DEVELOPING AN AUGMENTED REALITY SYSTEM FOR EMBODIED LEARNING OF VOLUME MEASUREMENT

DESARROLLANDO UN SISTEMA DE REALIDAD AUMENTADA PARA EL APRENDIZAJE INCORPORADO EN MATEMÁTICAS

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Volume measurement requires children to develop skills in the coordination of three dimensions, the mental structuring of space, and flexible volumetric reasoning. However, students often face difficulty in these areas. Augmented reality (AR) technology holds promise as a solution to this problem as it can create an environment that introduces a computer-generated layer into the visual environment of the user. In this paper, we introduce an AR smartphone application, MeoGeo, that we developed. This application enables students to use their perceptuomotor actions as pedagogical resources in forming a concept of volume. By employing a multimodal analysis of an elementary student's embodied interaction with this app, we revealed the moments that a student enacts volume measurement of a 3D object. This study suggests that coordinating perception and action in an AR-enabled environment can facilitate mathematical cognition.

Keywords: Technology and Learning Environment Design, Geometry and Measurement, Student Learning and Related Factors

Purpose of the Study

This Brief Research Report aims to demonstrate the potential of an augmented reality (AR) system as an embodied learning technology. AR allows users to augment their view of the real world with computer-generated information. In this paper, we illustrate the embodied repertoire of behavior of an elementary student in learning volume measurement, guided by AR smartphone application, *MeoGeo*, which was developed by the authors of this paper.

Children often struggle with volume measurement, as it requires coordination of three dimensions (3D; Battista & Clements, 1996). Previous research revealed four mental schemes of volume identified by young children: packing, building, filling, and comparing (see Van Dine, 2014; Curry & Outred, 2015). Each scheme can bring particular misconceptions to children. For example, children who use the packing approach (i.e., quantifying the space within a 3D container by iterating unit cubes) tend to miscount only visible faces (Ben-Haim et al., 1985; Panorkou, 2019). In general, mentally organizing the space and imposing structure for volume measurement can be challenging for children, and a carefully designed manipulative can be helpful (Ferrara & Mammana, 2014).

In this study, we introduce *MeoGeo.* This smartphone application was designed for children to *bodily* coordinate 3D models using virtual arrays of unit cubes in their everyday environment in real-time. For example, children can move their body (pre-symbolic register) in alignment with the virtual three axes of object. This experience constitutes situated action where mathematics meanings can arise (Varela, Thompson & Rosch, 1991; Morgan & Abrahamson, 2016). We extend literature on the positive impact of dynamic virtual manipulatives on volume learning (Panorkou, 2019; Rupnow, 2022) to the AR-incorporated embodied learning environment.

Given the increasing popularity of immersive technology across every sector of life, this study aims to report design principles of AR-enabled mathematics learning environments that emphasize pre-symbolic experience for children. We hypothesize that a student's coordination of action with *MeoGeo* can mobilize reasoning with respect to their concept of volume. Our research question asks, "What forms of volumetric reasoning can develop as a result of a child's guided engagement in/with an AR-enabled learning environment?"

Theoretical Framework

Our work is rooted in the theory of embodied cognition and sociocultural theory. Embodied cognition posits that cognitive processes are rooted in the body's interaction with the world, including perceptuomotor coordination and the spatial system (Wilson, 2002). With this framework, this study can analyze how students utilize their body with AR technology (Abrahamson, 2020; Walkington, 2023). Additionally, we also utilize recent scholarship that marries embodied cognition with sociocultural theory to study human-technology interaction (Danish, 2020; Kaptelinin & Nardi, 2017). This approach enables an analysis of the mutually constitutive role of a student within an AR system-applied learning environment (*MeoGeo*).

The Design Study as Mode of Inquiry

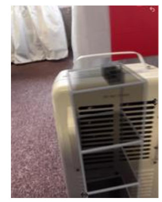
We conducted a design-based study with eight students in grades 2-6 to examine how the forms of volumetric reasonings emerge within the context of our AR system (Cobb et al., 2003). The students participated in a seven-week-long mathematics program in New England, with weekly sessions featuring a one-hour-long AR activity session. For this pilot study, we focused on Greg (pseudonym), as he demonstrated motor skills in operating an iPad. Greg showed limited understanding of volume before using the app. To address our research question, Greg was asked to explore how many virtual cubes make up real-objects of his interest. He chose to measure the air-conditioner (AC) (See Figure 1). Two data sources were used: screen recordings of the app and video recordings from an iPad placed in the classroom. These recordings captured Greg's verbal and non-verbal actions. We employed a microanalytic investigation to capture the evidence of how and why *MeoGeo* may help learning of volume (Erikson, 1992). We reviewed the video recordings and identified the forms of interactions linked to the features of the app (e.g, half-transparent virtual unit cubes) using the MAET framework focusing on a user's body position, gaze, and body movement (Walkington, 2023). Below, we illuminate three excerpts that show the moments of interaction between Greg and *MeoGeo*.

Results

Volume as Coordination of 3D

What if children could structure 3D models using virtual unit cubes that visualize hidden portions of shapes and navigate around to experience them in a real world (3D)? Simply observing a physical cube does not enhance children's mental coordination of 3D objects (Battista & Clements, 1996). The utilization of dynamic geometric software, such as *GeoGebra*, can assist students in coordinating 3D representations of volume, but it relies on a fixed computer screen (Rupnow, 2022). *MeoGeo* synchronizes the dimension of the object studied (3D) with the environment in which a child explores it (3D). Excerpt 1 demonstrates a student actively coordinating the 3D of an object, aided by *MeoGeo*. Greg, to measure the height of AC, adjusted his body positioning up and down (Excerpt 1.01). To ensure that the height of the unit cubes was the same as AC, he sat down to align his eye position with AC (Excerpt 1.02). He then moved on to measure the width (Excerpt 1.03), followed by examining the length of AC from another angle (Excerpt 1.04). He moved his body as if there were *x*, *y*, and *z* axes in space, aligning his body with these axes. When children employ volume as a packing or building method, they limit focus to visible faces (Van Dine, 2014). However, Greg's bodily maneuver, enhanced with the transparency of virtual unit cubes projected onto the AC, served to demonstrate the pedagogical utility of an AR environment in reasoning 3D properties.

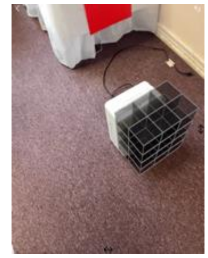
Table 1: Excerpt 1. Greg's spatial structuring of virtual unit cubes

01	Greg:	Hmm. And then, stack it! *(**Fig. 1**: Greg stands up and places the iPad face down)*	 **Fig. 1**  **Fig. 2**
02	Greg:	So, this. *(**Fig. 2**: Greg sits down, moves the iPad to change the camera angles, and then stacks up one more cube on the first column)*	 **Fig. 3**
03	Greg:	Yep. Let's go. Yeah! It's perfect. *(Greg taps iPad to add another column of virtual unit cubes.)*	
04	Greg:	I think there is another [column to stack up for width.] I can't quite decide. *(**Fig. 3**: Greg stands up and moves beside AC)*	

Volume as Unit Structuring Scheme

Children's flexible coordination of units and composite units (such as rows) can aid in volume reasoning, which later helps in the construction of volumetric calculation algorithms (Clements & Sarama, 2021; Rupnow, 2022). To emphasize the unit structuring scheme, we designed an app where children can build virtual unit cubes without gaps or overlaps, a mistake often made by children (Curry & Outhred, 2005). Drawing on gestalt psychology of continuity, we assumed children are more likely to create a row, column, or layer with cubes rather than randomly placing them. This design choice aimed to encourage students to flexibly manipulate units, units of units, and units of units of units for volumetric reasoning. Excerpt 2 shows evidence that children intuitively and bodily explored these unit construction schemes. Greg stacked up three layers with three cubes per row (Excerpt 2.01). Subsequently, he walked around the object to measure the length. He added three more columns (Excerpts 2.02-2.03).

Table 2: Excerpt 2. Greg's interaction with *MeoGeo* in real time

01 Greg: I think there is another [column to stack up for width.] I can't quite decide. *(Greg stands up and moves beside the AC)* 02 Greg: Ugh. Is this… Hoo.. Uhm. *(Greg* stacks up another column. ***Fig. 4****: Greg stands up and walks far away from the AC*) 03 Greg: How is it doing that? Alright. This one [column]. I figured it out. *(Greg* walks back closer to the AC. ***Fig. 5****: he stacks up three more columns, steps back, pauses, and observes his virtual cubes*)	 **Fig. 4**	 **Fig. 5**

Flexible use of multiple reasoning approaches to volume

A previous study found that children were likely to choose one volumetric reasoning strategy (e.g., packing) for a volumetric measurement problem (Vasilyeva et al., 2013). In contrast, our data shows Greg employed three interrelated reasoning strategies for volume. Greg stacks up virtual cubes *next* to the AC, which shows the volume as building approach (see Fig. 6). Then, he compared the size (comparing) using an allocentric frame of reference (Iachini et al., 2023). Then, Greg tried to 'fill' the virtual unit cubes, with the AC (Fig. 7). Although the filling strategy is often observed when measuring the volume of fluid (Van Dine, 2014), the affordances of AR, combining the digital and physical world, made the filling strategy available. It was also interesting that Greg used the AC (the object being measured) as the measuring unit itself. This finding shows that the use of AR can bring flexible volumetric reasoning strategies.

Table 3: Excerpt 3. Greg engaging in various reasoning strategies with *MeoGeo*

01 Greg: *(**Fig. 6**: Greg stacks up another virtual layer. He inserts his foot into the cubes. He circles around the AC and walks farther from it and come back where he stood.)* 02 Greg: (***Fig 7.*** *Greg* grabs the AC and then put inside virtual cubes.) 03 Greg: I am done.	 **Fig. 6**	 **Fig. 7**

Discussion and Conclusions

This study examined video recordings of a fifth grader's embodied interaction with *MeoGeo*. This coincides with the increasing popularity of immersive technology featured in the 2024 PME-NA conference theme. We tested our hypothesis that a student's coordination of action (body position, gaze, body movement) mobilized reasoning about volume by (a) coordinating 3D, (b) exploring unit structuring, and (c) flexibly employing three approaches on volume. As a pilot study, this research is limited to the interaction of only one student. However, it highlights the spontaneous and non-deterministic meaning-making process of a student performing volume measurements within an AR-enabled immersive setting. The virtual arrays of unit cubes, structured with no gaps and overlaps, seemed beneficial for students to arrive at reasonings of volume. We believe our study would be beneficial for educators and designers to attend to such design principles that value students' pre-symbolic register of the body in teaching abstract math concepts.

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