<u>Homework 2 (70pts)</u>
For full credit, show your work.

<u>Formatting (6pts)</u>
Single file, in pdf format.

<u>Definitions (8pts)</u>

a. Informed Player
   The player that knows the state of nature or a player that can choose an action contingent on the state of nature
b. Mixed Strategy
   A mixed strategy is a strategy that puts a positive probability on multiple pure strategies.
c. Strictly Dominated Strategy
   Strategy A is said to be strictly dominated by Strategy B if Strategy B always provides a higher payoff (not equal) than Strategy A for all the other player's strategies.
d. Explain why you wouldn't want to include a strictly dominated strategy in a mixed strategy.
   You could move any probability from the strictly dominated strategy to the strategy that dominates it and receives a strictly better payoff.

<u>Problem #1 Graphing Payouts When Strategies are Mixed (12 pts)</u>

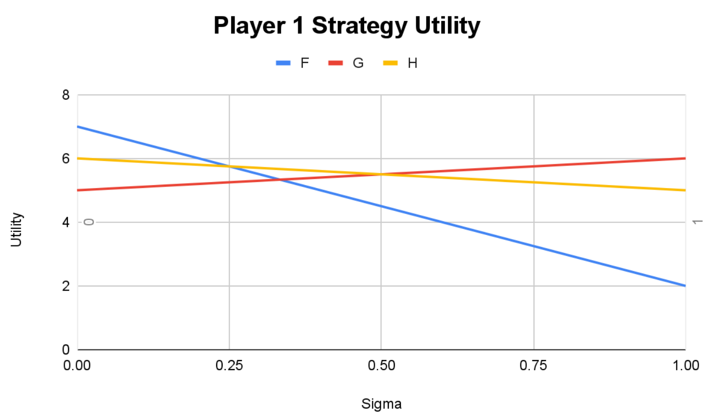
For each of the following game tables,

I. Eliminate any strictly dominated strategies for player 2. Also, state which strategy is dominated and by what.
II. Graph player 1's pure strategy payouts if player 2 uses a mixed strategy.

a)

		Player 2	
		S	T
Player 1	F	7, 3	2, 4
	G	5, 2	6, 1
	H	6, 1	5, 4

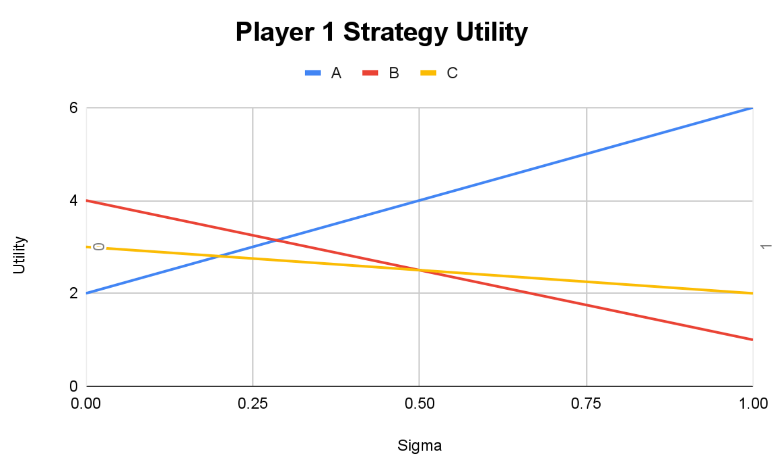
$u_1(s_1, (1-\sigma)S + \sigma T)$ $\forall s_1 \in \{(F), (G), (H)\}$



b)

		Player 2	
		X	Y
Player 1	A	2, 3	6, 1
	B	4, 2	1, 3
	C	3, 1	2, 4

$u_1(s_1, (1-\sigma)X + \sigma Y)$ $\forall s_1 \in \{(A), (B), (C)\}$

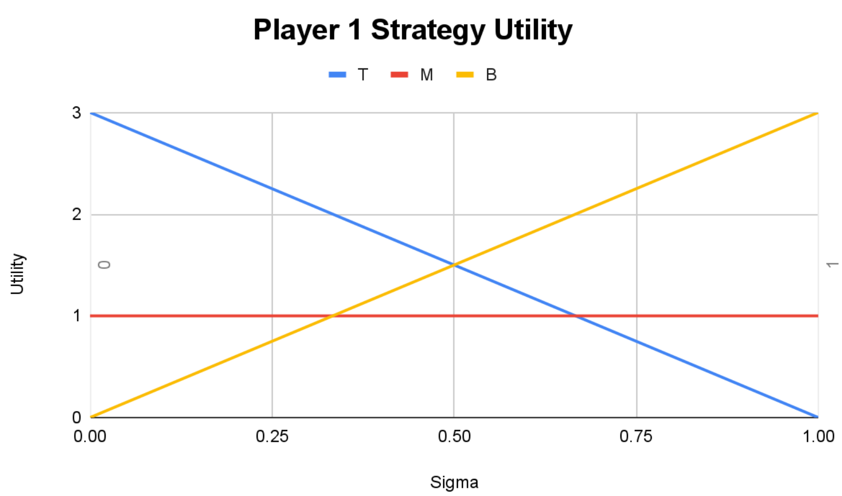


		Player 2		
		Left	Center	Right
Player 1	Top	3, 1	3, 1	0, 2
	Middle	1, 2	2, 1	1, 2
	Bottom	0, 2	3, 0	3, 1

c)

Center is strictly dominated by Right.

$u_1(s_1, (1 - \sigma)L + \sigma R)$ $\qquad \forall s_1 \in \{(T), (M), (B)\}$



Problem #2 Mixed-Strategy Nash Equilibrium (16 points)
For each of the following game tables, find all Nash equilibria.

a)

		Daffy	
		Duck	Rabbit
Bugs	Duck	-2, 1	0, 0
	Rabbit	0, 0	1, -2

Rabbit is strictly dominated by Duck for Daffy.
Duck is strictly dominated by Rabbit for Bugs.
NE:(Rabbit, Duck)

b)

		Buzz	
		Bail	Drive
Jim	Bail	0, 0	-1, 1
	Drive	1, -1	-10, -10

Step 1

Reduce the game by removing strictly dominated strategies

No strictly dominated strategies

Step 2

For each player, write down their mixed strategy.

Jim: $(1 - \theta)B + \theta D$

Buzz: $(1 - \sigma)B + \sigma D$

Step 3

For each player, write down the equations the mixed strategies must satisfy using the conditions in Theorem 1.

Utility for Jim

$U_J(B, (1 - \sigma)B + \sigma D) = (1 - \sigma)U_J(B, B) + \sigma U_J(B, D) = (1 - \sigma)0 + \sigma(-1) = -\sigma$

$U_J(D, (1 - \sigma)B + \sigma D) = (1 - \sigma)U_J(D, B) + \sigma U_J(D, D) = 1(1 - \sigma) + \sigma(-10) = 1 - 11\sigma$

Solve for values of delta such that these utilities fulfill Theorem 1. Any strategies with positive probability must have the same expected utility.

$1 - 11\sigma = -\sigma$

$10\sigma = 1$

$\sigma = 1/10$

What does this value represent?

The point on our pure strategy graph where B and D provide the same utility.

What does this tell us?

In order for Jim to mix between B and D, Buzz must play the mixed strategy:

$9/10\, B + 1/10\, D$

Utility for Buzz

$U_B((1 - \theta)B + \theta D, B) = (1 - \theta)U_B(B, B) + \theta U_B(D, B) = (1 - \theta)0 + \theta(-1) = -\theta$

$U_B((1 - \theta)B + \theta D, D) = (1 - \theta)U_B(B, D) + \theta U_B(D, D) = (1 - \theta)(1) + \theta(-10)) = 1 - 11\theta$

Solve for values of delta such that these utilities fulfill Theorem 1. Any strategies with positive probability must have the same expected utility.

$1 - 11\theta = -\theta$

$10\theta = 1$

$\theta = 1/10$

What does this value represent?

The point on our pure strategy graph where B and D provide the same utility.

What does this tell us?

In order for Buzz to mix between B and D, player 1 must play the mixed strategy:

$9/10\, B + 1/10\, D$

We can now conclude that the following mixed strategy profile is a Nash equilibrium.

$$(9/10\, B + 1/10\, D, 9/10\, B + 1/10\, D)$$

Step 4

Define best response functions. Use best response functions to find all Nash equilibrium.

Strategies

Jim: $(1 - \theta)B + \theta D$

Buzz: $(1 - \sigma)B + \sigma D$

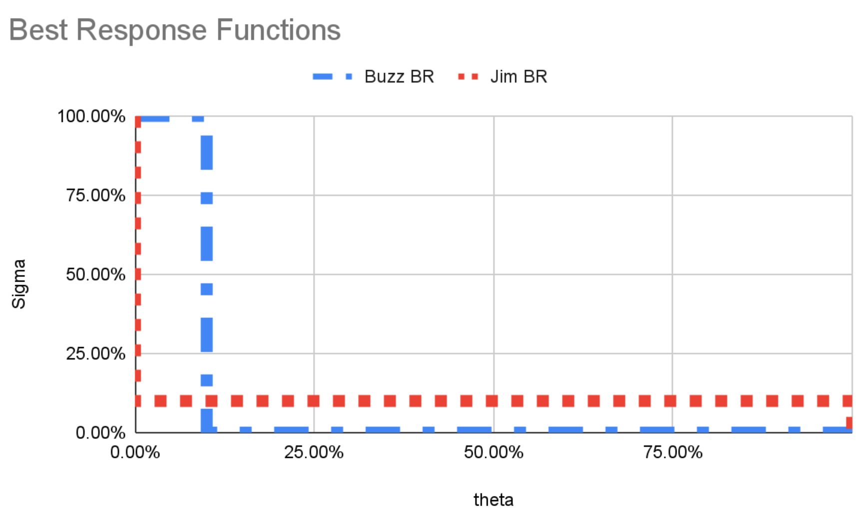
$BR_J(\sigma)$=

$$\theta = \begin{cases} 1 & when\ \sigma \in [0, 1/10) \\ [0, 1] & when\ \sigma = 1/10 \\ 0 & when\ \sigma \in (1/10, 1] \end{cases}$$

$BR_B(\theta)$=

$$\sigma = \begin{cases} 1 & when\ \theta \in [0, 1/10) \\ [0, 1] & when\ \theta = 1/10 \\ 0 & when\ \theta \in (1/10, 1] \end{cases}$$

Graph



NE: (B, D), $(9/10\,B + 1/10\,D, 9/10\,B + 1/10\,D)$, and (D, B)

Problem #3: States of nature (12 points)
Consider the following variant of the Prisoner's Dilemma: player 1 and player 2 work for a mob boss named Vito, who is unpredictable. Vito is not a player in this game. Still, when player 1 and player 2 are arrested, Vito may be Nice, with probability *p*, or Nasty, with probability 1 – *p*. Game tables for the two states are shown below:

Good (p)

		Player 2: T	Player 2: Q
Player 1	T	(-10,-10)	(0,-20)
	Q	(-20,0)	(-1,-1)

Bad (1-p)

		Player 2: T	Player 2: Q
Player 1	T	(-40,-40)	(-20,-20)
	Q	(-20,-20)	(-1,-1)

a) First, assume that *p* = 0.5, so that Vito is equally likely to be Nice or Nasty. Combine these two game tables into one table containing Guido and Luca's expected payoffs, then find all of the pure-strategy Nash equilibria in this game.

Utility vector: (player 1, player 2)

U(T, T, p = 0.5) = ½ U(T, T, nice) + ½ U(T, T, nasty)
= ½ (-10, -10) + ½ (-40, -40) = (-25,-25)

U(T, Q, p = 0.5) = ½ U(T, Q, nice) + ½ U(T, Q, nasty)
= ½ (0,-20) + ½ (-20, -20) = (-10, -20)

U(Q, T, p = 0.5) = ½ U(Q, T, nice) + ½ U(Q, T, nasty)
= ½ (-20, 0) + ½ (-20, -20) = (-20,-10)

U(Q, Q, p = 0.5) = ½ U(Q, Q, nice) + ½ U(Q, Q, nasty)
= ½ (-1, -1) + ½ (-1, -1) = (-1, -1)

		Player 2: T	Player 2: Q
Player 1	T	(-25, -25)	(-10, -20)
	Q	(-20, -10)	(-1, -1)

b) Now, assume that p is unknown. Once again, combine the two game tables into one table containing Guido and Luca's expected payoffs. **(Hint: These expected payoffs will have to be written in terms of *p*.)**

Utility vector: (player 1, player 2)

U(T, T, p) = p U(T, T, nice) + (1-p) U(T, T, nasty)
= p (-10, -10) + (1-p) (-40, -40) = (30p-40,30p-40)

U(T, Q, p) = p U(T, Q, nice) +(1-p) U(T, Q, nasty)
= p(0,-20) + (1-p) (-20, -20) = (20p-20, -20)

U(Q, T, p) = p U(Q, T, nice) + (1-p) U(Q, T, nasty)
= p (-20, 0) + (1-p) (-20, -20) = (-20,20p-20)

U(Q, Q, p) = p U(Q, Q, nice) + (1-p) U(Q, Q, nasty)
= p (-1, -1) + (1-p) (-1, -1) = (-1, -1)

		Player 2: T	Player 2: Q
Player 1	T	(30p-40, 30p-40)	(20p-20, -20)
	Q	(-20, 20p-20)	(-1, -1)

c) Based on your answer to b), for what values of p is (Quiet, Quiet) a Nash equilibrium?

player 1:

$U_1(Q, Q) > U_1(T, Q)$

-1> 20p-20

p<19/20

player 1:

$U_2(Q, Q) > U_2(Q, T)$

-1> 20p-20

p<19/20

Problem #4 Pure Strategy Bayesian Nash Equilibrium (16pts)

Suppose player 1 knows there exists a probability distribution over states with some unknown value p, and player 2 knows which state is going to be realized.

What are the possible pure strategy BNEs? Dr. Wu (University at Oregon)

		Good (p)				Bad (1-p)	
		player 2				player 2	
		C	D			C	D
player 1	A	(2,2)	(0,0)		A	(2,2)	(4,0)
	B	(0,0)	(3,3)		B	(0,4)	(3,3)

Uninformed PLAYER 1 strategies: {(A),(B)}

Informed PLAYER 2 strategies: {(CC),(CD),(DD),(DC)}

Step 1: suppose player 1 chooses A

Step 2: informed players best response: $BR_2(A) = (CC)$

		Good (p)				Bad (1-p)	
		player 2				player 2	
		C	D			C	D
player 1	A	(2,2)	(0,0)		A	(2,2)	(4,0)
	B						

Step 3

Expected utility for player1 strategies (A) and (B)

$$U_1(A, CC) = pU_1(A, C, Good) + (1 - p)U_1(A, C, Bad)$$

$$p * 2 + (1 - p) * 2 = 2$$

$$U_1(B, CC) = pU_1(B, C, Good) + (1 - p)U_1(B, C, Bad)$$

$$p * 0 + (1 - p) * 0 = 0$$

2 is always better than 0, therefore (A,CC) is a BNE for all p.

Step 1: suppose player 1 chooses B
Step 2: informed players best response: $BR_2(B) = (DC)$

		Good (p)				Bad (1-p)	
		player 2				player 2	
		C	D			C	D
player 1	A				A		
	B	(0,0)	(3,3)		B	(0,4)	(3,3)

Expected utility for player1 strategies (A) and (B)

$$U_1(A, DC) = pU_1(A, D, Good) + (1 - p)U_1(A, C, Bad)$$
$$p * 0 + (1 - p) * 2 = 2 - 2p$$

$$U_1(B, DC) = pU_1(B, D, Good) + (1 - p)U_1(B, C, Bad)$$
$$p * 3 + (1 - p) * 0 = 3p$$

We chose B to start, In order for the profile (B,DC) to be a BNE, B must be the best response to the best response of B. ie $BR_1(BR_2(B)) = B$. This means B must provide a higher utility.

$U_1(A, DC) < U_1(B, DC)$

2-2p<3p

p>⅖

Now (B,DC) is a BNE when p>⅖

To summarize
(B,DC) is a BNE when p>2/5 and (A,CC) is a BNE for all p