Testing of Shell Elements using Challenging Benchmark Problems

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ABSTRACT

This paper presents the author's experiences with testing the accuracy and convergence capability of the shell elements available in commercial software SAP2000 Ver. 11.0.0 and a variant of shell element developed by the author, namely the Kriging-based curved triangular shell element (K-Shell). A set of shell benchmark problems were utilized in the tests and two of them were selected here due to its difficulty in achieving a converged solution, i.e. the pinched cylinder with end diaphragms and the Raasch challenge problem. The performance of SAP2000 and the K-Shell elements were highlighted and compared to several shell elements from literature. The results showed that the convergence of the elements was relatively slow in the pinched cylinder problem and different shell elements converged to slightly different values in the Raasch challenge problem. The lesson learned is that a user of commercial finite element software must be cautious regarding the accuracy of the computational results when using shell elements. For SAP2000 users using its shell elements, the use of a very fine mesh of the quadrilateral shell elements are recommended in engineering practice. The performance of the K-Shell can be improved if its formulation could eliminate the shear and membrane locking. Future research for the K-Shell, therefore, should be directed on developing a locking-free K-Shell.

Key words: finite element, shell, benchmark, SAP2000, Raasch challenge problem

1. Introduction

Shell structures are widely used in various engineering products owing to its efficiently in carrying load. In order to study the mechanical behavior of shells and to assist their design, reliable and efficient analysis tools are needed. Nowadays the finite element method has become a practical tool for analysis of shell structures.

An enormous amount of shell elements have been developed for over a half century (see e.g. Alhazza & Alhazza, 2004; Bucalem & Bathe, 1997; Gilewski & Radwańska, 1991; Lomboy, 2007; Yang et al., 2000) and several of them have been integrated in different finite element commercial softwares for structural analyses. In developing shell elements, three approaches have been pursued (Cook et al., 2002, pp. 563): 1. curved shell elements based on classical shell theory, 2. degenerated-solid shell elements, based on degenerating the three-dimensional solid by imposing shell assumptions, 3. superposed shell elements, formed by combining a plane membrane element with a plate bending element.

Different shell elements have different accuracy and convergence capability depending on, among others, the formulation basis, treatment of shear and membrane locking, and treatment of shell normal (Knight, 1997; MacNeal et al., 1998). Therefore, it is imperative for a software user or a shell element developer to perform a series of numerical tests to assess the performance of shell elements in software or a newly developed shell element. According to Knight (1997), "the insight gain by examining the performance of a particular element by a gauntlet of test cases can be of considerable use to an analyst trying to verify and establish the reliability of a given finite element model". To a shell element developer, the tests are very important to detect the element shortcomings and to improve the element performance.

With the abovementioned motivation, the author and his team carried out a number of tests to the shell elements available in commercial software SAP2000 Ver 11.0.0 (Tanjoyo & Subianto, 2009) and a variant of shell element developed by the author (Wong, 2009), namely Kriging-based curved triangular shell element (K-Shell). The tests included a series of patch tests and a number of benchmark problems from literature (Belytschko et al., 1985; Knight, 1997; Ma, 1990; Macneal & Harder, 1985; White & Abel, 1989). The results were compared with those from different finite elements and mesh-free methods.

This paper presents two selected tests using two challenging benchmark problems. The first one is the pinched cylinder with end diaphragms. This problem was chosen because it is "one of the most severe tests for both inextensional bending modes and complex membrane states" (Belytschko et al., 1985). A lot of shell elements in literature display slow convergence in this problem, including shell elements used in this study. The second one is the Raasch challenge for shell elements. The problem was chosen because it was reported in Knight (1997) that the shell elements with transverse shear flexibility did not converge. Thus, it is interesting to use this problem to test the 'thick' element in SAP2000 Ver 11.0.0 and K-Shell, which are in the category of the shell element with transverse shear flexibility.

2. Shell Elements in SAP2000 Ver. 11.0.0

Commercial software SAP2000 is well known and widely used by structural engineers, produced by Computers and Strucures, Inc (http://www.csiberkeley.com/sap2000). This software is also used in the teaching-learning process of several courses in the Civil Engineering Study Program in the institution of the author. The two most widely-used elements in SAP2000 are frame and shell elements. For shell elements, two types of shell elements are available, namely the three-node triangular (T3) and four-node quadrilateral (Q4) shell elements. The Q4 does not have to be planar. These elements are in the category of the superposed shell elements with six degrees of freedom per node, developed by superposing membrane and plate bending elements as illustrated in Figure 1 (Computers and Structures, 2007; Wilson, 1995).

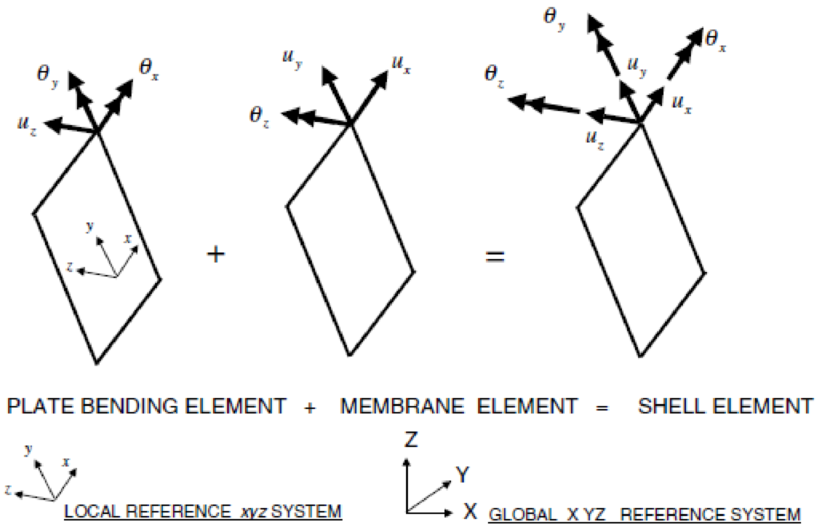


Figure 1. Formation of flat shell element (taken from Wilson, 1995)

For the plate bending element, the formulation basis employed is either Kirchhoff (without shear deformation) or Reissner-Mindlin plate theory (with shear deformation), which referred to as 'thin' plate and 'thick' plate in SAP2000. The assumed functions for the Q4 thick element are an enhanced cubic interpolation for the deflection and an enhanced quadratic interpolation for the rotation. In addition to the standard nodal deflection and rotation degrees of freedom (DOF) at the corner nodes, the DOF also includes what so-called hierarchical rotation at each mid-side of the element (Ibrahimbegović, 1993; Wilson, 1995). The mid-side rotations, however, are eliminated by static condensation and thus the element remains having 12 DOF. The shear locking is prevented using this interpolation scheme together with the assumed strain field. The Q4 thin element is what so called the discrete Kirchhoff quadrilateral plate element (see Ibrahimbegović (1993) and the references therein). The triangular plate bending elements used the same formulations as the quadrilateral elements (Wilson, 1995).

The Q4 membrane element has two in-plane displacement components and a normal rotation as the basic DOFs at each node and thus it has 12 DOF totally. According to Wilson (1995), the starting point in the development of the element is the eight-node quadrilateral element, 16 DOF (Figure 2a). The mid-side displacements is then rotated to be normal and tangential to each side and the tangential component is set to zero, reducing the element DOF to 12 (Figure 2b). The assumed functions for the in-plane displacement components are the natural bilinear interpolation functions from the nodal displacement components with the additional serendipity interpolation functions from the mid-side normal displacements. The mid-side normal displacements are then expressed in terms of the relative nodal rotations by assuming that the normal displacement of the side is parabolic (Figure 2c). Finally, the relative normal rotations are converted to the absolute values in the process of elimination of the spurious deformation mode (Figure 2d). The same method of formulation is applied to the membrane triangular element.

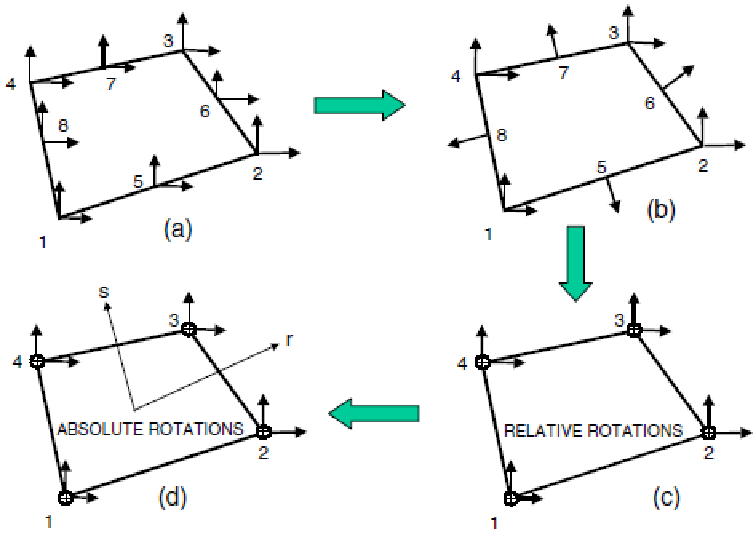


Figure 2. Development of the four-node quadrilateral membrane element used in SAP2000 (taken from Wilson, 1995)

3. Kriging-based Curved Triangular Shell Element

The K-Shell is formulated based on is the degenerated 3D elasticity theory and thus it is in the category of degenerated-solid shell element. The formulation follows closely to that presented by Hughes (1987). This shell element, however, is not a conventional finite element in the sense that its shape functions are not polynomial constructed using the element nodes but Kriging shape functions constructed using several nodes covering several layers of elements (Kanok-Nukulchai & Wong, 2008; Plengkhom & Kanok-Nukulchai, 2005; Wong & Kanok-Nukulchai, 2009; Wong, 2011).

To illustrate the concept of element layers, consider a 2D domain meshed with triangular elements as shown in Figure 3. For each element, Kriging shape functions are constructed based upon a set of nodes in a polygonal domain encompassing a predetermined number of layers of elements. Therefore, the more the number of predetermined layers, the more the nodes including in the construction of the shape functions. Aside from that, constructing Kriging shape functions requires a polynomial basis function and a model of covariance function. In the formulation of K-Shell, four layers of elements, quartic polynomial basis, and the quartic spline correlation function are employed. The reason to choose the high degree polynomial basis is to relieve the shear locking and membrane locking. This approach, whoever, is not able to completely eliminate the locking (Wong & Kanok-Nukulchai, 2006).

The Kriging shape functions are utilized to approximate both the displacement field and the geometry. In approximating the geometry, the parameterization of the shell mid-surface is performed element-by-element by mapping curved triangular elements onto flat planes passing through the element nodes. With this approach, K-Shell is applicable to smooth shells of any form. The displacement of a generic point in the shell is expressed in terms of Kriging interpolation of three nodal displacement components of the shell mid-surface and two nodal rotation components of the normal line. Therefore, there are five DOF at each node (without normal rotational DOF).

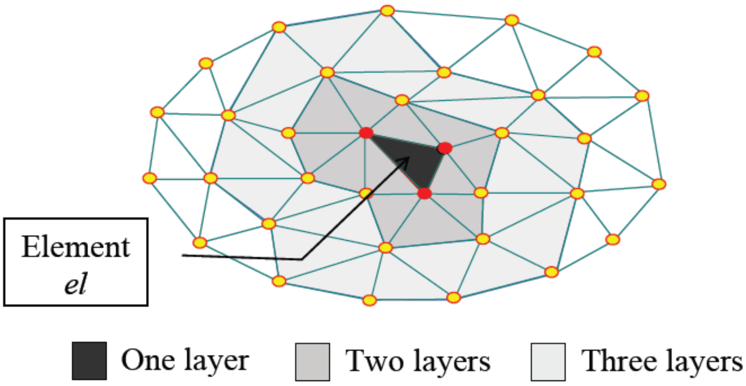


Figure 3. Domain of influence for element el with one, two and three layers of elements (Plengkhom & Kanok-Nukulchai, 2005)

4. Numerical Tests

As an independent assessment of a SAP2000 Ver 11.0.0 user, the author and his team tested the shell elements available in the software (Tanjoyo & Subianto, 2009). The tests included the patch tests (membrane, bending, shear), shear locking test, and the accuracy and convergence tests using a number of benchmark problems collected from literature (Belytschko et al., 1985; Knight, 1997; Ma, 1990; Macneal & Harder, 1985; White & Abel, 1989). The problems encompass Cook's tapered beam, MacNeal-Harder beams (straight, curved, twisted beams), Morley rhombic plate bending, the rectangular and circular plates, the cantilever quarter cylinder, the torsion of a Z section, the pinched hemispherical shell with 180° hole, Scordelis-Lo roof, the pinched cylinder with end diaphragms, and Raasch challenge problem.

The K-Shell was developed during the author's doctoral study under supervision of Prof. Worsak Kanok-Nukulchai (Wong, 2009). To evaluate the accuracy, convergence and versatility of the new element, a number of selected shell problems from the same literature were solved. The selected problems were those in the category of smooth shells, excluding shells in the form of folded plate such as the Z-section problem. This is because the K-Shell was formulated for smooth shells.

From the results the author confirmed that the pinched cylinder problem is the real challenging problem for shell finite elements, because all shell elements considered in the tests give inefficient convergence for this problem. Another interesting problem is the Raasch challenge problem. The description of these problems and the test results are presented here.

4.1. Pinched Cylinder with End Diaphragms

A short circular cylinder with rigid end diaphragms is subjected to two pinching forces as shown in Figure 4. This problem is one of the most severe tests of an element's ability to model both inextensional bending and complex membrane states of stress (Belytschko et al., 1985; Simo, Fox, & Rifai, 1989). Taking advantage of the symmetry, only one octant of the cylinder is analyzed. Meshes of 4x4 quadrilateral and triangular elements are shown in the figure.

The octant of the cylinder is analyzed using meshes with different degrees of refinement, that is from 4x4 to 32x32 elements. The resulting deflections at the node where the force acts are presented in Table 1 together with the reference solution as written in Belytschko et al. (1985). The table shows that the results of the SAP2000 quadrilateral shell elements and K-Shell for the finest mesh are nearly the same value, i.e. about 1.8E-5, which is about 2% lower than the reference solution. On the other hand, the SAP2000 triangular elements produce results about 17% lower even using the finest mesh of 32x32.

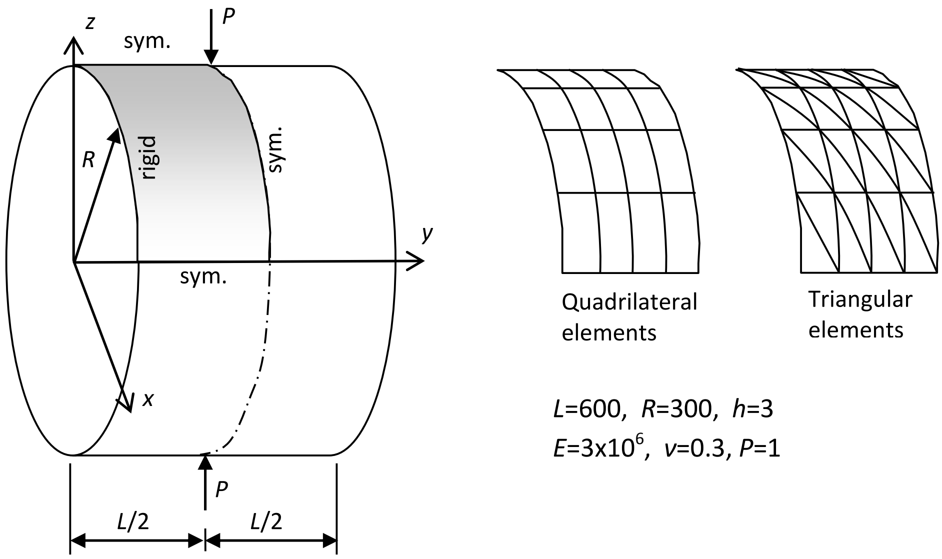


Figure 4. Pinched cylinder with diaphragms and its finite element models with mesh of 4x4 elements

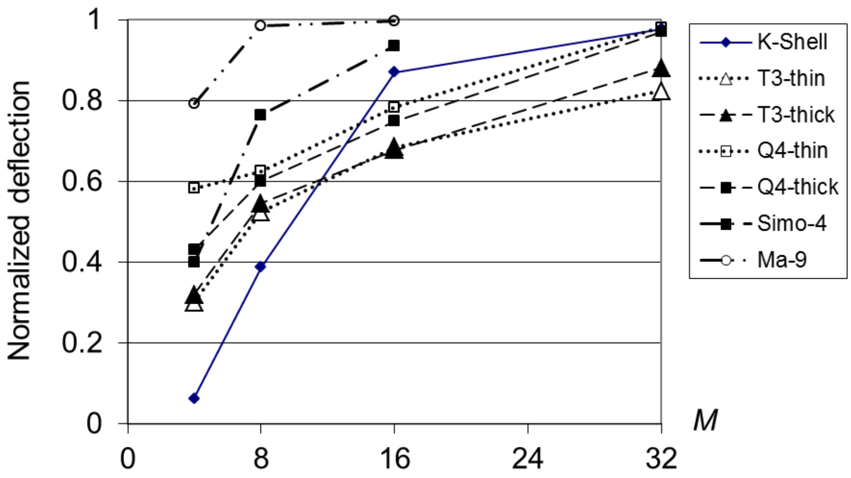
For a better comparison, the results are normalized by the reference solution and plotted in Figure 5 together with the results of two high performance elements, namely Simo-4 and Ma-9. Simo-4 is a four-node stress resultant geometrically exact shell element with the mixed formulation used for the membrane and bending stresses and with full 2-by-2 quadrature (Simo et al., 1989). To the author's knowledge, this element is the best 4-node quadrilateral shell element in terms of its performance. Ma-9 is a nine-node quadrilateral shell element based on assumed strain methods (Ma, 1990). It is seen that the convergent of the SAP2000 shell and K-Shell elements is quite slow in this demanding problem. The results with sufficiently fine mesh of 16x16 are still unsatisfactory. For very fine mesh (M>16), the performance of K-Shell is better than that of SAP2000 shell elements. For course meshes, however, the performance of K-Shell is very unsatisfactory. The reason for this is that in this problem, both membrane locking and shear locking are severe whereas the present method is based on pure displacement trial function, without special treatment to eliminate the shear and membrane locking. The performence of SAP2000 shell and K-Shell elements is inferior compared to high performance elements Simo-4 and Ma-9.

It is worth noting that the displacement field contains highly localized displacement in the vicinity of the load as iilustrated in Figure 6. This is another reason for the slow convergence of the most elements. It is similar to the case of poor performance of many finite elements in the benchmark rhombic plate problem (Wong, 2009) which is attributed to the singularity of bending moment (i.e. highly localized) in the vicinity of the obtuse angle.

Table 1. Deflections at the node where the force acts in the pinched cylinder problem obtained using different elements

M	T3-thin	T3-thick	Q4-thin	Q4-thick	K-Shell
4	5.480E-06	5.826E-06	1.062E-05	7.848E-06	1.139E-06
8	9.560E-06	9.960E-06	1.141E-05	1.095E-05	7.079E-06
16	1.250E-05	1.236E-05	1.430E-05	1.366E-05	1.587E-05
32	1.504E-05	1.608E-05	1.790E-05	1.770E-05	1.784E-05
Ref.	1.825E-05	1.825E-05	1.825E-05	1.825E-05	1.825E-05

M: Number of elements on each side



M: Number elements on each side

Figure 5. Normalized results with different shell elements

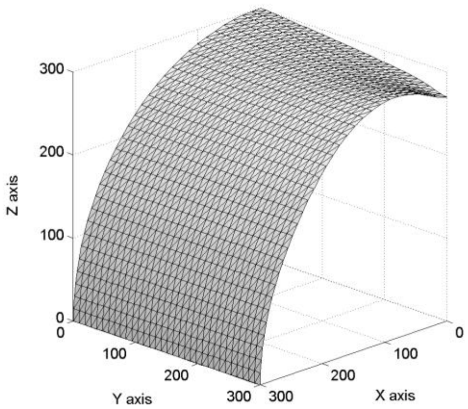


Figure 6. Deformation of the pinched cylinder for the K-Shell with mesh A of 32x32

4.2. Raasch Challenge Problem

The problem is a curved strip or a "hook", as shown in Figure 7, fixed at one end (all degrees of freedom are zero) and loaded by a force in the width direction at the other end. The hook consists of two different arc segments that are tangent at their point of intersection. Both segments have thicknesses h of 2 in and widths b of 20 in. Hence, the width-to-thickness ratio is 10 and it is in the category of "thick" shell.

According to Knight (1997), this problem had become a very interesting test problem for shell elements since the presentation of Harder at the Structures Technical Forum at the 1991 MSC World Users' Conference. It is a challenging problem because of the inherent coupling among three modes of deformation: bending, twisting, and shearing. It is called "Raasch challenge problem", after Ingo Raasch of BMW in Germany, who reported non-converging results in 1991 when he used shell elements in commercial finite element software MSC/NASTRAN (Knight, 1997; MacNeal et al., 1998). Knight (1997) reported very surprising findings: shell elements without transverse shear flexibility appear to converge to an appropriate value, whereas shell elements with *transverse shear flexibility* do *not* appear to converge. MacNeal's investigation of the trouble (Cook et al., 2002, pp. 584-585; MacNeal et al., 1998) concludes that the manner of transfer of twisting moment from element-to-element produces spurious transverse shear deformation. The remedy proposed involves proper treatment of the shell normal and drilling degrees of freedom.

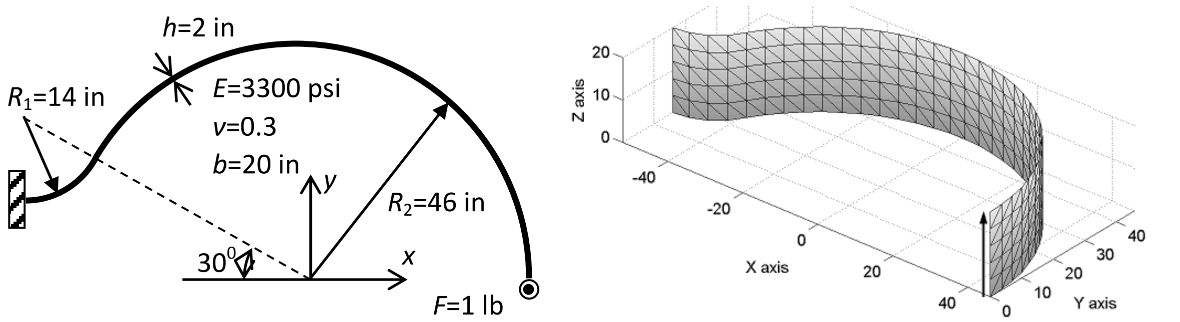


Figure 7. The Raasch challenge problem and its finite element model with mesh of 5x34 elements

In this test, two different thicknesses of the hook are considered, i.e. the original thickness of 2 in (b/h=10, thick hook) and a modified thickness of 0.02 in (b/h=1000, thin hook). The problem domain is modeled with different equally-spaced meshes following Knight (1997). The mesh is defined as the number of elements in the width direction by the number of elements along the arch length. The mesh of 5x34 is shown in Figure 7. Tip deflection in the direction of the in-plane force, which is defined as the average of the z-direction displacements at all nodes at the tip, is observed. The results are compared to those reported by Knight (1997) for the eight-node brick element based on assumed-stress hybrid formulation (8_HYB).

The Original Raasch Problem (Thick Hook)

The average tip deflections in the direction of the force for the thick hook are presented in Table 2. The overall deformation of the hook obtained using 5x36 K-Shell elements is illustrated in Figure 8. From practical engineering standpoint, with mesh 20x136 all the elements give acceptable results since there is no significant different among the results.

For a better comparison, the results are normalized by a solution obtained by using the 20x136x2 mesh of HYB_8 (written as 20x136 in Table 2), i.e. 4.9352 and shown in Figure 9. It is seen that the results of the SAP2000 shell elements without transverse shear flexibility, namely Q4-thin and T3-thin, converge to a nearly identical value, i.e. about 4.71, which is approximately 5% stiffer than the 8_HYB solution. This finding is the same as that reported by Knight (1997) for the shell elements without transverse shear flexibility. However, in contrast to the nonconverging results for the shell elements with transverse shear flexibility, the results of the Q4-thick converge to a value slightly greater than the converging Q4-thin result. In other words, the Q4-thick element predicts a slightly more flexible solution than the converged solution of Q4-thin, as expected. In contrast to the other elements, the convergence of the T3-thick results is not quite clear: It firstly decreases to a value slightly lower value than the 8_HYB solution but after further mesh refinement, the result increases again. The results of the K-Shell converge very well to a slightly higher value, i.e. about 2%, than the solution of the most refine mesh of 8_HYB.

Table 2. Resulting tip deflections for the hook with b/h=10 obtained using different elements

Mesh	T3-thin	T3-thick	Q4-thin	Q4-thick	K-Shell	8_HYB*
3x17	4.5989	5.0368	4.6670	4.6550	N.A.	4.6549
5x34	4.6764	4.9237	4.7023	4.7215	4.9360	4.8059
10x68	4.7037	4.9198	4.7107	4.7365	5.0137	4.8809
20x136	4.7110	4.9357	4.7128	4.7401	5.0265	4.9352

* taken from Knight (1997)

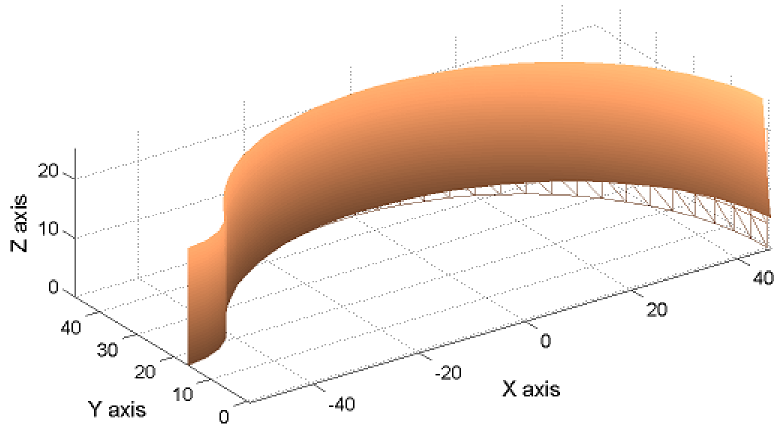
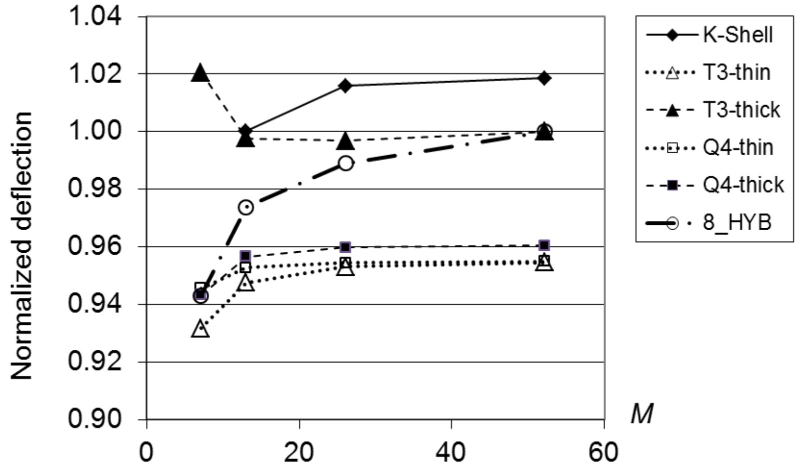


Figure 8. Deformation of the hook modeled by using mesh of 5x34 K-Shell elements



M: Number of equivalent elements on each side, i.e. the square root of the total number of elements

Figure 9. Normalized results for the hook with b/h=10

A Modified Raasch Problem (Thin Hook)

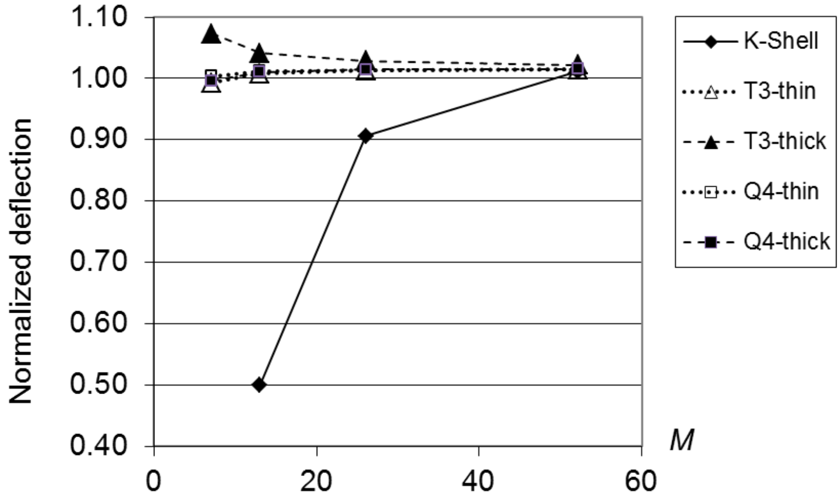
For the modified thin hook (b/h=1000), the results are presented in Table 3. The table shows that the converging results for different elements approach a nearly identical value, i.e. about 4.65×10^6.

For a better comparison, the results are normalized by a solution obtained by using the 20x136x2 mesh of HYB_8: 4,588,678.7 (which was mistakenly written as 45,886,787 in Knight (1997) and shown in Figure 10 with two different vertical axis ranges. It is seen that all the results converge to a value about 2% higher than that obtained using HYB-8. The SAP2000 shell elements can produce quite accurate results using course mesh of 5x34, while the K-Shell results produce too stiff solution for the same mesh. This is because the K-Shell formulation is based on pure displacement trial function, without special treatment to eliminate the shear and membrane locking.

Observing the results for the thick and thin hooks, it appears that the modified Raasch problem is less challenging than the original one.

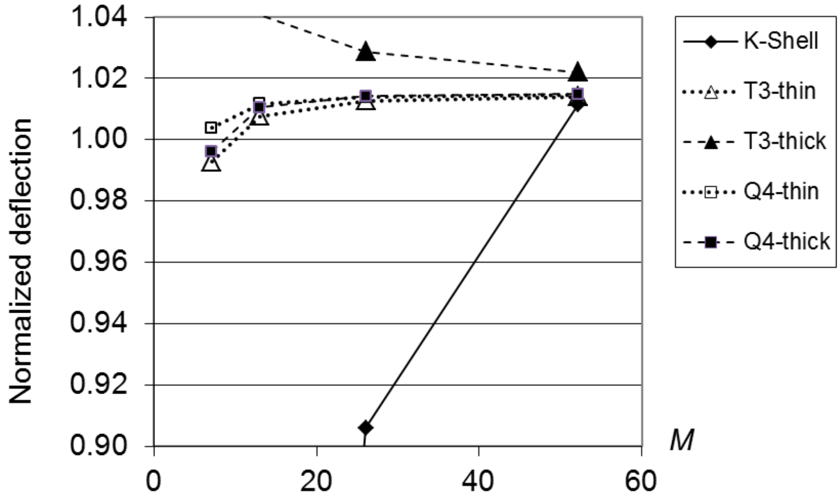
Table 3. Resulting tip deflections for the hook with b/h=1000 obtained using different elements

M	T3-thin	T3-thick	Q4-thin	Q4-thick	K-Shell
3x17	4556207	4924659	4606343	4571319	N.A.
5x34	4624150	4775795	4642247	4636799	2298549
10x68	4646797	4720528	4652675	4653794	4157512
20x136	4653344	4690005	4655529	4657228	4640841



M: Number of equivalent elements on each side, i.e. the square root of the total number of elements

(a) The range for vertical axis values 0.40-1.10



(b) The range for vertical axis values 0.90-1.04

Figure 10. Normalized results for the hook with b/h=1000

5. Conclusions

Numerical tests of the shell elements in SAP2000 Ver.11.0.0 and the K-Shell using two challenging benchmark problems, namely the pinched cylinder with end diaphragms and Raasch challenge problem, have been presented. Testing on the first problem showed that both SAP2000 shell elements and the K-Shell converged slowly and therefore a very fine mesh is needed to obtain accurate results. In the second problem, all the shell elements considered converged but to a slightly different values. The convergence of the SAP2000 thick triangular shell element, however, was not quite clear. In general, SAP2000 quadrilateral shell elements perform better than the triangular elements. The performance of K-Shell for thin shells is poor because it is not a locking free element.

It is clear that different shell elements have different performance. A user of finite element software must be cautious regarding the accuracy of the computational results when using shell elements. It is recommended that a software user to test shell elements available in the software, such as presented in this paper, in order to gain insight regarding the predictive capability of the shell elements. For SAP2000 users, the author recommends to use a very fine mesh of quadrilateral elements when modeling a shell structure, especially for shells subjected to concentrated forces. For the K-Shell to be able to compete with the present high performance shell elements, future research should be directed to eliminate the shear and membrane locking.

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