Principles of aerodynamics applied to common aircraft

Filippo Falezza
BSc Physics
School of Physics and Astronomy, University of Birmingham

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1 Introduction

The aim of this essay is to discuss the general principles of flight and how these have been applied to the two most used aircraft: aeroplanes and helicopters. Their main characteristics will be explained, focusing on how they have been made and how they can be controlled through the primary control system. Lastly, an overall comparison between the two will be made, including differences and similar aspects on some of their main features.

2 Principles

Any aircraft is essentially made by a main body attached to a system of wings. The former can vary from a simple metal bar to a more complex structure, like a cabin or a fuselage like aeroplanes. The latter is either composed of two wings, fixed on the sides of the fuselage, or of a rotary system of small wings, called blades.

To understand how an aircraft works, it is necessary to understand how lift is generated by a wing, despite its relative motion within the fluid. Therefore, wings should be considered as aerofoils, focusing on their two-dimensional asymmetrical profile, whereas their infinite length on the third dimension is neglected, as in Figures 1a and 1b.

The first fundamental principle of airflow is given by the Bernoulli Theorem as follows[6]:

$$p + \rho g y + \frac{1}{2}\rho v^2 = k, \tag{1}$$

stating that the sum of the pressure, of the potential and of the kinetic energy is constant for a compressible fluid at a given height and velocity. Since air is a compressible fluid, this law applies to aerodynamics and flight in general.

Moreover, Euler equations for a compressible fluid should be considered[6][8]:

$$\frac{\partial p}{\partial x} + \rho g \frac{\partial h}{\partial x} = -\rho \frac{dv_x}{dt} \tag{2}$$

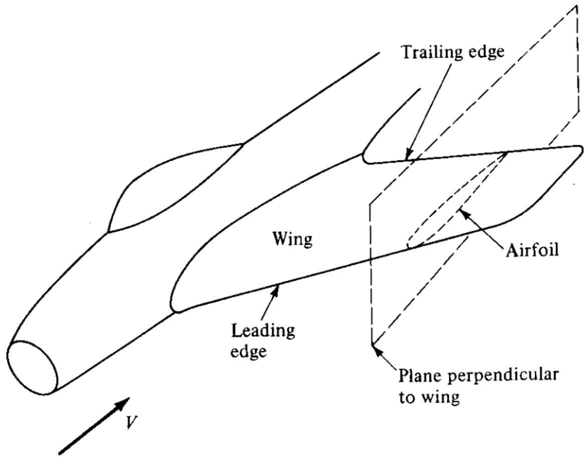
$$\frac{\partial p}{\partial y} + \rho g \frac{\partial h}{\partial y} = -\rho \frac{dv_y}{dt} \tag{3}$$

$$\frac{\partial p}{\partial z} + \rho g \frac{\partial h}{\partial z} = -\rho \frac{dv_z}{dt} \tag{4}$$

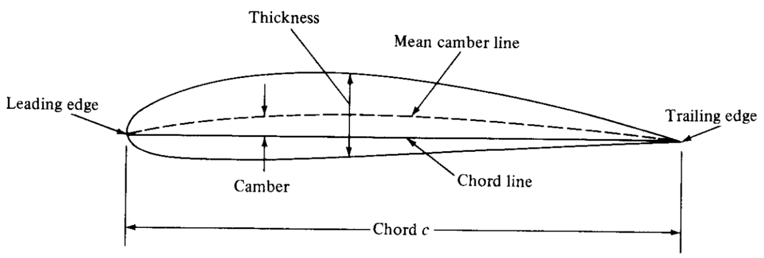
$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0. \tag{5}$$

Indeed, these equations explain how perfect fluids, which have no viscosity, behave in a limited space. Consequently, equations 2, 3 and 4 express the momentum conservation on the three axes, and equation 5 the continuity of fluids, so that their flow in a tube or in a defined volume does not vary with time when their motion is constant.

The direct consequence of Bernoulli and Euler equations is the possibility to analyse the behaviour of a wing downwind. As demonstrated by Prandtl (as cited in [6], pp.184-187), it is possible to consider a series of parallel aerofoils congruent one another with a constant vertical spacing a, as in Figure 2. Around each of these a rectangle $ABCD$ can be isolated so that both the incoming and outgoing wind velocity V_1 and V_2 are uniform along the two vertical segments



(a) Aerofoil from a wing([5]p.179)



(b) Aerofoil parts nomenclature([5]p.180)

Figure 1: Aerofoil description

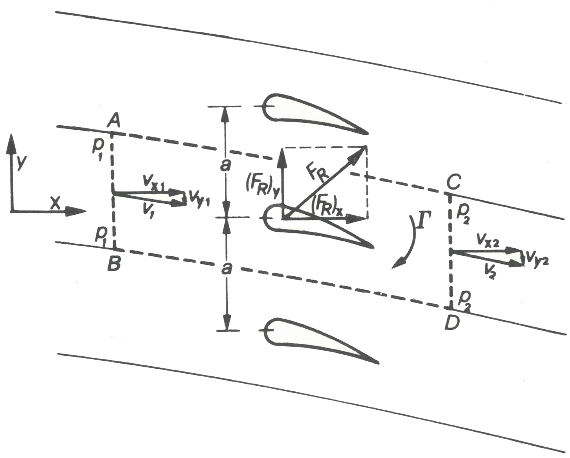


Figure 2: Array of asymmetrical fixed aerofoils([6]p.184)

AB and CD. Due to the continuity and Euler laws, the two velocities have the same horizontal component ($V_{x1} = V_{x2}$), and due to the Bernoulli principle, when the wind flows against the wing, a depression zone is created on the top of the wing, whereas the high pressure below push it upwards. The consequent pressure is

$$p_2 - p_1 = \frac{\rho}{2}(V_y 1^2 - V_y 2^2) \tag{6}$$

and therefore a certain circulation Γ is originated, commonly recognised as

$$\oint_{ABCD} \vec{V} ds. \tag{7}$$

In this situation, the resultant forces on the x and y axes are different one another, yet if we bring the aerofoil at infinite distance a one another the two velocities V_1 and V_2 are equal one another, so that

$$(F_R)_x = \rho\Gamma V_y \tag{8}$$

$$(F_R)_y = \rho\Gamma V_x \tag{9}$$

and the resulting lift L is the vector sum of the two components $L_x = (F_R)_x$ and $L_y = (F_R)_y$. Due to Pythagoras's theorem

$$L = (F_R) = \rho\Gamma V, \tag{10}$$

which is the Kutta-Žukovskij theorem. In particular, it is important to notice how Lift is always perpendicular to the wind and the perpendicular component V_y of the latter impresses a rotational moment M, implicitly expressed by equation 7.

An alternative to Kutta-Žukovskij's theorem is to calculate the force resulting from pressure difference between the top and the bottom sides of the aerofoil, as[5]

$$L = \int_{LE}^{TE} P_l \; dx - \int_{LE}^{TE} P_u \; dx. \tag{11}$$

Another way expresses lift by dimensional analysis, so that equation 10 can be rewritten as[5]

$$L = \frac{1}{2}\rho V^2 S C_l, \tag{12}$$

where ρ is the air density, V the velocity of the airflow, S the total area of the aerofoil and C_l the lift coefficient. This coefficient defines some characteristics of the wing, in particular its maximum angle of attack α, which is visible in Figure 3 and is the angle of incidence between the aerofoil and wind. As visible in Figures 4a and 4b, lift varies linearly till the maximum value of C_l, corresponding to the stalling angle; after this value, lift decreases quickly because of flow separation, visible in Figure 4b. Its importance is the physical limit that an aircraft has to respect when flying, for example an aeroplane would lose lift flying almost vertically.

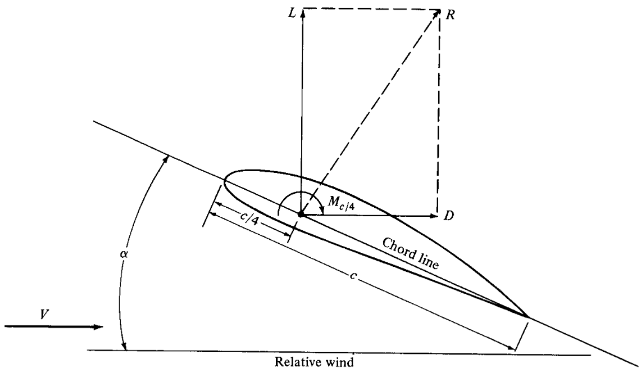
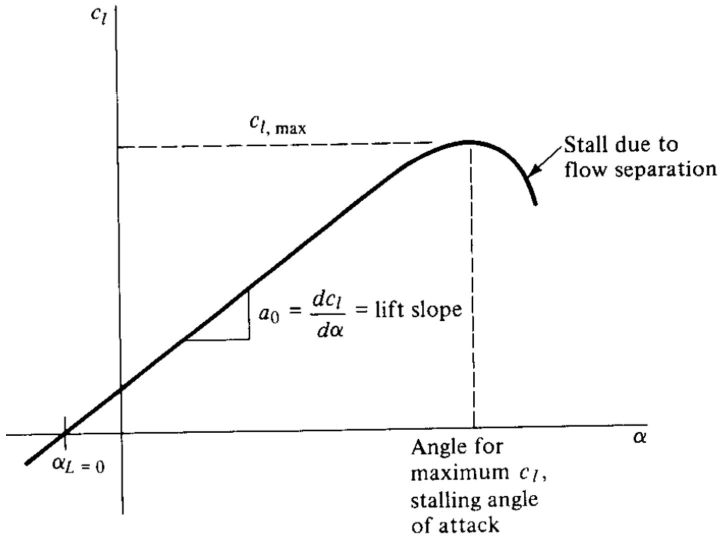
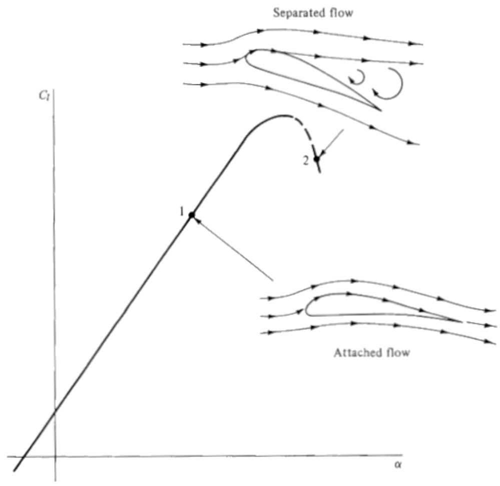


Figure 3: Aerofoil under incident wind V([5]p.180)



(a) Lift curve explained([5]p.187)

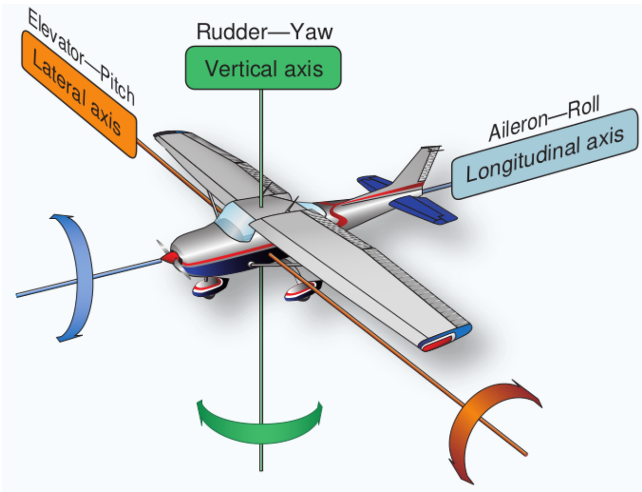


(b) Airflow difference after the stall point([5]p.188)

Figure 4: Typical lift curves

3 Aeroplanes

Aeroplanes are fixed-wing aircraft, formed of two wings, one on each side of the fuselage, which contains both the active parts of the aeroplane and the cargo. Hence, lift is generated by the wings moving in the fluid instead of the wind against them, so that the resulting relative motion acts like explained in section 2. The other fundamental parts of an aeroplane are the landing gear, the horizontal and vertical stabilizers and of course the propulsion engine. Stabilizers are generally made by a fixed aerofoil with a movable part at its very end and in aeroplanes these are divided on two planes: the elevator and the ailerons on the horizontal and the rudder on the vertical. All these parts are visible in Figure 5.
The elevator allows the movement on the vertical axis giving a rotation on the lateral axis and therefore changing the angle of attack α. Differently, the rudder spreads the rotation on the vertical axis to move the aircraft leftward or rightward. Ailerons act as a detachable extension of each wing, and hence are used in three different ways: to impress a rotation on the longitudinal axis, each moving independently; to generate more lift, when both upward i.e. when landing; to increase drag, when both downward i.e. during take-off.



Primary control surface	Airplane movement	Axes of rotation
Aileron	Roll	Longitudinal
Elevator/stabilator	Pitch	Lateral
Rudder	Yaw	Vertical

Figure 5: Aeroplane primary control system([2]p.3-3)

Consequently, the primary flight control system is composed of the engine speed regulator, two pedals connected to the rudder and the cloche[2]. This one controls both the elevator and the ailerons, however the latter is not part of the primary flight control system. The only purpose of engines is to provide the thrust required. Nonetheless, these can vary from common turbojets to simple fans.

4 Helicopters

While a fixed-wing aircraft flight is more straight-forward, a helicopter applies aerodynamic laws in a non-trivial way.
Helicopters use a rotor on the vertical axis to generate both lift and thrust, whereas a rear propulsion method maintains the helicopter on his axis, avoiding the torque induced by the rotation of the main rotor. This one generates lift similarly to aeroplanes: the principles applied are the same, yet the application is different. The Kutta-Žukovskij theorem sees the movement of the aerofoil rather than the airflow, so that lift is generated by the rotation of these only.
All the variations of altitude and horizontal speed are managed by the primary control system, more complex than the aeroplane's one[1][3]. Firstly, the throttle is used to vary the rotor angular velocity, then the collective regulates the pitch of the aerofoil and the consequent angle of attack, being able to move the helicopter upward and downward in relation to the resulting lift. Moreover, to generate thrust and move horizontally, the rotor can be tilted on the two horizontal axes (by a few degrees) using the cyclic, whereas to rotate the aircraft on the vertical axis, the two anti-torque pedals are used to control the rear propulsion method (often a rear rotor).
Obviously, the primary control system of a helicopter would limit some commands to avoid lift or thrust loss, like the pitch angle of the rotor or the maximum altitude reachable.

5 Some aircraft differences

After having highlighted the main characteristics of aeroplanes and helicopters, it is possible to trace some differences between them.
First of all, helicopters are capable of the so-called "hovering flight", that is stationary flight. Considering the first Newton law, this is possible only if

$$\sum F_i = 0;$$

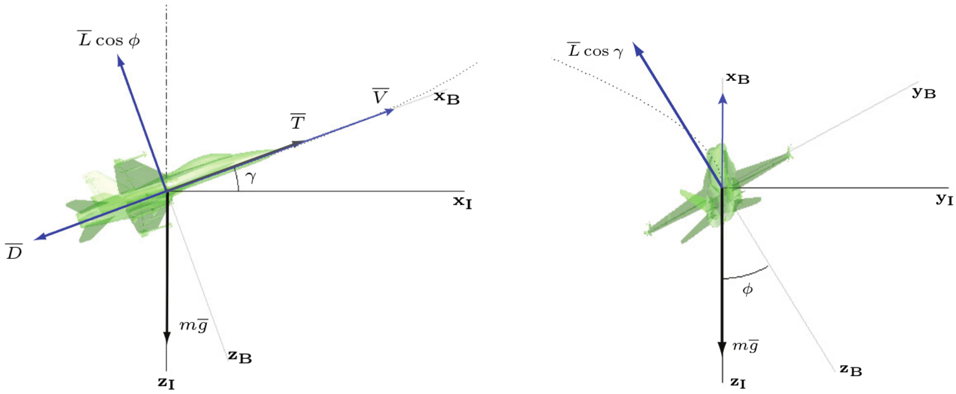
therefore, weight and lift should be balanced, as well as the forces acting on the two horizontal axes. Considering that weight varies in function of fuel consumed over time, a hoovering helicopter with no horizontal forces should satisfy $L - W(t) = 0$. On the other hand, the equivalent hoovering for an aeroplane can be found in the steady flight: again, considering Newton's laws, the aircraft should have all the forces balanced as elaborated by McClamroch[7]:

$$T \cos \alpha - D - W \sin \gamma = 0 \tag{13}$$

$$T \sin \alpha + L \cos \phi - W \cos \gamma = 0 \tag{14}$$

$$L \cos \gamma \sin \phi = \frac{W}{g} \frac{V^2}{R \cos^2 \gamma}. \tag{15}$$

As shown in Figure 6, T is the thrust of the aircraft, D the drag, W the weight, L the lift, α the angle of attack, γ the angle of the aircraft towards the longitudinal plane, ϕ the angle of the aircraft toward the lateral plane, V the wind velocity



(a) lateral view

(b) Rear view

Figure 6: Generic diagram of an aircraft in steady flight([7]p.128)

and R the radial component of lift onto the horizontal plane, which equals the mass of the aircraft times the radial component of its acceleration.
Steady flight often occurs on the horizontal plane only, so that some angles can be simplified. Moreover, it usually happens in fast wind conditions, which counterbalance the aircraft flight.
Since these conditions must be satisfied, the realisation of a full steady aeroplane flight is more difficult than a helicopter, where lift can be easily managed.
Safety is another main difference to be mentioned. An example is provided by the failure of the engine, which could be due to mechanical issues or fuel shortage. The outcome is that an aeroplane can glide whereas a helicopter would fall down instantly. Therefore, aeroplanes have the so called "glide ratio" or "lift to drag ratio"[4], a coefficient correspond to the horizontal space that can be travelled per meter of altitude, resulting in a high safety level.

6 Conclusions

The outcome of the analysis of the two aircraft is the essential understanding of the theory of flight. Nevertheless, the effective application sees air as a non-perfect fluid, so that other forces need to be considered as well as the Navier-Stokes equations, based on Euler's ones but much more complex. Drag force would need to be analysed separately as well, since it depends on many factors, such as the shape of the fuselage.

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