Signals and Systems
Fall 2013

Solution to Homework Assignment #5

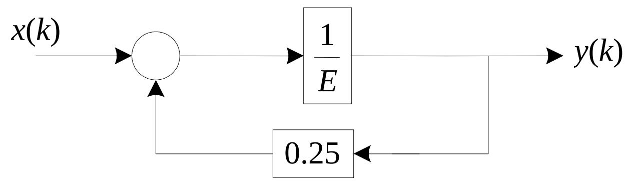
1.

(a) $y(k) = \dfrac{1}{E-0.25}\{x(k)\}$ [Ans: $h(0) = 0$; $h(k) = (0.25)^{k-1}$, $k = 1,2,3,\cdots$.]

From the input-output equation, the transfer function can be expressed as:

$$H(E) = \frac{1}{E-0.25} \Rightarrow y(k+1) = 0.25y(k) + x(k)$$

The diagram for the system is plotted below:



Impulse response:

$$y(0) = 0$$
$$y(1) = 0.25(0) + 1 = 1$$
$$y(2) = (0.25)(1) + 0 = 0.25$$
$$y(3) = (0.25)(0.25) = (0.25)^2$$
$$\vdots$$

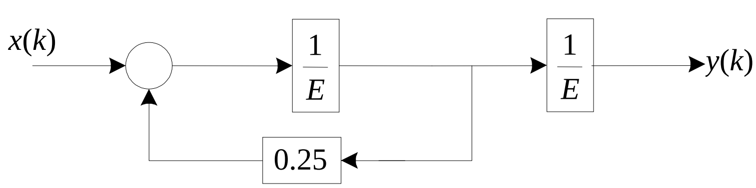
Therefore, the impulse response is: $h(k) = (0.25)^{k-1}u_s(k-1)$

(b) $y(k) = \dfrac{1}{E(E-0.25)}\{x(k)\}$

The transfer function is given as:

$$H(E) = \frac{1}{E\left(E-0.25\right)} \Rightarrow y(k+2) = 0.25y(k+1) + x(k)$$

The diagram for the system is plotted below:



Impulse response:

$y(0) = 0$

$y(1) = 0$

$y(2) = 1$

$y(3) = 0.25$

$\vdots$

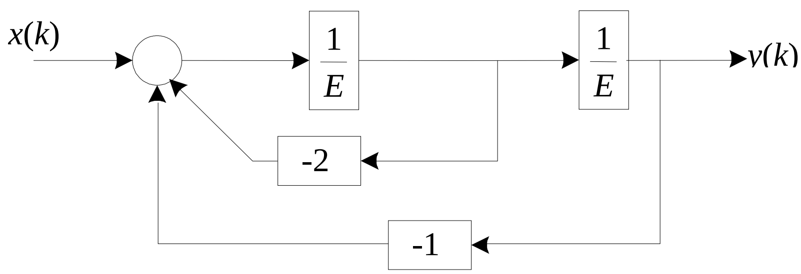
The impulse response is: $h(k) = (0.25)^{k-2} u_s(k-2)$

(c) $\quad y(k) = \dfrac{1}{E^2 + 2E + 1}\{x(k)\}$

From the input-output equation, the transfer function is:

$$H(E) = \frac{1}{E^2 + 2E + 1} \Rightarrow y(k+2) = -2y(k+1) - y(k) + x(k)$$

The diagram for the system is shown below:



Impulse response:

$y(0) = 0$

$y(1) = 0$

$y(2) = x(0) = 1$

$y(3) = -2y(2) = -2$

$y(4) = -2y(3) - y(2) = 3$

$y(5) = -2y(4) - y(3) = -4$

$\vdots$

The impulse response is: $h(k) = (k-1)(-1)^{k-2} u_s(k-1)$.

2.

(a) $h(k) = \begin{cases} 1, & k = 2 \\ 0, & k \neq 2 \end{cases}$

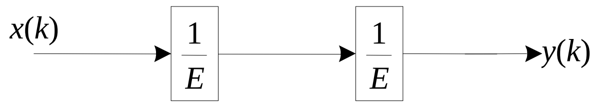
The impulse response can be written as:

$h(k) = \delta(k-2)$

It then follows that

$\Rightarrow y(k) = x(k-2)$

$\Rightarrow y(k) = \left(\frac{1}{E^2}\right)\{x(k)\}$



Hence, the operational transfer function can be expressed as:

$$\boxed{H(E) = \frac{1}{E^2}}$$

(The simulation diagram is depicted on the right)

(b) $h(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$

The above impulse response can be interpreted as follows:

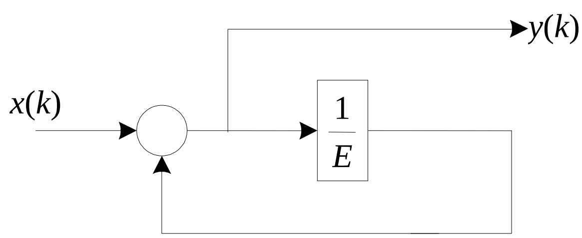
$h(k) = u_s(k) = \sum_{n=-\infty}^{k} \delta(n)$

Equivalently, $h(k) - h(k-1) = \delta(k)$

It then follows that:

$y(k) - y(k-1) = x(k)$

$\Rightarrow \left(1 - \frac{1}{E}\right)\{y(k)\} = x(k)$



The transfer function can then be expressed as:

$$\boxed{H(E) = \frac{1}{1 - 1/E} = \frac{E}{E-1}}$$

(The simulation diagram is depicted on the right).

(c) $h(k) = \begin{cases} 1, & k \geq 1 \\ 0, & k < 1 \end{cases}$

The impulse response can be interpreted as:

$h(k) = u_s(k-1)$

Note that $u_s(k-1) - u_s(k-2) = \delta(k-1)$

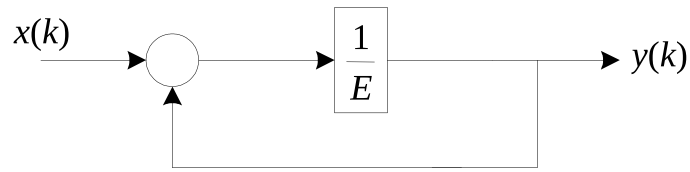
$$\Rightarrow h(k) - h(k-1) = \delta(k-1)$$

It then follows that:

$$y(k) - y(k-1) = x(k-1)$$

$$\Rightarrow \left(1 - \frac{1}{E}\right)\{y(k)\} = \frac{1}{E}x(k)$$

$$\Rightarrow (E-1)\{y(k)\} = x(k)$$



The transfer function of the system is:

$$\boxed{H(E) = \frac{1}{E-1}}$$

(The simulation diagram is depicted on the right).

(d) $h(k) = \begin{cases} 1, & k \geq 2 \\ 0, & k < 2 \end{cases}$

Similar to previous problem, we have: $h(k) = u_s(k-2)$

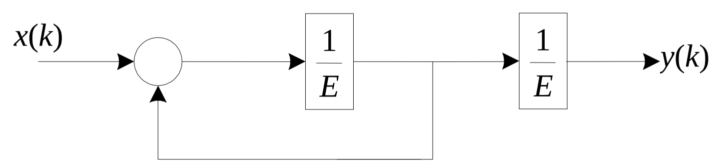
$$\Rightarrow h(k) - h(k-1) = \delta(k-2)$$

Therefore, we have:

$$y(k) - y(k-1) = x(k-2)$$

$$\Rightarrow \left(1 - \frac{1}{E}\right)\{y(k)\} = \frac{1}{E^2}x(k)$$

$$\Rightarrow (E-1)\{y(k)\} = \frac{1}{E}x(k)$$



The transfer function of the system is then expressed as:

$$\boxed{H(E) = \frac{1}{E(E-1)}}$$

(The simulation diagram is depicted on the right.)

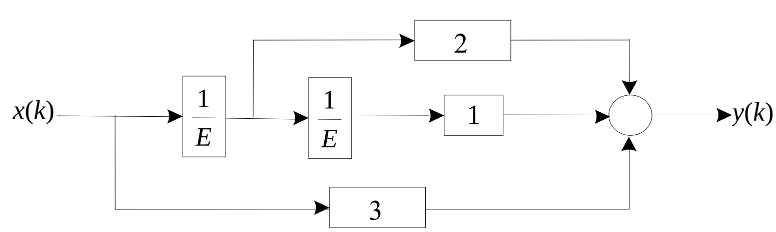
(e) $h(k) = \begin{cases} 3, & k = 0 \\ 2, & k = 1 \\ 1, & k = 2 \\ 0 & \text{otherwise} \end{cases}$

$$h(k) = 3\delta(k) + 2\delta(k-1) + \delta(k-2)$$

Hence,

$$y(k) = 3x(k) + 2x(k-1) + x(k-2)$$

$$= \left(3 + \frac{2}{E} + \frac{1}{E^2}\right)\{x(k)\}$$



The transfer function is given below:

$$\boxed{H(E) = \frac{3E^2 + 2E + 1}{E^2}}$$

(The simulation diagram is depicted on the right.)

3.

(a) $h(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$,

$h(k) = u_s(k),\ x(k) = u_s(k)$

Applying the convolution operator yields

$$y(k) = \sum_{m=0}^{k} h(k-m)x(m) = \sum_{m=0}^{k} (1).(1) = k+1 \text{ for } k \geq 0$$

Finally, we obtain:

$$\underline{y(k) = (k+1)u_s(k)}$$

(b)

$$h(k) = \begin{cases} 1, & k = 0 \\ 2, & k = 1 \\ 3, & k = 2 \\ 0 & \text{otherwise} \end{cases},$$

$h(k) = \delta(k) + 2\delta(k-1) + 3\delta(k-2)$

$x(k) = u_s(k)$

Applying the convolution operator yields

$$y(k) = \sum_{m=0}^{k} h(k-m)x(m)$$
$$= \sum_{m=0}^{k} h(k-m)$$

$k = 0:\ h(0) = 1$

$k = 1:\ h(0) + h(1) = 1 + 2 = 3$

$k = 2:\ h(0) + h(1) + h(2) = 1 + 2 + 3 = 6$

Finally, we have:

$$\underline{y(k) = u_s(k) + 2u_s(k-1) + 3u_s(k-2)}$$

$$h(k) = \begin{cases} 1, & k = 1 \\ 2, & k = 2 \\ 3, & k = 3 \\ 0 & \text{otherwise} \end{cases}$$

$h(k) = \delta(k-1) + 2\delta(k-2) + 3\delta(k-3)$

$x(k) = u_s(k)$

Applying the convolution operator yields

$$y(k) = \sum_{m=0}^{k} h(k-m)x(m)$$
$$= \sum_{m=0}^{k} h(k-m)$$

$k = 1:\ h(0) + h(1) = 0 + 1 = 1$

$k = 2:\ h(0) + h(1) + h(2) = 0 + 1 + 2 = 3$

$k = 3:\ h(0) + h(1) + h(2) + h(3) = 0 + 1 + 2 + 3 = 6$

Finally, we have:

$$\underline{y(k) = u_s(k-1) + 2u_s(k-2) + 3u_s(k-3)}$$

(c) $h(k) = \begin{cases} (0.9)^k, & k \geq 0 \\ 0, & k < 0 \end{cases}$. [Ans: $y(k) = 10 - 9(0.9)^k$, $k = 0,1,2,\cdots$.]

Applying the convolution operator yields

$$\begin{aligned} y(k) &= \sum_{m=0}^{k} h(k-m)x(m) \\ &= \sum_{m=0}^{k} \underbrace{x(k-m)}_{=1} h(m) \\ &= \sum_{m=0}^{k} (0.9)^m = \frac{1-(0.9)^{k+1}}{1-0.9} \\ &= \frac{1-(0.9)(0.9)^k}{0.1} = 10 - 9(0.9)^k, k \geq 0 \end{aligned}$$

It then follows that:

$$y(k) = \left[10 - 9(0.9)^k\right] u_s(k)$$

4.

(a) Given the operation transfer function

$$H_1(\mathrm{E}) = \frac{E}{E - 0.5}.$$

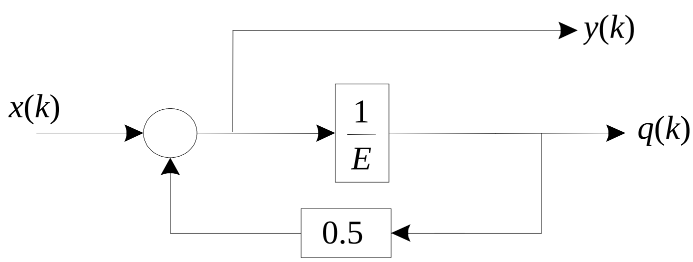
We have:

$$H_1(E) = \frac{1}{1 - 0.5(1/E)}$$

$$\Rightarrow \left(1 - 0.5 \times \frac{1}{E}\right)\{y(k)\} = x(k)$$

$$\Rightarrow y(k) - 0.5y(k-1) = x(k)$$

The corresponding simulation diagram is plotted below (Matlab codes to generate impulse and step responses are provided in the last page):



(b) Given the operation transfer function

$$H_2(\mathrm{E}) = \frac{E^4 + 0.5E^3 + 0.25E^2 + 0.125E + 0.0625}{E^4}.$$

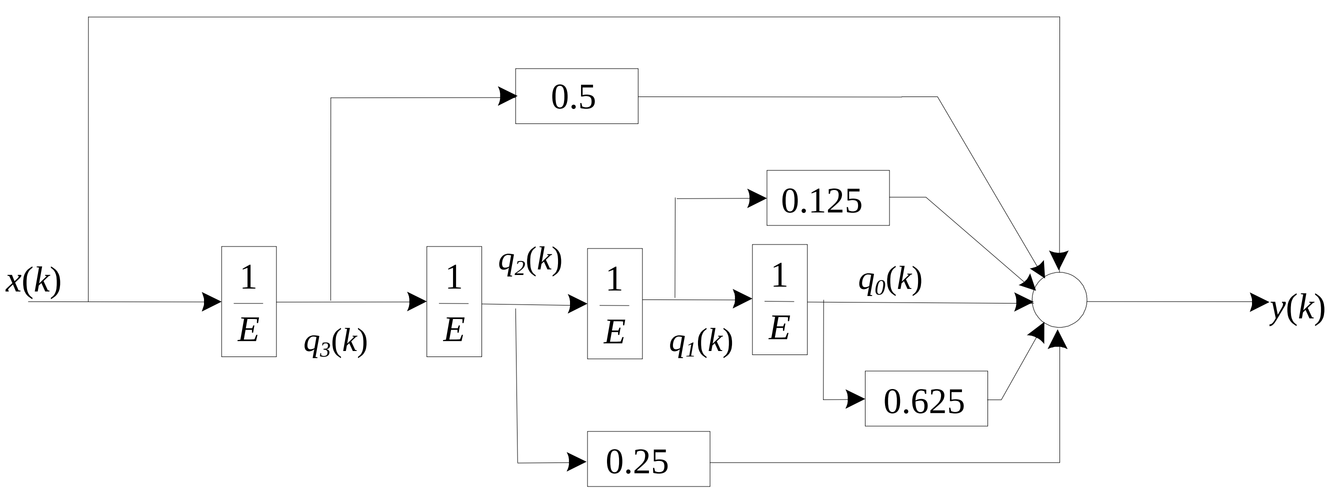
It is not hard to verify that:

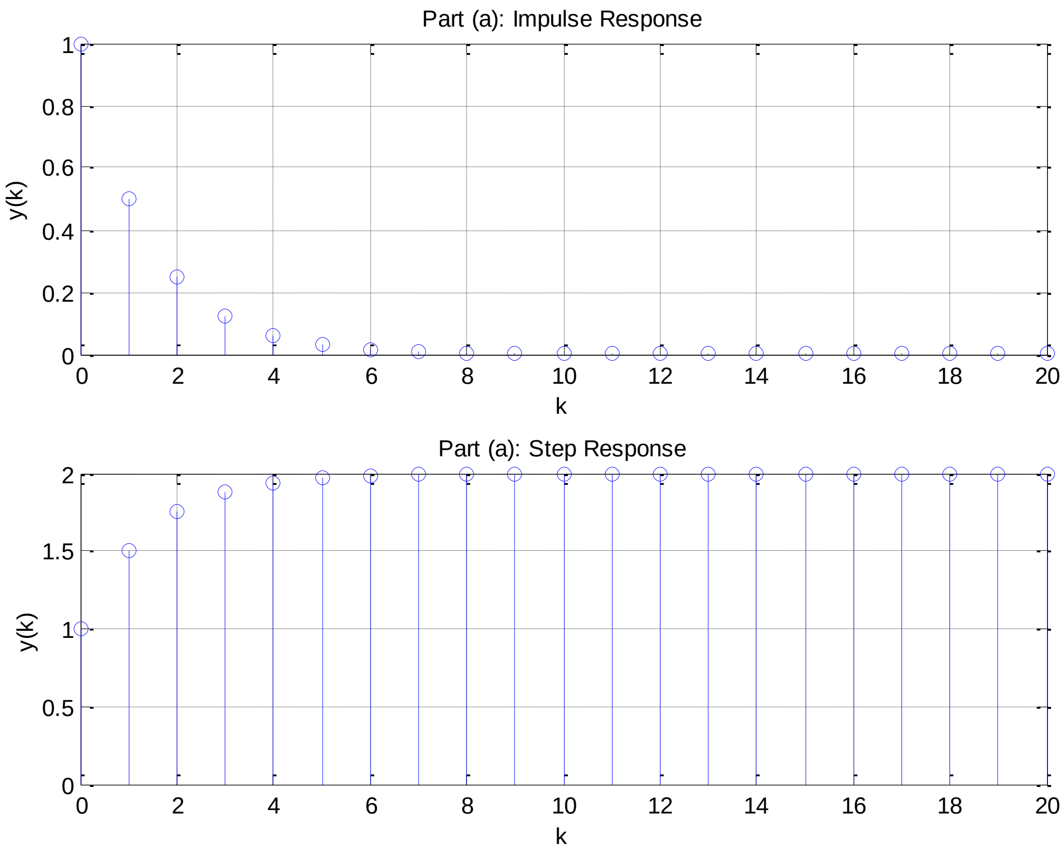
$$H_2(E) = 1 + 0.5\times\frac{1}{E} + 0.25\times\frac{1}{E^2} + 0.125\times\frac{1}{E^3} + 0.0625\times\frac{1}{E^4}$$

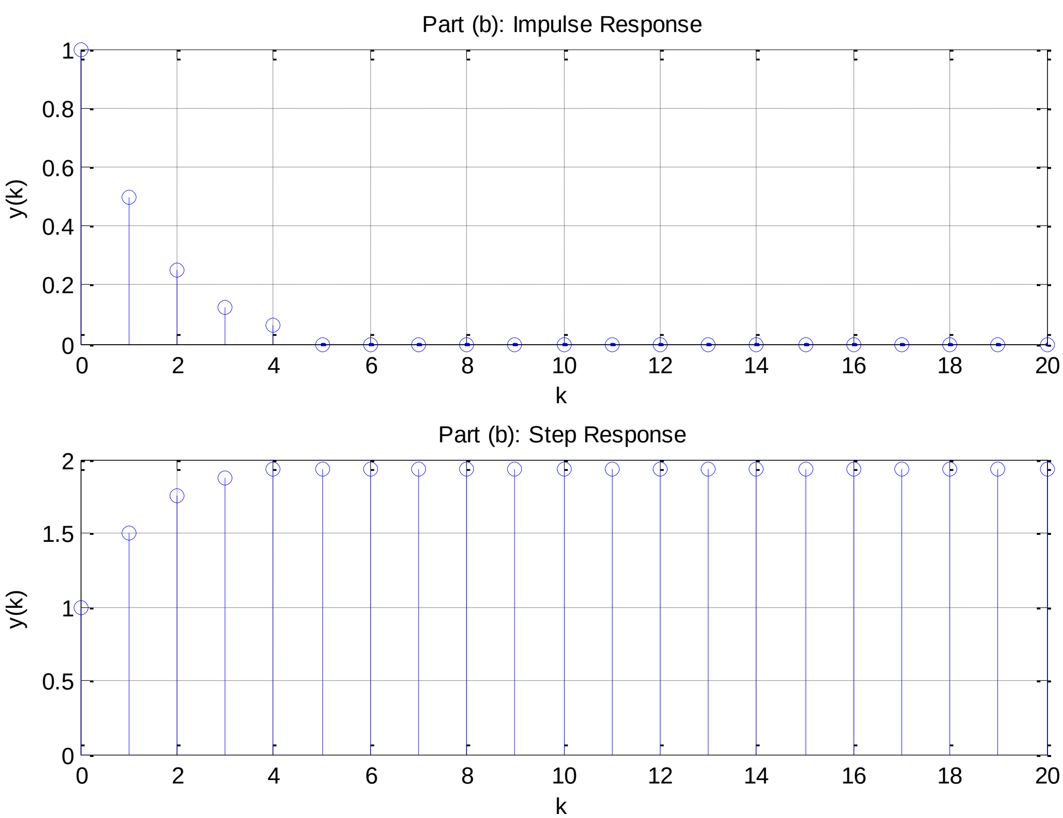
$$\Rightarrow y(k) = \left(1 + 0.5\times\frac{1}{E} + 0.25\times\frac{1}{E^2} + 0.125\times\frac{1}{E^3} + 0.0625\times\frac{1}{E^4}\right)\{x(k)\}$$

$$\Rightarrow y(k) = x(k) + 0.5x(k-1) + 0.25x(k-2) + 0.125x(k-3) + 0.0625x(k-4)$$

The simulation diagram is then given as:




Part (a): Impulse Response
y(k)
k
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20
Part (a): Step Response
y(k)
k
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
18
20


Part (b): Impulse Response
y(k)
k
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20
Part (b): Step Response
y(k)
k
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
18
20

```
%%%%%%%%%%%SIMULATION FOR IMPULSE RESPONSE AND STEP FUNCTION%%%%%%%%%%%%%%%
% Setup the variable and input
% xi = impulse input; xu = unit step input
for i = 1:21
    k(i)  = i-1;
    xu(i) = 1;
    xi(i) = 0;
end
xi(1) = 1;

% Part (a)
% Initial conditions
qi(1) = 0; qu(1) = 0;

%calculating the values
for i = 1:21
    %Impulse Response
    qi(i+1) = 0.5*qi(i) + xi(i);
    yi(i) = qi(i+1);
    %Step Response
    qu(i+1) = 0.5*qu(i) + xu(i);
    yu(i) = qu(i+1);
end

figure(1)
subplot(211),stem(k,yi,'o');grid on
title('Part (a): Impulse Response')
subplot(212),stem(k,yu,'o');grid on
title('Part (a): Step Response')

% Part (b)
% Initial conditions
q0(1) = 0; q1(1) = 0; q2(1) = 0; q3(1) = 0;
% Impulse Response
yi(1) = 0.0625*q0(1) + 0.125*q1(1) + 0.25*q2(1) + 0.5*q3(1) + xi(1);

% Calculating the values for Impulse Response
for i = 1:20
    q3(i+1) = xi(i);
    q2(i+1) = q3(i);
    q1(i+1) = q2(i);
    q0(i+1) = q1(i);

    %Impulse Response
    yi(i+1) = 0.0625*q0(i+1) + 0.125*q1(i+1) + 0.25*q2(i+1) + 0.5*q3(i+1) +
xi(i+1);
end

% Calculating the values for Step Response
for i = 1:20
    q3(i+1) = xu(i);
    q2(i+1) = q3(i);
    q1(i+1) = q2(i);
    q0(i+1) = q1(i);

    %Step Response
    yu(i+1) = 0.0625*q0(i+1) + 0.125*q1(i+1) + 0.25*q2(i+1) +...
        + 0.5*q3(i+1) + xu(i+1);
end

figure(2)
subplot(211),stem(k,yi,'o');grid on
```

```
title('Part (b): Impulse Response')
subplot(212),stem(k,yu,'o');grid on
title('Part (b): Step Response')


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```