CoDEx: Wilson coefficient calculator connecting SM EFT to UV theory☆

Supratim Das Bakshi[a,1], Joydeep Chakrabortty[a,*], Sunando Kumar Patra[a,2]

[a]*Department of Physics, Indian Institute of Technology, Kanpur-208016, India*

Abstract

We have developed a Wilson coefficient (WC) calculator "CoDEx" to connect the Standard Model effective field theory (SM EFT) and the ultraviolet Beyond Standard Models (UV-BSM). The basic working principle of "CoDEx" is based on integrating out the heavy propagators using covariant derivative expansion (CDE) technique suitably blended within functional methods. The WCs are computed in two different bases at the UV scale. There is a provision to perform the renormalisation group evolution of these Wilson coefficients to generate the operators with rescpective WCs at the electro-weak scale. Here, we have chosen few BSM scenarios as example models to demonstrate how this code works. Using "CoDEx", one can integrate out bosonic)both spin-0 and 1) and fermionic (spin-1/2) heavy degrees of freedom.

Keywords: keyword1; keyword2; keyword3; etc.

PROGRAM SUMMARY
Program Title: CoDEx
Licensing provisions: CC By 4.0
Programming language: Wolfram Language (Mathematica)

Supplementary material:

Nature of problem(approx. 50-250 words):
Solution method(approx. 50-250 words):
Additional comments including Restrictions and Unusual features (approx. 50-250 words):

[1] Reference 1
[2] Reference 2
[3] Reference 3

1. Introduction

We are in a very mysterious phase of particle physics. On the one side where we are cherishing the success of the Standard Model (SM) after the discovery of the SM-Higgs like particle, considered to be the pinnacle, on the other hand we have enough reason to believe the existence of theories beyond it (BSM). To address the shortcomings of the SM, many BSM scenarios are proposed at very different scale. It is believed that if there is any such theory which contains the SM as a part of it, will affect the electro-weak and the Higgs sector. Thus the precision observables are expected to carry the footprints of the *new* physics, unless it is in decoupling limit.

The ongoing and proposed future experiments are expected to improve the sensitivity at the per mille level on these precision observables. Thus even if we do not observe any resonance of new particles, indirectly we can estimate the allowed room left for the BSM physics. This motivates us to look into the BSM scenario through the eyes of Standard Model effective field theory (SM EFT). The basic idea of SM EFT is very clear: integrate out the heavy non-SM degrees of freedom and their impact will be captured through the higher mass dimensional operators: $\sum_i \frac{1}{\Lambda^{d_i-4}} C_i O_i$. Here, d_i is the mass dimensionality of the operators O_i, starting from 5, and C_i are their respective Wilson coefficients which are functions of BSM parameters. Λ is the cut off scale, at which all the WCs are computed as $C_i(\Lambda)$. In this EFT approach we rely on the validity of perturbative expansion of the S-matrix in the powers of Λ^{-1}, and expect that this series will pass the convergence test. As this scale is higher than the scale M_Z where the precision test is performed, dimension-6 operators are more suppressed than the dimension-5 ones and so. Now the query is where to truncate the $1/\Lambda$ series? This decision is made case by case based on the achieved and(or) expected precision level of the observables at the present and(or) future experiments. Now the questions that may be itching us: Why SM EFT instead of doing full calculation using the BSM Lagrangian which is supposed to be more exact and accurate? How one can ensure that the results computed in the SM EFT approach is using a truncated S-matrix will be very close (unnoticed in the precision test) to the results computed using full BSM theory? The computation with the full BSM is very involved, and tedious, and that also at the loop level. The cut-off Λ is chosen in such a way that the M_Z/Λ series is converging which ensures the truncation of this series at some finite order is safe and sufficient. Next question which must be addressed is: How do we connect the physics of two

*Corresponding author.
E-mail address: joydeep.chakrabortty@gmail.com
[1]sdbakshi13@gmail.com
[2]sunando.patra@gmail.com

different scales, namely UV and the M_Z? We need to recall that the WCs that we are computing using SM EFT are at the scale Λ, but the observables are measured at M_Z scale. So we need to evolve the $C_i(\Lambda)$ to obtain $C_i(M_Z)$ using the anomalous dimension matrix (γ). While performing the renormalisation group evolutions (RGEs) of the C_i's, we need to choose the γ carefully as the anomalous dimension matrix is basis dependent. Thus we should choose only those basis in which the precision observables are defined, and it is important to ensure the basis we are working with is the complete one. As the matrix γ contains non-zero off-diagonal elements, it is indeed possible to generate some new effective operators, which were absent at Λ scale, through RGEs.

2. CDE - Method of Integrating out

In this section, we will briefly motivate the need and methodology of covariant derivative expansion (CDE). This was introduced in [**?**], and then extended in [**?**]. As our goal is to match the UV theory onto the SM EFT and that too order by order in perturbation theory, we need to respect gauge invariance at each and every step. This requires to perform the perturbative expansion in terms of some gauge covariant quantities, and covariant derivative is the "chosen one". This CDE has much bigger impact and not only restricted to quantify the integrating out of heavy fields, see [**?**] for details.

Adjudging the status, and prospect of the present, and future experiments, we can safely restrict ourself to only dimension-6 operators including the tree level and one-loop parts of the effective action. The modus operandi of "CoDEx" is based on the method of CDE discussed in [**?**]. Here, we will work with a generalized Laplacian Operator ($\mathcal{D}^2 + U$), where $\mathcal{D}$ is the covariant derivative for a given gauge theory and U contains the information about the light degrees of freedom.

2.1. CDE and tree-level effective Lagrangian

In this section, we discuss the construction of tree level effective Lagrangian $\left(\mathcal{L}_{TL}^{(EFT)}\right)$ from BSM Lagrangian (say LBSM) using the CDE method. $\mathcal{L}_{TL}^{(EFT)}$ consists of the terms in LBSM which contains only light fields (fields which are physical in low energy limit) and higher dimension operators with their coefficients aka Wilson Coefficients (WC's) generated from tree level processes only. Of course, $\mathcal{L}_{TL}^{(EFT)} \subset \mathcal{L}^{(EFT)}$ where $\mathcal{L}^{(EFT)}$ is defined as the effective Lagrangian constructed by integrating out heavy degrees of freedom from all processes - tree, 1 loop, 2 loop and higher loop processes.

Tree level Wilson coefficients can be determined by using the solution of the heavy degrees of freedom in the LBSM. This is equivalent to replacing the degrees of freedom in LBSM by their solutions. The solutions of heavy fields can be determined by using the Euler-Lagrangian equation for the heavy fields on the given LBSM. Mathematically, this can be realised in following way.

Consider that we write down the LBSM or the UV Lagrangian,

$$\mathcal{L}^{(UV)} \equiv \mathcal{L}^{(UV)}(\phi, \Phi), \tag{1}$$

where ϕ represents all light fields and Φ represents all heavy fields present in the given UV theory. We can decompose

$$\mathcal{L}^{(UV)}(\phi, \Phi) = \mathcal{L}^{(\Phi)} + \mathcal{L}^{(\phi)} + \mathcal{L}_{int}^{(\phi,\Phi)}, \tag{2}$$

where $\mathcal{L}^{(\Phi)}, \mathcal{L}^{(\phi)}$and $\mathcal{L}_{int}^{(\phi,\Phi)}$ are defined as terms in UV Lagrangian containing heavy fields only, light fields only and heavy & light both respectively.

$\mathcal{L}_{int}^{(\phi,\Phi)}$ can be further decomposed in terms of powers of heavy field,

$$\mathcal{L}_{int}^{(\phi,\Phi)} = \mathcal{L}_{I}^{(\phi)} . \Phi^1 + \mathcal{L}_{II}^{(\phi)} . \Phi^2 + \dots \quad , \tag{3}$$

where $\mathcal{L}_{K}^{(\phi)}$'s are coefficients which contains light fields only and they can be vectors and matrices depending upon the representation of the concerned heavy multiplets. In order to get a more clear picture of this idea one can see section 4 where we consider a given BSM Lagrangian and show how this method is executed. As mentioned above, now we need the solution of the heavy fields which can determined by using the Euler-Lagrange equation,

$$\frac{\partial}{\partial \Phi}\mathcal{L}^{(UV)}(\phi, \Phi) = \mathcal{D}_\mu \, \frac{\partial}{\partial(\mathcal{D}_\mu \Phi)}\mathcal{L}^{(UV)}(\phi, \Phi). \tag{4}$$

Using, equation 2, 3and4, we get

$$\mathcal{L}_{I}^{(\phi)} = \; O_{\mathcal{D}} \, . \, \Phi, \tag{5}$$

keeping terms upto linear order in heavy fields. Here, $O_{\mathcal{D}}$ is an operator which contains covariant derivative operator and light fields, and takes different structure for multiplets with different spins. For example, $O_{\mathcal{D}}$ takes an elliptic structure ,$O_{\mathcal{D}} = \mathcal{D}^2 + m^2 + \mathcal{L}_{II}^{\phi}$, in cases where the heavy multiplet, Φ represents scalar multiplet. See section 4.1. Now, we recast equation 5,

$$\Phi_s = O_{\mathcal{D}}^{-1} \, . \, \mathcal{L}_{I}^{(\phi)}. \tag{6}$$

Thus, models in which $\mathcal{L}_{I}^{(\phi)}$ is missing, there will not be any possible tree level processes and hence there will not exist tree level Wilson coefficients. We use a subscript 's' on Φ in the above equation to remind ourself that it is the solution of the multiplet Φ.

2.1.1. Methodology: Construction of $\mathcal{L}_{TL}^{(EFT)}$ in the CoDEx package.

The most crucial task is to write down the LBSM. To write LBSM, we need the heavy fields and the SM fields. The SM fields are all provided in CoDEx and a Mathematica documentation for the SM fields is available within the package. But the representation of heavy fields are defined within CoDEx environment when we provide the following information in a file named as "UserDefined.m".

Information:.

1. Write the gauge quantum numbers for the SM gauge group ($SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$) in the "flrepre" in the "UserDefined.m" file. For example, for a heavy field which is color singlet, isospin triplet with hypercharge 1, we write

   `flrepre = {{1,3,1},{...},...}`

   in case where the mentioned heavy field is most massive. The dots represent other heavy multiplets if any.

2. Write the spin, mass and the hypercharge of the heavy field in "flch" in "UserDefined.m". For example, if the heavy field is scalar with mass m_Δ and hypercharge 0.

   `flch = {{0,mΔ,0},{...},...}`

   The dots stands for information of other heavy multiplets if any.

Defining "flrepre" and "flch" in the "UserDefined.nb" is necessary for tree level WC calculation. Next, we use "loadUserDefined[]" in the Mathematica frontend to load this information and if the package is loaded, then we are able to see the representation of the heavy fields. Next, we write the BSM Lagrangian, LBSM.

2.1.2. `trOut` :- Function for matching LBSM to WC's

After writing the LBSM, we create $O_{\mathcal{D}}$ operator as mentioned in equation 5. The structure of $O_{\mathcal{D}}$ is determined by the BSM model in hand. $O_{\mathcal{D}}$ is then inverted and operated on the tree level terms, $\mathcal{L}_I^{\phi}$, from which we get the solutions of heavy fields and then we replace the solutions back in LBSM for all heavy fields in LBSM. Next we search and collect the coefficients of the given higher dimension operators and we write the results.

All these sequential tasks are done by the function, trOut. So, once LBSM is created in the code, we need to write

`trOut[LBSM]`

and we will have the tree level Wilson coefficients as the output.

2.2. CDE and one-loop effective Lagrangian

In this section, we discuss the WC's generated in one loop processes. In the SMEFT regime, those diagrams only contribute to one loop WC's which have the SM fields as external legs and both heavy field and the SM field propagators in the loop. In this version of CoDEx, we calculate the contributions from one loop diagrams in which the fields constructing the loop are all heavy. We are not including contributions from the loop diagrams in which the loops consists of both heavy and light fields. Now we consider a BSM Lagrangian, LBSM and let the mass of the heavy field be m. We write the effective action,

$$\mathcal{S}^{(EFT)} = \int d^4x\, \mathcal{L}^{(EFT)}, \tag{7}$$

and, contributions to effective action from one loop processes, [3]

[3] see section 2.1.2 in [1]

$$\mathcal{S}^{(EFT)}_{1loop} = ic_s Tr[(\mathcal{D}_\mu)^2 + m^2 + U], \tag{8}$$

where for heavy field being a real scalar, complex scalar, fermion or gauge boson, $c_s = \frac{1}{2}, 1, -\frac{1}{2} \text{or} \frac{1}{2}$ respectively. U consists of light fields. Definition of $\mathcal{D}_\mu$ for the heavy field depends on the gauge quantum number of the heavy field under the SM gauge group. Following the analysis in the article [1], we reproduce

$$\begin{aligned}
\mathcal{L}^{(EFT)}_{1loop} \supset \mathcal{L}^{(EFT)}_{1loop,dim6} = \frac{c_s}{(4\pi)^2}\,\mathrm{tr}\Bigg\{ \\
&+ \frac{1}{m^2}\Bigg[-\frac{1}{60}(P_\mu G'_{\mu\nu})^2 - \frac{1}{90} G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu} - \frac{1}{12}(P_\mu U)^2 \\
&\qquad - \frac{1}{6}U^3 - \frac{1}{12} U G'_{\mu\nu}G'_{\mu\nu}\Bigg] \\
&+ \frac{1}{m^4}\Bigg[\frac{1}{24}U^4 + \frac{1}{12}U(P_\mu U)^2 + \frac{1}{120}(P^2 U)^2 + \frac{1}{24}\left(U^2 G'_{\mu\nu}G'_{\mu\nu}\right) \\
&\qquad - \frac{1}{120}[(P_\mu U),(P_\nu U)]G'_{\mu\nu} - \frac{1}{120}[U[U,G'_{\mu\nu}]]G'_{\mu\nu}\Bigg] \\
&+ \frac{1}{m^6}\Bigg[-\frac{1}{60}U^5 - \frac{1}{20}U^2(P_\mu U)^2 - \frac{1}{30}(UP_\mu U)^2\Bigg] \\
&+ \frac{1}{m^8}\Bigg[\frac{1}{120}U^6\Bigg]\Bigg\}.
\end{aligned} \tag{9}$$

where $\mathcal{L}^{(EFT)}_{1loop,dim6}$ represents all the terms in $\mathcal{L}^{(EFT)}_{1loop}$ which contain mass dimension six operators. $\mathcal{L}^{(EFT)}_{1loop}$ may contain mass dimension four operators which add to the couplings of the renormalizable operators in the $\mathcal{L}^{(EFT)}$. Here, $P_\mu = i\,\mathcal{D}_\mu$ *and* $G_{\mu,\nu} = [\mathcal{D}_\mu, \mathcal{D}_\nu]$. The above result is applicable when the mass squared matrix m^2 commutes with U.

2.2.1. `lOutput` :- $\mathcal{L}^{(EFT)}_{1loop}$ construction

Now, we construct the $\mathcal{L}^{(EFT)}_{1loop}$ within the package and determine the generated one loop Wilson coefficients of dimension six operators for a given BSM model. The information we put in the "UserDefined.m" for running the `trOut` function are all necessary for present case also. In addition to that, we also need to define the matrix structure of the generators of the isospin symmetry and color symmetry in cases where the heavy field are non singlet under these symmetries.[4]. So, given the symmetry generators, LBSM and the heavy field representation, we are able to construct $G'_{\mu\nu}$, U and $P_\mu = (i\mathcal{D}_\mu)$ which are needed to calculate $\mathcal{L}^{(EFT)}_{1loop,dim6}$ in equation 9. In CoDEx, we present two dimension six operator bases namely Warsaw and SILH. The operators definitions are tabulated in appendix []. For generating one loop WC's for some given BSM model with Lagrangian LBSM in SILH basis, we execute the following command in Mathematica front end,

```
lOutput[LBSM,SILH].
```

[4] Details on defining these matrices in CoDEx package are available in Mathematica documentation provided in CoDEx

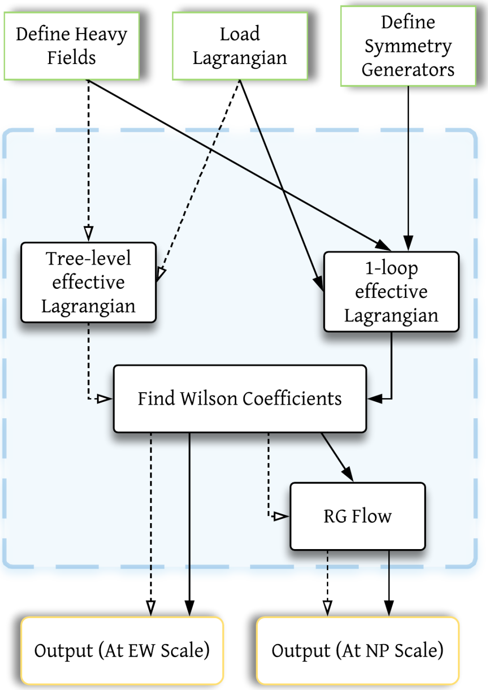

Figure 1: Flow-chart for CoDEx

2.3. *RGEs of WCs – Anomalous dimension matrix and choice of basis*

3. The Code

3.1. *Input details*

3.2. *Package details*

3.3. *Output details*

3.4. *RGEs of WCs*

4. Validation Examples

4.1. *Example Models with one BSM field.*

In this section, we consider models with one BSM field.

4.1.1. *Electro-weak $SU(2)_L$ Real Singlet Scalar*

$$\mathcal{L}_{BSM} = \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\,\phi^2 - A|H|^2\phi - \frac{1}{2}\kappa|H|^2\phi^2 - \frac{1}{3!}\,\mu\phi^3 - \frac{1}{4!}\,\lambda_\phi\phi^4 \tag{10}$$

4.1.2. *Electro-weak $SU(2)_L$ Triplet Scalar with hypercharge $Y = 0$*

$$\mathcal{L}_{BSM} = \mathcal{L}_{SM} + \frac{1}{2}(\mathcal{D}_\mu\Phi)^2 - \frac{1}{2}m_\Phi^2\,\Phi^a\,\Phi^a + 2\,\kappa\,H^\dagger\tau^a H\,\Phi^a - \eta\,|H|^2\,\Phi^a\,\Phi^a - \frac{1}{4}\,\lambda_\Phi\,(\Phi^a\Phi^a)^2 \tag{11}$$

[5]Electromagnetic charge, $Q = T_3 + Y$, where T_3 is isospin quantum number

Table 1: SM gauge quantum numbers of BSM fields

BSM Field	No. of field comp.s	$SU(3)_C$ quantum number	$SU(2)_L$ quantum number	$U(1)_Y$ charge[5]	spin	mass
ϕ	1	1	1	0	0	m_ϕ
Φ	3	1	3	0	0	m_Φ
Δ	3	1	3	1	0	m_Δ
Θ	4	1	4	3/2	0	m_Θ
N	1	1	1	0	1/2	m_N
Σ	3	1	3	0	1/2	m_Σ
Q	3	1	3	0	1	m_Q
φ	2	1	2	- 1/2	0	m_φ
K	1	1	1	0	1	m_K
$\tilde{Q}_{3,L}$	6	3	2	1/6	0	$m_{\tilde{Q}_3}$
$\tilde{t}_R$	3	3	1	2/3	0	$m_{\tilde{t}_R}$

Table 2: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 10

Dimension Six Ops.	Wilson Coefficients
O_H	$\frac{A^2}{m_\phi^4} + \frac{\lambda_\phi A^2}{16\pi^2 m_\phi^4} + \frac{\kappa^2}{192\pi^2 m_\phi^2}$
O_6	$-\frac{\kappa A^2}{2m_\phi^4} + \frac{\mu A^3}{6m_\phi^6} - \frac{\kappa\lambda_\phi A^2}{32\pi^2 m_\phi^4} - \frac{\kappa^3}{192\pi^2 m_\phi^2}$

Table 3: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 11

Dimension Six Ops.	Wilson Coefficients
O_H	$\frac{\eta^2}{16\pi^2 m_\Phi^2}$
O_T	$\frac{\kappa^2}{m_\Phi^4} + \frac{5\kappa^2\lambda_\Phi}{8\pi^2 m_\Phi^4}$
O_R	$\frac{2\kappa^2}{m_\Phi^4} + \frac{5\kappa^2\lambda_\Phi}{4\pi^2 m_\Phi^4}$
O_6	$-\frac{\eta\kappa^2}{m_\Phi^4} - \frac{5\eta\kappa^2\lambda_\Phi}{8m_\Phi^4\pi^2} - \frac{\eta^3}{8\pi^2 m_\Phi^2}$
O_{WW}	$\frac{\eta}{96\pi^2 m_\Phi^2}$
O_{2W}	$\frac{g_W^2}{480\pi^2 m_\Phi^2}$
O_{3W}	$\frac{g_W^2}{480\pi^2 m_\Phi^2}$

4.1.3. *Electro-weak $SU(2)_L$ Triplet Scalar with hypercharge $Y = 1$*

$$\begin{aligned}\mathcal{L}_{BSM} = \mathcal{L}_{SM} &+ Tr[(\mathcal{D}_\mu\Delta)^\dagger(\mathcal{D}^\mu\Delta)] - m_\Delta^2 Tr[\Delta^\dagger\Delta] \\ &+ [\mu(H^T i\sigma^2\Delta^\dagger H) + h.c.] + \lambda_1(H^\dagger H)Tr[\Delta^\dagger\Delta] \\ &+ \lambda_2(Tr[\Delta^\dagger\Delta])^2 + \lambda_3 Tr[(\Delta^\dagger\Delta)^2] + \lambda_4 H^\dagger\Delta\Delta^\dagger H \\ &- [y_\nu(L^C)^T(i\sigma^2\Delta)L + h.c.]\end{aligned} \tag{12}$$

Table 4: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 12

Dimension Five Op.	Wilson Coefficient
$ll\phi\phi$	$\frac{2\sqrt{2}\mu}{m_\Delta^2}$
Dimension Six Ops.	Wilson Coefficients
O_H	$\frac{\mu^2}{m_\Delta^4} + \frac{\lambda_4^2}{8\pi^2 m_\Delta^2}$
O_T	$\frac{\kappa^2}{m_\Phi^4} + \frac{5\kappa^2\lambda_\Phi}{8\pi^2 m_\Phi^4}$
O_R	$\frac{2\mu^2}{\mu_\Delta^4} + \frac{\lambda_5^2}{6m_\Delta^2\pi^2}$
O_6	$\frac{2(\lambda_4+\lambda_5)\mu^2}{m_\Delta^4} + \frac{\lambda_4^3}{4\pi^2 m_\Delta^2} + \frac{\lambda_4\lambda_5^2}{2\pi^2 m_\Delta^2}$
O_{WW}	$-\frac{\lambda_4}{48\pi^2 m_\Delta^2}$
O_{2W}	$\frac{g_W^2}{240\pi^2 m_\Delta^2}$
O_{3W}	$\frac{g_W^2}{240\pi^2 m_\Delta^2}$
O_{WB}	$\frac{\lambda_5}{24\pi^2 m_\Delta^2}$
O_{BB}	$-\frac{\lambda_4}{32\pi^2 m_\Delta^2}$
O_{2B}	$\frac{g_Y^2}{160\pi^2 m_\Delta^2}$

4.1.4. Electro-weak $SU(2)_L$ Quartet Scalar with $Y = 3/2$

$$\mathcal{L}_{BSM} = \mathcal{L}_{SM} + (\mathcal{D}_\mu\Theta)^\dagger(\mathcal{D}^\mu\Theta) - m_\Theta^2\,|\Theta|^2 - (\Theta^\dagger B + h.c), \tag{13}$$

where, 13

$$B = \begin{bmatrix} H_1^3 \\ \sqrt{3}H_1^2H_2 \\ \sqrt{3}H_1H_2^2 \\ H_2^3 \end{bmatrix}$$

,

and,
the SM Higgs H = $(H_1, H_2)^T$.

Table 5: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 13

Dimension Six Ops.	Wilson Coefficients
O_6	$\frac{\kappa^2}{m_\Theta^2}$
O_{2W}	$\frac{g_W^2}{96\pi^2 m_\Theta^2}$
O_{3W}	$\frac{g_W^2}{96\pi^2 m_\Theta^2}$
O_{2B}	$\frac{3g_Y^2}{160\pi^2 m_\Delta^2}$

4.1.5. $SU(2)_L$ singlet Heavy Right-handed neutrino

$$\mathcal{L}_{BSM} = \mathcal{L}_{SM} + \bar{\mathrm{N}}(i\partial\!\!\!/ - m_N)\mathrm{N} + [y_N\bar{L}\tilde{H}N + h.c.] \tag{14}$$

Table 6: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 14

Dimension Five Op.	Wilson Coefficient
$ll\phi\phi$	$\frac{y_N^2}{m_N}$

4.1.6. $SU(2)_L$ Real triplet Heavy fermion with $Y = 0$

$$\mathcal{L}_{BSM} = \mathcal{L}_{SM} + \bar{\Sigma}(i\mathcal{D}\!\!\!\!/ - m_\Sigma)\bar{\Sigma} + [y_\Sigma\tilde{H}^\dagger\Sigma L + h.c.] \tag{15}$$

Table 7: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 15

Dimension Five Op.	Wilson Coefficient
$ll\phi\phi$	$\frac{y_\Sigma^2}{m_\Sigma}$

4.1.7. Heavy $SU(2)$ gauge boson

$$\begin{aligned}\mathcal{L}_{BSM} = \mathcal{L}_{SM} &+ \frac{1}{2}\mathrm{Q}_\mu^a\left\{\mathcal{D}^2g^{\mu\nu} + m_\mathrm{Q}^2g^{\mu\nu} + 2[\mathcal{D}^\mu,\mathcal{D}^\nu]\right\}\mathrm{Q}_\nu^b \\ &+ \mathrm{Q}_\mu^a\left(\frac{g_1^4}{4(g_1^4+g_2^4)}|H|^2g^{\mu\nu}\right)\mathrm{Q}_\nu^b + \frac{g_1^2}{\sqrt{g_1^2+g_2^2}}\mathrm{Q}_\mu^a\mathcal{D}_\nu W^{a,\nu\mu}\end{aligned} \tag{16}$$

Table 8: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 16

Dimension Six Ops.	Wilson Coefficients
O_H	$\frac{g_1^8}{64(g_1^2+g_2^2)^2\pi^2 m_Q^2}$
O_6	$-\frac{g_1^{12}}{348(g_1^2+g_2^2)^3\pi^2 m_Q^2}$
O_{WW}	$\frac{g_1^4}{48(g_1^2+g_2^2)\pi^2 m_Q^2}$
O_{2W}	$\frac{g_1^4}{(g_1^2+g_2^2)m_Q^2} - \frac{37g_W^2}{480\pi^2 m_Q^2}$
O_{3W}	$\frac{g_W^2}{160\pi^2 m_Q^2}$

4.1.8. 2HDM

$$\begin{aligned}\mathcal{L}_{BSM} = \mathcal{L}_{SM} &+ |\mathcal{D}_\mu\,\varphi|^2 - m_\varphi^2\,|\varphi|^2 - \frac{\lambda_\varphi}{4}|\varphi|^4 \\ &+ (\eta_H|\tilde{H}|^2 + \eta_\varphi|\varphi|^2)(\tilde{H}^\dagger\varphi + \varphi^\dagger\tilde{H}) - \lambda_1|\tilde{H}|^2|\varphi|^2 - \lambda_2|\tilde{H}^\dagger\varphi|^2 \\ &- \lambda_3\left[(\tilde{H}^\dagger\varphi)^2 + (\varphi^\dagger\tilde{H})^2\right]\end{aligned} \tag{17}$$

4.2. Example Models with multiple BSM fields

In this section, we consider models with multiple BSM fields.
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Table 9: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 17

Dimension Six Ops.	Wilson Coefficients
O_H	$-\frac{3\eta_H\eta_\varphi}{8\pi^2 m_\varphi^2} + \frac{\lambda_1\lambda_2}{48\pi^2 m_\varphi^2} + \frac{\lambda_1^2}{48\pi^2 m_\varphi^2} + \frac{\lambda_2^2}{192\pi^2 m_\varphi^2} + \frac{\lambda_3^2}{48\pi^2 m_\varphi^2}$
O_T	$\frac{\lambda_2^2}{192\pi^2 m_\varphi^2} - \frac{\lambda_3^2}{48\pi^2 m_\varphi^2}$
O_R	$-\frac{3\eta_H\eta_\varphi}{8\pi^2 m_\varphi^2} + \frac{\lambda_2^2}{96\pi^2 m_\varphi^2} + \frac{\lambda_3^2}{24\pi^2 m_\varphi^2}$
O_6	$\frac{3\eta_H\eta_\varphi\lambda_1}{8\pi^2 m_\varphi^2} - \frac{\lambda 1^3}{48\pi^2 m_\varphi^2} + \frac{3\eta_H\eta_\varphi\lambda_2}{8\pi^2 m_\varphi^2} - \frac{\lambda_1^2\lambda_2}{32\pi^2 m_\varphi^2}$ $-\frac{\lambda_1\lambda_2^2}{32\pi^2 m_\varphi^2} - \frac{\lambda_2^3}{96\pi^2 m_\varphi^2} - \frac{\lambda_1\lambda_3^2}{8\pi^2 m_\varphi^2} - \frac{\lambda_2\lambda_3^2}{8\pi^2 m_\varphi^2} + \frac{3\eta_H^2\lambda_\varphi}{32\pi^2 m_\varphi^2}$
O_{WW}	$\frac{2\lambda_1+\lambda_2}{768\pi^2 m_\varphi^2}$
O_{2W}	$\frac{g_W^2}{960\pi^2 m_\varphi^2}$
O_{3W}	$\frac{g_W^2}{960\pi^2 m_\varphi^2}$
O_{WB}	$\frac{\lambda_2}{384\pi^2 m_\varphi^2}$
O_{BB}	$\frac{2\lambda_1+\lambda_2}{768\pi^2 m_\varphi^2}$
O_{2B}	$\frac{g_Y^2}{960\pi^2 m_\varphi^2}$

4.2.1. *Electro-weak $SU(2)_L$ Real Singlet Scalar and Electro-weak $SU(2)_L$ Triplet Scalar with hypercharge $Y = 0$*

$$
\begin{aligned}
\mathcal{L}_{BSM} = \mathcal{L}_{SM} \; &+ \; \frac{1}{2}\,(\partial_\mu\phi)^2 \; - \; \frac{1}{2}\, m_\phi^2\,\phi^2 - A|H|^2\phi \\
&- \frac{1}{2}\kappa_\phi|H|^2\phi^2 - \frac{1}{3!}\,\mu\phi^3 - \frac{1}{4!}\,\lambda_\phi\phi^4 \\
&+ \; \frac{1}{2}\,(\mathcal{D}_\mu\Phi)^2 \; - \; \frac{1}{2}\, m_\Phi^2\,\Phi^a\,\Phi^a + 2\,\kappa_\Phi\, H^\dagger\tau^a H\,\Phi^a \\
&- \; \eta\,|H|^2\,\Phi^a\,\Phi^a - \frac{1}{4}\,\lambda_\Phi\,(\Phi^a\Phi^a)^2 + 2\beta\,(H^\dagger\tau^a H)\,\Phi^a\,\phi
\end{aligned} \tag{18}
$$

Table 10: Non-zero Wilson Coefficients (upto one loop) for "SILH" basis dimension six operators for $\mathcal{L}_{BSM}$ in equation 18

Dimension Six Ops.	Wilson Coefficients
O_H	$\frac{A^2}{m_\phi^4} + \frac{\lambda_\phi A^2}{16\pi^2 m_\phi^4} + \frac{\kappa_\phi^2}{192\pi^2 m_\phi^2}$ $+\frac{\eta^2}{16\pi^2 m_\Phi^2} - \frac{\kappa_\phi\mu A}{96\pi^2 m_\Phi^2} + \frac{\mu^2 A^2}{192\pi^2 m_\Phi^6}$
O_T	$\frac{\kappa_\Phi^2}{m_\Phi^4} + \frac{\beta^2}{96\pi^2 m_\Phi^2} + \frac{5\kappa^2\lambda_\Phi}{8\pi^2 m_\Phi^4}$
O_R	$\frac{2\kappa_\Phi^2}{m_\Phi^4} + \frac{\beta^2}{48\pi^2 m_\Phi^2} + \frac{5\kappa^2\lambda_\Phi}{4\pi^2 m_\Phi^4}$
O_6	$-\frac{\kappa_\phi A^2}{2m_\phi^4} - \frac{\beta\kappa_\Phi A}{m_\Phi^2 m_\phi^2} - \frac{\eta\kappa^2}{m_\Phi^4} + \frac{\mu A^3}{6m_\phi^6} - \frac{\kappa\lambda_\phi A^2}{32\pi^2 m_\phi^4}$ $-\frac{\kappa^3}{192\pi^2 m_\phi^2} - \frac{5\eta\kappa^2\lambda_\Phi}{8m_\Phi^4\pi^2} - \frac{\eta^3}{8\pi^2 m_\Phi^2} - \frac{\kappa_\phi\lambda_\phi A^2}{32\pi^2 m_\phi^4}$ $-\frac{\kappa_\Phi^3}{192\pi^2 m_\Phi^2} - \frac{\eta^3}{8\pi^2 m_\Phi^2} - \frac{5\eta\kappa_\Phi^2\lambda_\Phi}{8\pi^2 m_\Phi^4}$
O_{WW}	$\frac{\eta}{96\pi^2 m_\Phi^2}$
O_{2W}	$\frac{g_W^2}{480\pi^2 m_\Phi^2}$
O_{3W}	$\frac{g_W^2}{480\pi^2 m_\Phi^2}$

4.2.2. *Supersymmetry : scalar tops (stops) and scalar quark doublets*

5. Summary

6. Acknowledgements

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References

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