Homework 3 (64pts)
An answer that shows understanding with some mistakes will be given partial credit, decided by the grader. A well-written response should receive more than 25%. Show all your work. Answers without work are not guaranteed credit.

Formatting (6pts)
Single file, in pdf format, etc.

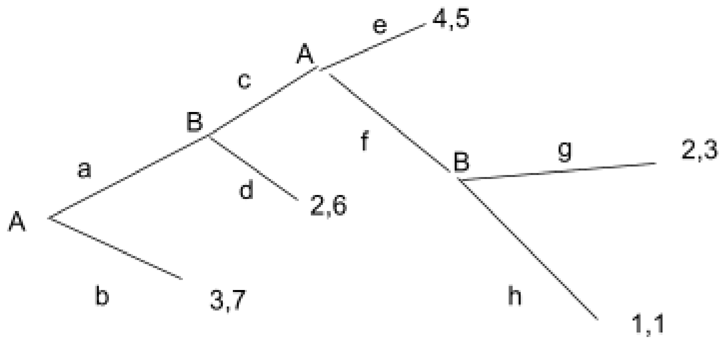
Definitions (6pts)

a. Subgame perfect Nash equilibria
   a refinement of Nash equilibria in which players must choose the best strategy in each subgame.
   Or
   a refinement of Nash equilibria in which players must choose the best response for every choice, even the ones not reached.

b. Signaling Strategy
   The informed player moves first and picks a different strategy for each state of nature.

c. Separating Equilibrium
   An equilibrium in which the informed player chooses a signaling strategy and all players are best responding to their opponent's strategy given the available information.

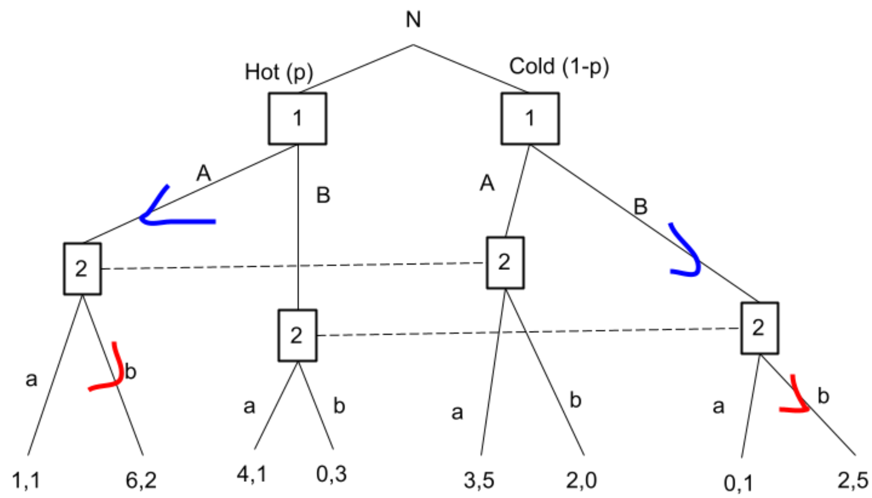
Problem #2: Backward Induction (12pts)



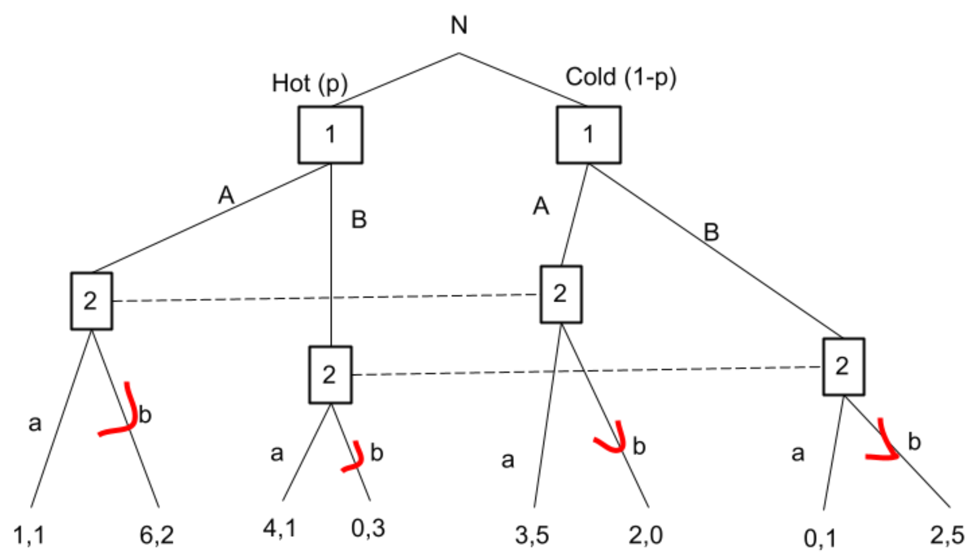
a. Find subgame-perfect Nash equilibria using backward induction.
   Notice that the players are: Player A and Player B.
   Clearly state the equilibria
   SPNE: (be , dg)
b. List all strategies for both players
   A: be, bf, ae, af
   B: dg, dh, cg, ch
c. Put in normal form

Player A \ Player B	cg	dg	ch	dh
ae	(4,5)	(2,6)	(4,5)	(2,6)
af	(2,3)	(2,6)	(1,1)	(2,6)
be	(3,7)	(3,7)	(3,7)	(3,7)
bf	(3,7)	(3,7)	(3,7)	(3,7)

Problem #2: Separating equilibria (20pts)
Let's start with (AB)

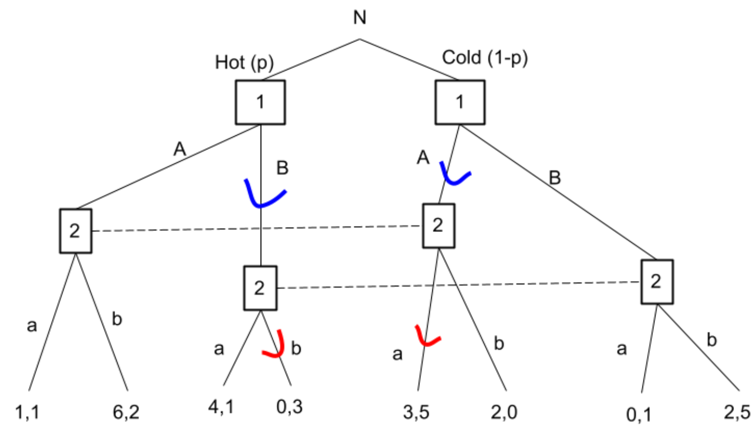


Now, player 2 can identify the state of Nature given this separating strategy. Therefore they can perfectly select their best response. which is (bb). Now, we must investigate if (AB) is the best response to (bb). Let's draw a new chart to help us.

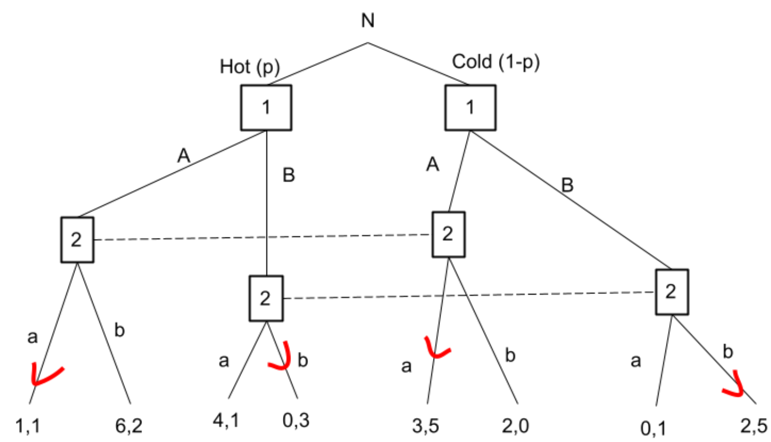


What would player 1 choose in the Hot state given players 2's choice of (bb)? (marked in red)
player 1 would prefer to choose A in the hot state because A provides 6 utility while a choice of B only provides 0.
player 1 would prefer either A or B in the cold state because both provide 2 utility.
Therefore, $BR_1(bb) = \{(AA), (AB)\}$. However we selected (AB) to start, and (bb) is the best response to (AB). ie $BR_2(AB) = bb \text{ and } BR_1(bb) = (AB)$
(AB,bb) is a separating equilibrium.

Now we will try (BA)



Now, player 2 can identify the state of Nature given this separating strategy. Therefore they can perfectly select their best response. which is (ab). Now, we must investigate if (BA) is the best response to (ba). Let's draw a new chart to help us.



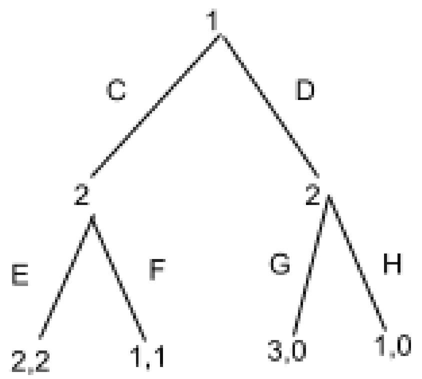
What would player 1 choose in the Hot state given players 2's choice of (ba)? (marked in red) player 1 would prefer to choose A in the hot state because A provides 1 utility while a choice of B only provides 0.

What would player 1 choose in the Cold state given players 2's choice of (ba)? (marked in red) player 1 would prefer to choose A in the hot state because A provides 3 utility while a choice of B only provides 2.

The best response to (ab) is AA. Therefore (BA,ba) is not a separating equilibrium.

Problem #3: Mixed-Strategy Subgame Perfect Nash Equilibria (20pts)

Find all subgame-perfect Nash equilibria



Notice that when we try to complete backward induction, the decision node where player 2 is choosing between G and H they are indifferent. However, we know they will choose E is their first decision node is reached.

When players are indifferent between the choices, we need to explore possible Nash equilibrium where a mixed strategy is played.

1. We know player 2 will always choose E at their first decision node. Therefore, their strategy can be expressed as such: $E\,(1-\alpha)G + \alpha H$
2. Now what are the possible choices for player 1?
   C, D, or a mix of the two: $(1-\beta)C + \beta D$
3. What's the expected utility of each strategy?
   a. $U_1(C, E\,(1-\alpha)G + \alpha H) = (1-\alpha)U_1(C, E\,G) + \alpha U_1(C, E\,H) = (1-\alpha)2 + \alpha 2 = 2$
   b. $U_1(D, E\,(1-\alpha)G + \alpha H) = (1-\alpha)U_1(D, E\,G) + \alpha U_1(D, E\,H) = (1-\alpha)3 + \alpha 1 = 3 - 2\alpha$
4. When will they choose each of these?
   a. Choose C if $U_1(C, E\,(1-\alpha)G + \alpha H) > U_1(D, E\,(1-\alpha)G + \alpha H)$
   b. Choose D if $U_1(C, E\,(1-\alpha)G + \alpha H) < U_1(D, E\,(1-\alpha)G + \alpha H)$
   c. Choose $(1-\beta)C + \beta D$ if $U_1(C, E\,(1-\alpha)G + \alpha H) = U_1(D, E\,(1-\alpha)G + \alpha H)$
5. We plug in the expected utility found in 3 into the inequalities shown in 4.
   a. $U_1(C, E\,(1-\alpha)G + \alpha H) > U_1(D, E\,(1-\alpha)G + \alpha H)$

      $2 > 3 - 2\alpha$

      $2\alpha > 1$

      $\alpha > 1/2$

Now we can say

(C, $E\,(1-\alpha)G + \alpha H$) is a subgame-perfect Nash equilibria when $\alpha > 1/2$

(D, $E\,(1-\alpha)G + \alpha H$) is a subgame-perfect Nash equilibria when $\alpha < 1/2$

($(1-\beta)C + \beta D$, $E\,(1-\alpha)G + \alpha H$) is a subgame-perfect Nash equilibrium when $\alpha = 1/2$