DISCRETE AND COMBINATORIAL MATHEMATICS

An Applied Introduction

THIRD EDITION

RALPH P. GRIMALDI

Rose-Hulman Institute of Technology



Addison-Wesley Publishing Company
Reading, Massachusetts • Menlo Park, California
New York • Don Mills, Ontario
Wokingham, England • Amsterdam • Bonn
Sydney • Singapore • Tokyo • Madrid
San Juan • Milan • Paris

Contents

PART 1 Fundamentals of Discrete Mathematics 1

Mathematics	
1	Fundamental Principles of Counting 3
1.1	The Rules of Sum and Product 3
1.2	Permutations 6
1.3	Combinations: The Binomial Theorem 19
1.4	Combinations with Repetition: Distributions 33
1.5	An Application in the Physical Sciences (Optional) 43
1.6	Summary and Historical Review 44
2	Fundamentals of Logic 51
2.1	Basic Connectives and Truth Tables 51
2.2	Logical Equivalence: The Laws of Logic 61
2.3	Logical Implication: Rules of Inference 77
2.4	The Use of Quantifiers 98
2.5	Quantifiers, Definitions, and the Proofs of Theorems 121
2.6	Summary and Historical Review 137
3	Set Theory 143
3.1	Sets and Subsets 143
3.2	Set Operations and the Laws of Set Theory 156
3.3	Counting and Venn Diagrams 169
3.4	A Word of Probability 172
3.5	Summary and Historical Review 176
4	Properties of the Integers: Mathematical Induction 183
4.1	The Well-Ordering Principle: Mathematical Induction 183
4.2	Recursive Definitions 201
4.3	The Division Algorithm: Prime Numbers 213
4.4	The Greatest Common Divisor: The Euclidean Algorithm 225
4.5	The Fundamental Theorem of Arithmetic 232

4.6 Summary and Historical Review 238

5	Relations	and	Functions	245
_	Nelations	allu	IUIICUOIIS	47.

- 5.1 Cartesian Products and Relations 246
- 5.2 Functions: Plain and One-To-One 251
- 5.3 Onto Functions: Stirling Numbers of the Second Kind 260
- 5.4 Special Functions 267
- 5.5 The Pigeonhole Principle 275
- 5.6 Function Composition and Inverse Functions 280
- 5.7 Computational Complexity 293
- 5.8 Analysis of Algorithms 297
- 5.9 Summary and Historical Review 308

6 Languages: Finite State Machines 315

- 6.1 Language: The Set Theory of Strings 316
- 6.2 Finite State Machines: A First Encounter 327
- 6.3 Finite State Machines: A Second Encounter 335
- 6.4 Summary and Historical Review 343

7 Relations: The Second Time Around 349

- 7.1 Relations Revisited: Properties of Relations 349
- 7.2 Computer Recognition: Zero-One Matrices and Directed Graphs 357
- 7.3 Partial Orders: Hasse Diagrams 371
- 7.4 Equivalence Relations and Partitions 382
- 7.5 Finite State Machines: The Minimization Process 388
- 7.6 Summary and Historical Review 394

PART 2 Further Topics in Enumeration 401

8 The Principle of Inclusion and Exclusion 403

- 8.1 The Principle of Inclusion and Exclusion 403
- 8.2 Generalizations of the Principle 413
- 8.3 Derangements: Nothing Is in Its Right Place 418
- 8.4 Rook Polynomials 420
- 8.5 Arrangements with Forbidden Positions 424
- 8.6 Summary and Historical Review 428

9 Generating Functions 433

- 9.1 Introductory Examples 433
- 9.2 Definition and Examples: Calculational Techniques 436
- 9.3 Partitions of Integers 445
- 9.4 The Exponential Generating Function 449
- 9.5 The Summation Operator 454
- 9.6 Summary and Historical Review 456

10	Recurrence Relations 461
10.1 10.2	The First-Order Linear Recurrence Relation 461 The Second-Order Linear Homogeneous Recurrence Relation with Constant Coefficients 471
10.3	The Nonhomogeneous Recurrence Relation 482
10.4	The Method of Generating Functions 493
10.5	A Special Kind of Nonlinear Recurrence Relation (Optional) 499
10.6	Divide-and-Conquer Algorithms (Optional) 511
10.7	Summary and Historical Review 521
PART 3 Graph Theo and Application	pry
11	An Introduction to Graph Theory 529
11.1	Definitions and Examples 529
	Subgraphs, Complements, and Graph Isomorphism 537
	Vertex Degree: Euler Trails and Circuits 550
	Planar Graphs 560
	Hamilton Paths and Cycles 578
11.6	Graph Coloring and Chromatic Polynomials 588
11.7	Summary and Historical Review 598
12	Trees 607
12.1	Definitions, Properties, and Examples 607
12.2	Rooted Trees 614
	Trees and Sorting 634
	Weighted Trees and Prefix Codes 638
	Biconnected Components and Articulation Points 644
12.6	Summary and Historical Review 650
13	Optimization and Matching 657
13.1	Dijkstra's Shortest-Path Algorithm 657
13.2	
13.3	Transport Networks: The Max-Flow Min-Cut Theorem 671
13.4	Matching Theory 683
13.5	Summary and Historical Review 694

PART 4 Modern Applied Algebra 699

Rings and Modular Arithmetic 701 14

- The Ring Structure: Definition and Examples 701 14.1
- 14.2 Ring Properties and Substructures 709

 14.3 The Integers Modulo n 717 14.4 Ring Homomorphisms and Isomorphisms 722 14.5 Summary and Historical Review 730
15 Boolean Algebra and Switching Functions 735
 15.1 Switching Functions: Disjunctive and Conjunctive Normal Forms 735 15.2 Gating Networks: Minimal Sums of Products: Karnaugh Maps 745 15.3 Further Applications: Don't-Care Conditions 756 15.4 The Structure of a Boolean Algebra (Optional) 762 15.5 Summary and Historical Review 772
16 Groups, Coding Theory, and Polya's Method of Enumeration 777
16.1 Definition, Examples, and Elementary Properties 777 16.2 Homomorphisms, Isomorphisms, and Cyclic Groups 784 16.3 Cosets and Lagrange's Theorem 791 16.4 Elements of Coding Theory 793 16.5 The Hamming Metric 798 16.6 The Parity-Check and Generator Matrices 801 16.7 Group Codes: Decoding with Coset Leaders 806 16.8 Hamming Matrices 810 16.9 Counting and Equivalence: Burnside's Theorem 812 16.10 The Cycle Index 820 16.11 The Pattern Inventory: Polya's Method of Enumeration 824 16.12 Summary and Historical Review 829
17 Finite Fields and Combinatorial Designs 835
 17.1 Polynomial Rings 835 17.2 Irreducible Polynomials: Finite Fields 843 17.3 Latin Squares 853 17.4 Finite Geometries and Affine Planes 859 17.5 Block Designs and Projective Planes 865 17.6 Summary and Historical Review 871
Appendix 1 Exponential and Logarithmic Functions A-1
Appendix 2 Matrices, Matrix Operations, and Determinants A-13
Appendix 3 Countable and Uncountable Sets A-27
Solutions S-1
Index I-1