EC 327 Spring 2023, Final Exam, 1A

Exam Grading Policy:

- If you don't know the answer, please write “I don't know” or leave it blank, and you will receive partial credit. 25%
- If you choose to attempt a question, you will be graded on the merit of your answer.
- An answer that shows understanding with some mistakes will be given partial credit, decided by the grader. A well-written response should receive more than 25%.
- Show all your work. Answers without work are not guaranteed credit.

Name________________ ID number:__________________

Problem #1: Expected Payout, 16 pts

Suppose you and your friend Tony like to gamble, but you are tired of the same dice and coin game given in previous exams. You both decide to gamble based on events in nature. You plan to observe the weather (either sunny, rainy, or cloudy) and the temperature (cold, moderate, or hot). Assume these events are independent and decided simultaneously when you both wake up. The probability of sunny is ½, cloudy is ¼, and rain is ¼. The probability of moderate is ½, cold is ¼, and hot is ¼.

You win $3 whenever it's sunny, and the temperature is moderate or hot. For all other outcomes, you lose $1.

1. How many outcomes are there?(4) 9 {SC, SM, SH, RC, RM, RH, CC, CM, CH}
2. What is your expected payoff? (4)
   E($) = p(SC)* $(SC) + p(SM)* $(SM)+ p(SH)* $(SH)+ p(RC)* $(RC)+ p(RM)* $(RM)+ p(RH)* $(RH)+ p(CC)* $(CC)+ p(CM)* $(CM)+ p(CH)* $(CH)
   = ½ *(¼*(-1)+ ½*(3) +¼ *(3)) + ¼ *(¼*(-1)+ ½*(-1) +¼ *(-1))
   + ¼ *(¼*(-1)+ ½*(-1) +¼ *(-1))
   = ½*(-¼ + 3/2 + ¾) + ¼(-¼ + -½ -¼) + ¼(-¼ + -½ -¼)
   = ½*(8/4) + ¼*(-1) + ¼*(-1)
   = 1-½
   = ½
3. What is your opponent's expected payoff?(4)
   The opponent's payoff is negative your payoff because of the structure of the game.
   = -½,
4. Is the game in your favor? (4)
   the game is in your favor.

Problem #2: Construct Game Matrix, 20 pts

Consider the following salary negotiation. If you work for a company, the value produced is 100. The company can hire someone else and receive a payoff equal to 50. You have chosen a degree with very few alternatives, and the next best job is in the service industry, which gives you a payoff equivalent to 15. Your hiring manager is a game theorist and uses the Nash bargaining solution to decide what wage to offer. Luckily, this person is fair and would like to split the surplus 50-50 (p = ½) with you.

Simultaneous Nash bargaining solution:
Employee receives: $a + (1 - p)(v - a - b)$
Company receives: $b + p(v - a - b)$

1. Specify players and strategies (2)
   Payers: You, Hiring Manager
   Strategies:
   You: Accept the job or take the alternative.
   Hiring Manager: Make an offer or take the alternative.
2. Construct a payoff **matrix** for this game (6)
   U(offer, accept) = $(a + (1 - p)(v - a - b), b + p(v - a - b))$
   = $(15 + (1 -.5)(100 - 15 - 50), 50 +.5(100 - 15 - 50))$
   = $(15 + (.5)(35), 50 +.5(35))$
   = $(32.5, 67.5)$
   Notice that all other combinations of strategies lead to an outcome of (15,50)

Player 2

		Make offer	alternative
You	Accept offer	(32.5, 67.5)	(15,50)
	alternative	(15,50)	(15,50)

3. Find pure strategy Nash equilibrium (2)
   (accept the offer, make an offer)
4. Give both definitions of a Nash equilibrium (10)
   a. A strategy profile in which each player is best responding to the strategies of all other players.
   b. A strategy profile in which all players cannot receive a higher payout if they deviate from their strategy unilaterally.

Problem #3: Verify Nash Equilibrium, 16pts

Player 2

Player 1		T	Q
	T	(4,3)	(1,3)
	Q	(2,4)	(4,0)

1. Verify whether the following strategy profile is Nash equilibrium using Theorem 1.
   (T, $\frac{4}{5}$T+ $\frac{1}{5}$ Q)
   (4)
   Player 1
   $U_1(A, \frac{4}{5}\ C + \ \frac{1}{5}\ D) = 4/5\, U_1(A,\ C) + 1/5\, U_1\,(A, D) = 4/5 * 4 + 1/5 * 1 = 17/5$
   $U_1(B, \frac{4}{5}\ C + \ \frac{1}{5}\ D) = 4/5U_1(B,\ C) + 1/5\, U_1\,(B, D) = 4/5 * 2 + 1/5 * 4 = 12/5$

   Player 2
   $U_2(A, C) = 3$
   $U_2(A, D) = 3$

   Nash Equilbirium is verified.

2. Clearly explain your findings.
   (12)
   For player 1, we have found that the strategy not included in the potential Nash equilibrium (B) has a lower payoff. Therefore, player 1 has no incentive to deviate from A.

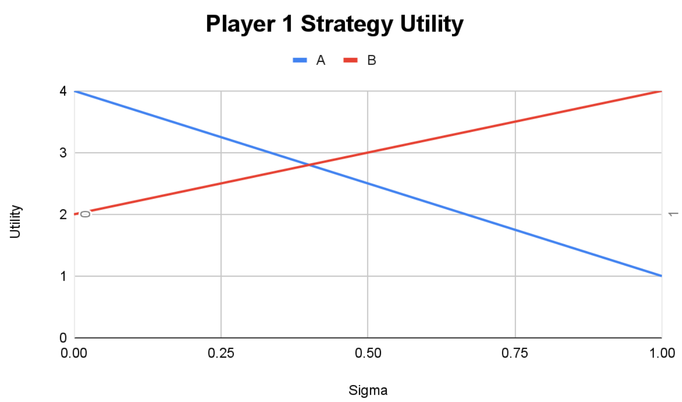
   For player 2, we have found that both actions in their mixed strategy give the same payout. They are indifferent between their strategies and have no reason to deviate.

Problem #4: Graphing Payouts When Strategies are Mixed, 16 pts

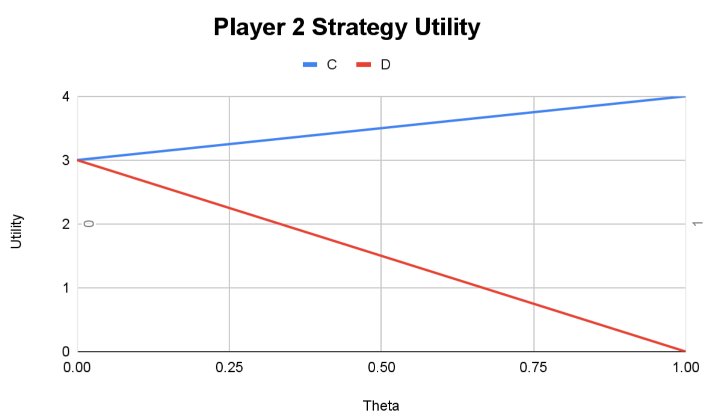
		Player 2	
		T	Q
Player 1	T	(4,3)	(1,3)
	Q	(2,4)	(4,0)

1. (8 pts) Graph both players' pure strategy payouts if the other uses a mixed strategy.

$U_1(s_1, (1 - \sigma)C + \sigma D) \qquad \forall s_1 \in \{(A), (B)\}$



$U_2((1 - \theta)A + \theta B, s_2) \qquad \forall s_2 \in \{(C), (D))\}$



2. (8 pts) Define a strictly dominated strategy and explain why we can eliminate these strategies when we are looking for a Nash equilibrium.
   Strategy A is said to be strictly dominated by Strategy B if Strategy B always provides a higher payoff (not equal) than Strategy A for all the other player's strategies.

   In any strategy profile in which a strictly dominated strategy is played (A), the player using that strategy would be better off if they unilaterally deviate to the dominating strategy (B). Therefore, a strictly dominated strategy will never be played in a NE, and we can safely eliminate it.

Problem #5: Mixed-Strategy Nash Equilibrium, 24 points

		Player 2: T	Player 2: Q
Player 1	T	(4,3)	(1,3)
	Q	(2,4)	(4,0)

(4pts)

Step 1: No strictly dominated strategies

Step 2: Define Strategies and Graph Pure Strategy Payouts

Player 1: $(1 - \theta)T + \theta Q$

Player 2: $(1 - \sigma)T + \sigma Q$

Step 3: Theorem 1

Utility for Player 1

$U_1(T, (1 - \sigma)T + \sigma Q) = (1 - \sigma)U_1(T, T) + \sigma U_1(T, Q) = (1 - \sigma)4 + \sigma 1 = 4 - 3\sigma$

$U_1(Q, (1 - \sigma)T + \sigma Q) = (1 - \sigma)U_1(Q, T) + \sigma U_1(Q, Q) = (1 - \sigma)2 + \sigma 4 = 2 + 2\sigma$

Solve for values of sigma such that these utilities fulfill Theorem 1.

$4 - 3\sigma = 2 + 2\sigma$

$2 = 5\sigma$

$\sigma = 2/5$

Utility for Player 2

$U_2((1 - \theta)T + \theta Q, T) = (1 - \theta)U_2(T, T) + \theta U_2(Q, T) = (1 - \theta)3 + \theta 4 = 3 + \theta$

$U_2((1 - \theta)T + \theta Q, Q) = (1 - \theta)U_2(T, Q) + \theta U_2(Q, Q) = (1 - \theta)3 + \theta 0 = 3 - 3\theta$

Solve for values of theta such that these utilities fulfill Theorem 1.

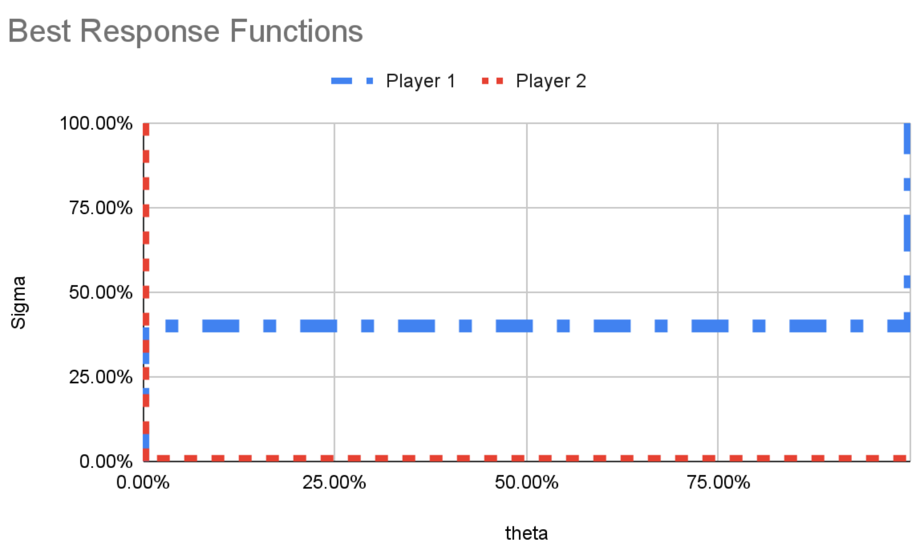
$3 + \theta = 3 - 3\theta$

$\theta = 0$

- Students may notice that $\theta = 0$ from the matrix, this is fine but some explanation must be given.

Step 4 (8pts) Define Best Response functions
Strategies
Player 1: $(1 - \theta)T + \theta Q$
Player 2: $(1 - \sigma)T + \sigma Q$

$BR_1(\sigma)$=
$\theta =$ 0 $when\,\sigma \in [0, 2/5)$
$[0, 1]$ $when\,\sigma = 2/5$
1 $when\,\sigma \in (2/5, 1]$
$BR_2(\theta)$=
$\sigma =$ $[0, 1]$ $when\,\theta = 0$
0 $when\,\theta \in (0, 1]$



NE (4pts):
$(T$, T)
$(T,\ 3/5T + 2/5Q)$
$(T, (1 - \sigma)T + \sigma Q\ \ where\,\sigma \in (0, 2/5)$

(8pts) Explain how you know what you just found is a Nash equilibrium
When we graph the best response functions, we know that the area they overlap is where both players are best responding to each other. This is the definition of a Nash equilibrium.

Problem #6: Pure Strategy Bayesian Nash Equilibrium, 16pts

Suppose player 1 knows a probability distribution over states with some unknown value p. Player 2 knows which state will be realized.

Good (p)

		Player 2: C	Player 2: D
Player 1	A	(2,1)	(0,0)
	B	(0,0)	(1,2)

Bad (1-p)

		Player 2: C	Player 2: D
Player 1	A	(2,0)	(0,2)
	B	(0,1)	(1,0)

Uninformed PLAYER 1 strategies: {(A),(B)}
Informed PLAYER 2 strategies:{(CC),(CD),(DD),(DC)}

a. (8pts) What are the possible pure strategy BNEs?

Step 1: suppose player 1 chooses B

Step 2: informed player's best response: $BR_2(B) = (DC)$

Step 3

Expected utility for player1 strategies (A) and (B)

$$U_1(A, DC) = pU_1(A, D, Good) + (1-p)U_1(A, C, Bad)$$
$$p * 0 + (1-p) * 2 = 2 - 2p$$

$$U_1(B, DC) = pU_1(B, D, Good) + (1-p)U_1(B, C, Bad)$$
$$p * 1 + (1-p) * 0 = p$$

We chose B to start, In order for the profile (B,DC) to be a BNE, B must be the best response to the best response of B. ie $BR_1(BR_2(B)) = B$. This means B must provide a higher utility.

$U_1(A, DC) < U_1(B, DC)$

2-2p<p

$2 < 3p$

$2/3 < p$

Now (B,DC) is a BNE when $p > 2/3$

Step 1: suppose player 1 chooses A

Step 2: informed players best response: $BR_2(A) = (CD)$

Step 3

Expected utility for player1 strategies (A) and (B)

$$U_1(A, CC) = pU_1(A, C, Good) + (1-p)U_1(A, D, Bad)$$
$$p * 2 + (1-p) * 0 = 2p$$

$U_1(B, CC) = pU_1(B, C, Good) + (1 - p)U_1(B, D, Bad)$
$p * 0 + (1 - p) * 1 = 1 - p$

We chose A to start, In order for the profile (A,DC) to be a BNE, A must be the best response to the best response of A. ie $BR_1(BR_2(A)) = A$. This means A must provide a higher utility.

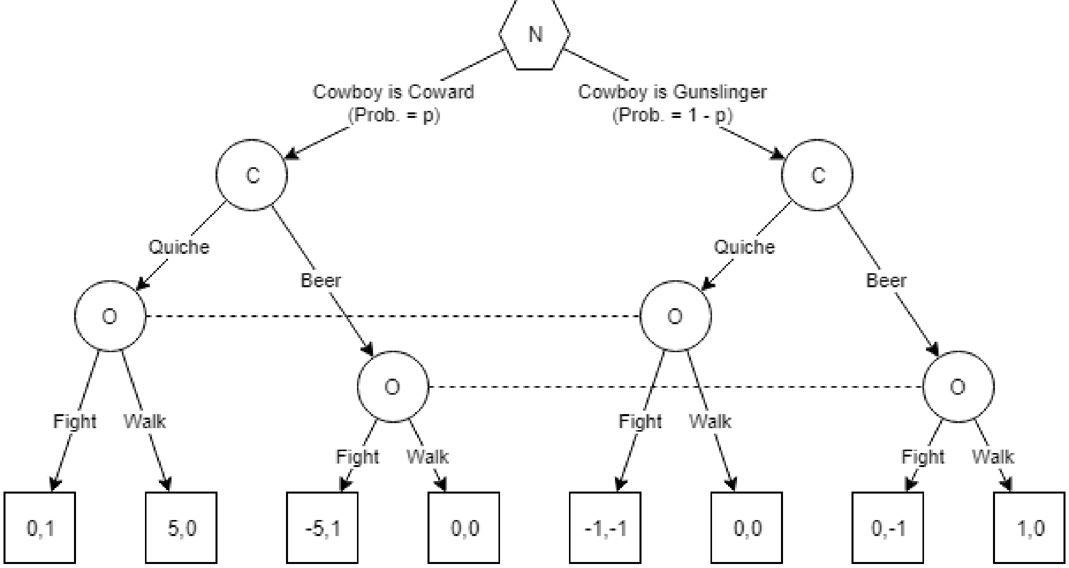
$U_1(A, CD) > U_1(B, CD)$
$2p > 1 - p$
$3p>1$
$p > 1/3$

(A,DC) is a BNE if $p > 1/3$

b. (8pts) Define a Bayesian Nash equilibrium and explain why what you just found qualifies.
Given the available information, all players are best responding to their opponent's strategy.

We went through and found a value for p in which player 1 is best responding to player 2's best response. Therefore, we know both players are best responding.

Problem #7: Separating Equilibrium, 16 pts



1. Define strategies for both players and exactly when the player does each action. 25%
   Cowboy: (action|Coward action|Gunslinger) (QQ),(QB),(BQ),(BB)
   Outlaw: (action|Quiche action|Beer)(FF),(FW),(WF),(WW)
2. Define signaling strategies. Why are these called signaling strategies? What do the signaling strategies allow us to do when finding separating Eq? 25%
   Signaling strategies are ones in which the informed player chooses a different action for each state of nature. They are called signaling strategies because they signal to the uniformed player the state of nature. Signaling strategies allow us to find the uniformed players' best response more easily.
3. Find separating equilibrium. 50%
   1. QB
   2. $BR_o(QB)$ = Fight | Quiche and Walk | Beer
   3. $BR_c(FW)$ ={QB, BB}
   4. We found a separating equilibrium (QB, FW) in which the cowboy's strategy is (Quiche|Coward Beer|Gunslinger) and the outlaw's strategy is Fight | Quiche and Walk | Beer

   1. BQ
   2. $BR_o(BQ)$ = Fight | Beer and Walk | Quiche
   3. $BR_c(WF)$ ={QB}
   4. We have not found a separating equilibrium

Problem #8: Principal-Agent Problem (20 pts)

Assume the role of the head of the Human resources department at a large corporation. You must design a contract for the next CEO. Your sources have given you the following quantitative data to help you decide. A typical CEO has a reservation wage of $150,000. Their cost of putting out a high effort is equivalent to $50,000, while the cost of low effort is 0. They are aggressive and think only of the expected outcome; in other words, they are risk neutral: U(wage, effort) = wage - c(e). The revenue of a successful project is $1,000,000, and the revenue from an unsuccessful project is zero. The probability of success with high effort is 0.7. With low effort, the probability of success is 0.3.

1. What is the pay to induce low effort? What is the contract structure for high effort (not the values)? Clearly explain how the agent understands this structure. 25%

   The pay to induce low effort is assumed to be a salary equal to the reservation wage.
   The contract structure is a salary and a bonus.
   The agent receives a salary always and a bonus if the project they oversee succeeds
2. Find high-effort contract (the values). 25%

   E(U, high) = p(success, high) * u(success, high) + p(failure, high) * u(failure, high)

   $= 0.7 * (s + b - 50,000) + 0.3 * (s - 50,000)$

   $= 0.7b + s - 50,000$

   E(U, low) = p(success, low) * u(success, low) + p(failure, low) * u(failure, low)

   $= 0.3 * (s + b) + 0.7 * (s)$

   $= 0.3b + s$

   First find the bonus to induce the high effort

   E(U, high) ≥ E(U, low)

   $0.7b + s - 50,000 \geq 0.3b + s$

   $0.4b \geq 50,000$

   $b \geq 125,000$

   Second, find the base amount to convince agent to accept position

   E(U, high) ≥ r

   $0.7\,(125,000) + s - 50,000 \geq 150,000$

   $s \geq 112,500$

   The agent must be offered a contract with a base salary of $112,00 and a bonus of $125,000 to induce a high level of effort.

3. Find expected profit for both effort levels. 50%

$\mathbf{\Pi}$(high effort) = p(success, high) * $\mathbf{\Pi}$(success, high)

+ p(success, high) * $\mathbf{\Pi}$(success, high)

$= 0.7 * (1{,}000{,}000 - 237{,}500) + 0.3 * (-112{,}500)$

= \$500,000

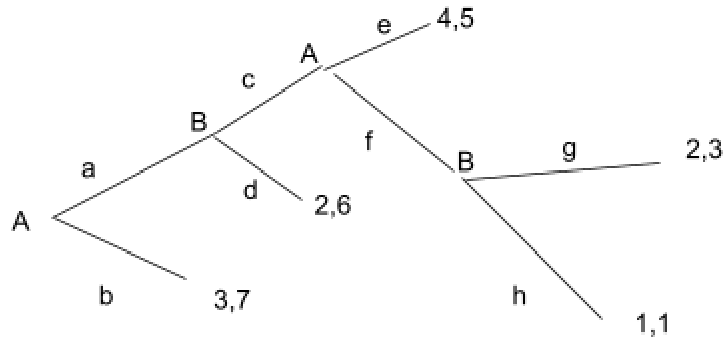
$\mathbf{\Pi}$(low effort) = p(success, low) * $\mathbf{\Pi}$(success, low)

+ p(success, low) * $\mathbf{\Pi}$(success, low)

$= 0.3 * (1{,}000{,}000 - 150{,}000) + 0.7 * (-150{,}000)$

= \$150,000

Problem #9: Backward Induction (12 pts)



1. What is a strategy? (Don't list them) 25%
   an information-contingent plan of action
2. Describe the process of backward induction. Why can we use it to find the sub-game perfect Nash equilibrium? 25%
   We start at the terminal nodes on the right and find the highest payoff for the player at that decision node. We can then move to the previous decision node; the player can now make their best choice assuming the subsequent player will make the best choice possible. We can repeat this process and find the best choice for all decision nodes. This is the subgame perfect Nash equilibrium concept: players make the optimal choice at all decision nodes, even those that are not reached.
3. What is the strategy profile of the SPNE? 50%
   (be, dg)