SLOPE AND PARALLEL LINES
N-GEN MATH® GEOMETRY

*We now move into a portion of geometry where we explore space and shapes using the tools of **coordinate geometry**. One of the main tools we will use is that of the **slope** of a line or line segment. Recall the following:*

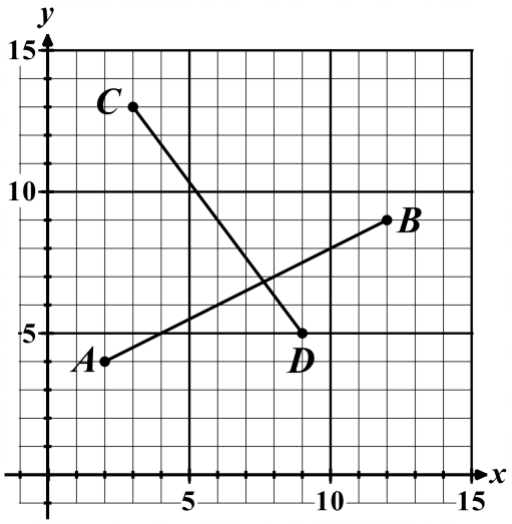
Slope of a Line (or Line Segment)

The **slope** of a line or segment is a **measurement** of how **much** its ***y*-coordinate changes** compared to its *x*-coordinate when moving from one point on the line to another point on the line. This **measurement** is expressed in the form of a **ratio** (expressed as a **fraction**) of the **change in *y* to the change in *x*.**

***Exercise* #1:** Two line segments are shown below, $\overline{AB}$ and $\overline{CD}$.

(a) Write down the coordinate points for each of the four endpoints.

$A(2,4), B(12,9), C(3,13), D(9,5)$



(b) For each line segment, find the following **graphically**. Express your slopes as fractions in simplest form.

$\overline{AB}$: $A(2,4), B(12,9)$

$\triangle x =$ 10

$\triangle y =$ 5

slope $= \frac{\triangle x}{\triangle y} =$ $\frac{5}{10} = \frac{1}{2}$

$\overline{CD}$: $C(3,13), D(9,5)$

$\triangle x =$ 6

$\triangle y =$ –8

slope $= \frac{\triangle x}{\triangle y} =$ $\frac{-8}{6} = -\frac{4}{3}$

(c) How could the change in *x* and change in *y* be calculated from the coordinates of the endpoints instead of found graphically? Illustrate.

You can simply subtract the two *x*-coordinates and subtract the two *y*-coordinates.

$\Delta x = 12 - 2 = 10$
$\Delta y = 9 - 4 = 5$

$\Delta x = 9 - 3 = 6$
$\Delta y = 5 - 13 = -8$

The Slope Formula

If a line passes through the two points (x_1, y_1) and (x_2, y_2), then its slope can be calculated by:

$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad = \frac{y_1 - y_2}{x_1 - x_2}$$

***Exercise* #2:** Use the slope formula above to calculate the slope of a line segment with endpoints given below. Express your slope in simplest form (fraction or whole number).

(a) *A*(–2, 4) and *B*(8, 10)

$m = \frac{10-4}{8-(-2)} = \frac{6}{10} = \frac{3}{5}$

(b) *C*(–10, 3) and *D*(11, –9)

$m = \frac{-9-3}{11-(-10)} = \frac{-12}{21} = -\frac{4}{7}$

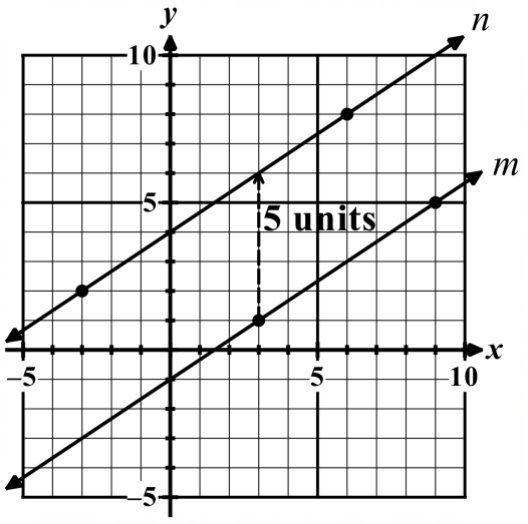
(c) *E*(2, 11) and *F*(–2, 3)

$m = \frac{3-11}{-2-2} = \frac{-8}{-4} = 2$

Answer Key - Unauthorized posting of key to any website is prohibited and a violation of copyright.

*The **slope** of a line gives us its **direction** or **motion** in a certain sense. Due to this, we can see a simple, and yet important, relationship between the **slopes** of **parallel lines** or **parallel line segments**.*

***Exercise* #3:** Two parallel lines *m* and *n* are shown graphed below.



(a) What is the vertical distance between the two lines for any points that share the same *x*-coordinate? Illustrate.

5 units

(b) Using the marked points, calculate the slope of each of the lines.

line *m*: $(3,1)$ and $(9,5)$ line *n*: $(-3,2)$ and $(6,8)$

$$m=\frac{5-1}{9-3}=\frac{4}{6}=\frac{2}{3} \qquad m=\frac{8-2}{6-(-3)}=\frac{6}{9}=\frac{2}{3}$$

(c) Why does it make sense that parallel lines must have slopes that are equal? Consider what would happen if they were not equal to the distance between the two lines.

Slope is a how much the *y* coordinate changes for a given change in the *x* coordinate. If this was different for two lines, then the vertical distance between them would change as *x* changes. So, the slopes must remain the same for the distance to remain the same between the two parallel lines.

Slope and Parallel Lines

1. Two non-vertical lines that are parallel will have equal slopes.
2. Two non-vertical lines that have equal slopes will be parallel.

***Exercise* #4:** Given the points *A*(–2, 1), *B*(6, 7), *C*(–4, –3), and *D*(8, 6) answer the following questions about segments that would connect these various points.

(a) Is $\overline{AB}$ parallel to $\overline{CD}$? Justify.

$$\overline{AB}: m=\frac{7-1}{6-(-2)}=\frac{6}{8}=\frac{3}{4}$$
$$\overline{CD}: m=\frac{6-(-3)}{8-(-4)}=\frac{9}{12}=\frac{3}{4}$$

These lines are parallel.

(b) Is $\overline{AC}$ parallel to $\overline{BD}$? Justify.

$$\overline{AC}: m=\frac{-3-1}{-4-(-2)}=\frac{-4}{-2}=2$$
$$\overline{BD}: m=\frac{6-7}{8-6}=\frac{-1}{2}=-\frac{1}{2}$$

These lines are not parallel.

(c) If $\overline{ED}$ is parallel to $\overline{AC}$ and point *E* has coordinates of (12, *k*), then what is the value of *k*? Show how you arrived at your answer.

$D(8,6)$
$E(12,k)$

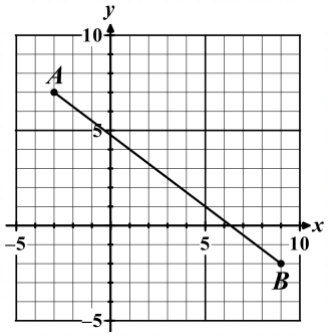
$$\overline{ED}: m=\frac{k-6}{12-8}=\frac{k-6}{4}$$

$$\frac{k-6}{4}=2$$
$$k-6=8$$
$$k=14$$

SLOPE AND PARALLEL LINES
N-GEN MATH® GEOMETRY HOMEWORK

FLUENCY

1. Which of the following is the slope of $\overline{AB}$ shown?



(1) $\frac{4}{3}$ (3) $-\frac{4}{3}$

(2) $\frac{3}{4}$ (4) $-\frac{3}{4}$

$A(-3,7), B(9,-2)$

$$m=\frac{-2-7}{9-(-3)}=\frac{-9}{12}=-\frac{3}{4}$$

(4)

2. Which of the following is the slope of the line that passes through the points (4, –5) and (7, 10)?

(1) $\frac{5}{3}$ (3) 5

(2) $-\frac{3}{5}$ (4) $\frac{1}{5}$

$$m=\frac{10-(-5)}{7-4}=\frac{15}{3}=5$$

(3)

3. In each case below, the endpoints of a line segment have been given. Calculate the slope of the lines segment in simplest form. Show the calculations that lead to your answers.

(a) $A(2, 1)$ and $B(12, 7)$

$$m=\frac{7-1}{12-2}=\frac{6}{10}=\frac{3}{5}$$

(b) $C(3, 8)$ and $D(15, 5)$

$$m=\frac{5-8}{15-3}=\frac{-3}{12}=-\frac{1}{4}$$

(c) $E(–1, –2)$ and $F(1, 4)$

$$m=\frac{4-(-2)}{1-(-1)}=\frac{6}{2}=3$$

(d) $G(9, –3)$ and $H(–6, 3)$

$$m=\frac{3-(-3)}{-6-9}=\frac{6}{-15}=-\frac{2}{5}$$

(e) $I(–2, 1)$ and $J(–10, –3)$

$$m=\frac{-3-1}{-10-(-2)}=\frac{-4}{-8}=\frac{1}{2}$$

(f) $K(–2, 4)$ and $L(6, 4)$

$$m=\frac{4-4}{6-(-2)}=\frac{0}{8}=0$$

4. It is known that $\overline{ST}$ is parallel to $\overline{CD}$, whose endpoints are at $C(–2, 1)$ and $D(4, 10)$. Which of the following is the slope of $\overline{ST}$?

(1) $\frac{2}{3}$ (3) $\frac{1}{4}$

(2) $\frac{3}{2}$ (4) $\frac{9}{2}$

Parallel lines have the same slope.

$$\overline{CD}: m=\frac{10-1}{4-(-2)}=\frac{9}{6}=\frac{3}{2}$$

(2)

5. Given the four points A(–3, 5), B(1, 13), C(4, 2), and D(10, 5), are $\overline{AB}$ and $\overline{CD}$ parallel? Justify.

$\overline{AB}: m = \frac{13-5}{1-(-3)} = \frac{8}{4} = 2$

$\overline{CD}: m = \frac{5-2}{10-4} = \frac{3}{6} = \frac{1}{2}$

⇨ **These lines are not parallel.**

6. Given the four points E(2, 5), F(7, 1), G(2, –3), and H(–8, 5), is $\overline{EF} \parallel \overline{GH}$? Justify.

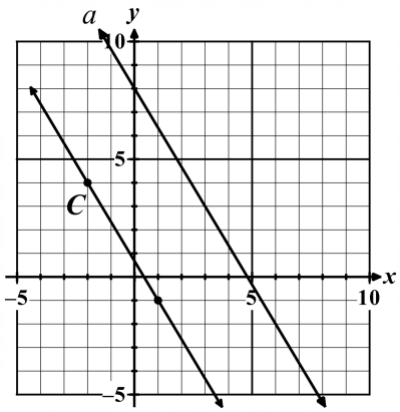
$\overline{EF}: m = \frac{1-5}{7-2} = \frac{-4}{5} = -\frac{4}{5}$

$\overline{GH}: m = \frac{5-(-3)}{-8-2} = \frac{8}{-10} = -\frac{4}{5}$

⇨ **These lines are parallel.**

REASONING

7. Given line *a* and point *C* shown, do the following:



(a) Draw a line through point *C* that is parallel to line *a*. Explain your method below.

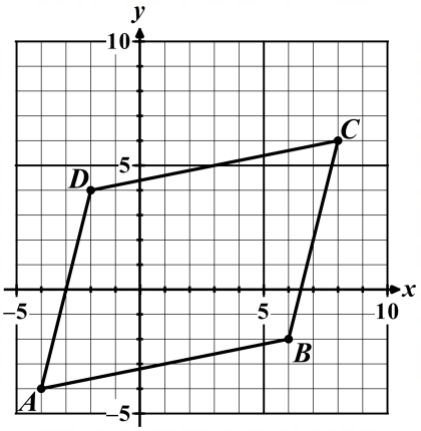
The slope of line *a* is $-\frac{5}{3}$. This tells you that to create a parallel line, you can start at point *C* and move right 3 units and down 5 units.

(b) Does the line you drew in (a) pass through the point *D*(16,–24)? Justify your yes/no response.

We can determine if *D* is on this line by calculating the slope of line segment $\overline{CD}$. ⇨ ***C*(–2, 4) and *D*(16, –24):** $m = \frac{-24-4}{16-(-2)} = -\frac{28}{18} = -\frac{14}{9}$ ⇨ **The slopes are not equal. Therefore, the line does not pass through point *D*.**

8. Quadrilateral *ABCD* has vertices at *A*(–4, –4), *B*(6, –2), *C*(8, 6), and *D*(–2, 4).

(a) Plot and label the four vertices. Then draw *ABCD*.



(b) Calculate the slope of each of the four sides:

$\overline{AB}$: $m = \frac{-2-(-4)}{6-(-4)} = \frac{2}{10} = \frac{1}{5}$

$\overline{BC}$: $m = \frac{6-(-2)}{8-6} = \frac{8}{2} = 4$

$\overline{CD}$: $m = \frac{4-6}{-2-8} = \frac{-2}{-10} = \frac{1}{5}$

$\overline{DA}$: $m = \frac{4-(-4)}{-2-(-4)} = \frac{8}{2} = 4$

(c) State the pairs of parallel sides based on (b).

$\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$

(d) What type of special quadrilateral is this? **This quadrilateral is a parallelogram.**