Structural Emergence of 4D Lorentzian Geometry from the Principle of Efficient Novelty

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Abstract

We derive quantum kinematics, four-dimensionality, Lorentzian signature, and Einstein dynamics from the Principle of Efficient Novelty (PEN) using the efficiency ratio $\rho = \nu/\kappa$. The argument is presented in a constructive mechanization-oriented form with explicit artifact support and consistent realization indexing across the companion PEN materials. The resulting claim is: (i) linear/quantum kinematics is selected because Cartesian cloning collapses higher homotopical novelty; (ii) $d = 4$ is selected by a native-associativity cost ceiling plus a 4D Hodge self-duality gain; (iii) Lorentzian signature is required to prevent univalent modal collapse of Flow and Cohesion; and (iv) Einstein–Hilbert dynamics is selected as the minimal abstract-syntax scalar curvature action.

1. Introduction

1.1 Architecture as a Selection Problem

Physics is formulated inside a very specific container: a 4D manifold, Lorentzian signature $(- + ++)$, and quantum state evolution. Standard presentations treat these as independent axioms. PEN instead treats them as outputs of one optimization criterion:

$$\rho = \frac{\nu \text{ (novel derivation schemas)}}{\kappa \text{ (specification and coherence cost)}}. \tag{1}$$

The key claim is not "anything is possible and we happened to land here," but "this architecture is selected because nearby alternatives have lower ρ."

1.2 Index Alignment with the Companion Paper

We follow realization numbering from *Deriving the Standard Model from PEN*: Frame/Metric is **R13**, Hilbert functional is **R14**, and Dynamical Cohesive Topos (DCT) is **R15**. All references below use that indexing consistently.

2. Quantum Kinematics from Efficient Novelty

2.1 No-Cloning Recast as a Type-Theoretic Constraint

A previous formulation conflated copying data with generating new derivation schemas. Here novelty remains strictly logical: ν counts genuinely new inference/program schemas. In that sense, unrestricted Cartesian copying is not "free novelty"; it is a degeneracy pressure.

In homotopy type theory, imposing strict global cloning coherence drives types toward h-sets (0-groupoids), suppressing higher path structure [5]. That collapse shrinks higher-geometric novelty channels and reduces long-run ν. The operational no-cloning pressure in quantum theory is classically captured by standard no-cloning results [3, 4], while resource-sensitive proof theory is rooted in linear logic [2].

2.2 Linearization and Stable Homotopy Gain

Moving from Cartesian product structure toward linear/smash-like composition increases available generalized cohomological constructions, i.e., more nontrivial derivation families at bounded coherence overhead. This is the PEN reason for quantum/linear kinematics: it preserves resource sensitivity while retaining high homotopical generativity.

Section conclusion. Quantum kinematics is selected because it prevents cloning-induced homotopy collapse and supports a larger stable family of derivation schemas per unit specification cost.

3. Dimensionality: Why Four

3.1 R13 Native Associativity Ceiling (Corrected Frame-Bundle Argument)

Pure differential geometry does *not* cap dimension by division algebras: $GL(n, \mathbb{R})$ exists for all n and is associative. The corrected PEN statement is about *native primitive cost*, not mathematical existence.

If the engine must synthesize full matrix infrastructure and coherence from scratch in high n, κ grows rapidly. PEN therefore prefers maximal reuse of already-realized associative rotational primitives (from the Hopf stage): $\mathbb{R}, \mathbb{C}, \mathbb{H}$ are strictly associative; $\mathbb{O}$ is not. Using octonionic-like multiplication as a native patching primitive breaks strict cocycle composition unless one adds explicit higher coherence towers (effectively A_∞ bookkeeping), which is costly.

Hence the native-associative ceiling is reached at quaternionic control, geometrically tied to

$$S^3 \hookrightarrow S^7 \to S^4, \tag{2}$$

with S^4 as the minimal high-yield base dimension supported by reusable associative primitives.

3.2 4D Hodge Endomorphism Bonus (Replacing Exotic-$\mathbb{R}^4$ Overclaim)

We remove the claim of a full formalized enumeration of exotic smooth structures. The tractable and sufficient 4D gain is algebraic:

$$\star : \Omega^2(M^d) \to \Omega^{d-2}(M^d). \tag{3}$$

Only at $d = 4$ does $\star$ close on 2-forms, giving a native endomorphism [6, 7]

$$\star : \Omega^2 \to \Omega^2, \tag{4}$$

and immediate splitting into self-dual/anti-self-dual sectors. This doubles gauge-curvature interaction channels without adding new primitive constructors, i.e., substantial ν gain at near-constant κ.

Section conclusion. Dimension four is selected by the intersection of a native-associativity cost ceiling and a unique Hodge self-duality novelty bonus.

4. Signature: Lorentzian as Modal Separation Geometry

4.1 R15 Modal Structure and Univalence

R15 (DCT) carries distinct modalities: Cohesion ($\flat$) for extended shape and Flow ($\bigcirc$) for temporal update. A physical metric must realize this distinction rather than quotienting it away.

By univalence, sufficiently coherent geometric paths induce type equivalences [5]. In Euclidean signature $(++++)$, $SO(4)$ isotropy enables continuous rotation between any unit directions. If one direction is interpreted as Flow and another as Cohesion, isotropy supplies a path that univalence promotes toward equivalence, yielding modal collapse $\bigcirc \simeq \flat$.

4.2 Light Cone as a Type-Theoretic Barrier

Lorentzian signature $(-+++)$ partitions tangent vectors into timelike/spacelike/null classes. Under $SO(1,3)$, timelike and spacelike regions are disconnected by the null cone; no continuous path crosses the barrier while preserving causal class [8, 9]. Therefore the specific univalent path needed to identify $\bigcirc$ with $\flat$ is blocked.

Section conclusion. Lorentzian anisotropy is selected because it is the minimal geometry that protects modal separation in R15.

5. Dynamics: Einstein–Hilbert as AST-Minimal Curvature Law

We replace an over-strong "fully formalized Lovelock uniqueness" claim with a computable minimality statement. Among local scalar curvature actions that yield second-order divergence-compatible equations in 4D, the Einstein–Hilbert term

$$S_{\mathrm{EH}} = \int (R - 2\Lambda)\sqrt{-g}\, d^4x \tag{5}$$

has minimal abstract syntax depth/size. Terms like R^2 or $R_{\mu\nu}R^{\mu\nu}$ require strictly larger constructor trees, hence larger κ unless forced by additional constraints; this is compatible with the classical Lovelock classification context in four dimensions [10].

6. Predictions

- **No accessible extra dimensions:** higher-dimensional proposals pay native coherence overhead without compensating novelty gain in this framework.
- **No Lorentz-violation regime:** breaking Lorentzian structure reopens modal-collapse channels.
- **Minimal gravity sector first:** leading UV-consistent corrections, if any, should appear as controlled high-κ deformations around the Einstein–Hilbert core rather than as an entirely different base geometry.

7. Mechanized Artifacts (Cubical Agda + Executable Checkers)

Artifact A: `Logic/ModalCollapse.agda`

This artifact is mechanized in Cubical Agda. It defines explicit Flow/Shape modal predicates over tangent data and proves Euclidean modal collapse via a univalence bridge while separately proving Lorentzian no-collapse under null-cone separation assumptions.

Artifact B: `Geometry/HopfCeiling.agda`

This artifact is mechanized in Cubical Agda. It encodes principal-bundle-style cocycle checking with a strictly associative operation interface and proves a 4-fold consistency lemma. A non-associative interface is mechanized separately and requires an explicit coherence payload to derive the analogous result.

Artifact C: `Haskell/HodgeEndomorphism.hs`

This executable Haskell artifact enumerates form-degree signatures by dimension and checks the Hodge map on 2-forms. The verifier reports that only $d = 4$ yields a native $\Omega^2 \to \Omega^2$ endomorphism and therefore supports self/anti-self-dual projectors without extra constructors.

Artifact D: `Haskell/MinimalAction.hs`

This executable Haskell artifact generates curvature-scalar candidates, computes AST node complexity, and filters by explicit constraints (local scalar invariant, presence of a curvature term, second-order equations, divergence compatibility). Under these constraints it certifies Einstein–Hilbert minimality.

Mechanization status. Artifacts A and B are mechanized and typechecked in Cubical Agda; Artifacts C and D are implemented as executable Haskell verifiers used to certify their stated computational claims. We do not claim formalization of the full exotic-$\mathbb{R}^4$ or full classical Lovelock theorem libraries at present.

8. Conclusion

With corrected geometry, consistent indexing, and realistic formalization scope, the PEN derivation becomes sharper: linear quantum kinematics maximizes resource-sensitive novelty, four dimensions maximize reusable structure plus Hodge gain, Lorentzian signature preserves modal logic, and Einstein–Hilbert dynamics minimizes syntactic cost. The architecture of spacetime is therefore treated as a constrained optimum, not an arbitrary backdrop.

References

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