Homework 4 (70 pts)

An answer that shows understanding with some mistakes will be given partial credit, decided by the grader. A well-written response should receive more than 25%. Show all your work. Answers without work are not guaranteed credit.

Formatting (6 pts)

Single file, in pdf format, etc.

Problem #1: Definitions (5 pts)

a. Define future discounting

This is the process of converting a value received in a future period to an equivalent value received immediately.

a. Why, for every value of p, is the Nash bargaining solution a Nash equilibrium?

First, we consider each player's strategies that are available in this setting: {accept the deal, reservation wage}

Accepting the deal (Nash bargaining solution) is always at least as good as the reservation wage, therefore there is no reason to deviate from accepting the deal for any value of p and accept, accept is always a nash equilibrium.

Problem #2: Principal-Agent Problem (12pts)

Assume the role of the head of the Human resources department at a large corporation. You must design a contract for the next CEO. Your sources have given you the following quantitative data to help make a decision. A typical CEO has a reservation wage of $150,000. Their cost of putting out a high effort is equivalent to $50,000, while the cost of low effort is 0. They are aggressive and think only of the expected outcome; in other words, they are risk neutral: U(wage, effort) = wage - c(e).

1. What contract should be offered so that the CEO will put in a high level of effort if you assume the low-level effort contract is a base salary equal to the reservation wage?

   Contract structure for high effort: Salary + Bonus.
   The salary is guaranteed, and a bonus is given if the project succeeds.

   E(U, high) = p(success, high) * u(success, high) + p(failure, high) * u(failure, high)

   $= 0.7 * (s + b - 50,000) + 0.3 * (s - 50,000)$

   $= 0.7b + s - 50,000$

   E(U, low) = p(success, low) * u(success, low) + p(failure, low) * u(failure, low)

   $= 0.3 * (s + b) + 0.7 * (s)$

   $= 0.3b + s$

   First find the bonus to induce the high effort

   E(U, high) ≥ E(U, low)

   $0.7b + s - 50,000 \geq 0.3b + s$

   $0.4b \geq 50,000$

   $b \geq 125,000$

   Second, find the base amount to convince agent to accept position

   E(U, high) ≥ r

   $0.7\,(125,000) + s - 50,000 \geq 150,000$

   $s \geq 112,500$

   The agent must be offered a contract with a base salary of $112,00 and a bonus of $125,000 to induce a high level of effort.

Now take on the role of the chief investor in the company. The revenue of a successful project is $1,000,000, and the revenue from an unsuccessful project is zero. The probability of success with high effort is 0.7. With low effort, the probability of success is 0.3.

2. What level of effort will give the highest expected profit?

Π(high effort) = p(success, high) * Π(success, high)
+ p(success, high) * Π(success, high)
$= 0.7 * (1,000,000 - 237,500) + 0.3 * (-112,500)$
= $500,000

Π(low effort) = p(success, low) * Π(success, low)
+ p(success, low) * Π(success, low)
$= 0.3 * (1,000,000 - 150,000) + 0.7 * (-150,000)$
= $150,000

Project with a high level of effort is most profitable.

Problem #3 Adverse Selection (10 pts)

Find the competitive equilibrium if types are not observable for the following labor market.

Types: $\theta_1 = 3, \theta_2 = 6, \theta_3 = 9$

Lottery: $p(\theta = \theta_1) = 1/3$, $p(\theta = \theta_2) = 1/3$, $p(\theta = \theta_3) = 1/3$

Reservation wages: $r(\theta_1) = 3, r(\theta_2) = 9, r(\theta_3) = 6$

Finding CE

1. Consider the case in which all workers accept employment
   $\Theta = \{\theta_1, \theta_2, \theta_3\}$
2. $E(\theta|\Theta)=1/3*3 + 1/3*6 + 1/3*9 = 18/3 = 6$
   If a firm offers a wage equal $E(\theta|\Theta)$, only type 1 accepts employment. Remember, we assumed agents with a reservation wage equal to the wage offered stay home.
   Therefore, no CE in which all types are employed.

1. Consider the case in which workers with the highest reservation wage are not employed.
   $\Theta = \{\theta_1, \theta_3\}$
2. $E(\theta|\Theta)=1/2*3 + 1/2*9 =12/2 = 6$
   If a firm offers a wage equal $E(\theta|\Theta)$, only type 1 accepts employment. Remember, we assumed agents with a reservation wage equal to the wage offered stay home.
   Therefore, no CE with types 1 and 3 employed

1. Consider the case in which workers with only the lowest reservation wage are employed.
   $\Theta = \{\theta_1\}$
2. $E(\theta|\Theta)= 3$
   If a firm offers a wage equal $E(\theta|\Theta)$, no types accept employment. Remember, we assumed agents with a reservation wage equal to the wage offered stay home.

Therefore, no CE with type 1 employed

Problem #4 Discount Rate (6 pts)

Using the following interest rates find the implied future discount rate, rounded to 2 decimal places.

a. 7.67% APR (current mortgage rate)
b. 3.2% APR (mortgage rate during Covid-19)
c. 24.24% APR (interest rate if I fail to pay the credit card on time)

Present value = future value * discount

a. $100 = 107.67 * \beta$
   $\beta = 0.93$
b. $100 = 103.2 * \beta$
   $\beta = 0.96$
c. $100 = 124.24 * \beta$
   $\beta = 0.8$