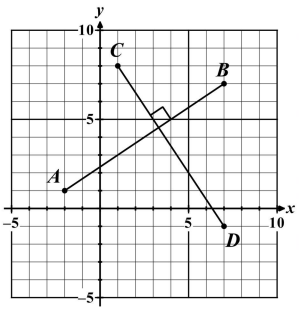
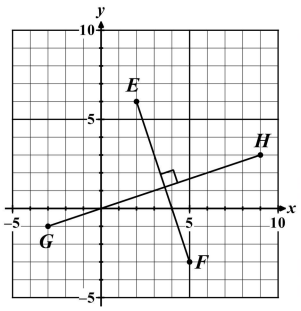
SLOPE AND PERPENDICULAR LINES
N-GEN MATH® GEOMETRY

*We have now seen the relationship between **slope** and **parallel lines**. There is a less obvious relationship between the slopes of two lines or line segments that are **perpendicular** to one another.*

***Exercise* #1:** In both graphs shown below, two perpendicular line segments have been graphed. Their endpoints are at $A(-2,1)$, $B(7,7)$, $C(1,8)$, $D(7,-1)$, $E(2,6)$, $F(5,-3)$, $G(-3,-1)$, and $H(9,3)$.



Graph #1



Graph #2

(a) Determine the slope of each of the line segments. Do so graphically or by using the slope formula. Express each slope as a fraction in simplest form.

Graph #1: **Graph #2:**

$\overline{AB}$: $\overline{CD}$: $\overline{EF}$: $\overline{GH}$:

$$m = \frac{7-1}{7-(-2)} = \frac{6}{9} = \frac{2}{3} \qquad m = \frac{-1-8}{7-1} = \frac{-9}{6} = -\frac{3}{2} \qquad m = \frac{-3-6}{5-2} = \frac{-9}{3} = -3 \qquad m = \frac{3-(-1)}{9-(-3)} = \frac{4}{12} = \frac{1}{3}$$

(b) Compare the slopes of segments that are perpendicular to one another. What observations can you make about them?

Slopes of lines that are perpendicular to each other have opposite signs. Also, the ratios are reciprocals of each other.

Slopes of Perpendicular Lines

Two lines, neither of which are horizontal nor vertical, are **perpendicular** to one another if **their slopes** are **negative reciprocals (neg. rec.)** of one another. If one slope is equal to $\frac{a}{b}$, then the other must be $-\frac{b}{a}$.

***Exercise* #2:** If $\overline{AB}$ has endpoints at $A(-2,7)$ and $B(4,15)$, then which of the following would be the slope of any line or line segment perpendicular to $\overline{AB}$?

(1) $\frac{4}{3}$ (3) $-\frac{3}{4}$

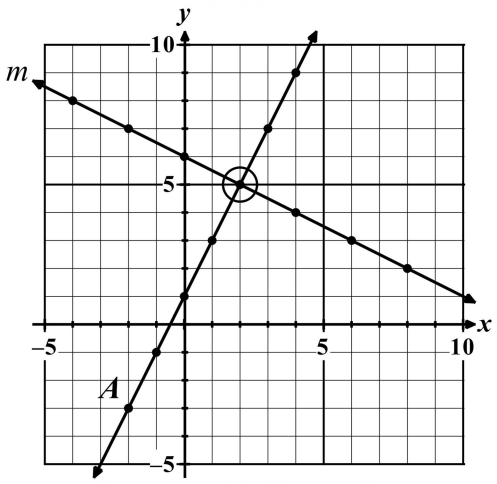
(2) $\frac{1}{4}$ (4) 4

$$m = \frac{15-7}{4-(-2)} = \frac{8}{6} = \frac{4}{3}$$

$$m_{\perp} = -\frac{3}{4}$$

(3)

***Exercise* #3:** A line can be drawn through point A perpendicular to line m shown below.



(a) What will be the slope of the perpendicular line? Justify.

Slope of line m :

$$m = \frac{5-6}{2-0} = \frac{-1}{2} = -\frac{1}{2}$$

⇨ **The negative reciprocal of $-\frac{1}{2}$ is 2.**

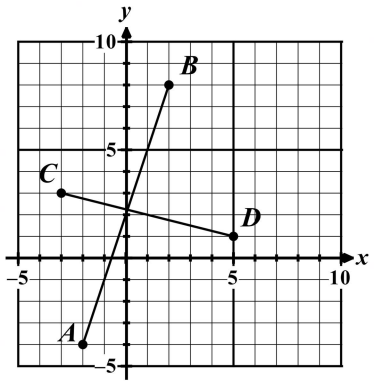
(b) At what point will this line intersect line m? Show how you found your answer.

Now, we draw a line through point A that has a slope of 2. The intersection point is $(2, 5)$.

We can use our knowledge of ***slopes of perpendicular lines*** *to determine if two lines or line segments are perpendicular or not.*

***Exercise* #4:** Two segments, $\overline{AB}$ and $\overline{CD}$, have endpoints at $A(-2,-4)$, $B(2,8)$, $C(-3,3)$, and $D(5,1)$.

Is $\overline{AB} \perp \overline{CD}$? Justify with numerical evidence.



$$\overline{AB}: m = \frac{8-(-4)}{2-(-2)} = \frac{12}{4} = 3$$

$$\overline{CD}: m = \frac{1-3}{5-(-3)} = \frac{-2}{8} = -\frac{1}{4}$$

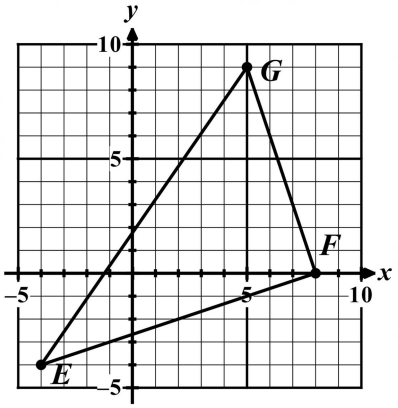
⇨ **$\overline{AB}$ is not perpendicular to $\overline{CD}$ because their slopes are not negative reciprocals of one another.**

We will be using our knowledge of slopes to help determine if given shapes in the coordinate plane fall into certain categories.

***Exercise* #5:** The vertices of ΔEFG are located at $E(-4,-4)$, $F(8,0)$, and $G(5,9)$.

(a) Plot and label ΔEFG.

(b) Use slopes to determine if ΔEFG is a right triangle.



$$\overline{EF}: m = \frac{0-(-4)}{8-(-4)} = \frac{4}{12} = \frac{1}{3}$$

$$\overline{FG}: m = \frac{9-0}{5-8} = \frac{9}{-3} = -3$$

$$\overline{EG}: m = \frac{9-(-4)}{5-(-4)} = \frac{13}{9}$$

⇨ **$\overline{EF}$ is perpendicular to $\overline{FG}$ because their slopes are negative reciprocals. Perpendicular segments form right angles. This means ΔEFG is a right triangle.**

SLOPE AND PERPENDICULAR LINES
N-GEN MATH® GEOMETRY HOMEWORK

FLUENCY

1. Each of the following represents the slope of a line. Give the slope of the line that would be perpendicular to the one with the given slope.

(a) $m=\frac{5}{2}$ $m=-\frac{2}{5}$

(b) $m=-\frac{3}{4}$ $m=\frac{4}{3}$

(c) $m=\frac{1}{4}$ $m=-4$

(d) $m=-3$ $m=\frac{1}{3}$

(e) $m=1$ $m=-1$

(f) $m=-\frac{5}{11}$ $m=\frac{11}{5}$

(g) $m=-\frac{1}{5}$ $m=5$

(h) $m=2$ $m=-\frac{1}{2}$

(i) $m=-\frac{8}{3}$ $m=\frac{3}{8}$

(j) $m=\frac{2}{7}$ $m=-\frac{7}{2}$

2. A line passes through the points $E(-1,4)$ and $F(3,-2)$. Which of the following is the slope of a line that is perpendicular to $\overleftrightarrow{EF}$?

(1) $\frac{2}{3}$ (3) -3

(2) $\frac{1}{3}$ (4) $-\frac{3}{2}$

$\overline{EF}: m=\frac{-2-4}{3-(-1)}=\frac{-6}{4}=-\frac{3}{2}$

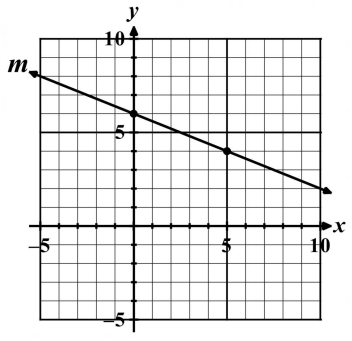
$m_{\perp}=\frac{2}{3}$

(1)

3. Which of the following would be the slope of a line that is perpendicular to line *m* shown graphed?

(1) $-\frac{3}{2}$ (3) $\frac{7}{3}$

(2) $\frac{5}{2}$ (4) $-\frac{3}{4}$



$m=-\frac{2}{5}$

$m_{\perp}=\frac{5}{2}$

(2)

4. A line segment whose endpoints are $(3,9)$ and $(7,k)$ is perpendicular to another segment whose slope is equal to -2. Which of the following is the value of *k*?

(1) 1
(2) -7
(3) 11
(4) -5

$m=-2$

$m_{\perp}=\frac{1}{2}$

$\frac{k-9}{7-3}=\frac{1}{2}$

$\frac{k-9}{4}=\frac{1}{2}$

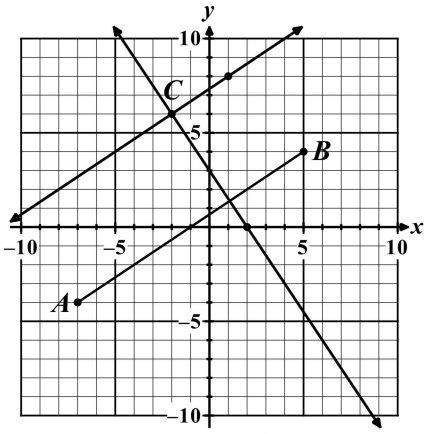
$k-9=\frac{1}{2}\cdot 4$

$k-9=2$

$k=11$

(3)

5. On the grid shown, $\overline{AB}$ and point C are plotted.



(a) Draw a line through C parallel to $\overline{AB}$. State a point below that lies on this line besides C.

$A(-7,-4)$ and $B(5,4)$

$\overline{AB}: m = \dfrac{4-(-4)}{5-(-7)} = \dfrac{8}{12} = \dfrac{2}{3}$

(1,8)

(b) Draw a line through C that is perpendicular to AB. State a point below that lies on this line besides point C.

$m_{\perp} = -\dfrac{3}{2}$

(2,0)

REASONING

6. If $\overleftrightarrow{AB}$ passes through the points $A(3,-2)$ and $B(6,13)$, and $\overleftrightarrow{CD}$ passes through the points $C(-5,8)$, and $D(5,6)$, are $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ perpendicular? Justify your answer.

$\overleftrightarrow{AB}: m = \dfrac{13-(-2)}{6-3} = \dfrac{15}{3} = 5$

$\overleftrightarrow{CD}: m = \dfrac{6-8}{5-(-5)} = \dfrac{-2}{10} = -\dfrac{1}{5}$

⇨ **These lines are perpendicular because their slopes are negative reciprocals of each other.**

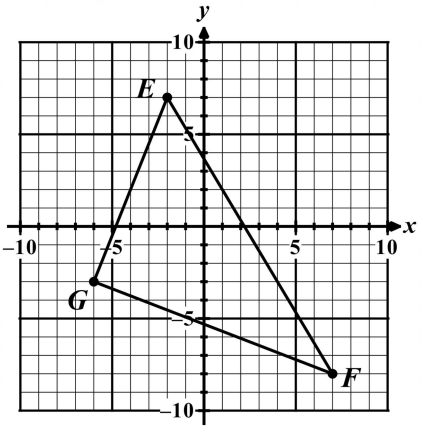
7. In ΔEFG, the vertices are located at $E(-2,7)$, $F(7,-8)$, and $G(-6,-3)$. Is ΔEFG a right triangle? Provide evidence for your yes/no answer. The use of the grid is encouraged but not required.

$\overline{EF}: m = \dfrac{-8-7}{7-(-2)} = \dfrac{-15}{9} = -\dfrac{5}{3}$

$\overline{FG}: m = \dfrac{-3-(-8)}{-6-7} = \dfrac{5}{-13} = -\dfrac{5}{13}$

$\overline{EG}: m = \dfrac{-3-7}{-6-(-2)} = \dfrac{-10}{-4} = \dfrac{5}{2}$

⇨ **This is not a right triangle because none of the sides have slopes that are negative reciprocals of each other.**



8. If point H lies on $\overline{EF}$ of the triangle from #7 such that $\overline{GH}$ is perpendicular to $\overline{EF}$, then what is the slope of $\overline{GH}$? Explain.

$\overline{EF}: m = \dfrac{-8-7}{7-(-2)} = \dfrac{-15}{9} = -\dfrac{5}{3}$

⇨ **The slope of $\overline{GH}$ would have to be $\dfrac{3}{5}$.**