

Hidden Action: Moral Hazard

In prior lectures we assumed productivity was fixed. What if a worker has hidden behavior that affects productivity?

We have asymmetric information between the worker (Agent) and the owner(s) of the firm that employs them (Principal). This hidden action case is known as Moral Hazard.

Strategic Interaction

The owner of a firm is considering hiring a manager to complete a one time project. The Revenue of this project is affected by the effort of the manager. How can we write a contract to induce the appropriate level of effort?

Agent (manager)

Reservation wage r

Cost of effort

Utility function

If agent is indifferent between options assume they will do whatever is requested

Principal (owner)

Probability project succeeds, function of Manager effort

Revenue from success and Revenue from failure

Reservation Revenue, usually assume this is zero

Only considered cost is the wage paid to manager

Utility function

Contract

Incentive Compatibility Constraint: Bonus to induce a high level of effort

- Pay to induce a high level of effort is the following scheme.
- Wage = Salary + bonus = $s + b$
- Where the bonus is only paid if the project succeeds.
- $w(\text{success}) = s + b$
- $w(\text{failure}) = s$

Participation Conditions

- The Principle chooses an outcome with the highest utility
- The Agent must receive at least reservation utility, salary
 - Pay to induce a low level of effort is a salary equal to the reservation wage with no bonus.
 - $s = r$
 - $b = 0$

Simple Example

Principal

The revenue of a successful project is \$1,000,000, the revenue from an unsuccessful project is zero.

The probability of success with good supervision is 0.50 with bad supervision the probability of success is 0.25.

The owner of the firm is risk neutral $U(\text{Revenue}, \text{wage}) = \text{Revenue} - \text{wage}$

Agent

The manager has a reservation wage of \$100,000.

The cost of high effort is \$50,000, while the cost of low effort is 0.

The manager is also risk neutral $U(\text{wage}, \text{effort}) = \text{wage} - c(e)$

Find the optimal contract

Incentive Constraints

- To induce a high level of effort, the expected utility of putting in a high level of effort must be greater than or equal to the expected utility of putting in a low level of effort.

Expected utility for low effort given a salary and bonus contract

$E(U(w, \text{low}))$

$$\begin{aligned} &= p(\text{success} | \text{low}) * u(\text{success} | \text{low}) + p(\text{failure} | \text{low}) * u(\text{failure} | \text{low}) \\ &= 0.25 * (w(\text{success}) - c(\text{low})) + 0.75 * (w(\text{failure}) - c(\text{low})) \\ &= 0.25 * (s + b - 0) + 0.75 * (s - 0) \\ &= s + 0.25b \end{aligned}$$

Expected utility for high effort given a salary and bonus contract

$E(U(w, \text{high}))$

$$\begin{aligned} &= p(\text{success} | \text{high}) * u(\text{success} | \text{high}) + p(\text{failure} | \text{high}) * u(\text{failure} | \text{high}) \\ &= 0.5 * (w(\text{success}) - c(\text{high})) + 0.5 * (w(\text{failure}) - c(\text{high})) \\ &= 0.5 * (s + b - 50,000) + 0.5 * (s - 50,000) \\ &= s + 0.5b - 50,000 \end{aligned}$$

To induce a high level of effort, the expected utility must be higher for a high level of effort.

$E(U(w, \text{low})) \leq E(U(w, \text{high}))$

$$s + 0.25b \leq s + 0.5b - 50,000$$

$$50,000 \leq 0.25b$$

$$\$200,000 \leq b$$

Notice we assumed indifference would go whatever way we want. Therefore a bonus equal to \$200,000 would be enough to induce high effort. $b = \$200,000$

Participation Conditions: Agent

Expected utility from reservation wage.

$$E(U(r)) = \$100,000$$

Expected utility from high level of effort

$$E(U(w, \text{high}))$$

$$= 0.5 \cdot (s + b - 50,000) + 0.5 \cdot (s - 50,000)$$

$$= 0.5 \cdot 200,000 + s - 50,000 = s + \$50,000$$

$$E(U(r)) \leq E(U(w, \text{high}))$$

$$100,000 \leq s + 50,000$$

$$50,000 \leq s$$

Notice we assumed indifference would go whatever way we want. Therefore a salary equal to \$50,000 would be enough to induce high effort.

Now we have a complete contract to induce high effort.

$$s = \$50,000$$

$$b = \$200,000$$

Participation Conditions: Principal

Maximizes utility by choosing between: no project, project with low effort and project with high effort.

No project

$$U = 0$$

Project with low effort

$$E(U(\text{Revenue} - \text{wage}(\text{low}))) = 0.25 \cdot (1,000,000 - 100,000) + 0.75 \cdot (0 - 100,000)$$

$$= 250,000 - 100,000 = \$150,000$$

Project with high effort

$$E(U(\text{Revenue} - \text{wage}(\text{high}))) = 0.5 \cdot (1,000,000 - 250,000) + 0.5 \cdot (0 - 50,000)$$

$$= 375,000 - 25,000 = \$350,000$$

Since a project with high effort gives the highest utility to the principal. The principal offers a contract to induce high effort.

$$s = \$50,000$$

$$b = \$200,000$$