

## EC 327 Spring 2023, Final Exam, 1A

### Exam Grading Policy:

- If you don't know the answer, please write "I don't know" or leave it blank, and you will receive partial credit. 25%
- If you choose to attempt a question, you will be graded on the merit of your answer.
- An answer that shows understanding with some mistakes will be given partial credit, decided by the grader. A well-written response should receive more than 25%.
- Show all your work. Answers without work are not guaranteed credit.

Name\_\_\_\_\_

ID number:\_\_\_\_\_

**Problem #1: Expected Payout, 20 pts**

Suppose you and your friend Tony like to gamble, but you are tired of the same dice and coin game given in previous exams. You both decide to gamble based on events in nature. You plan to observe the weather (either sunny, rainy, or cloudy) and the temperature (cold, moderate, or hot). Assume these events are independent and decided simultaneously when you both wake up. The probability of sunny is  $\frac{1}{2}$ , cloudy is  $\frac{1}{4}$ , and rain is  $\frac{1}{4}$ . The probability of moderate is  $\frac{1}{2}$ , cold is  $\frac{1}{4}$ , and hot is  $\frac{1}{4}$ .

You win \$3 whenever it's sunny and the temperature is moderate or hot. For all other outcomes, you lose \$1.

1. How many outcomes are there?
1. What is your expected payoff?
2. What is your opponent's expected payoff? Is the game in your favor?

**Problem #2: Construct Game Matrix, 20 pts**

Consider the following salary negotiation. If you work for a company, the value produced is 100. The company can hire someone else and receive a payoff equal to 50. You have chosen a degree with very few alternatives, and the next best job is in the service industry, which gives you a payoff equivalent to 15. Your hiring manager is a game theorist and uses the Nash bargaining solution to decide what wage to offer. Luckily, this person is fair and would like to split the surplus 50-50 ( $p = \frac{1}{2}$ ) with you.

Simultaneous Nash bargaining solution:

Employee receives:  $a + (1 - p)(v - a - b)$

Company receives:  $b + p(v - a - b)$

1. Specify players and strategies
2. Construct a payoff matrix for this game
3. Find pure strategy Nash equilibrium
4. Give both definitions of a Nash equilibrium

**Problem #3: Verify Nash Equilibrium, 12pts**

		Player 2	
		T	Q
Player 1	T	(4,3)	(1,3)
	Q	(2,4)	(4,0)

1. Verify if the following strategy profile is Nash equilibrium or not using Theorem 1.  
(A,  $\frac{4}{5}$  C +  $\frac{1}{5}$  D)
2. Clearly explain your findings.

**Problem #4: Graphing Payouts When Strategies are Mixed, 12 pts**

		Player 2	
		T	Q
Player 1	T	(4,3)	(1,3)
	Q	(2,4)	(4,0)

1. Graph both players' pure strategy payouts if the other uses a mixed strategy.
2. Define a strictly dominated strategy and explain why we can eliminate these strategies when we are looking for a Nash equilibrium.

**Problem #5: Mixed-Strategy Nash Equilibrium, 24 points**

		Player 2	
		T	Q
Player 1	T	(4,3)	(1,3)
	Q	(2,4)	(4,0)

1. Find all Nash equilibria.
2. Explain how you know what you just found is a Nash equilibrium.

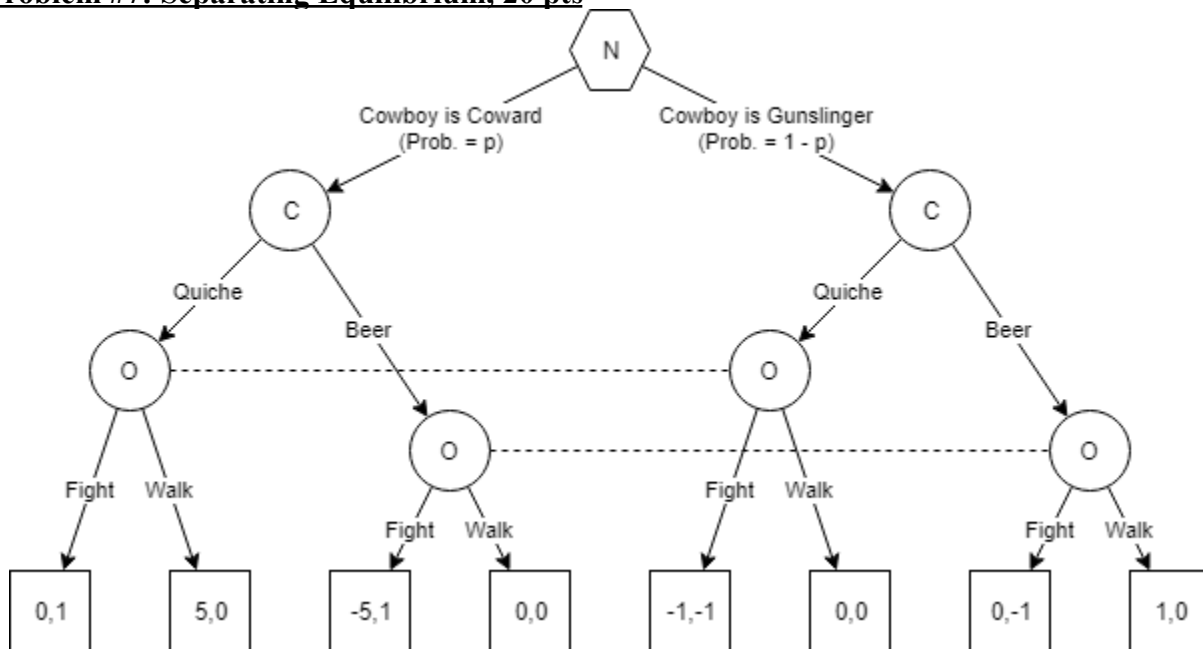
**Problem #6: Pure Strategy Bayesian Nash Equilibrium, 16pts**

Suppose player 1 knows a probability distribution over states with some unknown value  $p$ . Player 2 knows which state will be realized.

		Good ( $p$ )		Bad ( $1-p$ )	
		Player 2		Player 2	
		C	D	C	D
Player 1	A	(2,1)	(0,0)	(2,0)	(0,2)
	B	(0,0)	(1,2)	(0,1)	(1,0)

1. What are the possible pure strategy BNEs?
2. Define a Bayesian Nash equilibrium and explain why what you just found qualifies.

**Problem #7: Separating Equilibrium, 20 pts**



1. Define strategies for both players and exactly when the player does each action. 25%
2. Define signaling strategies. Why are these called signaling strategies? What do the signaling strategies allow us to do when finding separating Eq? 25%
3. Find separating equilibrium. 50%

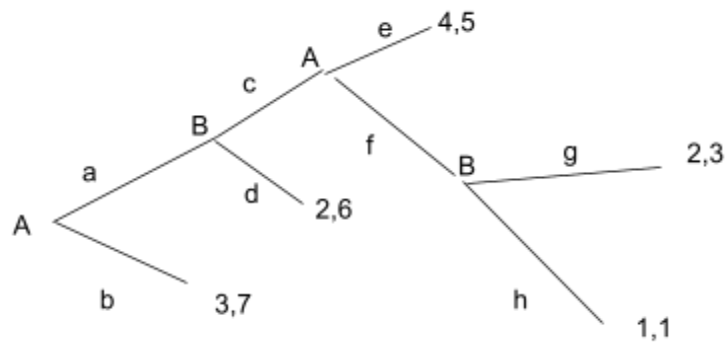


**Problem #8: Principal-Agent Problem (20 pts)**

Assume the role of the head of the Human resources department at a large corporation. You must design a contract for the next CEO. Your sources have given you the following quantitative data to help you decide. A typical CEO has a reservation wage of \$150,000. Their cost of putting out a high effort is equivalent to \$50,000, while the cost of low effort is 0. They are aggressive and think only of the expected outcome; in other words, they are risk-neutral:  $U(\text{wage}, \text{effort}) = \text{wage} - c(e)$ . The revenue of a successful project is \$1,000,000, and the revenue from an unsuccessful project is zero. The probability of success with high effort is 0.7. With low effort, the probability of success is 0.3.

1. What is the pay to induce low effort? What is the contract structure for high effort (not the values)? Clearly explain how the agent understands this structure. 25%
2. Find high-effort contract (the values). 25%
3. Find expected profit for both effort levels. 50%

**Problem #9: Backward Induction (12 pts)**



1. What is a strategy? (Don't list them) 25%
2. Describe the process of backward induction. Why can we use it to find the sub-game perfect Nash equilibrium? 25%
3. What is the strategy profile of the sub-game perfect Nash equilibrium? 50%