



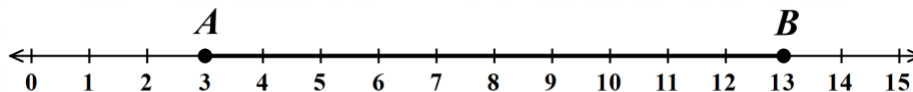
THE MIDPOINT FORMULA

N-GEN MATH® GEOMETRY



The **midpoint** of a line segment is an important feature of any segment. It is the **unique point** on a line segment that divides (or partitions) the line segment in half. It is a simple feature to find in the **coordinate plane**. First, we start with a line segment that lies on a one-dimensional number line.

Exercise #1: \overline{AB} is shown below with endpoints at 3 and 13.

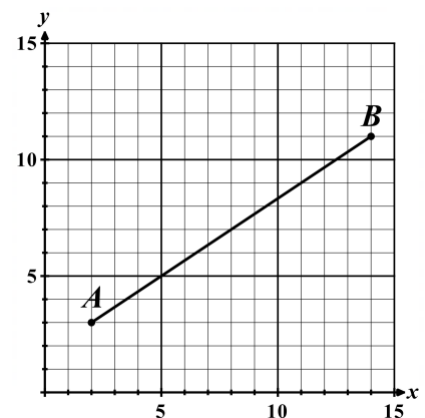


- (a) What is the length of \overline{AB} ? How can you find it using a calculation rather than counting?
- (b) Where is the midpoint of the segment? Plot it as point C. How do you know it is the midpoint?
- (c) Find the average (or mean) of the positions of points A and B. Show your calculation below.
- (d) What do you notice about the average of the two endpoints?

The **average** or **mean** of two numbers will **always** fall **halfway** between the two numbers since the average always balances the number of units above and below it. This also works with line segments in the coordinate plane.

Exercise #2: On the grid below, \overline{AB} has been plotted and has endpoints at A(2, 3) and B(14, 11).

- (a) Find the average of the x and y coordinate of the endpoints. Show your calculations below.



- (b) Plot point C using the average x and y coordinates from (a).
- (c) Give an argument for why C must be the midpoint of \overline{AB} .



The Midpoint Formula

If (x_1, y_1) and (x_2, y_2) are the two **endpoints** of a line segment, then the **midpoint**, M , lies at the **average coordinates** of the two endpoints:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Exercise #3: Find the coordinates of the midpoint of a line segment whose endpoints have the coordinates shown.

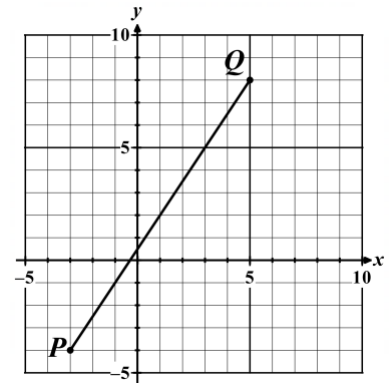
- (a) (9, 3) and (13, 7) (b) (-4, 8) and (2, 12) (c) (-7, -2) and (1, 14) (d) (8, -4) and (3, -10)

Exercise #4: Does the line whose equation is $y = 5x - 32$ bisect \overline{EF} whose endpoints lie at the points $E(4, 3)$ and $F(8, -7)$? Justify your yes/no response.

Exercise #5: On the grid below, \overline{PQ} is plotted with endpoints at $P(-3, -4)$ and $Q(5, 8)$.

- (a) Find the coordinates of the midpoint of \overline{PQ} . Plot it as point M .

- (b) Find the slope of \overline{PQ} in simplest form.



- (c) Draw the perpendicular bisector of \overline{PQ} . State an equation for it below in point-slope form.

- (d) Pick a point on the line you drew in (c) that is not point M . Label it as point R . Using the distance formula, find the distance from R to both endpoints of \overline{PQ} . What do you observe about the distances?

Coordinates of R :

Distance from R to P :

Distance from R to Q :

Observation: _____



Name: _____

Date: _____

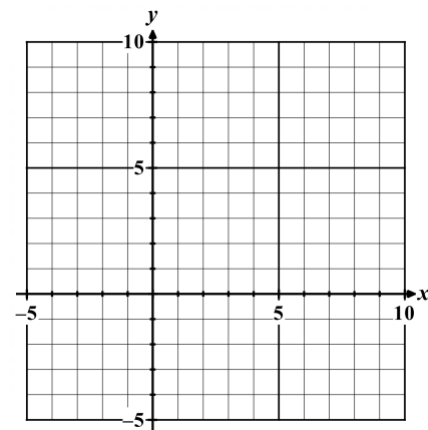
THE MIDPOINT FORMULA
N-GEN MATH[®] GEOMETRY HOMEWORK

FLUENCY

1. For the two sets of points below, calculate three quantities about the line segments that would connect them: the slope of the segment, the midpoint of the segment, and the length (distance) of the segment. Show all calculations and express your answers in simplest form.
- | | |
|-------------------------------|-------------------------------|
| (a) $A(-4, 10)$ and $B(8, 6)$ | (b) $F(-1, 3)$ and $G(9, -3)$ |
| Slope: | Slope: |
| Midpoint: | Midpoint: |
| Length/distance: | Length/distance: |
2. If two points, R and T , have coordinates of $R(-5, 8)$ and $T(3, 14)$, then which of the following points lies at the midpoint of \overline{RT} ?
- (1) $(-2, 22)$
 - (2) $(-5, 14)$
 - (3) $(-1, 11)$
 - (4) $(2, 11)$
3. Which of the following would be true about the perpendicular bisector of \overline{EF} if \overline{EF} has endpoints at $E(-9, -1)$ and $F(3, 15)$?
- (1) It has a slope of $-\frac{3}{4}$ and passes through $(-6, 14)$.
 - (2) It has a slope of $\frac{3}{4}$ and passes through $(-3, 7)$.
 - (3) It has a slope of $-\frac{4}{3}$ and passes through $(-6, 14)$.
 - (4) It has slope of $-\frac{3}{4}$ and passes through $(-3, 7)$.



4. Determine an equation for the perpendicular bisector of \overline{JK} , which has endpoints at $J(-4, 9)$ and $K(6, 1)$. The use of the grid is optional (but recommended).



REASONING

5. In the following diagram, $\triangle ABC$ is drawn with coordinates at $A(-4, 8)$, $B(-2, -2)$, and $C(6, 4)$.

- (a) Find the midpoints of \overline{AB} and \overline{AC} . Label them as points D and E respectively.

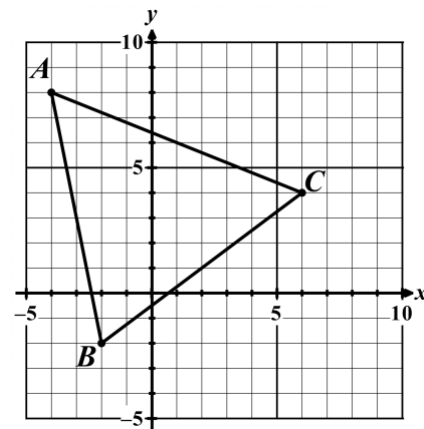
Midpoint of \overline{AB} :
(Point D)

Midpoint of \overline{AC} :
(Point E)

- (b) Draw \overline{DE} on the graph and find its slope and length. Show your calculations below.

Slope of \overline{DE} :

Length of \overline{DE} :



- (c) Find the slope and length of \overline{BC} . Show your calculations below.

Slope of \overline{BC} :

Length of \overline{BC} :

- (d) Give two observations you can make about how \overline{DE} and \overline{BC} relate to each other.

