12/11/2021 Friday Midterm Duration: 80 minutes

Name: Student No:

P1 [30 pts] Suppose that there is a class of 21 students. They want to form three basketball teams, each having 5 playing players and 2 substitutes.

(a) In how many ways can they form these teams? (b) Suppose that there are two strong players who cannot be substitutes. (c) Suppose that there are two strong players who cannot be substitutes and also two weak players who have to be substitutes.

(a)	(b)	(c)

P2 [20 points]

(a) How many non-negative integer solutions are there to the equation

$$x_1 + x_2 + x_3 + \ldots + x_8 = 47,$$
$$x_6 + x_7 + x_8 = 17, \forall i : 0 \leq x_i$$

(b) How many non-negative integer solutions are there to the equation

$$x_1 + x_2 + x_3 + \ldots + x_8 = 47, \forall i : 0 \leq x_i \leq 15$$

P3 [20 points]

(a) Establish the validity of the argument:

$p \wedge q$
$p \to (r \wedge q)$
$r \to (s \vee t)$
$\neg s$
$\therefore t$

	Steps	Reasons
1.	$p \wedge q$	Premise.
2.	$p \to (r \wedge q)$	Premise.
3.	$r \to (s \vee t)$	Premise.
4.	$\neg s$	Premise.
5.	$\ldots$	

(b) Give a counterexample for the argument:

$r \vee \neg p$
$q \to r$
$\neg p \to q$
$\therefore p$

p:
q:
r:
(Hint: Note that an argument is false when all premises are true the conclusion can be false.)

P4 [30 points] Prove the following statement by using mathematical induction:

(a) Recall that Fibonacci numbers are defined as $F_0 = 0, F_1 = 1$ and for $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$. Prove that $\forall n \geq 0 : F_{5n}$ is divisible by 5.

(b) $\forall n \in \mathbb{Z}^+, n \geq 60$:

$$\exists i, j \in \mathbb{N}\ [n = 7i + 11j]$$

(For all integers $n \geq 60$, there exists natural numbers i and j such that n can be written as $7i + 11j$. $\mathbb{N} = \{0, 1, 2, \ldots\}$)

P5 [20 points] (In this question, define your pigeons and pigeonholes clearly.)

(a) Show that, in the world, there must be at least two people having the same number of friends.

(b) Let S be a set of five positive integers the maximum of which is at most 9. Prove that the sums of the elements in all the nonempty subsets of S cannot all be distinct. [Hint: You may consider thinking about only subsets of certain sizes, not all of them]