Sequential Bayesian Game

What if one player knows the state of nature but the other doesn't?

- The **informed player** may choose to take different actions depending on the state of nature.
- The **uninformed player** is forced to take the same action in both states of nature since they can't tell the states apart. However, they can observe the opponent's move which leads us to the idea of information sets.

Information Sets

Since the uninformed player observes the opponent's strategy, but not the state of nature. We group their information sets by the opponent's strategies.

Separating equilibrium

- The informed player moves first and picks a different strategy for each state of nature. This is called a **signaling strategy**.
- The uninformed player observes the move made by the informed player and then chooses an action. They do not see the state of nature. But can correctly infer the state of nature given the informed player is using a separating equilibrium.
- All players are best responding to their opponent's strategy given the available information.

Finding Separating Equilibrium

To search for separating equilibria, we will follow these steps:

1. Pick a signaling strategy for the **informed** player.
2. Find the uninformed player's best response to that strategy.
3. Use these best responses to find the informed player's payoffs in each state of nature and check that they could not get a larger payoff by changing their move in any state.
4. If, in step 3, the informed player could not get any larger payoffs, this is a separating equilibrium.

To find all separating equilibria, we must do this for every possible separating strategy.

Example

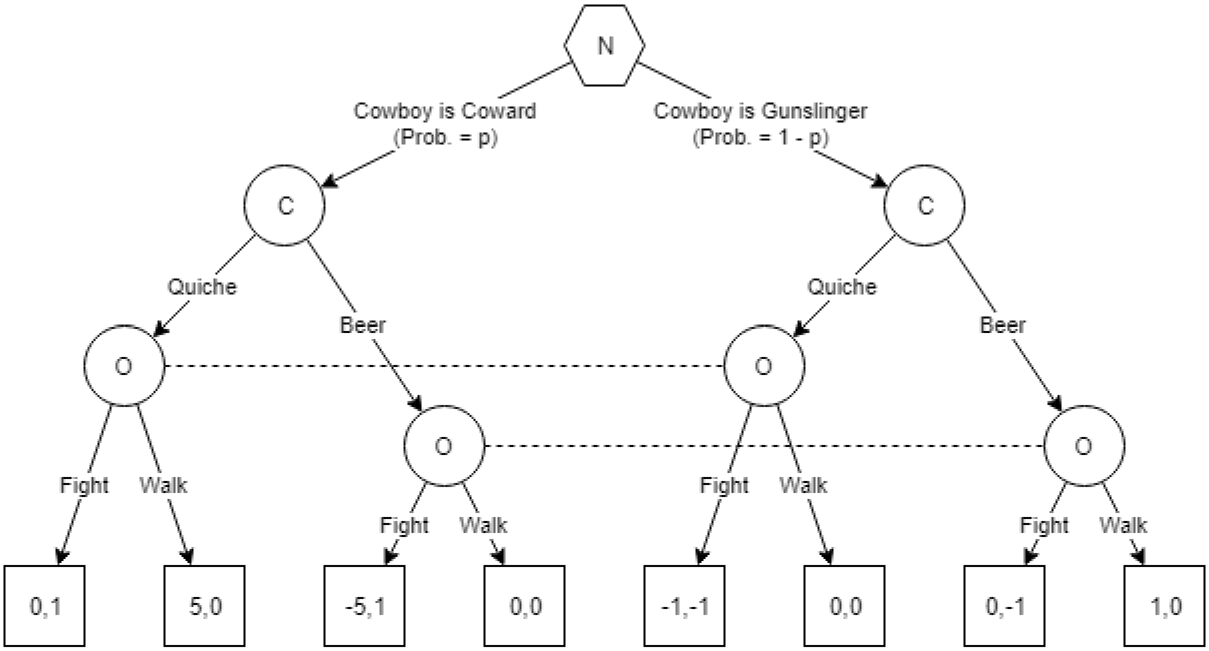
In this game, the players are a Cowboy who wanders into a town in the Wild West, and an Outlaw who terrorizes the town. The Cowboy may be either a Coward or a Gunslinger; the probability of each state is 0.5 (i.e. $p = 0.5$). The Cowboy knows which type they are but the Outlaw does not. The Cowboy's payoff is listed first and the Outlaw's second in each terminal node.

The Cowboy wants breakfast, but the hotel serves only two things for breakfast: Quiche and Beer.

The Outlaw enjoys picking on the weak, and would like to pick a fight with a Cowboy who is a Coward, but is not actually a good shot and would lose a fight to a Gunslinger.

The Outlaw can observe what the Cowboy orders for breakfast before deciding whether to start a Fight or Walk away.

1. What are the information sets of the outlaw?
   Cowboy plays Beer and Cowboy plays Quiche
2. How do we show this in sequential form?



3. What are the strategies for each player?

   Cowboy: (action|Coward action|Gunslinger) {(QQ),(QB),(BQ),(BB)}
   Outlaw: (action|Quiche action|Beer){(FF),(FW),(WF),(WW)}
   a. What do these strategies represent?
      Cowboy:(QB) (Quiche|Coward Beer|Gunslinger)
      i. the Cowboy will order Quiche (Q) if they are a Coward
      ii. the Cowboy will order Beer (B) if they are a Gunslinger
      Outlaw:(FW) (Fight|Quiche Walk|Beer)
      iii. the Outlaw will Fight (F) is they observe the gunslinger eating Quiche
      iv. the Outlaw will Walk (W) is they observe the gunslinger drinking Beer
   b. What is the information contingent plan?
      The Outlaw's information set is what the Cowboy is eating or drinking. The Cowboy's information is if they are a Coward or a Gunslinger.

4. Which of those strategies are signaling strategies?
   Cowboy:{(QB),(BQ)}
   a. How does the Outlaw interpret what he observes under each of these strategies?
      (QB): if the Cowboy chooses Quiche they are a coward
      If the Cowboy chooses Beer they are a gunslinger

   b. (BQ): if the Cowboy chooses Quiche they are a gunslinger
      If the Cowboy chooses Beer they are a coward

Finding Separating Equilibrium

1. QB
2. $BR_o(QB)$ = Fight | Quiche and Walk | Beer
3. $BR_c(FW)$ ={QB, BB}
4. We found a separating equilibrium (QB, FW) in which the cowboy's strategy is (Quiche|Coward Beer|Gunslinger) and the outlaw's strategy is Fight | Quiche and Walk | Beer

1. BQ
2. $BR_o(BQ)$ = Walk | Quiche and Fight | Beer
3. $BR_c(WF)$ ={QB}
4. We have not found a separating equilibrium