Signals and Systems
Fall 2012

Solution to Homework Assignment #14

1. $X(j\omega) = \dfrac{j\omega}{\omega^2 - 3j\omega - 2} = F\{x(t)\}$

(a) $y(t) = x(t+1) \Rightarrow Y(j\omega) = e^{j\omega} X(j\omega)$

$z(t) = y(2t) = x(2t+1) \Rightarrow Z(j\omega) = \dfrac{1}{2} Y\left(\frac{j\omega}{2}\right) = \dfrac{1}{2} e^{j\omega/2} X\left(\frac{j\omega}{2}\right)$

$$\Rightarrow \boxed{F\{x(2t+1)\} = \frac{1}{2} e^{j\omega/2} \frac{-\frac{j\omega}{2}}{\left(\frac{j\omega}{2}\right)^2 + 3\left(\frac{j\omega}{2}\right) + 2}}$$

(b) $y(t) = \dfrac{dx(t)}{dt}$

$$\Rightarrow \boxed{F\{dx/dt\} = F\{y(t)\} = j\omega X(j\omega) = \frac{-(j\omega)^2}{(j\omega)^2 + 3(j\omega) + 2}}$$

(c) $y(t) = x(t)\cos(t) = x(t)\left[\dfrac{1}{2} e^{jt} + \dfrac{1}{2} e^{-jt}\right]$

$$Y(j\omega) = \frac{1}{2} X(j(\omega+1)) + \frac{1}{2} X(j(\omega-1))$$

$$= \frac{1}{2}\left[\frac{-j(\omega+1)}{\left(j(\omega+1)^2 + 3j(\omega+1) + 2\right)} + \frac{-j(\omega-1)}{\left(j(\omega-1)^2 + 3j(\omega-1) + 2\right)}\right]$$

$$\Rightarrow \boxed{F\{x(t)\cos(t)\} = \frac{-2j\omega\left[(j\omega)^2 + 3\right]}{(j\omega)^4 + 6(j\omega)^3 + 15(j\omega)^2 + 18(j\omega) + 9}}$$

2.

(a) $F(j\omega) = e^{-\omega^2} = \dfrac{a}{\sqrt{\pi}} \dfrac{\sqrt{\pi}}{a} e^{-(\omega^2/4a^2)},\ a = \dfrac{1}{2}$

$$\Rightarrow \boxed{f(t) = \frac{a}{\sqrt{\pi}} e^{-a^2 t^2} = \frac{1}{2\sqrt{\pi}} e^{-t^2/4}}$$

(b) $F(j\omega)=\dfrac{e^{j\omega}}{-j\omega+1}\Rightarrow F(-j\omega)=\dfrac{e^{-j\omega}}{j\omega+1}$

$\Rightarrow\ f(-t)=e^{-(t-1)}u_s(t-1)$

$\Rightarrow\ \boxed{f(t)=e^{(t+1)}u_s(-(t+1))}$

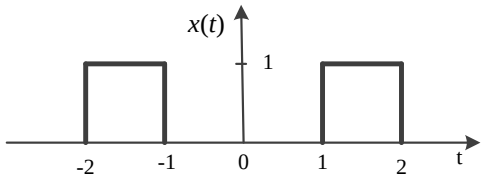
(c) $F(j\omega)=\dfrac{j\omega}{j\omega+1}$

$\Rightarrow\ f(t)=\dfrac{\mathrm{d}}{\mathrm{d}t}\left[e^{-t}u_s(t)\right]=-e^{-t}u_s(t)+e^{-t}\delta(t)$

$\Rightarrow\ \boxed{f(t)=-e^{-t}u_s(t)+\delta(t)}$

3.

(a) $x(t)=\begin{cases}1 & -2<t<-1\\ 1 & 1<t<2\\ 0 & \text{otherwise}\end{cases}$



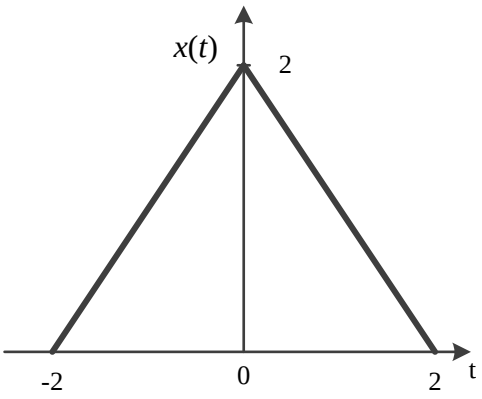
Let $y(t)=\begin{cases}1, & -1/2<t<1/2\\ 0, & \text{otherwise}\end{cases}$ $\Rightarrow Y(j\omega)=\dfrac{\sin(\omega/2)}{(\omega/2)}$

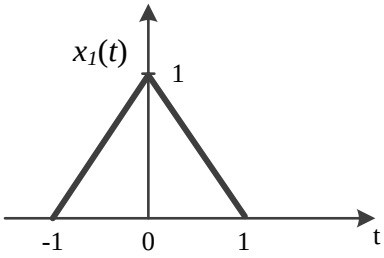
Then $x(t)=y(t+1.5)+y(t-1.5)$

$\Rightarrow X(j\omega)=e^{j1.5\omega}Y(j\omega)+e^{-j1.5\omega}Y(j\omega)$

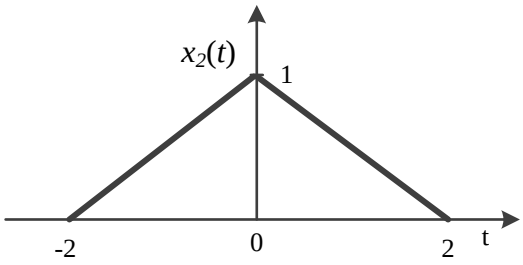
Finally, $\boxed{X(j\omega)=\dfrac{2\cos(1.5\omega)\sin(0.5\omega)}{0.5\omega}}$

(b) $x(t)=\begin{cases}2+t, & -2<t<0\\ 2-t, & 0<t<2\\ 0 & \text{otherwise}\end{cases}$

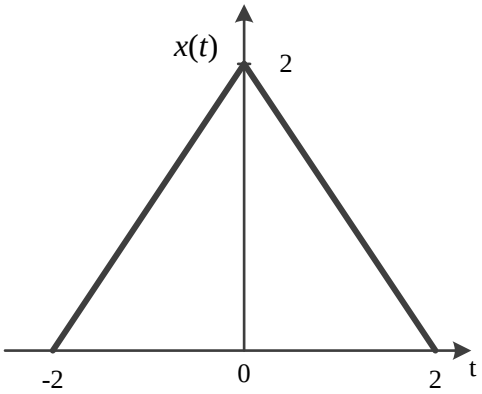




$$\Rightarrow X_1(j\omega)=\left(\frac{\sin(\omega/2)}{(\omega/2)}\right)^2$$



$$\Rightarrow X_2(j\omega)=2\left(\frac{\sin(\omega)}{\omega}\right)^2$$



$$\Rightarrow X(j\omega)=4\left(\frac{\sin(\omega)}{\omega}\right)^2$$