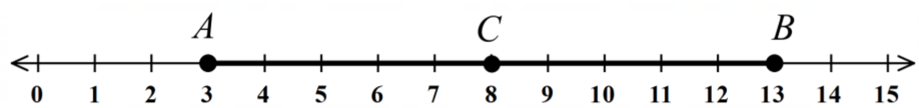
THE MIDPOINT FORMULA
N-GEN MATH® GEOMETRY

*The **midpoint** of a line segment is an important feature of any segment. It is the **unique point** on a line segment that divides (or partitions) the line segment in half. It is a simple feature to find in the **coordinate plane**. First, we start with a line segment that lies on a one-dimensional number line.*

***Exercise* #1:** $\overline{AB}$ is shown below with endpoints at 3 and 13.



(a) What is the length of $\overline{AB}$? How can you find it using a calculation rather than counting?

It is 10 units long. We can subtract $13-3$.

(b) Where is the midpoint of the segment? Plot it as point *C*. How do you know it is the midpoint?

The midpoint is at 8 because it is 5 units from 13 and 3.

(c) Find the average (or mean) of the positions of points *A* and *B*. Show your calculation below.

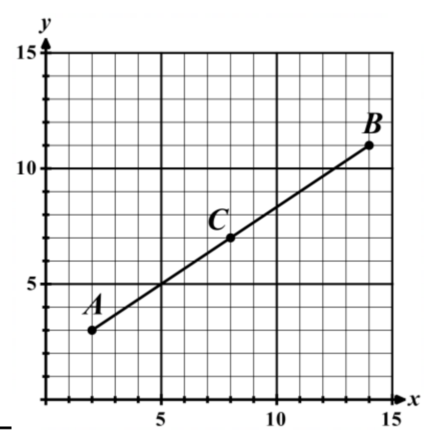
$$\frac{13+3}{2}=\frac{16}{2}=8$$

(d) What do you notice about the average of the two endpoints?

The average of the two endpoints is at the midpoint.

*The **average** or **mean** of two numbers will **always** fall **halfway** between the two numbers since the average always balances the number of units above and below it. This also works with line segments in the coordinate plane.*

***Exercise* #2:** On the grid below, $\overline{AB}$ has been plotted and has endpoints at *A*(2, 3) and *B*(14, 11).



(a) Find the average of the *x* and *y* coordinate of the endpoints. Show your calculations below.

$$x_{avg}=\frac{2+14}{2}=\frac{16}{2}=8 \qquad y_{avg}=\frac{3+11}{2}=\frac{14}{2}=7$$

(b) Plot point *C* using the average *x* and *y* coordinates from (a).

(c) Give an argument for why *C* must be the midpoint of $\overline{AB}$.

$$AC=\sqrt{(2-8)^2+(3-7)^2}$$
$$AC=\sqrt{(-6)^2+(-4)^2}$$
$$AC=\sqrt{36+16}$$
$$AC=\sqrt{52}=2\sqrt{13}$$

$$CB=\sqrt{(8-14)^2+(7-11)^2}$$
$$CB=\sqrt{(-6)^2+(-4)^2}$$
$$CB=\sqrt{36+16}$$
$$CB=\sqrt{52}=2\sqrt{13}$$

Since point *C* divides $\overline{AB}$ into two segments of the same length, it must be the midpoint.

The Midpoint Formula

If (x_1, y_1) and (x_2, y_2) are the two **endpoints** of a line segment, then the **midpoint**, *M*, lies at the **average coordinates** of the two endpoints:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

***Exercise* #3:** Find the coordinates of the midpoint of a line segment whose endpoints have the coordinates shown.

(a) (9, 3) and (13, 7)

$$\left(\frac{9+13}{2}, \frac{3+7}{2}\right)$$
$$\left(\frac{22}{2}, \frac{10}{2}\right) \Rightarrow (11, 5)$$

(b) (–4, 8) and (2, 12)

$$\left(\frac{-4+2}{2}, \frac{8+12}{2}\right)$$
$$\left(\frac{-2}{2}, \frac{20}{2}\right) \Rightarrow (-1, 10)$$

(c) (–7, –2) and (1, 14)

$$\left(\frac{-7+1}{2}, \frac{-2+14}{2}\right)$$
$$\left(\frac{-6}{2}, \frac{12}{2}\right) \Rightarrow (-3, 6)$$

(d) (8, –4) and (3, –10)

$$\left(\frac{8+3}{2}, \frac{-4+(-10)}{2}\right)$$
$$\left(\frac{11}{2}, \frac{-14}{2}\right) \Rightarrow (5.5, -7)$$

***Exercise* #4:** Does the line whose equation is $y = 5x - 32$ bisect $\overline{EF}$ whose endpoints lie at the points *E*(4, 3) and *F*(8, –7)? Justify your yes/no response.

$$\left(\frac{4+8}{2}, \frac{3+(-7)}{2}\right)$$

$(6,-2)$ midpoint of $\overline{EF}$

$\Rightarrow$

We need to see if the midpoint $(6,-2)$ lies on the line $y = 5x - 32$

$\Rightarrow$

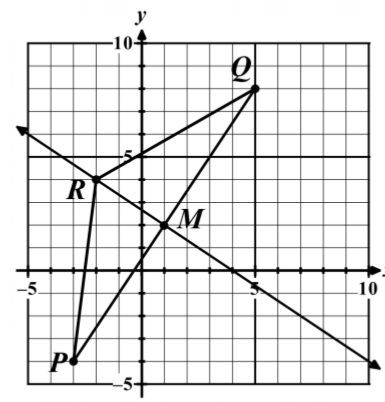
$$y = 5x - 32$$
$$-2 = 5 \cdot 6 - 32$$
$$-2 = 30 - 32$$
$$-2 = -2 \text{ True}$$

Yes, the line bisects $\overline{EF}$ since it passes through its midpoint.

***Exercise* #5:** On the grid below, $\overline{PQ}$ is plotted with endpoints at *P*(–3, –4) and *Q*(5, 8).

(a) Find the coordinates of the midpoint of $\overline{PQ}$. Plot it as point *M*.

$$M\left(\frac{-3+5}{2}, \frac{-4+8}{2}\right) \Rightarrow M\left(\frac{2}{2}, \frac{4}{2}\right) \Rightarrow M(1, 2)$$



(b) Find the slope of $\overline{PQ}$ in simplest form.

$$m = \frac{8-(-4)}{5-(-3)} = \frac{12}{8} = \frac{3}{2}$$

(c) Draw the perpendicular bisector of $\overline{PQ}$. State an equation for it below in point-slope form.

$m_\perp = -\frac{2}{3}$ $\Rightarrow$ **Using the midpoint $(1,2)$.** $\Rightarrow$ $y - 2 = -\frac{2}{3}(x-1)$

(d) Pick a point on the line you drew in (c) that is not point *M*. Label it as point *R*. Using the distance formula, find the distance from *R* to both endpoints of $\overline{PQ}$. What do you observe about the distances?

Coordinates of R: $(-2,4)$

Distance from *R* to *P*: $RP = \sqrt{(-2-(-3))^2 + (4-(-4))^2} = \sqrt{1+64} = \sqrt{65}$

Distance from *R* to *Q*: $RQ = \sqrt{(-2-5)^2 + (4-8)^2} = \sqrt{49+16} = \sqrt{65}$

Observation: **Point *R* is equidistant from the endpoints of the line segment.**

THE MIDPOINT FORMULA
N-GEN MATH® GEOMETRY HOMEWORK

FLUENCY

1. For the two sets of points below, calculate three quantities about the line segments that would connect them: the slope of the segment, the midpoint of the segment, and the length (distance) of the segment. Show all calculations and express your answers in simplest form.

(a) $A(-4, 10)$ and $B(8, 6)$

Slope: $$\boxed{m=\frac{6-10}{8-(-4)}=\frac{-4}{12}=-\frac{1}{3}}$$

Midpoint: $$\boxed{\left(\frac{-4+8}{2},\frac{10+6}{2}\right)\Rightarrow\left(\frac{4}{2},\frac{16}{2}\right)\Rightarrow(2,8)}$$

Length/distance:
$$D=\sqrt{(8-(-4))^2+(6-10)^2}$$
$$D=\sqrt{(12)^2+(-4)^2}$$
$$D=\sqrt{144+16}$$
$$D=\sqrt{160}=4\sqrt{10}$$

(b) $F(-1, 3)$ and $G(9, -3)$

Slope: $$\boxed{m=\frac{-3-3}{9-(-1)}=\frac{-6}{10}=-\frac{3}{5}}$$

Midpoint: $$\boxed{\left(\frac{-1+9}{2},\frac{3+-3}{2}\right)\Rightarrow\left(\frac{8}{2},\frac{0}{2}\right)\Rightarrow(4,0)}$$

Length/distance:
$$D=\sqrt{(9-(-1))^2+(-3-3)^2}$$
$$D=\sqrt{(10)^2+(-6)^2}$$
$$D=\sqrt{100+36}$$
$$D=\sqrt{136}=\sqrt{4}\sqrt{34}=2\sqrt{34}$$

2. If two points, R and T, have coordinates of $R(-5, 8)$ and $T(3, 14)$, then which of the following points lies at the midpoint of $\overline{RT}$?

(1) $(-2, 22)$
(2) $(-5, 14)$
(3) $(-1, 11)$
(4) $(2, 11)$

$$\boxed{\left(\frac{-5+3}{2},\frac{8+14}{2}\right)\Rightarrow\left(\frac{-2}{2},\frac{22}{2}\right)\Rightarrow(-1,11)}$$

(3)

3. Which of the following would be true about the perpendicular bisector of $\overline{EF}$ if $\overline{EF}$ has endpoints at $E(-9, -1)$ and $F(3, 15)$?

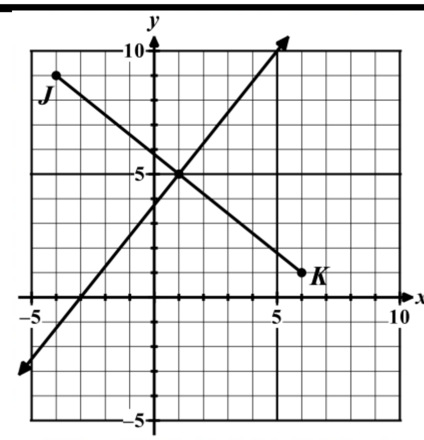
(1) It has a slope of $-\frac{3}{4}$ and passes through $(-6, 14)$.
(2) It has a slope of $\frac{3}{4}$ and passes through $(-3, 7)$.
(3) It has a slope of $-\frac{4}{3}$ and passes through $(-6, 14)$.
(4) It has slope of $-\frac{3}{4}$ and passes through $(-3, 7)$.

$$\boxed{\overline{EF}: m=\frac{-1-15}{-9-3}=\frac{-16}{-12}=\frac{4}{3}\Rightarrow m_\perp=-\frac{3}{4}}$$

$$\boxed{\left(\frac{-9+3}{2},\frac{-1+15}{2}\right)\Rightarrow\left(\frac{-6}{2},\frac{14}{2}\right)\Rightarrow(-3,7)}$$

(4)

4. Determine an equation for the perpendicular bisector of $\overline{JK}$, which has endpoints at $J(-4, 9)$ and $K(6, 1)$. The use of the grid is optional (but recommended).



$\overline{JK}$: $m=\frac{1-9}{6-(-4)}=\frac{-8}{10}=-\frac{4}{5}$

$m_{\perp}=\frac{5}{4}$

midpoint of $\overline{JK}$:

$\left(\frac{-4+6}{2},\frac{9+1}{2}\right)\Rightarrow\left(\frac{2}{2},\frac{10}{2}\right)\Rightarrow(1,5)$

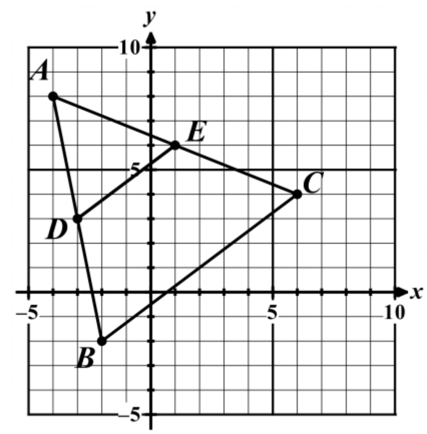
Equation of Perpendicular Bisector:

$y-5=\frac{5}{4}(x-1)$

REASONING

5. In the following diagram, $\triangle ABC$ is drawn with coordinates at $A(-4, 8)$, $B(-2, -2)$, and $C(6, 4)$.

(a) Find the midpoints of $\overline{AB}$ and $\overline{AC}$. Label them as points D and E respectively.



Midpoint of $\overline{AB}$:
(Point D)

$D\left(\frac{-4+-2}{2},\frac{8+-2}{2}\right)\Rightarrow D(-3,3)$

Midpoint of $\overline{AC}$:
(Point E)

$E\left(\frac{-4+6}{2},\frac{8+4}{2}\right)\Rightarrow E(1,6)$

(b) Draw $\overline{DE}$ on the graph and finds its slope and length. Show your calculations below.

Slope of $\overline{DE}$:

$m=\frac{6-3}{1-(-3)}=\frac{3}{4}$

Length of $\overline{DE}$:

$DE=\sqrt{(1-(-3))^2+(6-3)^2}$

$DE=\sqrt{(4)^2+(3)^2}=\sqrt{25}=5$

(c) Find the slope and length of $\overline{BC}$. Show your calculations below.

Slope of $\overline{BC}$:

$m=\frac{4-(-2)}{6-(-2)}=\frac{6}{8}=\frac{3}{4}$

Length of $\overline{BC}$:

$BC=\sqrt{(6-(-2))^2+(4-(-2))^2}$

$BC=\sqrt{(8)^2+(6)^2}=\sqrt{64+36}=\sqrt{100}=10$

(d) Give two observations you can make about how $\overline{DE}$ and $\overline{BC}$ relate to each other.

1. Segments $\overline{DE}$ and $\overline{BC}$ are parallel because their slopes are equal.

2. Segment $\overline{DE}$ is half the length of segment $\overline{BC}$ as shown by the distance formula.