Mixed Strategy

Intro
In a single-move simultaneous game, can players mix between strategies?
Not really; they must choose a single strategy to play.
However, they can choose a mixed strategy that puts a positive probability on multiple pure strategies.

They then calculate the expected utility using the probability they have assigned to each pure strategy and the utility from that strategy. (See Von Neumann-Morgenstern expected utility function in 01 Background lecture)

Definition
Given players i's pure strategy set S_i, a mixed strategy for player i, $L_i: S_i \rightarrow [0, 1]$, assigns to each pure strategy $s_i \in S_i$ a probability $L_i(s_i) \geq 0$ that it will be played, where $\sum_{s_i \in S_i} L_i(s_i) = 1$

- Now we can define a profile of mixed strategies for players i's opponents.
  $L_{-i} = [L_1, L_2, \ldots, L_{i-1}, L_{i+1}, \ldots L_n]$

- Player i's set of all mixed strategies is $L(S_i)$

Mixed strategy profile
L = (L_i, L_{-i})

Examples of mixed strategy
0.5 R + 0.5 L

Method for Verifying Mixed Strategy Nash Equilibrium

Theorem 1

A mixed strategy profile L^* is a Nash equilibrium if and only if, for each player i, given other players' mixed strategy ${L^*}_{-i}$, the following conditions are met.

A. All actions that ${L^*}_{i}$ assigns positive probability produce the same expected payoff

B. Every action that ${L^*}_{i}$ assigns zero probability produces an expected payoff less than or equal to the expected payoff of any action that ${L^*}_{i}$ assigns positive probability.

Example: Matching Pennies

		Player 2	
		H	T
Player 1	H	(-1,1)	(1,-1)
	T	(1,-1)	(-1,1)

Let us verify that (½H + ½T, ½H + ½T) is indeed a Nash equilibrium in the matching Pennies game

Given player 2's mixed strategy, payoff to player 1's pure strategies are:
U(H,½H + ½T) = ½ *-1 + ½ *1 = 0
U(T,½H + ½T) = ½ *1 + ½ *-1 = 0

Given player 1's mixed strategy, payoff to player 2's pure strategies are:
U(½H + ½T,H) = ½ *1 + ½ *-1 = 0
U(T,½H + ½T,T) = ½ *-1 + ½ *1 = 0

We see that the two conditions in the theorem are met: given player 2's mixed strategy ½H + ½T), player 1's two actions H and T yields him the same payoff, so condition (A) is satisfied; Condition (B) is also satisfied trivially because there is not an action that receives zero probability. The conditions also hold for player 2. Therefore the strategy profile is a Nash equilibrium.

Additional Example

		Player 2		
		L	C	R
	T	(3,2)	(3,3)	(1,1)
Player 1	M	(2,1)	(0,3)	(2,2)
	B	(3,4)	(5,1)	(0,7)

Verify the strategy profile: (¾ T+ ¼B, ⅓C + ⅔R) is a Nash equilibria

Player 1

$U_1(T, \frac{1}{3}C\ +\ \frac{2}{3}R)$ = 0 + 1 + ⅔ = 5/3

$U_1(M, \frac{1}{3}C\ +\ \frac{2}{3}R)$ = 0 + 0 + ⅘ = 4/5

$U_1(B, \frac{1}{3}C\ +\ \frac{2}{3}R)$ = 0 + 5/3 + 0 = 5/3

Player 2

$U_1(\frac{3}{4}T\ +\ \frac{1}{4}B, L)$ = 6/4 + 0+ 1 = 2.5

$U_1(\frac{3}{4}T\ +\ \frac{1}{4}B, C)$ = 9/4 + 0+ ¼ = 2.5

$U_1(\frac{3}{4}T\ +\ \frac{1}{4}B, R)$ = ¾ + 0+ 7/4 = 2.5

Conditions for Nash equilibrium are satisfied.