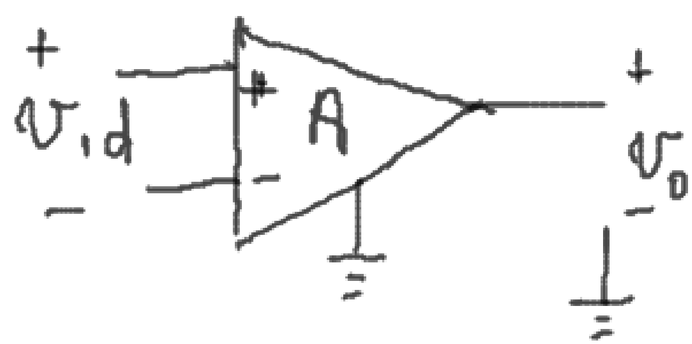
OPERATIONAL AMPLIFIERS

- SUMMATION
- INTEGRATION
- DIFFERENCE
- SCALING

DIFFERENTIAL AMPLIFIER

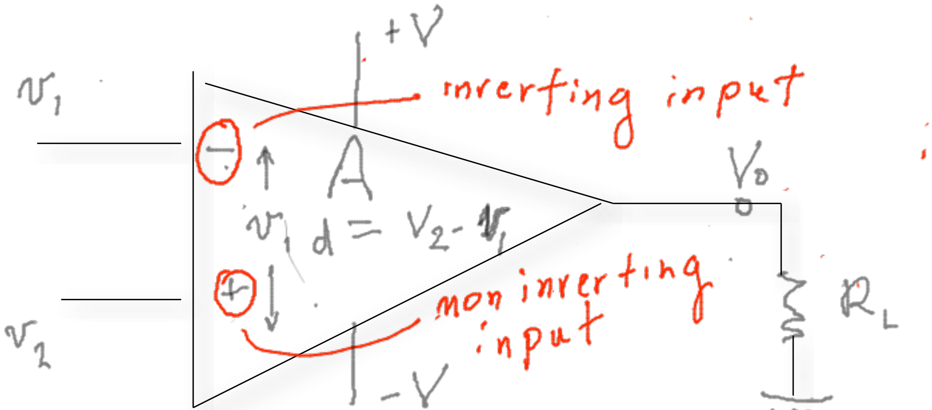
It responds to the difference of two input signals



$$v_0 = A v_{id}$$

Differential Amplifiers

General Consideration:



It amplifies the difference of two input signals

$v_{id} = v_2 - v_1$ = voltage to be amplified

v_1 = the voltage applied to the inverting input

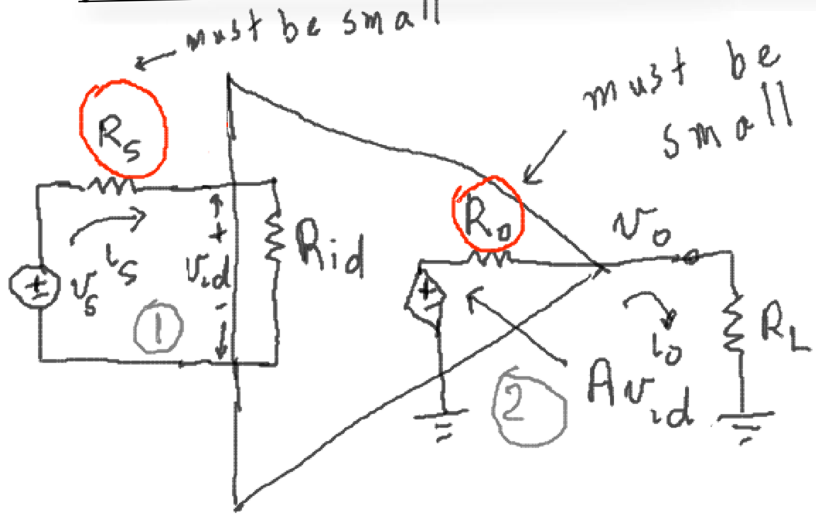
v_2 = the voltage applied to the non inverting input

The output of the Amp depends from:

1. Gain of the Amp
2. The polarity relationship between v_1 & v_2
3. The values of the supply voltage $+V$, $-V$
4. The load resistance, R_L

A = voltage gain, open-circuit voltage gain.
This is the gain without feedback path from the output to the op input

DIFFERENTIAL AMPLIFIER MODEL



R_s = source resistance

v_s = Thevenin equivalent voltage

A = voltage gain

v_{id} = differential input signal voltage

R_{id} = amplifier input resistance

R_o = amplifier output resistance.

We must prove that:

$$\boxed{A_v = \frac{v_o}{v_s} = A \frac{R_{id}}{R_s + R_{id}} \frac{R_L}{R_o + R_L}}$$

an ideal amplifier should be independent of R_s, R_L so that

$R_s \ll R_{id}$

$R_o \ll R_L$

We look at circuits 1 & 2

Goal: find $A_v = \frac{v_o}{v_s}$

Steps: Find v_s, v_o

1

$V_{id} = i_s R_s$ (1)

Eliminate i_s

$v_s = V_{R_s} + V_{id}$

$= i_s (R_s + R_{id})$

$i_s = \frac{V_s}{R_s + R_{id}}$

Since $V_{id} = i_s R_{id}$ (1)

$\Rightarrow$ $V_{id} = \frac{V_s}{(R_s + R_{id})} R_{id}$ (2)

From circuit 2

$V_o = i_o R_L$ (3)

Eliminate i_o

$A v_{id} = V_{R_o} + V_o$

$= i_o (R_o + R_L) \Rightarrow i_o = \frac{A v_{id}}{R_o + R_L}$ (4)

Therefore:

$$V_o = \frac{A v_{id}}{(R_o + R_L)} R_L \qquad (5)$$

Combining (2) & (5)

$$A_v = \frac{v_o}{v_s} = \frac{A \cancel{v_{id}}\, R_L}{(R_o + R_L)} \; \frac{R_{id}}{\cancel{v_{id}}\,(R_s + R_{id})}$$

$$= A \underbrace{\frac{R_{id}}{(R_s + R_{id})}}_{\text{input circuit parameters}} \; \underbrace{\frac{R_L}{(R_o + R_L)}}_{\text{output circuit parameters}} \qquad (6)$$

IDEAL DIFFERENTIAL AMPLIFIERS

$$\boxed{\left.\begin{aligned} R_{ID} &= \infty \\ R_{out} &= 0 \end{aligned}\right\} \text{conditions}} \qquad (7)$$

looking at (6)

$$\boxed{\left.\begin{aligned} A_v &= A \\ v_o &= A\, v_{id} \end{aligned}\right\}} \qquad (8)$$

{since

Realistically speaking, max v_o can be achieved when

$$\boxed{\left.\begin{aligned} R_{id} &\gg R_s \\ R_o &\ll R_L \end{aligned}\right\} \text{fully mismatched resistance condition}} \qquad (9)$$

Conditions of an Ideal Op Amp

special case of the ideal difference Amp.

$$\left.\begin{aligned} R_{id} &= \infty \\ R_o &= 0 \\ \text{voltage gain, } A &= \infty \end{aligned}\right\} \quad 10$$

$$v_{id} = \frac{v_o}{A} \quad \& \quad \lim_{A \to \infty} v_{id} = 0 \qquad (11)$$

$$\text{if } A = \infty \Rightarrow v_{id} = 0 \qquad (12)$$

ASSUMPTION 1

$$\text{If } R_{id} = \infty \Rightarrow i_+, i_- = 0 \qquad (13)$$

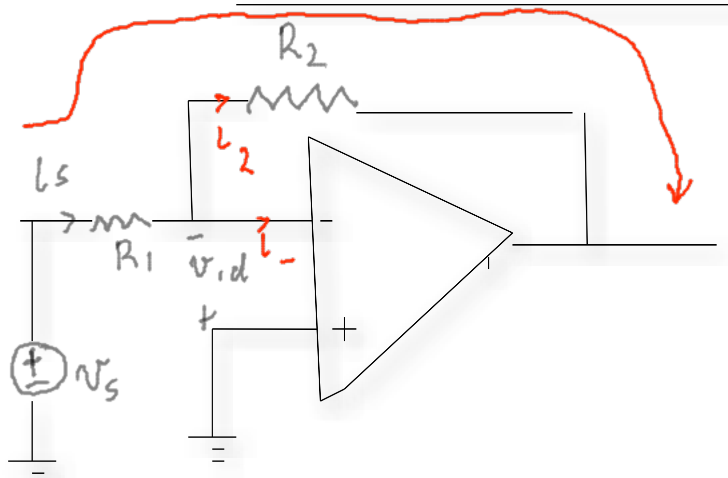
Assumption 2

These two assumptions form the basis of the ideal Op Amp.

Additional Assumptions

Infinite common-mode rejection
Infinite output voltage range
Infinite open-loop bandwidth
Infinite slew-rate
Zero output resistance

INVERTING AMPLIFIER



- Ground the positive input of the Amp.
- introduce a feedback network consisting of 2 resistors R_1, R_2

Goal

find $A_v = \frac{v_o}{v_s}$

$$v_s = i_s R_1 + i_2 R_2 + v_o \qquad (1)$$

Now $\boxed{i_s = i_- + i_2 = i_2 \text{ since } i_- = 0}$ (2) according to assumption 2

$\Rightarrow$

$$v_s = i_s (R_1 + R_2) + v_o \qquad (3)$$

$$i_s = \frac{v_s - v_-}{R_1} \qquad (4)$$

v_-: voltage at the inverting input

but $\boxed{v_{id} = v_+ - v_- = 0}$

since $v_+ = 0$

$\Rightarrow v_- = 0$

therefore $\boxed{i_s = \frac{v_s}{R_1}}$ (5)

$5 \to 3$

$$v_s = v_s + \frac{v_s}{R_1} R_2 + v_o \Rightarrow \frac{v_s}{R_1} R_2 + v_o = 0$$

or

$$V_0 = -V_S \frac{R_2}{R_1}$$

$\Rightarrow$

$$\boxed{A_v = \text{voltage gain} = -\frac{R_2}{R_1}} \qquad (6)$$

- Negative gain indicates 180° phase shift between input & output signa

A_v depends from the choice of R_1 & R_2

if $R_2 \geq R_1$ $\quad A_v \geq 1$

$R_1 > R_2$ $\quad A_v < 1$

The input resistance (from Eq 5)

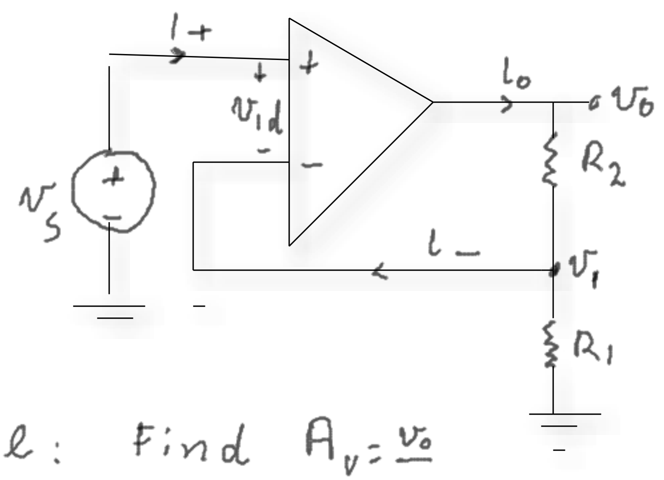
$$R_{IN} = \frac{V_S}{i_S} = R_1 \qquad (7)$$

output resistance

$$R_o = 0 \qquad (8)$$

NON INVERTING AMPLIFIER

The input signal is applied to the noninverting input. Portion of the output signal is fed back to the negative input terminal



Goal: Find $A_v = \frac{v_o}{v_s}$

$$\left.\begin{array}{l} i_- = 0 \\ i_+ = 0 \end{array}\right\} \text{Assumption 2}$$

$$v_1 = v_o \frac{R_1}{R_1 + R_2} \qquad (1)$$

$$v_s = v_{id} + v_1 \qquad (2)$$

but $v_{id} = 0$ (assumption 1)

$$\Rightarrow v_s = v_1 \qquad (3)$$

3 -> 1

$$v_s = v_o \frac{R_1}{R_1 + R_2}$$

$A_v \geq 1$

$$\Rightarrow \boxed{A_v = \frac{v_o}{v_s} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}} \quad (4)$$