A Sequential Monte Carlo Approach to Endogenous Time Varying Parameter Models

My Contribution

- I adapt the Sequential Monte Carlo filter to handle a nonlinear state space model with endogenous explanatory variables.
- A method to estimate a TVP model with nonlinearities, where the time-varying parameter is on an endogenous variable and instrumental variables are and are not available

Importance

- Complex models are becoming more prevalent and essential to understanding macroeconomic dynamics.
- Many of these models allow for nonlinearities and shifts in parameters.

Need

- Researchers in the empirical macroeconomic literature have investigated conditionally linear TVP models with endogenous regressors
- Peersman and Pozzi (2004) and Kim (2006) rely on a method that substitutes the fitted value from the instrumenting equation for the actual value, creating an issue of heteroscedasticity.
- Kim and Kim (2011) address this issue using a Kalman filter for linear models. This method has not been extended to nonlinear models.

Outline of the remaining section

1. State space model with endogenous variable
2. Fundamental ideas of particle filter
3. Extending the particle filter
4. Evidence that technique works

State Space Model

Consider the following conditionally linear state space model with a random walk time varying parameter:

$$y_t = h(X_t, \delta, \lambda_t) + \nu_t,$$

$$\nu_t \backsim IN(0, \sigma_\nu^2)$$

$$\lambda_t = \lambda_{t-1} + \omega_t, \qquad \omega_t \backsim IN(0, \sigma_\omega^2).$$

State Space Model

To incorporate endogeneity into the model, I assume that there exists a joint distribution between ν, ξ. Later, I test with a T-distribution. Here, I make a joint normality assumption:

$$X_t = MZ_t + \xi_t,$$

$$\nu, \xi \backsim IN(0, \Sigma), \qquad \Sigma = \begin{bmatrix} \sigma_\nu^2 & \sigma_{\nu,\xi} \\ \sigma_{\nu,\xi} & \sigma_\xi^2 \end{bmatrix}$$

State Space Mode: Joint Normality Assumption

This assumption adds alot of structure to the model. However, it is commonly made in the instrument variable literature. I also test a different distribution for a robusteness check.

- A static IV regression model in structural form with this assumption is used by Hausman (1983) and Zivot and Kleibergen (2003)
- Time series versions that use this assumption are Peersman and Pozzi (2004) , Kim (2006), and Kim and Kim (2011).
- Both the Kalman filter and the Particle filter can use a different distribution but it must have an explicit form.

Fundamentals of Particle Filter

Given a suggestion of the static parameters, the particle filter

1. Estimates N paths of the time-varying parameter,
2. Uses these paths to find the likelihood of the data conditioned on the suggestion of static parameters.

Fundamentals of Particle Filter

The particle filter estimates the parameter path using the following steps at each period

1. Uses the static parameters to create a naive sample of the possible values for the parameter.
2. Takes an informed sample of states from the naive sample using the relative probability of the each state.

Extending the Particle Filter

My contribution to the particle filter method is in the informed sample stage and the likelihood of the static parameters.

Extending the Particle Filter: Resampling Stage

- Using the static parameters we can calculate the implied values of ξ_t.
- We can then pair the value of ξ_t with each of the error terms $\nu_{t,i}$, implied by our sample of states.
- I use the assumed joint normal distribution here instead of the univariate assumption.
- The new probability of state i is

$$p_{i,t} = \frac{g(\xi_t, \nu_{t,i}|\theta)}{\sum_{j=1}^{N} g(\xi_t, \nu_{t,i}|\theta)}$$

Extending the Particle Filter: Likelihood

Using the error terms from the final sample of the states and the instrumental error, we can calculate the per period likelihood as the average likelihood between the instrumental error and the final sample state errors:

$$g(\nu_t, \xi_t|\theta) = \frac{1}{N}\sum_{i=0}^{N} g(\nu_{t,i}, \xi_t|\theta),$$

Independence still holds across periods, allowing us to take the product of the per-period likelihood to find a likelihood that will give unbiased estimates of model parameters:

$$L(Y^T|\theta) = \prod_{t=1}^{T} g(\nu_t, \xi_t|\theta).$$

Benefits of an Extended Filter

To analyze the benefits of the particle filter, I will compare the output of an extended particle filter to a traditional particle filter. The comparison is made using a single model with different levels of covariance.

Benefits of an Extended Filter

For estimation, I choose to implement a Bayesian Markov Chain Monte Carlo. In this estimation method, we collect samples of the parameters of interest to specify a posterior distribution of the parameters.

Benefits of an Extended Filter

It is beneficial for the accuracy of the time-varying parameter and the accuracy of the static parameters to include instruments when there is endogeniety. When covariance is zero, the extended particle filter makes slightly less accurate estimates.

Model I

The first test will use a nonlinear model with an additive error. This sample model was chosen because of its relation to a sticky information Phillips curve with drift in the rate of attentiveness. Endogeneity is known to be an issue when estimating the Phillips curve.

Model I

The model has the following form:
measurement equation:

$$y_t = \frac{1-\lambda_t}{\lambda_t} x_{1,t} + \beta x_{2,t} + \nu_t,$$

transition equation:

$$\lambda_t = \lambda_{t-1} + \omega_t,$$

instrumental equation:

$$x_{1,t} = m z_t + \xi_t.$$

Absolute Deviation of TVP

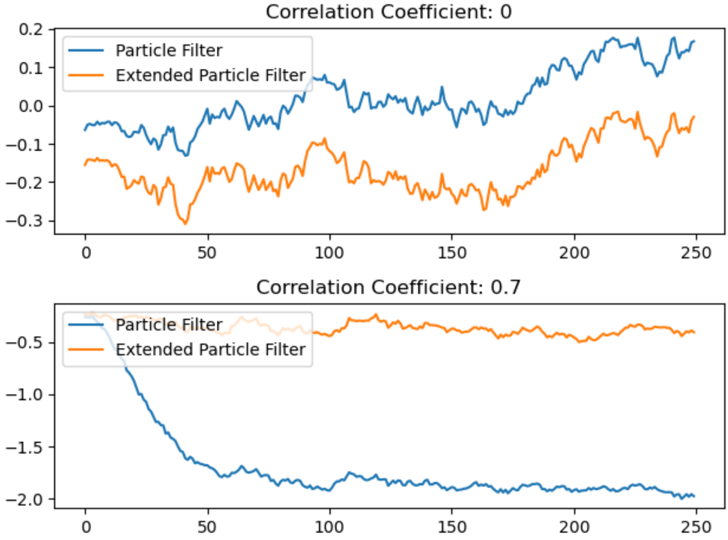

Figure: Absolute Deviation of λ_t, Model I.

Zero Covariance

	Extended Particle Filter	Particle Filter
β =1	0.959 (0.054)	0.976 (0.045)
$\sigma_\nu = 1$	0.957 (0.049)	0.963 (0.043)
λ_0 =4	3.845 (0.384)	3.934 (0.239)
σ_ω =0.1	0.167 (0.025)	0.151 (0.021)
σ_ξ =1	1.005 (0.049)	-
$\rho = 0$	-0.066 (0.074)	-
$m = 1$	0.981 (0.049)	-
Acceptance Rate	0.400	0.578

Table: Metropolis-Hastings Sampler Results for Model 1

Model I

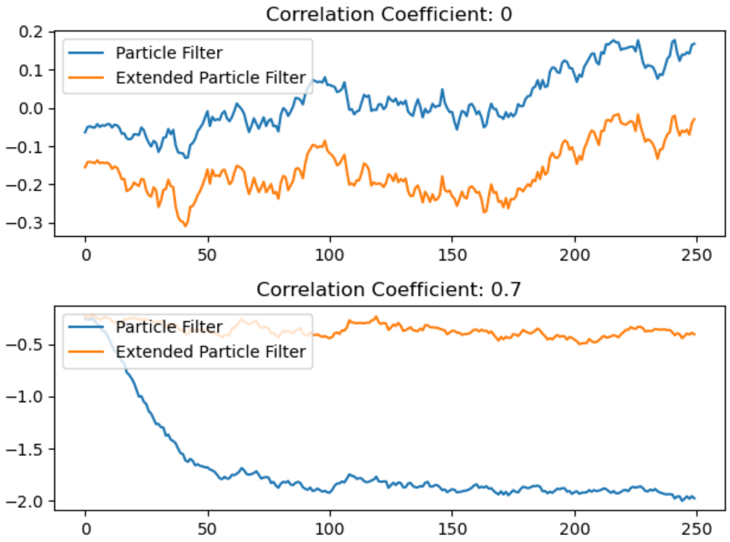

Figure: Absolute Deviation of λ_t, Model I.

Strong Covariance

	Extended Particle Filter	Particle Filter
$\beta = 1$	0.971 (0.041)	0.841 (0.036)
$\sigma_\nu = 1$	0.897 (0.054)	0.784 (0.036)
$\lambda_0 = 4$	3.735 (0.311)	3.736 (0.125)
$\sigma_\omega = 0.1$	0.181 (0.018)	0.167 (0.014)
$\sigma_\xi = 1$	1.000 (0.048)	-
$\rho = 0.7$	0.685 (0.041)	-
$m = 1$	1.055 (0.048)	-
Acceptance Rate	0.344	0.310

Table: Metropolis-Hastings Sampler Results for Model 1

Model I

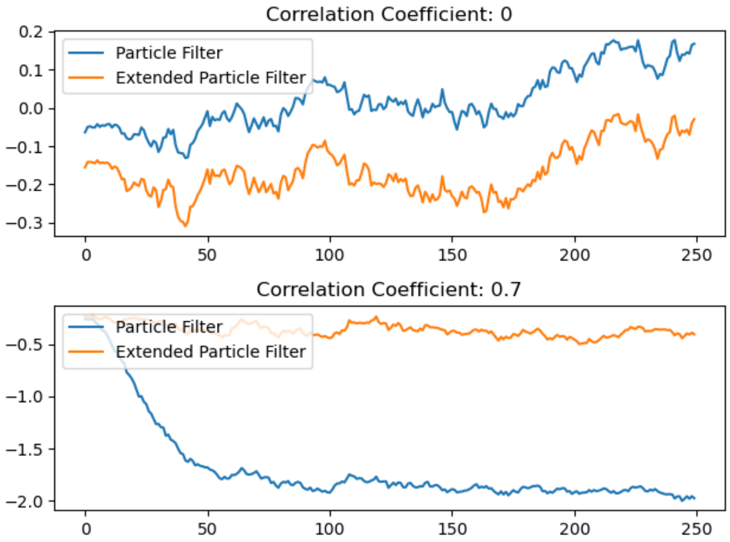

Figure: Absolute Deviation of λ_t, Model I.

The Role of Instruments

The joint normal assumption furnishes the necessary structure for estimating a model and enables us to directly model a relationship between the endogenous variable and the measurement error.

The Role of Instruments

The test will use a linear model with an additive error. This sample model was chosen because of its simplicity. The model has the following form:
measurement equation:

$$y_t = \lambda_t * x_{1,t} + \beta * x_{2,t} + \nu_t,$$

transition equation:

$$\lambda_t = \lambda_{t-1} + \omega_t,$$

instrumental equation:

$$x_{1,t} = mz_t + \xi_t.$$

The Role of Instruments

In this experiment, there actually is an instrument (m does not equal zero), but the researcher assumes that m equals zero. A useful tool if a researcher doesn't have access to instruments or available instruments are weak.

The Role of Instruments

	With Instruments	Without Instruments
$\sigma_\nu = 1$	0.933 (0.027)	0.995 (0.023)
β =1	1.001 (0.024)	1.000 (0.023)
λ_0 =1	1.000 (0.024)	0.999 (0.023)
σ_ω =1	1.009 (0.024)	1.002 (0.023)
σ_ξ =1	1.001 (0.024)	1.011 (0.023)
$\rho = 0.7$	0.823 (0.040)	0.707 (0.016)
m=1	1.000 (0.024)	-
Acceptance Rate	0.368	0.348

Table: Metropolis-Hastings Sampler Results for Linear Model

TVP Mean Squared Error

Let's denote:

- λ as the vector of target values (TVP),
- $\hat{\lambda}_i$ is sample path i,
- N as the number of sample paths.

Mean Squared Error

The equations to compute the Mean Squared Error (MSE) are:

1. Compute the squared errors:

$$\mathrm{E_i} = (\hat{\lambda}_i - \lambda)^2$$

2. Mean of the squared errors across sample paths:

$$\mathrm{MSE}(t) = \frac{1}{N} \sum_{i=1}^{N} E_i(t)$$

The Role of Instruments

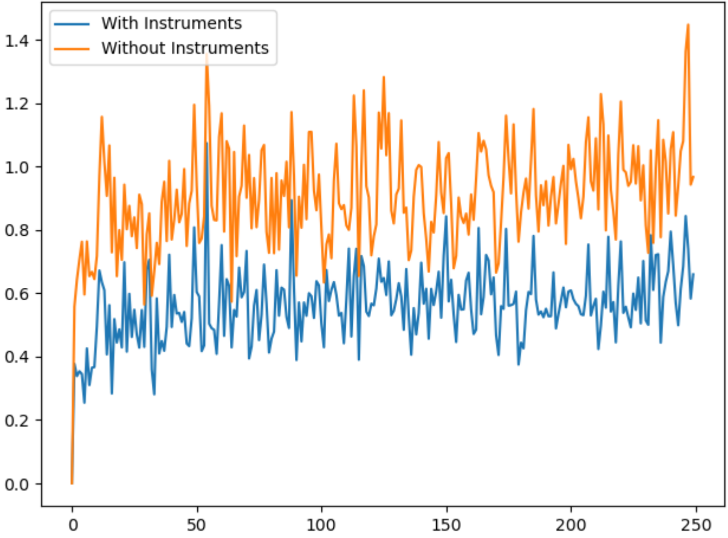

Figure: Per-Period Mean Squared Error of λ_t

The Role of Instruments

Utilizing instruments if they are available will lower the MSE of the TVP. However, if instruments are not available or are weak, the extended particle filter would be a robust choice.

Robustness to Misspecification

The method relies heavily on the joint normal assumption between the variables. To test the robustness of the extended particle filter to misspecification, I will compare particle filters when the true error has a joint t-distribution with three degrees of freedom. One particle filter will use the normal assumption, while the other correctly specifies a t-distribution with three degrees of freedom.

Robustness to Misspecification

The model has the following form:
measurement equation:

$$y_t = \lambda_t * x_{1,t} + \beta * x_{2,t} + \nu_t,$$

instrumental equation:

$$x_1 = mz_t + \xi_t,$$

$$\nu, \xi \backsim T(0, \Sigma, df = 3), \qquad \Sigma = \begin{bmatrix} \sigma_\nu^2 & \rho * \sigma_\nu * \sigma_\xi \\ \rho * \sigma_\nu * \sigma_\xi & \sigma_\xi^2 \end{bmatrix}$$

transition equation:

$$\lambda_t = \lambda_{t-1} + \omega_t.$$

TVP Mean Squared Error

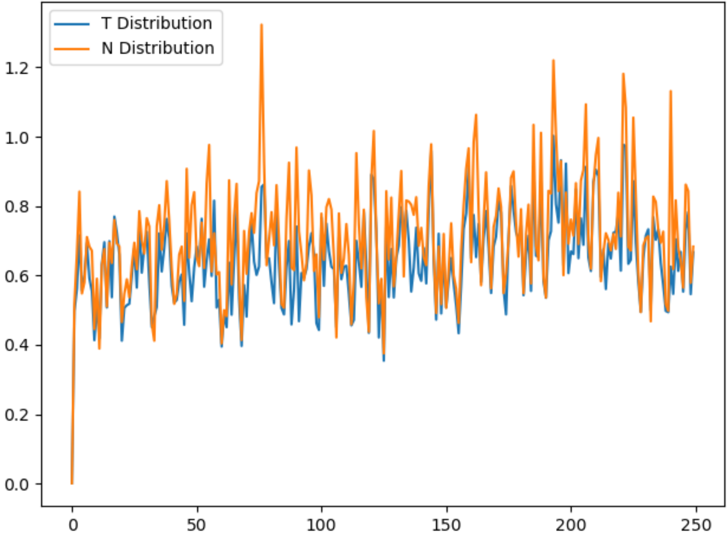

Figure: Per-Period Mean Squared Error of λ_t

Robustness to Misspecification

	T Distribution	N Distribution
$\sigma_\nu = 1$	0.955 (0.026)	0.979 (0.023)
β =1	1.000 (0.024)	0.998 (0.023)
λ_0 =1	1.000 (0.024)	0.998 (0.023)
σ_ω =1	1.013 (0.024)	1.009 (0.023)
σ_ξ =1	1.012 (0.024)	1.052 (0.027)
$\rho = 0.7$	0.777 (0.031)	0.739 (0.021)
m=1	1.002 (0.024)	0.998 (0.023)
Acceptance Rate	0.268	0.129

Table: Metropolis-Hastings Sampler Results for Non-Normal Model

Robustness to Misspecification

While the estimates are fairly accurate, the acceptance rate is very low, and the variation between the acceptance rates of independent draws is large. With accurate starting values, the extended particle filter is robust to misspecification.

Conclusion

A method to estimate a TVP model with nonlinearities, where the time-varying parameter is on an endogenous variable and instrumental variables may or may not be available, is not previously defined in the literature. I fill this gap in the literature by adapting the particle filter to run a joint estimation procedure. The IV estimation was shown to correct for bias in the TVP. This bias affects the estimation of static parameters.