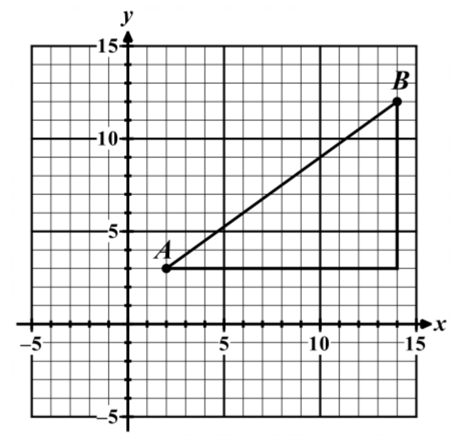
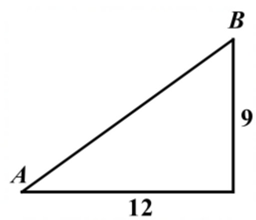
THE DISTANCE FORMULA
N-GEN MATH® GEOMETRY

In our last lesson, we saw how the ***Pythagorean Theorem*** *could be used to find the* ***length*** *of a* ***line segment****. This is the same as finding the* ***distance*** *between* ***two points*** *in the coordinate plane.*

***Exercise* #1:** A segment is shown below with endpoints at $A(2, 3)$ and $B(14, 12)$.

(a) Draw a right triangle on the grid that has horizontal and vertical legs and that has $\overline{AB}$ as its hypotenuse. Reproduce it below with the lengths of its legs labeled.





(b) Determine the length of $\overline{AB}$ (or the distance between A and B) using the Pythagorean Theorem. Show your calculations.

$$9^2+12^2=c^2$$
$$81+144=c^2$$

$$c^2=225$$
$$c=\sqrt{225}=15$$

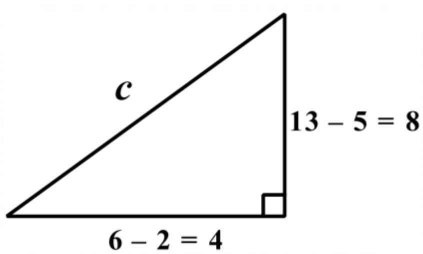
(c) How could have we found the lengths of the legs using the coordinates of the endpoints? Illustrate below.

We can find the lengths of the legs by subtracting the two *x*-coordinates and two *y*-coordinates as shown.

$$\Delta x=14-2=12$$

$$\Delta y=12-3=9$$

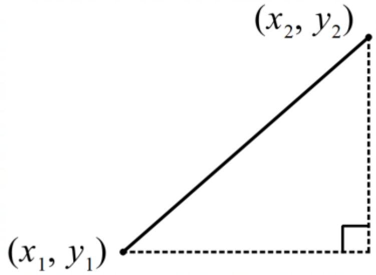
***Exercise* #2:** Find the distance between the points (2, 5) and (6, 13) in simplest radical form. Draw a right triangle to justify your approach.



$$4^2+8^2=c^2$$
$$16+64=c^2$$
$$c^2=80$$

$$c=\sqrt{80}$$
$$c=\sqrt{16}\sqrt{5}$$
$$c=4\sqrt{5}$$

***Exercise* #3:** Given two arbitrary points in the xy-plane (x_1, y_1) and (x_2, y_2), determine a formula that can be used to find the distance between them based on the Pythagorean Theorem.



$$(x_2-x_1)^2+(y_2-y_1)^2=c^2$$

$$c=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

The Distance Formula

If (x_1, y_1) and (x_2, y_2) are two points in the plane, then the **distance**, ***D***, between them can be found using:

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

The ***distance formula*** *will be very important in this course and in other courses. Never forget, though, that it is simply the* ***Pythagorean Theorem*** *applied in the coordinate plane.*

***Exercise* #4:** Find the distance between each set of points shown below using the distance formula.

(a) (3, 12) and (15, 7)

$$D=\sqrt{(15-3)^2+(7-12)^2}$$
$$D=\sqrt{(12)^2+(-5)^2}$$
$$D=\sqrt{144+25}$$
$$D=\sqrt{169}=13$$

(b) (–3, 4) and (5, 10)

$$D=\sqrt{(5-(-3)^2+(10-4)^2}$$
$$D=\sqrt{(8)^2+(6)^2}$$
$$D=\sqrt{64+36}$$
$$D=\sqrt{100}=10$$

***Exercise* #5:** Find the distance between each set of points shown below using the distance formula. Express each answer in simplest radical form.

(a) (4, 1) and (–2, 10)

$$D=\sqrt{(-2-4)^2+(10-1)^2}$$
$$D=\sqrt{(-6)^2+(9)^2}$$
$$D=\sqrt{36+81}$$

$\Rightarrow$

$$D=\sqrt{117}$$
$$D=\sqrt{9}\sqrt{13}$$
$$D=3\sqrt{13}$$

(b) (–2, –5) and (8, 0)

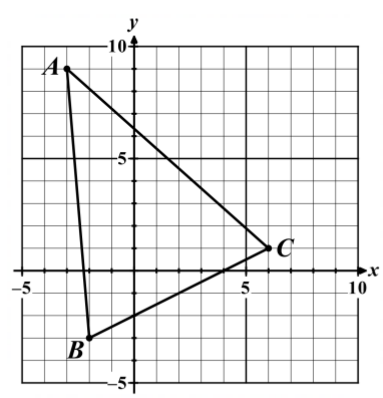
$$D=\sqrt{(8-(-2))^2+(0-(-5))^2}$$
$$D=\sqrt{(10)^2+(5)^2}$$
$$D=\sqrt{100+25}$$

$\Rightarrow$

$$D=\sqrt{125}$$
$$D=\sqrt{25}\sqrt{5}$$
$$D=5\sqrt{5}$$

We will use the distance formula often to tell whether two segments in the coordinate plane have the same length.

***Exercise* #6:** $\triangle ABC$ is shown below with vertices at $A(-3, 9)$, $B(-2, -3)$, and $C(6, 1)$. We would like to determine if $\triangle ABC$ is isosceles.



(a) Which two sides appear to be the same length?

$\overline{AB}$ **and** $\overline{AC}$

(b) Use the distance formula to see if $\triangle ABC$ is isosceles.

$\overline{AB}$:

$$D=\sqrt{(-3-9)^2+(-2-(-3))^2}$$
$$D=\sqrt{(-12)^2+(1)^2}$$
$$D=\sqrt{144+1}=\sqrt{145}$$

$\overline{AC}$:

$$D=\sqrt{(1-9)^2+(6-(-3))^2}$$
$$D=\sqrt{(-8)^2+(9)^2}$$
$$D=\sqrt{64+81}=\sqrt{145}$$

$\Rightarrow$

ΔABC is an isosceles triangle.

THE DISTANCE FORMULA
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FLUENCY

1. For each set of points below, find the distance between them using the distance formula. Each of your answers will be an integer. Show the work that leads to your answers.

(a) (–2, 7) and (10, 12)

$$D=\sqrt{(10-(-2))^2+(12-7)^2}$$
$$D=\sqrt{(12)^2+(5)^2}$$
$$D=\sqrt{144+25}$$
$$D=\sqrt{169}=13$$

(b) (10, –5) and (–6, 7)

$$D=\sqrt{(-6-10)^2+(7-(-5))^2}$$
$$D=\sqrt{(-16)^2+(12)^2}$$
$$D=\sqrt{256+144}$$
$$D=\sqrt{400}=20$$

2. For each set of points below, find the distance between them using the distance formula. Express each answer in simplest radical form. Show the work that leads to your answers.

(a) (2, –4) and (6, 4)

$$D=\sqrt{(6-2)^2+(4-(-4))^2}$$
$$D=\sqrt{(4)^2+(8)^2}$$
$$D=\sqrt{16+64}$$
$$D=\sqrt{80}=\sqrt{16}\sqrt{5}$$
$$D=4\sqrt{5}$$

(b) (5, 7) and (–1, 17)

$$D=\sqrt{(-1-5)^2+(17-7)^2}$$
$$D=\sqrt{(-6)^2+(10)^2}$$
$$D=\sqrt{36+100}$$
$$D=\sqrt{136}=\sqrt{4}\sqrt{34}$$
$$D=2\sqrt{34}$$

3. If the endpoints of $\overline{MN}$ have coordinates of $M(2, 7)$ and $N(-3, 1)$, then which of the following is closest to the length of $\overline{MN}$?

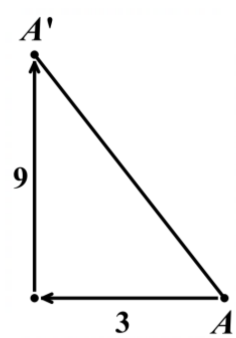
(1) 6.1
(2) 6.7
(3) 7.5
(4) 7.8

$$MN=\sqrt{(-3-2)^2+(1-7)^2}$$
$$MN=\sqrt{(-5)^2+(-6)^2}$$
$$MN=\sqrt{25+36}$$
$$MN=\sqrt{61}\approx 7.8$$

(4)

4. A point A is translated three units to the left and nine units up to produce its image point A'. Which of the following is the length of $\overline{AA'}$?

(1) 12
(2) $2\sqrt{13}$
(3) $3\sqrt{10}$
(4) 18



$$AA'=\sqrt{(-3)^2+(9)^2}$$
$$AA'=\sqrt{9+81}$$
$$AA'=\sqrt{90}=\sqrt{9}\sqrt{10}$$
$$AA'=3\sqrt{10}$$

(3)

5. Which of the following would represent the distance between the x-intercept and y-intercept of a line whose equation is $y = 3x + 12$?

(1) $\sqrt{120}$

(2) $\sqrt{160}$

(3) $\sqrt{200}$

(4) $\sqrt{224}$

The y-intercept of this line is the point $(0,12)$.
The x-intercept is the point $(-4,0)$:

$$D=\sqrt{(-4-0)^2+(0-12)^2}$$
$$D=\sqrt{(-4)^2+(-12)^2}$$
$$D=\sqrt{16+144}=\sqrt{160}$$

(2)

6. Two segments, $\overline{AB}$ and $\overline{CD}$, have endpoints at $A(-5, 11)$, $B(3, 5)$, $C(9, 3)$, and $D(2, 10)$. Which of the two segments is longer? Provide evidence to support your answer.

$\overline{AB}$:
$$AB=\sqrt{(3-(-5))^2+(5-11)^2}$$
$$AB=\sqrt{(8)^2+(-6)^2}$$
$$AB=\sqrt{64+36}$$
$$AB=\sqrt{100}=10$$

$\overline{CD}$:
$$CD=\sqrt{(2-9)^2+(10-3)^2}$$
$$CD=\sqrt{(-7)^2+(7)^2}$$
$$CD=\sqrt{49+49}$$
$$CD=\sqrt{98}\approx 9.9$$

Segment $\overline{AB}$ is slightly longer than $\overline{CD}$.

REASONING

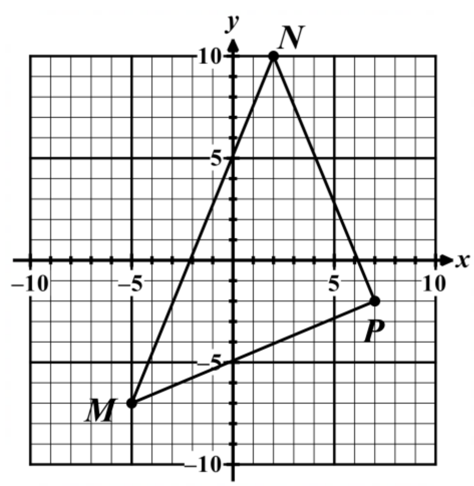
7. If $\triangle MNP$ has vertices at $M(-5, -7)$, $N(2, 10)$, and $P(7, -2)$, is $\triangle MNP$ isosceles? Show evidence to support your conclusion. Use of the grid is optional.

$$MN=\sqrt{(2-(-5))^2+(10-(-7))^2}$$
$$MN=\sqrt{(7)^2+(17)^2}=\sqrt{338}\approx 18.4$$

$$NP=\sqrt{(2-7)^2+(10-(-2))^2}$$
$$NP=\sqrt{(-5)^2+(12)^2}=\sqrt{169}=13$$

$$PM=\sqrt{(7-(-5))^2+(-2-(-7))^2}$$
$$PM=\sqrt{(12)^2+(5)^2}=\sqrt{169}=13$$

Yes, $\triangle MNP$ is isosceles. After calculating the length of each side, we see that two sides, $\overline{PM}$ and $\overline{NP}$, are the same length. Any triangle with at least two sides of equal length is isosceles.



8. A circle has a center located at the point $(-2, 4)$ and a radius of 10. Does the point $(4, -4)$ lie on this circle? Justify your yes/no answer.

$$D=\sqrt{(-2-4)^2+(4-(-4))^2}$$
$$D=\sqrt{(-6)^2+(8)^2}$$
$$D=\sqrt{36+64}$$
$$D=\sqrt{100}=10$$

Yes, this point does lie on the circle since the distance it is away from the center is equal to the radius of the circle.