Introduction

- I'm a PH.D. student at UO
- I enjoy game theory but am sympathetic to those that struggle. I did not have an easy time in my first game theory course.
- Mental health is important; please come see me if you feel overwhelmed about your future. University resource on the syllabus.
- It will serve you well to think of this like a math course.

Preferences

Rational Preferences

- Completeness
  - For $\forall\, x, y \in X$ we have $x \succcurlyeq y$ or $x \preccurlyeq y$ or both
- Transitive
  - For $\forall\, x, z, y \in X$ if $x \succcurlyeq y$ and $y \succcurlyeq z$ then $x \succcurlyeq z$

Vectors

- A vector is a list.
- We can have vectors of all sorts of things. Numbers or even choices.

Examples

(Tacos,The Notebook), (H,T), (1,2)

Utility Functions

- A utility function $U: X \rightarrow \Re$ is a utility function representing preference relation $\succcurlyeq$ if $\forall\, x, y \in X$ we have $x \succcurlyeq y$ iff $u(x) \geq u(y)$
    - Substitutes
        - $U(x, y) = ax + by$
    - Perfect complements
        - $U(x, y) = Min\{x,\ y\}$
        - $U(x, y) = Min\{ax,\ by\}$
    - Risk neutral
        - $U(x) = ax$
    - Risk averse
        - $U(x) = x^{0.5}$

Substitutes $u(x, y) = 5x + y$
Notice that a single unit of x provides 5 times the utility as y.

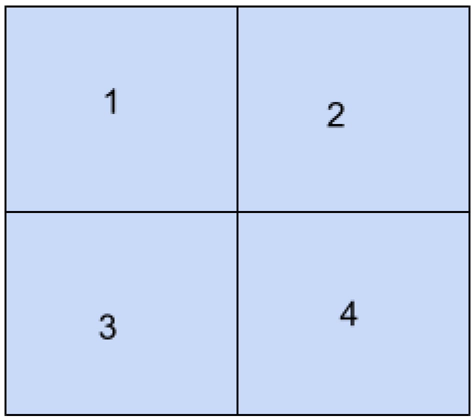
Examples

1. Choosing what to eat between dinner options
2. Choosing a pair of shoes
    a. X represents the quality of the left shoe
    b. Y represents the quality of the right shoe
3. Choosing how much to spend between dinner and dessert.

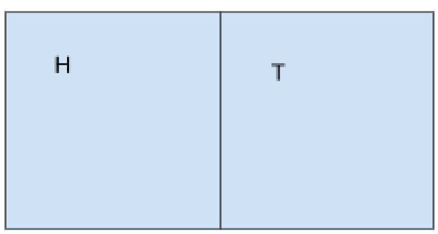
Probability

Venn-Diagram: we can use a diagram to illustrate each outcome

Suppose we roll a fair four-sided die



Probabilities
P(1) = P(2) = P(3) = P(4) = ¼



Probabilities
P(H) = P(T) =½

Probability of Multiple Independent Events

P(event A and event B) = P(event A) * P(event B)
P(event A and event B and event C) = P(event A) * P(event B)* P(event C)

Examples
Three coins

1. Possible outcomes: (HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)
2. The probability of each outcome is ½ * ½ *½ = 1/8

Two 4-sided die

1. Possible outcomes: (1,1); (1,2);(1,3); (1,4);
   (2,1); (2,2);(2,3); (2,4);
   (3,1); (3,2);(3,3); (3,4);
   (4,1); (4,2);(4,3); (4,4);
2. The probability of each outcome is ¼ *¼ = 1/16

Expected Payout / Utility

Lotteries

- A vector of probabilities over n outcomes.

  $L = (p_1, p_2, \ldots, p_n)$ with $p_i \geq 0 \; \forall i$ and $\sum_{i=1}^{n} p_i = 1$

Von Neumann-Morgenstern expected utility function

- $u(L) = p_1 * u_1 + p_2 * u_2 + \ldots + p_n * u_n$

Utility Matrices

Suppose you are flipping a coin. If it is Heads you win a dollar, if it is Tails you lose five dollars

H	T
1	-5

Expected payoff = ½ *1 + ½ *-5 = -1.5

Suppose you roll a fair four sided die the payoff from landing each number is as follows

U(rolling 1) = 20
U(rolling 2) = 12
U(rolling 3) = 4
U(rolling 4) = 8

Expected utility = ¼ * 20 + ¼ * 12 + ¼ * 4 + ¼ * 8= 11

Understanding risk aversion

Risk diminishes some peoples' utility
Consider a lottery that pays \$100 with probability ½ and \$0 with probability ½
The expected return of this lottery is \$50.

How does the lottery affect utility?
Expected utility from lottery

- Risk neutral
  - $U(x) = x$
  - E U(x) = ½ *0 + ½ *100 = 50
- Risk averse
  - $U(x) = x^{0.5}$
  - E U(x) = ½ * $0^{0.5}$ + ½ *$100^{0.5}$ = 5

Let's compare to the utility of a guaranteed \$50:

- Risk neutral
  - $U(x) = x$
  - E U(50) = 50
- Risk averse
  - $U(x) = x^{0.5}$
  - E U(50) = $50^{0.5}$ = 7.07
  - That's a 40% increase in happiness!