Structural Emergence of 4D Lorentzian Geometry from the Principle of Efficient Novelty

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Abstract

We derive quantum kinematics, four-dimensionality, Lorentzian signature, and Einstein dynamics from the Principle of Efficient Novelty (PEN) using the spectral efficiency ratio $\rho = \nu/\kappa$ with $\nu = \nu_G + \nu_H + \nu_C$. The argument is presented in a constructive mechanization-oriented form with explicit artifact support and consistent Step 13/14/15 indexing across companion PEN materials. The resulting claim is: (i) linear/quantum kinematics is selected because Cartesian cloning collapses higher homotopical novelty ($\nu_H \to 0$); (ii) $d = 4$ is selected by a native-associativity cost ceiling plus a 4D Hodge endomorphism spike in ν_C; (iii) Lorentzian signature is required to prevent univalent modal collapse of Flow and Cohesion; and (iv) Einstein–Hilbert dynamics is selected as the MBTT-shortest curvature action.

1. Introduction

1.1 Scope: Kinematic Library vs Parametric Instantiation

The foundational text, *The Principle of Efficient Novelty: The Algorithmic Origin of the Kinematic Framework of Physics*, is explicit (Remark 2.1) that PEN does not directly derive fixed physical constants such as a unique spacetime dimension or equations of motion at the library-construction stage. This is not a contradiction with the present paper: the foundational work derives a *Kinematic Library* (generic reusable operators, metrics, cohesion, Hilbert functionals, and DCT), while this companion evaluates a *Parametric Instantiation* of that library under the same selection rule $\max \rho$.

In that second-stage instantiation problem, candidate architectures are scored by

$$\rho = \frac{\nu_G + \nu_H + \nu_C}{\kappa}, \tag{1}$$

where ν_G counts new Introduction schemas, ν_H new homotopy/computation schemas, ν_C new Elimination schemas, and κ MBTT specification complexity. Under this unchanged PEN mechanics, the physical optimum isolates $d = 4$, Lorentzian signature, and Einstein–Hilbert dynamics.

1.2 Index Alignment with the Companion Paper

We use the foundational indexing directly: **Step 13** (Metric/Frame), **Step 14** (Hilbert functional), and **Step 15** (Dynamical Cohesive Topos). This standardization is used throughout.

2. Quantum Kinematics from Efficient Novelty

2.1 No-Cloning as a Type-Theoretic Constraint

In homotopy type theory, imposing strict global cloning coherence drives types toward h-sets (0-groupoids), suppressing higher path structure [5]. In PEN spectral language this is exactly the collapse channel identified by Proposition 9.2 (*Inefficiency of Discrete Structures*): forcing discreteness sends topological novelty to

$$\nu_H = 0, \tag{2}$$

and the branch cannot compete once the efficiency threshold rises.

The operational no-cloning pressure in quantum theory is classically captured by standard no-cloning results [3, 4], while resource-sensitive proof theory is rooted in linear logic [2]. PEN's point is algorithmic: no-cloning preserves higher-path channels, so ν_H can remain superlinear in representational depth instead of collapsing to zero.

2.2 Linearization and the $d = 2$ Coherence Window

By Theorem 4.2 (the $d = 2$ Coherence Window in the foundational paper), native foundations cannot resolve 3-dimensional coherence obligations without explicit overhead. In discrete/h-set collapse regimes, integration debt follows the Fibonacci barrier $\Delta_n = F_n$, so purely discrete encodings cannot outpace this cost. Linear/quantum kinematics is therefore a survival condition: avoid cloning-induced modal flattening to preserve the homotopy amplification channel ($\nu_H \sim \mathcal{O}(d^2)$ in the non-collapsed regime) while paying only bounded incremental κ.

Section conclusion. Quantum kinematics is selected because it prevents cloning-induced $\nu_H = 0$ collapse and preserves high-yield homotopy amplification relative to coherence cost.

3. Dimensionality: Why Four

3.1 The Native Associativity Ceiling and the Quaternionic Base

Pure differential geometry does *not* cap dimension by division algebras: $GL(n, \mathbb{R})$ exists for all n and is associative. PEN's selection statement is about primitive synthesis cost: if high-n patching is built from scratch, κ rises sharply.

Theorem 4.2 (Coherence Window $d = 2$) makes this precise for non-associative patching. Octonionic patching requires explicit higher coherence towers (Stasheff A_∞ data) because associators cannot be discharged natively in a 2-dimensional coherence window. That explicit coherence payload is AST-heavy and induces a large κ penalty. By contrast, quaternionic S^3-controlled fibrations are associative and remain within native coherence handling.

Hence the native associativity ceiling is reached at quaternionic control, geometrically tied to

$$S^3 \hookrightarrow S^7 \to S^4, \tag{3}$$

with S^4 as the maximal high-yield base dimension before non-native coherence costs dominate.

3.2 Algebraic Optimality of the 4D Hodge Endomorphism

The tractable and sufficient 4D gain is algebraic:

$$\star : \Omega^2(M^d) \to \Omega^{d-2}(M^d). \tag{4}$$

Only at $d = 4$ does $\star$ close on 2-forms,

$$\star : \Omega^2 \to \Omega^2, \tag{5}$$

producing self-dual/anti-self-dual projectors [6, 7]. In PEN accounting this is a sharp ν_C spike: Step 13 already pays the metric/Hodge infrastructure cost, so the 4D closure unlocks many new elimination channels at marginal $\Delta\kappa \approx 0$ (instanton sector decompositions, chirality-sensitive gauge decompositions, and projector-based curvature factorizations).

Artifact C (`Haskell/HodgeEndomorphism.hs`) computes this exactly and reports that among tested dimensions only $d = 4$ yields native $\Omega^2 \to \Omega^2$ closure, confirming the concentrated ν_C gain at fixed constructor budget.

Section conclusion. Dimension four is selected at the intersection of a strict κ bound (associativity/coherence ceiling) and a unique 4D ν_C bonus from Hodge self-duality.

4. Signature: Lorentzian as Modal Separation Geometry

4.1 Step 15 Modal Structure, Tangent Time, and Univalence

Step 15 (DCT) carries distinct modalities: Cohesion ($\flat$) for extended shape and Flow ($\bigcirc$) for temporal update. The Kinematic-Dynamic Equivalence Lemma (Lemma 5.10 in the foundational text) identifies time-internalization as

$$\bigcirc X \simeq X^{\mathbb{D}}, \tag{6}$$

so Flow is geometrically bound to tangent data. A physical metric must preserve this modal distinction rather than quotienting it away.

By univalence, coherent geometric paths induce type equivalences [5]. In Euclidean signature $(++++)$, $SO(4)$ isotropy continuously rotates a timelike-designated tangent vector into a spacelike direction. Via Lemma 5.10, that smooth geometric rotation lifts to a universe path in $\mathcal{U}$ identifying the Flow generator with a spatial/cohesive direction, driving modal collapse $\bigcirc \simeq \flat$.

4.2 Light Cone as a Type-Theoretic Firewall

Lorentzian signature $(- + ++)$ partitions tangent vectors into timelike/spacelike/null classes. Under $SO(1,3)$, timelike and spacelike components are separated by the null cone; no causal-class-preserving path crosses this barrier [8, 9]. The univalent identification path needed for $\bigcirc \simeq \flat$ is therefore topologically absent, and Step 15 modal structure remains protected.

Artifact A (`Logic/ModalCollapse.agda`) implements both branches: Euclidean collapse is derivable under rotational isotropy, whereas Lorentzian no-collapse is verified under null-cone separation hypotheses.

Section conclusion. Lorentzian anisotropy is selected as the minimal geometric firewall that preserves Step 15 modal separation.

5. Dynamics: Einstein–Hilbert as MBTT-AST Minimum

Rather than relying on non-constructive classical Lovelock uniqueness theorems, PEN evaluates a computable AST minimality statement. Under the prefix-free Minimal Binary Type Theory (MBTT) grammar from the foundational paper, among local scalar curvature actions yielding second-order divergence-compatible 4D dynamics, the Einstein–Hilbert term

$$S_{\mathrm{EH}} = \int (R - 2\Lambda)\sqrt{-g}\, d^4x \tag{7}$$

has strictly shortest encoding and lowest constructor depth.

Higher-curvature terms such as R^2 or $R_{\mu\nu}R^{\mu\nu}$ add AST nesting and increase κ without introducing comparably new elimination families ν_C for the second-order sector, so ρ drops. Artifact D (`Haskell/MinimalAction.hs`) computes candidate AST node counts and verifies this strict minimality under the explicit filter set (local scalar invariant, curvature dependence, second-order equations, divergence compatibility), in compatibility with Lovelock context [10].

6. Predictions

- **No accessible extra dimensions:** higher-dimensional proposals cross the associativity/coherence κ ceiling without compensating spectral gain.
- **No Lorentz-violation regime:** breaking Lorentzian structure reopens univalent modal-collapse channels.
- **Minimal gravity sector first:** leading UV-consistent corrections, if any, should appear as controlled high-κ deformations around the Einstein–Hilbert core.

7. Mechanized Artifacts (Cubical Agda + Executable Checkers)

Artifact A: `Logic/ModalCollapse.agda`

Mechanized in Cubical Agda. Defines explicit Flow/Shape modal predicates over tangent data and proves Euclidean collapse via a univalence bridge while separately proving Lorentzian no-collapse under null-cone separation assumptions.

Artifact B: `Geometry/HopfCeiling.agda`

Mechanized in Cubical Agda. Encodes principal-bundle cocycle checking with a strictly associative operation interface and proves a 4-fold consistency lemma. A non-associative interface is mechanized separately and requires explicit coherence payload to derive analogous consistency.

Artifact C: `Haskell/HodgeEndomorphism.hs`

Executable Haskell checker. Enumerates form-degree signatures by dimension and verifies that only $d = 4$ yields native $\Omega^2 \to \Omega^2$ closure.

Artifact D: `Haskell/MinimalAction.hs`

Executable Haskell checker. Generates curvature-scalar candidates, computes AST node complexity, and filters by explicit constraints, certifying Einstein–Hilbert minimality in the admissible set.

Mechanization status. Artifacts A and B are mechanized and typechecked in Cubical Agda; Artifacts C and D are executable Haskell verifiers used for quantitative certification of the 4D Hodge closure and MBTT-AST minimality claims.

8. Conclusion

PEN's foundational paper derives the reusable kinematic language; this companion solves the physical instantiation optimization in that language. Under the same spectral metrics $(\nu_G, \nu_H, \nu_C, \kappa)$ and selection rule $\max \rho$, quantum kinematics survives by preventing ν_H collapse, $d = 4$ is pinned by associativity/coherence bounds plus a unique ν_C Hodge spike, Lorentzian signature protects Step 15 modal logic, and Einstein–Hilbert is MBTT-minimal among admissible second-order curvature laws. Spacetime architecture is thus a constrained efficiency optimum, not an arbitrary backdrop.

References

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