Problem 2

$$G(s) = \frac{s+1}{(s+3)(s+12)} = \frac{-2/9}{s+3} + \frac{11/9}{s+12}$$

$$g(t) = \left(\frac{-2}{9} e^{-3t} + \frac{11}{9} e^{-12t} \right) u_s(t)$$

$$h(k) = \frac{-2}{9} T e^{-3kT} + \frac{11}{9} T e^{-12kT}$$

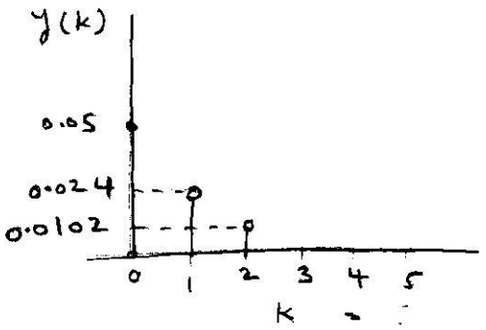
Choose $T = 0.05$, $k = 0, 1, 2$

$$h(k) = \begin{cases} 0.05 & k = 0 \\ 0.0240 & k = 1 \\ 0.0102 & k = 2 \\ 0 & \text{otherwise.} \end{cases}$$

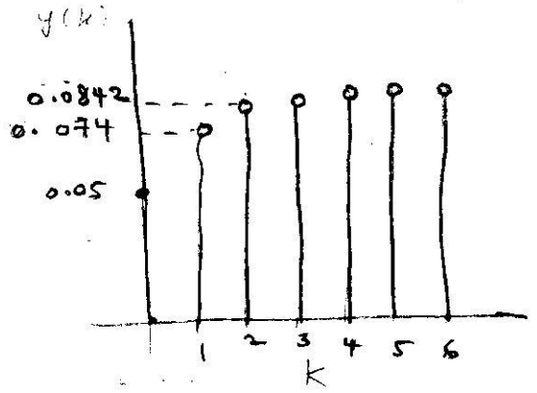
$$H(z) = 0.05 + \frac{0.024}{z} + \frac{0.0102}{z^2} = \frac{0.05z^2 + 0.024z + 0.0102}{z^2}.$$

$$y(k+2) = 0.05\,x(k+2) + 0.024\,x(k+1) + 0.0102\,x(k)$$

Impulse Response



Step Response



Problem # 3

$$Y(s) = \frac{G(s)}{s} = \frac{1}{s} \cdot \frac{(s+1)}{(s+3)(s+12)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+12}$$

$$A = \frac{1}{36}, \quad B = \frac{2}{27}, \quad C = \frac{-11}{108}$$

$$Y(s) = \left(\frac{1}{36}\right)\frac{1}{s} + \left(\frac{2}{27}\right)\left(\frac{1}{s+3}\right) - \frac{11}{108}\left(\frac{1}{s+12}\right)$$

$$y(t) = \frac{1}{36}u_s(t) + \frac{2}{27}e^{-3t}u_s(t) - \frac{11}{108}e^{-12t}u_s(t)$$

$$y(kT) = \frac{1}{36}u_s(t) + \frac{2}{27}(e^{-3T})^k u_s(t) - \frac{11}{108}(e^{-12T})^k u_s(t)$$

$$\Rightarrow Y_d(z) = \frac{1}{36}\left(\frac{z}{z-1}\right) + \frac{2}{27}\left(\frac{z}{z-e^{-3T}}\right) - \frac{11}{108}\left(\frac{z}{z-e^{-12T}}\right)$$

$$H(z) = \frac{z-1}{z} \cdot Y_d(z)$$

$$= \frac{1}{36} + \frac{2}{27}\left(\frac{z-1}{z-e^{-3T}}\right) - \frac{11}{108}\left(\frac{z-1}{z-e^{-12T}}\right)$$

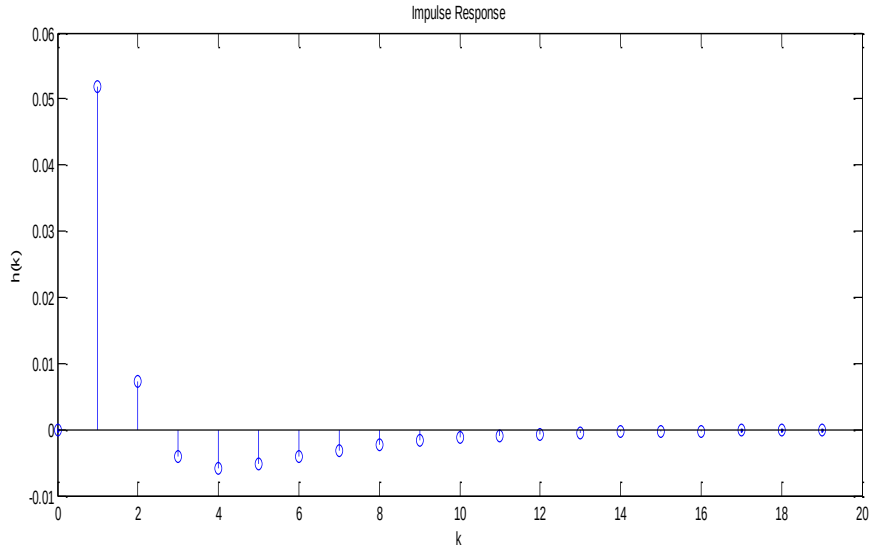
choosing $T = 0.1$

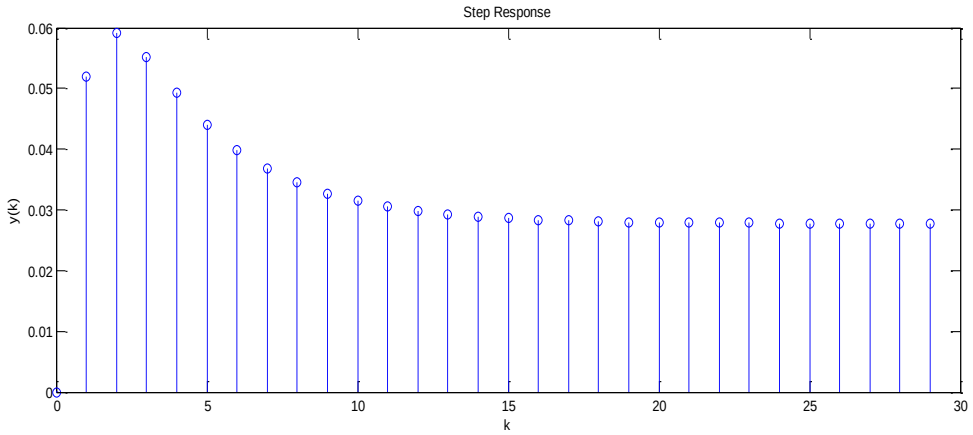
$$H(z) = \frac{5,613z - 5,07}{108z^2 - 112,54z + 24,098}$$

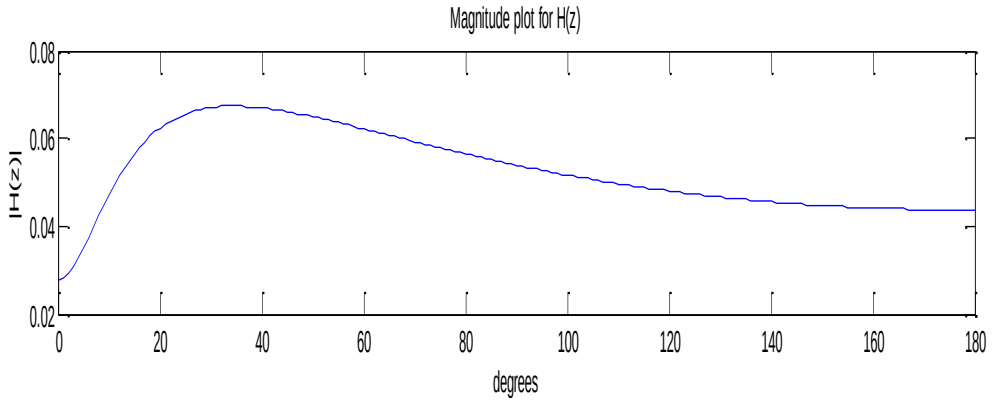
```
% HW#11 , problem 3c
% Matlab code for impulse response

N=20;
time=0:N-1;

h(1)=0;
h(2)=5.613/108;

x=zeros(N,1);
x(1)=1;

for k=3:N
    h(k)=(1/108)*(5.613*x(k-1)-5.07*x(k-2)+112.54*h(k-1)-24.098*h(k-2));
end

figure(1)
stem(time,h)
title('Impulse Response')
ylabel('h(k)')
xlabel('k')
```

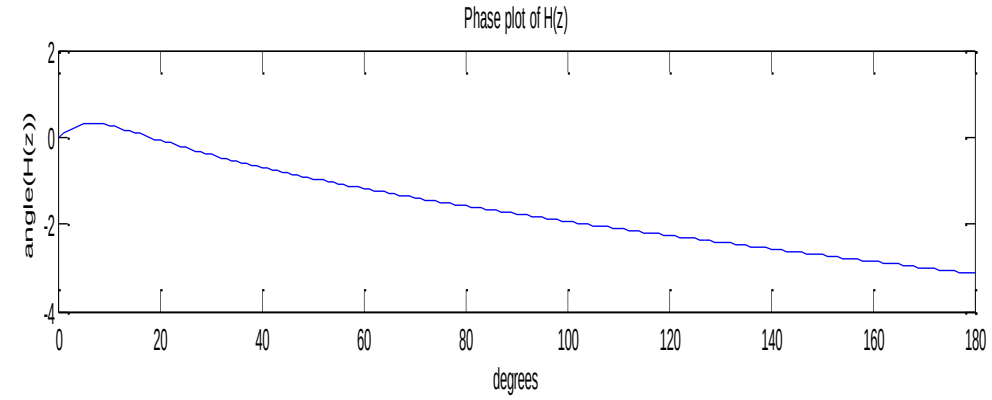
```
% HW#11 , problem 3d
% Matlab code for step response

N=30;
time=0:N-1;

y(1)=0;
y(2)=5.613/108;

x=ones(N,1);

for k=3:N
    y(k)=(1/108)*(5.613*x(k-1)-5.07*x(k-2)+112.54*y(k-1)-24.098*y(k-2));
end

figure(2)
stem(time,y)
title('Step Response')
ylabel('y(k)')
xlabel('k')
```

```
% HW#11 , problem 3e
% Matlab code for frequency response


theta=0:180;
thrad=theta*pi/180;
j=sqrt(-1);
z=exp(j*thrad);

for k=1:181
    H(k)=(5.613*z(k)-5.07)/(108*z(k)^2-112.54*z(k)+24.098);
end

figure(1)
subplot(2,1,1)
plot(theta,abs(H))
title('Magnitude plot for H(z)')
ylabel('|H(z)|')
xlabel('degrees')

subplot(2,1,2)
plot(theta,angle(H))
title('Phase plot of H(z)')
ylabel('angle(H(z))')
xlabel('degrees')
```




Step Response
y(k)
k
0.06
0.05
0.04
0.03
0.02
0.01
0
0
5
10
15
20
25
30


Magnitude plot for H(z)
|H(z)|
degrees
0.08
0.06
0.04
0.02
0
20
40
60
80
100
120
140
160
180


Phase plot of H(z)
angle(H(z))
degrees
2
0
-2
-4
0
20
40
60
80
100
120
140
160
180