Signals and Systems 14 November 2012
Fall 2012 Quiz: Wednesday 21 November 2012

Homework Assignment #13

1. Compute the coefficients of the exponential Fourier series expansion for the functions given on the interval [0,T]. For each case, sketch the amplitude spectrum $|\alpha_n|$ and sketch the function represented by the Fourier series for $-T \le t \le 2T$.

(a) $$x(t) = \begin{cases} 2At, & 0 \le t \le T/2 \\ -2A(t-T), & T/2 \le t \le T \end{cases}$$ (Triangular wave)

(b) $$x(t) = \begin{cases} A\sin(2\pi t/T), & 0 \le t \le T/2 \\ 0, & T/2 \le t \le T \end{cases}$$ (Half-wave rectified sinusoid)

2. Let $x(t)$ be a square wave. Use a digital computer to plot an approximation to $x(t)$ consisting of the first 5, 10, and 15 terms of the Fourier series for a square wave, as given in Table 3.1.

3. Repeat problem 2 if $x(t)$ is a triangular wave. For which wave (square or triangular) do the approximations seem closer to the actual wave?

4. The Fourier spectrum (exponential Fourier series coefficients) of a certain periodic waveform is given by

$$\alpha_n = \begin{cases} \dfrac{2}{\pi}, & n = 0 \\ \dfrac{1}{2}, & n = \pm 1 \\ \dfrac{-2\cos(n\pi/2)}{\pi(n^2-1)}, & n = \pm 2, \pm 3, \ldots \end{cases}$$

[Note that the spectrum is real -- that is, the amplitude spectrum is as given, and the phase spectrum is zero.]

(a) Write the trigonometric Fourier series for the corresponding function $x(t)$.

(b) Use a computer program to calculate and plot an approximation to the function $x(t)$. Use as many terms of the series as seems necessary.