Answers

24 October 2023 Tuesday 18:54

1-) 4-5 hours

2-) a) $x[m, z]$

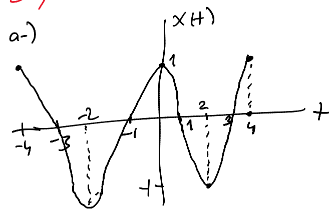
digital image function

coordinates

function value is RGB value.

3-) $x(t) = \cos\left(\frac{\pi t}{2}\right)$

a-)



b-) $x(t)$ is 1D

c-) $x(t)$ is deterministic we know output value for any given input.

d-) $x(t)$ is periodic

$$T_0 : \frac{2\pi}{\pi/2} = 2\cancel{\pi} \cdot \frac{2}{\cancel{\pi}} = 4$$

$$f_0 = \frac{1}{4}$$

e-) $x(t)$ is even signal

$$x(t) = x(-t)$$

f-) causal $x(t) = 0$ for $t < 0$

$x[n] = 0$ for $n \leq -1$

$x(t)$ is noncausal

g-) $$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$x(t)$ is periodic so its energy is ∞.

$x(t)$ is not a energy signal.

h-) $$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

for periodic signal

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} \left|\cos\left(\frac{\pi t}{2}\right)\right|^2 dt$$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} \cos^2\left(\frac{\pi t}{2}\right) dt$$

$$\cos 2x = 2\cos^2 x - 1$$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} \frac{\cos \pi t + 1}{2} dt$$

$$\cos 2x + 1 = 2\cos^2 x$$

$$\frac{\cos 2x + 1}{2} = \cos^2 x$$

$$P_x = \frac{1}{2T}\left(\int_{-T/2}^{T/2} \cos \pi t + \int_{-T/2}^{T/2} 1\right)$$

$$P_x = \frac{1}{2T}\left(\frac{1}{\pi}\left(\sin w \Big|_{-\frac{\pi T}{2}}^{\frac{\pi T}{2}}\right) + \frac{T}{2} + \frac{T}{2}\right)$$

$$\pi t = w$$

$$\pi = \frac{dw}{dt}$$

$$dt = \frac{dw}{\pi}$$

$$P_x = \frac{1}{2T}\left(\frac{\sin\left(\frac{\pi T}{2}\right)}{\pi} + \frac{\sin\left(\frac{\pi T}{2}\right)}{\pi} + T\right)$$

$$T = 4$$

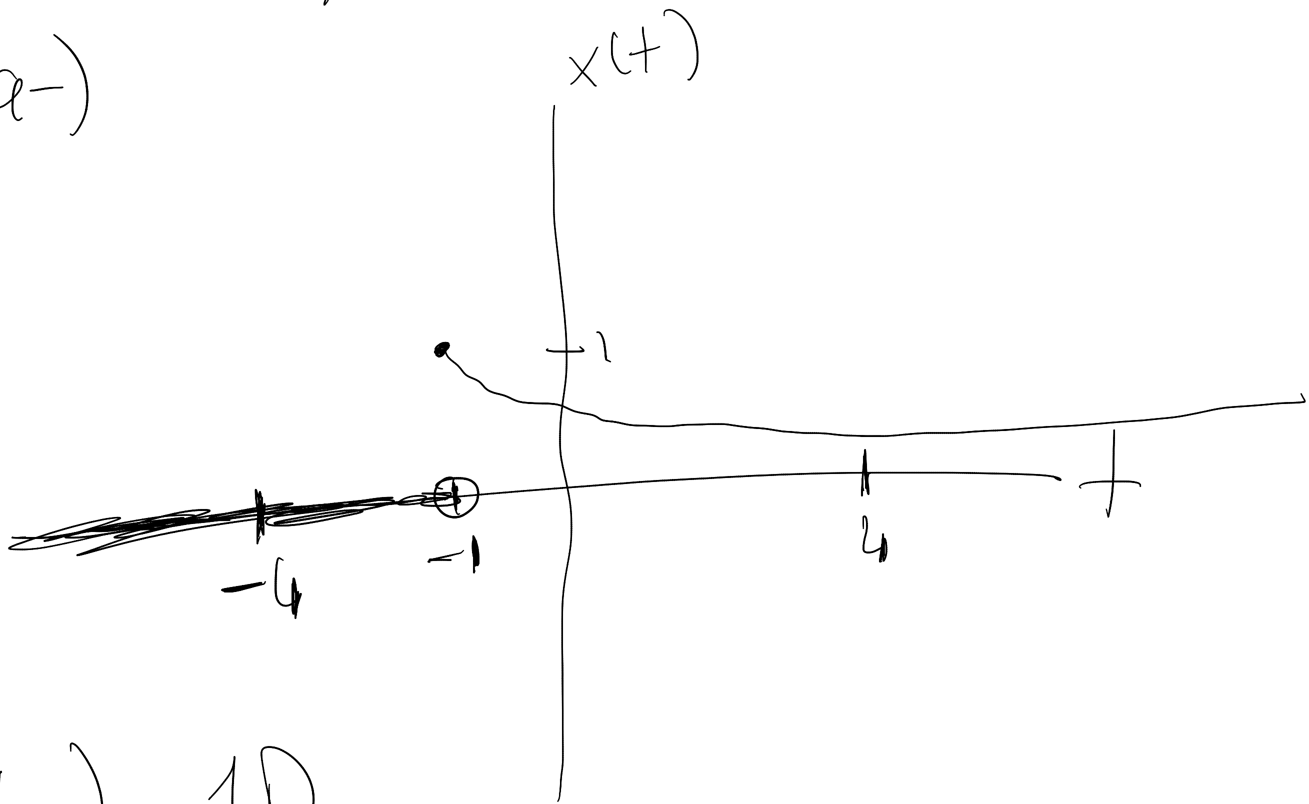
$$P_x = \frac{1}{8}\left(\cancel{\frac{\overset{0}{\sin 2\pi}}{\pi}} + \cancel{\frac{\overset{0}{\sin(2\pi)}}{\pi}} + 4\right)$$

$$P_x = \frac{4}{8} = \frac{1}{2}$$

i-) Since, $0 < P_x < \infty$
$x(t)$ is a power signal,

4-) $$x(t) = \begin{cases} 0 & t < -1 \\ e^{-(t+1)} & t \geq -1 \end{cases}$$

a-)



b-) 1D

c-) $x(t)$ is deterministic

d-) $x(t)$ is not periodic

e-) Neither

f-) x(t) is non-causal

g-) $= \int_{-1}^{\infty} \left| e^{-(t+1)} \right|^2 dt$

$$E_x = \int_{-1}^{\infty} e^{-2t-2} dt$$

$-2t-2 = w$

$-2dt = dw$

$dt = \frac{dw}{-2}$

$$E_x = \int -e^{w} \frac{dw}{2}$$

$$E_x = -\frac{1}{2} \int_{0}^{-\infty} e^{w} dw$$

$$E_x = -\frac{1}{2} \left(e^{w} \Big|_{0}^{-\infty} \right)$$

$$E_x = -\frac{1}{2} \left(e^{-\infty} - e^{0} \right) = -\frac{1}{2} (0 - 1)$$

$$E_x = \frac{1}{2}$$

h-) $P_x = \lim_{T \to \infty} \frac{1}{T} \underbrace{\int_{-1}^{T/2} e^{-2t-2} dt}_{\frac{1}{2}}$

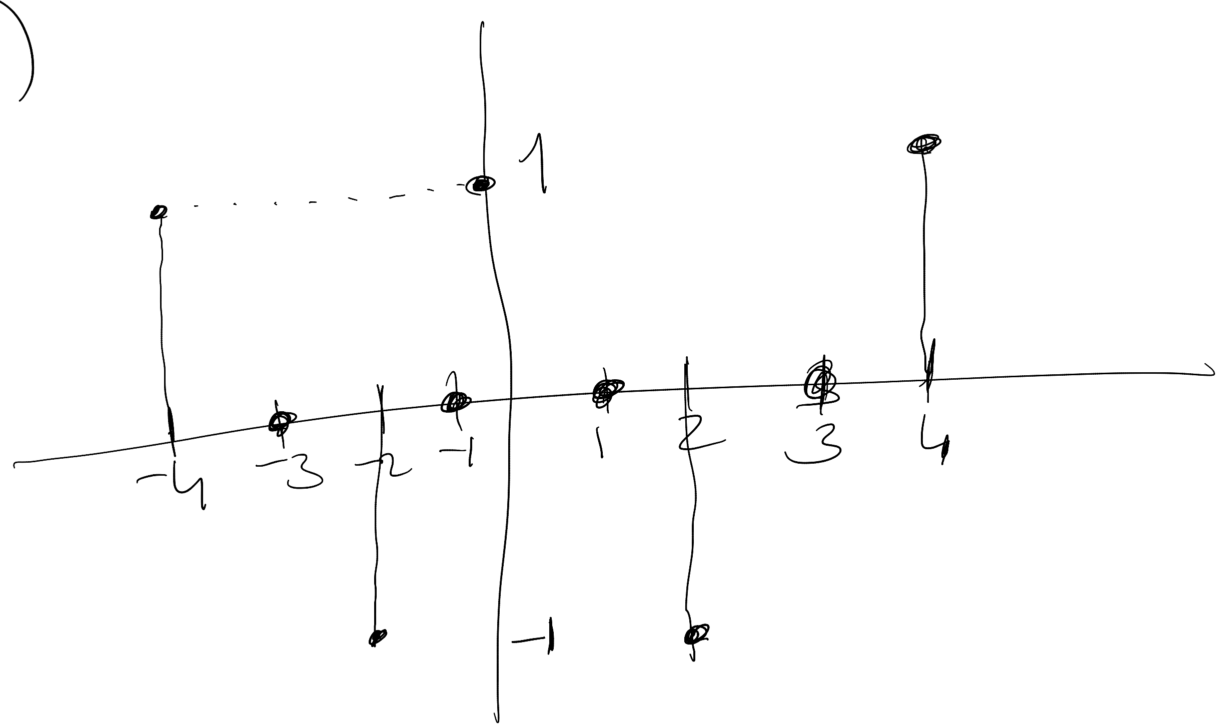
$$P_x = \lim_{T \to \infty} \frac{1/2}{T} = \underline{\underline{0}}$$

i-) $x(t)$ is a energy signal.

Q5

$$x[n] = \cos\left[\frac{\pi n}{2}\right]$$

a-)



b-) 1D

c-) $x[n]$ is deterministic.

d-) $\frac{2\pi}{\pi/2} = 4$

$N_0 = 4 \quad f_0 = \frac{1}{4}$

e-) $x[n] = x[-n]$

so $x[n]$ is even

f-) $x[n]$ is noncausal.

g-) $E_x = \sum_{n=\infty}^{\infty} |x[n]|^2$

$E_x = \sum_{n=-\infty}^{\infty} \cos^2\left[\frac{\pi n}{2}\right] = \infty$

h-) $P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n]$

$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi n}{2}\right)$

value of $\cos^2\left(\frac{\pi n}{2}\right) \Rightarrow$

n: ... -3 -2 -1 0 1 2 3 ......

0 1 0 1 0 1 0

$$\sum_{n=-N}^{N} \cos^2\left(\frac{\pi n}{2}\right) = N + \frac{1}{2}$$

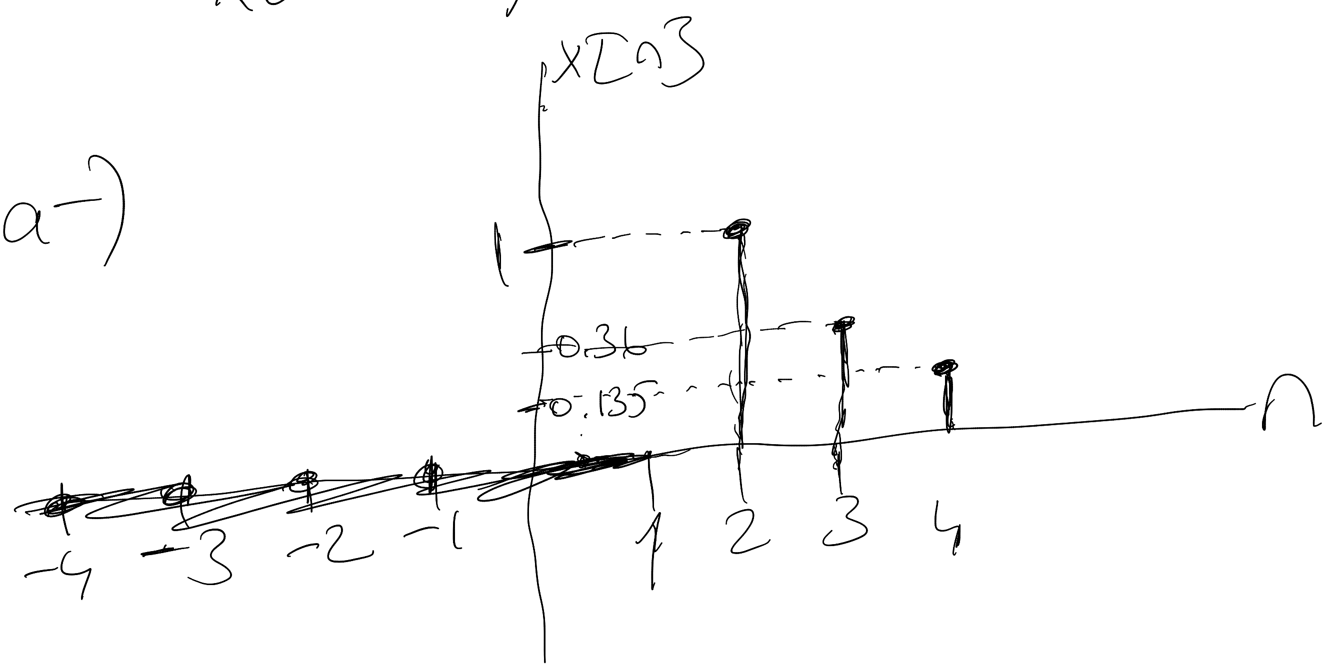
$$\text{So, } P_x = \lim_{N \to \infty} \frac{N + 1/2}{2N+1} = \frac{1}{2}$$

i-) $0 < P_x < \infty \Rightarrow x[n]$ is a power signal.

Q6

$$x[n] = \begin{cases} 0 & n < 2 \\ e^{-(n-2)} & n \geq 2 \end{cases}$$

a-)



b-) 1D

c-) $x[n]$ is deterministic

d-) $x[n]$ is not periodic

e-) $x[n]$ is neither even nor odd.

f-) $x[n]$ is noncausal.

g-) $E_x = \sum_{n=2}^{\infty} |e^{-n+2}|^2$

$E_x = \sum_{n=2}^{\infty} e^{-2n+4}$

$E_x = \sum_{n=2}^{\infty} e^{-2n+4} = e^0 + e^{-2} + e^{-4} \ldots\ldots$

$E_x = \dfrac{1}{1-\frac{1}{e^2}}$ $\qquad S = \dfrac{a}{1-r}$

$$E_x = \frac{1}{\frac{e^2-1}{e^2}} = \frac{e^2}{e^2-1}$$

h-) $P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} e^{-2n+4}$

$$P_x = \lim_{N \to \infty} \frac{1}{2N+1} \left(\frac{e^2}{e^2-1} \right) \quad P_x = \infty$$

i-) Since $0 < E_x < \infty$

$x[n]$ is an energy signal.