Multi-path Propagation
Theoretical Laboratory Session

WIRELESS COMMUNICATIONS 371-1-1903

SPRING 2020

Part 1 – General Theoretical Information

Path Loss

Path loss, or path attenuation, is the reduction in power density (attenuation) of an electromagnetic wave as it propagates through space. Path loss is a major component in the analysis and design of the link budget of a telecommunication system.

Path loss may be due to many effects, such as **free-space loss**, refraction, diffraction, reflection, aperture-medium coupling loss, and absorption. Path loss is also influenced by terrain contours, environment (urban or rural, vegetation and foliage), propagation medium (dry or moist air), the distance between the transmitter and the receiver, and the height and location of antennas.

Free Space Path Loss

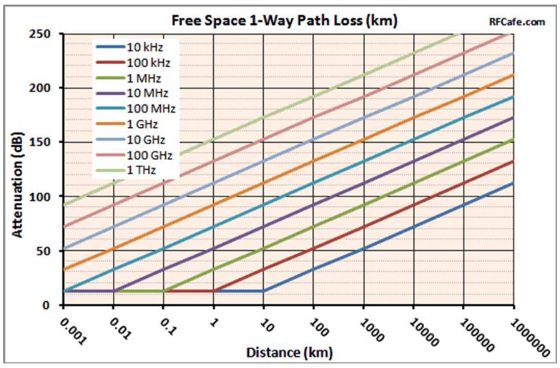
Free-space path loss (FSPL) is the attenuation of radio energy between the feedpoints of two isotropic antennas that results from the combination of the receiving antenna's capture area plus the obstacle free, line-of-sight path through free space (usually air).

The free-space path loss (FSPL) formula derives from the Friis transmission formula and states:

$$\frac{P_r}{P_t} = D_t D_r \left(\frac{\lambda}{4\pi d}\right)^2$$

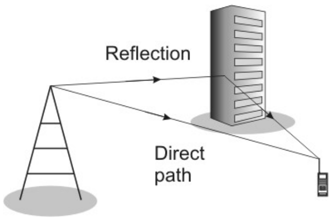
- λ is the signal wavelength
- d is the distance between the antennas
- D_t is the isotropic directivity of the transmitting antenna in the direction of the receiving antenna
- D_r is the isotropic directivity of the receiving antenna in the direction of the transmitting antenna

Substitute $\frac{c}{f}$ for λ to calculate from frequency



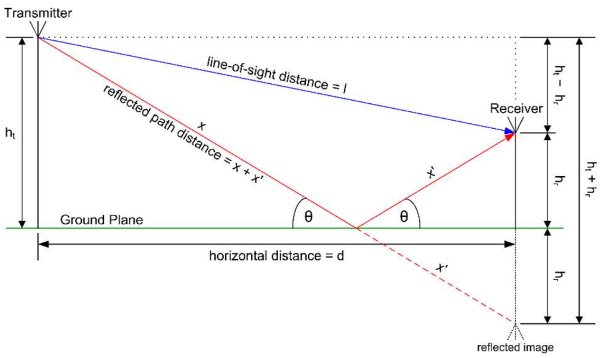
Multipath

Multipath results from that fact that in most communication environments, we don't have a single path for the signal to travel from the transmitter to the receiver. Any time there is an object that is reflective to the signal, a new path can be established between the two nodes. Surfaces like buildings, signs, trees, people, cats, etc. can all produce signal reflections. Each of these reflective paths will show up at the receiver at different times based on the length of the path. Summing these together at the receiver causes distortions, both constructively and destructively.



Two Ray Model

The Two-Rays Ground Reflected Model is a radio propagation model which predicts the path losses between a transmitting antenna and a receiving antenna when they are in LOS (line of sight). Generally, the two antenna each have different height. The received signal having two components, the LOS component and the multipath component formed predominantly by a single ground reflected wave.



From the geometry of the figure above, yields the LOS distance:

$$l = \sqrt{(h_t - h_r)^2 + d^2}$$

and the ground reflected ray distance:

$$x + x' = \sqrt{(h_t + h_r)^2 + d^2}$$

From these distances we can obtain the LOS propagation time delay:

$$t_{LOS} = \frac{l}{C} = \frac{\sqrt{(h_t - h_r)^2 + d^2}}{C}$$

and the ground reflected ray propagation time delay:

$$t_{GR} = \frac{x + x'}{C} = \frac{\sqrt{(h_t + h_r)^2 + d^2}}{C}$$

Resources
https://en.wikipedia.org/wiki/Radio_propagation_model
https://en.wikipedia.org/wiki/Free-space_path_loss
https://en.wikipedia.org/wiki/Two-ray_ground-reflection_model
Wireless communications, Andrea Goldsmith, Stanford University, California, 2005, 9780511841224

Theoretical Questions

1. Under a free space path loss model, find the transmit power required to obtain a received power of 1 dBm for a wireless system with isotropic antennas ($G_l = 1$) and a carrier frequency $f = 5\,GHz$, assuming a distance $d = 10m$. Repeat for $d = 100m$.

2. Consider an indoor wireless LAN with $f_c = 900\,MHz$, cells of radius $100\,m$, and nondirectional antennas. Under the free-space path loss model, what transmit power is required at the access point such that all terminals within the cell receive a minimum power of $10\,\mu W$. How does this change if the system frequency is $5\,GHz$?

Part 2 – Two-Ray Simulation

In this experiment we will construct a communication system using the GNU Radio freeware. During the sending of the signal we will change the channel model to include more and more reflections to create a multipath environment.

1. Change the samp_rate to 512e9.

2. Add "Import" component and enter "import numpy" in the Import field.

3. Add another "Import" component and enter "import scipy.constants" in the Import field.

   (If not installed, write in terminal: sudo apt-get install python-scipy)

4. Add "QT GUI Range" for the frequency – from 1Hz to 6GHz in 100 Hz steps, default value of 900MHz.

5. Add another "QT GUI Range" for the distance – from 10m to 15Km in 10m steps, default value of 10m.

6. Add 4 "Variable" components:
   - ant_gain_r – 1
   - ant_gain_t – 1
   - ant_hight_t – 10m
   - ant_hight_r – 10m

7. Add "Variable" wavelength and enter the value scipy.constants.c/frequency

8. Add “Variable” LOS_distance and enter the value

   numpy.sqrt(numpy.square(ant_gain_t-ant_gain_r)+numpy.square(distance))

9. Add “Variable” GR_distance and enter the value

   numpy.sqrt(numpy.square(ant_hight_t+ant_hight_r)+numpy.square(distance))

10. Add “Variable” LOS_delay and enter the value LOS_distance/scipy.constants.c

11. Add “Variable” GR_delay and enter the value GR_distance/scipy.constants.c

12. Add a float Sine “Signal source” with and amplitude of 1V and a Frequency of frequency

13. Add 2 float “Delay” components:
    - Delay of int(LOS_delay*samp_rate)
    - Delay of int(GR_delay*samp_rate)

    Connect both input ports to the signal source’s output port.

14. Add 2 float “Multiply Const” components:
    - Multiply by a constant of ant_gain_t*ant_gain_r*numpy.square(wavelength/(4*(numpy.pi)*LOS_distance))
    - Multiply by a constant of ant_gain_t*ant_gain_r*numpy.square(wavelength/(4*(numpy.pi)*GR_distance))

    Connect both input ports to the Delay output ports respectively.

15. Add a float “Add” component and connect both of the Multiply Const” components output ports to it’s input ports.

16. Add a float “QT GUI Time Sink” and enter the following preferences:

    General tab:

    - Type: float
    - Y Axis Label: Power (W)
    - Grid = Yes
    - Autoscale = Yes
    - Number of inputs: 3

    Trigger tab:
    - Trigger mode = Auto

    Config tab:

    - Control Panel = Yes
    - Line 1 Label: LOS
    - Line 2 Label: Ground Reflected
    - Line 3 Label: Combined

Connect the In0 input port to the LOS Multiply Const output port.

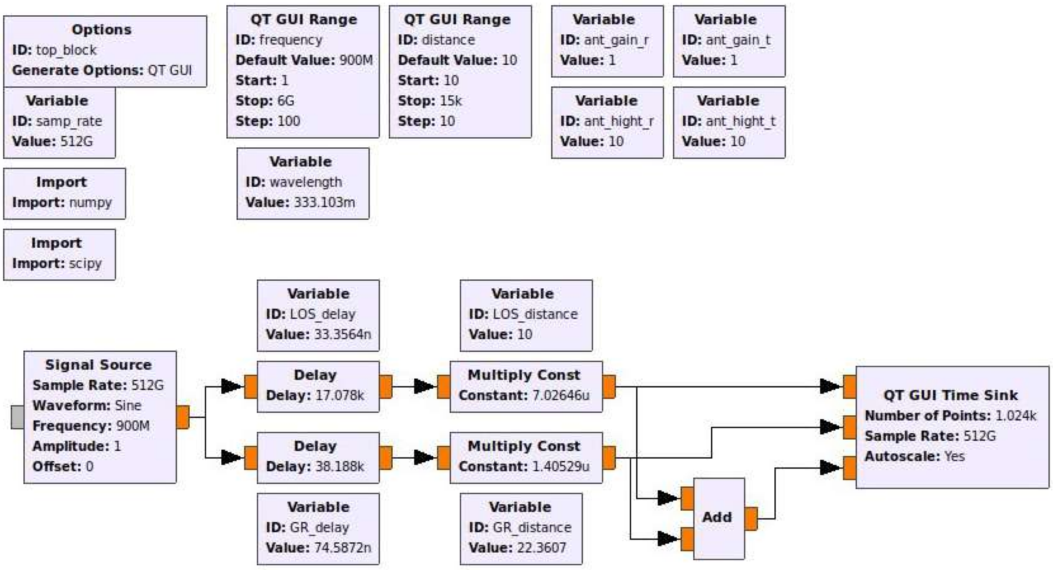
Connect the In1 input port to the GR Multiply Const output port.

Connect the In2 input port to the Add output port.

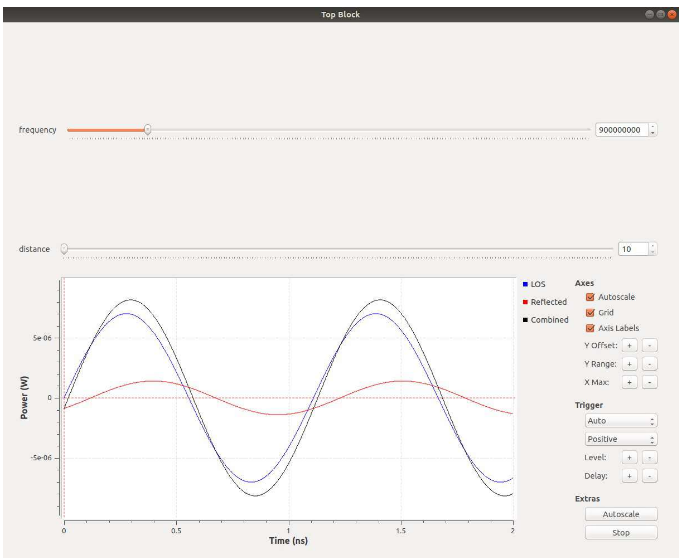
17. Save and run your code.

Save your code and add it to your submission!

If you preformed all the stages correctly, your system's code should look like this:



This are the signals that you should see in your oscilloscopes:



Theoretical Questions

3. (a) Change the frequency while the distance is 10m, what is the power difference you witness for the combined signal?
(b) Set the frequency (as you wish) while changing the distance. At which distance does the path loss effect the signals most? Why is that? Include snapshots of your results.

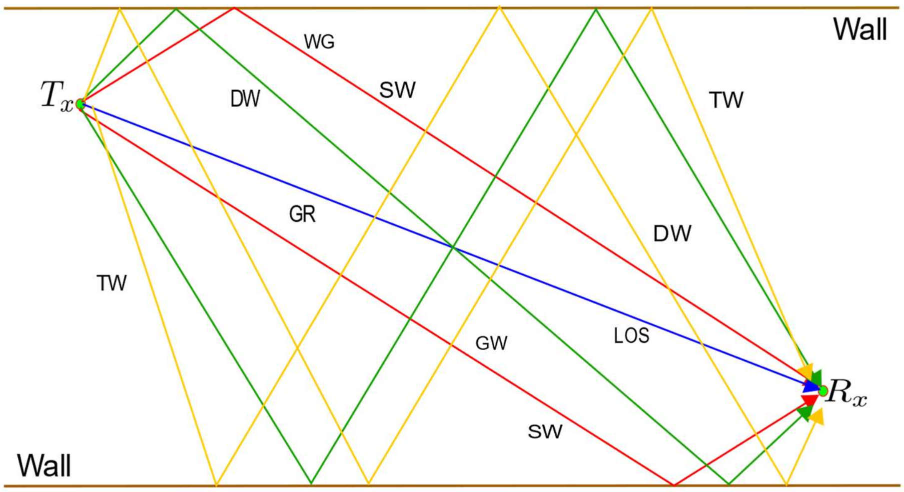
4. Find a frequency and a distance at which there is a constructive interference and a frequency and distance at which there is a destructive interference. Show simulation proof of the phenomena and explain why it accrues.

5. For $\Gamma = -1$ (complete reflection) derive an appropriate expression for the location of the signal nulls at the receiver.

6. For a two-path propagation model with transmitter-receiver separation $d = 80m$, $h_t = 10m$, and $h_r = 1m$, $f = 900MHz$ find the delay spread between the two signals (Show both mathematically and in simulation).

Part 3 – Ten-Ray Simulation

The ten-ray model is a model applied to the transmissions in the urban area, to generate a model of ten rays typically four rays more are added to the two-ray model. This incorporate paths from one to three reflections: specifically, there is the LOS (Line of sight), GR (ground reflected), SW (single-wall reflected), DW (double-wall reflected), TW (triple-wall reflected), WG (wall-ground reflected) and GW (ground-wall reflected paths). Where each one of the paths bounces on both sides of the wall.



Experimentally, it has been demonstrated that the ten ray model simulates or can represent the propagation of signals through a dielectric canyon, in it which the rays that travel from a transmitter point to a receiver point bounce many times.

Let's expand out simulation of the two ray model bay adding more rays.

***Save your code as a new project so not to overwrite the two-ray simulation.**

1. Expand the number of inputs in both your “Add” component and the “QT GUI Time Sink" to 10.

2. In the ten-ray model we have 2 walls so set 4 “Variable”: distance from wall 1 to the transmitting antenna and the receiving and distance from wall 2 to the transmitting antenna and the receiving. Set them as you wish.

3. Now we need to calculate all the ray’s distances. Set a “Variable” for the distance for each of the following rays:

- LOS

$$\sqrt{d^2 + (h_t - h_r)^2}$$

$$\mathrm{numpy.sqrt(numpy.square(ant\_hight\_r} - \mathrm{ant\_hight\_t)} + \mathrm{numpy.square(distance))}$$

- GR

$$\sqrt{d^2 + (h_t + h_r)^2}$$

$$\mathrm{numpy.sqrt(numpy.square(ant\_hight\_r} + \mathrm{ant\_hight\_t)} + \mathrm{numpy.square(distance))}$$

- SW (X2)

it is necessary find the angle between the direct distance d and the distance of line of view $\sqrt{d^2 + (h_t - h_r)^2}$

$$\cos\theta = \frac{d}{\sqrt{d^2 + (h_t - h_r)^2}}$$

Viewing the model with a side view, it is necessary to find a flat distance between the transmitter and receiver called d'.

$$d' = \sqrt{\mathrm{d}^2 - (\mathrm{w}_{\mathrm{r2}} - \mathrm{w}_{\mathrm{t2}})^2}$$

Now we deduce the remaining height of the wall from the height of the receiver called z by the similarity of triangles:

$$\frac{z}{a} = \frac{h_t}{d'} = \frac{h_t}{\sqrt{\mathrm{d}^2 - (\mathrm{w}_{\mathrm{r2}} - \mathrm{w}_{\mathrm{t2}})^2}}$$

$$z = \frac{h_t a}{d'}$$

By likeness of triangles we can deduce the distance from where collides the ray to wall until the perpendicular of the receiver called a, getting:

$$\frac{a'}{d} = \frac{w_{r2}}{w_{t2} + w_{r2}}$$

$$a = \frac{d' w_{r2}}{w_{t2} + w_{r2}}$$

The third ray is defined as a model of two-rays, SW, by which is:

$$\frac{\sqrt{(h_t - h_r - z)^2 + (d' - a)^2} + \sqrt{z^2 + a^2}}{\cos\theta}$$

$$(\text{numpy.sqrt(numpy.square(ant\_hight\_r} - \text{ant\_hight\_t} - \text{z}) + \text{numpy.square(d} - \text{a))} + \text{numpy.sqrt(numpy.square(z)} + \text{numpy.square(a)))/cos\_theta}$$

As **exist two rays that collide once on the wall**, then is find the fifth ray, equating it to the third.

- WG (X2)

Taking a side view it is achieves to evidence the reflected ray that there in SW and is find as following manner:

$$\frac{\sqrt{(h_t - h_r - z)^2 + (d' - a)^2} + \sqrt{(2h_r + z)^2 + a^2}}{\cos\theta}$$

$$((\text{numpy.sqrt(numpy.square(ant\_hight\_r} - \text{ant\_hight\_t} - \text{z}) + \text{numpy.square(d} - \text{a)))} + \text{numpy.sqrt(numpy.square(z} + 2 * \text{ant\_hight\_r)} + \text{numpy.square(a)))/cos\_theta}$$

As **exist two rays that collide once on the wall and then once on ground,** the sixth ray with the fourth ray are the same, since they have the same characteristics.

- DW (X2)

To model the rays that collide with the wall twice, is used the Pythagoras theorem because of the direct distance d and the sum of the distances between the receiver to each wall with double of distance of the transmitter to the wall w_{t2}, this divides on the angle formed between the direct distance and the reflected ray.

$$\frac{\sqrt{d^2 + (w_{r2} + 2w_{t2} + w_{r1})^2}}{\cos\theta}$$

$$(\text{numpy.sqrt(numpy.square(distance)} + \text{numpy.square(wall\_2\_distance\_r} + 2 * \text{wall\_2\_distance\_t} + \text{wall\_1\_distance\_r)))/cos\_theta}$$

As **exist two rays that collide twice on the wall**, then is find the ninth ray, equating it to the seventh.

- TW (X2)

  For the eighth ray is calculate a series of variables that allow to deduce the complete equation, which is composed by distances and heights that were found by likeness of triangles.

  In first instance is take the flat distance between the wall of the second shock and the receiver:

$$x = \frac{d' w_{r1}}{w_{r2} + w_{r1}}$$

  Is found the flat distance between the transmitter and the wall in the first shock.

$$y = \frac{d' w_{t2}}{w_{r2} + w_{r1}}$$

  Finding the distance between the height of the wall of the second shock with respect to the first shock, is obtain:

$$z_1 = \frac{h_t\big(d' - (x + y)\big)}{d'}$$

  Deducing also the distance between the height of the wall of the second shock with respect to the receiver:

$$z_2 = \frac{h_t x}{d'}$$

  Calculating the height of the wall where occurs the first hit:

$$h_{p1} = h_r + z_1 + z_2$$

  Calculating the height of the wall where occurs the second shock:

$$h_{p2} = h_r + z_2$$

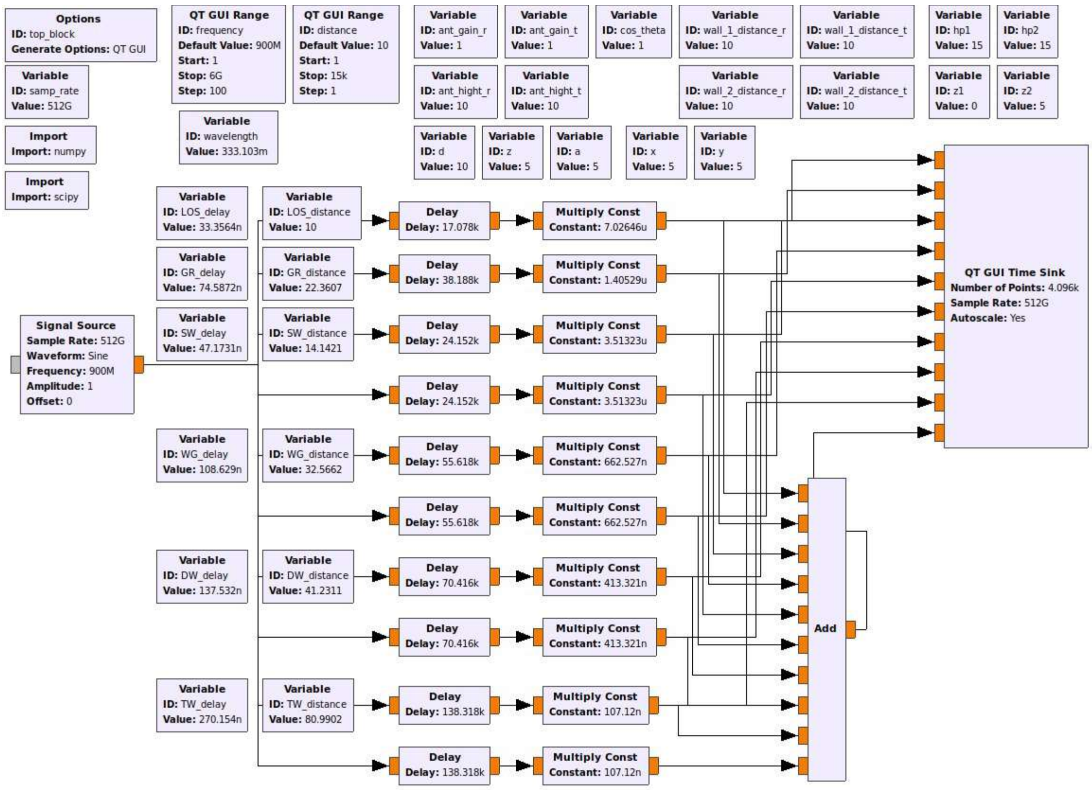
  With these parameters is calculate the equation for the eighth ray:

$$\frac{\sqrt{y^2 + \left(h_t + h_{p1}\right)^2} + \sqrt{\big(d' - (x + y)\big)^2 + \left(h_{p1} + h_{p2}\right)^2} + \sqrt{x^2 + \left(h_r + h_{p2}\right)^2}}{\cos\theta}$$

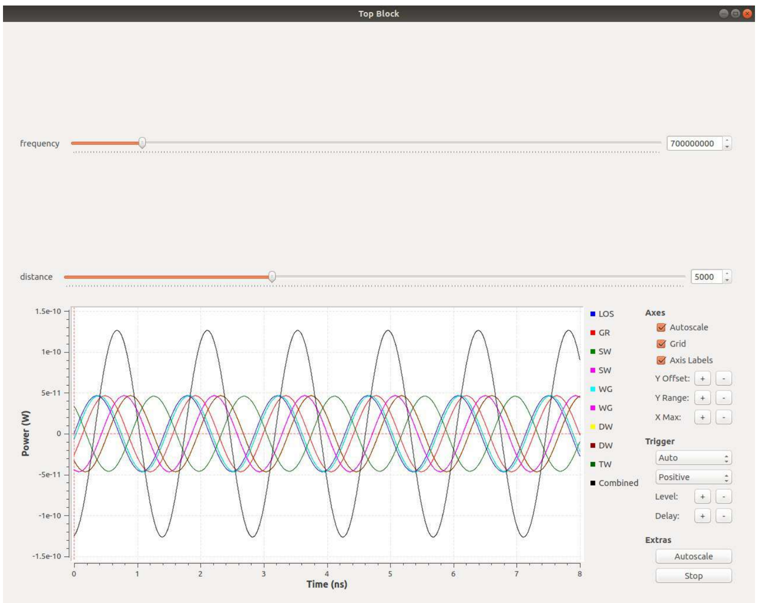
$$\begin{aligned}(\mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{y}) + \mathrm{numpy.square}(\mathrm{ant\_hight\_t} + \mathrm{hp1})))\\ + (\mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{d} - (\mathrm{x} + \mathrm{y})) + \mathrm{numpy.square}(\mathrm{hp2}\\ + \mathrm{hp1}))) + (\mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{x})\\ + \mathrm{numpy.square}(\mathrm{ant\_hight\_r} + \mathrm{hp2})))/\mathrm{cos\_theta}\end{aligned}$$

For the tenth ray, the equation is the same as the eighth ray due to its reflected ray shape.

If you preformed all the stages correctly, your system's code should look like this:



This are the signals that you should see in your oscilloscopes:



Save your code and add it to your submission!

Resources
https://en.wikipedia.org/wiki/Ten_rays_model
Wireless communications, Andrea Goldsmith, Stanford University, California, 2005, 9780511841224

Theoretical Questions

7. Change the frequency and the distance, can you locate a place of destructive interference? Why is it more difficult to find than with the two-ray model simulation? Explain and show proof from your simulation.

8. For the 10-ray model, assume the transmitter and receiver are in the middle of a street of width $50m$ and are at the same height. The transmitter-receiver separation is $500m$. Find the delay spread for this model. Find the power of the strongest and the weakest ray to arrive at the receiver relative to the transmitter power P.

Part 4 – Power Delay Profile

The received signal is the sum of the line-of-sight (LOS) path and all resolvable multipath components:

$$r(t) = \Re\{\sum_{n=0}^{N(t)} \alpha_n(t)u\big(t-\tau_n(t)\big)e^{j(2\pi f_c(t-\tau_n(t))+\phi D_n}\}$$

where $n = 0$ corresponds to the LOS path. The unknowns in this expression are the number of resolvable multipath components $N(t)$, and for the LOS path and each multipath component, its path length $r_n(t)$ and corresponding delay $\tau_n(t) = r_n(t)/c$, Doppler phase shift $\phi D_n(t)$ and amplitude $\alpha_n(t)$.

Since the parameters $\alpha_n(t)$, $r_n(t)$, and $\phi D_n(t)$ associated with each resolvable multipath component change over time, they are characterized as random processes which we assume to be both stationary and ergodic. Thus, the received signal is also a stationary and ergodic random process.

The received signal $r(t)$ is obtained by convolving the baseband input signal $u(t)$ with the equivalent lowpass time-varying channel impulse response $c(\tau, t)$ of the channel.

If the channel is time-invariant then the time-varying parameters in $c(\tau, t)$ become constant, and $c(\tau, t) = c(\tau)$ is just a function of τ:

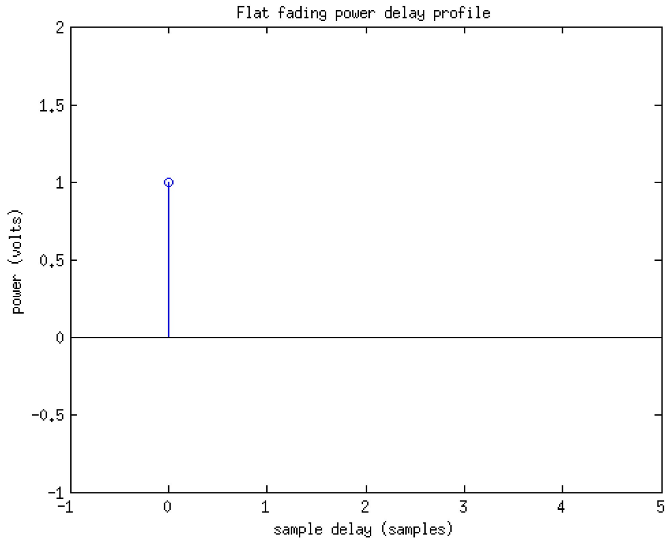
$$c(\tau) = \sum_{n=0}^{N} \alpha_n e^{-j\phi_n}\delta(\tau - \tau_n)$$

For stationary channels the response to an impulse at time t_1 is just a shifted version of its response to an impulse at time t_2, $t_1 \neq t_2$.

We can describe a fadeless channel, where the signal is not attenuated or delayed, as:

$$c(\tau) = \delta(\tau)$$

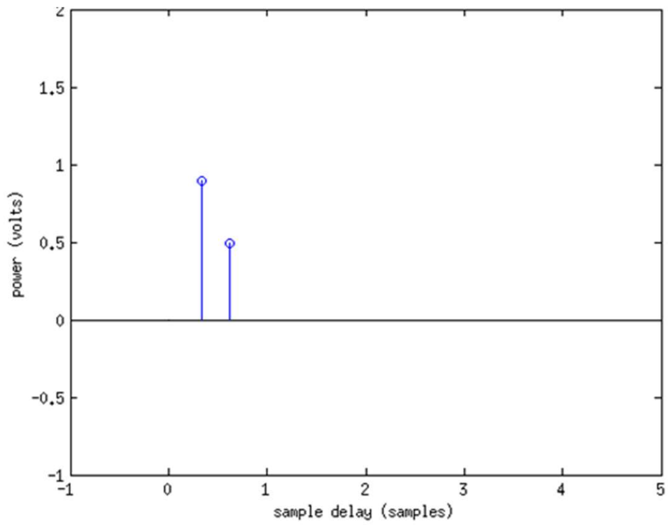
Where $N = 1$, $\alpha_0 = 1$, $\phi_0 = 0$, $\tau_0 = 0$



For a two-ray model we get:

$$c(\tau) = \alpha_0 e^{-j\phi_0}\delta(\tau - \tau_0) + \alpha_1 e^{-j\phi_1}\delta(\tau - \tau_1)$$

Where $N = 2$, $\alpha_0 = \left(\frac{\lambda\sqrt{G_l}}{4\pi}\right)^2$, $\phi_0 = \frac{2\pi l}{\lambda}$, $\tau_0 = \frac{l}{c}$, $\alpha_1 = \left(\frac{\lambda\sqrt{RG_l}}{4\pi(x+x')}\right)^2$, $\phi_1 = \frac{2\pi(x+x')}{\lambda}$, $\tau_1 = \frac{x+x'}{c}$



This is discrete description of the channel is called Power Delay Profile (PDP).

The power delay profile $A_c(\tau)$, also called the multipath intensity profile, is defined as the autocorrelation with $\Delta t = 0$: $A_c(\tau) \triangleq A_c(\tau, 0)$.

Theoretical Questions

9. Consider a two-path channel with impulse response $h(t) = \alpha_1\delta(\tau) + \alpha_2\delta(\tau - 0.05\mu sec)$. Find the distance separating the transmitter and receiver, as well as α_1 and α_2, assuming free space path loss on each path with a reflection coefficient of -1. Assume the transmitter and receiver are located 10 meters above the ground and the carrier frequency is 900MHz.

10. Consider a wireless LAN operating in a factory near two conveyor belts. The transmitter and receiver have a LOS path between them with gain α_0, phase ϕ_0 and delay τ_0. Every T_0 seconds a metal item comes down the first conveyor belt, creating an additional reflected signal path in addition to the LOS path with gain α_1, phase ϕ_1 and delay τ_1. Every $2T_0$ seconds another metal item comes down the second conveyor belt, creating an additional reflected signal path in addition to the LOS path with gain α_2, phase ϕ_2 and delay τ_2.
    Find the time-varying impulse response $c(\tau, t)$ of this channel.

11. Calculate and sketch a PDP for the Ten-Ray model, N=10 (All rays amplitudes, arrival times and phases should be relative to the original signal). Explain your work.

12. Bonus: Assume $N \to \infty$, how would the PDP look like? What is the distribution? Explain.

Resources

Wireless communications, Andrea Goldsmith, Stanford University, California, 2005, 9780511841224

https://www.site.uottawa.ca/~sloyka/elg4179/Lec_5_ELG4179.pdf

Rappaport Wireless Communications Principles And Practice 2[Nd] Edition, Prentice Hall, 2002 Theodore S. Rappaport, 978-0130422323

Good luck!