Mixed Strategy

Finding Nash equilibrium

Step 1

Reduce the game by removing strictly dominated strategies

Step 2

For each player, write down their mixed strategy.

Step 3

For each player, write down the equations the mixed strategies must satisfy using the conditions in Theorem 1.

Step 4

Define best response functions. Use best response functions to find all nash equilibrium

You and your date Michel were planning to meet at one of two restaurants (Bistro LeRoux or Pizza Hut) but your phone died before you could pick which. Michel prefers Pizza Hut and you prefer the Bistro. Though neither of you want to eat alone since eating alone would give you zero enjoyment.

a. State the players and pure strategies
b. Find all the Nash equilibrium

a. Players: Michel, You
   Strategies: (Bistro) and (Pizza)

(You, Michel)		Michel: Bistro	Michel: Pizza
You	Bistro	(2,1)	(0,0)
	Pizza	(0,0)	(1,2)

Mixed strategy Nash equilibrium
Step 1: No strictly dominated strategies

Step 2: Define Strategies
You: $(1 - \theta)B + \theta P$
Michel $(1 - \sigma)B + \sigma P$

Step 3: Theorem 1
Utility for You
Bistro: $U_y(B, (1 - \sigma)B + \sigma P)$ = $2(1 - \sigma)$ + 0 σ= $2(1 - \sigma)$
Pizza: $U_y(P, (1 - \sigma)B + \sigma P)$ = $0(1 - \sigma)$ + 1 σ= σ

Solve for values of delta such that these utilities fulfill Theorem 1. Any strategies with positive probability must have the same expected utility.
$2(1 - \sigma) = \sigma$
$2 - 2\sigma = \sigma \rightarrow 2 = 3\sigma$
$\sigma = 2/3$

What does this value represent?
The point on our pure strategy graph where Bistro and Pizza provide the same utility.

What does this tell us?
In order for you to mix between Bistro and Pizza, Michele must play the mixed strategy:

$$1/3B + 2/3P$$

Utility for Michel
Bistro: $U_m((1 - \theta)B + \sigma P,\ B)$ = $1(1 - \theta)$ + 0 θ= $1 - \theta$
Pizza: $U_m((1 - \theta)B + \theta P,\ P)$ = $0(1 - \theta)$ + 2 θ= 2 θ

Solve for values of delta such that these utilities fulfill Theorem 1. Any strategies with positive probability must have the same expected utility.
$1 - \theta = 2\,\theta$
$\theta = 1/3$

What does this tell us?
In order for Michel to mix between Bistro and Pizza, you must play the mixed strategy:

$$2/3B + 1/3P$$

We can now conclude that the following mixed strategy profile is a Nash equilibrium.

$$(2/3B + 1/3P, 1/3B + 2/3P),$$

Is it the only Nash equilibrium?
No

Step 4 Define Best Response functions
You: $(1 - \theta)B + \theta P$
Michel $(1 - \sigma)B + \sigma P$

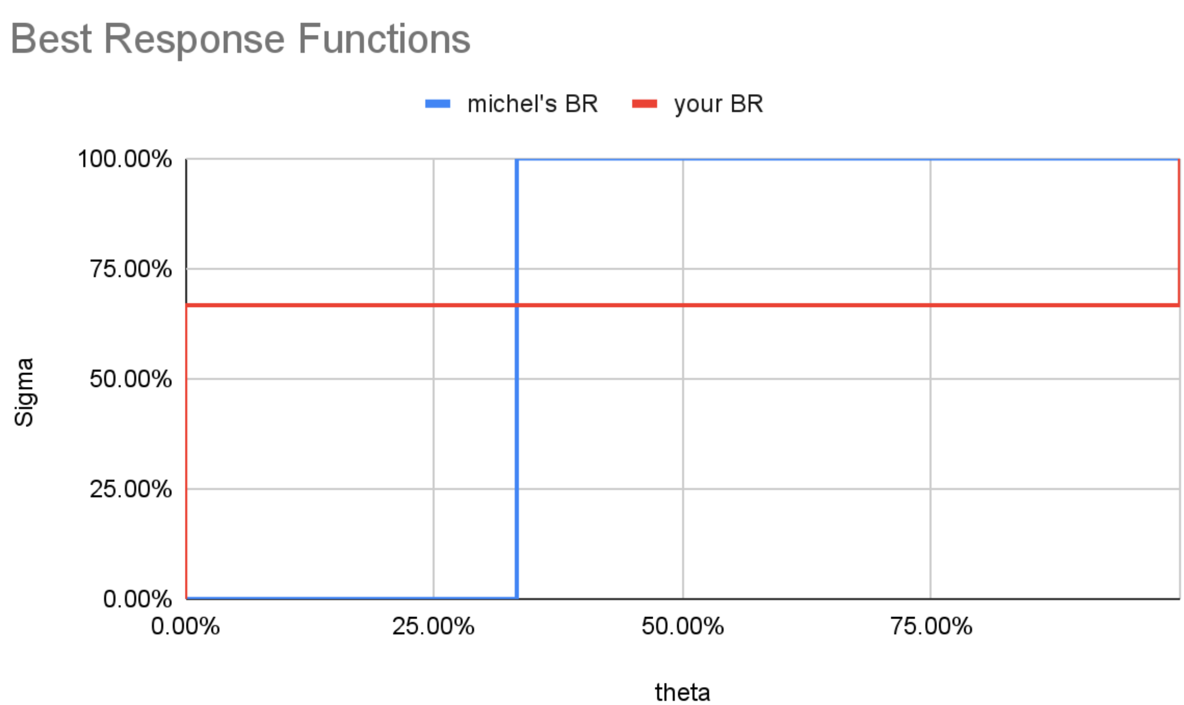
Your Best Response To Michel
$BR_{you}(\sigma)$=

$$\theta = \begin{cases} 0 & when\, \sigma \in [0, 2/3) \\ [0, 1] & when\, \sigma = 2/3 \\ 1 & when\, \sigma \in (2/3, 1] \end{cases}$$

Michel Best Response To You
$BR_{michel}(\theta)$=

$$\sigma = \begin{cases} 0 & when\, \theta \in [0, 1/3) \\ [0, 1] & when\, \theta = 1/3 \\ 1 & when\, \theta \in (1/3, 1] \end{cases}$$

Graph



NE: $(2/3B + 1/3P$, $1/3B + 2/3P)$, (B,B) and (P,P)

Example 2

		Player 2	
		T	P
Player 1	H	(0,1)	(0,2)
	K	(2,2)	(0,1)

Find All Nash Equilibrium
Step 1: No strictly dominated strategies
Step 2: Define Strategies and Graph Pure Strategy Payouts
Player 1: $(1 - \theta)H + \theta K$
Player 2: $(1 - \sigma)T + \sigma P$

Step 3: Theorem 1
Utility for Player 1
$U_1(H, (1 - \sigma)T + \sigma P)$ = $(1 - \sigma)0 + \sigma 0$= 0
$U_1(K, (1 - \sigma)T + \sigma P)$ =$(1 - \sigma)2 + \sigma 0$= $2(1 - \sigma)$

Solve for values of delta such that these utilities fulfill Theorem 1. Any strategies with positive probability must have the same expected utility.
$2(1 - \sigma) = 0$
$\sigma = 1$

What does this value represent?
The point on our pure strategy graph where H and K provide the same utility.

What does this tell us?
In order for player 1 to mix between H and K, Player 2 must play the mixed strategy:

$$0T + P$$

Utility for Player 2

$U_2((1-\theta)H+\theta K, T)$ = $(1-\theta)1+\theta 2$= $1+\theta$

$U_2((1-\theta)H+\theta K, P)$ =$(1-\theta)2+\theta 1$= $2-\theta$

Solve for values of delta such that these utilities fulfill Theorem 1. Any strategies with positive probability must have the same expected utility.

$1+\theta = 2-\theta$

$2\theta = 1$

$\theta = 1/2$

What does this value represent?
The point on our pure strategy graph where T and P provide the same utility.

What does this tell us?
In order for player 2 to mix between T and P, player 1 must play the mixed strategy:

$$1/2H + 1/2K$$

We can now conclude that the following mixed strategy profile is a Nash equilibrium.

$$(1/2H + 1/2K, \text{P})$$

Is it the only Nash equilibrium?
No, we need to find best response functions

Step 4 Define Best Response functions

Strategies

Player 1: $(1 - \theta)H + \theta K$

Player 2: $(1 - \sigma)T + \sigma P$

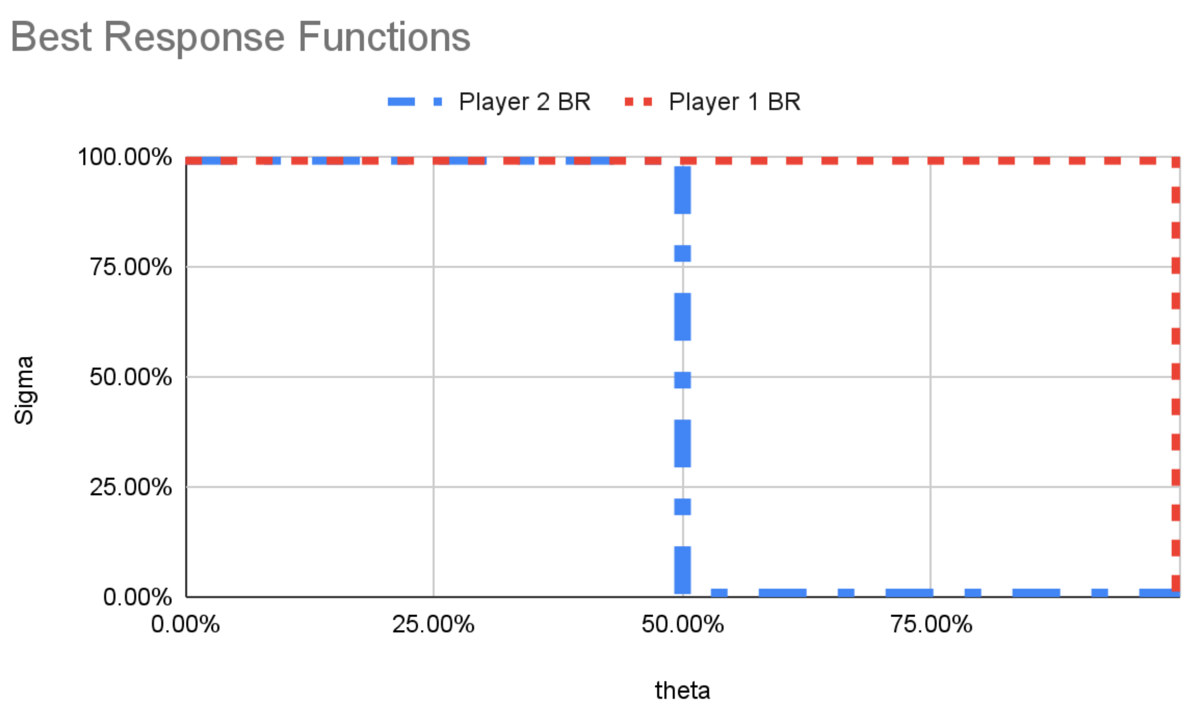
$BR_1(\sigma)$=

$$\theta = \begin{cases} 1 & when\, \sigma \in [0, 1) \\ [0, 1] & when\, \sigma = 1 \end{cases}$$

$BR_2(\theta)$=

$$\sigma = \begin{cases} 1 & when\, \theta \in [0, 1/2) \\ [0, 1] & when\, \theta = 1/2 \\ 0 & when\, \theta \in (1/2, 1] \end{cases}$$

Graph



NE: (H,P),(K,T) and $(1 - \theta)H + \theta K$,P) $where\, \theta \in (0, 1/2]$