Game Theory

Game theory extends the framework of strategic choice to deal with multiple decision-makers.

Necessary things for a game

- Players
- Strategies
- The payoffs (only used to solve the game, not the answer)
- Strategy Profile (the outcomes)

Definitions

Players

- We will be playing games with strictly two players for the majority of the class

Strategy

Def. an information-contingent plan of action:

- that is, it defines what a player would do any time they would have to make a decision under any possible circumstances.
- Generally, the set of strategies is not the same as the set of actions.

The payoffs (only used to solve the game, not the answer)

- Von Neumann-Morgenstern expected utility function
  $u(L) = p_1 * u_1 + p_2 * u_2 + \ldots + p_n * u_n$
- More is always better

Strategy Profile (Solution)

Def. a list of strategies, one for each player.

Assumptions

Players are rational.
This means that:

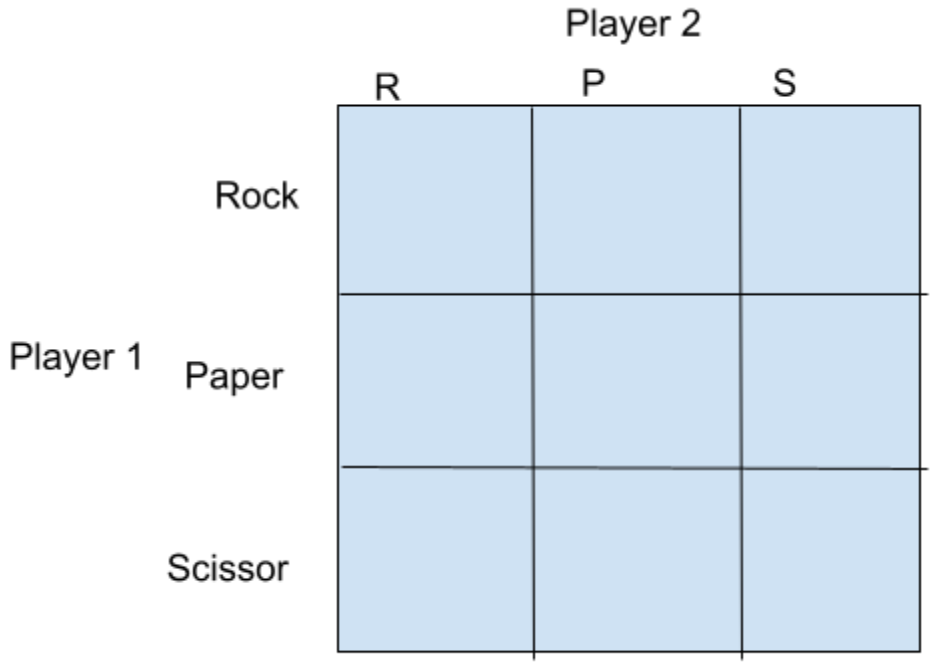
- Each player acts to maximize their own payoff (or expected payoff) and is capable of choosing the best course of action without error.
- Each player's payoffs remain consistent over the course of the game. That is, they do not change their minds about what they want partway through the game.

Common Knowledge of Rationality
We will also assume that, not only is each player rational, but they also know that the other player is rational...And that each player knows that the other player knows they are rational...And that each player knows that the other player knows that they know the other players rational...Etc.

Example: Rock-Paper-Scissors

Define the following:

- Players
- Strategies



Find the following with the class:

- Strategy Profiles
- The payoffs: Can we put payoffs in the matrix that make sense?

Example: Prisoner’s Dilemma

There are two criminals, who have each committed murder, but the police have no way of proving it. However, the police can prove that they have lied on their taxes, which is a much lesser offense.

The police get the criminals into separate rooms so that they can't talk, and offer them a choice: Testify against the other, or stay Quiet. If they Testify, they will receive lenient sentencing, and the police will be able to convict the other criminal. If they both keep quiet, they are convicted of tax evasion and receive short prison sentences. If they both testify, then both are convicted of tax evasion and murder, but only receive medium prison sentences due to their cooperation.

Find the following with the class:

- Players
- Strategies
- Strategy Profiles
- The payoffs: Can we put payoffs in the matrix that make sense?

Analyzing the Game

Strictly Dominated Strategy

Strategy A is said to be strictly dominated by strategy B if, for any strategy played by the other player, strategy B always provides a **higher** payoff (not equal) than strategy A.

- Find any strictly dominated strategies in the previous two examples.

Best Response

Given a specific strategy chosen by the other player, the set of best strategies that give the highest payout. Notice multiple strategies may give this highest payout.

- Circle the best responses for each player in the previous two examples.

Nash Equilibrium

A strategy profile in which each player is best responding to the strategies of all other players.

- Find all the pure Nash Equilibrium in the previous two examples.