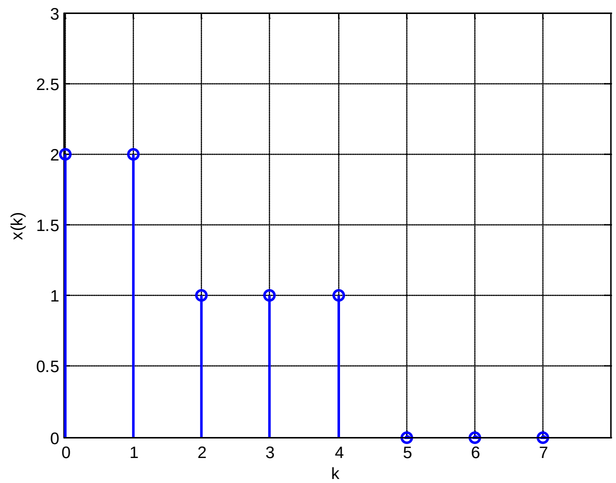
Signals and Systems
Fall 2012

Solution to Homework Assignment #2

1. Sketch of the following discrete-time signals.

(a) $x(k) = 2u_s(k) - u_s(k-2) - u_s(k-5)$

It is not hard to see that when either k<0 or k>4, $x(k)$=0. The signal $x(k)$ is plotted below:



(b) $x(k) = \delta(k) + \delta(k-1) + u_s(k) - u_s(k-5)$

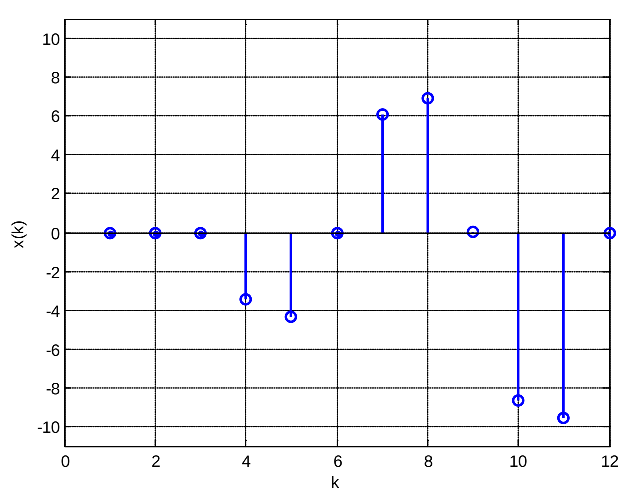
Same plot as in (a).

(c) $x(k) = 2[u_s(k) - u_s(k-2)] + [u_s(k-2) - u_s(k-5)]$

Same plot as in (a).

(d) $x(k) = k\sin(k\pi/3) \cdot [u_s(k-3) - u_s(k-12)]$

It is straightforward to observe that when either k<4 or k>12, $x(k)$=0. The signal $x(k)$ is plotted below:



(e) $x(k) = \mathrm{Re}\{(0.4 + j0.6)^k \cdot u_s(k)\}$

Let $z = 0.4 + j0.6$. Using polar transform, we have

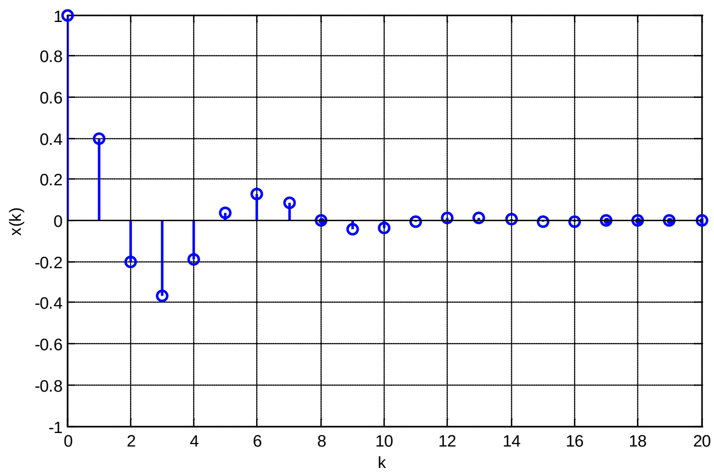
$$z = 0.4 + j0.6 = 0.7211\angle 0.9828 \text{ rad } = 0.7211 e^{j0.9828 \text{ rad}}$$

(Note: Unit of phase is rad).

It then follows that:

$$z^k = (0.7211)^k e^{j(0.9828 \text{ rad})k} = \underbrace{(0.7211)^k \cos 0.9828k}_{\mathrm{Re}\{(0.4+j0.6)^k\}} + j(0.7211)^k \sin 0.9828k$$

The real part of $x(k)$ is sketched below. Note that $x(k)$=0 when k<0.



It can be observed that the signal is exponentially decaying. It is because the magnitude 0.7211 is smaller than 1 (as similar to what we studied in class).

2. For each of the input-output relations, determine whether the system is linear or nonlinear, time-invariant or time-varying, causal or non-causal.

(a) $y(t) = 2x(t) + \int_{-\infty}^{t} 5x(\tau)d\tau$: linear, time-invariant, causal

(b) $y(t) = 2tx(t) + \int_{-\infty}^{t} 5x(\tau)d\tau$: linear, <u>time-varying</u>, causal

(c) $y(t) = \int_{t-2}^{t+2} 5x(\tau)d\tau$: linear, time-invariant, <u>non-causal</u>

(d) $y(t) = \begin{cases} \int_{2}^{t} 5x(\tau)d\tau, & t \geq 2 \\ 0, & t < 2 \end{cases}$: linear, <u>time-varying</u>, causal

(e) $y(t) = \int_{-\infty}^{t} \tau\, |\, x(\tau)\, |\, d\tau$: <u>nonlinear</u>, <u>time-varying</u>, causal

3. Use fixed-point iterations of the form

$$x(k+1) = f(x(k)).$$

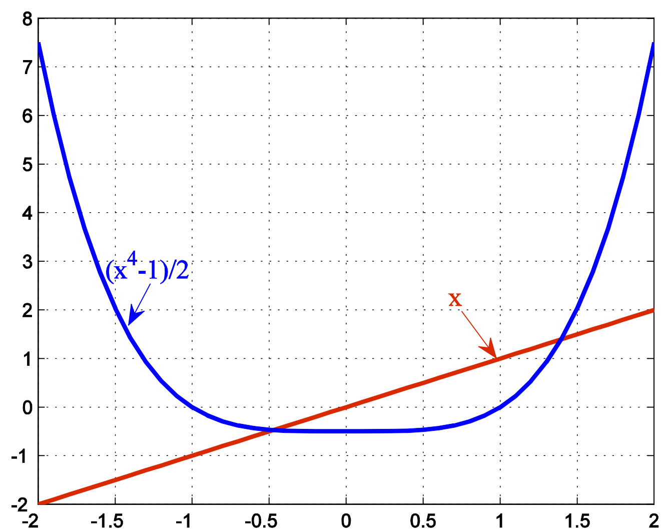
to find <u>both</u> real solutions of the nonlinear equation

$$x^4 - 2x - 1 = 0 \Rightarrow x^4 = 2x + 1.$$

In order to isolate one instance of x, we first consider the following equation:

$$x = \frac{x^4 - 1}{2}$$

The first step one should do is to draw a graph to narrow down some possible solutions. In the plot below, the two functions x and $\frac{x^4 - 1}{2}$ are drawn.



Obviously, we have two solutions. Now, consider the following system: $x_{k+1} = \frac{x_k^4 - 1}{2}$ and let $x_0 = 1$. We then have:

$x_1 = \left(x_0^4 - 1\right)/2 = 0$

$x_2 = \left(x_1^4 - 1\right)/2 = -1/2$

$x_3 = \left(x_2^4 - 1\right)/2 = -0.4688$

$x_4 = \left(x_3^4 - 1\right)/2 = -0.4759$

$x_5 = \left(x_4^4 - 1\right)/2 = -0.4744$

$x_6 = \left(x_5^4 - 1\right)/2 = -0.4747$

$x_7 = \left(x_6^4 - 1\right)/2 = -0.4746$

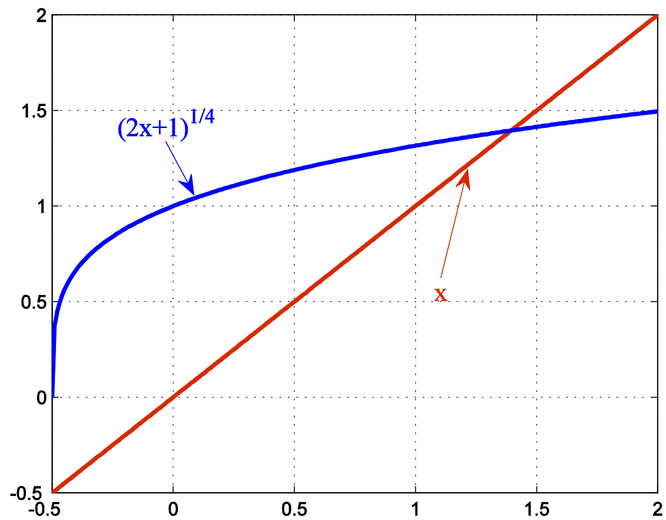
$x_8 = \left(x_7^4 - 1\right)/2 = -0.4746$

Hence, one real solution is $\underline{x = -0.4746}$

You may try different initial values, but you will see that either they end up with the same solution we obtained above or they diverge. As such, we need to check another equation to find the second solution. In particular, we can isolate the other instance of x as follows:

$$x = \sqrt[4]{2x+1}$$

The two functions x and $\sqrt[4]{2x+1}$ are sketched below:



Now, consider the system $x_{k+1} = \sqrt[4]{2x_k + 1}$ and let $x_0 = 1$. We have:

$x_1 = \sqrt[4]{2x_0 + 1} = 1.3161$

$x_2 = \sqrt[4]{2x_1 + 1} = 1.3805$

$x_3 = \sqrt[4]{2x_2 + 1} = 1.3926$

$x_4 = \sqrt[4]{2x_3 + 1} = 1.3948$

$x_5 = \sqrt[4]{2x_4 + 1} = 1.3952$

$x_6 = \sqrt[4]{2x_5 + 1} = 1.3953$

$x_7 = \sqrt[4]{2x_6 + 1} = 1.3953$

We finally obtain another real solution, which is $\underline{x = 1.3953}$