MAT 222 Linear Algebra – Assignment 1

IMPORTANT: In each of the following problems, only do the part whose number is equal to the last digit of your student id. If the last digit is 0, attempt part (x).

In all three problems you can use a software or web page in order to aid you in some calculations, e.g. to calculate eigenvalues and eigenvectors, to find the inverse of $\mathbf{P}$ or to carry out matrix multiplication. In this case, you must include the print/screenshot of the calculations in your work.

Problem 1. Find the general term of the sequence whose recurrence and first two terms are given by the following. (Note: This can be achieved in many ways. However, in this assignment you are expected to use the method that relies on expressing the recurrence by means of a matrix relation and calculating the eigenvalues and eigenvectors of the involved matrix.)

(i) $x_0 = 0, x_1 = 1,\ x_{n+2} = 5x_{n+1} + 14x_n$
(ii) $x_0 = 1, x_1 = 1,\ x_{n+2} = x_{n+1} + 2x_n$
(iii) $x_0 = 1, x_1 = 0,\ x_{n+2} = x_{n+1} + 6x_n$
(iv) $x_0 = 1, x_1 = 2,\ x_{n+2} = 3x_{n+1} - 2x_n$
(v) $x_0 = 2, x_1 = 1,\ x_{n+2} = -x_{n+1} + 2x_n$
(vi) $x_0 = 0, x_1 = 1,\ x_{n+2} = 5x_{n+1} + 24x_n$
(vii) $x_0 = 1, x_1 = 1,\ x_{n+2} = 2x_{n+1} + 15x_n$
(viii) $x_0 = 1, x_1 = 0,\ x_{n+2} = x_{n+1} + 12x_n$
(ix) $x_0 = 1, x_1 = 2,\ x_{n+2} = -2x_{n+1} + 8x_n$
(x) $x_0 = 2, x_1 = 1,\ x_{n+2} = -3x_{n+1} + 4x_n$

Problem 2. Consider a discrete dynamical model about the population of two animal species in the same ecological system, where x_k and y_k denote the predator and prey population at the end of k-th month, respectively. The model is of the form $\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_k \\ y_k \end{bmatrix}$, where $\mathbf{A}$ is a 2×2 matrix. In each part below, use the given matrix $\mathbf{A}$ and the initial populations to determine the long-term behavior of both the predator and the prey populations. Your answer should include the following:

- What is $\lim_{k\to\infty} x_k$? What is $\lim_{k\to\infty} y_k$?
- What is $\lim_{k\to\infty} \frac{x_k}{y_k}$?
- A comment on the long-term behavior of both populations (Do they grow indefinitely or do they perish in the long-term, or do they approach a certain limit?)

(i) $\mathbf{A} = \begin{bmatrix} 0.5 & 0.4 \\ -0.2 & 1.1 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 22 \\ 14 \end{bmatrix}$.

(ii) $\mathbf{A} = \begin{bmatrix} 0.4 & 0.3 \\ -0.5 & 1.2 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$.

(iii) $\mathbf{A} = \begin{bmatrix} 0.4 & 0.3 \\ -0.325 & 1.2 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 34 \\ 67 \end{bmatrix}$.

(iv) $\mathbf{A} = \begin{bmatrix} 0.5 & 0.4 \\ -0.125 & 1.1 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 48 \\ 52 \end{bmatrix}$.

(v) $\mathbf{A} = \begin{bmatrix} 0.3 & 0.4 \\ -0.3 & 1.1 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 30 \\ 35 \end{bmatrix}$.

(vi) $\mathbf{A} = \begin{bmatrix} 0.4 & 0.5 \\ -0.4 & 1.3 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 60 \\ 50 \end{bmatrix}$.

(vii) $\mathbf{A} = \begin{bmatrix} 0.5 & 0.6 \\ -0.3 & 1.4 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 35 \\ 20 \end{bmatrix}$.

(viii) $\mathbf{A} = \begin{bmatrix} 0.8 & 0.3 \\ -0.4 & 1.5 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 21 \\ 25 \end{bmatrix}$.

(ix) $\mathbf{A} = \begin{bmatrix} 0.86 & 0.08 \\ -0.12 & 1.14 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \end{bmatrix}$.

(x) $\mathbf{A} = \begin{bmatrix} 0.7 & 0.1 \\ -0.5 & 1.3 \end{bmatrix}$, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \end{bmatrix}$.

Problem 3. Find the unique polynomial of degree less than or equal to 3 that passes through the given points. (Again, this can be done in various ways. In this assignment you are expected to calculate the Lagrange basis polynomials $L_0(x), L_1(x), L_2(x)$ and $L_3(x)$ first.)

(i) (-1,9), (1,5), (2,15), (4,89)
(ii) (-2,0), (0,6), (1,4.5), (3,4.5)
(iii) (-3,-35), (-2,-5), (-1,3), (1,1)
(iv) (-1,-5.25), (0,-3), (1,0.25), (2,3)
(v) (-4,-26), (-3,-15), (1,9), (2,10)
(vi) (-2,-14), (-1,-3.5), (1,2.5), (2,10)
(vii) (-1,-9), (0,-8), (1,-7), (2,0)
(viii) (-3,-10), (-2,5), (1,2), (4,53)
(ix) (-3,-51), (-1,-7), (1,-3), (2,14)
(x) (-2,-5.6), (-1,-0.2), (0,2), (1,2.2)