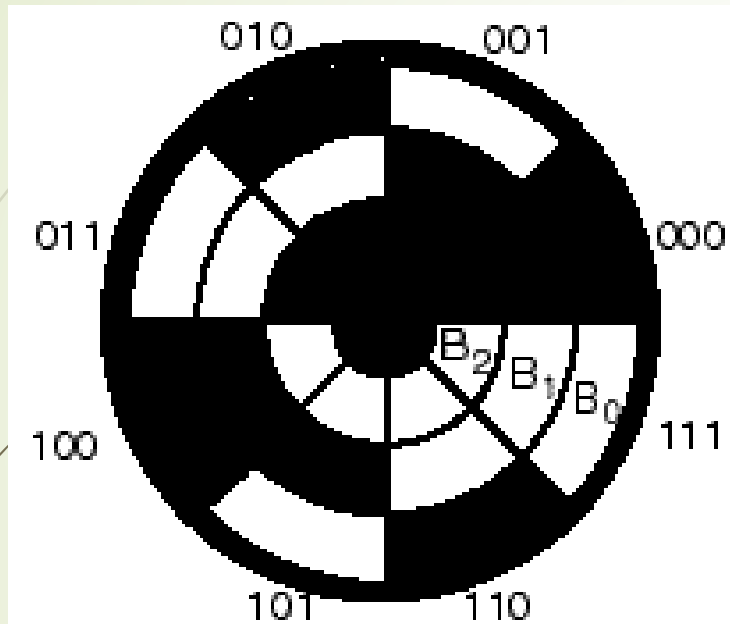
- The BCD code is the 8,4,2,1 code.
- This code is the simplest, most intuitive binary code for decimal digits and uses the same weights as a binary number, but only encodes the first ten values from 0 to 9.
- Example: $1001 (9) = 1000 (8) + 0001 (1)$
- *How many “invalid” code words are there?*
- *What are the “invalid” code words?*

Gray Code

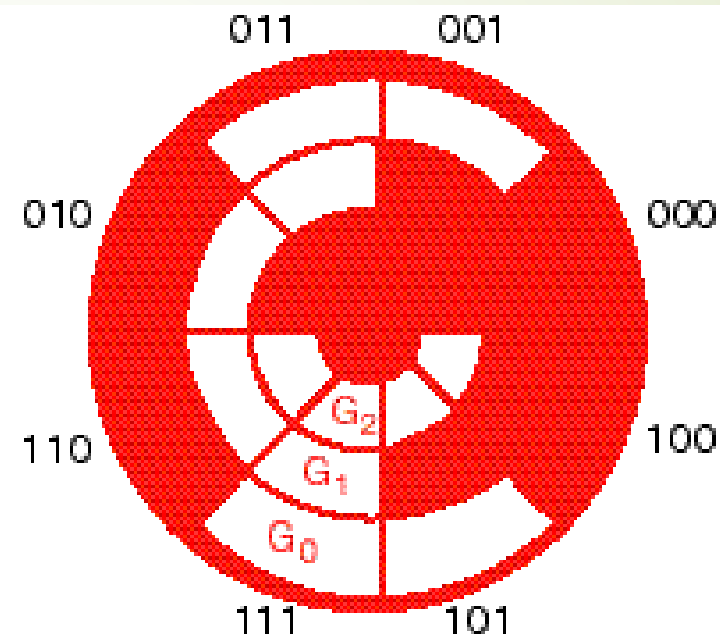
- What property does this Gray code have?
 - Counting up or down changes only one bit at a time (including counting between 9 and 0)

Decimal	Binary	Gray
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Gray Code: Optical Shaft Encoder

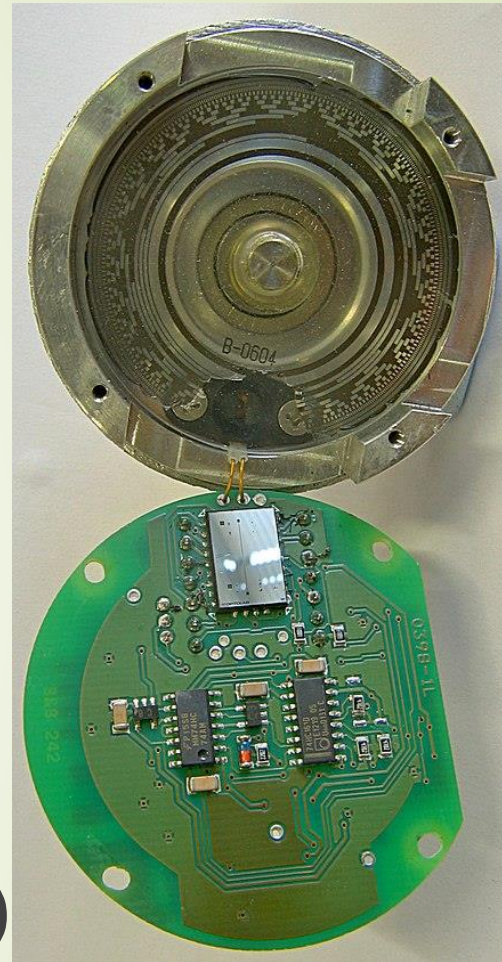


(a) Binary Code for Positions 0 through 7



(b) Gray Code for Positions 0 through 7

- Shaft encoder: Capture angular position (e.g., compass)
- For binary code, what values can be read if the shaft position is at boundary of “3” and “4” (011 and 100) ?
- For Gray code, what values can be read ?



Warning: Conversion or Coding?

- ▶ Do NOT mix up conversion of a decimal number to a binary number with coding a decimal number with a BINARY CODE.
- ▶ $13_{10} = 1101_2$ (This is conversion)
- ▶ $13 \Leftrightarrow 0001 \mid 0011$ (This is coding)