Signals and Systems 24 October 2012
Fall 2012 Quiz: Tuesday 30 October 2012

Homework Assignment #10

1. Let $x(k+1) = Ax(k) + Bu(k)$ and $y(k) = Cx(k) + Du(k)$, with

$$A = \begin{pmatrix} 0.25 & 0.3 & 0.75 \\ 0.8 & -0.2 & 0.8 \\ 0 & 0 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \; D = 0.$$

   (a) Draw the system simulation diagram.

   (b) Determine the poles of the system transfer function.

   (c) Simulate the system on a digital computer to find the response $y(k)$ to a unit step input.

2. A study of student enrollment records for the Colleges of Engineering and Business at a certain university yields the following observations:

   - Among entering students, 35% enroll in Engineering, and 65% enroll in Business.
   - Among Engineering students at the end of each year, 65% will re-enroll for the following year in Engineering, 8% will enroll in Business, and 20% will graduate.
   - Among Business students at the end of each year, 78% will re-enroll for the following year in Business, and 20% will graduate.

   (a) Draw the simulation diagram of a discrete-time system to model the student enrollments in Engineering and Business. The output $y(k)$ should represent the total number of graduating students in year k, while the input $u(k)$ represents the total number of entering students in year k.

   (The variable $u(k)$ denotes the system input, not necessarily the unit step sequence.)

   (b) Write the dynamic (state) equations in the matrix form

$$x(k+1) = Ax(k) + Bu(k)$$

   and the output equation in the form

$$y(k) = Cx(k).$$

   (You simply need to identify the matrices A, B, and C.)

   (c) Find the z-domain transfer function relating $Y(z)$ and $U(z)$.

3. (a) Determine whether the enrollment model in Problem 2 is stable.

   (b) If the university has been admitting exactly 100 new students per year for many years, determine the number of students that graduate each year according to the model.