Homework 1 (76pts)

An answer that shows understanding with some mistakes will be given partial credit, decided by the grader. A well-written response should receive more than 25%. Show all your work. Answers without work are not guaranteed credit.

Formatting (6pts)

Single file, in pdf format.

Definitions (6pts)

a. Strategy
   An information-contingent plan of action.
b. Strategy profile
   A list of strategies, one for each player.
c. Nash Equilibrium (either definition)
   A strategy profile in which each player is best responding to the strategies of all other players.

Problem #1: Expected Payout (12 pts)

Suppose you roll a fair die and your opponent flips a fair coin simultaneously (independent events). You win $4 whenever a Head appears and the number of dots on the top face of the dice is either one or six. For all other outcomes, you lose $1. (Dr Rui Zhao, University at Albany)

a. How many possible outcomes are there?

12 (1,H) (2,H)(3,H)(4,H)(5,H)(6,H)

(1,T) (2,T)(3,T)(4,T)(5,T)(6,T)

b. What is your expected payoff?

1/12 * (4 -1 -1 -1-1 +4-1-1-1-1-1-1) = 1/12*(-2) = -⅙ = -0.17

c. What is your opponent's expected payoff? Is the game in your favor?

1/12 * (-4 +1 +1 +1+1 -4+1+1+1+1+1+1) = 1/12*(2) = ⅙ = 0.17

The game is not in your favor

Problem #2: Best Response (16 pts)

Your favorite candy is Reese's. However, you substitute between Reeses and any other type of candy (denoted candy). A single unit of Reeses provides double the utility as a single unit of candy.

a. Rank your preferences over the following bundles.
   Bundle = {Candy,Reese's}

   A= {0.1}   B= {1,0}   C= {1,1}   D = {4,1}   E = {1,3}

   E>D>C>A>B

b. Form a utility function to describe your preferences

   U(candy,Reese's) = 1 candy + 2 Reeses

c. State the maximization problem given an allowance of $40, price for Reese's is $10 and the cheapest alternative candy has a price of $4

   Max_{x,y} U(candy, Reeses)

   st

   candy*4 + Reeses *10 <= 40

d. Find the best response for the state of the world described in part c.

   Utility per dollar spent on Reeses is 2/10

   Utility per dollar spent on Candy is 1/4

   The candy provides more utility per dollar spent. Spend all money on Candy for a bundle {10,0} a

Problem #3: Best Response (8 pts)

Assume the role of the manager of a research division of an organization in the biomedical/health sector. Below is data on ratings from a third-party scientific panel regarding potential research projects you could fund. Each proposal has received a rating on a scale from 1 to 5 (with 5 being the top rating) by seven scientific experts unaffiliated with the projects under consideration (reviewers 1 through 7) on your advisory board. All have the same cost.

Richard T. Carson & Joshua Graff Zivin & Jordan J. Louviere & Sally Sadoff & Jeffrey G. Shrader, 2022. "The Risk of Caution: Evidence from an Experiment," Management Science, vol 68(12), pages 9042-9060.

A. Find the average rank of each project.
   E(A) = 1/7 *(1+1+1+1+1+5+5) = 1/7*(15) = 15/7
   E(B) = 1/7 *(1+1+2+2+2+2+2) = 1/7*(12) = 12/7
   E(C) = 1/7 *(2+2+2+5+5+5+1) = 1/7*(22) = 22/7
   E(D) = 1/7 *(3+3+3+3+3+3+3) = 1/7*(21) = 21/7

B. Rank projects to fund.
   C>D>A>B

Reviewer	Project A	Project B	Project C	Project D
1	1	1	2	3
2	1	1	2	3
3	1	2	2	3
4	1	2	5	3
5	1	2	5	3
6	5	2	5	3
7	5	2	1	3

Problem #4: Construct Game Matrix (12 pts)

The Ducks and Beavers football teams are involved in a recruiting battle to add new talent to their programs. Both can simultaneously choose whether to spend their recruiting efforts on trying to attract in-state or out-of-state recruits. Because of their recent history of success, the Ducks hold an advantage and can attract better recruits than the Beavers if they both recruit from the same pool (in or out of state). If they both recruit in-state, the Ducks will have a 10-2 season and the Beavers 5-5; if they both recruit out-of-state Ducks will have an 11-1 season and the Beavers 4-6. If the Beavers recruit in-state and the Ducks out-of-state, the Ducks will have an 8-4 season and the Beavers 7-5. If the Beavers recruit out-of-state and the Ducks in-state, the Beavers will have a 6-6 season and the Ducks 9-3. The objective of both teams is to maximize wins. (Dr. Christopher Ellis, University of Oregon)

Notice that the season outcomes denote (Wins, Losses). Teams don't care about losses and should not be included in your matrix.

a. Explicitly state players and strategies for both players.
   Players: Ducks and Beavers
   Strategies: (In) and (Out)
b. Construct a payoff matrix to illustrate this game
   Payoff matrix given the ordering: (Ducks, Beavers)

		Beavers	
		Out	In
Ducks	Out	(11,4)	(8,7)
	In	(9,6)	(10,5)

c. Find pure strategy Nash equilibrium using best responses.
   Must show best responses on matrix. There is no Nash equilibrium in this game.

Problem #5 Verify Nash Equilibrium 8pts

Verify if the following strategy profiles are Nash equilibrium or not using Theorem 1.
Dr. Zhao (University at Albany)

Player 2

Player 1		L	C	R
	T	(3,2)	(0,1)	(1,0)
	B	(1,0)	(2,1)	(2,2)

a. player 1's mixed strategy (½T + ½B), player 2's mixed strategy (⅓L + ⅔R)

Player 1

$U_1(T, ⅓L\ +\ ⅔R) = ⅓U_1(T, L)\ +\ ⅔U_1(T, R)\ = ⅓ * 3\ +\ ⅔\ * 1\ =\ 5/3$

$U_1(B, ⅓L\ +\ ⅔R) = ⅓U_1(B, L)\ +\ ⅔U_1(B, R)\ = ⅓ * 1\ +\ ⅔\ * 2\ =\ 5/3$

Player 2

$U_2(½T\ +\ ½B, L) = ½\,U_2(T, L)\ +\ ½\,U_2(B, L)\ = ½ * 2\ +\ ½\ * 0\ =\ 2/2$

$U_2(½T\ +\ ½B, C) = ½\,U_2(T, C)\ +\ ½\,U_2(B, C)\ = ½ * 1\ +\ ½ * 1\ =\ 2/2$

$U_2(½T\ +\ ½B, R) = ½\,U_2(T, R)\ +\ ½\,U_2(B, R)\ = ½ * 0\ +\ ½\ * 2\ =\ 2/2$

Conditions for Nash equilibrium are satisfied.

b. player 1's mixed strategy (½T + ½B), player 2's mixed strategy (⅓L + ⅓C + ⅓R)

Player 1

$U_1(T, ⅓L\ +\ ⅓C\ +\ ⅓R) = ⅓U_1(T, L)\ + ⅓U_1(T, C) +\ ⅓U_1(T, R) =$
$⅓ * 3\ +\ ⅓\ * 0\ +\ ⅓ * 1\ =\ 4/3$

$U_1(B, ⅓L\ +\ ⅓C\ +\ ⅓R) = ⅓U_1(B, L)\ + ⅓U_1(B, C) +\ ⅓U_1(B, R) =$
$⅓ * 1\ +\ ⅓\ * 2\ +\ ⅓ * 2\ =\ 5/3$

Conditions for Nash equilibrium are not satisfied, B provides a higher payoff than T against player 2's mixed strategy.