Signals and Systems 17 October 2012
Fall 2012 Quiz: Wednesday 24 October 2012

Homework Assignment #9

1. Consider the DT, LTI system with the z-domain transfer function

$$H(z) = \frac{z}{z^2 - 2 \cdot (0.95)\cos(\pi/6)z + (0.95)^2}$$

(a) Plot the magnitude and phase of the frequency response of the system.

(b) Program the corresponding difference equation, and iterate to determine the responses to the inputs

$$x_1(k) = \cos(k\pi/12) \cdot u_s(k),$$

$$x_2(k) = \cos(k\pi/6) \cdot u_s(k),$$

$$x_3(k) = \cos(k\pi/3) \cdot u_s(k).$$

(c) Explain the amplitudes of the steady-state responses from part (b) in terms of the system frequency response.

2. Consider the sequence

$$x_1(k) = \cos(k\pi/6).$$

(a) Find the transfer function and the difference equation for a 2nd-order FIR filter that has unity d.c. gain, but for which the given input $x(k)$ produces zero forced response. (Your filter will have a "notch" at the given frequency, and $H(1) = 1$.)

(b) Find the transfer function and the difference equation for a stable 2nd-order filter that has unity d.c. gain, but for which the given input $x(k)$ produces a large forced response. (Your filter will have a lightly damped resonance at the given frequency, and $H(1) = 1$.)

Plot the frequency response of each filter.

Plot the response of each filter to the input sequence

$$x_2(k) = \cos(k\pi/6) + 1.$$

Make sure the time-domain results are consistent with the frequency-response plots.

3. Find the eigenvalues and eigenvectors of each given matrix A. If A is nonsingular, find A^{-1}, and the eigenvalues and eigenvectors of A^{-1}. (How are the eigenvalues and eigenvectors of A^{-1} related to those of A?) If A is singular, find the rank of A.

(a) $A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$ (b) $A = \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$ (c) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{pmatrix}$

(See Problem 4 on reverse.)

4. A dairy farmer wishes to have a model for the cow population of his farm. At the beginning of each year, the cows are divided into three groups. Young heifers (aged 0 to 1 year) make up the first group. The second group consists of cows aged 1 to 2 years, and the third group consists of all cows aged 2 years or older. From past statistical data, the following assumptions are made:

(i) Each year, 50% of the third group and 80% of the second group of cows produce a single calf. Half are male and half are female. The males are sold shortly after birth.

(ii) Each year, 20% of the third group die of natural causes.

(iii) Some cows from the third group are sold for slaughter each year.

Assume that the number of cows sold for slaughter is the input and the population of group 3 is the output of the system.

(a) Write the state equations for the system and draw a corresponding simulation diagram. Determine the state-variable matrix parameters (A, B, C, D) for this simulation diagram.

(b) Find the operational transfer function for the system. Is the system stable? Would the farmer want a stable system?