Review

Solving for unknown variables: Solve for conditions on x and y st (A, C) is a NE.

Player 2

Player 1	C	D
A	(x,y)	(9,10)
B	(13,6)	(6,7)

Introduction

Up to this point, we have assumed players know all the relevant information about each other and their payoffs. These types of games are known as games of complete information. This assumption is very strong. In reality, players have incomplete information.

Nature

The most basic way to model uncertainty is to add a player: Nature. Nature does not receive a payoff. We use a probability distribution for choices made by nature.

Good (p)

Player 1 \ Player 2	T	Q
T	(1,0)	(0,0)
Q	(1,1)	(0,1)

Bad (1-p)

Player 1 \ Player 2	T	Q
T	(0,1)	(1,1)
Q	(0,0)	(1,0)

Combine this game table in terms of p using vector multiplication.

Bayesian Nash Equilibria

Given the available information, all players are best responding to their opponent's strategy.

Informed Player

The player knows the state of nature or can choose an action contingent on the state of nature.

Uninformed Player

The player doesn’t know the state of nature or can choose only a single action for both states.

Find Bayesian Nash Equilibria

Steps to find BNE when we have an informed and uninformed agent.

1. Choose a strategy for the uninformed player. s_u
2. Find the best response for the informed player. $BR_i(s_u)$
3. Calculate the expected utility for each of the uninformed player's strategies. The strategy or strategies with the highest payoff are the informed player's best response.
   $BR_u(BR_i(s_u))$
4. If the strategy you picked in Step 1 is equal to the best response then you have found a BNE.
   $s_u = BR_u(BR_i(s_u))$

Repeat these steps for all uniformed player's strategies.

Example 1

Player 1 knows which state is going to be realized. Player 2 knows the probability distribution over states. Given that p = ½, what are all the BNE?

Good (p)

Player 1 \ Player 2	T	Q
T	(-15,-15)	(0,-30)
Q	(-30,0)	(-2,-2)

Bad (1-p)

Player 1 \ Player 2	T	Q
T	(-50,-50)	(-30,-30)
Q	(-30,-30)	(-2,-2)

For all $s_2 \in \{(T), (Q)\}$ we'll follow the steps outlined above.

1. Player 2 chooses Testify.
   s_2 = (Testify).
2. Player 1's best response to testify when Vito is nice is to Testify. Player 1's best response to testify when Vito is nasty is to remain Quiet. Therefore player 1's strategy (TQ) is the best response to player 2's strategy (T)
   $BR_1(Testify)$= TQ
3. Now we calculate the expected utility of Luca's strategies using the distribution over the states.
   $U_2(TQ, T) = p * U_2(T, T, Nice) + (1-p) * U_2(Q, T, Nasty)$
   =p(-15) + (1-p)(-30)= 15p-30
   $U_2(TQ, Q) = p * U_2(T, Q, Nice) + (1-p) * U_2(Q, Q, Nasty)$
   =p (-30) + $(1-p)$(-2) = $-2-28p$
4. (TQ,T) is a BNE if $U_2(TQ, T)$ > $U_2(TQ, Q)$, lets check.
   $U_2(TQ, T)$ =15p-30
   $U_2(TQ, Q) = -2-28p$
   15p-30 > $-2-28p$?
   43p > 28 ?
   p>28/43 ?
   No, p = ½. But when is (TQ, T) a BNE?

   We can conclude: $U_2(TQ, T)$ < $U_2(TQ, Q)$
   Quiet = $BR_2(BR_1(Testify))$
   Therefore (TQ,T) is not a BNE.

Repeat steps for player 2 chooses Quiet.

1. Luca chooses Quiet.
   s_2 = (Quiet).
2. Guido's best response to testify when Vito is nice is to Testify. Guido's best response to testify when Vito is nasty is to remain Quiet. Therefore Guido’s strategy (TQ) is the best response to Luca’s strategy (T)
   $BR_1(Quiet)$= TQ
3. Now we calculate the expected utility of Luca’s strategies using the distribution over the states.
4. $U_2(TQ, T)$ < $U_2(TQ, Q)$
   Quiet = $BR_2(BR_1(Testify))$

Therefore (TQ, Q) is a BNE.