EC 327 Spring 2023, Midterm Exam, 1A

Exam Grading Policy:

- If you don't know the answer, please write “I don't know” or leave it blank, and you will receive partial credit. 25%
- If you choose to attempt a question, you will be graded on the merit of your answer.
- An answer that shows understanding with some mistakes will be given partial credit, decided by the grader. A well-written response should receive more than 25%.
- Show all your work. Answers without work are not guaranteed credit.

Name________________ ID number:__________________

Problem #1: Expected Payout, 16 pts

Suppose you roll a fair die, and your opponent flips an unfair coin simultaneously (independent events). The die has a ¼ chance of coming up Heads and a ¾ chance of coming up Tails. The die places a ⅙ chance on all values.

You win \$3 whenever a Tail appears, and the number of dots on the top face of the dice is either one or six. For all other outcomes, you lose \$1. (Dr. Rui Zhao, University at Albany)

a. (8) What is your expected payoff?

$$E(\$) = \tfrac{1}{4} * \tfrac{1}{6} * (-1) + \tfrac{1}{4} * \tfrac{1}{6} * (-1) + \tfrac{1}{4} * \tfrac{1}{6} * (-1) + \tfrac{1}{4} * \tfrac{1}{6} * (-1) +$$
$$\tfrac{1}{4} * \tfrac{1}{6} * (-1) + \tfrac{1}{4} * \tfrac{1}{6} * (-1) + \tfrac{3}{4} * \tfrac{1}{6} * (-1) + \tfrac{3}{4} * \tfrac{1}{6} * (-1) +$$
$$\tfrac{3}{4} * \tfrac{1}{6} * (-1) + \tfrac{3}{4} * \tfrac{1}{6} * (-1) + \tfrac{3}{4} * \tfrac{1}{6} * (3) + \tfrac{3}{4} * \tfrac{1}{6} * (3)$$
$$= 1/24*(-1-1-1-1-1-1) + 3/24*(-1-1-1-1) + 3/24*(3+3)$$
$$= -6/24 - 12/24 + 18/24 = 0$$

b. (8) What is your opponent's expected payoff? Is the game in your favor?

$$E(\$) = \tfrac{1}{4} * \tfrac{1}{6} * (1) + \tfrac{1}{4} * \tfrac{1}{6} * (1) + \tfrac{1}{4} * \tfrac{1}{6} * (1) + \tfrac{1}{4} * \tfrac{1}{6} * (1) +$$
$$\tfrac{1}{4} * \tfrac{1}{6} * (1) + \tfrac{1}{4} * \tfrac{1}{6} * (1) + \tfrac{3}{4} * \tfrac{1}{6} * (1) + \tfrac{3}{4} * \tfrac{1}{6} * (1) +$$
$$\tfrac{3}{4} * \tfrac{1}{6} * (1) + \tfrac{3}{4} * \tfrac{1}{6} * (1) + \tfrac{3}{4} * \tfrac{1}{6} * (-3) + \tfrac{3}{4} * \tfrac{1}{6} * (-3)$$
$$= 1/24*(+1+1+1+1+1+1) + 3/24*(+1+1+1+1) + 3/24*(-3-3)$$
$$= +6/24 + 12/24 - 18/24 = 0$$

The game is fair; it is in no player's favor.

Problem #2: Construct Game Matrix, 20 pts

Two fishing firms, "Seafood Unlimited" and "Critter Catchers," catch Dungeness Crab off the Oregon coast. Each chooses between using a large vessel or a small one. It costs \$10,000 and \$20,000 to run a small and large vessel, respectively, for a fishing season. If both run small vessels, they will each catch 2,500 pounds of crab per season, and the market price will be \$10 a pound. If one uses a small vessel and the other a large one, the market price will be \$8, and the large vessel will catch 4,000 pounds of crab and the small vessel 2,500 pounds of crab (because they fish at different distances offshore). If both choose large vessels, they will both catch 3,500 pounds of crab, and the market price of crab will be \$2. The fishing firms must make their vessel size choices simultaneously. Firms measure their utility in profit. Recall profit = revenue - cost. (Dr. Ellis, University of Oregon)

a. (4) Specify players and strategies
   Players: Seafood Unlimited and Critter Catchers
   Strategies: a large vessel or a small one
b. (8) Construct a payoff matrix for this game
   $\pi_L(L, L) = 3500 * 2 - 20000 = -13,000$
   $\pi_S(S, S) = 2500 * 10 - 10000 = 15,000$
   $\pi_S(L, S) = 2500 * 8 - 10000 = 10,000$
   $\pi_L(L, S) = 4000 * 8 - 20000 = 12,000$

Payoff: (CC,SF)

		SF: L	SF: S
CC	L	(-13000, -13000)	(12000, 10000)
CC	S	(10000, 12000)	(15000, 15000)

c. (4) Find pure strategy Nash equilibrium
   NE: (S, S)
d. (8) Give both definitions of a Nash equilibrium
   i. A strategy profile in which each player is best responding to the strategies of all other players.
   ii. A strategy profile in which all players cannot receive a higher payout if they deviate from their strategy unilaterally.

Problem #3: Verify Nash Equilibrium, 16pts

		Player 2: C	Player 2: D
Player 1	A	(4, 3)	(1, 3)
	B	(2, 4)	(4, 0)

I. (8) Verify if the following strategy profile is Nash equilibrium or not using Theorem 1.
(A, ⅘ C+ ⅕ D)
Player 1
$U_1(A, ⅘\ C + ⅕\ D) = 4/5\, U_1(A,\ C) + 1/5\, U_1\,(A, D) = 4/5 * 4 + 1/5 * 1 = 17/5$
$U_1(B, ⅘\ C + ⅕\ D) = 4/5 U_1(B,\ C) + 1/5\, U_1\,(B, D) = 4/5 * 2 + 1/5 * 4 = 12/5$

Player 2
$U_2(A, C) = 3$
$U_2(A, D) = 3$

Nash Equilbirium is verified.

II. (8) Clearly explain your findings.
For player 1, we have found that the strategy not included in the potential Nash equilibrium (B) has a lower payoff. Therefore, player 1 has no incentive to deviate from A.

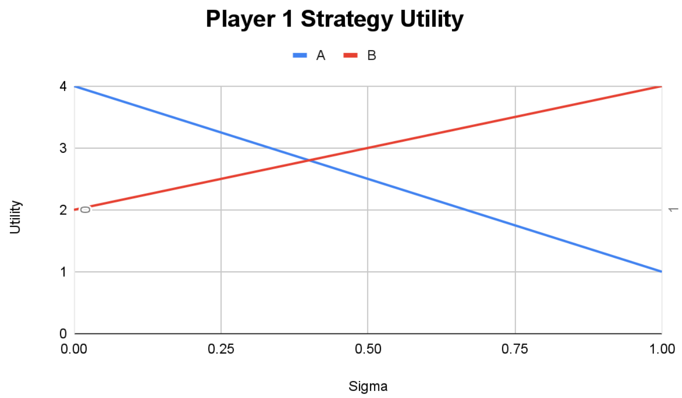
For player 2, we have found that both actions in their mixed strategy give the same payout. They are indifferent between their strategies and have no reason to deviate.

Problem #4: Graphing Payouts When Strategies are Mixed, 16 pts

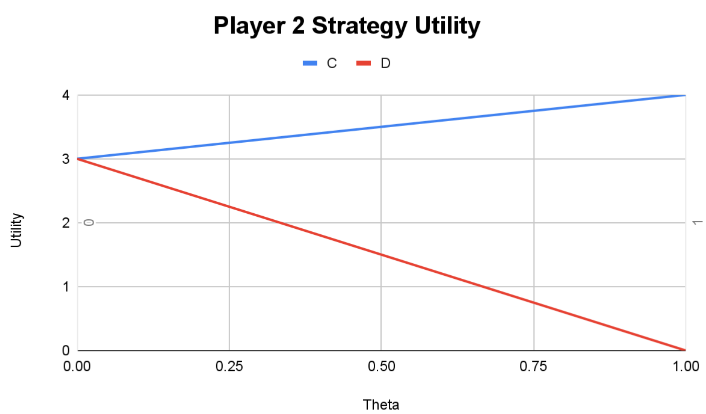
		Player 2	
		C	D
Player 1	A	(4, 3)	(1, 3)
	B	(2, 4)	(4, 0)

I. (8 pts) Graph both players' pure strategy payouts if the other uses a mixed strategy.

$U_1(s_1, (1-\sigma)C + \sigma D) \quad \forall s_1 \in \{(A), (B)\}$



$U_2((1-\theta)A + \theta B, s_2) \quad \forall s_2 \in \{(C), (D)\}$



II. (8 pts) Define a strictly dominated strategy and explain why we can eliminate these strategies when we are looking for a Nash equilibrium.

Strategy A is said to be strictly dominated by Strategy B if Strategy B always provides a higher payoff (not equal) than Strategy A for all the other player's strategies.

In any strategy profile in which a strictly dominated strategy is played (A), the player using that strategy would be better off if they unilaterally deviate to the dominating strategy (B). Therefore, a strictly dominated strategy will never be played in a NE, and we can safely eliminate it.

Problem #5: Mixed-Strategy Nash Equilibrium, 24 points

		Player 2	
		T	Q
Player 1	T	(4, 3)	(1, 3)
	Q	(2, 4)	(4, 0)

a. Find all Nash equilibria.

(8pts)

Step 1: No strictly dominated strategies

Step 2: Define Strategies and Graph Pure Strategy Payouts

Player 1: $(1 - \theta)T + \theta Q$

Player 2: $(1 - \sigma)T + \sigma Q$

Step 3: Theorem 1

Utility for Player 1

$U_1(T, (1 - \sigma)T + \sigma Q) = (1 - \sigma)U_1(T, T) + \sigma U_1(T, Q) = (1 - \sigma)4 + \sigma 1 = 4 - 3\sigma$

$U_1(Q, (1 - \sigma)T + \sigma Q) = (1 - \sigma)U_1(Q, T) + \sigma U_1(Q, Q) = (1 - \sigma)2 + \sigma 4 = 2 + 2\sigma$

Solve for values of sigma such that these utilities fulfill Theorem 1.

$4 - 3\sigma = 2 + 2\sigma$

$2 = 5\sigma$

$\sigma = 2/5$

Utility for Player 2

$U_2((1 - \theta)T + \theta Q, T) = (1 - \theta)U_2(T, T) + \theta U_2(Q, T) = (1 - \theta)3 + \theta 4 = 3 + \theta$

$U_2((1 - \theta)T + \theta Q, Q) = (1 - \theta)U_2(T, Q) + \theta U_2(Q, Q) = (1 - \theta)3 + \theta 0 = 3 - 3\theta$

Solve for values of theta such that these utilities fulfill Theorem 1.

$3 + \theta = 3 - 3\theta$

$\theta = 0$

- Students may notice that $\theta = 0$ from the matrix, this is fine but some explanation must be given.

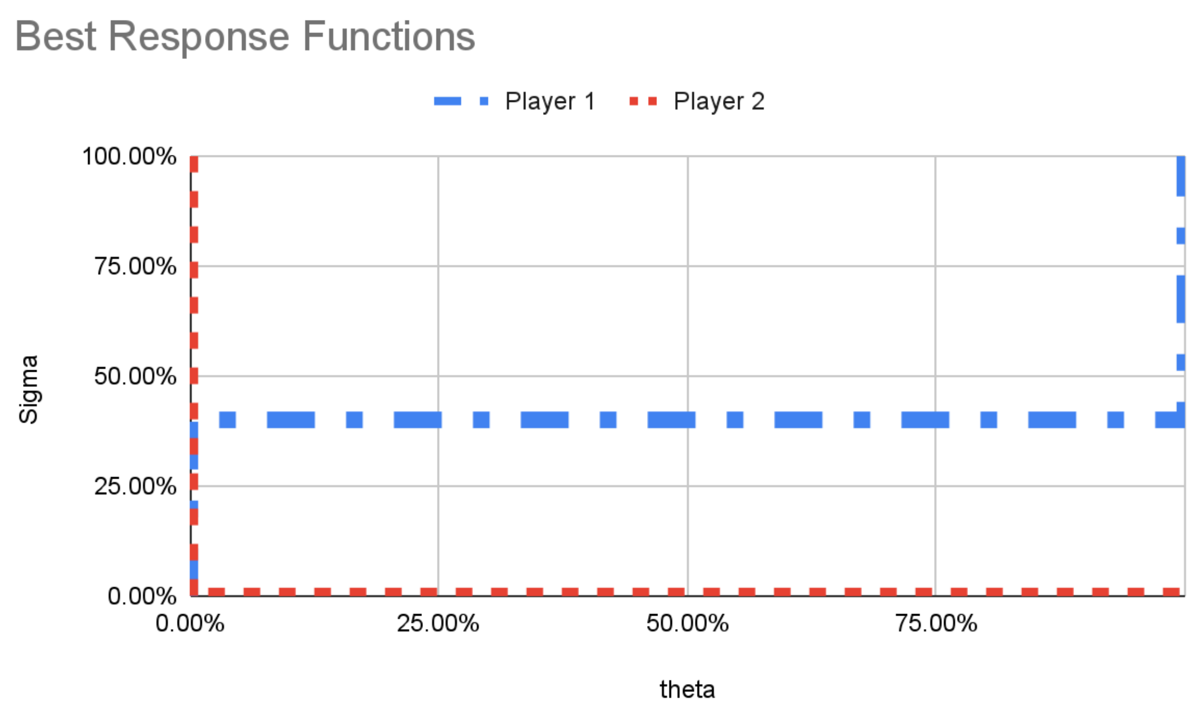
Step 4 (8pts) Define Best Response functions
Strategies
Player 1: $(1 - \theta)T + \theta Q$
Player 2: $(1 - \sigma)T + \sigma Q$

$BR_1(\sigma)$=

$$\theta = \begin{cases} 0 & when\, \sigma \in [0, 2/5) \\ [0, 1] & when\, \sigma = 2/5 \\ 1 & when\, \sigma \in (2/5, 1] \end{cases}$$

$BR_2(\theta)$=

$$\sigma = \begin{cases} [0, 1] & when\, \theta = 0 \\ 0 & when\, \theta \in (0, 1] \end{cases}$$



NE (4pts):
(T, T)
$(T,\ 3/5T + 2/5Q)$
$(T, (1 - \sigma)T + \sigma Q\ \ where\, \sigma \in (0, 2/5)$

b. (4pts) Explain how you know what you just found is a Nash equilibrium
When we graph the best response functions, we know that the area they overlap is where both players are best responding to each other. This is the definition of a Nash equilibrium.

Problem #6: States of Nature, 16 points

Good (p)

		Player 2	
		T	Q
Player 1	T	(2,2)	(0,0)
	Q	(0,0)	(3,3)

Bad (1-p)

		Player 2	
		T	Q
Player 1	T	(2,2)	(4,0)
	Q	(0,4)	(3,3)

a) Assume that *p* is unknown. Combine the two-game tables into one table containing player 1's and player 2's expected payoffs.
(8pts)
Utility vector: (player 1, player 2)
U(T, T, p) = p U(T, T, good) + (1-p) U(T, T, bad)
= p (2, 2) + (1-p) (2, 2) = (2, 2)

U(T, Q, p) = p U(T, Q, good) +(1-p) U(T, Q, bad)
= p(0,0 + (1-p) (4, 0) = (4-4p, 0)

U(T, Q, p) = p U(T, Q, good) +(1-p) U(T, Q, bad)
= p(0,0 + (1-p) (4, 0) = (0, 4-4p)

U(Q, Q, p) = p U(Q, Q, good) + (1-p) U(Q, Q, bad)
= p (3, 3) + (1-p) (3, 3) = (3, 3)

		Player 2	
		T	Q
Player 1	T	(2,2)	(4-4p, 0)
	Q	(0, 4-4p)	(3,3)

b) Based on your answer to b), for what values of *p* is (Quiet, Quiet) a Nash equilibrium?
(8pts)
player 1:
$U_1(Q, Q) > U_1(T, Q)$
3> 4-4p
p>1/4

player 2:
$U_2(Q, Q) > U_2(Q, T)$
3> 4-4p
p>1/4

Problem #7: Pure Strategy Bayesian Nash Equilibrium, 16pts

Suppose player 1 knows there exists a probability distribution over states with some unknown value p. Player 2 knows which state is going to be realized.

Player 1 knows there exists a probability distribution over states with some unknown value p.

Player 2 knows which state is going to be realized.
What are the possible pure strategy BNEs?
Dr. Wu (University at Oregon)

Good (p)

		Player 2: C	Player 2: D
Player 1	A	(2,1)	(0,0)
	B	(0,0)	(1,2)

Bad (1-p)

	Player 2: C	Player 2: D
A	(2,0)	(0,2)
B	(0,1)	(1,0)

Uninformed PLAYER 1 strategies: {(A),(B)}
Informed PLAYER 2 strategies:{(CC),(CD),(DD),(DC)}

a. (8pts) What are the possible pure strategy BNEs?
Step 1: suppose player 1 chooses B
Step 2: informed players best response: $BR_2(B) = (DC)$
Step 3
Expected utility for player1 strategies (A) and (B)

$$U_1(A, DC) = pU_1(A, D, Good) + (1-p)U_1(A, C, Bad)$$
$$p * 0 + (1-p) * 2 = 2 - 2p$$

$$U_1(B, DC) = pU_1(B, D, Good) + (1-p)U_1(B, C, Bad)$$
$$p * 1 + (1-p) * 0 = p$$

We chose B to start, In order for the profile (B,DC) to be a BNE, B must be the best response to the best response of B. ie $BR_1(BR_2(B)) = B$. This means B must provide a higher utility.

$U_1(A, DC) < U_1(B, DC)$
2-2p<p
$2 < 3p$
$2/3 < p$

Now (B,DC) is a BNE when $p > 2/3$

Step 1: suppose player 1 chooses A
Step 2: informed players best response: $BR_2(A) = (CD)$

Step 3
Expected utility for player1 strategies (A) and (B)

$U_1(A, CC) = pU_1(A, C, Good) + (1 - p)U_1(A, D, Bad)$

$p * 2 + (1 - p) * 0 = 2p$

$U_1(B, CC) = pU_1(B, C, Good) + (1 - p)U_1(B, D, Bad)$

$p * 0 + (1 - p) * 1 = 1 - p$

We chose A to start, In order for the profile (A,DC) to be a BNE, A must be the best response to the best response of A. ie $BR_1(BR_2(A)) = A$. This means A must provide a higher utility.

$U_1(A, CD) > U_1(B, CD)$

$2p > 1 - p$

$3p$>1

$p > 1/3$

(A,DC) is a BNE if $p > 1/3$

b. (8pts) Define a Bayesian Nash equilibrium and explain why what you just found qualifies.
Given the available information, all players are best responding to their opponent's strategy.

We went through and found a value for p in which player 1 is best responding to player 2's best response. Therefore, we know both players are best responding.