Hanyang Univ. - 과탑사이허접

1 Math

1.1 Gauss-Jordan Elimination

```
#include <bits/stdc++.h>
using namespace std;
typedef struct _Matrix{
   int N:
    vector<vector<double>> matrix;
    Matrix(int X) {
       N = X;
        matrix.resize(N, vector<double>(N + 1));
   } // N by N + 1 matrix
} Matrix;
void row_swap(Matrix& A, int i) {
   vector<double> temp = A.matrix[i];
   A.matrix.erase(A.matrix.begin() + i);
   A.matrix.push_back(temp);
}
//Gauss-Jordan Elmination
void gauss jordan(Matrix& A) {
    for(int i = 0; i < A.N; ++i) {
        while(A.matrix[i][i] == 0) row_swap(A, i); //check diagonal components
         are non-zero, when if, rotate row(swap)
        for(int j = 0; j < A.N; ++j) { //make RREF
            if(i != j) {
                double ratio = A.matrix[j][i] / A.matrix[i][i];
                for(int k = 0; k \le A.N; ++k) {
                    A.matrix[j][k] = A.matrix[j][k] - ratio * A.matrix[i][k];
        }
}
```

1.2 Miller-Rabin Prime Test

```
#include <bits/stdc++.h>
using namespace std;

using ull = unsigned long long;

vector<ull> prime_list = {2, 7, 61};
//~ int range
//{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}
//~ long long range
//use __int128_t instead of ull
//O(klog^3(x))
```

```
ull mod pow(ull a, ull b, ull M) {
    if(b == 0) return 1;
    ull temp = mod pow(a, b / 2, M);
    if(b & 1) return (((temp % M) * (temp % M)) % M * (a % M)) % M;
    else return ((temp % M) * (temp % M)) % M;
}
bool miller_rabin(ull x) {
    if(x < 2) return false;</pre>
    bool ret = true;
    for(auto& p : prime_list) {
        if(x == p) return true;
        ull k = x - 1;
        while(true) {
            ull val = mod_pow(p, k, x) % x;
            if(val == x - 1) {
                ret = true;
                break;
            if(k & 1) {
                ret = (val == 1 || val == x - 1);
                break;
            }
            k /= 2;
        if(!ret) break;
    return ret;
1.3 Fast-Fourier-Transform
#include <bits/stdc++.h>
```

```
#include <bits/stdc++.h>
using namespace std;
using cpx = complex<double>;
//Cooley-Tukey FFT
void FFT(vector<cpx>& A, cpx w) {
   int n = (int)A.size();
   if(n == 1) return;

   vector<cpx> even(n / 2), odd(n / 2);
   for(int i = 0; i < n; ++i) {
      if(i & 1) odd[i / 2] = A[i];
      else even[i / 2] = A[i];
   }

   FFT(even, w * w);</pre>
```

```
FFT(odd, w * w);
    cpx w_e(1, 0);
    for(int i = 0; i < n / 2; ++i) {
        A[i] = even[i] + w e * odd[i];
        A[i + n / 2] = even[i] - w_e * odd[i];
        w_e *= w;
   }
}
void product(vector<cpx>& A, vector<cpx>& B) {
    int n = (A.size() <= B.size()) ? ceil(log2((double)B.size())) : ceil(log2((</pre>
      double)A.size()));
    n = pow(2, n + 1);
    A.resize(n);
    B.resize(n);
    vector<cpx> C(n);
    cpx \ w(cos(2 * acos(-1) / n), sin(2 * acos(-1) / n));
    FFT(A, w);
    FFT(B, w);
    for(int i = 0; i < n; ++i) C[i] = A[i] * B[i];
    FFT(C, cpx(1, 0) / w);
    for(int i = 0; i < n; ++i) {
        C[i] /= cpx(n, 0);
        C[i] = cpx(round(C[i].real()), round(C[i].imag()));
}
void FFT(vector<cpx>& A, bool invert) {
    int n = (int)A.size();
    for(int i = 1, j = 0; i < n; ++i) {
        int bit = n \gg 1;
        while(j >= bit) {
            i -= bit:
            bit >>= 1;
        j += bit;
        if(i < j) swap(A[i], A[j]);
   for(int length = 2; length <= n; length <<= 1) {</pre>
        double ang = 2 * PI / length * (invert ? -1 : 1);
        cpx w(cos(ang), sin(ang));
        for(int i = 0; i < n; i += length) {
            cpx w_i(1, 0);
```

```
for(int j = 0; j < length / 2; ++j) {
            cpx u = A[i + j], v = A[i + j + length / 2] * w_i;
            A[i + j] = u + v, A[i + j + length / 2] = u - v;
            w_i *= w;
        }
    }
}
if(invert) {
    for(int i = 0; i < n; ++i) {
        A[i] /= cpx(n, 0);
        A[i] = cpx(round(A[i].real()), round(A[i].imag()));
    }
} // referenced from https://blog.myungwoo.kr/54
*/ //faster version of FFT</pre>
```

1.4 수학 관련 노트

- 1. 모듈러 역원은 M이 소수일 때, $a^{M-2} mod M$ 이다.
- 2. a + b = c 인 (a, b, c) 개수 찾는 문제면 FFT 시도해보기.
- 3. 모든 케이스에 대한 경우의 수도 FFT 생각해보기.
- 4. 순서가 있는 쌍을 찾으라고 하면 정렬한 것과 비교하는 식으로 스위핑일 수 있음. 스위핑이면 보통 세그먼트 트리로 최적화해서 풀 수 있음.
- 5. 최적화 문제에서 뭔가 안되면, 이분탐색 / 삼분탐색 떠올려보기.
- 6. 가장 가까운 두 쌍 찾기는 거리를 기준으로 한 분할정복으로 찾을 수 있음.
- 7. 누가 이기는 건지 묻는걸로 바꿀 수 있으면, 게임이론 -> 스프라그 그런디 정리 쓸수 있는 지 생각해보기
- 8. dp[i] = max/min(a[i]b[j] + c[j]) + d[i] 꼴에, b가 단조증가 단조감소이면 CHT임.
- 9. 카탈란 수 : 길이가 2n인 올바른 괄호 문자열 수, n+2각형을 n개 삼각형으로 분할 하는 경우의 수 등으로 나올 수 있음.

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

$$C_{n+1} = \sum_{i=0}^n C_i C_{n-i}$$

$$C_{n+1} = \frac{2(2n+1)}{n+2} C_n$$

2 DP optimization

2.1 Convex Hull Trick

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
struct line {
    11 m, n;
    long double s;
};
ll dp[100'001];
line line stack[100'001];
long double get_intersection(const line& a, const line& b) {
    return (long double)(a.n - b.n) / (long double)(b.m - a.m);
11 solve(vector<11>& a, vector<11>& b, int n) {
    int top = 0, cur = 0;
    for(int i = 2; i <= n; ++i) {
        line g = \{b[i - 1], dp[i - 1], 0\};
        while(top > 0) {
            g.s = get_intersection(line_stack[top - 1], g);
            if(line_stack[top - 1].s < g.s) break;</pre>
            if(--top == cur) cur--;
        }
        line_stack[top++] = g;
        11 x = a[i];
        while(cur + 1 < top && line stack[cur + 1].s < x) cur++;</pre>
        dp[i] = line_stack[cur].m * a[i] + line_stack[cur].n;
    return dp[n];
}
DP 아닌데 저런 꼴이 보여도 적용할 수 있음.
만약 a가 단조 증가하지 않으면 구간을 이분탐색으로 찾아주면 됨.
```

3 Segment Tree

3.1 Segment Tree with Lazy Propagation

```
#include <bits/stdc++.h>
using namespace std;
typedef long long l1;
```

```
class segment_tree {
   private:
        vector<ll> tree;
        vector<ll> lazy;
   public:
        segment_tree(int N) {
            tree.resize(N * 4);
            lazy.resize(N * 4);
        void propagate(int start, int end, int node) {
            if(lazy[node] == 0) return;
            tree[node] += (ll)(end - start + 1) * lazy[node];
            if(start != end) {
                lazy[node * 2] += lazy[node];
                lazy[node * 2 + 1] += lazy[node];
            }
            lazy[node] = 0;
        void update(int start, int end, int node, int left, int right, ll diff)
            propagate(start, end, node);
            if(start > right || end < left) return;</pre>
            if(left <= start && end <= right) {</pre>
                tree[node] += (ll)(end - start + 1) * diff;
                if(start != end) {
                    lazy[node * 2] += diff;
                    lazy[node * 2 + 1] += diff;
                }
                return;
            int mid = (start + end) / 2;
            update(start, mid, node * 2, left, right, diff);
            update(mid + 1, end, node * 2 + 1, left, right, diff);
            tree[node] = tree[node * 2] + tree[node * 2 + 1];
        11 query(int start, int end, int node, int left, int right) {
            propagate(start, end, node);
            if(start > right || end < left) return 0;</pre>
            if(left <= start && end <= right) return tree[node];</pre>
```

```
int mid = (start + end) / 2;
            return query(start, mid, node * 2, left, right) + query(mid + 1, end
              , node * 2 + 1, left, right);
};
      Fenwick Tree
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
class fenwick tree {
    private:
        vector<11> tree;
        int max_size;
    public:
        fenwick tree(int N) {
            tree.resize(N + 1);
            max size = N;
        void update(int idx, ll diff) {
            for(int i = idx; i <= max_size; i += (i & (-i)))</pre>
                tree[idx] += diff;
        11 query(int idx) {
            11 \text{ ret} = 0;
            for(int i = idx; i > 0; i -= (i & (-i)))
                ret += tree[i];
            return ret;
};
class fenwick tree 2d {
    private:
        vector<vector<ll>> tree;
        int max_size;
    public:
        fenwick tree 2d(int N) {
            tree.resize(N + 1);
            for(int i = 1; i <= N; ++i)
                tree[i].resize(N + 1);
            max_size = N;
        void update(int y, int x, ll diff) {
            for(int i = x; i <= max_size; i += (i & (-i))) {</pre>
                for(int j = y; j <= max_size; j += (j & (-j))) {</pre>
                    tree[i][j] += diff;
```

```
11 query(int y, int x) {
            11 \text{ ret} = 0;
             for(int i = x; i > 0; i -= (i & (-i))) {
                 for(int j = y; j > 0; j -= (j & (-j))) {
                     ret += tree[i][j];
             }
             return ret;
};
```

Dynamic Segment Tree

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
using pii = pair<int, int>;
using ppii = pair<int, pii>;
struct node {
    node *1, *r;
    11 s;
    node() {
        1 = r = NULL;
        s = 0;
};
class dynamic_segment_tree {
    private:
        node* root;
        void delete nodes(node* v) {
            if(!v) return;
            delete_nodes(v->1);
            delete_nodes(v->r);
            delete v;
    public:
        dynamic_segment_tree() {
            root = new node();
        node* get_root() {
            return root;
        void update(int start, int end, node* cur_node, int idx, ll val) {
```

};

```
if(idx < start || idx > end) return;
    if(start == end) {
        cur node->s = val;
        return;
    int mid = (start + end) / 2;
    if(idx <= mid) {</pre>
        if(!cur_node->1) cur_node->1 = new node();
        update(start, mid, cur_node->1, idx, val);
    else {
        if(!cur_node->r) cur_node->r = new node();
        update(mid + 1, end, cur_node->r, idx, val);
    }
    11 1_val = (cur_node->l ? cur_node->l->s : 0);
    11 r_val = (cur_node->r ? cur_node->r->s : 0);
    cur node->s = 1 val + r val;
11 query(int start, int end, node* cur_node, int left, int right) {
    if(!cur_node) return 0;
    if(right < start || left > end) return 0;
    if(left <= start && end <= right) return cur node->s;
    int mid = (start + end) / 2;
    return query(start, mid, cur_node->l, left, right) + query(mid + 1,
     end, cur node->r, left, right);
~dynamic_segment_tree() {
    delete_nodes(root);
```

3.4 ___gnu_pbds Ordered Set (can be replaced by k-th segtree)

```
int main() {
    ordered_set pbds_set;

int X; cin >> X;
    pbds_set.insert(X); //insert
    cout << pbds_set.order_of_key(X) << '\n'; //Number of elements smaller than
        X : O(log N)
    cout << *pbds_set.find_by_order(X) << '\n'; //X-th element in a set (0-based
        ) : O(log N)
    //based on red-black tree.

return 0;
}</pre>
```

4 Graph

4.1 Bellman-Ford

```
#include <bits/stdc++.h>
using namespace std;
const long long INF = 1e18;
vector<long long> dist(501, INF);
void solve(vector<pair<int, int>> (&adj)[501], int& N) {
    dist[1] = 0;
    for(int i = 0; i < (N - 1); ++i) {
        for(int j = 1; j <= N; ++j) {
            for(auto edge : adj[j]) {
                if(dist[j] != INF) {
                     dist[edge.first] = min(dist[j] + edge.second, dist[edge.
                      first]);
    }
    for(int i = 1; i <= N; ++i) {</pre>
        for(auto edge : adj[i]) {
            if(dist[i] != INF && dist[edge.first] > dist[i] + edge.second) {
                cout << "-1\n";
                return;
            }
    for(int i = 2; i <= N; ++i) {</pre>
        if(dist[i] != INF) {
            cout << dist[i] << '\n';
        else cout << "-1\n";</pre>
```

4.2 SCC(Kosaraju's Algorithm)

```
#include <bits/stdc++.h>
void forward DFS(int node, vector<vector<int>>& adj, stack<int>& stk, vector
 bool>& visit) {
   for(auto& next : adj[node]) {
        if(visit[next]) continue;
        visit[next] = true;
        forward DFS(next, adj, stk, visit);
    stk.push(node);
}
void backward DFS(int node, vector<vector<int>>& adj, vector<bool>& visit,
 vector<int>& group, int& group number) {
   group[node] = group number;
   for(auto& next : adj[node]) {
        if(visit[next]) continue;
        visit[next] = true;
        backward DFS(next, adj, visit, group, group number);
}
void get_SCC(int N, vector<vector<int>>& f_adj, vector<vector<int>>& b_adj,
    vector<int>& group, vector<vector<int>>& scc adj) {
    stack<int> stk;
    vector<bool> visit(N + 1, false);
    for(int i = 1; i <= N; ++i) {</pre>
        if(visit[i]) continue;
        visit[i] = true;
        forward_DFS(i, f_adj, stk, visit);
   }
   fill(visit.begin(), visit.end(), false);
   int group_number = 1;
    while(!stk.empty()) {
        int node = stk.top();
        stk.pop();
       if(visit[node]) continue;
        visit[node] = true;
        backward_DFS(node, b_adj, visit, group, group_number);
        group_number++;
    scc adj.resize(group number);
    vector<int> in_degree(group_number);
    for(int i = 1; i <= N; ++i) {</pre>
        for(auto& j : f_adj[i]) {
            if(group[i] != group[j]) {
```

4.3 Heavy-Light Decomposition

```
#include <bits/stdc++.h>
#define MAX 100'000
#define INF 987654321
using namespace std;
using ll = long long;
using pii = pair<int, int>;
using ppii = pair<int, pii>;
class segment_tree { /*implementation here*/ };
class heavy_light_decomposition {
    private:
        int group cnt;
        int tree_size[MAX], depth[MAX], parent[MAX], top_chain[MAX], in[MAX],
         out[MAX];
        bool visit[MAX];
        vector<int> child[MAX];
        vector<int> adj[MAX];
        segment_tree tree;
    public:
        void init() {
            fill(tree size, tree size + MAX, 0);
            fill(depth, depth + MAX, 0);
            fill(parent, parent + MAX, 0);
            fill(top_chain, top_chain + MAX, 0);
            fill(in, in + MAX, 0);
            fill(out, out + MAX, 0);
            fill(visit, visit + MAX, false);
            group_cnt = 0;
            int N; cin >> N;
            tree.resize(MAX);
            for(int i = 0; i < N - 1; ++i) {
                int u, v; cin >> u >> v;
                adj[u].push_back(v);
                adj[v].push back(u);
            dfs_child_set();
            dfs size();
            dfs_grouping();
```

};

```
void dfs_child_set(int v = 1) {
    visit[v] = true;
    for(auto& next : adj[v]) {
        if(visit[next]) continue;
        visit[next] = true;
        child[v].push_back(next);
        dfs_child_set(next);
}
void dfs size(int v = 1) {
    tree size[v] = 1;
    for(auto& next : child[v]) {
        depth[next] = depth[v] + 1;
        parent[next] = v;
        dfs size(next);
        tree_size[v] += tree_size[next];
        if(tree_size[next] > tree_size[child[v][0]]) swap(child[v][0],
}
void dfs_grouping(int v = 1) {
    in[v] = ++group cnt;
    for(auto& next : child[v]) {
        top_chain[next] = (next == child[v][0] ? top_chain[v] : next);
        dfs grouping(next);
    out[v] = group_cnt;
void update(int v, int w) {
    tree.update(1, MAX, 1, in[v], w);
int query(int a, int b) {
    int ret = 0;
    while(top_chain[a] != top_chain[b]) {
        if(depth[top_chain[a]] < depth[top_chain[b]]) swap(a, b);</pre>
        int v = top chain[a];
        ret += tree.query(1, MAX, 1, in[v], in[a]);
        a = parent[v];
    if(depth[a] > depth[b]) swap(a, b);
    ret += tree.query(1, MAX, 1, in[a], in[b]);
    return ret;
```

5 Geometry

5.1 선분교차 3 (선분교차 여부 + 교점 좌표 출력)

```
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
const long double eps = 1e-7;
typedef struct _Point {
    long double x;
    long double y;
} Point;
Point getVector(Point& A, Point& B) {
    Point vec = \{B.x - A.x, B.y - A.y\};
    return vec;
}
int ccw(Point& v, Point& u) {
    ll val = v.x * u.y - v.y * u.x;
    if(val > 0) return 1;
    else if(val < 0) return -1;</pre>
    else return 0;
bool operator==(const Point& A, const Point& B) {
    return (abs(A.x - B.x) < eps && abs(A.y - B.y) < eps);
Point& operator/=(Point& A, const long double div) {
    A = \{A.x / div, A.y / div\};
    return A;
}
Point operator*(const Point& A, const long double mul) {
    Point X = A;
    X = {X.x * mul, X.y * mul};
    return X;
}
bool operator<=(const Point& A, const Point& B) {</pre>
    if(A.x < B.x) return true;</pre>
    else if(A.x == B.x && A.y <= B.y) return true;</pre>
    else return false;
}
int is_cross(Point& A, Point& B, Point& C, Point& D) {
    Point vecAB = getVector(A, B);
    Point vecCD = getVector(C, D);
    Point vecBC = getVector(B, C);
```

```
Point vecBD = getVector(B, D);
    Point vecDA = getVector(D, A);
    Point vecDB = getVector(D, B);
    if(ccw(vecAB, vecBC) * ccw(vecAB, vecBD) == 0 && ccw(vecCD, vecDA) * ccw(
      vecCD, vecDB) == 0) {
        if(B \le A) swap(A, B);
        if(D \leftarrow C) swap(C, D);
        if(A <= D && C <= B) return 1;
        else return -1;
    else if(ccw(vecAB, vecBC) * ccw(vecAB, vecBD) <= 0 && ccw(vecCD, vecDA) *</pre>
      ccw(vecCD, vecDB) <= 0) return 2;</pre>
    else return -1;
}
long double get_size(Point& vec) {
    return sqrt(vec.x * vec.x + vec.y * vec.y);
}
Point get_meet(Point& A, Point& B, Point& C, Point& D) {
    Point ret;
   long double a, b, c, d, e, f;
    a = A.y - B.y;
    b = B.x - A.x;
    c = A.x * a + A.v * b;
    d = C.y - D.y;
    e = D.x - C.x;
   f = C.x * d + C.y * e;
   long double dn = a * e - b * d;
    ret = \{((e * c - b * f) / dn), ((a * f - c * d) / dn)\};
    return ret;
}
int main() {
   Point A, B, C, D;
    cin >> A.x >> A.y >> B.x >> B.y;
    cin >> C.x >> C.y >> D.x >> D.y;
    cout << fixed;</pre>
    cout.precision(15);
    if(is_cross(A, B, C, D) == 1) {
        cout << "1\n";
        Point unit_dir_vec_AB = {B.x - A.x, B.y - A.y};
        long double sz_AB = get_size(unit_dir_vec_AB);
        unit dir vec AB /= sz AB;
```

```
Point unit_dir_vec_CD = {D.x - C.x, D.y - C.y};
        long double sz_CD = get_size(unit_dir_vec_CD);
        unit dir vec CD /= sz CD;
        if(unit dir vec AB == unit dir vec CD || unit dir vec AB ==
          unit dir vec CD * (-1)) {
            if(B == C) cout << B.x << ' ' << B.y << '\n';
            else if(A == D) cout << A.x << ' ' << A.y << '\n';
        else {
            if(A == C | | A == D) cout << A.x << ' ' << A.y << '\n';
            else if(B == C || B == D) cout << B.x << ' ' << B.y << '\n';
    else if(is_cross(A, B, C, D) == 2) {
        cout << "1\n";
        Point X = get_meet(A, B, C, D);
        cout << X.x << ' ' << X.y << '\n';
    else cout << "0\n";</pre>
    return 0;
}
5.2 Convex Hull
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
typedef struct Point {
    int x;
    int y;
} Point;
//Standard Point to Sort
Point S;
Point get_vector(const Point& A, const Point& B) {
    Point v = \{B.x - A.x, B.y - A.y\};
    return v;
}
//ccw test
int ccw(const Point& v, const Point& u) {
    11 \text{ val} = (11)v.x * u.y - (11)v.y * u.x;
    if(val > 0) return 1;
    else if(val < 0) return -1;</pre>
    else return 0;
}
int ccw(const Point& A, const Point& B, const Point& C) {
```

```
Point v = get_vector(A, B);
                                                                                          if(convex_hull.size() == 2) return get_dist(convex_hull[0], convex_hull[1]);
    Point u = get_vector(B, C);
    return ccw(v, u);
                                                                                          11 \text{ ret} = 0;
}
                                                                                          int a idx = 1, b idx = 2;
//to sort by ccw
                                                                                          Point a start = convex hull[0], a end = convex hull[1], b start =
bool comp(const Point& A, const Point& B) {
                                                                                            convex hull[1], b end = convex hull[2];
    Point v = get_vector(S, A);
                                                                                          while(true) {
    Point u = get_vector(S, B);
                                                                                              ret = max(ret, get dist(a start, b start));
    if(ccw(v, u) > 0) return true;
                                                                                              Point v1 = get_vector(a_start, a_end);
                                                                                              Point v2 = get_vector(b_start, b_end);
    else if(ccw(v, u) < 0) return false;
    return (v.x == u.x)? (v.y < u.y): (v.x < u.x);
                                                                                              if(ccw(v1, v2) > 0) {
}
                                                                                                  b_idx = (b_idx + 1) % convex_hull.size();
                                                                                                  b start = b end;
bool operator<(const Point& A, const Point& B) {</pre>
                                                                                                  b end = convex hull[b idx];
    return (A.x == B.x)? (A.y < B.y): (A.x < B.x);
                                                                                              else {
                                                                                                  a idx++;
//Graham's Scan Method
vector<Point> get convex hull(vector<Point>& A) {
                                                                                                  if(a idx == convex hull.size()) {
    S = *min_element(A.begin(), A.end());
                                                                                                      a_start = a_end;
    sort(A.begin(), A.end(), comp);
                                                                                                      a_end = convex_hull[0];
    int n = (int)A.size();
                                                                                                  else if(a_idx == convex_hull.size() + 1) break; //end condition
    vector<Point> convex hull;
                                                                                                  a_start = a_end;
    //get Convex Hull
                                                                                                  a_end = convex_hull[a_idx];
    for(int i = 0; i < n; ++i) {</pre>
                                                                                          }
        while((int)convex_hull.size() > 1
        && ccw(convex_hull[(int)convex_hull.size() - 2], convex_hull.back(), A[i
         1) <= 0) {</pre>
                                                                                          return ret;
            convex_hull.pop_back();
                                                                                     }
        convex_hull.push_back(A[i]);
                                                                                      5.4 Point in Convex Hull Test
    }
                                                                                      #include <bits/stdc++.h>
    return convex_hull;
}
                                                                                      using namespace std;
     Rotating Calipers
                                                                                      typedef struct _Point {
                                                                                          int x;
#include <bits/stdc++.h>
                                                                                          int y;
using namespace std;
                                                                                      } Point;
using ll = long long;
struct Point { double x, y; };
                                                                                      Point get_vector(const Point& A, const Point& B);
int ccw(Point, Point);
                                                                                      int ccw(const Point& v, const Point& u);
                                                                                      int ccw(const Point& A, const Point& B, const Point& C);
Point get_vector(Point, Point);
11 get_dist(Point& A, Point& B) { return (B.x - A.x) * (B.x - A.x) + (B.y - A.y)
                                                                                      //convext_hull size >= 3
   * (B.y - A.y); }
                                                                                      bool isInside(vector<Point>& convext_hull, Point& A) {
                                                                                          int 0 = 0;
11 max_dist(vector<Point>& convex_hull) {
                                                                                          int L = 1, R = (int)convext_hull.size() - 1;
    if(convex hull.size() == 1) return 0;
                                                                                          int M = (L + R) / 2;
```

```
Point vecOL = get_vector(convext_hull[0], convext_hull[L]);
Point vecOA = get_vector(convext_hull[0], A);
Point vecOR = get_vector(convext_hull[0], convext_hull[R]);
Point vecOM = get_vector(convext_hull[0], convext_hull[M]);

if(ccw(vecOL, vecOA) < 0) return false;
if(ccw(vecOR, vecOA) > 0) return false;

while(L + 1 != R) {
    M = (L + R) / 2;
    vecOM = get_vector(convext_hull[0], convext_hull[M]);

    if(ccw(vecOM, vecOA) > 0) L = M;
    else R = M;
}

if(ccw(convext_hull[L], A, convext_hull[R]) <= 0) return true;
else return false;</pre>
```

6 String

6.1 KMP

```
#include <bits/stdc++.h>
using namespace std;
void get_fail(string& P, vector<int>& fail) {
    for(int i = 1, j = 0; i < (int)P.length(); ++i) {
        while(j && P[i] != P[j]) j = fail[j - 1];
        if(P[i] == P[j]) fail[i] = ++j;
}
void KMP(string& T, string& P) { //O(N + M)
    int T len = (int)T.length();
    int P_len = (int)P.length();
    vector<int> fail(P len);
    get_fail(P, fail);
    vector<int> ret;
    for(int i = 0, j = 0; i < T_len; ++i) {</pre>
        while(j && T[i] != P[j]) j = fail[j - 1];
        if(T[i] == P[j]) {
            if(j == P_len - 1) { //matching success
                ret.push_back(i - P_len + 2);
                j = fail[j];
            else j++;
```

```
}

cout << (int)ret.size() << '\n';
for(auto& element : ret) cout << element << '\n';
}</pre>
```

7 Optimization

7.1 GCC pragma

```
#pragma GCC optimize ("03")
#pragma GCC optimize ("0fast")
#pragma GCC optimize ("unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse4,avx,avx2")
```

8 LCA

8.1 LCA

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define PRECISION 0
#define fr first
#define sc second
using 11 = long long;
using ld = long double;
typedef pair<ll, ll> pll;
typedef pair<int,int> pii;
const 11 MOD = 1e9+7;
const int MAX_DEPTH = 17; //ceil(log base2(n))
int n,q;
int table[MAX_DEPTH+1][100005];
vector<int>edges[100005];
int parent[100005];
int depth[100005];
bool vis[100005];
void dfs(int cur, int d){
    depth[cur] = d;
    vis[cur] = 1;
    for(int &nxt:edges[cur]){
```

```
if(vis[nxt]) continue;
    parent[nxt] = cur;
    dfs(nxt, d+1);
}

void fill_table(){
    dfs(1,1);
    for(int i=1; i<=n; i++){
        table[0][i] = parent[i];
    }
    for(int k=1; k<=MAX_DEPTH; k++){
        for(int i=1; i<=n; i++){
            table[k][i] = table[k-1][table[k-1][i]];
        }
    }
}</pre>
```

9 Mo's

9.1 Best Student

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define PRECISION 0
#define fr first
#define sc second
using ll = long long;
using ld = long double;
typedef pair<11,11> pll;
typedef pair<int,int> pii;
const 11 MOD = 1e9+7;
const int MAX_DEPTH = 17; //ceil(log base2(n))
int n,q;
int table[MAX_DEPTH+1][100005];
vector<int>edges[100005];
int parent[100005];
int depth[100005];
bool vis[100005];
void dfs(int cur, int d){
    depth[cur] = d;
    vis[cur] = 1;
    for(int &nxt:edges[cur]){
```

```
if(vis[nxt]) continue;
    parent[nxt] = cur;
    dfs(nxt, d+1);
}

void fill_table(){
    dfs(1,1);
    for(int i=1; i<=n; i++){
        table[0][i] = parent[i];
    }
    for(int k=1; k<=MAX_DEPTH; k++){
        for(int i=1; i<=n; i++){
            table[k][i] = table[k-1][table[k-1][i]];
        }
}</pre>
```