

CHAPTER 1—PROBLEM SOLUTIONS

1.1 (a) $I = \frac{V}{R} = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA}$

(b) $R = \frac{V}{I} = \frac{10 \text{ V}}{1 \text{ mA}} = 10 \text{ k}\Omega$

(c) $V = IR = 10 \text{ mA} \times 10 \text{ k}\Omega = 100 \text{ V}$

(d) $I = \frac{V}{R} = \frac{10 \text{ V}}{100 \text{ }\Omega} = 0.1 \text{ A}$

Note: Volts, millamps, and kilo-ohms constitute a consistent set of units.

1.2 (a) $P = I^2R = (30 \times 10^{-3})^2 \times 1 \times 10^3 = 0.9 \text{ W}$

Thus, R should have a 1-W rating.

(b) $P = I^2R = (40 \times 10^{-3})^2 \times 1 \times 10^3 = 1.6 \text{ W}$

Thus, the resistor should have a 2-W rating.

(c) $P = I^2R = (3 \times 10^{-3})^2 \times 10 \times 10^3 = 0.09 \text{ W}$

Thus, the resistor should have a $\frac{1}{8}$ -W rating.

(d) $P = I^2R = (4 \times 10^{-3})^2 \times 10 \times 10^3 = 0.16 \text{ W}$

Thus, the resistor should have a $\frac{1}{4}$ -W rating.

(e) $P = V^2/R = 20^2/(1 \times 10^3) = 0.4 \text{ W}$

Thus, the resistor should have a $\frac{1}{2}$ -W rating.

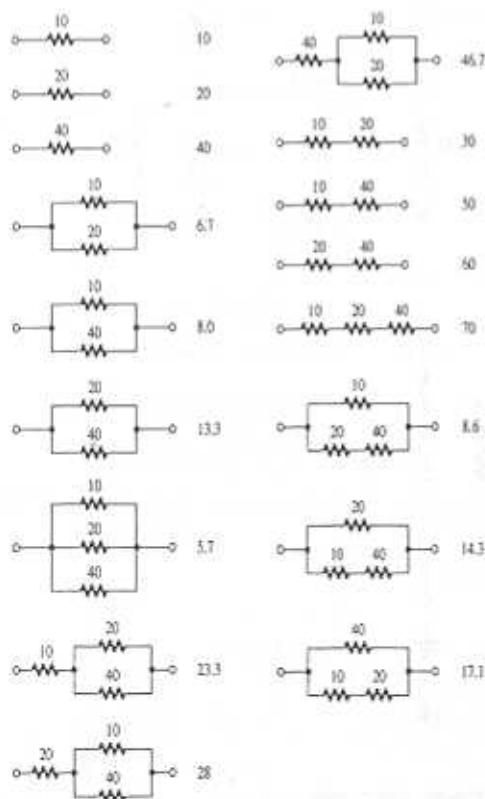
(f) $P = V^2/R = 11^2/(1 \times 10^3) = 0.121 \text{ W}$

Thus, a rating of $\frac{1}{8}$ W should theoretically suffice though $\frac{1}{4}$ W would be prudent to allow for consistent tolerances and measurement errors.

- 1.3 (a) $V = IR = 10 \text{ mA} \times 1 \text{ k}\Omega = 10 \text{ V}$
 $P = I^2 R = (10 \text{ mA})^2 \times 1 \text{ k}\Omega = 100 \text{ mW}$
- (b) $R = V/I = 10 \text{ V}/1 \text{ mA} = 10 \text{ k}\Omega$
 $P = VI = 10 \text{ V} \times 1 \text{ mA} = 10 \text{ mW}$
- (c) $I = P/V = 1 \text{ W}/10 \text{ V} = 0.1 \text{ A}$
 $R = V/I = 10 \text{ V}/0.1 \text{ A} = 100 \Omega$
- (d) $V = P/I = 0.1 \text{ W}/10 \text{ mA} = 100 \text{ mW}/10 \text{ mA} = 10 \text{ V}$
 $R = V/I = 10 \text{ V}/10 \text{ mA} = 1 \text{ k}\Omega$
- (e) $P = I^2 R \Rightarrow I = \sqrt{P/R}$
 $I = \sqrt{1000 \text{ mW}/1 \text{ k}\Omega} = 31.6 \text{ mA}$
 $V = IR = 31.6 \text{ mA} \times 1 \text{ k}\Omega = 31.6 \text{ V}$

Note: V, mA, kΩ, and mW constitute a consistent set of units.

1.4



Thus, there are 17 possible resistance values.

1.5 Shunting the 10 kΩ by a resistor of value R result in the combination having a resistance R_{eq} .

$$R_{eq} = \frac{10R}{R+10}$$

Thus, for a 1% reduction,

$$\frac{R}{R+10} = 0.99 \Rightarrow R = 990 \text{ k}\Omega$$

For a 5% reduction,

$$\frac{R}{R+10} = 0.95 \Rightarrow R = 190 \text{ k}\Omega$$

For a 10% reduction,

$$\frac{R}{R+10} = 0.90 \Rightarrow R = 90 \text{ k}\Omega$$

For a 50% reduction,

$$\frac{R}{R+10} = 0.50 \Rightarrow R = 10 \text{ k}\Omega$$

Shunting the 10 kΩ by

(a) 1 MΩ result in

$$R_{eq} = \frac{10 \times 1000}{1000 + 10} = \frac{10}{1.01} = 9.9 \text{ k}\Omega, \text{ a } 1\% \text{ reduction};$$

(b) 100 kΩ results in

$$R_{eq} = \frac{10 \times 100}{100 + 10} = \frac{10}{1.1} = 9.09 \text{ k}\Omega, \text{ a } 9.1\% \text{ reduction};$$

(c) 10 kΩ results in

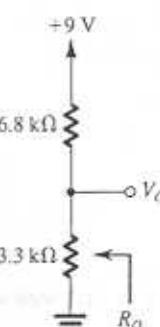
$$R_{eq} = \frac{10}{10 + 10} = 5 \text{ k}\Omega, \text{ a } 50\% \text{ reduction},$$

$$1.6 V_O = V_{DD} \frac{R_2}{R_1 + R_2}$$

To find R_O , we short circuit V_{DD} and look back into node X,

$$R_O = R_2 // R_1 = \frac{R_1 R_2}{R_1 + R_2}$$

1.7



$$V_O = 9 \frac{3.3}{3.3 + 6.8}$$

$$= 2.94 \text{ V}$$

$$R_O = 2.22 \text{ k}\Omega$$

For $\pm 5\%$ resistor tolerance the extreme values of V_O are

$$V_{O,\text{low}} = 9 \frac{3.3(1 - 0.05)}{3.3(1 - 0.05) + 6.8(1 + 0.05)}$$

$$= 2.75 \text{ V}$$

$$V_{O,\text{high}} = 9 \frac{3.3(1 + 0.05)}{3(1 + 0.05) + 6.8(1 - 0.05)}$$

$$= 3.14 \text{ V}$$

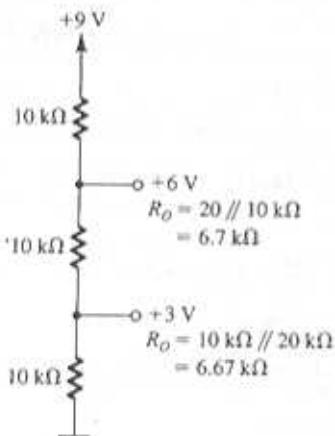
The extreme values of R_O are

$$R_{O,\text{low}} = \frac{3.3(1 - 0.05) \times 6.8(1 - 0.05)}{3.3(1 - 0.05) + 6.8(1 - 0.05)}$$

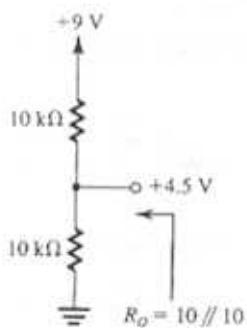
$$= 2.22(1 - 0.05) = 2.11 \text{ k}\Omega$$

$$R_{O,\text{high}} = 2.22(1 + 0.05) = 2.33 \text{ k}\Omega$$

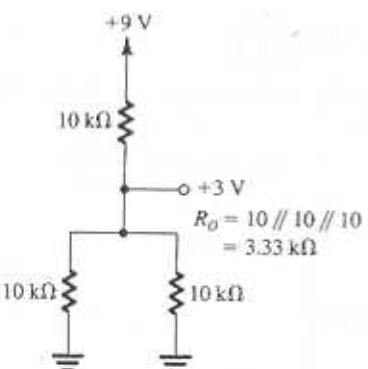
1.8



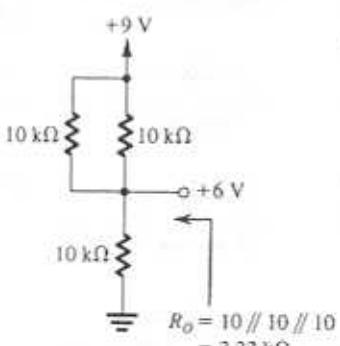
(a)



(b)



(c)



(d)

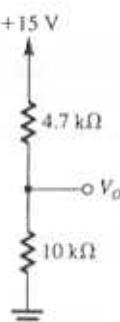
Voltages generated:

+3 V (two ways: (a) and (c) with (c) having lower output resistance)

+4.5 V (b)

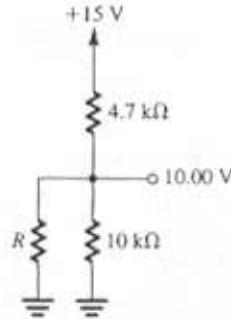
+6 V (two ways: (a) and (d) with (d) having a lower output resistance)

1.9



$$V_O = 15 \frac{10}{10 + 4.7} = 10.2 \text{ V}$$

To reduce V_O to 10.00 V we shunt the 10-kΩ resistor by a resistor R whose value is such that $10 \parallel R = 2 \times 4.7$.



Thus

$$\frac{1}{10} + \frac{1}{R} = \frac{1}{9.4}$$

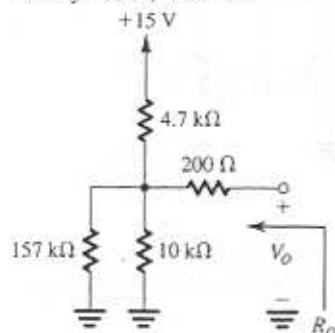
$$\Rightarrow R = 156.7 \approx 157 \text{ k}\Omega$$

Now,

$$R_O = 10 \text{ k}\Omega \parallel R \parallel 4.7 \text{ k}\Omega$$

$$= 9.4 \parallel 4.7 = \frac{9.4}{3} = 3.133 \text{ k}\Omega$$

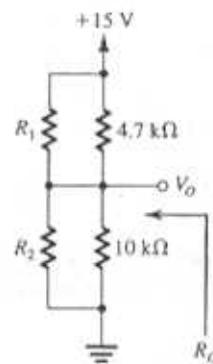
To make $R_O = 3.33$ we add a series resistance of approximately 200 Ω, as shown.



To obtain $V_O = 10.00 \text{ V}$ and $R_O = 3 \text{ k}\Omega$ we have to shunt both the 4.7-kΩ and the 10-kΩ resistors as shown. To yield an output voltage $V_O = 10.00 \text{ V}$ we must have

$$\frac{(R_2 \parallel 10)}{R'_2} = \frac{2(R_1 \parallel 4.7)}{R'_1}$$

$$R'_2 = 2R'_1 \quad (1)$$



For $R_O = 3 \text{ k}\Omega$ we must have

$$R'_1 \parallel R'_2 = 3 \quad (2)$$

Solving (1) and (2) yields

$$R'_1 = 4.5 \text{ k}\Omega$$

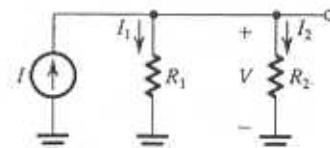
$$R'_2 = 9.0 \text{ k}\Omega$$

which can be used to find R_1 and R_2 respectively,

$$R_1 = 157 \text{ k}\Omega$$

$$R_2 = 90 \text{ k}\Omega$$

1.10



$$V = I(R_1 \parallel R_2)$$

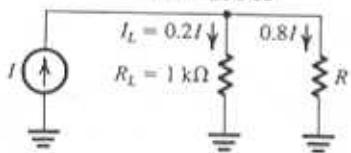
$$= I \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} = I \frac{R_2}{R_1 + R_2}$$

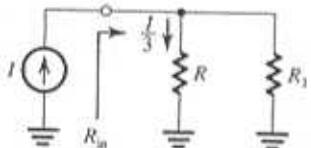
$$I_2 = \frac{V}{R_2} = I \frac{R_1}{R_1 + R_2}$$

1.11 Connect a resistor R in parallel with R_L . To make $I_L = 0.2I$ (and thus the current through R , $0.8I$), R should be such

$$0.2I \times 1 \text{ k}\Omega = 0.8I R \\ \Rightarrow R = 250 \text{ }\Omega$$



1.12



To make the current through R equal to $I/3$ we shunt R by a resistance R_1 of value such that the current through it will be $2I/3$; thus

$$\frac{1}{3}R = \frac{2I}{3}R_1 \Rightarrow R_1 = \frac{R}{2}$$

The input resistance of the divider, R_{in} , is

$$R_{in} = R \parallel R_1 = R \parallel \frac{R}{2} = \frac{1}{3}R$$

Now if R_1 is 10% too high, i.e.,

$$R_1 = 1.1 \frac{R}{2}$$

the problem can be solved in two ways:

(a) Connect a resistor R_2 across R_1 of value such that $R_1 \parallel R_2 = R/2$, thus

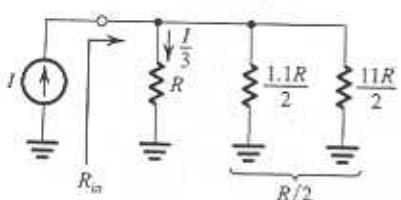
$$\frac{R_2(1.1R/2)}{R_2 + (1.1R/2)} = \frac{R}{2}$$

$$1.1R_2 = R_2 + \frac{1.1R}{2}$$

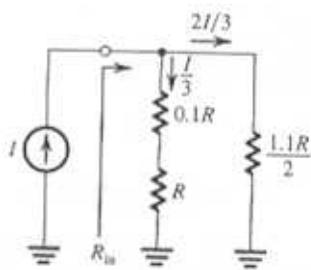
$$\Rightarrow R_2 = \frac{11R}{2} = 5.5R$$

$$R_{in} = R \parallel \frac{1.1R}{2} \parallel \frac{11R}{2}$$

$$= R \parallel \frac{R}{2} = \frac{R}{3}$$



(b) Connect a resistor in series with the load resistor R so as to raise the resistance of the load branch by 10%, thereby restoring the current division ratio to its desired value. The added series resistance must be 10% of R i.e., $0.1R$.

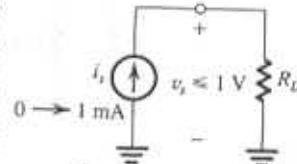


$$R_{in} = 1.1R \parallel \frac{1.1R}{2} \\ = \frac{1.1R}{3}$$

i.e., 10% higher than in case (a).

1.13 If $R_L = 10 \text{ k}\Omega$

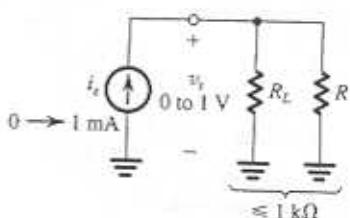
then a voltage of 0 to 10 V may develop across the source. To limit the voltage to the specified maximum of 1 V, we have to shunt R_L with a resistor R whose value is such that the parallel combination of R_L and R is $\leq 1 \text{ k}\Omega$. Thus,



$$\frac{RR_L}{R + R_L} \leq 1$$

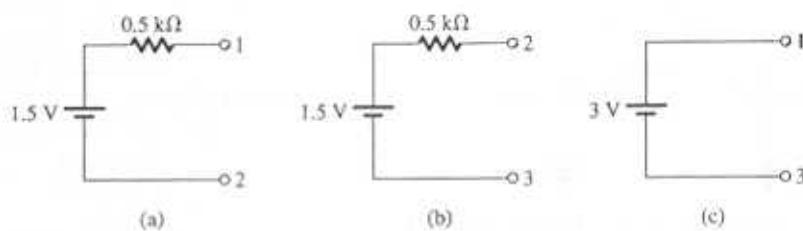
$$R \leq 1.111 \text{ k}\Omega$$

$$\Rightarrow R = 1.1 \text{ k}\Omega$$

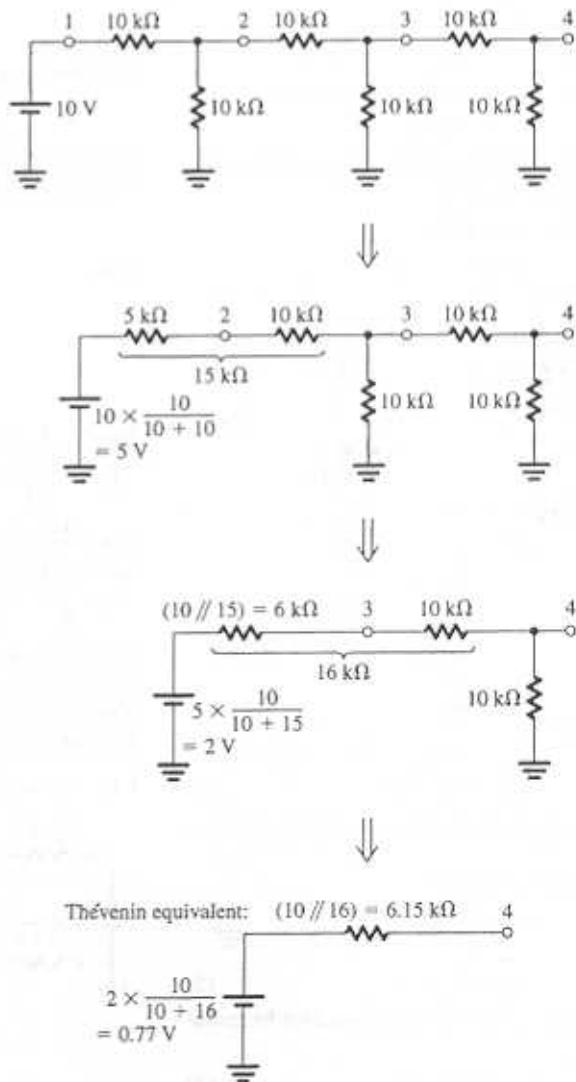


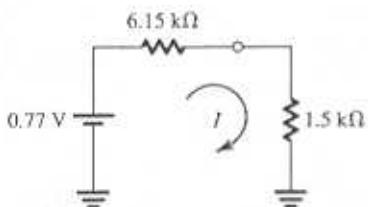
The resulting circuit, utilizing only one additional resistor of value $1.1 \text{ k}\Omega$ creates a current divider across the source.

1.14



1.15





Now, when a resistance of $1.5 \text{ k}\Omega$ is connected between 4 and ground,

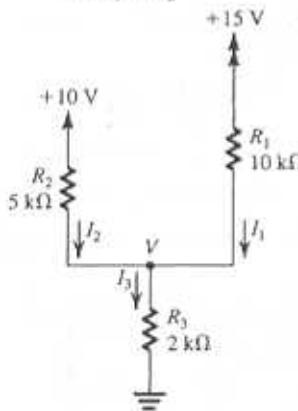
$$I = \frac{0.77}{6.15 + 1.5} \\ = 0.1 \text{ mA}$$

1.16 (a) Node equation at the common node yields

$$I_3 = I_1 + I_2$$

Using the fact that the sum of the voltage drops across R_1 and R_3 equals 15 V, we write

$$15 = I_1 R_1 + I_3 R_3 \\ = 10I_1 + (I_1 + I_2) \times 2 \\ = 12I_1 + 2I_2$$



That is,

$$12I_1 + 2I_2 = 15 \quad (1)$$

Similarly, the voltage drops across R_2 and R_3 add up to 10 V, thus

$$10 = I_2 R_2 + I_3 R_3 \\ = 5I_2 + (I_1 + I_2) \times 2$$

which yields

$$2I_1 + 7I_2 = 10 \quad (2)$$

Equations (1) and (2) can be solved together by multiplying (2) by 6.

$$12I_1 + 42I_2 = 60 \quad (3)$$

Now, subtracting (1) from (3) yields

$$40I_2 = 45 \\ \Rightarrow I_2 = 1.125 \text{ mA}$$

Substituting in (2) gives

$$2I_1 = 10 - 7 \times 1.125 \text{ mA} \\ \Rightarrow I_1 = 1.0625 \text{ mA}$$

$$I_3 = I_1 + I_2 \\ = 1.0625 + 1.125 \\ = 1.1875 \text{ mA}$$

$$V = I_3 R_3 \\ = 1.1875 \times 2 = 2.3750 \text{ V}$$

To summarize:

$$I_1 = 1.06 \text{ mA} \quad I_2 = 1.13 \text{ mA} \\ I_3 = 1.19 \text{ mA} \quad V = 2.38 \text{ V}$$

(b) A node equation at the common node can be written in terms of V as

$$\frac{15 - V}{R_1} + \frac{10 - V}{R_2} = \frac{V}{R_3}$$

Thus,

$$\frac{15 - V}{10} + \frac{10 - V}{5} = \frac{V}{2} \\ \Rightarrow 0.8V = 3.5 \\ \Rightarrow V = 2.375 \text{ V}$$

Now, I_1 , I_2 , and I_3 can be easily found as

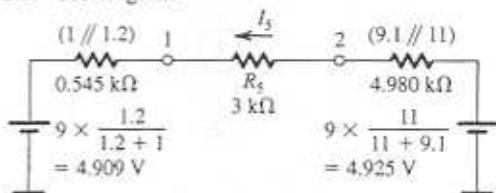
$$I_1 = \frac{15 - V}{10} = \frac{15 - 2.375}{10} = 1.0625 \text{ mA} = 1.06 \text{ mA}$$

$$I_2 = \frac{10 - V}{5} = \frac{10 - 2.375}{5} = 1.125 \text{ mA} = 1.13 \text{ mA}$$

$$I_3 = \frac{V}{R_3} = \frac{2.375}{2} = 1.1875 \text{ mA} = 1.19 \text{ mA}$$

Method (b) is much preferred; faster, more insightful and less prone to errors. In general, one attempts to identify the least possible number of variables and write the corresponding minimum number of equations.

1.17 See diagram

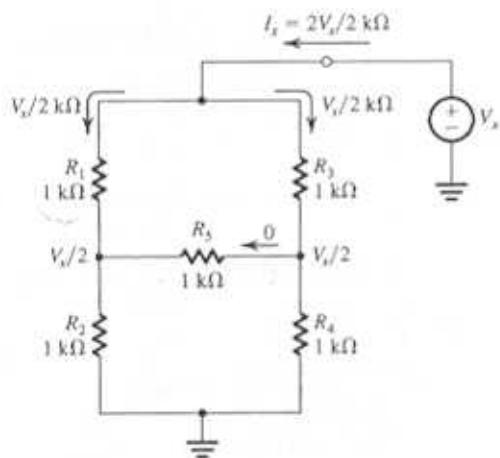


$$I_5 = \frac{4.925 - 4.909}{4.98 + 3 + 0.545} = 1.88 \mu\text{A}$$

$$V_5 = 1.88 \mu\text{A} \times 3 \text{ k}\Omega = 5.64 \text{ mV}$$

1.18 From the symmetry of the circuit, there will be no current in R_5 . (Otherwise the symmetry would be violated.) Thus each branch will carry a current $V_x/2 \text{ k}\Omega$ and I_x will be the sum of the two currents.

$$I_x = \frac{2V_x}{2 \text{ k}\Omega} = \frac{V_x}{1 \text{ k}\Omega}$$



Thus,

$$R_{eq} \equiv \frac{V_x}{I_x} = 1 \text{ k}\Omega$$

Now, if R_4 is raised to $1.2 \text{ k}\Omega$ the symmetry will be broken. To find I_x we use Thévenin's theorem as follows:

$$I_5 = \frac{0.545V_x - 0.5V_x}{0.5 + 1 + 0.545} = 0.022V_x$$

$$V_1 = \frac{V_x}{2} + 0.022V_x \times 0.5 \\ = 0.5V_x \times 1.022 = 0.511V_x$$

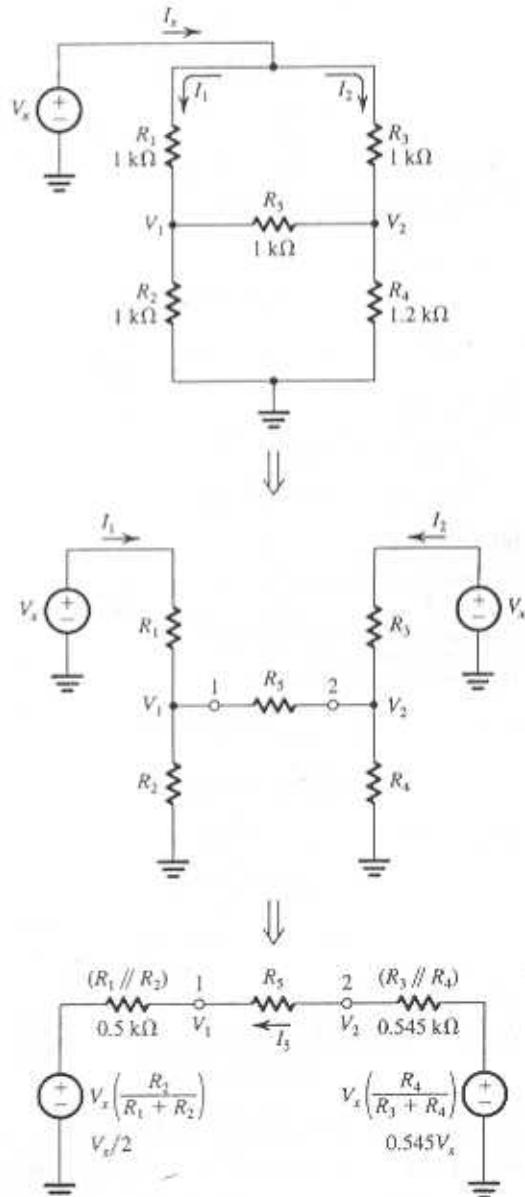
$$V_2 = V_1 + I_5 R_5 = 0.533V_x$$

$$I_1 = \frac{V_x - V_1}{1 \text{ k}\Omega} = 0.489V_x$$

$$I_2 = \frac{V_x - V_2}{1 \text{ k}\Omega} = 0.467V_x$$

$$I_x = I_1 + I_2 = 0.956V_x$$

$$\Rightarrow R_{eq} \equiv \frac{V_x}{I_x} = 1.05 \text{ k}\Omega$$



1.19 (a) $T = 10^{-4} \text{ ms} = 10^{-7} \text{ s}$

$$f = \frac{1}{T} = 10^7 \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^7 \text{ rad/s}$$

(b) $f = 1 \text{ GHz} = 10^9 \text{ Hz}$

$$T = \frac{1}{f} = 10^{-9} \text{ s}$$

$$\omega = 2\pi f = 6.28 \times 10^9 \text{ rad/s}$$

(c) $\omega = 6.28 \times 10^2 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 10^{-2} \text{ s}$$

(d) $T = 10 \text{ s}$

$$f = \frac{1}{T} = 10^{-1} \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^{-1} \text{ rad/s}$$

(e) $f = 60 \text{ Hz}$

$$T = \frac{1}{f} = 1.67 \times 10^{-2} \text{ s}$$

$$\omega = 2\pi f = 3.77 \times 10^2 \text{ rad/s}$$

(f) $\omega = 1 \text{ krad/s} = 10^3 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = 1.59 \times 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 6.28 \times 10^{-3} \text{ s}$$

(g) $f = 1900 \text{ MHz} = 1.9 \times 10^9 \text{ Hz}$

$$T = \frac{1}{f} = 0.526 \times 10^{-9} \text{ s}$$

$$\omega = 2\pi f = 1.194 \times 10^9 \text{ rad/s}$$

1.20 (a) $Z = 1 \text{ k}\Omega$ at all frequencies

(b) $Z = 1/j\omega C = -j \frac{1}{2\pi f \times 10 \times 10^{-12}}$

$$\text{At } f = 60 \text{ Hz}, \quad Z = -j265 \text{ k}\Omega$$

$$\text{At } f = 100 \text{ kHz}, \quad Z = -j159 \text{ }\Omega$$

$$\text{At } f = 1 \text{ GHz}, \quad Z = -j0.016 \text{ }\Omega$$

(c) $Z = 1/j\omega C = -j \frac{1}{2\pi f \times 2 \times 10^{-12}}$

$$\text{At } f = 60 \text{ Hz}, \quad Z = -j1.33 \text{ G}\Omega$$

$$\text{At } f = 100 \text{ kHz}, \quad Z = -j0.8 \text{ M}\Omega$$

$$\text{At } f = 1 \text{ GHz}, \quad Z = -j79.6 \text{ }\Omega$$

(d) $Z = j\omega L = j2\pi f L = j2\pi f \times 10 \times 10^{-3}$

$$\text{At } f = 60 \text{ Hz}, \quad Z = j3.77 \text{ }\Omega$$

$$\text{At } f = 100 \text{ kHz}, \quad Z = j6.28 \text{ k}\Omega$$

$$\text{At } f = 1 \text{ GHz}, \quad Z = j62.8 \text{ M}\Omega$$

1.21 (a) $Z = R + \frac{1}{j\omega C}$

$$= 10^3 + \frac{1}{j2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}} \\ = (1 - j1.59) \text{ k}\Omega$$

(b) $Y = \frac{1}{R} + j\omega C$

$$= \frac{1}{10^3} + j2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6} \\ = 10^{-3}(1 + j0.628) \text{ S}$$

$$Z = \frac{1}{Y} = \frac{1000}{1 + j0.628}$$

$$= \frac{1000(1 - j0.628)}{1 + 0.628^2} \\ = (717.2 - j450.4) \text{ }\Omega$$

(c) $Y = \frac{1}{R} + j\omega C$

$$= \frac{1}{100 \times 10^3} + j2\pi \times 10 \times 10^3 \times 100 \times 10^{-12} \\ = 10^{-5}(1 + j0.628)$$

$$Z = \frac{10^5}{1 + j0.628}$$

$$= (71.72 - j45.04) \text{ k}\Omega$$

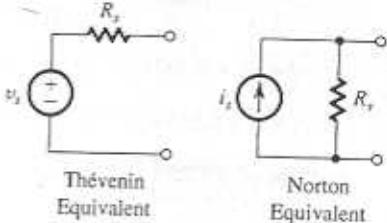
(d) $Z = R + j\omega L$

$$= 100 + j2\pi \times 10 \times 10^3 \times 10 \times 10^{-3}$$

$$= 100 + j6.28 \times 100$$

$$= (100 + j628) \text{ }\Omega$$

1.22



$$v_{oc} = v_i$$

$$i_{sc} = i_i$$

$$v_i = i_i R_i$$

Thus,

$$R_i = \frac{v_{oc}}{i_{sc}}$$

(a) $v_i = v_{oc} = 10 \text{ V}$

$$i_i = i_{sc} = 100 \mu\text{A}$$

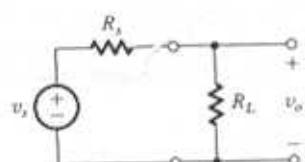
$$R_i = \frac{v_{oc}}{i_{sc}} = \frac{10 \text{ V}}{100 \mu\text{A}} = 0.1 \text{ M}\Omega = 100 \text{ k}\Omega$$

(b) $v_i = v_{oc} = 0.1 \text{ V}$

$$i_i = i_{sc} = 10 \mu\text{A}$$

$$R_i = \frac{v_{oc}}{i_{sc}} = \frac{0.1 \text{ V}}{10 \mu\text{A}} = 0.01 \text{ M}\Omega = 10 \text{ k}\Omega$$

1.23



$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_s}$$

$$v_o = v_i \left(1 + \frac{R_s}{R_L} \right)$$

Thus,

$$\frac{v_i}{1 + \frac{R_s}{100}} = 30$$

and

$$\frac{v_i}{1 + \frac{R_s}{10}} = 10$$

Dividing (1) by (2) gives

$$\frac{1 + (R_s/10)}{1 + (R_s/100)} = 3$$

$$\Rightarrow R_s = 28.6 \text{ k}\Omega$$

Substituting in (2) gives

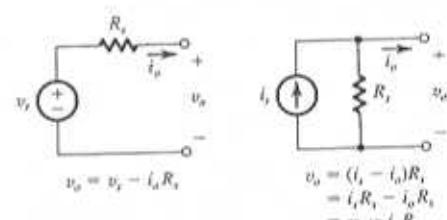
$$v_i = 38.6 \text{ mV}$$

The Norton current i_i can be found as

$$i_i = \frac{v_i}{R_i} = \frac{38.6 \text{ mV}}{28.6 \text{ k}\Omega} = 1.35 \mu\text{A}$$

1.24 The observed output voltage is 1 mV/°C which is one half the voltage specified by the sensor, presumably under open-circuit conditions that is without a load connected. It follows that that sensor internal resistance must be equal to R_L , i.e., 10 kΩ.

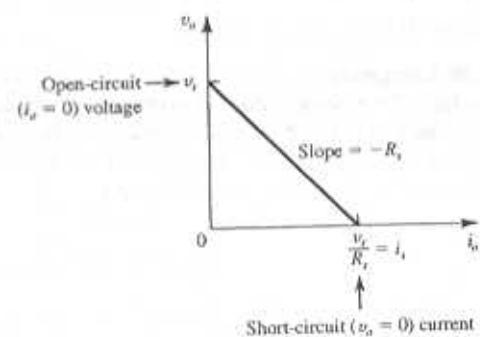
1.25



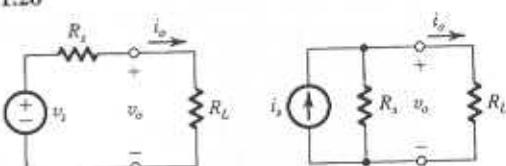
$$v_o = v_i - i_i R_s$$

$$v_o = (i_i - i_o) R_i$$

$$v_o = i_i R_i - i_o R_i$$



1.26



R_s represents the input resistance of the processor

For $v_o = 0.9 v_i$,

$$0.9 = \frac{R_L}{R_s + R_L} \Rightarrow R_L = 9R_s$$

For $i_o = 0.9 i_i$,

$$0.9 = \frac{R_s}{R_s + R_L} \Rightarrow R_L = R_s/9$$

1.27

Case	ω [rad/s]	f (Hz)	T (s)
a	6.28×10^9	1×10^9	1×10^{-9}
b	1×10^9	1.59×10^8	6.28×10^{-9}
c	6.28×10^{10}	1×10^{10}	1×10^{-10}
d	3.77×10^2	60	1.67×10^{-2}
e	6.28×10^3	1×10^3	1×10^{-3}
f	6.28×10^6	1×10^6	1×10^{-6}

1.28 (a) $V_{\text{peak}} = 117 \times \sqrt{2} = 165$ V

(b) $V_{\text{rms}} = 33.9 / \sqrt{2} = 24$ V

(c) $V_{\text{peak}} = 220 \times \sqrt{2} = 311$ V

(d) $V_{\text{peak}} = 220 \times \sqrt{2} = 311$ kV

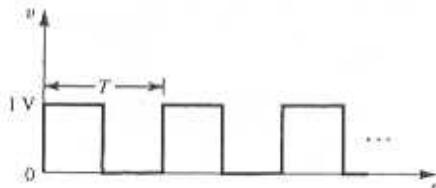
1.29 (a) $v = 10 \sin(2\pi \times 10^4 t)$, V

(b) $v = 120\sqrt{2} \sin(2\pi \times 60)$, V

(c) $v = 0.1 \sin(1000t)$, V

(d) $v = 0.1 \sin(2\pi \times 10^4 t)$, V

1.30 Comparing the given waveform to that described by Eq. 1.2 we observe that the given waveform has an amplitude of 0.5 V (1 V peak-to-peak) and its level is shifted up by 0.5 V (the first term in the equation). Thus the waveform look as follows.



Average value = 0.5 V

Peak-to-peak value = 1 V

Lowest value = 0 V

Highest value = 1 V

Period $T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 10^{-3}$ s

1.31 The two harmonics have the ratio $126/98 = 9/7$. Thus, these are the 7th and 9th harmonics. From Eq. 1.2 we note that the amplitudes of these two harmonics will have the ratio 7 to 9, which is confirmed by the measurement reported. Thus the fundamental will have a frequency of $98/7$ or 14 kHz and peak amplitude of $63 \times 7 = 441$ mV. The rms value of the fundamental will be $441/\sqrt{2} = 312$ mV. To find the peak-to-peak amplitude of the square wave we note

that $4V/\pi = 441$ mV. Thus,

$$\text{Peak-to-peak amplitude} = 2V = 441 \times \frac{\pi}{2} = 693 \text{ mV}$$

$$\text{Period } T = \frac{1}{f} = \frac{1}{14 \times 10^3} = 71.4 \mu\text{s}$$

1.32 To be barely audible by a relatively young listener, the 5th harmonic must be limited to 20 kHz; thus the fundamental will be 4 kHz. At the low end, hearing extends down to about 20 Hz. For the fifth and higher to be audible the fifth must be no lower than 20 Hz. Correspondingly, the fundamental will be at 4 Hz.

1.33 If the amplitude of the square wave is V_{sq} then the power delivered by the square wave to



a resistance R will be V_{sq}^2/R . If this power is to equal that delivered by a sine wave of peak amplitude \hat{V} then

$$\frac{V_{\text{sq}}^2}{R} = \frac{(\hat{V}/\sqrt{2})^2}{R}$$

Thus, $V_{\text{sq}} = \hat{V}/\sqrt{2}$. This result is independent of frequency.

1.34 Decimal Binary

0	0
5	101
8	1000
25	11001
57	111001

1.35 $b_3 \ b_2 \ b_1 \ b_0$ Value Represented

0 0 0 0	+0
0 0 0 1	+1
0 0 1 0	+2
0 0 1 1	+3
0 1 0 0	+4
0 1 0 1	+5
0 1 1 0	+6
0 1 1 1	+7
1 0 0 0	-0
1 0 0 1	-1
1 0 1 0	-2
1 0 1 1	-3
1 1 0 0	-4
1 1 0 1	-5
1 1 1 0	-6
1 1 1 1	-7

Note that there are two possible representation of zero: 0000 and 1000. For a 0.5-V step size, analog signals in the range ± 3.5 V can be represented

Input	Steps	Code
+2.5 V	+5	0101
-3.0 V	-6	1110
+2.7	+5	0101
-2.8	-6	1110

1.36 (a) For N bits there will be 2^N possible levels, from 0 to V_{FS} . Thus there will be $(2^N - 1)$ discrete steps from 0 to V_{FS} with the step size given by

$$\text{Step size} = \frac{V_{FS}}{2^N - 1}$$

This is the analog change corresponding to a change in the LSB. It is the value of the resolution of the ADC.

(b) The maximum error in conversion occurs when the analog signal value is at the middle of a step. Thus the maximum error is

$$\frac{1}{2} \times \text{step size} = \frac{1}{2} \frac{V_{FS}}{2^N - 1}$$

This is known as the quantization error.

$$(c) \frac{10 \text{ V}}{2^N - 1} \leq 5 \text{ mV}$$

$$2^N - 1 \geq 2000$$

$$2^N \geq 2001 \Rightarrow N = 11$$

For $N = 11$,

$$\text{Resolution} = \frac{10}{2^{11} - 1} = 4.9 \text{ mV}$$

$$\text{Quantization error} = \frac{4.9}{2} = 2.4 \text{ mV}$$

1.37 When $b_i = 1$, the i th switch is in position 1 and a current ($V_{ref}/2^i R$) flows to the output. Thus i_o will be the sum of all the currents corresponding to "1" bits, i.e.,

$$i_o = \frac{V_{ref}}{R} \left(\frac{b_1}{2^1} + \frac{b_2}{2^2} + \cdots + \frac{b_N}{2^N} \right)$$

(b) b_N is the LSB
 b_1 is the MSB

$$(c) i_{O\max} = \frac{10 \text{ V}}{5 \text{ k}\Omega} \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} \right)$$

$$= 1.96875 \text{ mA}$$

Corresponding to the LSB changing from 0 to 1 the output changes by $10/5 \times 1/2^6 = 0.03125 \text{ mA}$.

1.38 There will be 44,100 samples per second with each sample represented by 16 bits. Thus the throughput or speed will be $44,100 \times 16 = 7.056 \times 10^5$ bits per second.

$$1.39 (a) A_v = \frac{v_o}{v_i} = \frac{10 \text{ V}}{100 \text{ mV}} = 100 \text{ V/V}$$

or, $20 \log 100 = 40 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{v_i / i_i} = \frac{10 \text{ V} / 100 \Omega}{100 \mu\text{A}} = \frac{0.1 \text{ A}}{100 \mu\text{A}}$$

$$= 1000 \text{ A/A}$$

or, $20 \log 1000 = 60 \text{ dB}$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i} = 100 \times 1000$$

$$= 10^5 \text{ W/W}$$

or $10 \log 10^5 = 50 \text{ dB}$

$$(b) A_v = \frac{v_o}{v_i} = \frac{2 \text{ V}}{10 \mu\text{V}} = 2 \times 10^5 \text{ V/V}$$

or, $20 \log 2 \times 10^5 = 106 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{v_i / i_i} = \frac{2 \text{ V} / 10 \text{ k}\Omega}{100 \text{ nA}} = \frac{0.2 \text{ mA}}{100 \text{ nA}} = \frac{0.2 \times 10^{-3}}{100 \times 10^{-9}} = 2000 \text{ A/A}$$

or $20 \log A_i = 66 \text{ dB}$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i} = 2 \times 10^5 \times 2000$$

$$= 4 \times 10^8 \text{ W/W}$$

or $10 \log A_p = 86 \text{ dB}$

$$(c) A_v = \frac{v_o}{v_i} = \frac{10 \text{ V}}{1 \text{ V}} = 10 \text{ V/V}$$

or, $20 \log 10 = 20 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{v_i / i_i} = \frac{10 \text{ V} / 10 \Omega}{1 \text{ mA}} = \frac{1 \text{ A}}{1 \text{ mA}} = 1000 \text{ A/A}$$

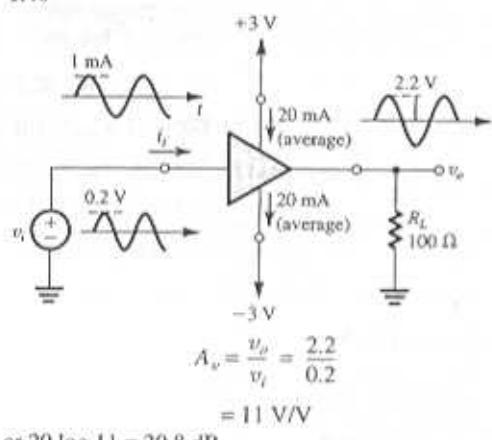
or, $20 \log 1000 = 60 \text{ dB}$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i}$$

$$= 10 \times 1000 = 10^4 \text{ W/W}$$

or $10 \log_{10} A_p = 40 \text{ dB}$

1.40



or $20 \log 11 = 20.8 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{2.2 \text{ V}/100 \Omega}{1 \text{ mA}}$$

$$= \frac{22 \text{ mA}}{1 \text{ mA}} = 22 \text{ A/A}$$

or, $20 \log A_i = 26.8 \text{ dB}$

$$A_p = \frac{P_o}{P_i} = \frac{(2.2/\sqrt{2})^2 / 100}{\frac{0.2}{\sqrt{2}} \times 10^{-3}}$$

$$= 242 \text{ W/W}$$

or, $10 \log A_p = 23.8 \text{ dB}$

Supply power = $2 \times 3 \text{ V} \times 20 \text{ mA} = 120 \text{ mW}$

$$\text{Output power} = \frac{v_{o,\text{rms}}^2}{R_L} = \frac{(2.2/\sqrt{2})^2}{100 \Omega} = 24.2 \text{ mW}$$

$$\text{Input power} = \frac{24.2}{242} = 0.1 \text{ mW} \quad (\text{negligible})$$

Amplifier dissipation = Supply power - Output power
 $= 120 - 24.2 = 95.8 \text{ mW}$

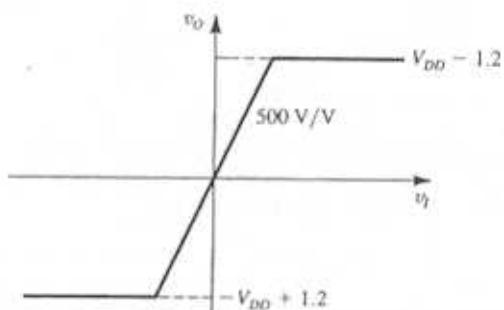
$$\text{Amplifier efficiency} = \frac{\text{Output power}}{\text{Supply power}} \times 100$$

$$= \frac{24.2}{120} \times 100 = 20.2\%$$

1.41 For $V_{DD} = 5 \text{ V}$:

The largest undistorted sine-wave output is of 3.8-V peak amplitude or $3.8/\sqrt{2} = 2.7 \text{ V}_{\text{rms}}$. Input needed is $5.4 \text{ mV}_{\text{rms}}$.

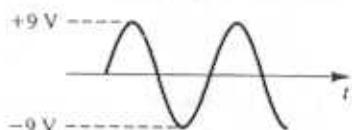
Supplies are V_{DD} and $-V_{DD}$



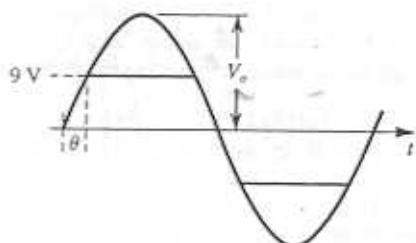
For $V_{DD} = 10 \text{ V}$, the largest undistorted sine-wave output is of 8.8-V peak amplitude or $6.2 \text{ V}_{\text{rms}}$. Input needed is $12.4 \text{ mV}_{\text{rms}}$.

For $V_{DD} = 15 \text{ V}$, the largest undistorted sine-wave output is of 13.8-V peak amplitude or $9.8 \text{ V}_{\text{rms}}$. The input needed is $9.8 \text{ V}/500 = 19.6 \text{ mV}_{\text{rms}}$.

1.42 (a) For an output whose extremes are just at the edge of clipping, i.e., an output of $9 \text{ V}_{\text{peak}}$, the input must have $9 \text{ V}/1000 = 9 \text{ mV}_{\text{peak}}$.



(b) For an output that is clipping 90% of the time, $\theta = 0.1 \times 90^\circ = 9^\circ$ and $V_p \sin 9^\circ = 9 \text{ V} \Rightarrow V_p = 57.5 \text{ V}$ which of course does not occur as the output saturates at $\pm 9 \text{ V}$. To produce this result, the input peak must be $57.5/1000 = 57.5 \text{ mV}$.



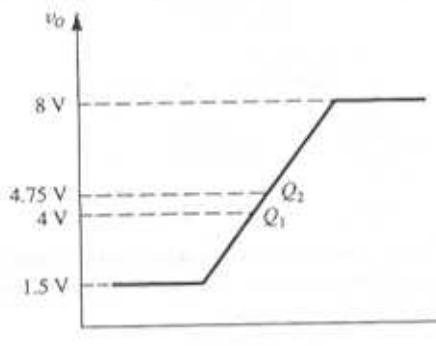
(c) For an output that is clipping 99% of the time,
 $\theta = 0.01 \times 90^\circ = 0.9^\circ$

$$V_p \sin 0.9^\circ = 9 \text{ V}$$

$$\Rightarrow V_p = 573 \text{ V}$$

and the input must be $573 \text{ V} / 1000$ or $0.573 \text{ V}_{\text{peak}}$.

1.43 When the amplifier is biased at 4 V (i.e., at point Q_1), the maximum possible amplitude of a sine-wave output without clipping is $(4 - 1.5) = 2.5 \text{ V}_{\text{peak}}$.

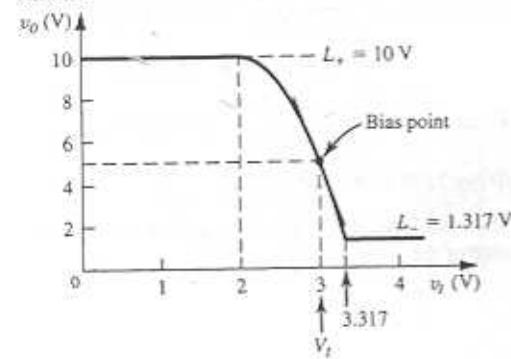


To obtain the largest undistorted sine wave output possible with this amplifier, it must be biased halfway between the saturation levels, i.e., at $v_o = (8 + 1.5)/2 = 4.75 \text{ V}$ (point Q_2) and the resulting output will have a peak value of $8 - 4.75 = 4.75 - 1.50 = 3.25 \text{ V}$.

1.44 $v_o = 10 - 5(v_i - 2)^2, \quad 2 \leq v_i \leq v_o + 2, \quad v_o \geq 0$

(a) For $v_i \leq 2 \text{ V}, v_o = 10 \text{ V}$.

The upper limit on v_i is found by substituting $v_i = v_o + 2$, that is, $v_o = v_i - 2$ in the transfer characteristic. The result is $v_o = 10 - 5v_o^2$, whose solution is $v_o = 1.317 \text{ V}$ and the corresponding $v_i = 3.317 \text{ V}$. To obtain a sketch of v_o versus v_i , we evaluate v_o for values of v_i in the range 2 V to 3.317 V . The result is the following sketch:



(b) To obtain $v_o = 5 \text{ V}$ we bias at $v_i = 3 \text{ V}$.

$$(c) \text{ Small-signal gain at bias point } = \left. \frac{\partial v_o}{\partial v_i} \right|_{v_i=3 \text{ V}} = -5 \times 2(v_i - 2) = -10 \text{ V/V}$$

(d) $v_i = 3 + V_i \cos \omega t$

$$v_o = 10 - 5(3 + V_i \cos \omega t - 2)^2$$

$$= 10 - 5(1 + 2V_i \cos \omega t + V_i^2 \cos^2 \omega t)$$

$$= 5 - 10V_i \cos \omega t - 5V_i^2 \left(\frac{1}{2} + \frac{1}{2} \cos 2\omega t \right)$$

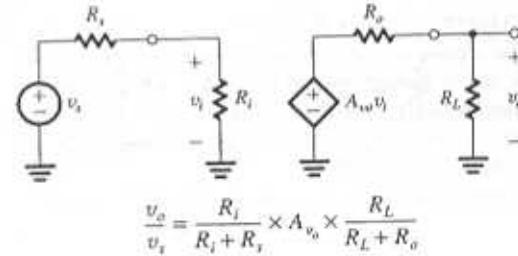
$$= \underbrace{(5 - 2.5V_i^2)}_{\text{dc}} - \underbrace{10V_i \cos \omega t}_{\text{Fundamental}} - \underbrace{2.5V_i^2 \cos 2\omega t}_{\text{2nd harmonic}}$$

For 1% second-harmonic distortion: $2.5V_i^2/10V_i = 0.01$

Thus,

$$V_i = \frac{10 \times 0.01}{2.5} \text{ V} = 40 \text{ mV}$$

1.45



$$\frac{v_o}{v_i} = \frac{R_o}{R_i + R_o} \times A_{v_o} \times \frac{R_L}{R_L + R_o}$$

$$(a) \frac{v_o}{v_i} = \frac{10R_o}{10R_i + R_o} \times A_{v_o} \times \frac{10R_o}{10R_o + R_o}$$

$$= \frac{10}{11} \times 10 \times \frac{10}{11} = 8.26 \text{ V/V}$$

or, $20 \log 8.26 = 18.3 \text{ dB}$

$$(b) \frac{v_o}{v_i} = \frac{R_o}{R_i + R_o} \times A_{v_o} \times \frac{R_o}{R_o + R_o}$$

$$= 0.5 \times 10 \times 0.5 = 2.5 \text{ V/V}$$

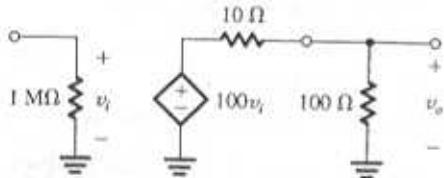
or, $20 \log 2.5 = 8 \text{ dB}$

$$(c) \frac{v_o}{v_i} = \frac{R_o/10}{(R_i/10) + R_o} \times A_{v_o} \times \frac{R_o/10}{(R_o/10) + R_o}$$

$$= \frac{1}{11} \times 10 \times \frac{1}{11} = 0.083 \text{ V/V}$$

or $20 \log 0.083 = -21.6 \text{ dB}$

1.46 $20 \log A_{v_o} = 40 \text{ dB} \Rightarrow A_{v_o} = 100 \text{ V/V}$



$$\begin{aligned} A_v &= \frac{v_o}{v_i} \\ &= 100 \times \frac{100}{100 + 10} \\ &= 90.9 \text{ V/V} \end{aligned}$$

or, $20 \log 90.9 = 39.1 \text{ dB}$

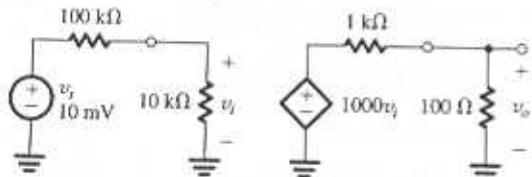
$$A_p = \frac{v_o^2 / 100 \Omega}{v_i^2 / 1 \text{ M}\Omega} = A_v^2 \times 10^4 = 8.3 \times 10^7 \text{ W/W}$$

or $10 \log (8.3 \times 10^7) = 79.1 \text{ dB}$.

For a peak output sine-wave current of 100Ω , the peak output voltage will be $100 \text{ mA} \times 100 \Omega = 10 \text{ V}$. Correspondingly v_i will be a sine wave with a peak value of $10 \text{ V}/A_v = 10/90.9$ or an rms value of $10/(90.9 \times \sqrt{2}) = 0.08 \text{ V}$.

$$\begin{aligned} \text{Corresponding output power} &= (10/\sqrt{2})^2 / 100 \Omega \\ &= 0.5 \text{ W} \end{aligned}$$

1.47

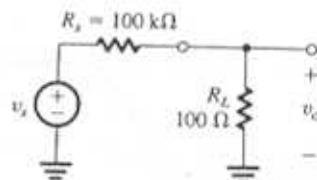


$$\begin{aligned} \frac{v_o}{v_i} &= \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 100 \text{ k}\Omega} \times 1000 \times \frac{100 \Omega}{100 \Omega + 1 \text{ k}\Omega} \\ &= \frac{10}{110} \times 1000 \times \frac{100}{1100} = 8.26 \text{ V/V} \end{aligned}$$

The signal loses about 90% of its strength when connected to the amplifier input (because $R_i = R_s/10$). Also, the output signal of the amplifier loses approximately 90% of its strength when the load is connected

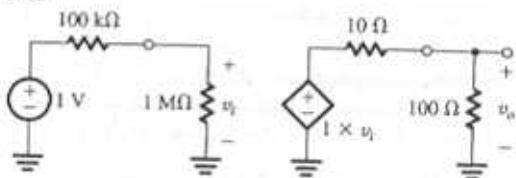
(because $R_L = R_o/10$). Not a good design! Nevertheless, if the source were connected directly to the load,

$$\begin{aligned} \frac{v_o}{v_i} &= \frac{R_L}{R_L + R_s} \\ &= \frac{100 \Omega}{100 \Omega + 100 \text{ k}\Omega} \\ &= 0.001 \text{ V/V} \end{aligned}$$



which is clearly a much worse situation. Indeed inserting the amplifier increases the gain by a factor $8.3/0.001 = 8300$.

1.48



$$\begin{aligned} v_o &= 1 \text{ V} \times \frac{1 \text{ M}\Omega}{1 \text{ M}\Omega + 100 \text{ k}\Omega} \times 1 \times \frac{100 \Omega}{100 \Omega + 10 \Omega} \\ &= \frac{1}{1.1} \times \frac{100}{110} = 0.83 \text{ V} \end{aligned}$$

$$\text{Voltage gain} = \frac{v_o}{v_i} = 0.83 \text{ V/V} \quad \text{or} \quad -1.6 \text{ dB}$$

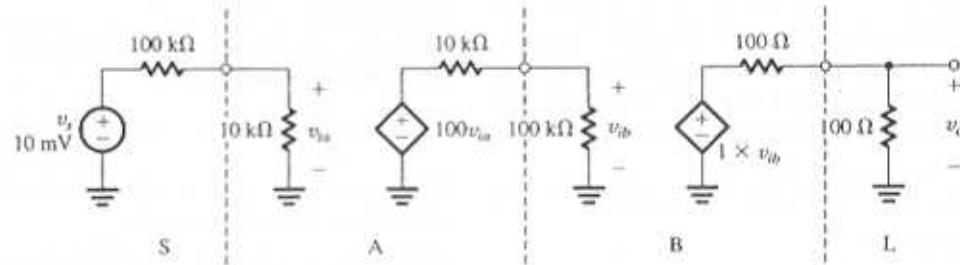
$$\begin{aligned} \text{Current gain} &= \frac{v_o/100 \Omega}{v_i/1.1 \text{ M}\Omega} = 0.83 \times 1.1 \times 10^4 \\ &= 9091 \text{ A/A} \quad \text{or} \quad 79.2 \text{ dB} \end{aligned}$$

$$\text{Power gain} = \frac{v_o^2/100 \Omega}{v_i^2/1.1 \text{ M}\Omega} = 7578 \text{ W/W}$$

or $10 \log 7578 = 38.8 \text{ dB}$

(This takes into acct. the power dissipated in the internal resistance of the source.)

1.50 Case (a) S-A-B-L

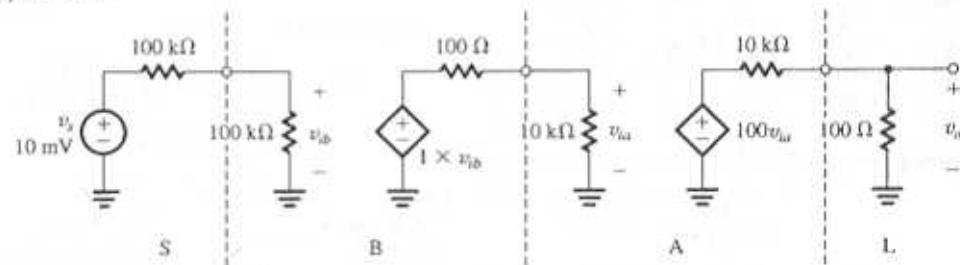


$$\frac{v_o}{v_s} = \frac{10}{10+100} \times 100 \times \frac{100}{100+10} \times 1 \times \frac{100}{100+100}$$

$$= 4.1 \text{ V/V}$$

or $20 \log 4.1 = 12.3 \text{ dB}$

Case (b) S-B-A-L



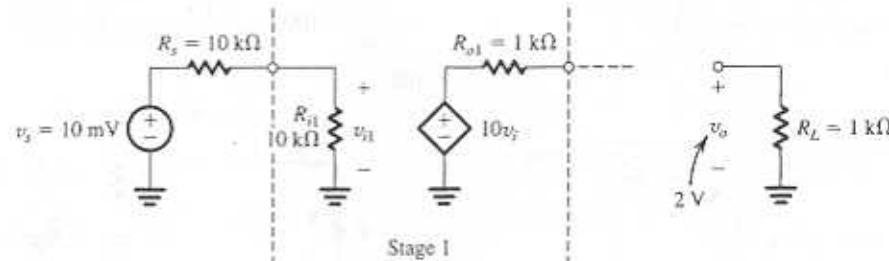
$$\frac{v_o}{v_s} = \frac{100}{100+100} \times 1 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 100 \text{ }\Omega} \times 100 \times \frac{100 \text{ }\Omega}{100 \text{ }\Omega + 10 \text{ k}\Omega}$$

$$= 0.5 \times \frac{10}{10.1} \times 100 \times \frac{0.1}{10.1}$$

$$= 0.5 \text{ V/V or } -6 \text{ dB}$$

Thus, obviously case (a) i.e., SABL is preferred.

1.51

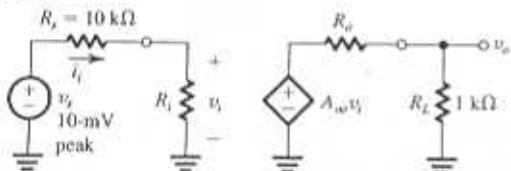


Required overall voltage gain = $2 \text{ V} / 10 \text{ mV} = 200 \text{ V/V}$. Each stage is capable of providing a maximum voltage gain of 10 (the open-circuit gain value). For n stages in cascade the maximum (unattainable) voltage gain is 10^n . We thus see that we need at least 3 stages. For 3 stages, the overall voltage gain obtained is

$$\frac{v_o}{v_i} = \frac{10}{10+10} \times 10 \times \frac{10}{1+10} \times 10 \times \frac{10}{1+10} \times 10 \times \frac{1}{1+1} \\ = 206.6 \text{ V/V}$$

Thus, three stages suffice and provide a gain slightly larger than required. The output voltage actually obtained is $10 \text{ mV} \times 206.6 = 2.07 \text{ V}$.

1.53



$$(a) \text{ Required voltage gain} = \frac{v_o}{v_i} \\ = \frac{3 \text{ V}}{0.01 \text{ V}} = 300 \text{ V/V}$$

(b) The smallest R_i allowed is obtained from

$$0.1 \mu\text{A} = \frac{10 \text{ mV}}{R_s + R_i} \Rightarrow R_s + R_i = 100 \text{ k}\Omega$$

Thus $R_i = 90 \text{ k}\Omega$.

For $R_i = 90 \text{ k}\Omega$, $i_i = 0.1 \mu\text{A}$ peak, and

$$\text{Overall current gain} = \frac{v_o/R_L}{i_i} \\ = \frac{3 \text{ mA}}{0.1 \mu\text{A}} = 3 \times 10^4 \text{ A/A}$$

$$\text{Overall power gain} = \frac{v_{o(\text{rms})}^2 / R_L}{v_{i(\text{rms})} \times i_{i(\text{rms})}} \\ = \frac{\left(\frac{3}{\sqrt{2}}\right)^2 / 1000}{\left(\frac{10 \times 10^{-3}}{\sqrt{2}}\right) \times \left(\frac{0.1 \times 10^{-6}}{\sqrt{2}}\right)} \\ = 9 \times 10^6 \text{ W/W}$$

(This takes into account the power dissipated in the internal resistance of the source.)

(c) If $(A_{v_o} v_i)$ has its peak value limited to 5 V, the largest value of R_o is found from

$$5 \times \frac{R_L}{R_L + R_o} = 3 \Rightarrow R_o = \frac{2}{3} R_L = 667 \Omega$$

(If R_o were greater than this value, the output voltage across R_L would be less than 3 V.)

(d) For $R_i = 90 \text{ k}\Omega$ and $R_o = 667 \Omega$, the required value of A_{v_o} can be found from

$$300 \text{ V/V} = \frac{90}{90+10} \times A_{v_o} \times \frac{1}{1+0.667} \\ \Rightarrow A_{v_o} = 555.7 \text{ V/V}$$

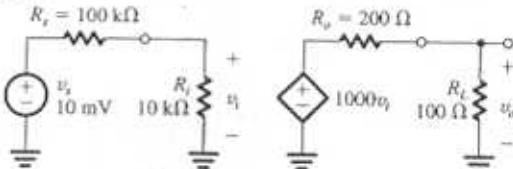
$$(e) R_i = 100 \text{ k}\Omega (1 \times 10^5 \Omega)$$

$$R_o = 100 \Omega (1 \times 10^2 \Omega)$$

$$300 = \frac{100}{100+10} \times A_{v_o} \times \frac{1000}{1000+100}$$

$$\Rightarrow A_{v_o} = 363 \text{ V/V}$$

1.54

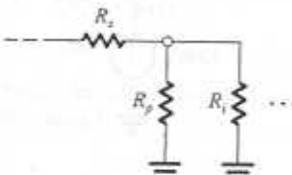


$$(a) v_o = 10 \text{ mV} \times \frac{10}{10+100} \times 1000 \times \frac{100}{100+200} \\ = 303 \text{ mV}$$

$$(b) \frac{v_o}{v_i} = \frac{303 \text{ mV}}{10 \text{ mV}} = 30.3 \text{ V/V}$$

$$(c) \frac{v_o}{v_i} = 1000 \times \frac{100}{100+200} = 333.3 \text{ V/V}$$

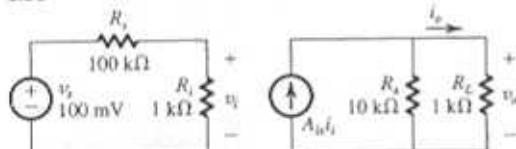
(d)



Connect a resistance R_p in parallel with the input and select its value from

$$\begin{aligned}\frac{(R_p \parallel R_i)}{(R_p \parallel R_i) + R_i} &= \frac{1}{2} \frac{R_i}{R_i + R_p} \\ \Rightarrow 1 + \frac{R_i}{R_p \parallel R_i} &= 22 \Rightarrow R_p \parallel R_i = \frac{R_i}{21} = \frac{100}{21} \\ \Rightarrow \frac{1}{R_p} + \frac{1}{R_i} &= \frac{21}{100} \\ R_p &= \frac{1}{0.21 - 0.1} = 9.1 \text{ k}\Omega\end{aligned}$$

1.55

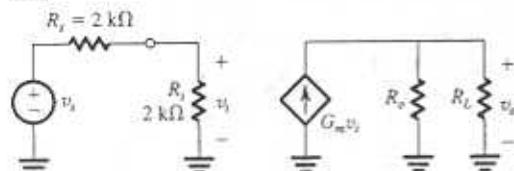


$$\begin{aligned}\text{(a) Current gain} &= \frac{i_o}{i_i} \\ &= A_{i_o} \frac{R_o}{R_o + R_L} \\ &= 100 \frac{10}{11} \\ &= 90.9 \frac{\text{A}}{\text{A}} = 39.2 \text{ dB}\end{aligned}$$

$$\begin{aligned}\text{(b) Voltage gain} &= \frac{v_o}{v_i} \\ &= \frac{i_o}{i_i} \frac{R_o}{R_o + R_i} \\ &= 90.9 \times \frac{1}{101} \\ &= 0.9 \text{ V/V} = -0.9 \text{ dB}\end{aligned}$$

$$\begin{aligned}\text{(c) Power gain} &= A_p = \frac{v_o i_o}{v_i i_i} \\ &= 0.9 \times 90.9 \\ &= 81.8 \text{ W/W} = 19.1 \text{ dB}\end{aligned}$$

1.56



$$G_m = 40 \text{ mA/V}$$

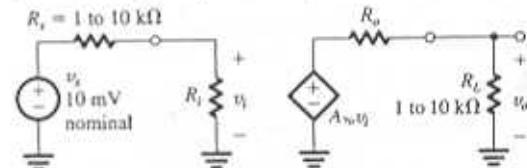
$$R_o = 20 \text{ k}\Omega$$

$$R_L = 1 \text{ k}\Omega$$

$$\begin{aligned}v_i &= v_t \frac{R_i}{R_i + R_f} \\ &= v_t \frac{2}{2+2} = \frac{v_t}{2} \\ v_o &= G_m v_i (R_L \parallel R_o) \\ &= 40 \frac{20 \times 1}{20+1} v_t \\ &= 40 \frac{20}{21} \frac{v_t}{2}\end{aligned}$$

$$\text{Overall voltage gain} = \frac{v_o}{v_t} = 19.05 \text{ V/V}$$

1.57 A voltage amplifier is required.



To limit the change in v_o to 10% as R_i varies from 1 to 10 kΩ we select R_i sufficiently large:

$$R_i \geq 10R_{i\max}$$

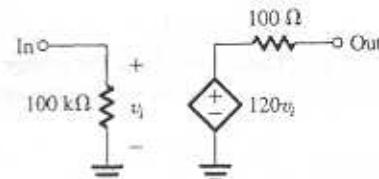
Thus $R_i = 100 \text{ k}\Omega$.

To limit the change in v_o corresponding to R_L varying in the range 1 to 10 kΩ, to 10%, we select R_o sufficiently small:

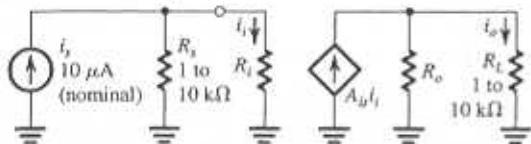
$$R_o \leq R_{L\min}/10$$

Thus,

$$\begin{aligned}R_o &= 100 \Omega \\ v_{o\min} &= v_t \frac{R_i}{R_i + R_{i\max}} A_{v_o} \frac{R_{L\min}}{R_{L\min} + R_o} \\ 1 &= 0.01 \frac{100}{100+10} A_{v_o} \frac{1000}{1000+100} \\ \Rightarrow A_{v_o} &= 121 \text{ V/V}\end{aligned}$$



1.58 Current amplifier.



To limit the change in i_o resulting from R_i varying over the range 1 to 10 kΩ, to 10% we select R_i sufficiently low so that,

$$R_i \leq R_{i,\min} / 10$$

Thus, $R_i = 100 \Omega$.

To limit the change in i_o as R_L changes from 1 to 10 kΩ, to 10% we select R_o sufficiently large;

$$R_o \geq 10R_{L,\max}$$

Thus, $R_o = 100 \text{ k}\Omega$.

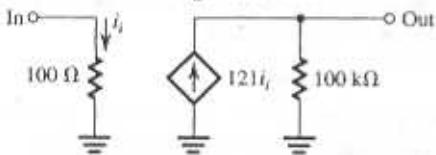
Now for $i_s = 10 \mu\text{A}$,

$$i_{o,\min} = 10^{-5} \frac{R_{i,\min}}{R_{i,\min} + R_i} A_{i,i} \frac{R_o}{R_o + R_{L,\max}}$$

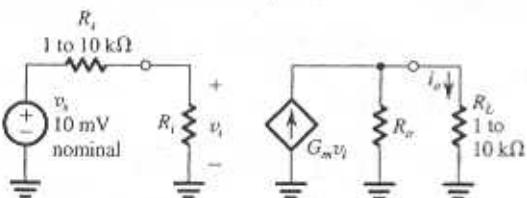
Thus,

$$10^{-3} = 10^{-5} \frac{1000}{1000 + 100} A_{i,i} \frac{100}{100 + 10}$$

$$\Rightarrow A_{i,i} = 121 \text{ A/A}$$



1.59 Transconductance amplifier.



For R_i varying in the range 1 to 10 kΩ, and Δi_o limited to 10% we have to select R_i sufficiently large;

$$R_i \geq 10R_{i,\max}$$

$$R_i = 100 \Omega$$

For R_L varying in the range 1 to 10 kΩ, the change in i_o can be kept to 10% if R_o is selected sufficiently large;

$$R_o \geq R_{L,\max}$$

Thus $R_o = 100 \text{ k}\Omega$.

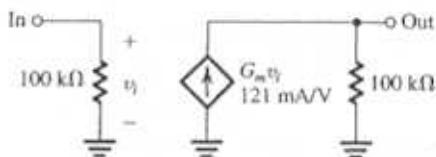
For $v_t = 10 \text{ mV}$,

$$i_{o,\min} = 10^{-2} \frac{R_i}{R_i + R_{i,\max}} G_m \frac{R_o}{R_o + R_{L,\max}}$$

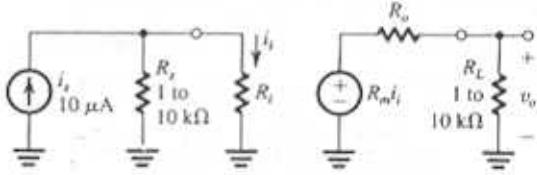
$$10^{-3} = 10^{-2} \frac{100}{100 + 10} G_m \frac{100}{100 + 10}$$

$$G_m = 1.21 \times 10^{-1} \text{ A/V}$$

$$= 121 \text{ mA/V}$$



1.60 Transresistance amplifier



To limit Δv_o to 10% corresponding to R_i varying in the range 1 to 10 kΩ, we select R_i sufficiently low;

$$R_i \leq \frac{R_{i,\min}}{10}$$

Thus, $R_i = 100 \Omega$.

To limit Δv_o to 10% while R_L varies over the range 1 to 10 kΩ, we select R_o sufficiently low;

$$R_o \leq R_{L,\min} / 10$$

Thus, $R_o = 100 \Omega$.

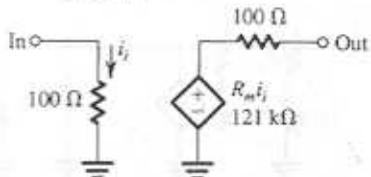
Now, for $i_s = 10 \mu\text{A}$,

$$v_{o,\min} = 10^{-5} \frac{R_{i,\min}}{R_{i,\min} + R_i} R_m \frac{R_{L,\min}}{R_{L,\min} + R_o}$$

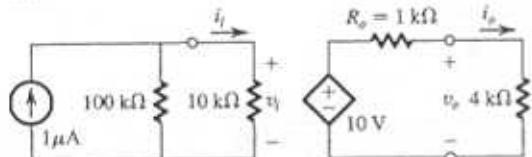
$$1 = 10^{-5} \frac{1000}{1000 + 100} R_m \frac{1000}{1000 + 100}$$

$$\Rightarrow R_m = 1.21 \times 10^5$$

$$= 121 \text{ k}\Omega$$



1.64



$$R_o = \frac{\text{Open-circuit output voltage}}{\text{Short-circuit output current}} = \frac{10 \text{ V}}{10 \text{ mA}} = 1 \text{ k}\Omega$$

$$v_o = 10 \times \frac{4}{1+4} = 8 \text{ V}$$

$$A_v = \frac{v_o}{v_i} = \frac{8}{1 \times 10^{-3} \times (100 // 10) \times 10^3} = 888 \text{ V/V or } 58.9 \text{ dB}$$

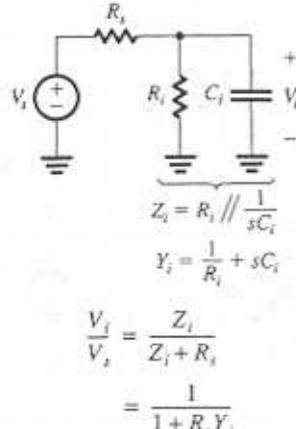
$$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{10^{-3} \times \frac{100}{100+10}} = \frac{8/(4 \times 10^3)}{10^{-3} \times \frac{100}{110}} = 2200 \text{ A/A or } 66.8 \text{ dB}$$

$$A_t = \frac{v_o^2 / R_L}{i_i^2 R_i} = \frac{8^2 / (4 \times 10^3)}{\left(10^{-3} \times \frac{100}{100+10}\right)^2 10 \times 10^3} = 19.36 \times 10^5 \text{ W/W or } 62.9 \text{ dB}$$

$$\text{Overall current gain} = \frac{i_o}{1 \mu\text{A}}$$

$$= \frac{v_o / R_L}{1 \mu\text{A}} = \frac{8 / (4 \times 10^3)}{10^{-3}} = 2000 \text{ A/A or } 66 \text{ dB}$$

1.65 Using the voltage divider rule



$$= \frac{1}{1 + R_s \left(\frac{1}{R_i} + sC_i \right)} = \frac{1}{1 + \frac{R_i}{R_s} + sC_i R_s} = \frac{1}{1 + sC_i \frac{R_i}{R_s} \frac{1}{1 + \frac{R_i}{R_s}}} = \frac{1}{1 + sC_i (R_i // R_s)}$$

$$= \frac{1}{1 + \frac{R_i}{R_s}} \frac{1}{1 + sC_i \left(\frac{R_i R_s}{R_i + R_s} \right)} = \frac{R_s}{R_s + R_i} \frac{1}{1 + sC_i (R_s // R_i)}$$

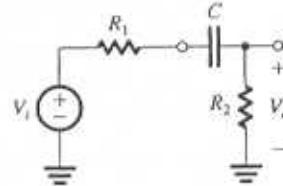
This transfer function is of the STC low-pass type with a dc gain $K = R_s / (R_s + R_i)$ and a 3-dB frequency $\omega_0 = 1 / C_i (R_s // R_i)$.

For $R_s = 20 \text{ k}\Omega$, $R_i = 80 \text{ k}\Omega$, and $C_i = 5 \text{ pF}$,

$$\omega_0 = \frac{1}{5 \times 10^{-12} \times \frac{20 \times 80}{20+80} \times 10^3} = 1.25 \times 10^7 \text{ rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1.25 \times 10^7}{2\pi} = 2 \text{ MHz}$$

1.66 Using the voltage-divider rule,



$$T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 + \frac{1}{sC}} = \frac{R_2}{R_1 + R_2} \frac{s}{s + \frac{1}{C(R_1 + R_2)}}$$

which from Table 1.2 is of the high-pass type with

$$K = \frac{R_2}{R_1 + R_2}$$

and

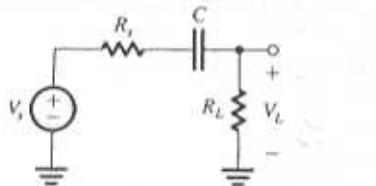
$$\omega_0 = \frac{1}{C(R_1 + R_2)}$$

As a further verification that this is a high-pass network and $T(s)$ is a high-pass transfer function we observe as at $s = 0$, $T(s) = 0$; and that as $s \rightarrow \infty$, $T(s) = R_2/(R_1 + R_2)$. Also, from the circuit observe as at $s \rightarrow \infty$, $(1/sC) \rightarrow 0$ and $V_o/V_i = R_2/(R_1 + R_2)$. Now, for $R_1 = 10 \text{ k}\Omega$, $R_2 = 40 \text{ k}\Omega$, and $C = 0.1 \mu\text{F}$,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 0.1 \times 10^{-6} (10 + 40) \times 10^3} = 31.8 \text{ Hz}$$

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{\sqrt{10 + 40}} = 0.57 \text{ V/V}$$

1.67 Using the voltage divider rule,



$$\begin{aligned} \frac{V_o}{V_i} &= \frac{R_L}{R_L + R_i + \frac{1}{sC}} \\ &= \frac{R_L}{R_L + R_i} \frac{s}{s + \frac{1}{C(R_L + R_i)}} \end{aligned}$$

which is of the high-pass STC type (see Table 1.2) with

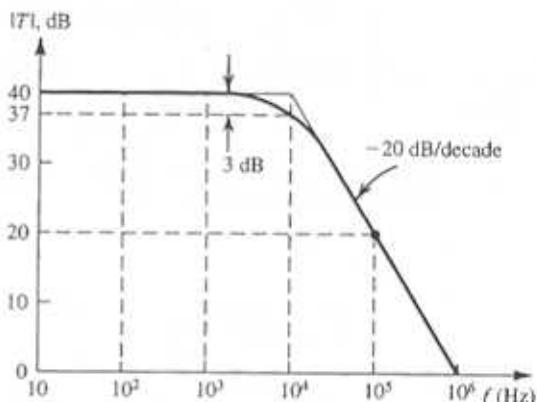
$$K = \frac{R_L}{R_L + R_i}, \quad \omega_0 = \frac{1}{C(R_L + R_i)}$$

For $f_0 \leq 10 \text{ Hz}$

$$\begin{aligned} \frac{1}{2\pi C(R_L + R_i)} &\leq 10 \\ \Rightarrow C &\geq \frac{1}{2\pi \times 10(20 + 5) \times 10^3} \end{aligned}$$

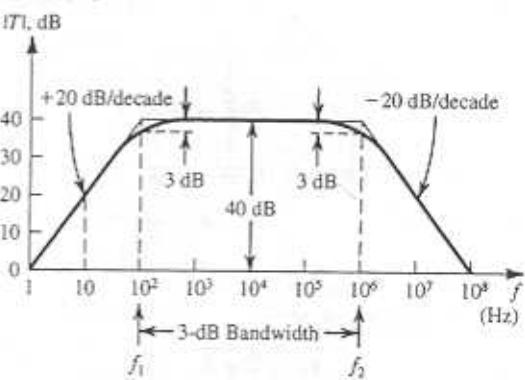
Thus, the smallest value of C that will do the job is $C = 0.64 \mu\text{F}$.

1.68 The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of 10^4 Hz . From our knowledge of the Bode plots for low-pass STC networks (Figure . . .) we can complete the Table entries and sketch the amplifier frequency response.



$f(\text{Hz})$	$ T (\text{dB})$	$\angle T(\text{degrees})$
1000	40	-5.7°
10^4	37	-45°
10^5	20	-84.3°
10^6	0	-90°

1.69 From our knowledge of the Bode plots of STC low-pass and high-pass networks we see that this amplifier has a mid-band gain of 40 dB, a low-frequency response of the high-pass STC type with $f_{3\text{dB}} = 10^2 \text{ Hz}$, and a high-frequency response of the low-pass STC type with $f_{3\text{dB}} = 10^6 \text{ Hz}$. We thus can sketch the amplifier frequency response and complete the table entries as follows



$f(\text{Hz})$	1	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
$ T (\text{dB})$	0	20	37	40	40	40	37	20	0

1.72 Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects since the buffer amplifiers have input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

where

$$\omega_0 = 1/CR$$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_{1\text{dB}}}{\omega_0}\right)^2}} = -1$$

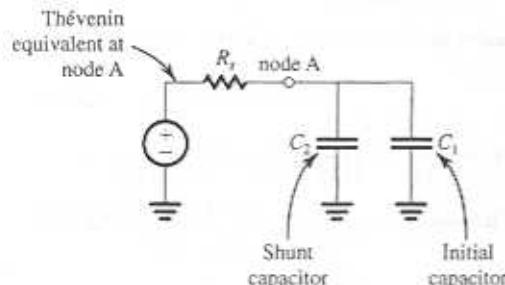
$$\Rightarrow 1 + \left(\frac{\omega_{1\text{dB}}}{\omega_0}\right)^2 = 10^{0.1}$$

$$\omega_{1\text{dB}} = 0.51\omega_0$$

$$\omega_{1\text{dB}} = 0.51/CR$$

1.73 $R_i = 100 \text{ k}\Omega$, since the 3-dB frequency is reduced by a very high factor (from 6 MHz to 120 kHz) C_2 must be much larger than C_1 . Thus, neglecting C_1 we find C_2 from

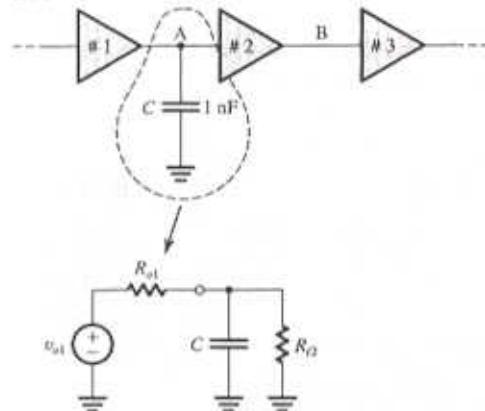
$$\begin{aligned} 120 \text{ kHz} &= \frac{1}{2\pi C_2 R_i} \\ &= \frac{1}{2\pi C_2 \times 10^5} \\ \Rightarrow C_2 &= 13.3 \text{ pF} \end{aligned}$$



If the original 3-dB frequency (6 MHz) is attributable to C_1 then

$$\begin{aligned} 6 \text{ MHz} &= \frac{1}{2\pi C_1 R_i} \\ \Rightarrow C_1 &= \frac{1}{2\pi \times 6 \times 10^6 \times 10^5} \\ &= 0.26 \text{ pF} \end{aligned}$$

1.74



Since when C is connected the 3-dB frequency is reduced by a large factor, the value of C must be much larger than whatever parasitic capacitance originally existed at node A (i.e., between A and ground). Furthermore, it must be that C is now the dominant determinant of the amplifier 3-dB frequency (i.e., it is dominating over whatever may be happening at node B or anywhere else in the amplifier). Thus, we can write

$$\begin{aligned} 150 \text{ kHz} &= \frac{1}{2\pi C(R_{o1} \parallel R_{i2})} \\ \Rightarrow (R_{o1} \parallel R_{i2}) &= \frac{1}{2\pi \times 150 \times 10^3 \times 1 \times 10^{-9}} \\ &= 1.06 \text{ k}\Omega \end{aligned}$$

Now $R_{i2} = 100 \text{ k}\Omega$,

Thus $R_{o1} = 1.07 \text{ k}\Omega$

Similarly, for node B,

$$\begin{aligned} 15 \text{ kHz} &= \frac{1}{2\pi C(R_{o2} \parallel R_{i3})} \\ \Rightarrow R_{o2} \parallel R_{i3} &= \frac{1}{2\pi \times 15 \times 10^3 \times 1 \times 10^{-9}} \\ &= 10.6 \text{ k}\Omega \end{aligned}$$

$$R_{o2} = 11.9 \text{ k}\Omega$$

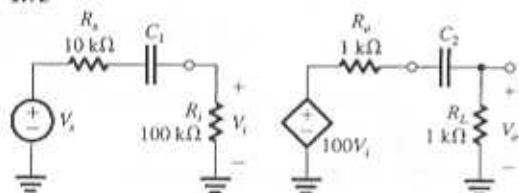
She should connect a capacitor of value C_p to node B where C_p can be found from,

$$10 \text{ kHz} = \frac{1}{2\pi C_p (R_{o2} \parallel R_{i3})}$$

$$\Rightarrow C_p = \frac{1}{2\pi \times 10 \times 10^3 \times 10.6 \times 10^3} = 1.5 \text{ nF}$$

Note that if she chooses to use node A she would need to connect a capacitor 10 times larger!

1.75



For the input circuit, the corner frequency f_{01} is found from

$$f_{01} = \frac{1}{2\pi C_1 (R_i + R_t)}$$

For $f_{01} \leq 100 \text{ Hz}$,

$$\frac{1}{2\pi C_1 (10 + 100) \times 10^3} \leq 100$$

$$\Rightarrow C_1 \geq \frac{1}{2\pi \times 110 \times 10^3 \times 10^2} = 4.4 \times 10^{-8}$$

Thus we select $C_1 = 1 \times 10^{-7} \text{ F} = 0.1 \mu\text{F}$. The actual corner frequency resulting from C_1 will be

$$f_{01} = \frac{1}{2\pi \times 10^{-7} \times 110 \times 10^3} = 14.5 \text{ Hz}$$

For the output circuit,

$$f_{02} = \frac{1}{2\pi C_2 (R_o + R_L)}$$

For $f_{02} \leq 100 \text{ Hz}$,

$$\frac{1}{2\pi C_2 (1 + 1) \times 10^3} \leq 100$$

$$\Rightarrow C_2 \geq \frac{1}{2\pi \times 2 \times 10^3 \times 10^2} = 0.8 \times 10^{-6}$$

Select $C_2 = 1 \times 10^{-6} = 1 \mu\text{F}$

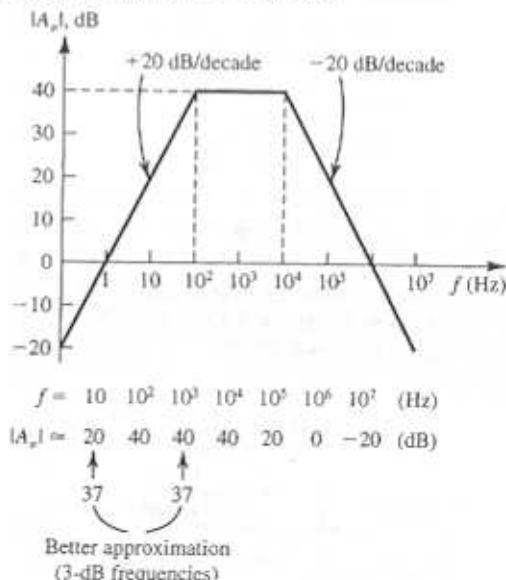
This will place the corner frequency at

$$f_{02} = \frac{1}{2\pi \times 10^{-6} \times 2 \times 10^3} = 80 \text{ Hz}$$

$$T(s) = 100 \frac{s}{\left(1 + \frac{s}{2\pi f_{01}}\right) \left(1 + \frac{s}{2\pi f_{02}}\right)}$$

1.76 The LP factor $1/(1 + jf/10^4)$ results in a Bode plot like that in Fig. 1.23(a) with the 3 dB frequency $f_0 = 10^4 \text{ Hz}$. The high-pass factor $1/(1 + 10^4/jf)$ results in a Bode plot like that in Fig. 1.24(a) with the 3 dB frequency $f_0 = 10^4 \text{ Hz}$.

The Bode plot for the overall transfer function can be obtained by summing the dB values of the two individual plots and then raising the resulting plot vertically by 40 dB (corresponding to the factor 100 in the numerator). The result is as follows:



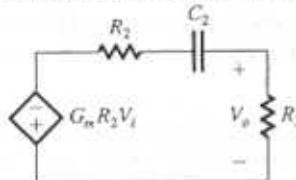
Bandwidth = $10^4 - 10^2 = 9900 \text{ Hz}$

1.77

$$T_1(s) = \frac{V_i(s)}{V_o(s)} = \frac{1/s C_1}{1/s C_1 + R_1} = \frac{1}{s C_1 R_1 + 1} \quad \text{LP}$$

$$3 \text{ dB frequency} = \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi 10^{-11} 10^6} = 15.9 \text{ Hz}$$

For $T_o(s)$, the following equivalent circuit can be used:



$$T_o(s) = -G_m R_2 \frac{R_3}{R_2 + R_3 + 1/sC_2}$$

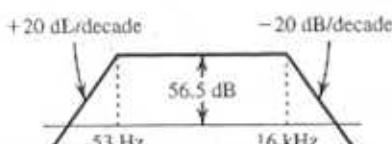
$$= -G_m (R_2 \parallel R_3) \frac{s}{s + \frac{1}{C_2(R_2 + R_3)}}$$

$$3 \text{ dB frequency} = \frac{1}{2\pi C_2(R_2 + R_3)}$$

$$= \frac{1}{2\pi 100 \times 10^{-9} \times 30 \times 10^3} = 53 \text{ Hz}$$

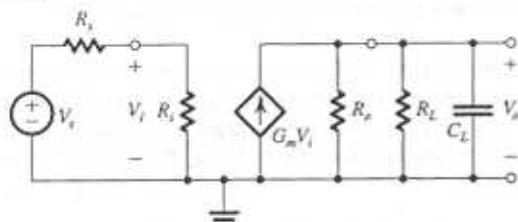
$$\therefore T(s) = T_o(s) T_p(s)$$

$$= \frac{1}{1 + \frac{s}{2\pi \times 15.9 \times 10^3}} \times -666.7 \times \frac{s}{s + (2\pi \times 53)}$$



$$\text{Bandwidth} = 16 \text{ kHz} - 53 \text{ Hz} = 16 \text{ kHz}$$

1.78



$$V_f = V_i \frac{R_i}{R_i + R_f} \quad (1)$$

To satisfy constraint (1), namely

$$V_f \geq \left(1 - \frac{x}{100}\right) V_i$$

We substitute in Eq.(1) to obtain

$$\frac{R_i}{R_i + R_f} \geq 1 - \frac{x}{100}$$

Thus

$$\frac{R_i + R_f}{R_i} \leq \frac{1}{1 - \frac{x}{100}}$$

$$\frac{R_f}{R_i} \leq \frac{1}{1 - \frac{x}{100}} - 1 = \frac{\frac{x}{100}}{1 - \frac{x}{100}}$$

which can be expressed as

$$\frac{R_f}{R_i} \geq \frac{1 - \frac{x}{100}}{\frac{x}{100}}$$

resulting in

$$R_f \geq R_i \left(\frac{100}{x} - 1 \right) \quad (1)$$

The 3-dB frequency is determined by the parallel RC circuit at the output,

$$f_0 = \frac{1}{2\pi} \omega_0 = \frac{1}{2\pi} \frac{1}{C_L (R_L \parallel R_o)}$$

Thus,

$$f_0 = \frac{1}{2\pi C_L} \left(\frac{1}{R_L} + \frac{1}{R_o} \right)$$

To obtain a value for f_0 greater than a specified value f_{3dB} we select R_o so that

$$\frac{1}{2\pi C_L} \left(\frac{1}{R_L} + \frac{1}{R_o} \right) \geq f_{3dB}$$

$$\frac{1}{R_L} + \frac{1}{R_o} \geq 2\pi C_L f_{3dB}$$

$$\frac{1}{R_o} \geq 2\pi C_L f_{3dB} - \frac{1}{R_L}$$

$$R_o \leq \frac{1}{2\pi f_{3dB} C_L - \frac{1}{R_L}} \quad (2)$$

To satisfy constraint (3), we first determine the dc gain as

$$\text{dc gain} = \frac{R_i}{R_i + R_f} G_m (R_o \parallel R_L)$$

For the dc gain to be greater than a specified value A_0 ,

$$\frac{R_i}{R_i + R_f} G_m (R_o \parallel R_L) \geq A_0$$

The first factor on the LHS is (from constraint (1))

greater or equal to $(1 - x/100)$. Thus

$$G_m \geq \frac{A_0}{\left(1 - \frac{x}{100}\right)(R_o \parallel R_L)} \quad (3)$$

Substituting $R_o = 10 \text{ k}\Omega$ and $x = 20\%$ in (1) results in

$$R_i \geq 10 \left(\frac{100}{20} - 1 \right) = 40 \text{ k}\Omega$$

Substituting $f_{\text{cav}} = 3 \text{ MHz}$, $C_L = 10 \text{ pF}$ and $R_L = 10 \text{ k}\Omega$ in Eq. (2) results in

$$R_o \leq \frac{1}{2\pi \times 3 \times 10^6 \times 10 \times 10^{-12} - \frac{1}{10^4}} = 11.3 \text{ k}\Omega$$

Substituting $A_0 = 80$, $x = 20\%$, $R_L = 10 \text{ k}\Omega$, and $R_o = 11.3 \text{ k}\Omega$, eq. (3) results in

$$G_m \geq \frac{80}{\left(1 - \frac{20}{100}\right)(10 \parallel 11.3) \times 10^3} = 18.85 \text{ mA/V}$$

If the more practical value of $R_o = 10 \text{ k}\Omega$ is used then

$$G_m \geq \frac{80}{\left(1 - \frac{20}{100}\right)(10 \parallel 10) \times 10^3} = 20 \text{ mA/V}$$

1.79 Using the voltage-divider rule we obtain

$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_1 = R_1 \parallel \frac{1}{sC_1} \quad \text{and} \quad Z_2 = R_2 \parallel \frac{1}{sC_2}.$$

It is obviously more convenient to work in terms of admittances. Therefore we express V_o/V_i in the alternate form

$$\frac{V_o}{V_i} = \frac{Y_1}{Y_1 + Y_2}$$

and substitute $Y_1 = (1/R_1) + sC_1$ and $Y_2 = (1/R_2) + sC_2$ to obtain

$$\begin{aligned} \frac{V_o}{V_i} &= \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + \frac{1}{R_2} + s(C_1 + C_2)} \\ &= \frac{C_1}{C_1 + C_2} \frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{(C_1 + C_2)(R_1 + R_2)}} \end{aligned}$$

This transfer function will be independent of frequency (s) if the second factor reduces to unity. This in turn will happen if

$$\frac{1}{C_1 R_1} = \frac{1}{C_1 + C_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

which can be simplified as follows

$$\frac{C_1 + C_2}{C_2} = R_1 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (1)$$

$$1 + \frac{C_2}{C_1} = 1 + \frac{R_1}{R_2}$$

or

$$C_1 R_1 = C_2 R_2$$

When this condition applies, the attenuator is said to be compensated, and its transfer function is given by

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

which, using Eq. (1) above can be expressed in the alternate form

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

Thus when the attenuator is compensated ($C_1 R_1 = C_2 R_2$) its transmission can be determined either by its two resistors R_1 , R_2 or by its two capacitors, C_1 , C_2 , and the transmission is *not* a function of frequency.

1.80 The HP STC circuit whose response determines the frequency response of the amplifier in the low-frequency range has a phase angle of 11.4° at $f = 100 \text{ Hz}$. Using the equation for $\angle T(j\omega)$ from Table 1.2 we obtain

$$\tan^{-1} \frac{f_0}{100} = 11.4^\circ \Rightarrow f_0 = 20.16 \text{ Hz.}$$

The LP STC circuit whose response determines the amplifier response at the high-frequency end has a phase angle of -11.4° at $f = 1 \text{ kHz}$. Using the relationship for $\angle T(j\omega)$ given in Table 1.2 we obtain for the LP STC circuit

$$-\tan^{-1} \frac{10^3}{f_0} = -11.4^\circ \Rightarrow f_0 = 4959.4 \text{ Hz}$$

At $f = 100$ Hz the drop in gain is due to the HP STC network, and thus its value is

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{20.16}{100}\right)^2}} = -0.17 \text{ dB}$$

Similarly, at $f = 1$ kHz the drop in gain is caused by the LP STC network. The drop in gain is

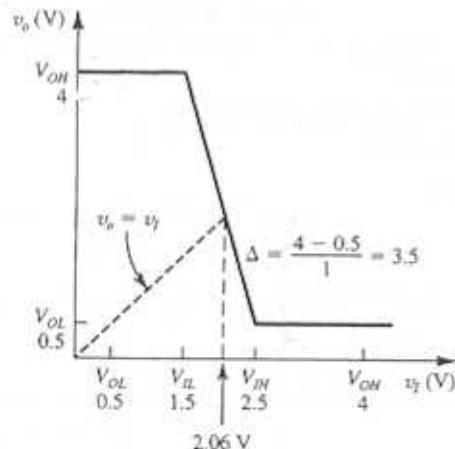
$$20 \log \frac{1}{\sqrt{1 + \left(\frac{1000}{4959.4}\right)^2}} = -0.17 \text{ dB}$$

The gain drops by 3 dB at the corner frequencies of the two STC networks, that is, at $f = 20.16$ Hz and $f = 4959.4$ Hz.

$$1.81 \quad NM_H = V_{OH} - V_{IH} = 3.3 - 1.7 = 1.6 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 1.3 - 0 = 1.3 \text{ V}$$

1.82



$$(a) \quad NM_H = V_{OH} - V_{IH} = 4 - 2.5 = 1.5 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 1.5 - 0.5 = 1 \text{ V}$$

(b) In the transition region

$$\begin{aligned} V_O &= 4 - 3.5(V_I - 1.5) \\ &= 9.25 - 3.5V_I \end{aligned}$$

If

$$V_O = V_I \Rightarrow 4.5V_O = 9.25$$

$$V_O = V_I = 2.06 \text{ V}$$

(c) Slope = -3.5 V/V

1.83

$$NM_H = V_{OH} - V_{IH} = 0.8 V_{DD} - 0.6 V_{DD} = 0.2 V_{DD}$$

$$NM_L = V_{IL} - V_{OL} = (0.4 - 0.1)V_{DD} = 0.3 V_{DD}$$

$$\text{width of transition region} = V_{IH} - V_{IL} = 0.2 V_{DD}$$

for a minimum NM of 1 V $\Rightarrow 0.2 V_{DD} = 1$

$$V_{DD} = 5 \text{ V}$$

1.84

$$(a) \quad \text{Worse case } NM_H = V_{OH,\min} - V_{IH} = 2.4 - 2 = 0.4 \text{ V}$$

$$\text{Worse case } NM_L = V_{IL,\max} - V_{OL} = 0.8 - 0.4 = 0.4 \text{ V}$$

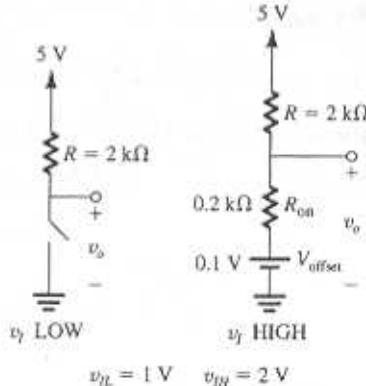
$$(b) \quad P_{D_{av}} = \frac{1}{20} [5 \times 3 + 5 \times 1] = 10 \text{ mW}$$

$$(c) \quad \text{Dynamic power dissipation} = f_c V_{DD}^2 = 10^6 \times 45 \times 10^{-12} \times 25 = 1.13 \text{ mW}$$

$$(d) \quad t_p(\text{typical}) = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(7 + 11) = 9 \text{ ns}$$

$$t_p(\text{maximum}) = \frac{1}{2}(15 + 22) = 18.5 \text{ ns}$$

1.85



$$v_L = 1 \text{ V} \quad v_H = 2 \text{ V}$$

$$(a) \quad V_{OL} = \frac{5 - 0.1}{2.2} = 0.2 + 0.1 = 0.545 \text{ V}$$

$$V_{OH} = 5 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = 3 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 0.455 \text{ V}$$

$$(b) \quad V_{OH} = 5 - N(0.2 \times 10^{-3})R = 5 - 0.4N$$

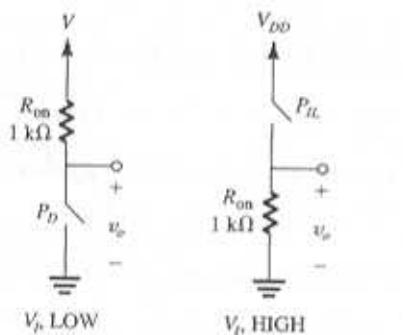
$$NM_H = 5 - 0.4N - 2 = 3 - 0.4N = 0.455 \quad \therefore N = 6$$

$$(c) (i) P_{D_{v_L} \text{LOW}} = (5 - 0.1)^2 / 2.2 \text{ k}\Omega = 10.9 \text{ mW}$$

$$(ii) P_{D_{v_L} \text{HIGH}} = 5 \times (0.2 \times 6) = 6 \text{ mW}$$

$$1.86 \text{ (a)} V_{OL} = 0 \quad V_{OH} = 5 \quad NM_L = V_{IL} - V_{OL} = 2.5 - 0 = 2.5 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = 5 - 2.5 = 2.5 \text{ V}$$



$$(b) V_O(t) = 0 - (0 - 5)e^{-t/R_{on}C} = 5e^{-t/R_{on}C}$$

$$\text{For } t_{PLH} \Rightarrow V_O(t) = 5e^{-t_{PLH}/R_{on}C} = \frac{1}{2}(5) = 2.5$$

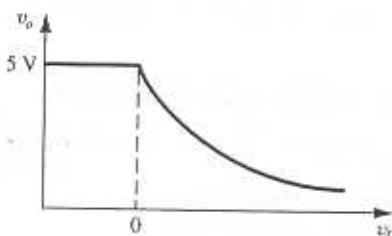
$$t_{PLH} = -(10^3)(10^{-9}) \ln \frac{2.5}{5} = 0.69 \text{ ns}$$

$$\text{For } t_{THL} \quad V_O(t) = 5e^{-t_{THL}/R_{on}C} = 4.5 \text{ V}$$

$$t_1 = 0.01 \text{ ns} \quad V_O(t) = 5e^{-t_1/R_{on}C} = 0.5 \text{ V}$$

$$t_2 = 2.3 \text{ ns}$$

$$\therefore t_{THL} = t_2 - t_1 = 2.2 \text{ ns}$$



$$(c) V_O(t) = 5 - (5 - 0)e^{-t/R_{on}C} = 5 - 5e^{-t/R_{on}C}$$

$$V_O = 5 - 5e^{-t_{PLH}/R_{on}C} = 2.5 \text{ ns}$$

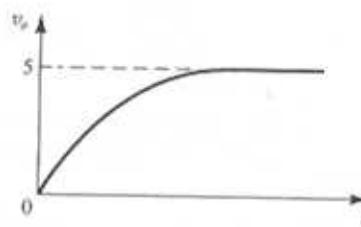
$$t_{PLH} = 0.69 \text{ ns}$$

For t_{TLH} ,

$$V_O(t) = 5 - 5e^{-t_1/R_{on}C} = 0.5 \Rightarrow t_1 = 0.10 \text{ ns.}$$

$$V_O(t) = 5 - 5e^{-t_2/R_{on}C} = 4.5 \Rightarrow t_2 = 2.3 \text{ ns.}$$

$$t_{TLH} = 2.3 - 0.1 = 2.2 \text{ ns}$$



1.87

$$V_{OH} = 5 \text{ V}$$

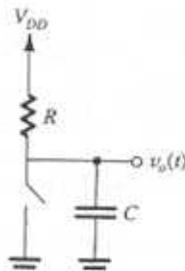
$$V_{OL} = 5 - 2 \times 1 = 3 \text{ V}$$

$$1.88 \quad P_{\text{dynamic}} = f C V_{DD}^2 = 100 \times 10^6 \times 10 \times 10^{-12} \times 25 = 25 \text{ mW}$$

$$P = V_{DD} I_{avg} = 5 I_{avg} = 25 \text{ mW}$$

$$I_{avg} = 5 \text{ mA}$$

1.89



$v_o(t)$ begins at V_{OL} and rises toward V_{OH} (in this case $V_{OH} = V_{DD}$) according to

$$v_o(t) = v_{ss} - (v_{ss} - v_{ol}) e^{-t/CR} \\ = V_{OH} - (V_{OH} - V_{OL}) e^{-t/CR}$$

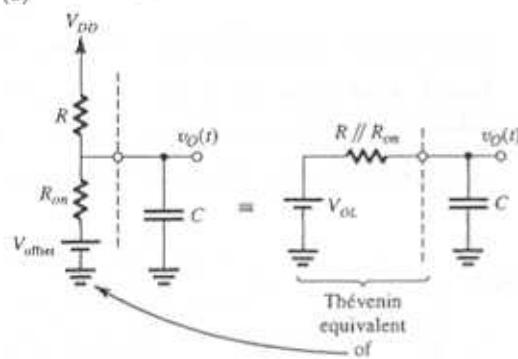
$$= V_{OH} - (V_{OH} - V_{OL}) e^{-t/\tau_1}, \quad \tau_1 = CR \quad \text{Q.E.D.}$$

$$v_o(t) \text{ reaches } \frac{1}{2}(V_{OH} + V_{OL}) \text{ at } t = t_{PLH},$$

$$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OH} - (V_{OH} - V_{OL}) e^{-t_{PLH}/\tau_1}$$

$$\Rightarrow t_{PLH} = \tau_1 \ln 2 = 0.69 CR \quad \text{Q.E.D.}$$

(b)



$$V_{OL} = V_{DD} \frac{R_{on}}{R + R_{on}} + V_{offset} \frac{R}{R + R_{on}}$$

$$v_O(t) = v_{in} - (v_{in} - v_{OL}) e^{-t/\tau_2}, \quad \tau_2 = C(R // R_{on})$$

$$= V_{OL} - (V_{OL} - V_{OH}) e^{-t/\tau_2}$$

$$= V_{OL} + (V_{OH} - V_{OL}) e^{-t/\tau_2} \quad \text{Q.E.D.}$$

$$v_O(t_{PLH}) = \frac{1}{2}(V_{OL} + V_{OH})$$

Thus,

$$t_{PLH} = \tau_2 \ln 2 = 0.69 C (R // R_{on})$$

$$= 0.69 CR_{on}, \quad R_{on} \ll R \quad \text{Q.E.D.}$$

(c) $\tau_p = \frac{1}{2}(t_{PLH} + t_{PDL}) = \frac{1}{2} \times 0.69(CR + CR_{on})$

$$= 0.35CR, \quad \text{for } R_{on} \ll R \quad \text{Q.E.D.}$$

(d) Static power is dissipated only when the output is low, in which case.

$$P = \frac{V_{DD}^2}{R + R_{on}} \approx \frac{V_{DD}^2}{R}, \quad \text{for } R_{on} \ll R,$$

and assuming that $V_{offset} \ll V_{DD}$. Thus if the inverter spends only half the time in this low-output state,

$$P = \frac{1}{2} \frac{V_{DD}^2}{R}$$

(e) Large R results in low P but high τ_p and vice-versa. For $V_{DD} = 5$ V and $C = 10$ pF,

- $\tau_p \leq 10$ ns $\Rightarrow 0.35CR \leq 10$ ns
 $\Rightarrow R \leq 2875 \Omega$
- $P \leq 10$ mW $\Rightarrow \frac{1}{2} \times \frac{25}{R} \leq 10 \times 10^{-3}$
 $\Rightarrow R \geq 1250 \Omega$

Thus,

$$1250 \leq R \leq 2875 \Omega$$

Selecting $R = 2 \text{ k}\Omega$,

$$\tau_p = 0.35 \times 10 \times 10^{-12} \times 2 \times 10^3 = 7 \text{ ns}$$

$$P = \frac{1}{2} \times \frac{25}{2} = 6.25 \text{ mW}$$

Unnumbered 1.48

$$\frac{v_{i1}}{v_i} = 0.5 \quad \frac{v_{i2}}{v_{i1}} = 100 \quad \frac{1000}{1001} \approx 100$$

$$\frac{v_{i2}}{v_{i1}} = 10 \times \frac{10}{11} = 9.09$$

$$\frac{v_L}{v_{i3}} = \frac{10}{11} = 0.909$$

$$A_v = 826.3$$

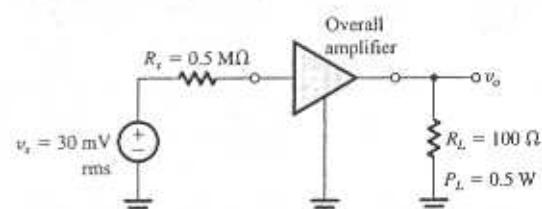
$$\frac{v_o}{v_i} = 413.1$$

$$0.5 \times 100 \times 10 \times \frac{10}{11} = 450$$

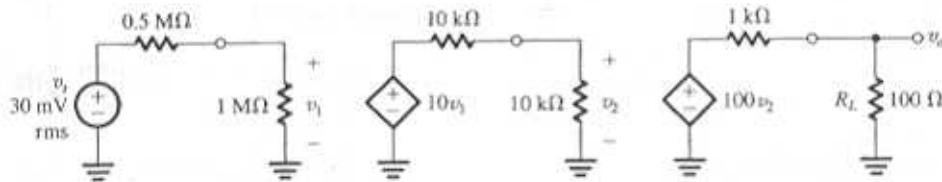
$$P_L = 0.5 \text{ W} = \frac{v_o^2}{R_L}$$

$$v_o = \sqrt{0.5 \times 100} = 7.07 \text{ V}$$

Unnumbered 1.50

Required overall voltage gain = $7.07/0.03 = 235.7 = 235 \text{ V/V}$.

In order to avoid the loss of more than 2/3 of the signal strength in coupling the source to the first stage of the amplifier, we have used the type (1), high-input resistance amplifier as the input stage. Both type 2 and type 3 do not satisfy this requirement. The type (1) amplifier has an open-circuit voltage gain of 10 and thus connects by itself to satisfy the overall gain requirement.



We next consider cascading the type (1) input stage with the type (2), high-gain stages. The result would be

Overall voltage gain

$$= \frac{1}{1+0.5} \times 10 \times \frac{10}{10+10} \times 100 \times \frac{100}{100+1000}$$

$$= 30.3 \text{ V/V}$$

Thus, this cascade amplifier does not meet specs. The reason is obvious from the gain calculation: the output resistance, $1 \text{ k}\Omega$, is too high for feeding a load of 100Ω and indeed results in a loss of gain by a factor of 11!

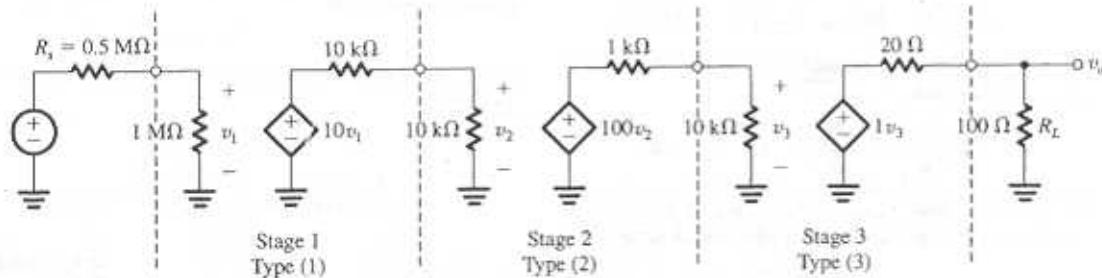
That's where the type (3) amplifier stage can be beneficial. While its open-circuit gain is only 1, its output resistance is 20Ω , five times lower than the 100Ω load. The overall amplifier would then look as follows:

$$\text{Overall voltage gain} \equiv \frac{v_o}{v_i}$$

$$= \frac{1}{1+0.5} \times 10 \times \frac{10}{10+10} \times 100 \times \frac{10}{10+1} \times 1 \times \frac{100}{100+20}$$

$$= 252.5 \text{ V/V}$$

which meets the specified gain of 235 V/V.



Chapter 2 - Problems

2.1

The minimum number of pins required by dual-op-amp is 8. Each op-amp has 2 input terminals (4 pins) and one output terminal (2 pins). Another 2 pins are required for power.

Similarly, the minimum number of pins required by quad-opamp is 14:

$$4 \times 2 + 4 \times 1 + 2 = 14$$

2.2

Refer to Fig. P2.2. $V_+ = V_3 \frac{1KR_2}{1MR_2 + 1KR_2} = \frac{4}{1001} V$
 $V_o = A V_+ \Rightarrow A = \frac{4}{4/1001} = 1001$

2.3

The voltage at the positive input has to be $-3.000V$.

$$V_+ = -3.020V, A = \frac{V_o}{(V_+ - V_2)} = \frac{-2}{-3.020 - (-3)} = 100$$

2.4

#	V_1	V_2	$\frac{V_{ds}}{V_2 - V_1}$	V_o	V_o/V_d
1	0.00	0.00	0.00	0.00	-
2	1.00	1.00	0.00	0.00	-
3	②	1.00	③	1.00	-
4	1.00	1.10	0.10	10.1	101
5	2.01	2.00	-0.01	-0.99	99
6	1.99	2.00	0.01	1.00	100
7	5.10	④	⑤	-510	-

experiments 4, 5, 6 show that the gain is

approximately $100 V/V$. The missing entry for experiment #3 can be predicted as follows:

$$\textcircled{b} \quad V_d = \frac{V_2}{A} = \frac{1.00}{100} = 0.01 V.$$

$$\textcircled{c} \quad V_1 = V_2 - V_d = 1.00 - 0.01 = 0.99 V$$

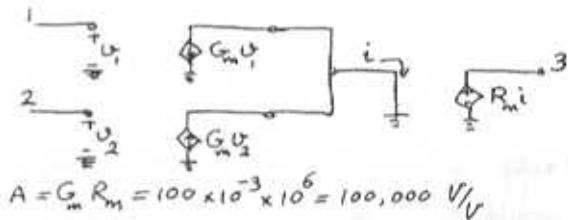
The missing entries for experiment #7:

$$\textcircled{d} \quad V_d = \frac{-5.10}{100} = -0.051 V$$

$$\textcircled{e} \quad V_2 = V_1 + V_d = 5.10 - 0.051 = 5.049 V$$

All the results seem to be reasonable.

2.5



$$A = G_m R_m = 100 \times 10^{-3} \times 10^6 = 100,000 V/V$$

2.6

$$V_{CM} = 10 \sin(2\pi 60t) = \frac{1}{2} (V_1 + V_2)$$

$$V_d = 0.01 \sin(2\pi 1000t) = V_1 - V_2$$

$$V_1 = V_{CM} - V_d/2 = \sin(120\pi t) - 0.005 \sin 2000\pi t$$

$$V_2 = V_{CM} + V_d/2 = \sin 120\pi t + 0.005 \sin 2000\pi t$$

2.7

$$V_d = R(G_{m2}V_2 - G_{m1}V_1) \quad \text{Refer to Fig. 2.4.}$$

$$V_o = V_3 = \mu V_d = \mu R(G_{m2}V_2 - G_{m1}V_1)$$

$$V_o = \mu R(G_{m2}V_2 + \frac{1}{2} \Delta G_m V_2 - G_{m1}V_1 + \frac{1}{2} \Delta G_m V_1)$$

$$V_o = \mu R G_m \underbrace{(V_2 - V_1)}_{V_{Id}} + \frac{1}{2} \mu R \Delta G_m \underbrace{(V_1 + V_2)}_{2V_{ICM}}$$

$$\text{we have } V_o = A_d V_{Id} + A_{CM} V_{ICM}$$

$$\Rightarrow A_d = \mu R G_m, \quad A_{CM} = \mu R \Delta G_m$$

$$CMRR = 20 \log |A_d| / |A_{CM}| = 20 \log \frac{G_m}{\Delta G_m}$$

cont.

$$20 \log_{10} A_d = 80 \text{ dB} \Rightarrow A_d = 10^4$$

$$A_{CM} = \frac{\Delta G_m}{G_m} \Rightarrow A_{CM} = 10^4 \times \frac{0.1}{100} = 10$$

$$CMRR = 20 \log \frac{G_m}{\Delta G_m} = 20 \log \frac{1}{0.1/100} = 60$$

2.8

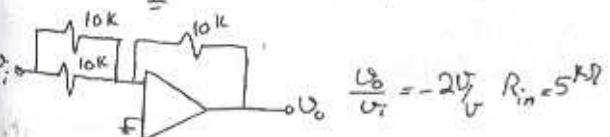
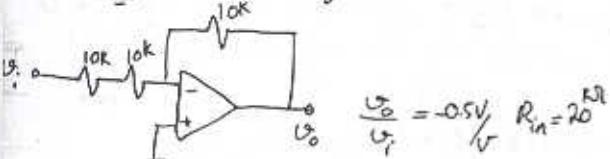
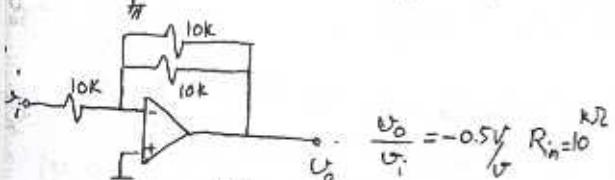
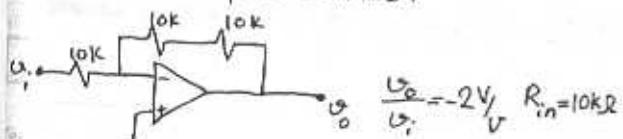
circuit	$\frac{U_o}{U_i}$ (V/V)	R_{in} (kΩ)
a	$\frac{-100}{10} = -10$	10
b	-10	10
c	-10	10
d	-10	10 no current in 10kΩ

2.9

Closed loop gain = -1 V/V . For $U_i = 5 \text{ V} \Rightarrow U_o = -5 \text{ V}$
 Gain would be in the range of $\frac{-0.95}{1.05}$ to $\frac{1.05}{0.95} : -0.9 < G < -1.1$
 For $U_i = 5 \Rightarrow 4.5 < U_o < -5.5 \text{ V}$

2.10

There are four possibilities:



2.11

- a. $G = 1 \text{ V/V}$
 c. $G = 0.1 \text{ V/V}$
 e. $G = 10 \text{ V/V}$

- b. $G = 10 \text{ V/V}$
 d. $G = 100 \text{ V/V}$

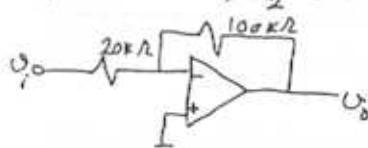
2.12

- a. $G = -1 \text{ V/V} = -\frac{R_2}{R_1} \Rightarrow R_1 = R_2 = 10 \text{ k}\Omega$
 b. $G = -2 \text{ V/V} = -\frac{R_2}{R_1} \Rightarrow R_1 = 10 \text{ k}\Omega, R_2 = 20 \text{ k}\Omega$
 c. $G = -0.5 \text{ V/V} = -\frac{R_2}{R_1} \Rightarrow R_1 = 20 \text{ k}\Omega, R_2 = 10 \text{ k}\Omega$
 d. $G = -100 \text{ V/V} = -\frac{R_2}{R_1} \Rightarrow R_1 = 10 \text{ k}\Omega, R_2 = 1 \text{ M}\Omega$

2.13

$$\frac{U_o}{U_i} = -5 = -\frac{R_2}{R_1} \Rightarrow R_2 = 5R_1$$

$$R_1 + R_2 = 120 \text{ k}\Omega \Rightarrow 5R_1 + R_1 = 120 \text{ k}\Omega \Rightarrow R_1 = 20 \text{ k}\Omega \Rightarrow R_2 = 100 \text{ k}\Omega$$



2.14

$$20 \log |G| = 26 \text{ dB} \Rightarrow G = -19.95 \text{ V/V} = \frac{U_o}{U_i} = -\frac{R_2}{R_1}$$

$$\Rightarrow R_2 = 19.95 R_1 \leq 10 \text{ M}\Omega$$

For largest possible input resistance, select
 $R_2 = 10 \text{ M}\Omega \Rightarrow R_1 \approx 500 \text{ k}\Omega$
 $R_{in} = 500 \text{ k}\Omega$

2.15

$$G = \frac{U_o}{U_i} = \frac{-R_2}{R_1} = \frac{-100}{10} = -10$$

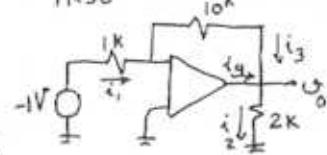
$$U_{low} = -10 \text{ V}, U_{high} = 0, U_{avg} = -5 \text{ V} \dots$$

2.16

$$\frac{U_o}{U_i} = -\frac{R_2}{R_1} \Rightarrow U_o = -1 \times \frac{-10k\Omega}{1k\Omega} = 10V$$

$$i_2 = \frac{U_o}{2k\Omega} = 5mA$$

$$i_1 = i_3 = \frac{U_o}{10k\Omega} = 1mA$$



$i_4 = i_2 - i_3 = 4mA$ This additional current comes from the output of the op-amp.

2.17

$$|Gain| = \frac{R_2}{R_1} = \frac{R_2 (1 + X/100)}{R_1 (1 + X/100)} \approx \frac{R_2}{R_1} (1 \pm \frac{2X}{100})$$

$\Rightarrow 2X\%$ is the tolerance on the closed loop gain (G).

$$G = -100V/V, X = 5 \Rightarrow -110 < G < -90$$

$$\text{or more precisely: } -100 \times \frac{105}{95} < G < -100 \times \frac{95}{105}$$

$$-110.5 < G < -90.5$$

2.18

$$G = \frac{U_o}{U_i} = -\frac{R_2}{R_1} \Rightarrow R_2 = \frac{5}{15} R_1$$

$$U_i = 0V, U_2 = U_o = 5V$$

$$\text{For } \pm 1\% \text{ on } R_1, R_2: R_1 = 15 \pm 0.15 k\Omega$$

$$R_2 = 5 \pm 0.05 k\Omega$$

$$U_o = U_i \frac{R_2}{R_1} = 15 \frac{R_2}{R_1} \Rightarrow 15 \times \frac{4.95}{15.15} < U_o < 15 \times \frac{5.05}{14.85}$$

$$\Rightarrow 4.9V \leq U_o \leq 5.1V$$

$$\text{For } U_i = -15 \pm 0.15 V \quad 14.85 \pm \frac{4.95}{15.15} < U_o < 15.15 \pm \frac{5.05}{14.85}$$

$$\Rightarrow 4.85V \leq U_o \leq 5.15V$$

2.19

$$\frac{U_o}{U_i} = \frac{-U_o}{A} = -\frac{U_o}{200}$$

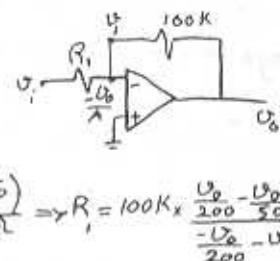
$$\frac{U_2}{U_i} = 50V/V$$

$$\frac{U_i - (-\frac{U_2}{A})}{R_1} = \frac{(-\frac{U_2}{A} - U_o)}{100k\Omega} \Rightarrow R_1 = 100k\Omega \times \frac{\frac{U_2}{200} - \frac{U_o}{50}}{\frac{-U_2}{200} - U_o}$$

$$\Rightarrow R_1 = 100k \times \frac{3}{201} = 1.49k\Omega$$

shunt Resistor R_a : $R_a \parallel 2k\Omega = 1.49k\Omega$

$$\frac{R_a \times 2}{R_a + 2} = 1.49 \Rightarrow R_a = 5.84k\Omega$$



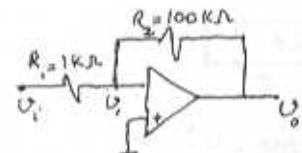
2.20

(a)

$$\frac{U_o}{U_i} = -\frac{R_2}{R_1} \Rightarrow -100V/V = -\frac{R_2}{1k\Omega} \Rightarrow R_2 = 100k\Omega$$

$$(b) A = 1000V/V$$

$$U_i = \frac{-U_o}{A}$$



$$\frac{U_i - U_o}{R_1} = \frac{U_i - U_o}{R_2}$$

$$\frac{U_o}{U_i} = \frac{-R_2/R_1}{1 + (1 + R_2/A)} = \frac{-100}{1 + \frac{101}{1000}} = -90.8V/V$$

$$\Rightarrow \frac{U_o}{U_i} = 90.8V/V$$

(b) Assume $R'_1 = R_x \parallel R_1$ when $R_1 = 1k\Omega$

$$\frac{U_o}{U_i} = 100V/V$$

$$\frac{U_i - U_o}{R'_1} = \frac{U_i - U_o}{R_2} \Rightarrow R'_1 = R_2 \times \left(\frac{U_o}{100} - \frac{-U_o}{1000} \right) / \left(\frac{-U_o}{1000} \right)$$

$$R'_1 = \frac{1 - 0.1}{1.001} = 0.899k\Omega = \frac{R_1 R_x}{R_1 + R_x} = \frac{R_x}{1 + R_x}$$

$$\Rightarrow R_x = 8.9k\Omega \approx 8.87k\Omega \pm 1\%$$

2.21

Voltage of the inverting input terminal

Cont.

will vary from $\frac{-10V}{1000}$ to $\frac{10V}{1000}$. Thus the virtual ground will depart from the ideal voltage of zero by a maximum of $\pm 10mV$.

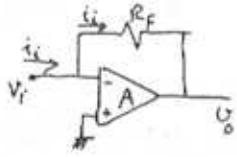
2.22

a) For $A = \infty$: $V_i = 0$

$$V_o = -i_c R_F$$

$$R_m = \frac{V_o}{i_c} = -R_F$$

$$R_{ia} = \frac{V_o}{i_c} = 0$$



b) For $A = \text{Finite}$: $V_i = -\frac{V_o}{A}$, $V_o = V_i + i_c R_F$

$$\Rightarrow V_o = -\frac{V_o}{A} - i_c R_F \Rightarrow R_m = \frac{V_o}{i_c} = -\frac{R_F}{A}$$

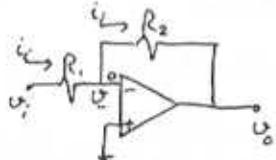
$$R_i = \frac{V_i}{i_c} = \frac{R_F}{1+A} \quad R_l > \frac{V_i}{i_c} = \frac{-V_o/A}{i_c} = \frac{-V_o}{A} + i_c R_F$$

2.23

$$V_o = -AV_+ = V_- - i_c R_2$$

$$i_c R_2 = (1+A) V_-$$

$$V_- = \frac{i_c R_2}{1+A}$$



$$\text{Now: } V_i = i_c R_1 + V_- = i_c R_1 + i_c R_2$$

$$R_{in} = \frac{V_i}{i_c} = R_1 + \frac{R_2}{1+A}$$

2.24

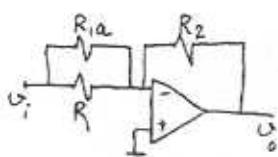
$$G = \frac{-R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}}$$

$$\text{Gain Error } \epsilon = \left(1 + \frac{R_2}{R_1}\right) \times 100$$

$$\epsilon \mid \begin{array}{ccc} 0.1\% & 1\% & 10\% \\ 1000(1 + \frac{R_2}{R_1}) & 100(1 + \frac{R_2}{R_1}) & 10(1 + \frac{R_2}{R_1}) \end{array}$$

Gain with R_{ia} :

$$G = \frac{R_2}{R_1} \left(1 + \frac{R_1}{R_{ia}}\right)$$



where we have neglected the effect of R_{ia} on

the error on the denominator. To restore the gain to its nominal value of R_2/R_1 , we use:

$$\frac{R_i}{R_{ia}} = \frac{1 + R_2/R_1}{A} = \frac{\epsilon}{100} \rightarrow R_{ia} = \frac{100R_1}{\epsilon}$$

$$\epsilon \mid \begin{array}{ccc} 0.1\% & 1\% & 10\% \\ 1000R_1 & 100R_1 & 10R_1 \end{array}$$

2.25

$$R'_i = R_i \parallel R_C \quad G' = \frac{-R_2/R'_i}{1 + \frac{1 + R_2/R'_i}{A}}$$

$$G = \frac{-R_2}{R_i}$$

$$\text{In order for } G' = G, \quad G = \frac{-R_2/R'_i}{1 + \frac{1 + R_2/R'_i}{A}} = \frac{-R_2}{R_i}$$

$$R'_i = \frac{R_i R_C}{R_i + R_C}$$

$$\Rightarrow \frac{R_i + R_C}{R_i R_C} = \frac{1}{R_i} \left(1 + \frac{1 + \frac{R_2(R_i + R_C)}{R_i R_C}}{A}\right)$$

$$(R_i + R_C) A = AR_C + R_C + \frac{R_2}{R_i} (R_i + R_C)$$

$$R_i A = R_C + GR_i + GR_C$$

$$\frac{R_C}{R_i} = \frac{A - G}{1 + G}$$

2.26

$$G = \frac{-R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}} \quad G_{\text{Nominal}} = \frac{-R_2}{R_1}$$

$$\epsilon = \left| \frac{G - G_{\text{Nominal}}}{G_{\text{Nominal}}} \right| = \left| \frac{G}{G_{\text{Nominal}}} - 1 \right|$$

$$\epsilon = \left| \frac{1}{1 + \frac{1 + R_2/R_1}{A}} - 1 \right| = \left| \frac{\frac{1 + R_2/R_1}{A}}{1 + \frac{1 + R_2/R_1}{A}} \right| = \frac{1}{\frac{A}{1 + R_2/R_1} + 1}$$

which can be rearranged to yield:

$$\frac{A}{1 + \frac{R_2}{R_1}} + 1 = \frac{1}{\epsilon} \Rightarrow A = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{\epsilon} - 1\right)$$

$$\text{or } A = \left(1 - \frac{G_{\text{Nominal}}}{\epsilon}\right) \left(\frac{1}{\epsilon} - 1\right)$$

For $G_{\text{Nominal}} = 100 V/V$ and $\epsilon = 0.1\% = 0.1$

$$A = \left(1 + 100\right) \left(\frac{1}{0.1} - 1\right) = 909 V/V$$

This is the minimum required value for A.

2.27

$$|G| = \frac{R_2/R_1}{1 + \frac{R_2}{R_1}} \quad A \rightarrow A(1 - \frac{x}{100})$$

$$|G'| = \frac{R_2/R_1}{1 + \frac{R_2}{R_1} R_x} \quad G \rightarrow G'$$

$$\text{For } |G'| = |G| (1 - \frac{x}{100})$$

$$\frac{R_2/R_1}{1 + \frac{R_2}{R_1}} = \frac{R_2/R_1}{1 + \frac{R_2}{R_1} (1 - \frac{x}{100})}$$

$$\frac{1 + \frac{R_2}{R_1}}{A(1 - \frac{x}{100})} = (1 + \frac{R_2/R_1}{A}) / (1 - \frac{x}{100})$$

$$1 - \frac{x}{100} + \frac{1 + R_2/R_1}{A} \cdot \frac{1 - x/100}{1 - x/100} = 1 + \frac{1 + R_2/R_1}{A}$$

$$\frac{1 + R_2/R_1}{A} \cdot \frac{1 - x/100}{1 - x/100} = \frac{x}{100}$$

$$A = \frac{1 + K}{1 - \frac{x}{100}} (1 + R_2/R_1) = (\frac{K+1}{1 - \frac{x}{100}}) (1 + \frac{R_2}{R_1})$$

$$\text{For } \frac{R_2}{R_1} = 100, x = 50, K = 100; A = \frac{99}{0.5} \times 101 = 19998$$

$$A \approx 2 \times 10^4 \text{ V/V}$$

Thus for $A = 2 \times 10^4 \text{ V/V}$, a reduction of 50% results in only 0.5% reduction of the closed loop gain whose nominal value is $\frac{R_2}{R_1}$ (100).

2.28

From the results of example 2.2, the gain of the circuit in fig. 2.8 is given by:

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} (1 + \frac{R_4}{R_2} + \frac{R_4}{R_3})$$

$$\text{For } R_1 = R_2 = R_4 = 1 \text{ M}\Omega \Rightarrow \frac{V_o}{V_i} = -(1 + 1 + \frac{1}{R_3}) \text{ K}\Omega$$

$$\text{a) } \frac{V_o}{V_i} = -10 \text{ V/V} \Rightarrow 10 = 2 + \frac{1}{R_3} \Rightarrow R_3 = \frac{1}{10} \text{ M}\Omega = 125 \text{ k}\Omega$$

$$\text{b) } \frac{V_o}{V_i} = -100 \text{ V/V} \Rightarrow 100 = 2 + \frac{1}{R_3} \Rightarrow R_3 = \frac{1}{100} \text{ M}\Omega = 10.2 \text{ k}\Omega$$

$$\text{c) } \frac{V_o}{V_i} = -2 \text{ V/V} \Rightarrow 2 = 2 + \frac{1}{R_3} \Rightarrow R_3 = \infty; \text{ eliminate } R_3.$$

2.29

$$R_2/R_1 = 1000, R_2 = 100 \text{ k}\Omega \Rightarrow R_1 = 100 \Omega$$

$$\text{a) } R_{in} = R_1 = 100 \Omega$$

$$\text{b) } \frac{V_o}{V_i} = \frac{-R_2}{R_1} (1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}) = -1000$$

$$\text{If } R_2 = R_1 = R_4 = 100 \text{ k}\Omega \Rightarrow R_3 = \frac{100 \text{ k}\Omega}{1000 - 2} \approx 100 \Omega$$

$$R_{in} = R_1 = 100 \text{ k}\Omega$$

2.30

$$V_x = 0 - i_1 R_2, i_1 = \frac{V_i}{R_1} \Rightarrow V_x = -\frac{V_i}{R_1} R_2$$

$$\frac{V_x}{V_i} = -\frac{R_2}{R_1}$$

$$V_x = V_o \frac{R_2 || R_3}{R_2 || R_3 + R_4} = \frac{V_o R_2 R_3}{R_2 R_3 + R_4 R_2 + R_4 R_3}$$

$$\frac{V_o}{V_x} = \frac{R_2 R_3 + R_2 R_4 + R_3 R_4}{R_2 R_3} = 1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}$$

$$\frac{V_o}{V_x} = \frac{V_o}{V_i} = \frac{(1 + R_4/R_3 + R_4/R_2)}{-R_1/R_2} \Rightarrow$$

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} (1 + \frac{R_4}{R_3} + \frac{R_4}{R_2})$$

2.31

$$\text{a) } R_1 = R$$

$$R_2 = R || R + \frac{R}{2} = \frac{R}{2} + \frac{R}{2} = R$$

$$R_3 = R_2 || R + \frac{R}{2} = R || R + \frac{R}{2} = R$$

$$R_4 = R_3 || R + \frac{R}{2} = R || R + \frac{R}{2} = R$$

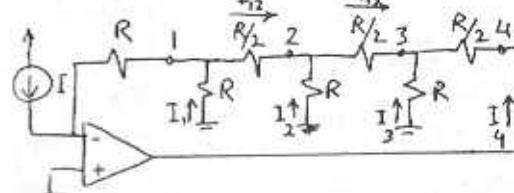
$$\text{b) } V_o = RI = R I_1 \Rightarrow I_1 = \frac{V_o}{R}$$

$$I_{12} = I + I_1 = 2I \Rightarrow \frac{V_o}{R} + 2I \times \frac{R}{2} = RI_2$$

$$RI + RI_1 = RI_2 \Rightarrow I_2 = 2I$$

$$I_3 = I_2 + I_{12} = 4I \Rightarrow \frac{V_o}{R} + 4I \times \frac{R}{2} = RI_3$$

$$R \times 2I + 4I \times \frac{R}{2} = RI_3 \Rightarrow I_3 = 4I, I_4 = -(4I + 4I) \Rightarrow I_4 = -8I$$



Cont.

2.34

$$\begin{aligned}
 a) V_1 &= I_1 R = IR \\
 V_2 &= I_2 R = 2IR \\
 V_3 &= -I_3 R = -4IR \\
 V_4 &= -I_3 R + I_4 \frac{R}{2} = -4IR - 8I \frac{R}{2} = -8IR
 \end{aligned}$$

2.32

$$\begin{aligned}
 a) I_1 &= \frac{1V}{10k\Omega} = 0.1mA \\
 I_2 &= 0.1mA, I_2 \times 10k\Omega = I_3 \times 100\Omega \Rightarrow I_3 = 10mA \\
 V_x &= 10mA \times 100\Omega = 1V
 \end{aligned}$$

$$\begin{aligned}
 b) V_x &= R_L I_L + V_o, I_1 = I_2 + I_3 = 10.1mA \\
 1V &= R_L \times 10.1mA + V_o \\
 R_L &= \frac{1-V_o}{10.1} \Rightarrow R_{L\max} = \frac{1-V_{o\min}}{10.1} = \frac{14}{10.1} = 1.38k\Omega \\
 R_{L\min} &=
 \end{aligned}$$

$$\begin{aligned}
 c) 100\Omega < R_L < 1k\Omega \\
 I_L \text{ stays fixed at } 10.1mA \\
 V_o = V_x - R_L I_L = 1 - R_L \times 10.1 \Rightarrow -9.1 \leq V_o \leq -0.01V
 \end{aligned}$$

2.33

$$\begin{aligned}
 a) \frac{i_L}{i_I} &= 20 \Rightarrow i_L = 20i_I \\
 -10k\Omega \times i_I &= R(i_I - i_L) \\
 R &= \frac{10k\Omega \times i_I}{20i_I - i_I} = 0.53k\Omega
 \end{aligned}$$

$$\begin{aligned}
 b) R_L &= 1k\Omega, -12 \leq V_o \leq 12V \\
 V_o &= R_L i_L + 10k\Omega \times i_I = i_I (1k\Omega \times \frac{i_L}{i_I} + 10k\Omega) \\
 V_o &= i_I (1 \times 20 + 10) = 30i_I \\
 i_I &= \frac{V_o}{30} \Rightarrow -12 \leq i_I \leq 12mA \Rightarrow -0.4 \leq i \leq 0.4mA
 \end{aligned}$$

$$\begin{aligned}
 c) R_L &= \frac{V_o}{i_I} = \frac{0}{i_I} = 0 \\
 V_o &= 0 \Rightarrow i = 0 \\
 \Rightarrow i_L &= 1mA \\
 \text{From part a: } i_L &= 20 \times i_I = 20mA
 \end{aligned}$$

$$R_2 \gg R_3, \text{ if we ignore the current across } R_3 : V_A = \frac{V_o R_3}{R_3 + R_4}$$

$$\frac{V_T}{R_1} = \frac{0 - V_A}{R_2} \Rightarrow V_A = -\frac{R_2 V_T}{R_1}$$

$$\frac{V_o}{R_3 + R_4} = -\frac{R_2}{R_1} \times V_I \Rightarrow \frac{V_o}{V_I} = -\frac{R_2}{R_1} \left(1 + \frac{R_4}{R_3} \right)$$

Now if we recalculate V_A considering that there is a voltage divider between R_4 and $R_3 \parallel R_2$:

$$V_A = V_o \frac{R_3 \parallel R_2}{R_4 + R_3 \parallel R_2} = V_o \frac{R_3 R_2}{R_4 (R_3 + R_2) + R_2 R_3}$$

$$V_A = V_o \frac{R_2 R_3}{R_3 R_4 + R_2 R_4 + R_2 R_3}$$

$$V_A = V_o \frac{1}{R_4/R_2 + R_4 + 1}$$

$$V_A = -\frac{R_2}{R_1} \cdot V_I \Rightarrow \frac{V_o}{V_I} = -\frac{R_2}{R_1} \left(\frac{R_4}{R_2} + \frac{R_4}{R_3} + 1 \right)$$

same as example 2.2.

2.35

$$R_I = 100k\Omega \quad -10 \leq \frac{V_o}{V_I} \leq -1 \frac{V}{V}$$

$$R_I = R_1 = 100k\Omega$$

$$\frac{V_o}{V_I} = -\frac{R_2}{R_1} \left(\frac{R_4}{R_3} + \frac{R_4}{R_2} + 1 \right)$$

$$R_4 = 0 \Rightarrow \frac{V_o}{V_I} = -\frac{R_2}{R_1} = -1 \Rightarrow R_2 = 100k\Omega$$

$$R_4 = 10k\Omega \Rightarrow \frac{V_o}{V_I} = -10 = -1 \times \left(\frac{10k\Omega}{R_3} + \frac{10k\Omega}{100k\Omega} + 1 \right)$$

$$+10 = \left(\frac{10}{R_3} + 1.1 \right) \Rightarrow R_3 = 1.12k\Omega$$

Potentiometer in the middle: $\frac{V_o}{V_I} = -1 \left(\frac{5}{5+R_3} + \frac{5}{100} + 1 \right)$

$$\frac{V_o}{V_I} = -1.875 \frac{V}{V}$$

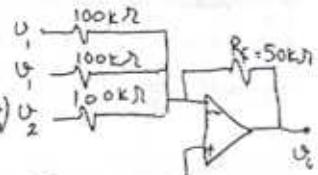
2.36

According to eq. 2.7:

$$U_o = -\left(\frac{R_F}{R_1} U_1 + \frac{R_F}{R_2} U_2 + \frac{R_F}{R_3} U_3\right)$$

$$U_o = -\left(\frac{50k}{100k} U_1 + \frac{50k}{100k} U_2 + \frac{50k}{100k} U_3\right)$$

$$U_o = -(U_1 + U_2) \quad U_1 = 3, U_2 = -3 \Rightarrow U_o = -1.5V$$

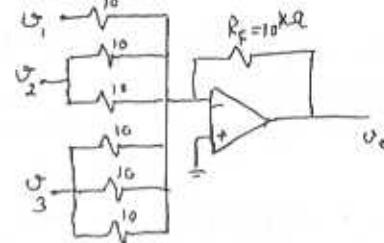


$$\frac{R_F}{R_3} = 3 \Rightarrow R_3 = \frac{10}{3} k\Omega$$

$$R_{I1} = 10k\Omega$$

$$R_{I2} = 5k\Omega$$

$$R_{I3} = 3.3k$$



$$b) U_o = -(U_1 + U_2 + 2U_3 + 2U_4)$$

$$\frac{R_F}{R_1} = 1 \Rightarrow R_1 = 10k\Omega$$

$$\frac{R_F}{R_2} = 1 \Rightarrow R_2 = 10k\Omega$$

$$\frac{R_F}{R_3} = 2 \Rightarrow R_3 = \frac{10}{2} k\Omega$$

$$\frac{R_F}{R_4} = 2 \Rightarrow R_4 = \frac{10}{2} k\Omega$$

$$R_{I1} = R_{I2} = 10k\Omega$$

$$R_{I3} = R_{I4} = 5k\Omega$$

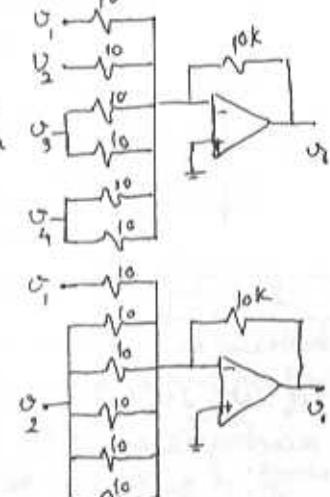
$$c) U_o = -(U_1 + 5U_2)$$

$$R_1 = 10k$$

$$R_2 = \frac{10k}{5}$$

$$R_{I1} = 10k\Omega$$

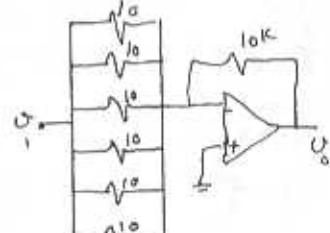
$$R_{I2} = 2k\Omega$$



$$d) U_o = -6U_1$$

$$R_1 = 10k$$

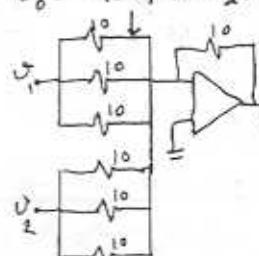
$$R_{I1} = 1.67k\Omega$$



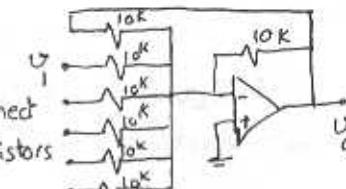
Suggested configurations:

$$U_o = -(2U_1 + 2U_2 + 2U_3) \rightarrow$$

$$U_o = -(3U_1 + 3U_2)$$



In order to have coefficient = 0.5, connect one of the input resistors to U_o . $\frac{U_o}{U_1} = 0.5$



2.38

$$U_o = -(2U_1 + 4U_2 + 8U_3)$$

$$R_1, R_2, R_3 \geq 10k\Omega$$

$$\frac{R_F}{R_1} = 2, \frac{R_F}{R_2} = 4, \frac{R_F}{R_3} = 8$$

$$R_3 = 10k\Omega \Rightarrow R_F = 80k\Omega$$

$$R_2 = 20k\Omega$$

$$R_1 = 40k\Omega$$

2.39

$$a) U_o = -(U_1 + 2U_2 + 3U_3)$$

$$\frac{R_F}{R_1} = 1 \Rightarrow R_1 = 10k\Omega, \frac{R_F}{R_2} = 2 \Rightarrow R_2 = 5k\Omega$$

2.40

The output signal should be:

$$V_o = -5 \sin \omega t - 5$$

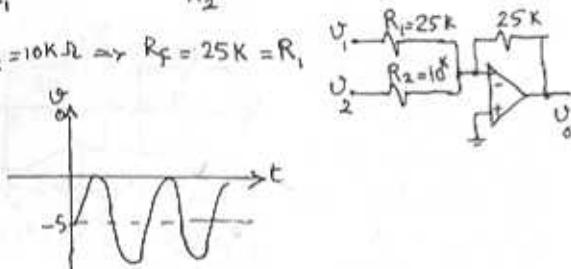
if we assume: $V_1 = 5 \sin \omega t$

$$V_2 = 2V \quad \{ V_o = V_1 + 2.5V_2 \}$$

In a weighted summer configuration:

$$\frac{R_f}{R_1} = +1 \quad \frac{R_f}{R_2} = 2.5$$

$$R_2 = 10k\Omega \Rightarrow R_f = 25k\Omega = R_1$$



2.41

$$V_o = V_1 + 2V_2 - 3V_3 - 4V_4 : \text{Consider Fig. 2.11.}$$

According to eq. 2.8 For a weighted summer circuit:

$$V_o = V_1 \frac{R_a}{R_1} \frac{R_c}{R_b} + V_2 \frac{R_a}{R_2} \frac{R_c}{R_b} - V_3 \frac{R_c}{R_3} - V_4 \frac{R_c}{R_4}$$

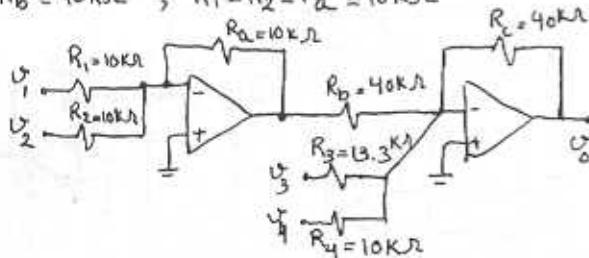
$$\frac{R_a}{R_1} \frac{R_c}{R_b} = 1, \quad \frac{R_a}{R_2} \frac{R_c}{R_b} = 1, \quad \frac{R_c}{R_3} = 3, \quad \frac{R_c}{R_4} = 4$$

assume:

$$R_4 = 10k\Omega \Rightarrow R_c = 40k\Omega \Rightarrow R_3 = \frac{40}{3} = 13.3k\Omega$$

$$\frac{R_a}{R_1} \frac{40}{R_b} = 1 \quad \frac{R_a}{R_2} \frac{40}{R_b} = 1$$

$$R_b = 40k\Omega, \quad R_1 = R_2 = R_a = 10k\Omega$$



2.42

$$V_1 = 3 \sin(2\pi \times 60t) + 0.01 \sin(2\pi \times 1000t)$$

$$V_2 = 3 \sin(2\pi \times 60t) - 0.01 \sin(2\pi \times 1000t)$$

we want to have: $V_o = 10V_1 - 10V_2$

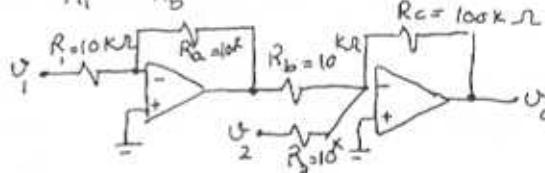
we use the circuit in Fig. 2.11.

According to Eq. 2.8:

$$V_o = V_1 \frac{R_a}{R_1} \frac{R_c}{R_b} - V_2 \frac{R_c}{R_3}$$

$$\frac{R_a}{R_1} \frac{R_c}{R_b} = 10, \quad \frac{R_c}{R_3} = 10, \text{ if } R_3 = 0 \Rightarrow R_c = 100k\Omega$$

$$\Rightarrow \frac{R_a}{R_1} \times \frac{100k\Omega}{R_b} = 10 \Rightarrow R_a = R_1 = R_b = 10k\Omega$$



$$V_o = 10V_1 - 10V_2 = 10 \times 0.02 \sin(2\pi \times 1000t)$$

$$V_o = 0.2 \sin(2\pi \times 1000t) \quad -0.2 \leq V_o \leq 0.2$$

2.43

This is a weighted summer circuit:

$$V_o = -\left(\frac{R_f}{R_0} V_0 + \frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3\right)$$

$$\text{we may write: } V_0 = 5^0 \times a_0, \quad V_1 = 5^1 \times a_1, \quad V_2 = 5^2 \times a_2, \quad V_3 = 5^3 \times a_3$$

$$V_o = -R_f \left(\frac{5a_0}{80k} + \frac{5}{40k} a_1 + \frac{5}{20k} a_2 + \frac{5}{10k} a_3 \right)$$

$$V_o = -R_f \left(\frac{a_0}{16} + \frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{2} \right)$$

$$V_o = -\frac{R_f}{16} (2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3)$$

$$-12 \leq V_o \leq 0 \Rightarrow \frac{R_f}{16} (2 \times 1 + 2 \times 1 + 2^2 \times 1 + 2^3 \times 1) =$$

$$= \frac{15 R_f}{16} = 12 \quad \text{when } a_0 = a_1 = a_2 = a_3 = 1 \text{ we have the peak value at } V_o.$$

$$\Rightarrow R_f = 12.8k\Omega$$

2.44

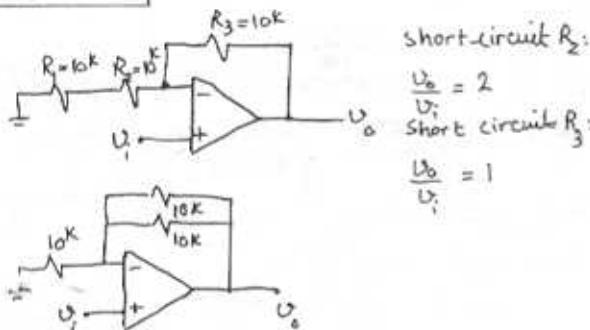
$$a) \frac{V_o}{V_1} = 1 = 1 + \frac{R_2}{R_1} \Rightarrow R_2 = 0, R_1 = 10k\Omega$$

$$b) \frac{V_o}{V_1} = 2 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = R_2 = 10k\Omega$$

Cont.

c) $\frac{U_o}{U_i} = 101 \frac{V}{V} = 1 + \frac{R_2}{R_1} \Rightarrow \text{if } R_1 = 10k\Omega \Rightarrow R_2 = 100k\Omega$
d) $\frac{U_o}{U_i} = 100 \frac{V}{V} = 1 + \frac{R_2}{R_1} \Rightarrow \text{if } R_1 = 10k\Omega \Rightarrow R_2 = 990k\Omega$

2.45



2.46

$$U_+ = U_- = V = R \times i, i = 100 \mu A \text{ when } V = 10^5$$

$$\Rightarrow R = \frac{10}{0.1mA} = 100k\Omega$$

As indicated, i only depends on R and V and the meter resistance does not affect i .

2.47

Refer to the circuit in P2.47:

a) using superposition, we first set $U_p = U_{p2} = \dots = 0$. The output voltage that results in response to $U_{N1}, U_{N2}, \dots, U_{Nn}$ is:

$$U_{ON} = -\left[\frac{R_F}{R_{N1}} U_{N1} + \frac{R_F}{R_{N2}} U_{N2} + \dots + \frac{R_F}{R_{Nn}} U_{Nn} \right]$$

Then we set $U_{N1} = U_{N2} = \dots = 0$, then:

$$R_N = R_{N1} \parallel R_{N2} \parallel R_{N3} \parallel \dots \parallel R_{Nn}$$

The circuit simplifies to:

$$U_{Op} = \left(1 + \frac{R_F}{R_N}\right) \times$$

$$\left(\frac{U_p}{R_{P1}} + \frac{U_p}{R_{P2}} + \dots + \frac{U_p}{R_{Pn}} + \frac{U_{p2}}{R_{P1}} + \frac{U_{p2}}{R_{P2}} + \dots + \frac{U_{p2}}{R_{Pn}} + \dots + \frac{U_{pn}}{R_{P1}} + \dots + \frac{U_{pn}}{R_{Pn}} \right)$$

$$U_{Op} = \left(1 + \frac{R_F}{R_N}\right) \left(U_p \frac{R_P}{R_{P1}} + U_{p2} \frac{R_P}{R_{P2}} + \dots + U_{pn} \frac{R_P}{R_{Pn}} \right)$$

where:
 $R_P = R_{P1} \parallel R_{P2} \parallel \dots \parallel R_{Pn}$

when all inputs are present:

$$U_o = U_{ON} + U_{Op} = -\left(\frac{R_F}{R_N} U_{N1} + \frac{R_F}{R_N} U_{N2} + \dots + \left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_{P1}} U_{P1} + \frac{R_P}{R_{P2}} U_{P2} + \dots \right) \right)$$

$$\text{b) } U_o = -2U_{N1} + U_{P1} + 2U_{P2}$$

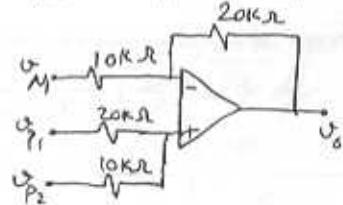
$$\frac{R_F}{R_N} = 2 \quad R_{N1} = 10k\Omega \Rightarrow R_F = 20k\Omega$$

$$\left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_{P1}} \right) = 1 \Rightarrow 3 \frac{R_P}{R_{P1}} = 1 \Rightarrow R_{P2} = \frac{R_{P1}}{2}$$

$$\left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_{P2}} \right) = 2 \Rightarrow 3 \frac{R_P}{R_{P2}} = 2 \Rightarrow R_{P2} = \frac{R_{P1}}{2}$$

where $R_P = \frac{R_{P1} \times R_{P2}}{R_{P1} + R_{P2}}$ (ignoring R_{P3})

Note that if the results from the last 2 constraints differ, we would use an additional resistor connected from the positive input to ground. (R_{P3})



2.48

$$U_o = U_{I1} + 3U_{I2} - 2(U_{I3} + 3U_{I4})$$

Refer to P2.47.

$$\frac{R_F}{R_{N3}} = 2 \quad \text{if } R_{N3} = 10k\Omega \Rightarrow R_F = 20k\Omega$$

$$\frac{R_F}{R_{N4}} = 6 \Rightarrow R_{N4} = \frac{20}{6} = 3.3k\Omega$$

$$R_N = R_{N3} \parallel R_{N4} = 10k \parallel 3.3k = 2.48k\Omega$$

$$\left(1 + \frac{R_F}{R_N}\right) \frac{R_P}{R_{P1}} = 1 \Rightarrow \left(1 + \frac{20}{2.48}\right) \frac{R_P}{R_{P1}} = 1 \Rightarrow 9.06 \frac{R_P}{R_{P1}} = R_{P1}$$

$$R_P = R_{P1} \parallel R_{P2} \parallel R_{P3} \Rightarrow R_P = \frac{1}{\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P3}}}$$

$$\left(1 + \frac{R_F}{R_N}\right) \frac{R_P}{R_{P2}} = 3 \Rightarrow 9.06 \frac{R_P}{R_{P2}} = 3 \Rightarrow R_{P2} \approx 3R_{P1}$$

$$R_{P1} \parallel R_{P2} = \frac{9 \times 3R_{P1}}{9+3} = 2.25R_{P1}, R_P = 2.25R_{P1} \parallel R_{P3}$$

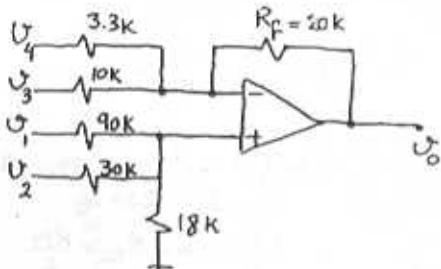
Cont.

$$2.25 R_p + R_{P_0} = 2.25 R_{A_0} \Rightarrow R_{P_0} = 1.8 R_p$$

if $R_p = 10\text{ k}\Omega \Rightarrow R_{P_0} = 18\text{ k}\Omega$

$$R_{P_1} = 9 \times 10\text{ k} = 90\text{ k}\Omega$$

$$R_{P_2} = 3 \times 10\text{ k} = 30\text{ k}\Omega$$



2.49

$$U_+ = U_i \frac{R_4}{R_3 + R_4} = U_i$$

$$\frac{U_o}{R_1} = \frac{U_+ - U_-}{R_2} \rightarrow U_o = U_- (1 + \frac{R_2}{R_1})$$

from the two above equations :

$$\frac{U_o}{U_i} = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) = \frac{1 + R_2/R_1}{1 + R_3/R_4}$$

2.50

Refer to Fig. 2.50. Setting $U_2 = 0$, we obtain the output component due to U_1 as:

$$U_{o1} = -20U_1$$

Setting $U_1 = 0$, we obtain the output component due to U_2 as:

$$U_{o2} = U_2 \left(1 + \frac{20R}{R}\right) \left(\frac{20R}{20R + R}\right) = 20U_2$$

The total output voltage is:

$$U_o = U_{o1} + U_{o2} = 20(U_2 - U_1)$$

$$\text{For } U_1 = 10\sin 2\pi \times 60t - 0.1\sin(2\pi \times 1000t)$$

$$U_2 = 10\sin 2\pi \times 60t + 0.1\sin(2\pi \times 1000t)$$

$$U_o = 4\sin(2\pi \times 1000t)$$

2.51

$$\frac{U_o}{U_i} = 1 + \frac{R_2}{R_1} = 1 + \frac{(1-x)}{x} = 1 + \frac{1}{x} - 1 = \frac{1}{x}$$

$$0 < x \leq 1 \Rightarrow 1 + \frac{1}{x} \leq \infty$$

if we add a resistor on the ground path:

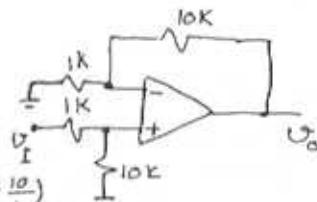
$$\frac{U_o}{U_i} = 1 + \frac{(1-x)\times 10\text{ k}}{x \times 10\text{ k} + R}$$

$$(\text{Gain}_{\max} = 21 \text{ when } x=0)$$

$$x=0 \Rightarrow 21 = 1 + \frac{10\text{ k}}{R} \Rightarrow R = \frac{10\text{ k}}{20} = 0.5\text{ k}\Omega$$



2.52



$$U_o = U_i \frac{10}{1+10} (1 + \frac{10}{1})$$

$$U_o = 10U_i$$

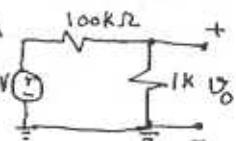
2.53

a) Source is connected directly,

$$U_o = 10 \times \frac{1}{1+10} = 0.99V$$

$$i_L = \frac{U_o}{1\text{ k}} = \frac{0.99}{1\text{ k}} = 0.99\text{ mA}$$

current supplied by the 10V source is 0.99mA.

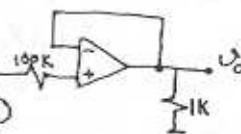


b) inserting a buffer

$$U_o = 10V$$

$$i_L = \frac{10V}{1\text{ k}} = 10\text{ mA}$$

current supplied by the 10V source is 0.



The load current i_L comes from the power supply of the op-amp.

2.54

$$U_o = U_i - \frac{U_o}{A}$$

$$\frac{U_o}{U_i} = \frac{1}{1 + \frac{1}{A}}$$

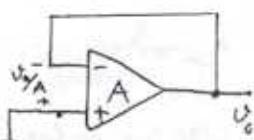
error of Gain magnitude

$$\left| \frac{U_o}{U_i} - 1 \right| = - \frac{1}{A+1}$$

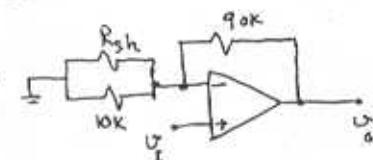
 $A(V/V)$ $\frac{U_o}{U_i}(V/V)$

Gain Error

	1000	100	10
0.999	0.99	0.909	
-0.1%	-1%	-9.1%	

we can shunt a resistor with R_s .

Compensated:



$$R_{sh} : 10 = \frac{1 + \frac{90}{10} + \frac{90}{R_{sh}}}{1 + \frac{90}{10} + \frac{90}{R_{sh}}} \Rightarrow$$

$$10 \times (510 R_{sh} + 90 R_{sh} + 900) = 50 \times (10 R_{sh} + 90 R_{sh} + 900)$$

$$100 R_{sh} = 3600 \Rightarrow R_{sh} = 36 \text{ k}\Omega$$

if $A = 100$ then :

$$G_{uncompensated} = \frac{1 + \frac{90}{10}}{1 + \frac{90}{10} + \frac{90}{10}} = \frac{10}{1.1} = 9.09 V/V$$

$$G_{compensated} = \frac{1 + \frac{90}{10} + \frac{90}{36}}{1 + \frac{90}{10} + \frac{90}{36}} = \frac{12.5}{1.125} = 11.1 V/V$$

2.55

for an inverting amplifier:

$$R_i = R_s, G = -\frac{R_2}{R_s}$$

for a non-inverting amplifier:

$$R_i = \infty, G = 1 + \frac{R_2}{R_s}$$

Case	Gain %	R_{in}	R_s	R_2
a	-10	10K	10K	100K
b	-1	100K	100K	100K
c	-2	50K	50K	100K
d	+1	∞	10K	10K
e	+2	∞	10K	10K
f	+11	∞	10K	100K
g	-0.5	10K	10K	5K

2.56

$$A = 50 V/V, 1 + \frac{R_2}{R_s} = 10 V/V$$

$$\text{if } R_s = 10 \text{ k}\Omega \Rightarrow R_2 = 90 \text{ k}\Omega$$

$$\text{According to Eq. 2.11: } G = \frac{U_o}{U_i} = \frac{1 + \frac{R_2}{R_s}}{1 + \frac{1 + R_2/R_s}{A}}$$

$$G = \frac{1 + \frac{90}{10}}{1 + \frac{1 + 90/10}{50}} = \frac{10}{1.2} = 8.33 V/V$$

In order to compensate the gain drop,

2.57

$$G = \frac{G_o}{1 + \frac{G_o}{A}}, \frac{G_o - G}{G_o} \times 100 = \frac{G_o/A \times 100}{1 + \frac{G_o}{A}} \times \chi$$

$$\text{or } \frac{1 + \frac{G_o}{A}}{\frac{G_o}{A}} \geq \frac{100}{\chi} \Rightarrow \frac{A}{G_o} \geq \underbrace{\frac{100}{\chi} - 1}_{F}$$

$$\Rightarrow A \geq G_o F \text{ where } F = \frac{100}{\chi} - 1 \leq \frac{100}{\chi} F$$

$$\frac{x}{F} \mid \begin{array}{ccccc} 0.01 & 0.1 & 1 & 10 & 100 \\ \hline 10^4 & 10^3 & 10^2 & 10 & \end{array}$$

Thus for :

$$x = 0.01: G_o(V/V) \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4$$

$$A(V/V) \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8$$

too high to be practical

$$x = 0.1: G_o(V/V) \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4$$

$$A(V/V) \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7$$

$$x = 1: G_o(V/V) \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4$$

$$A(V/V) \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6$$

$$x = 10: G_o(V/V) \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4$$

$$A(V/V) \quad 10 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5$$

2.58

for non-inverting amplifier, Eq. 2.11:

$$G = \frac{G_0}{1 + \frac{G_0}{A}}, E = \frac{G_0 - G}{G_0} \times 100$$

for inverting amplifier, Eq. 2.5:

$$G = \frac{G_0}{1 + \frac{1 - G_0}{A}}, E = \frac{G_0 - G}{G_0} \times 100$$

Case	$G_0(V_{in})$	$A(V_{in})$	$G(V_{in})$	$E\%$
a	-1	10	-0.83	16
b	1	10	0.91	9
c	-1	100	-0.98	2
d	10	10	5	50
e	-10	100	9	10
f	-10	1000	-9.89	1.1
g	+1	2	0.67	33

2.59

Refer to Fig. P2.59, when potentiometer is set to the bottom:

$$V_o = V_+ = -15 + \frac{30 \times 20}{20 + 100 + 20} = -10.714V$$

$$\text{when set to the top: } V_o = -15 + \frac{30 \times 120}{20 + 100 + 20} = 10.714V$$

$$\Rightarrow -10.714 \leq V_o \leq +10.714$$

pot has 20 turns, each turn: $\Delta V = \frac{2 \times 10.714}{20} = 1.07V$

2.60

Refer to Fig. 2.16. Notice that similar to eq. 2.15 we have: $\frac{R_4}{R_3} = \frac{R_2}{R_1} = \frac{100}{10}$. therefore according to 2.16:

$$V_o = \frac{R_2}{R_1} V_{id} \Rightarrow A = \frac{R_2}{R_1} = 10 \frac{V}{V}$$

According to 2.20: $R_{id} = 2R_1 = 20k\Omega$

If $\frac{R_2}{R_1}, \frac{R_4}{R_3}$ were different by $\frac{1}{10}$:

$$\frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$$

$$\text{Refer to eq. 2.19: } A_{cm} = \frac{R_4}{R_4 + R_3} \left(1 - \frac{R_2}{R_1} \cdot \frac{R_3}{R_4} \right)$$

$$A_{cm} = \frac{100}{100 + 10} (1 - 0.99) = 0.009$$

$$\text{CMRR} = 20 \log \frac{|A_d|}{|A_{cm}|}, \text{ so let's calculate } A_d$$

$$A_d = \frac{V_o}{V_{id}} \text{ if we apply superposition:}$$

$$V_{o1} = -\frac{R_2}{R_1} V_+, \quad V_{o2} = \frac{V_+}{I_2} \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_2}{R_1} \right)$$

$$V_o = V_{o2} + V_{o1} = \frac{V_+}{I_2} \frac{R_4 / R_3}{1 + \frac{R_4}{R_3}} \left(1 + \frac{R_2}{R_1} \right) - \frac{R_2}{R_1} V_+$$

$$\text{Replace } \frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3} \Rightarrow \frac{R_2}{R_1} = 0.99 \times \frac{100}{10} = 9.9$$

$$V_o = V_{i2} \frac{10}{1 + 10} (1 + 9.9) - V_{i1} 9.9 = 9.9 (V_{i2} - V_{i1})$$

$$\frac{V_o}{V_{id}} = 9.9 = A_d \Rightarrow \text{CMRR} = 20 \log \frac{9.9}{0.009} = 60.8$$

$$\text{CMRR} = \underline{60.8}$$

2.61

If we assume $R_3 = R_1, R_4 = R_2$, then

$$\text{eq. 2.20: } R_{id} = 2R_1 \Rightarrow R_1 = \frac{20}{2} = 10k\Omega$$

(Refer to Fig. 2.16)

$$\text{a) } A_d = \frac{R_2}{R_1} = 1 \frac{V}{V} \Rightarrow R_2 = 10k\Omega$$

$$R_1 = R_2 = R_3 = R_4 = 10k\Omega$$

$$\text{b) } A_d = \frac{R_2}{R_1} = 2 \frac{V}{V} \Rightarrow R_2 = 20k\Omega = R_4$$

$$R_1 = R_3 = 10k\Omega$$

$$\text{c) } A_d = \frac{R_2}{R_1} = 100 \frac{V}{V} \Rightarrow R_2 = 1M\Omega = R_4$$

$$R_1 = R_3 = 10k\Omega$$

$$\text{d) } A_d = \frac{R_2}{R_1} = 0.5 \frac{V}{V} \Rightarrow R_2 = 5k\Omega = R_4$$

$$R_1 = R_3 = 10k\Omega$$

2.62

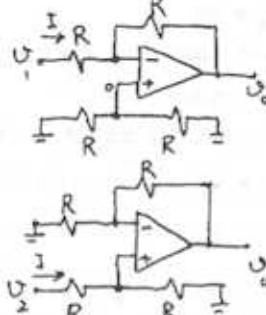
Refer to Fig P2.62 :

Cont.

Considering that $U_+ = U_-$:

$$U_i + \frac{U_o - U_i}{2} = \frac{U_o}{2} \Rightarrow U_o = U_2 - U_1$$

$$U_1 \text{ only: } R_I = \frac{U_1}{I} = R$$

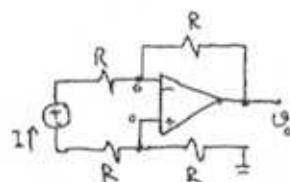


$$U_2 \text{ only: } R_I = \frac{U_2}{I} = 2R$$

U_3 between 2 terminals:

$$R_I = \frac{U}{I} = 2R$$

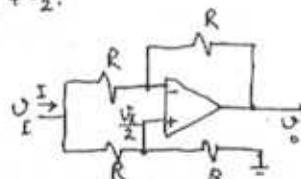
$$U_+ = U_- = 0$$



U_3 connected to both U_+ & U_- :

$$R_I = \frac{U}{I} = R$$

$$U_+ = U_- = \frac{U_2}{2}$$



2.63

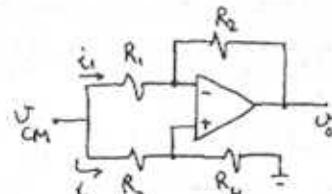
$$U_+ = U_{CM} \frac{R_4}{R_3 + R_4}$$

$$U_+ = U_-$$

$$i_2 = \frac{U_{CM}}{R_3 + R_4}$$

$$i_1 = \frac{U_{CM}}{R_1} - \frac{U_{CM} R_4}{R_3 + R_4} \times \frac{1}{R_1} = \frac{U_{CM} R_3}{R_1 (R_3 + R_4)}$$

$$i = i_1 + i_2 = \frac{U_{CM}}{R_1} \frac{R_3}{R_3 + R_4} + \frac{U_{CM}}{R_3 + R_4}$$



$$\frac{1}{R_I} = \frac{i}{U_{CM}} = \frac{1}{R_1} \frac{1}{\frac{R_4}{R_3} + 1} + \frac{1}{R_3 + R_4}$$

$$\text{if we replace } \frac{R_4}{R_3} \text{ with } \frac{R_2}{R_1}: \left(\frac{R_4}{R_3} + \frac{R_2}{R_1} \right)$$

$$\frac{1}{R_I} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} \Rightarrow R_I = (R_1 + R_2) \parallel (R_3 + R_4)$$

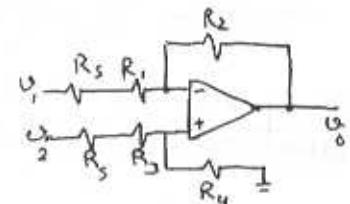
2.64

In order to have an ideal differential amp:

$$\frac{R_S + R_1}{R_2} = \frac{R_S + R_3}{R_4}$$

$$\frac{R_S/R_1 + 1}{R_2/R_1} = \frac{R_S/R_3 + 1}{R_4/R_3}$$

$$\text{Since } \frac{R_2}{R_1} = \frac{R_4}{R_3}:$$



$$\frac{R_S}{R_1} + 1 = \frac{R_S}{R_3} + 1 \Rightarrow R_1 = R_3 \Rightarrow R_2 = R_4$$

2.65

Refer to eq. 2.19 and Fig. P2.62:

$$A_{CM} = \frac{U_o}{U_{CM}} = \frac{R_4}{R_3 + R_4} \left(1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

The worst case is when A_{CM} has its maximum value.

$$A_{CM} = \frac{1}{\frac{R_3}{R_4} + 1} \left(1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

Max $A_{CM} \Rightarrow \frac{R_3}{R_4}$ has to be at its minimum value and also $\frac{R_4}{R_2}$ has to be minimum.

$$\frac{100-x}{100+x} \leq \frac{R_3}{R_4} \leq \frac{100+x}{100-x}, \quad \frac{100-x}{100+x} \leq \frac{R_2}{R_1} \leq \frac{100+x}{100-x}$$

$$\text{so if } \frac{R_3}{R_4} = \frac{100-x}{100+x} \text{ & } \frac{R_2}{R_1} = \frac{100-x}{100+x}$$

$$A_{CM Max} = \frac{1}{\frac{100-x}{100+x} + 1} \left(1 - \frac{100-x}{100+x} \frac{100-x}{100+x} \right)$$

$$A_{CM Max} = \frac{1}{200} \frac{(100+x)^2 - (100-x)^2}{100+x} = \frac{2x}{100+x} \approx \frac{x}{50}$$

$$\begin{array}{c|ccc} x & 0.1 & 1 & 5 \\ \hline A_{CM Max} & 0.002 & 0.02 & 0.1 \end{array}$$

CHRR = $20 \log \left| \frac{A_{d}}{A_{CM}} \right|$. Now we have to calculate A_{CM} based on values we chose for $R_1 - R_4$ that gave us $A_{CM Max}$.

$$R_2 = R_3 = 100-x \quad R_1 = R_4 = 100+x$$

$U_o = U_{O1} + U_{O2}$ by applying superposition

$$U_o = -\frac{R_2}{R_1} U_+ + U_2 \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_2}{R_1} \right)$$

$$U_o = -\frac{100-x}{100+x} U_+ + U_2 \frac{100+x}{200} \left(1 + \frac{100-x}{100+x} \right)$$

$$U_o = -\frac{100-x}{100+x} U_+ + U_2$$

$$\text{if we consider } \frac{100-x}{100+x} \ll 1 \Rightarrow \frac{U_o}{U_{id}} \approx 1$$

Cont.

2.67

$$CMRR = 20 \log \frac{A_d}{A_{cm}} = 20 \log \frac{1}{X_{S0}} = 20 \log \frac{50}{X}$$

X	0.1	1	5
CMRR	54db	34db	20db

2.66

Refer to Fig. 2.16 and eq. 2.19:

$$A_{cm} = \frac{R_4}{R_3 + R_4} \left(1 - \frac{R_2}{R_1} \frac{R_3}{R_4}\right)$$

In order to calculate A_d , we use superposition principle:

$$V_o = V_{o1} + V_{o2} = -\frac{R_2}{R_1} V_1 + V_2 \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_2}{R_1}\right)$$

$$\text{then replace } V_1 = V_{cm} - \frac{V_d}{2}$$

$$V_2 = V_{cm} + \frac{V_d}{2}$$

$$V_o = -\frac{R_2}{R_1} V_{cm} + \frac{R_2}{R_1} \frac{V_d}{2} + V_{cm} \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_2}{R_4}} + \frac{V_d}{2} \frac{1 + \frac{R_2}{R_4}}{1 + \frac{R_2}{R_4}}$$

$$V_o = \underbrace{\frac{R_2}{2R_1} \left[1 + \frac{R_2}{R_4} + \frac{R_2}{R_1} \frac{R_4}{R_3 + R_4}\right]}_{A_d} V_d + \frac{R_2}{R_1} \left[1 + \frac{R_2}{R_4} + \frac{R_2}{R_1} \frac{R_4}{R_3 + R_4}\right] V_{cm}$$

$$CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right| = 20 \log \frac{\frac{R_2}{2R_1} \left[1 + \frac{R_2}{R_4} + \frac{R_2}{R_1} \frac{R_4}{R_3 + R_4}\right]}{\frac{R_2}{R_1} \left[1 + \frac{R_2}{R_4} + \frac{R_2}{R_1} \frac{R_4}{R_3 + R_4}\right]}$$

$$CMRR = 20 \log \left| \frac{1 + \frac{R_2}{R_4} + \frac{R_2}{R_1} \frac{R_4}{R_3 + R_4}}{\frac{R_2}{R_1} - \frac{R_2}{R_4}} \right|$$

For worst case, or minimum CMRR we have to maximize the denominator, which means:

$$R_1 = R_{in} (1 + \epsilon) \quad R_3 = R_{out} (1 - \epsilon)$$

$$R_2 = R_{in} (1 - \epsilon) \quad R_4 = R_{out} (1 + \epsilon)$$

$$\text{also: } \frac{R_{in}}{R_{out}} = \frac{R_{in}}{R_{out}} = K$$

$$CMRR = 20 \log |K| \frac{1 + \frac{1}{2K} \frac{1+\epsilon}{1-\epsilon} + \frac{1}{2K} \frac{1-\epsilon}{1+\epsilon}}{1 - \frac{1-\epsilon}{1+\epsilon}}$$

$$CMRR = 20 \log |K (1 - \epsilon^2) + (1 + \epsilon^2)| \approx 20 \log |K + 1|$$

for $\epsilon^2 \ll 1$.

$$\text{if } K = A_d (\text{ideal}) = 100, \epsilon = 0.01$$

$$CMRR = 20 \log \frac{101}{0.01} \approx 68 \text{ db}$$

$$A_d = 100$$

$$\text{we assume } \frac{R_2}{R_1} = \frac{R_4}{R_3} \text{ then } A_d = \frac{R_2}{R_1} = K$$

$$K \leq 100$$

$$R_{id} = 2R_1 = 20 \text{ k}\Omega \Rightarrow R_1 = 10 \text{ k}\Omega$$

$$CMRR = 80 \text{ db} = 20 \log \frac{A_d}{A_{cm}} \Rightarrow \frac{A_d}{A_{cm}} = 10^4$$

$$\Rightarrow A_{cm} = 0.01$$

$$A_d = 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 1 \text{ M}\Omega$$

$$\text{Refer to P2.66: } CMRR = 20 \log \frac{K+1}{4\epsilon}$$

$$CMRR = 10^4$$

$$\Rightarrow \epsilon = 10^{-2} \times 0.25$$

$$\text{we assumed earlier } \frac{R_2}{R_1} = \frac{R_4}{R_3} \text{ then}$$

$$\frac{R_4}{R_3} \leq 100 \Rightarrow \text{if } R_3 = 10 \text{ k}\Omega \pm \epsilon$$

$$\Rightarrow R_4 = 1 \text{ M}\Omega \pm \epsilon$$

$$R_2 = 1 \text{ M}\Omega \pm \epsilon$$

$$\epsilon = 0.25\%$$

$$R_1 = 10 \text{ k}\Omega \pm \epsilon$$

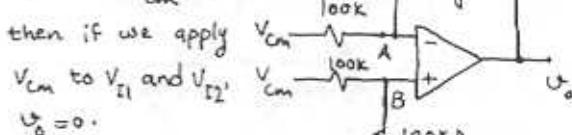
2.68

Refer to Fig. P2.68 and Eq. 2.19.

$$A_{cm} = \frac{R_1}{R_3 + R_4} \left(1 - \frac{R_2}{R_1} \frac{R_3}{R_4}\right) = \frac{100}{100+100} \left(1 - \frac{100 \cdot 100}{100 \cdot 100}\right)$$

$$A_{cm} = 0 \quad \text{Refer to 2.17: } \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$\Rightarrow A_d = \frac{R_2}{R_1} = 1$$

b) Since $A_{cm} = 0$,

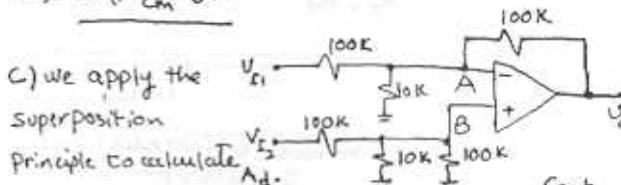
$$\text{Therefore, } V_A = V_{cm} \frac{100}{100+100} =$$

$$V_A = \frac{V_{cm}}{2}$$

$$\text{Similarly, } V_B = \frac{V_{cm}}{2}$$

$$\text{We know } V_A = V_B \text{ and } -2.5 \leq V_A \leq 2.5$$

$$\Rightarrow -5 \leq V_{cm} \leq 5$$



Superposition

principle to calculate A_d .

Cont.

U_{01} is the output voltage when $i_2 = 0$

U_{02} is the output voltage when $V_{I1} = 0$

$$U_0 = U_{01} + U_{02}$$

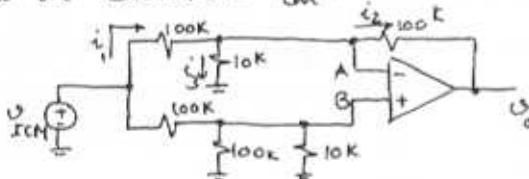
$$i_1 = -\frac{R_2}{R_1} i_2 \quad V_{I1} = -U_{I1}$$

$$U_{02} = \frac{V_{I2}}{100K} \cdot \frac{100K || 10K}{100K || 10K + 100} \left(1 + \frac{100K}{100K || 10K} \right)$$

$$U_{02} = \frac{V_{I2}}{100K} \cdot 1$$

$$\Rightarrow U_0 = U_{01} + U_{02} = -U_{I1} + U_{I2} \Rightarrow A_d = 1$$

Now we calculate A_{cm} :



$$U_B = U_{ICM} \cdot \frac{100K || 10K}{100K || 10K + 100K}, \quad U_A = U_B$$

$$i_1 = \frac{U_{ICM} - U_A}{100K}$$

$$U_0 = U_A - 100K \cdot i_2 \quad \text{and} \quad i_2 = i_1 - i_3 = i_1 - \frac{U_A}{10K}$$

$$U_0 = U_A - 100K \cdot i_1 + 10 \cdot U_A$$

$$U_0 = U_A - U_{ICM} + U_A + 10 \cdot U_A$$

$$U_A = U_B \Rightarrow U_0 = U_{ICM} \left(-1 + 12 \frac{100K || 10K}{100K || 10K + 100K} \right)$$

$$\frac{U_0}{U_{ICM}} = A_{cm} = 0$$

Now we calculate U_{ICM} range:

$$-2.5 \leq U_B \leq 2.5 \Rightarrow -2.5 \cdot U_{ICM} \cdot \frac{100K || 10K}{100K || 10K + 100K} \leq 2.5$$

$$-30 \leq U_{ICM} \leq 30$$

2.69

Refer to Fig. P2.69; we use superposition:

$$U_0 = U_{01} + U_{02}$$

$$\text{calculate } U_{01}: \quad U_+ = \frac{\beta U_{01}}{2} = U_-$$

$$\frac{U_+ - \frac{\beta U_{01}}{2}}{R} = \frac{\frac{\beta U_{01}}{2} - U_0}{R} \Rightarrow U_{01} = \frac{U_0}{\beta - 1}$$

calculate U_{02} :

$$U_2 = \frac{U_{02}}{2} = U_+ \Rightarrow U_2 - \frac{U_{02}}{2} = \frac{U_{02}}{2} - \beta U_{02} \\ \Rightarrow U_{02} = \frac{U_2}{1-\beta}$$

$$U_0 = U_{01} + U_{02} = \frac{U_1}{\beta-1} + \frac{U_2}{1-\beta} = \frac{1}{1-\beta} (U_2 - U_1)$$

$$A_d = \frac{U_0}{U_2 - U_1} = \frac{1}{1-\beta}$$

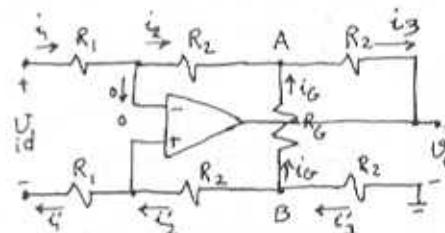
$$A_d = 10 \Rightarrow \beta = 0.9 = \frac{R_S}{R_S + R_6}$$

$$R_{id} = 2R = 2M\Omega \Rightarrow R = 1M\Omega$$

$$R_S + R_6 \leq \frac{R}{100} \Rightarrow R_S + R_6 \leq 10K\Omega$$

$$R_S = 6.8K\Omega \quad R_6 = 680\Omega \quad \Rightarrow \beta = \frac{6.8}{6.8 + 0.68} \approx 0.9$$

2.70



$U_+ = U_-$ so we can consider U_+, U_- a virtual short:

$$i_1 = U_{id}/2R_1 \Rightarrow i_2 = \frac{U_{id}}{2R_1}$$

$$i'_1 = i'_2 = \frac{U_{id}}{2R_1}$$

$$\text{then: } i'_2 R_2 + U_{AB} + i'_2 R_2 = 0 \Rightarrow U_{AB} = -\frac{U_{id}}{R_1} R_2$$

$$i'_G = \frac{U_{id}}{R_G} \times \frac{R_2}{R_1}$$

$$i_3 = i_2 + i'_G = \frac{U_{id}}{2R_1} + \frac{U_{id}}{R_G} \frac{R_2}{R_1}$$

$$i'_3 = i_3 + i'_2 = i_3$$

$$\Rightarrow U_0 = -[i'_3 R_2 + U_{BA} + i_3 R_2]$$

$$U_0 = -[2i_3 R_2 + U_{BA}]$$

$$U_0 = -[\frac{2U_{id}R_2}{2R_1} + 2\frac{U_{id}R_2}{R_1} \frac{R_2}{R_G} + \frac{U_{id}R_2}{R_1}]$$

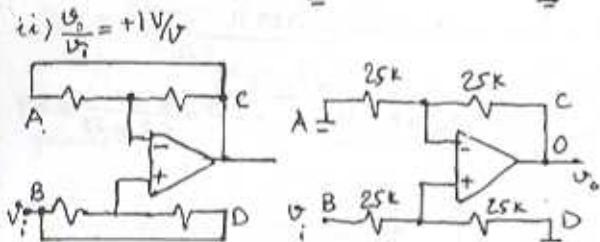
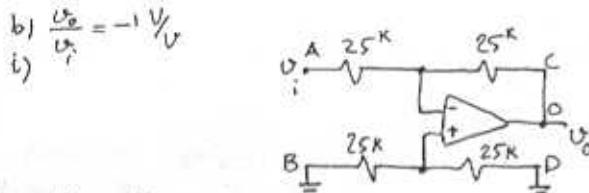
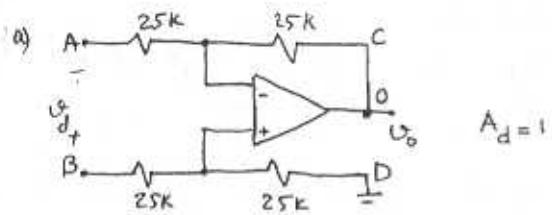
$$\frac{U_0}{U_{id}} = A_d = -2 \frac{R_2}{R_1} \left[1 + \frac{R_2}{R_G} \right]$$

2.71

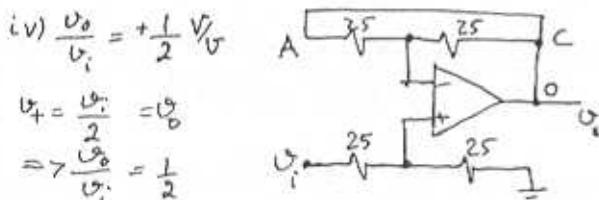
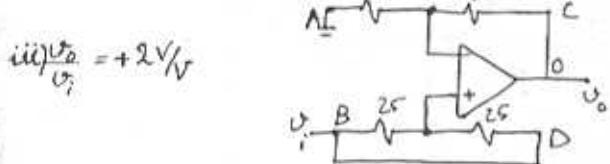
a)

Refer to Eq. 2.17: $A_d = \frac{R_2}{R_1} = 1$. Connect C and D together.

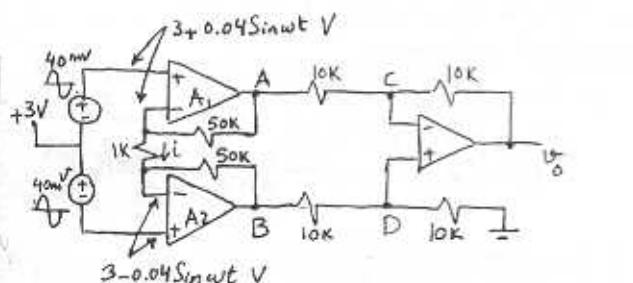
Cont.



The circuit on the left ideally has infinite input resistance



2.72



$$i = 3 + 0.04 \sin \omega t - (3 - 0.04 \sin \omega t) = 0.08 \sin \omega t, \text{ mA}$$

$$V_A = 3 + 0.04 \sin \omega t + 50k \times i = 3 + 4.04 \sin \omega t, \text{ V}$$

$$V_B = 3 - 0.04 \sin \omega t - 50k \times i = 3 - 4.04 \sin \omega t, \text{ V}$$

$$V_C = V_D = \frac{1}{2} V_B = 1.5 - 2.02 \sin \omega t, \text{ V}$$

$$V_o = V_B - V_A = -8.08 \sin \omega t, \text{ V}$$

2.73

Refer to Fig. 2.20.a.

The gain of the first stage is: $(1 + \frac{R_2}{R_1}) = 101$

If the opamps of the first stage saturate at $\pm 14 \text{ V}$: $-14 \leq V_1 \leq +14 \text{ V} \Rightarrow -14 \leq 101 V_{icm} \leq +14$
 $\Rightarrow -0.14 \leq V_{icm} \leq 0.14$

As explained in the text, the disadvantage of circuit in Fig. 2.20.a is that V_{icm} is amplified by a gain equal to $v_{id}(1 + \frac{R_2}{R_1})$ in the first stage and therefore a very small V_{icm} range is acceptable to avoid saturation.

b) In Fig. 2.20.b, when V_{icm} is applied, U_+ for both A_1 & A_2 is the same and therefore no current flows through ZR_1 . This means voltage at the output of A_1 and A_2 is the same as V_{icm} .

$$-14 \leq V_o \leq 14 \Rightarrow -14 \leq V_{icm} \leq 14$$

This circuit allows for bigger range of V_{icm} .

2.74

$$V_{i1} = V_{cm} - U_{d1/2}$$

$$V_{i2} = V_{cm} + U_{d2/2}$$

Refer to Fig. 2.20.a.

output of the first stage: $(1 + \frac{R_2}{R_1})(V_{cm} - U_{d1/2})$

$$V_{o1} = (1 + \frac{R_2}{R_1})(V_{cm} - U_{d1/2})$$

$$V_{o2} = (1 + \frac{R_2}{R_1})(V_{cm} + U_{d2/2})$$

$$V_{o2} - V_{o1} = (1 + \frac{R_2}{R_1}) U_d \Rightarrow A_{d(1)} = 1 + \frac{R_2}{R_1}$$

$$\frac{V_{o2} + V_{o1}}{2} = (1 + \frac{R_2}{R_1}) V_{cm} \Rightarrow A_{cm(1)} = 1 + \frac{R_2}{R_1}$$

$$\text{CMRR} = 20 \log |\frac{A_d}{A_{cm}}| = 0 \quad (\text{First stage})$$

Now Consider Fig. 2.20.b

$$V_{o1} = V_{i1} + R_2 \times \frac{(V_{i1} - V_{i2})}{2R_1}$$

$$V_{o1} = V_{cm} - \frac{V_{i2}}{2} + \frac{R_2}{2R_1} (-U_d)$$

Cont.

$$V_{O1} = V_{CM} - \frac{V_d}{2} \left(1 + \frac{R_2}{R_1} \right)$$

$$V_{O2} = V_{i2} - R_2 \times \frac{V_{i1} - V_{i2}}{2R_1} = V_{CM} + \frac{V_d}{2} + R_2 \cdot \frac{V_d}{2R_1}$$

$$V_{O2} = V_{CM} + \frac{V_d}{2} \left(1 + \frac{R_2}{R_1} \right)$$

$$V_{O2} - V_{O1} = V_d \left(1 + \frac{R_2}{R_1} \right) \Rightarrow A_{d(1)} = 1 + \frac{R_2}{R_1}$$

$$\frac{V_{O2} + V_{O1}}{2} = V_{CM} \Rightarrow A_{CM(1)} = 1$$

$$CMRR = 20 \log \left| \frac{A_d}{A_{CM}} \right| = 20 \log \left(1 + \frac{R_2}{R_1} \right)$$

In 2.20.b, the common mode voltage is not amplified and it is only propagated to the outputs of the first stage.

2.75

Refer to eq. 2.2.2 :

$$A_d = \frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1} \right) = \frac{100K}{100K} \left(1 + \frac{100K}{5K} \right) = 21 \text{ V/V}$$

$$A_{CM} = 0$$

$$CMRR = 20 \log \left| \frac{A_d}{A_{CM}} \right| = \infty$$

If all resistors are $\pm 1\%$:

$$A_d \approx 21$$

In order to calculate A_{CM} , apply V_{CM} to both inputs and note that V_{CM} will appear at both output terminals of the first stage.

Now we can evaluate V_o by analyzing the second stage as was done in problem 2.65.

In P2.65 we showed that if each 100k resistor has $\pm x\%$ tolerance, A_{CM} of the differential amplifier is : $A_{CM} = \frac{V_o}{V_{CM}} = \frac{x}{50}$. Therefore the overall A_{CM} is also $\frac{x}{50}$.

$$x = 1 \Rightarrow A_{CM} = \frac{1}{50} = 0.02$$

$$CMRR = 20 \log \frac{21}{0.02} = 60 \text{ db}$$

$$\text{If } 2R_1 = 1K\Omega : A_d = \frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1} \right) = 201 \text{ V/V}$$

$$A_{CM} = 0.02 \text{ unchanged}$$

$$CMRR = 20 \log \frac{201}{0.02} = 80 \text{ db}$$

Conclusion: Large CMRR can be achieved by

having relatively large A_d in the first stage.

2.76

$$A_d \text{ of the second stage is } \frac{R_4}{R_3} = 0.5$$

$$R_4 = 100K\Omega, R_3 = 200K\Omega$$

We use a series configuration of R_F and R_1 (pot): $R_1 = 100K \text{ pot}$

$$\text{Minimum gain} = 0.5 \left(1 + \frac{R_2}{R_1} \right) = 0.5 \left(1 + \frac{R_2}{100K+R_1} \right)$$

$$1 \leq A_d \leq 100 \Rightarrow 1 = 0.5 \left(1 + \frac{2R_2}{R_F + 100K} \right)$$

$$\Rightarrow R_F + 100 = 2R_2 \quad \textcircled{1}$$

$$\text{Maximum gain} = 100 = 0.5 \left(1 + \frac{R_2}{R_F/2} \right) \Rightarrow$$

$$2R_2 = 199R_F \quad \textcircled{2}$$

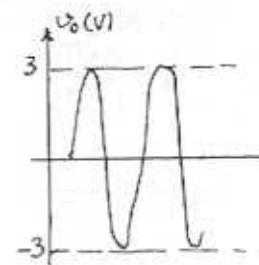
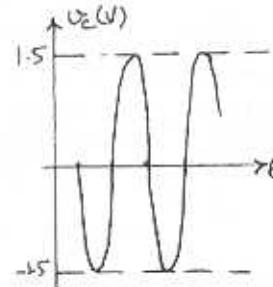
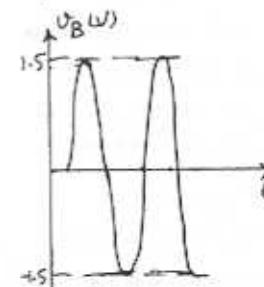
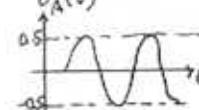
$$\textcircled{1}, \textcircled{2} \Rightarrow R_F = 0.505K\Omega \approx 0.5K\Omega$$

$$R_2 = 50.25K\Omega \approx 50K\Omega$$

2.77

$$\text{a) } \frac{V_B}{V_A} = 1 + \frac{20}{10} = 3 \text{ V/V}, \frac{V_C}{V_A} = -\frac{30}{10} = -3 \text{ V/V}$$

$$\text{b) } V_o = V_B - V_C = 6 \text{ V} \Rightarrow \frac{V_o}{V_A} = 6 \text{ V/V}$$



c) V_B and V_C can be $\pm 1V$ or $28V$ p-p.

$$-28 \leq V_o \leq 28 \text{ or } \frac{56}{\sqrt{2}}$$

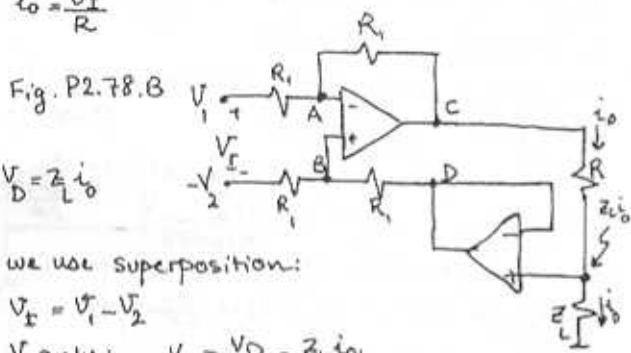
$$V_{o,\text{rms}} = \frac{19.8}{\sqrt{2}} = \frac{28}{\sqrt{2}}$$

2.78

Refer to Fig. P.2.78.a.

Since the inputs of the op-amp don't draw any current, V_i appears across R_1 .

$$i_o = \frac{V_i}{R_1}$$



We use superposition:

$$V_B = V_1 - V_2$$

$$V_1 \text{ only: } V_B = \frac{V_D}{2} = \frac{Z_L i_{o1}}{2}$$

$$\frac{V_1 - \frac{Z_L i_{o1}}{2}}{R_1} = \frac{\frac{Z_L i_{o1}}{2} - i_{o1}(Z_L + R)}{R_1}$$

$$\Rightarrow V_1 = i_{o1} R \Rightarrow i_{o1} = \frac{V_1}{R}$$

Now if only $(-V_2)$ is applied:

$$V_B = \frac{-V_2 + Z_L i_{o2}}{2}, V_A = \frac{i_{o2} \times (R + Z_L)}{2}$$

$$V_A = V_B \Rightarrow -V_2 + Z_L i_{o2} = i_{o2} R + i_{o2} Z_L$$

$$-V_2 = i_{o2} R \Rightarrow i_{o2} = \frac{-V_2}{R}$$

The total current due to both sources is:

$$i_o = i_{o1} + i_{o2} = \frac{V_1}{R} - \frac{V_2}{R} = \frac{V_1 - V_2}{R}$$

The circuit in Figure P.2.78(a) has ideally infinite input resistance, and it requires that both terminals of Z_L be available, while the other circuit has finite input resistance with one side of Z_L grounded.

2.79

A_o	f_b (Hz)	f_c (Hz)	eq. 2.28:
10^5	10^2	10^7	$\omega_c = A_o \omega_b$
10^6	1	10^6	$\Rightarrow f_c = A_o f_b$
10^5	10^3	10^8	
10^7	10^1	10^6	
2×10^5	10	2×10^6	

2.80

$$\text{Eq. 2.25: } A = \frac{A_o}{1 + j \frac{\omega}{\omega_b}} \Rightarrow |A| = \frac{|A_o|}{\sqrt{1 + (\frac{f}{f_b})^2}}$$

$$A_o = 86 \text{ dB}, A = 40 \text{ dB} @ f = 100 \text{ kHz}$$

$$20 \log \sqrt{1 + (\frac{f}{f_b})^2} = 20 \log \frac{|A_o|}{|A|} = 20 \log 40 - 20 \log A$$

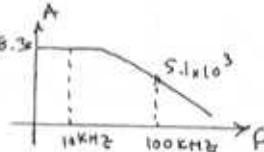
$$1 + (\frac{100 \text{ kHz}}{f_b})^2 = (199.5)^2 \Rightarrow f_b = 0.501 \text{ kHz}$$

$$f_c = A_o f_b = \underbrace{1.995 \times 10^4}_{86 \text{ dB}} \times 501 = 9.998 \text{ MHz} \approx 10 \text{ MHz}$$

2.81

$$A_o = 8.3 \times 10^3 \text{ V/V}$$

$$\text{Eq. 2.25: } A = \frac{A_o}{1 + j \frac{f}{f_b}}$$



$$f_c = A_o f_b$$

$$5.1 \times 10^3 = \frac{8.3 \times 10^3}{\sqrt{1 + (\frac{100 \text{ kHz}}{f_b})^2}} \Rightarrow 1 + (\frac{100 \text{ kHz}}{f_b})^2 = 2.65$$

$$f_b = 60.7 \text{ kHz}$$

$$f_c = A_o f_b = 8.3 \times 10^3 \times 60.7 = 50.3 \text{ MHz}$$

2.82

we have:

$$A_o = 20 \text{ dB} + A_{(dB)} \quad 20 \text{ dB} = 20 \log 10 \Rightarrow A_o = 10 \text{ A}$$

$$\text{a) } A_o = 10 \times 3 \times 10^5 = 3 \times 10^6 \text{ Hz} \text{ V/V}$$

$$A = \frac{A_o}{1 + j \frac{f}{f_b}} \Rightarrow |1 + j \frac{f}{f_b}| = \frac{A_o}{A} = 10 \Rightarrow \frac{10^{12}}{f_b} = \sqrt{99} \Rightarrow f_b = 1 \text{ Hz}$$

$$f_c = A_o f_b = 50 \text{ MHz}$$

$$\text{c) } A = 1500 \text{ V/V} \Rightarrow A_o = 15000 \text{ V/V}$$

Cont.

$$|1 + \frac{jF}{f_b}| = 10 \Rightarrow \frac{0.1 \times 10^6}{f_b} = \sqrt{99} \Rightarrow f_b = 10 \text{ kHz}$$

$$f_t = 15000 \times 10^6 = 150 \text{ MHz}$$

$$b) |1 + \frac{R_2}{R_1}| = 100 \text{ V/V} \quad f_{3db} = 100 \text{ kHz}$$

$$f_t = f_{3db} (1 + \frac{R_2}{R_1}) = 10 \text{ MHz}$$

$$d) A_v = 10 \times 100 = 1000 \text{ V/V}$$

$$|1 + \frac{jF}{f_b}| = 10 \Rightarrow \frac{0.1 \times 10^6}{f_b} = \sqrt{99} \Rightarrow f_b = 10 \text{ MHz}$$

$$f_t = 1000 \times 10 \text{ MHz} = 100 \text{ GHz}$$

$$c) |1 + \frac{R_2}{R_1}| = 2 \text{ V/V} \quad f_{3db} = 10 \text{ kHz}$$

$$f_t = 10 \text{ MHz} \times 2 = 20 \text{ MHz}$$

$$e) A_v = 25 \text{ V/mV} \times 10 = 25 \times 10^4 \text{ V/V}$$

$$|1 + \frac{jF}{f_b}| = 10 \Rightarrow \frac{25 \text{ kHz}}{f_b} = \sqrt{99} \Rightarrow f_b = 2.51 \text{ kHz}$$

$$f_t = A_v f_b = 25 \times 10^4 \times 2.51 \times 10^3 = 62.75 \text{ MHz}$$

$$d) |1 + \frac{R_2}{R_1}| = -2 \text{ V/V} \quad f_{3db} = 10 \text{ kHz}$$

$$f_t = 10 \text{ MHz} (1+2) = 30 \text{ MHz}$$

$$e) |1 + \frac{R_2}{R_1}| = -1000 \text{ V/V} \quad f_{3db} = 20 \text{ kHz}$$

$$f_t = 20 \text{ kHz} (1+1000) = 20.02 \text{ MHz}$$

2.83

$$G_{hom} = -\frac{R_2}{R_1} = -20 \quad A_v = 10^4 \text{ V/V} \quad f_b = 10^6 \text{ Hz}$$

$$\text{Eq. 2.35: } \omega_{3db} = \frac{\omega_b}{1 + R_2/R_1} = \frac{2\pi \times 10^6}{1 + 20} = 2\pi \times 47.6 \text{ kHz}$$

$$f_{3db} = 47.6 \text{ kHz}$$

$$\text{Eq. 2.34: } \frac{V_o}{V_i} \approx \frac{-R_2/R_1}{1 + \frac{s}{\omega_b/(1+R_2/R_1)}} = \frac{-20}{1 + \frac{215}{2\pi \times 10^6}}$$

$$f = 0.1 f_{3db} \Rightarrow \left| \frac{V_o}{V_i} \right| = \frac{-20}{\sqrt{1 + (0.1)^2}} = 19.9 \text{ V/V}$$

$$f = 10 f_{3db} \Rightarrow \left| \frac{V_o}{V_i} \right| = \frac{-20}{\sqrt{1 + 100}} = 1.99 \text{ V/V}$$

$$f) |1 + \frac{R_2}{R_1}| = 1 \text{ V/V} \quad f_{3db} = 1 \text{ kHz}$$

$$f_t = 1 \text{ M} \times 1 = 1 \text{ MHz}$$

$$g) |1 + \frac{R_2}{R_1}| = -1 \quad f_{3db} = 1 \text{ kHz}$$

$$f_t = 1 \text{ M} (1+1) = 2 \text{ MHz}$$

2.86

$$|1 + \frac{R_2}{R_1}| = 100 \text{ V/V} \quad f_{3db} = 8 \text{ kHz}$$

$$f_t = 8 \times 100 = 800 \text{ kHz}$$

$$\text{for } f_{3db} = 20 \text{ kHz} : G_0 = \frac{800}{20} = 40 \text{ V/V}$$

2.87

$$f_{3db} = f_t = 1 \text{ MHz}$$

$$|G| = \frac{1}{\sqrt{1 + (\frac{f}{f_{3db}})^2}} = \frac{1}{\sqrt{1 + f^2}} \quad f \text{ in MHz}$$

$$|G| = 0.99 \Rightarrow f = 0.142 \text{ MHz}$$

The follower behaves like a low-pass STC circuit with a time constant $\tau = \frac{1}{\omega_{3db}}$

$$\text{Thus: } \tau = \frac{1}{2\pi \times 10^6} = \frac{1}{2\pi} \mu\text{s}$$

$$t_r = 2.2\tau = 0.35 \mu\text{s} \quad (\text{Refer to Appendix F})$$

2.85

$$a) |1 + \frac{R_2}{R_1}| = 100 \text{ V/V} \quad f_{3db} = 100 \text{ kHz}$$

$$\text{Eq. 2.35: } \omega_t = \omega_{3db} (1 + \frac{R_2}{R_1}) \Rightarrow f_t = 100 \times 100 = 10 \text{ MHz}$$

2.88

$$1 + \frac{R_2}{R_1} = 10 \text{ V/V} \quad R_1 = 1 \text{ k}\Omega \quad R_2 = 9 \text{ k}\Omega$$

If we consider 5% the time that it takes for the output voltage to reach 99% of its final value, then: $5\% = 100 \text{ ns} \Rightarrow T = 20 \text{ ns}$

$$T = \frac{1}{\omega_{3\text{db}}} \Rightarrow \omega_{3\text{db}} = 50 \times 10^6 \Rightarrow f_{3\text{db}} = 7.96 \text{ MHz}$$

$$f_t = \left(1 + \frac{R_2}{R_1}\right) f_{3\text{db}} = 10 \times 7.96 = 79.6 \text{ MHz}$$

2.89

a) Assume two identical stages, each with a gain function: $G_i = \frac{G_o}{1 + j \frac{f}{f_i}} = \frac{G_o}{1 + j F/f_i}$

$$G_i = \frac{G_o}{\sqrt{1 + \left(\frac{F}{f_i}\right)^2}}$$

Overall gain of the cascade is $\frac{G_o}{1 + \left(\frac{F}{f_i}\right)^2}$
The gain will drop by 3dB when: $1 + \left(\frac{f_{3\text{db}}}{f_i}\right)^2 = \sqrt{2}$

$$1 + \left(\frac{f_{3\text{db}}}{f_i}\right)^2 = \sqrt{2}, \text{ Note } 3\text{dB} = 20 \log \sqrt{2}$$

$$f_{3\text{db}} = f_i \sqrt{\sqrt{2} - 1}$$

b) $40 \text{ dB} = 20 \log G_o \Rightarrow G_o = 100 = 1 + \frac{R_2}{R_1}$

$$f_{3\text{db}} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{1 \text{ MHz}}{100} = 10 \text{ kHz}$$

c) Each stage should have 20dB gain or $1 + \frac{R_2}{R_1} = 10$ and therefore a 3dB frequency of: $f_i = \frac{10^6}{10} = 10^5 \text{ Hz}$.
The overall $f_{3\text{db}} = 10^5 \sqrt{\sqrt{2} - 1} = 64.35 \text{ kHz}$ which is 6 times greater than the bandwidth achieved using single op-amp. (case b above)

2.90

$$f_t = 100 \times 5 = 500 \text{ MHz} \text{ if single op-amp is used.}$$

With op-amp that has only $f_t = 40 \text{ MHz}$, the possible closed loop gain at 5MHz is:
 $K = \frac{10}{5} = 8 \text{ V/V}$

To obtain an overall gain of 100, three such amplifiers cascaded, would be required.

Now, if each of the 3 stages, has a low-frequency (d) closed loop gain K, then its 3dB frequency will be $\frac{40}{K} \text{ MHz}$. Thus for each stage the closed loop gain is: $|G| = \frac{K}{\sqrt{1 + \left(\frac{F}{K}\right)^2}}$

which at $F = 5 \text{ MHz}$ becomes:

$$|G_{5\text{MHz}}| = \frac{K}{\sqrt{1 + \left(\frac{5}{K}\right)^2}}$$

$$\text{The overall gain of 100: } 100 = \left[\frac{K}{\sqrt{1 + \left(\frac{5}{K}\right)^2}} \right]^3$$

$$K = 5.7 \quad \text{Thus for each cascade stage: } f_{3\text{db}} = \frac{40}{5.7} \\ f_{3\text{db}} = 7 \text{ MHz}$$

The 3-dB frequency of the overall amplifier, f_i , can be calculated as:

$$\left[\frac{5.7}{\sqrt{1 + \left(\frac{F}{f_i}\right)^2}} \right]^3 = \frac{(5.7)^3}{\sqrt{2}} \Rightarrow f_i = 3.6 \text{ MHz}$$

2.91

a) $\frac{R_2}{R_1} = K \quad f_{3\text{db}} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{1 + K}$
GBP = Gain $\times f_{3\text{db}}$

$$\text{GBP} = K \frac{f_t}{1 + K}$$

b) $1 + \frac{R_2}{R_1} = K \quad f_{3\text{db}} = \frac{f_t}{K}$
GBP = $K \frac{f_t}{K} = f_t$

The non-inverting amplifier realizes a higher GBP and it's independent of K.

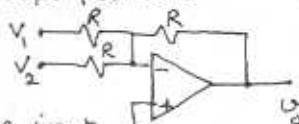
2.92

To find $f_{3\text{db}}$ we use superposition:

Set $V_2 = 0$

Now Using Thevenin's

Theorem to Simplify the input circuit results in:



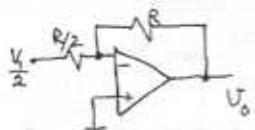
Cont.

$$\frac{V_o}{V_{1/2}} = \frac{-R/R/2}{1 + S \frac{1+R/R/2}{w_c}}$$

which gives:

$$\frac{V_o}{V_i} = \frac{-1}{1 + S \frac{1}{w_c}}$$

$F_{3dB} = \frac{f_E}{3}$. Similar results can be obtained for $\frac{V_o}{V_2}$.



Thevenin's equivalent

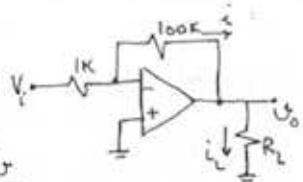
2.93

The peak value of the largest possible sine wave that can be applied at the input without output clipping is: $\frac{\pm 12V}{100} = 0.12V = 120mV$
rms value = $\frac{120}{\sqrt{2}} = 85mV$

2.94

a) $R_L = 1k\Omega$

for $V_{Omax} = 10V$: $V_p = \frac{10}{100} = 0.1V$



when output is at its peak, $i_L = \frac{10}{1k} = 10mA$
 $i = \frac{10}{100k} = 0.1mA$. therefore $i_o = 10 + 0.1 = 10.1mA$
is well under $i_{omax} = 20mA$.

b) $R_L = 100\Omega$

If output is at its peak: $i_L = \frac{10}{0.1} = 100mA$

which exceeds $i_{omax} = 20mA$. Therefore V_o cannot go as high as 10V. instead:

$$20mA = \frac{V_o}{100\Omega} + \frac{V_o}{100k} \Rightarrow V_o = \frac{20}{10.01} = 2V$$

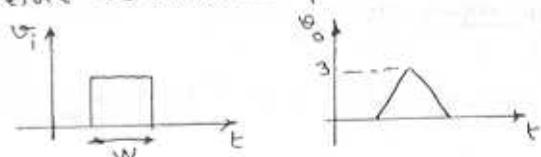
$$V_p = \frac{2}{100} = 0.02V = 20mV$$

c) $R_L = ?$ $i_{omax} = 20mA = \frac{10V}{R_{min}} + \frac{10V}{100k}$
 $20 - 0.1 = \frac{10}{R_{min}} \Rightarrow R_{min} = 502\Omega$

2.95

The output is triangular with the slew rate

of $20V/\mu s$. In order to reach 3V, it takes $\frac{3}{20} \mu s = 0.15 \mu s = 150ns$. Therefore the minimum pulse width is $150ns$.



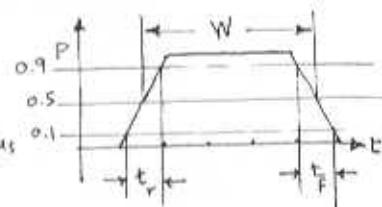
2.96

$w = 2\mu s$

$t_r + t_f = 0.2W = 0.4\mu s$

$t_r = t_f = 0.2\mu s$

$$SR = \frac{(0.9 - 0.1)P}{t_r} = \frac{0.8 \times 10}{0.2} = 40V/\mu s$$



2.97

Slope of the triangle wave = $\frac{20V}{T/2} = SR$

thus $\frac{20}{T} \times 2 = 10V/\mu s$

$\Rightarrow T = 4\mu s$ or $f = \frac{1}{T} = 250kHz$

For a Sine wave $V_o = \hat{V}_o \sin(2\pi \times 250 \times 10^3 t)$

$$\left| \frac{dV_o}{dt} \right|_{max} = 2\pi \times 250 \times 10^3 \hat{V}_o = SR$$

$$\Rightarrow \hat{V}_o = \frac{10 \times 10^6}{2\pi \times 10^3 \times 250} = 6.37V$$

2.98

$$V_o = 10 \sin(\omega t) \Rightarrow \left| \frac{dV_o}{dt} \right| = 10\omega \cos(\omega t) \Rightarrow \left| \frac{dV_o}{dt} \right|_{max} = 10\omega$$

The highest frequency at which this wave output is possible is that for which:

$$\left| \frac{dV_o}{dt} \right|_{max} = SR \Rightarrow 10\omega_{max} = 60 \times 10^6 \Rightarrow \omega_{max} = 6 \times 10^6$$

$$\Rightarrow f_{max} = 45.5 kHz.$$

2.99

a) $V_i = 0.5 \Rightarrow V_o = 10 \times 0.5 = 5V$

Contd.

Output distortion will be due to slew rate limitation and will occur at the frequency for which $\frac{dV_o}{dt}|_{\text{max}} = SR$

$$\omega_{\text{max}} \times 5 = \frac{1}{10^{-8}} = 2\pi \times 10^5 \text{ rad/s} \Rightarrow f_{\text{max}} = 31.8 \text{ kHz}$$

b) The output will distort at the value of V_i that results in $\frac{dV_o}{dt}|_{\text{max}} = SR$.

$$V_o = 10 \text{ V}, \sin 2\pi \times 20 \times 10^3$$

$$\frac{dV_o}{dt}|_{\text{max}} = 10 V_i \times 2\pi \times 20 \times 10^3$$

$$\text{Thus } V_i = \frac{1/10^6}{10 \times 2\pi \times 20 \times 10^3} = 0.795 \text{ V}$$

$$c) V_i = 50 \text{ mV} \quad V_o = 500 \text{ mV} = 0.5 \text{ V}$$

Slew rate begins at the frequency for which $\omega \times 0.5 = SR$

$$\text{which gives } \omega = \frac{1/10^6}{0.5} = 2 \times 10^6 \text{ rad/s or } f = 31.8 \text{ kHz}$$

However the small signal 3db frequency is $f_{3\text{db}} = \frac{f_0}{1 + \frac{R_2}{R_1}} = \frac{2 \times 10^6}{10} = 200 \text{ kHz}$

Thus the useful frequency range is limited at 200 kHz.

d) for $f = 5 \text{ kHz}$, the slew rate limitation occurs at the value of V_i given by

$$\omega \times 10 V_i = SR \Rightarrow V_i = \frac{1/10^6}{2\pi \times 5 \times 10^3 \times 10} = 3.18 \text{ V}$$

Such an input voltage, however would ideally result in an output of 31.8 V which exceeds V_{max} . Thus $V_{i\text{max}} = \frac{V_{\text{max}}}{10} = 1 \text{ V peak}$.

2.102

Output DC offset, $V_{o5} = 3 \text{ mV} \times 1000 = 3 \text{ V}$

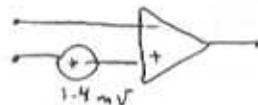
Therefore the maximum amplitude of an input sinusoid is the one that results in an output peak amplitude of $13 - 3 = 10 \text{ V} \Rightarrow V_i = \frac{10}{1000} = 10 \text{ mV}$

If the amplifier is capacity coupled, then:

$$V_{i\text{max}} = \frac{13}{1000} = 13 \text{ mV}$$

2.103

$$V_{o5} = \frac{1.4}{100} = 1.4 \text{ mV}$$



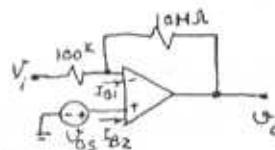
2.104

$$a) I_B = (I_{B1} + I_{B2})/2$$

open input:

$$V_o = V_+ + R_2 I_{B1} = V_{o5} + R_2 I_{B1}$$

$$9.31 = V_{o5} + 10000 I_{B1} \quad (1)$$



input connected to ground:

$$V_o = V_+ + R_2 (I_{B1} + \frac{V_{o5}}{R_1}) = V_{o5} (1 + \frac{R_2}{R_1} + R_2 I_{B1})$$

$$9.09 = V_{o5} \times 101 + 10000 I_{B1} \quad (2)$$

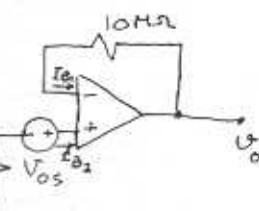
$$(1), (2) \Rightarrow 100 V_{o5} = -0.22 \Rightarrow V_{o5} = -2.2 \text{ mV}$$

$$\Rightarrow I_{B1} = 930 \text{ nA}$$

$$I_B \approx I_{B1} = 930 \text{ nA}$$

$$b) V_{o5} = -2.2 \text{ mV}$$

c) In this case, since R is too large, we may ignore V_{o5} compared to the voltage drop $R_2 I_{B1}$ across R .



$V_{o5} \ll R_2 I_{B1}$, Also Eq. 2.46 holds: $R_3 = R_1 // R_2$
therefore from Eq. 2.47: $V_o = I_{o5} R_2 \Rightarrow I_{o5} = \frac{0.8}{10^4}$
 $I_{o5} = -80 \text{ nA}$

2.100

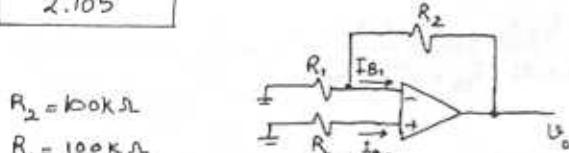
$$V_o = V_{o5} (1 + \frac{R_2}{R_1}) \Rightarrow -0.3 = V_{o5} (1 + \frac{100}{1}) = 3 \text{ mV}$$

2.101

$$V_{o5} = \pm 2 \text{ mV}$$

$$V_o = 0.01 \sin \omega t \times 200 + V_{o5} \times 200 = 2 \sin \omega t \pm 0.4 \text{ V}$$

2.105



$$R_2 = 100k\Omega$$

$$R_1 = 100k\Omega$$

$$R_3 = 5k\Omega$$

$$I_{B1} = 1 \pm 0.05 \mu A, V_{OS} = 0$$

$$I_{B2} = 1 \pm 0.05 \mu A$$

$$\text{From Eq. 2.45: } V_o = -I_{B2}R_3 + R_2(I_{B1} - I_{B2}\frac{R_2}{R_1})$$

$$\text{For } I_{B1} = 1.05 \mu A, I_{B2} = 0.95 \mu A$$

$$V_o = -0.95 \times 5 + 100(1.05 - 0.95 \times \frac{5}{100} \times 9) = 57.5 \text{ mV}$$

b) For $I_{B1} = 0.95 \mu A, I_{B2} = 1.05 \mu A$

$$V_o = -1.05 \times 5 + 100(0.95 - 1.05 \times \frac{5}{100} \times 9) = 42.5 \text{ mV}$$

$$\Rightarrow 42.5 \leq V_o \leq 57.5 \text{ mV}$$

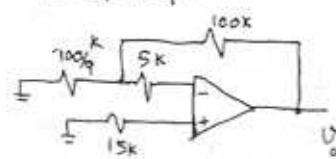
From the discussion in the text we know that to minimize the dc output voltage resulting from the input bias current, we should make the total DC resistance in the inputs of the op-amp equal. Currently, the negative input sees a resistance of $R_1 || R_2 = \frac{100}{9} || 100 = 10k\Omega$ while the positive input terminal sees $5k\Omega$ source resistance. Therefore we should add $5k\Omega$ series resistor to the positive input terminal to make the effective resistance $5k\Omega + 5k\Omega = 10k\Omega$. The resulting V_o can be found as follows:

$$V_o = -I_{B2} \times 10 + 100(I_{B1} - I_{B2} \frac{10}{100}) = (I_{B1} - I_{B2}) \times 100$$

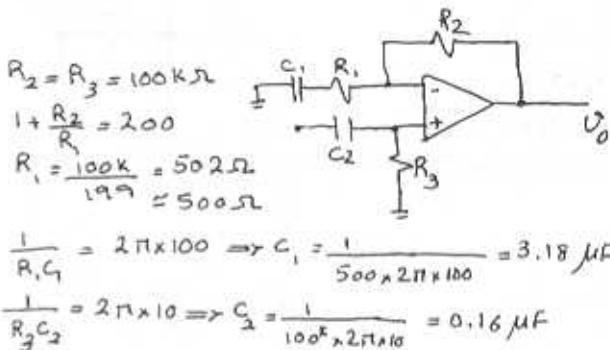
$$V_o = I_{OS} \times 100 = \pm 0.1 \times 100 = \pm 10 \text{ mV}$$

$$V_o = \pm 10 \text{ mV}$$

If the signal source resistance is $15k\Omega$, then the resistances can be equalized by adding a $5k\Omega$ resistor in series with the negative input load of the op-amp.



2.106



$$R_2 = R_3 = 100k\Omega$$

$$1 + \frac{R_2}{R_1} = 200$$

$$R_1 = \frac{100k}{199} = 50.2 \Omega \approx 50k\Omega$$

$$\frac{1}{R_1 C_1} = 2\pi \times 100 \Rightarrow C_1 = \frac{1}{500 \times 2\pi \times 100} = 3.18 \mu F$$

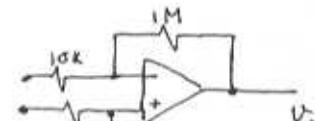
$$\frac{1}{R_2 C_2} = 2\pi \times 10 \Rightarrow C_2 = \frac{1}{100 \times 2\pi \times 10} = 0.16 \mu F$$

2.107

The output component due to V_{OS} is:

$$V_{O1} = V_{OS} \left(1 + \frac{1M}{10k}\right)$$

$$V_{O1} = 4(1+100) = 404 \text{ mV}$$



The output component due to I_B or input bias current is:

$$I_{B1} = I_B + \frac{I_{OS}}{2}, I_{B2} = I_B - \frac{I_{OS}}{2}$$

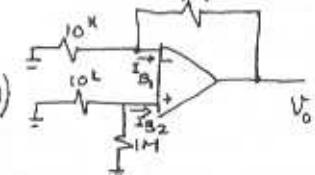
$$I_{B1} = 0.3 + \frac{0.05}{2} = 0.325 \mu A \quad I_{B2} = 0.275 \mu A$$

$$V_+ = -I_{B2} \times (10k \parallel 1M)$$

$$V_+ = -2.72 \text{ mV}$$

$$V_{O2} = V_+ + \left(1M \times \left(I_{B1} + \frac{V_+}{10k}\right)\right)$$

$$V_{O2} = 50 \text{ mV}$$



The worst case (largest) DC offset voltage at the output is $404 + 50 = 454 \text{ mV}$

2.108

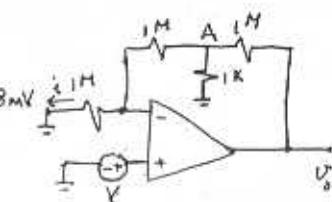
$$V_+ = V_+ = V_{OS} \Rightarrow V_A = 2V_{OS} = 8 \text{ mV}$$

$$i = \frac{V_{OS}}{1M} = V_{OS} (\mu A)$$

$$V_o = V_A + 1M \times \left(i + \frac{V_A}{1k}\right)$$

$$V_o = 2V_{OS} + 1M \left(\frac{V_{OS}}{1M} + 2\frac{V_{OS}}{1k}\right) = 2003V_{OS} = 2003 \times 4 = 8 \text{ V}$$

$$V_o = 8 \text{ V}$$



Cont.

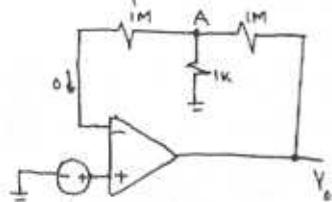
for capacitively coupled input:

$$V_f = V_- = V_{OS}$$

$$V_A = V_{OS}$$

$$V_B = V_A + 1M \times \frac{V_{OS}}{1k} = \pm 4.004V$$

$$V_B = 100I_{OS} = \pm 4.004V$$



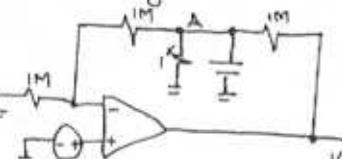
for capacitively coupled 1k to ground:

$$V_f = V_- = V_{OS}$$

$$V_A = 2V_{OS}$$

$$V_B = 3V_{OS} = \pm 12mV$$

This is much smaller than capacitively coupled input case.



2.109

At $0^\circ C$, we expect $\pm 10 \times 25 \times 1000 \mu = \pm 250mV$

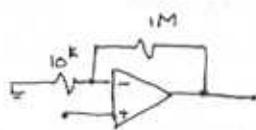
At $75^\circ C$, we expect $\pm 10 \times 50 \times 1000 \mu = \pm 500mV$

We expect these quantities to have opposite polarities.

2.110

$$100 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = 10k\Omega$$

$$a) V_o = 100 \times 10^{-9} \times 1 \times 10^6 = 0.1V$$



b) Largest output offset is:

$$V_D = 1mV \times 100 + 0.1V = 200mV = 0.2V$$

c) For bias current compensation we connect a resistor R_3 in series with the positive input terminal of the op-amp, with: $R_3 = R_1 \parallel R_2$

$$I_{OS} = \frac{100}{10} = 10nA$$

$$R_3 = 10 \parallel 1M = 10k\Omega$$

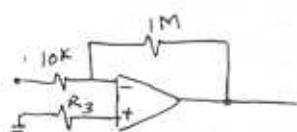
The offset current alone results in an output offset voltage of $I_{OS} \times R_3 = 10 \times 10^{-9} \times 1 \times 10^6 = 10mV$

$$d) V_o = 100mV + 10mV = 110mV$$

2.111

$$R_3 = R_1 \parallel R_2 = 9.9k\Omega$$

(Refer to 2.46)

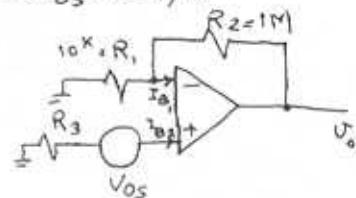


$$V_o = I_{OS} R_2 \quad \text{Eq. 2.47}$$

$$V_o = 0.21 = I_{OS} \times 1M \Rightarrow I_{OS} = 0.21 \mu A$$

$$\text{If } V_{OS} = 1mV$$

$$V_o = -I_{OS} R_3 + V_{OS}$$



$$I_{B_1} = \frac{R_3 I_{B_2} + V_{OS}}{R_1} + \frac{0.21 + R_3 I_{B_2} + V_{OS}}{R_2}$$

$$I_{B_1} = R_3 I_{B_2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + V_{OS} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow I_{B_1} - I_{B_2} = \pm V_{OS} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\Rightarrow I_{OS} = \pm \frac{1mV}{9.9k} = \pm 0.1 \mu A$$

If we apply the same current as I_{OS} to the other end of R_3 , then it will cancel out the offset current effect on the output. $\pm 0.1 \mu A$

Now if we use $\pm 15V$ supplies:

2.112

$$\frac{V_o}{V_i} = -\frac{1}{SCR} = -\frac{1}{j\omega CR} = \frac{1}{-j\omega \times 10 \times 10^{-9} \times 100 \times 10^3}$$

$$\frac{V_o}{V_i} = -\frac{10^3}{j\omega}$$

$$a) \frac{V_o}{V_i} = 1 \Rightarrow \omega = 1 \text{ rad/s} \Rightarrow f = 159 \text{ Hz}$$

b) $\frac{1}{j\omega}$ indicates 90° lag, but since its $\frac{-1}{j\omega}$, it results in output leading the input by 90° .

c) $\frac{V_o}{V_i} = -\frac{10^3}{j\omega}$ if frequency is lowered by a factor of 10, then the output would increase by a factor of 10.

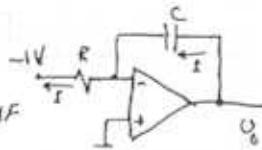
Cont.

d) The phase does not change and the output still leads the input by 90°

2.113

$$R_{in} = R = 100\text{ k}\Omega$$

$$CR = 15 \Rightarrow C = \frac{1}{100 \times 10^3} = 10\text{ nF}$$

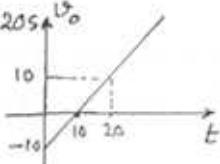


with a -1 V DC input applied, the capacitor charges with a constant current:

$I = \frac{1\text{ V}}{R} = 0.01\text{ mA}$ and its voltage rises linearly:

$$V_o(t) = -10 + \frac{1}{C} \int_0^t I dt = -10 + \frac{I}{C} t = -10 + \frac{t}{RC}$$

the voltage reaches 0 V at $t = 10\text{ RC} = 10\text{ s}$ and it reaches 10 V at $t = 20\text{ s}$



2.114

$$|T| = \frac{1}{\omega RC} \quad \text{IF } |T| = 100 \text{ V/V for } f = 1\text{ kHz}, \text{ then}$$

for $|T| = 1\text{ V/V}$, f has to be $1\text{ kHz} \times 100 = 100\text{ kHz}$.

$$\text{Also } RC = \frac{1}{\omega T} = \frac{1}{2\pi \times 10^3 \times 100} = 1.59\text{ }\mu\text{s}$$

2.115

$R_{in} = R$, Thus $R = 100\text{ k}\Omega$.

$$|T| = \frac{1}{\omega RC} = 1 \quad \text{at } \omega = \frac{1}{RC}$$

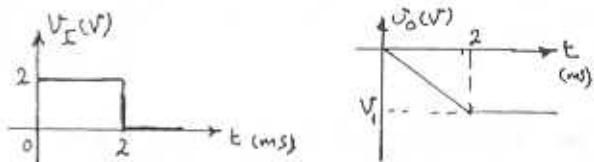
$$\omega = 1000 = \frac{1}{RC} \Rightarrow C = \frac{1}{1000 \times 10^3} = 10\text{ nF}$$

with a $2\text{ V}-2\text{ ms}$ pulse at the input, the output falls linearly until $t = 2\text{ ms}$ at which

$$V_o = V_i, \quad V_o = \frac{-I}{C} t = \frac{-2}{RC} t = -2t \text{ Volts}$$

where t in ms

Thus $V_o = -4\text{ V}$



with $V_i = 2\sin 1000t$ applied at the input,

$$V_o(t) = 2 \cdot \frac{1}{1000 \times 10^3} \sin(1000t + 90^\circ)$$

$$V_o(t) = 2 \sin(1000t + 90^\circ)$$

2.116

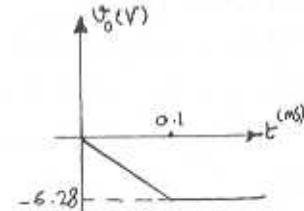
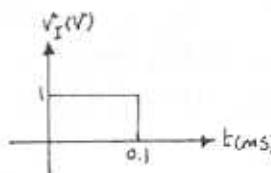
$$R_{in} = R = 20\text{ k}\Omega$$

$$|T| = \frac{1}{\omega RC} = 1 \quad \text{at } \omega = 2\pi \times 10^3 \text{ rad/s} = \frac{1}{C} \Rightarrow C = \frac{1}{2\pi \times 10^3 \times 20} \text{ F}$$

Refer to discussion in page 110:

$\frac{V_o}{V_i} = \frac{R_f/R}{1 + sCR_f}$ and the finite DC gain is $\frac{-R_f}{R}$. Therefore for 40dB gain or equivalently 100 V/V we have: $\frac{-R_f}{R} = -100 \frac{\text{V}}{\text{V}}$
 $\Rightarrow R_f = 100 \times 20\text{ k} = 2\text{ M}\Omega$

The corner frequency $\frac{1}{CR_f}$ is: $\frac{1}{0.796 \times 2 \times 10^6} = 628$ Hz



a) when no R_f

$$V_o(t) = -\frac{1}{RC} \int_0^t i dt = -62.8t \quad \text{if } t \leq 0.1\text{ ms}$$

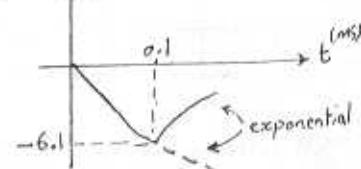
$$V_o(0.1) = -6.28\text{ V}$$

$$\text{b) with } R_f: \quad V_o(t) = V_o(\infty) (1 - e^{-t/CR_f})$$

(Refer to pg. 112)

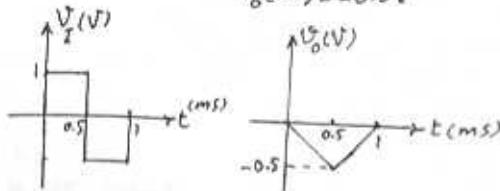
$$V_o(\infty) = -I \times R_f = -\frac{1\text{ V}}{20\text{ k}} \times 2^4 = -100\text{ V}$$

$$V_o(t) = -100(1 - e^{-t/0.1})$$



2.117

For $0 \leq t \leq 0.5 \text{ ms}$: $V_o(t) = V_o(0) - \frac{1}{RC} \int_0^t V_i dt$
 $V_o(t) = 0 - \frac{t}{RC} = -\frac{t}{1 \text{ ms}}$
 $V_o(0.5) = -0.5 \text{ V}$



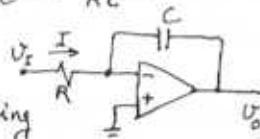
For $0.5 \leq t \leq 1 \text{ ms}$: $V_o(t) = V_o(0.5) - \frac{1}{RC} \int_{0.5}^t V_i dt$
 $V_o(t) = -0.5 + \frac{1}{RC} (t - 0.5)$
 $V_o(1 \text{ ms}) = -0.5 + \frac{0.5}{1} = 0 \text{ V}$

Another way of thinking about this circuit is as follows:

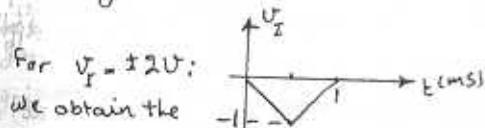
for $0 \leq t \leq 0.5 \text{ ms}$ a current $I = \frac{V_i}{R}$ flows through R and C in the direction indicated on the diagram. At time t we write:

$$I \cdot t = -C(V_o(t)) \Rightarrow V_o(t) = -\frac{I}{C} t = -\frac{1}{RC} t$$

which indicates that the output voltage is linearly decreased, reaching -0.5 V at $t = 0.5 \text{ ms}$.



Then for $0.5 \leq t \leq 1 \text{ ms}$, the current flows in the opposite direction and V_o rises linearly reaching 0 V at $t = 1 \text{ ms}$.



Following waveform: (assuming time constant is the same)

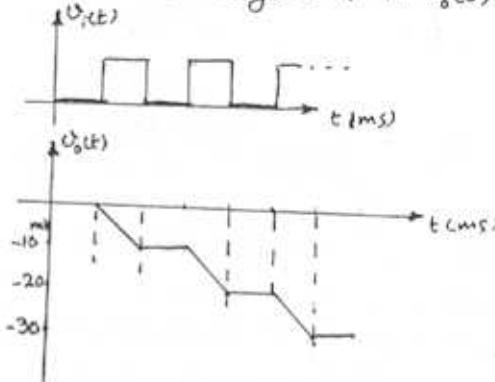
If AC is also doubled, then the waveform becomes the same as the first case where $V_i = \pm 1 \text{ V}$ and $RC = 1 \text{ ms}$.

2.118

Each pulse lowers the output voltage by:

$$\Delta V_o = \frac{1}{RC} \int_0^{10 \text{ ms}} 1 \cdot dt = \frac{10 \mu\text{s}}{RC} = \frac{10 \mu\text{s}}{1 \text{ ms}} = 10 \text{ mV}$$

Therefore a total of 100 pulses are required to cause a change of 1V in $V_o(t)$.



2.119

Refer to Fig. P2.119.

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{Y_1}{Y_2 + SC} = -\frac{R_2/R_1}{1 + SCR_2}$$

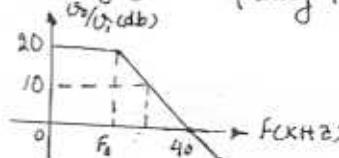
which is an STC LP circuit with a dc gain of $-\frac{R_2}{R_1}$ and a 3-dB frequency $\omega_0 = \frac{1}{CR_2}$. The input resistance equal to R_1 . So for:

$$R_1 = 1 \text{ k} \Omega \Rightarrow R_1 = 1 \text{ k} \Omega \text{ and for dc gain of } 20 \text{ dB}$$

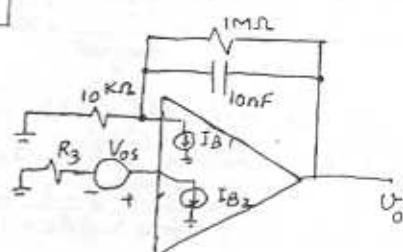
$$10 : \frac{R_2}{R_1} = 10 \Rightarrow R_2 = 10 \text{ k} \Omega$$

$$\text{For } 3 \text{ dB Frequency of } 4 \text{ kHz: } \omega_0 = 2\pi \times 4 \times 10^3 = \frac{1}{CR_2} \Rightarrow C = \frac{1}{4\pi \times 10^3 R_2}$$

the unity gain frequency is (dB) is 40 kHz

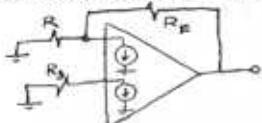


2.120



Cont.

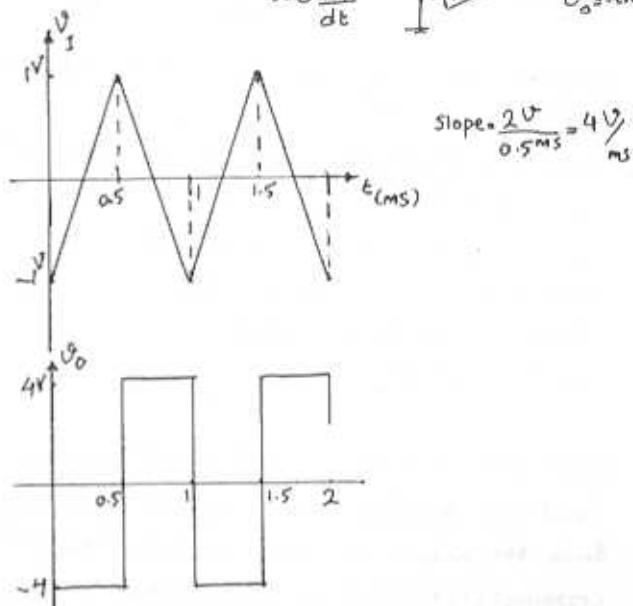
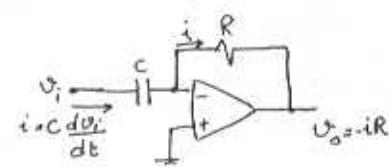
a) To compensate for the effect of dc bias current I_B , we can consider the following model



Similar to the discussion leading to equation (2.46) we have: $R_3 = R \parallel R_F = 10\text{k}\Omega \parallel 1\text{M}\Omega \Rightarrow R_3 = 9.9\text{k}\Omega$

(b) As discussed in Section 2.8.2 the dc output voltage of the integrator when the input is grounded is: $V_o = V_{os} \left(1 + \frac{R_F}{R}\right) + I_{os} R_F$
 $V_o = 3\text{mV} \left(1 + \frac{1\text{M}\Omega}{10\text{k}\Omega}\right) + 10\text{nA} \times 1\text{M}\Omega = 0.303\text{V} + 0.01\text{V}$
 $V_o = 0.313\text{V}$

2.123



2.121

$$\frac{V_o}{V_i} = -SRC = -5 \times 0.01 \times 10^{-6} \times 10 \times 10^3 = -10^{-5}$$

$$\frac{V_o}{V_i} (j\omega) = -j\omega \times 10^{-4} \Rightarrow \left| \frac{V_o}{V_i} \right| = \omega \times 10^{-4} \Rightarrow$$

$$\left| \frac{V_o}{V_i} \right| = 1 \text{ when } \omega = 10^4 \text{ Rad/s or } f = 1.59 \text{ kHz}$$

For an input 10 times this frequency, the output will be 10 times as large as the input: 10V peak-to-peak. The (-j) indicates that the output lags the input by 90°. Thus $V_o(t) = -5 \sin(10^5 t + 90^\circ)$ Volts

Thus the peak value of the output square wave is $0.4 \text{mA} \times 10^4 \Omega = 4\text{V}$. The frequency of the output is the same as the input (1kHz).

The average value of the output is 0. To increase the value of the output to 10V, R has to be increased to $\frac{10}{4} = 2.5$, i.e. $25\text{k}\Omega$.

When a 1-kHz, 1V peak input sine wave is applied $V_i = \sin(2\pi \times 1000t)$

a sinusoidal signal appears at the output.

It can be determined by one of the following methods:

$$a) V_o(t) = -RC \frac{dV_i}{dt} = -0.1 \times 10^{-6} \times 10 \times 10^3 \frac{dV_i}{dt} = -10 \frac{dV_i}{dt}$$

$$V_o(t) = -10^3 \times 2\pi \times 1000 \times C_{av} (2\pi \times 1000t)$$

$$V_o(t) = -2\pi \cos(2\pi \times 1000t)$$

Thus the peak amplitude is 6.28V and the negative peaks occur at $t = 0, \frac{2\pi}{2\pi \times 1000}, \dots$

2.122

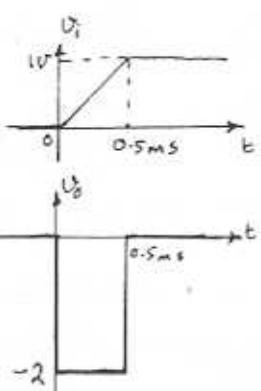
$$V_o = -CR \frac{dV_i}{dt}$$

therefore:

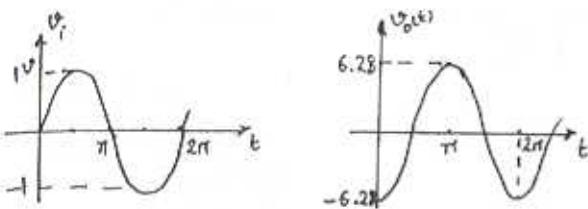
for $0 \leq t \leq 0.5$:

$$V_o = -1\text{ms} \times \frac{1\text{V}}{0.5\text{ms}} = -2\text{V}$$

and $V_o = 0$ otherwise



Cont.



2.125

Refer to Fig. P2.125:

$$\frac{U_0}{U_i} = -\frac{Z_2}{Z_1} = \frac{-R_2}{R_1 + \frac{1}{jC}} = \frac{-(R_2)S}{S + \frac{1}{R_1 C}}, \text{ which is the}$$

transfer function of an STC HP filter with a high frequency gain $K = -\frac{R_2}{R_1}$ and a 3-dB frequency $\omega_0 = \frac{1}{R_1 C}$.

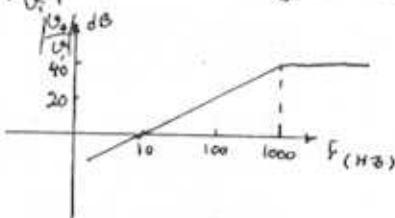
The high-frequency input impedance approaches R_1 . (as $\frac{1}{j\omega C}$ becomes negligibly small) So we can select $R_1 = 10k\Omega$

To obtain a high-frequency gain of 40dB (i.e. 100) : $\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 1M\Omega$.

For a 3-dB frequency of 1000 Hz:

$$\frac{1}{R_1 C} = 2\pi \times 1000 \Rightarrow C = 15.9 \text{nF}$$

From the Bode-diagram below, we see that $\left|\frac{U_0}{U_i}\right|$ reduces to unity at $F = 0.01 F_0 = 10 \text{ Hz}$



2.124

$$RC = 10^3 \text{ when } C = 10^{-6} \text{ F} \Rightarrow R = 100k\Omega$$

$$\frac{U_0}{U_i} = -SRC \quad \frac{U_0}{U_i}(j\omega) = -j\omega RC \quad \phi = -90^\circ$$

always

$$\left|\frac{U_0}{U_i}\right| = 1 \Rightarrow \omega = \frac{1}{RC} = 10 \text{ rad/s} \text{ Gain is 10 times the unity}$$

gain, when the frequency is 10 times the unity gain frequency. Similarly for $\omega = \frac{1}{10} \text{ rad/s}$, gain is $0.1V/V$. (for $\omega = 10 \text{ rad/s}$, gain = $10V/V$)

for high frequencies C is short-circuited, R is in series with the output.

$$\frac{U_0}{U_i} = \frac{R}{R_1} = 100 \Rightarrow R_1 = 1k\Omega$$

$$\frac{U_0}{U_i} = \frac{-RCS}{R_1 CS + 1} = \frac{-10^3}{10^{-5}S + 1} \Rightarrow \omega_{3\text{db}} = 100 \text{ rad/s or } f = 15.9 \text{ kHz}$$

for unity gain: $|10^3| = |10^3 + 1| \Rightarrow \omega_H = 1.01 \text{ rad/s}$

$$\text{if } \omega = 10.1 \text{ rad/s} : \left|\frac{U_0}{U_i}\right| = \frac{10.1}{1.01} = 10, \quad \phi = -95.77^\circ$$



2.126

Refer to the circuit in Fig. P2.126:

$$\frac{U_0}{U_i} = -\frac{Z_2}{Z_1} = -\frac{1}{Z_1 Y_2} = -\frac{1}{(R_1 + \frac{1}{j\omega C_1})(\frac{1}{R_2} + j\omega C_2)}$$

$$\frac{U_0}{U_i} = -\frac{R_2/R_1}{(1 + \frac{1}{R_1 S}) (1 + j\omega R_2 C_2)}$$

$$\frac{U_0}{U_i}(j\omega) = \frac{-R_2/R_1}{(1 + \frac{1}{j\omega R_1 C_1}) (1 + j\omega R_2 C_2)} = \frac{-R_2/R_1}{(1 + \frac{\omega_1}{j\omega}) (1 + \frac{\omega_2}{j\omega})}$$

$$\text{where } \omega_1 = \frac{1}{R_1 C_1}, \quad \omega_2 = \frac{1}{R_2 C_2}$$

a) For $\omega \ll \omega_1 \ll \omega_2$

$$\frac{U_0}{U_i}(j\omega) \approx \frac{-R_2/R_1}{(1 + \frac{\omega_1}{j\omega})} \approx \frac{-R_2/R_1}{\omega_1/j\omega} = -j \frac{R_2}{R_1} \frac{\omega}{\omega_1}$$

Cont.

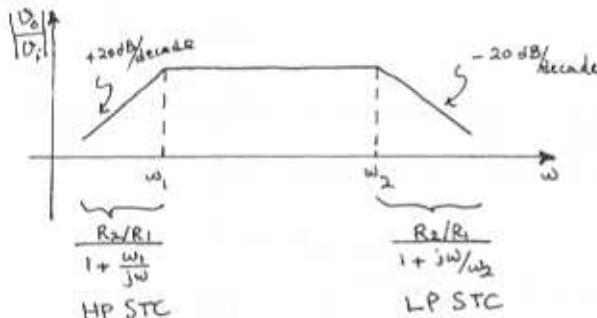
b) for $\omega \ll \omega_1 \ll \omega_2$

$$\frac{V_o}{V_i} (j\omega) \approx -\frac{R_2}{R_1}$$

c) for $\omega \gg \omega_2$ and $\omega_2 \gg \omega_1$:

$$\frac{V_o}{V_i} (j\omega) \approx \frac{-R_2/R_1}{1 + j\omega/\omega_2} \approx \frac{-R_2/R_1}{j\omega/\omega_2} = j \left(\frac{R_2}{R_1} \right) \left(\frac{\omega_2}{\omega} \right)$$

from the results of a), b) and c) we can draw
the Bode-plot:



Design: $\frac{R_2}{R_1} = 1000$ (60dB gain in the mid-frequency range)

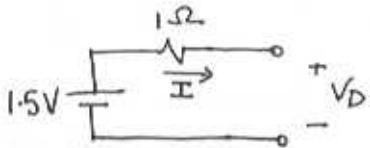
Rin for $\omega \gg \omega_1$ = $R_1 = 1\text{ k}\Omega$ $\Rightarrow R_2 = 1\text{ M}\Omega$

$$f_1 = 100\text{ Hz} \Rightarrow \omega_1 = 2\pi \times 100 = \frac{1}{R_1 C_1} \Rightarrow C_1 = 1.59\text{ }\mu\text{F}$$

$$f_2 = 10\text{ kHz} \Rightarrow \omega_2 = 2\pi \times 10 \times 10^3 = \frac{1}{R_2 C_2} \Rightarrow C_2 = 15.9\text{ pF}$$

CHAPTER 3 - PROBLEMS

3.1



The diode can be reverse-biased and thus no current would flow, or forward-biased where current would flow.

(a) Reverse biased $I = 0\text{A}$ $V_D = 1.5\text{V}$

(b) Forward biased $I = 1.5\text{A}$ $V_D = 0\text{V}$

3.2

(a) Diode is conducting and thus has a 0V drop across it. Consequently

$$V = \underline{\underline{3V}}$$

$$I = \frac{3 - (-3)}{10k\Omega} = \underline{\underline{0.6\text{mA}}}$$

(b) Diode is cut off.

$$V = \underline{\underline{3V}} \quad I = \underline{\underline{0\text{A}}}$$

(c) Diode is conducting

$$V = \underline{\underline{3V}}$$

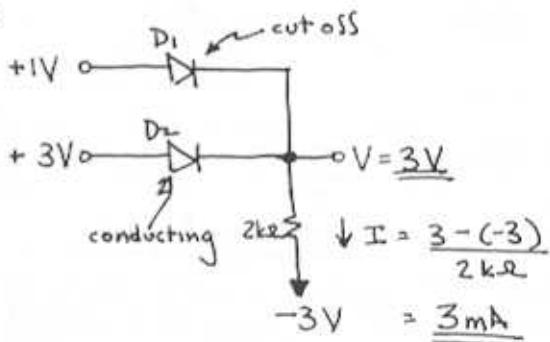
$$I = \frac{3 - (-3)}{10k\Omega} = \underline{\underline{0.6\text{mA}}}$$

(d) Diode is cut off.

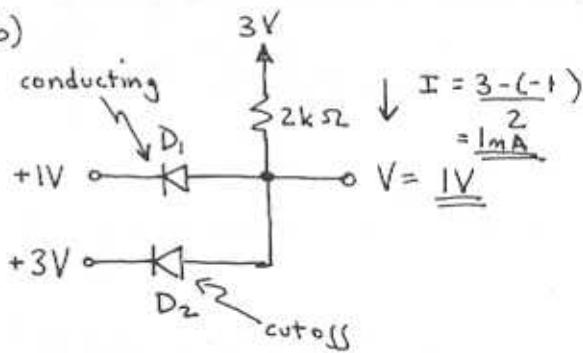
$$V = \underline{\underline{-3V}} \quad I = \underline{\underline{0\text{A}}}$$

3.3

(a)

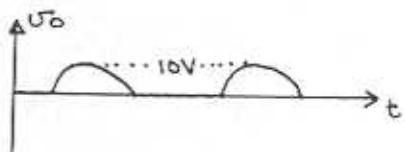


(b)



3.4

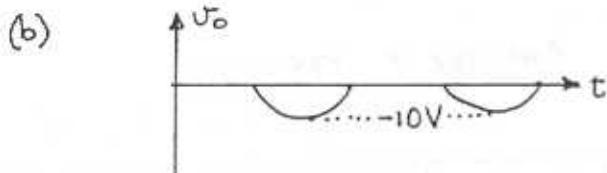
(a)



$$V_{P+} = \underline{\underline{10V}} \quad V_{P-} = \underline{\underline{0V}}$$

$$f = 1\text{kHz}$$

CHAPTER 3 PROBLEMS



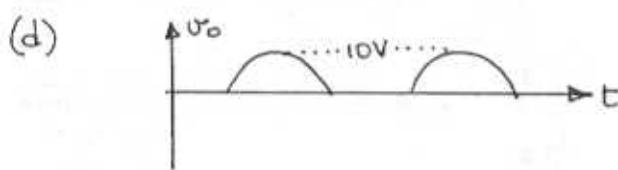
$$V_{p+} = \underline{0V} \quad V_{p-} = \underline{-10V}$$

$$f = 1\text{kHz}$$



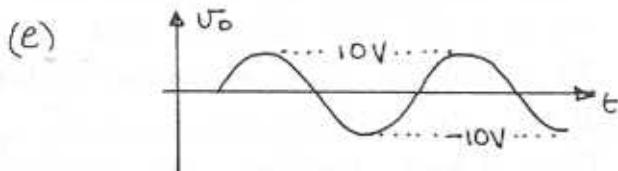
$$V_o = \underline{0V}$$

Neither D_1 nor D_2 conducts so there is no output.



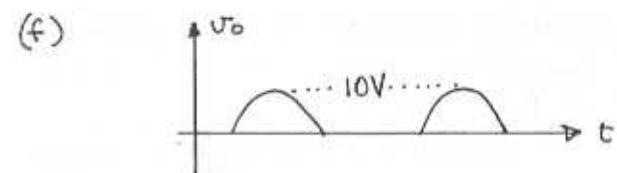
$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{0V} \quad f = 1\text{kHz}$$

Both D_1 and D_2 conduct when $U_I > 0$



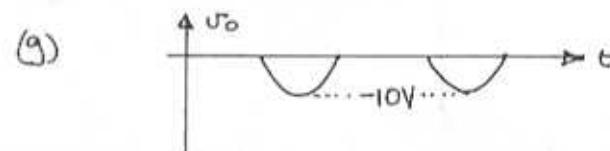
$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{-10V} \quad f = 1\text{kHz}$$

D_1 conducts when $U_I > 0$ and D_2 conducts when $U_I < 0$. Thus the output follows the input.



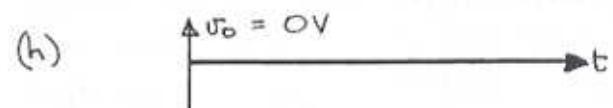
$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{0V} \quad f = 1\text{kHz}$$

$-D_1$ is cut off when $U_I < 0$

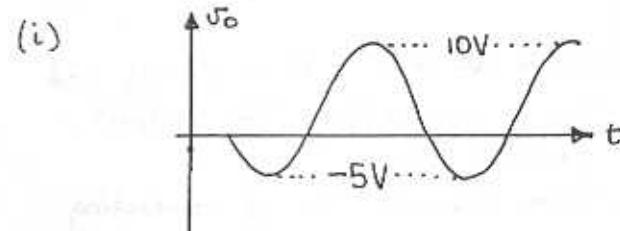


$$V_{p+} = \underline{0V} \quad V_{p-} = \underline{-10V} \quad f = 1\text{kHz}$$

D_1 shorts to ground when $U_I > 0$ and is cut off when $U_I < 0$ whereby the output follows U_I .



$V_o = \underline{0V}$ ~ The output is always shorted to ground as D_1 conducts when $U_I > 0$ and D_2 conducts when $U_I < 0$.

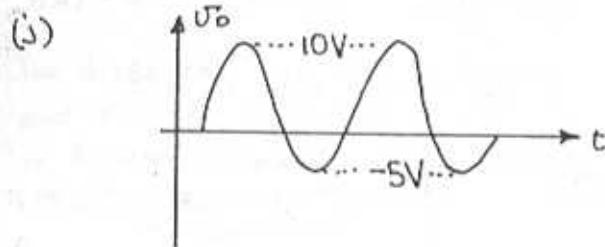


$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{-5V} \quad f = 1\text{kHz}$$

When $U_I > 0$, D_1 is cut off and V_o follows U_I .

-When $U_I < 0$, D_1 is conducting and the circuit becomes a voltage divider where the negative peak is

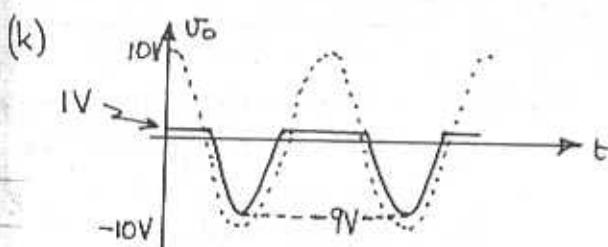
$$\frac{1k\Omega}{1k\Omega + 1k\Omega} \cdot -10V = -5V$$



$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{-5V} \quad f = 1k\text{Hz}$$

-When $U_I > 0$, the output follows the input as D_1 is conducting.

-When $U_I < 0$, D_1 is cut off and the circuit becomes a voltage divider.



$$V_{p+} = \underline{1V} \quad V_{p-} = \underline{-9V} \quad f = 1k\text{Hz}$$

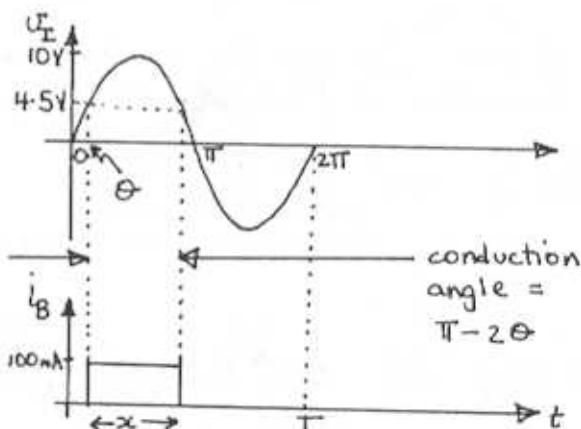
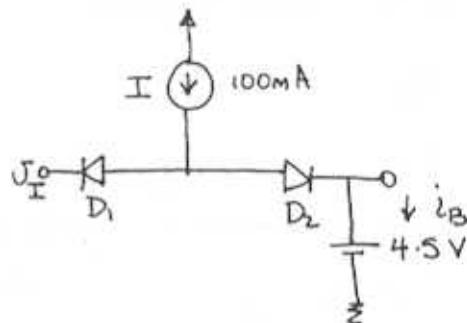
-When $U_I > 0$, D_1 is cut off and D_2 is conducting. The output becomes 1V.

-When $U_I < 0$, D_1 is conducting and D_2 is cut off. The output becomes :-

$$U_o = U_I + 1V .$$

CHAPTER 3 PROBLEMS

3.5



-When $U_I < 4.5V$ D_1 conducts and D_2 is cut off so $i_B = 0A$. For $U_I > 4.5V$ D_2 conducts and D_1 is cut off thus disconnecting the input U_I . All of the current then flows through the battery.

$$10 \sin \theta = 4.5V$$

$$\theta = \sin^{-1}(4.5/10)$$

$$\text{conduction angle} = \pi - 2\theta$$

Fraction of cycle that $i_B = \underline{100mA}$ is given by :-

$$x = \frac{\pi - 2\theta}{2\pi} = 0.35$$

CHAPTER 3 PROBLEMS

$$i_{B\text{avg}} = \frac{1}{T} \int_T i_B dt$$

$$= \frac{1}{T} \left[100 + 0.35T \right]$$

$$= \underline{\underline{35 \text{ mA}}}$$

If V_x is reduced by 10% the peak value of i_B remains the same

$$i_{B\text{peak}} = \underline{\underline{100 \text{ mA}}}$$

but the fraction of the cycle for conduction changes

$$x = \frac{\pi - 2\theta}{2\pi} = \frac{\pi - 2 \sin^{-1}(4.5/9)}{2\pi}$$

$$= \frac{1}{3}$$

Thus:

$$i_{B\text{avg}} = \frac{1}{T} \left[100 \cdot \frac{1}{3} \right]$$

$$= \underline{\underline{33.3 \text{ mA}}}$$

$-x$ and y are the same for $A = B$
 $-x$ and y are opposite if $A \neq B$

3.7

$$\frac{5-O}{R} \leq 0.1 \text{ mA}$$

$$R \geq \frac{5}{0.1} = \underline{\underline{50 \text{ k}\Omega}}$$

3.8

The maximum input current occurs when one input is low and the other two are high.

$$\frac{5-O}{R} \leq 0.1 \text{ mA}$$

$$R \geq \underline{\underline{50 \text{ k}\Omega}}$$

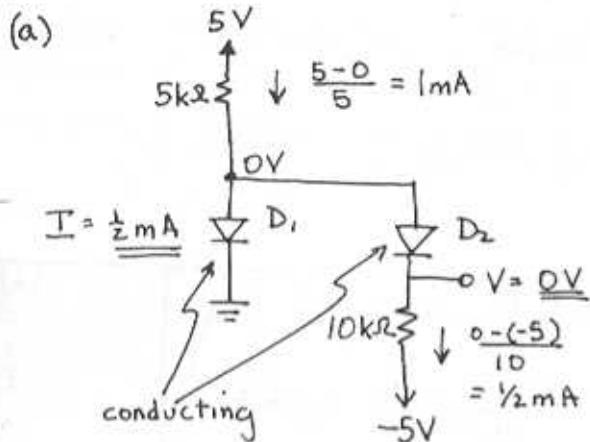
3.6

A	B	x	y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

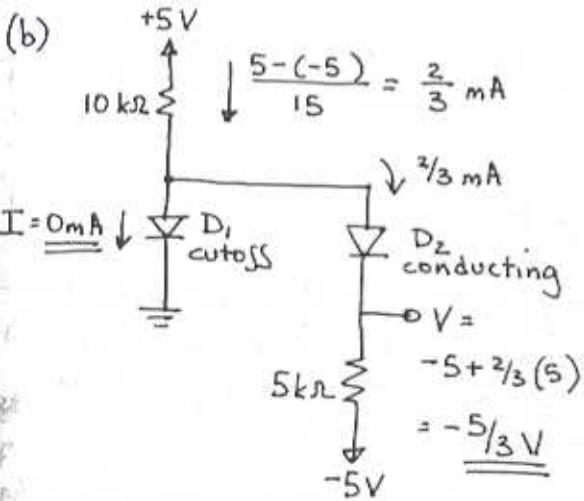
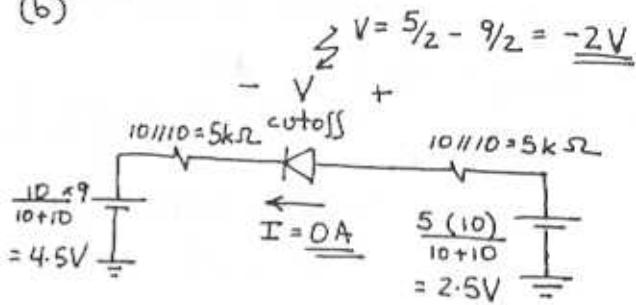
$$x = \underline{\underline{A \cdot B}}$$

$$y = \underline{\underline{A + B}}$$

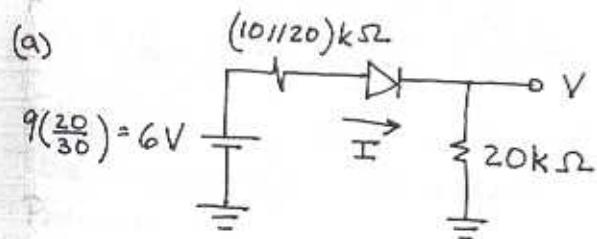
3.9



(b)



3.10



$$V = \frac{20}{(10//20)+20} \times 6 = 4.5\text{V}$$

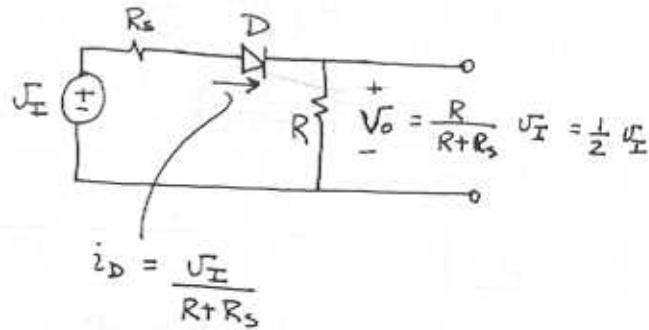
3.11

$$R \geq \frac{120\sqrt{2}}{50} \geq 3.4\text{k}\Omega$$

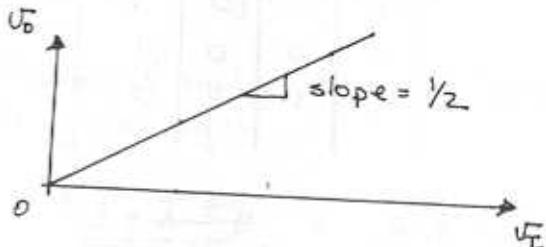
The largest reverse voltage appearing across the diode is equal to the peak input voltage

$$120\sqrt{2} = 169.7\text{V}$$

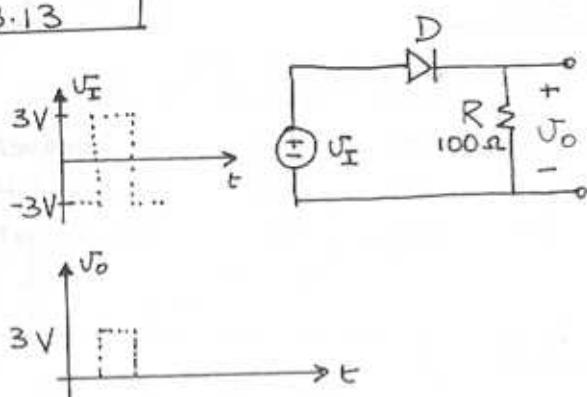
3.12



D starts to conduct when $V_I > 0$



3.13



$$U_{O, \text{peak}} = 3V$$

$$\begin{aligned} U_{O, \text{avg}} &= \frac{1}{T} \int U_O dt \\ &= \frac{1}{T} \left[3 \frac{T}{2} \right] = \underline{\underline{1.5V}} \end{aligned}$$

$$i_{D, \text{peak}} = \frac{3}{100} = \underline{\underline{30mA}}$$

$$i_{D, \text{avg}} = \frac{1.5}{100} = \underline{\underline{15mA}}$$

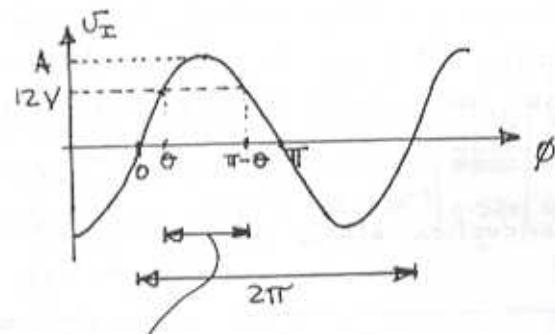
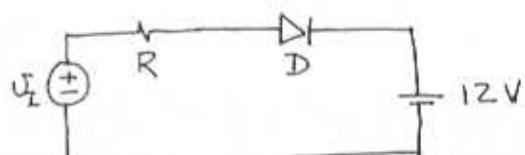
The maximum reverse diode voltage is 3V

$$i_{D, \text{peak}} = \frac{U_{O, \text{peak}}}{100} = \underline{\underline{50mA}}$$

$$i_{D, \text{avg}} = i_{D, \text{peak}}/2 = \underline{\underline{25mA}}$$

maximum reverse voltage = 1V

3.15



conduction occurs

$$U_I = A \sin \theta = 12 \sim \text{conduction across } D \text{ occurs}$$

For a conduction angle $(\pi - 2\theta)$ that is 20% of a cycle

$$\frac{\pi - 2\theta}{2\pi} = \frac{1}{5}$$

$$\theta = 0.3\pi$$

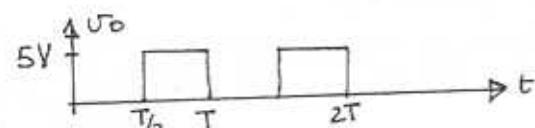
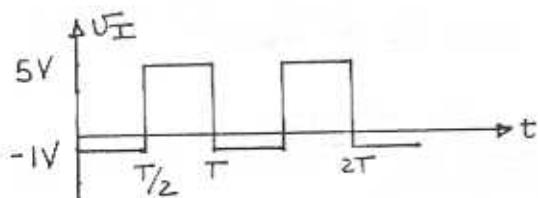
$$A = 12 / \sin \theta = 14.83V$$

$$\begin{aligned} \textcircled{o} \text{ Peak-to-peak sine wave voltage} \\ = 2A = \underline{\underline{29.67V}} \end{aligned}$$

Given the average diode current to be

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{A \sin \phi - 12}{R} d\phi = 100mA$$

3.14



$$U_{O, \text{peak}} = \underline{\underline{5V}}$$

$$U_{O, \text{avg}} = \underline{\underline{2.5V}}$$

$$\frac{1}{2\pi} \left[\frac{-14.83 \cos \phi - 12 \phi}{R} \right]_{\phi=0.3\pi}^{\phi=0.7\pi} = 0.1$$

$$R = \underline{3.75 \Omega}$$

$$\text{Peak diode current} = \frac{A-12}{R} = \underline{0.75A}$$

$$\text{Peak reverse voltage} = A+12 = \underline{26.83V}$$

For resistors specified to only one significant digit and peak-to-peak voltage to the nearest volt then choose $A = 15$ so the peak-to-peak sine wave voltage = 30V and $R = \underline{3\Omega}$

$$\begin{aligned} \text{Conduction starts at } U_E &= A \sin \theta = 12 \\ 15 \sin \theta &= 12 \\ \theta &= 0.93 \text{ rad} \end{aligned}$$

Conduction stops at $\pi - \theta$

$$\therefore \text{Fraction of cycle that current flows is } \frac{\pi - 2\theta}{2\pi} \times 100 = 20.5 \approx \underline{20\%}$$

Average diode current =

$$\frac{1}{2\pi} \left[\frac{-15 \cos \phi - 12 \phi}{3} \right]_{\phi=0.93}^{2.21} = \underline{136mA}$$

Peak diode current

$$= \frac{15-12}{3} = \underline{1A}$$

Peak reverse voltage =

$$A+12 = \underline{27V}$$

3.16

V	RED	GREEN	
3V	ON	OFF	- D_1 conducts
0	OFF	OFF	- No current flows
-3V	OFF	ON	- D_2 conducts

3.17

$$V_T = \frac{kT}{q} \quad \text{where } k = 1.38 \times 10^{-23} \text{ J/K}$$

$$T = 273 + x^\circ \text{C}$$

$$q = 1.60 \times 10^{-19} \text{ C}$$

x [°C]	V _T [mV]
-40	20
0	23.5
40	27
150	36.5

$$\begin{aligned} \text{for } V_T &= 25 \text{ mV} \\ T &= \underline{16.8^\circ \text{C}} \end{aligned}$$

3.18

$$i = I_s e^{\frac{U}{2 \times 0.025}}$$

$$\therefore 1000 I_s = I_s e^{\frac{U}{0.05}}$$

$$U = \underline{0.345V}$$

$$\text{at } U = 0.7V$$

$$i = I_s e^{\frac{0.7}{0.05}} = \underline{1.2 \times 10^6 I_s}$$

3.19

$$i_1 = I_s e^{-V_1/nV_T} = 10^{-3}$$

$$i_2 = I_s e^{-V_2/nV_T}$$

$$\frac{i_2}{i_1} = \frac{i_2}{10^{-3}} = e^{\frac{0.5 - 0.7}{0.025}}$$

$$i_2 = \underline{0.335 \mu A}$$

3.20

$$i = I_s e^{V_1/nV_T} = I_s e^{0.7/0.025} = 5(10^{-3})$$

$$I_s = 5(10^{-3}) e^{-0.7/0.025} = \underline{3.46 \times 10^{-5} A}$$

V	i
0.71 V	7.46 mA
0.8 V	273.21 mA
0.69 V	3.35 mA
0.6 V	91.65 μA

$$\text{Let } i_1 = I_s e^{V_1/0.025}$$

$$i_2 = 10i_1 = I_s e^{V_2/0.025}$$

$$\frac{i_2}{i_1} = 10 = e^{\frac{V_2 - V_1}{0.025}}$$

$$\therefore \Delta V = V_2 - V_1 = \underline{57.56 mV}$$

3.21

To calculate I_s use

$$I_s = I e^{-V_1/nV_T} = I e^{-V_1/n \times 0.025}$$

To calculate the voltage at 1% of the measured current use

$$i_2 = 0.01 i_1 \quad \text{so,}$$

$$\frac{i_2}{i_1} = 0.01 = e^{\frac{V_2 - V_1}{nV_T}}$$

$$\begin{aligned} V_2 &= V_1 + nV_T \ln 0.01 \\ &= V + n(0.025) \ln(0.01) \end{aligned}$$

V [V]	I [A]	I_s		V [V]	V [V]
		$n=1$ [A]	$n=2$ [A]		
0.7	1 A	6.91×10^{-13}	8.32×10^{-7}	0.585	0.470
0.650	1 mA	5.11×10^{-15}	2.26×10^{-9}	0.535	0.420
0.650	10 μA	5.11×10^{-17}	2.26×10^{-11}	0.535	0.420
0.7	10 μA	6.91×10^{-15}	8.32×10^{-9}	0.584	0.470

3.22

Let $I_1 = I_s e^{V_1/nV_T}$ and

$$I_2 = I_s e^{V_2/nV_T} = I_1/10$$

Calculate n by :-

$$\frac{I_2}{I_1} = e^{\frac{V_2 - V_1}{nV_T}}$$

$$n = \frac{1}{V_T} \left[\frac{V_2 - V_1}{\ln \frac{I_2}{I_1}} \right] = \frac{1}{0.025} \left[\frac{V_2 - V_1}{\ln 0.1} \right]$$

Calculate I_s by :-

$$I_s = I_1 e^{-V_1/nV_T}$$

Calculate the diode voltage at 10I_s by :- $V_3 = nV_T / \ln \frac{10I_1}{I_s}$

I	V_1 [V]	V_2 [V]	n	I_s [A]	V_3 [V]
10mA	0.7	0.6	1.737	10^{-9}	0.8
1mA	0.7	0.6	1.737	10^{-10}	0.8
100A	0.8	0.7	1.737	10^{-7}	0.9
1mA	0.7	0.58	2.085	1.47×10^{-9}	0.82
10μA	0.7	0.64	1.042	2.15×10^{-17}	0.7

through it is

$$I = I_s e^{\frac{V_1}{nV_T}}$$

With two diodes in parallel, the current splits between each diode so that the diodes each has half the current

$$\frac{I}{2} = I_s e^{\frac{V_2}{nV_T}}$$

$$\therefore \frac{I/2}{I} = e^{\frac{V_2 - V_1}{nV_T}}$$

The change in voltage is

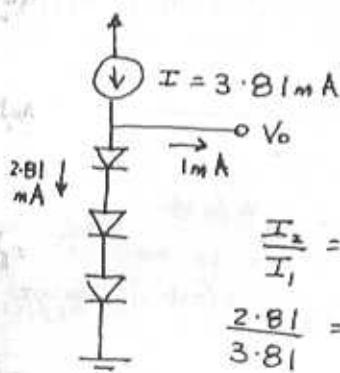
$$\Delta V = V_2 - V_1 = nV_T \ln\left(\frac{1}{2}\right) = -17.3 \text{ mV}$$

3.23

o The voltage across each diode is $V_0/3$

$$I = I_s e^{\frac{V_0/3}{nV_T}} = 10^{-14} e^{\frac{2/3}{0.025}}$$

$$\Rightarrow \underline{3.81 \text{ mA}}$$

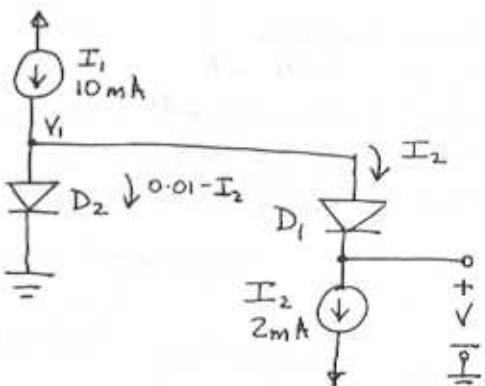


$$\frac{I_2}{I_1} = e^{\frac{(V_2 - V_1)/3}{0.025}}$$

$$\frac{2.81}{3.81} = e^{\frac{(V_2 - 2)/3}{0.025}}$$

$$\Delta V = V_2 - 2 = -22.8 \text{ mV}$$

3.25



The current through D_1 is

$$10 I_s e^{\frac{V_1 - V}{nV_T}} = I_2 \quad (A)$$

The current through D_2 is

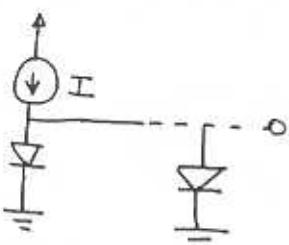
$$I_s e^{\frac{V_1}{nV_T}} = 0.01 - I_2$$

$$I_s = (0.01 - I_2) e^{\frac{V_1}{nV_T}} \quad (B)$$

(B) \rightarrow (A)

$$10 (0.01 - I_2) e^{\frac{-V}{nV_T}} = I_2$$

3.24



With one diode the current

$$V = -V_T \ln\left(\frac{I_2}{10(0.01 - I_2)}\right)$$

$$= 0.025 \ln\left(\frac{2}{10(8)}\right) = \underline{\underline{92.2 \text{ mV}}}$$

For $V = 50 \text{ mV}$

$$-V_T \ln\left(\frac{I_2}{10(10 - I_2)}\right) = 50 \times 10^{-3}$$

$$I_2 = 10(10 - I_2) e^{-2}$$

$$I_2 (1 + 10e^{-2}) = 100e^{-2}$$

$$I_2 = \underline{\underline{5.75 \text{ mA}}}$$

3.27

AT A CONSTANT TEMPERATURE, THE DIODE VOLTAGE DROP CHANGES WITH CURRENT ACCORDING TO

$$\Delta V = V_T \ln\left(\frac{I_2}{I_1}\right)$$

WHERE

$$V_T = \frac{kT}{q} = \frac{1.38 \times 10^{-23} (273 + \text{Temp. } ^\circ\text{C})}{1.6 \times 10^{-19}}$$

THUS:

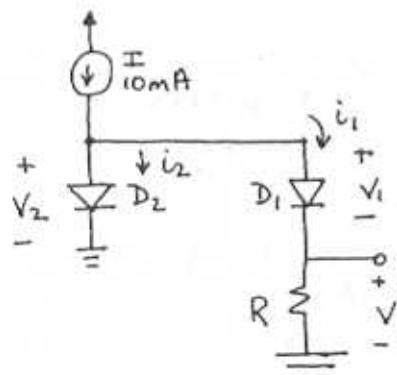
TEMP (°C)	0	50	75	100	-50
V _T (mV)	23.5	27.9	30	32.2	19.2

AT A CONSTANT CURRENT, THE DIODE VOLTAGE DROP CHANGES WITH TEMPERATURE ACCORDING TO

$$\Delta V = -2 \text{ (mV)} \times \text{TEMPERATURE CHANGE (°C)}$$

THUS:

- (a) 620 mV AT $10 \mu\text{A}$ AND 0°C
- 728 mV AT 1 mA AND 0°C
- 678 mV AT 1 mA AND 25°C



Given for each diode
 $i = I_s e^{\frac{V}{nV_T}} \Rightarrow 10 \times 10^{-3} = I_s e^{0.7/n \times 0.025} \quad \text{①}$
 $100 \times 10^{-3} = I_s e^{0.8/n \times 0.025} \quad \text{②}$

$$\text{②/①} \quad 10 = e^{0.1/n(0.025)}$$

$$n = 1.737$$

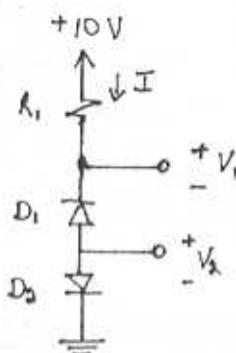
$$V = V_2 - V_1 = nV_T \ln\left(\frac{i_2}{i_1}\right) = 80 \text{ mV}$$

$$1.737 (25 \times 10^{-3}) \ln\left(\frac{0.01 - i_1}{i_1}\right) = 80$$

$$i_1 = 1.4 \text{ mA}$$

$$R = 80/i_1 = 80/1.4 = \underline{\underline{57.1 \Omega}}$$

3.28

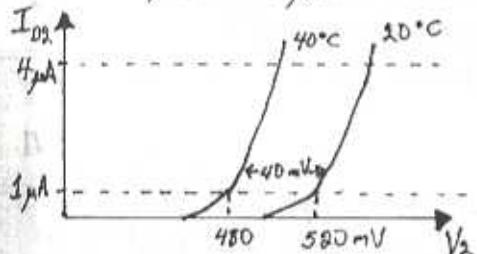


At 20°C :

$$V_{R1} = V_d = 520 \text{ mV}$$

$$R_1 = 520 \text{ k}\Omega$$

$$I = \frac{520 \text{ mV}}{520 \text{ k}\Omega} = 1 \mu\text{A}$$

At 40°C, $I = 4 \mu\text{A}$ 

$$V_d = 480 + 2.3 \times 1 \times 25 \log 4 \\ = 514.6 \text{ mV}$$

$$V_{R1} = 4 \mu\text{A} \times 520 \text{ k}\Omega = \underline{\underline{2.08 \text{ V}}}$$

$$\text{At } 0^\circ\text{C}, I = \frac{1}{4} \mu\text{A}$$

$$V_d = 560 - 2.3 \times 1 \times 25 \log 4 \\ = \underline{\underline{525.4 \text{ mV}}}$$

$$V_{R1} = \frac{1}{4} \times 520 = \underline{\underline{0.13 \text{ V}}}$$

3.29

The voltage drop = $700 - 580 = 120 \text{ mV}$
Since the diode voltage decreases by approximately 2 mV for every 1°C increase in temperature, the junction temperature must have increased by

$$\frac{120}{2} = \underline{\underline{60^\circ\text{C}}}$$

Power being dissipated =

$$580 \times 10^{-3} \times 15 = \underline{\underline{8.7 \text{ W}}}$$

$$\begin{aligned} \text{Thermal Resistance} &= \text{temperature rise}/\text{watt} \\ &= 60/8.7 = \underline{\underline{6.9^\circ\text{C/W}}} \end{aligned}$$

3.30

$$\begin{aligned} i &= I_s e^{V/nkT} \\ 10 &= I_s e^{0.8/2(0.025)} \\ I_s &= 1.12 \times 10^{-6} \text{ A} \end{aligned}$$

For current varying between $i_1 = 0.5 \text{ mA}$ to $i_2 = 1.5 \text{ mA}$, the voltage varies from

$$V_1 = 2(0.025) \ln \left(\frac{0.5 \times 10^{-3}}{1.12 \times 10^{-6}} \right) = \underline{\underline{0.305 \text{ V}}}$$

to:

$$V_2 = 2(0.025) \ln \left(\frac{1.5 \times 10^{-3}}{1.12 \times 10^{-6}} \right) = \underline{\underline{0.360 \text{ V}}}$$

∴ the voltage decreases by approximately 2 mV for every 1°C increase in temperature, the voltage may vary by $\pm 50 \text{ mV}$ for the $\pm 25^\circ\text{C}$ temperature variation.

3.31

$$i = I_s e^{V/nV_T}$$

$$\frac{I_{s2}}{I_{s1}} = \frac{1}{0.1 \times 10^{-3}} = 10^4$$

For identical currents

$$I_{s1} e^{V_1/nV_T} = I_{s2} e^{V_2/nV_T}$$

$$e^{\frac{V_1 - V_2}{nV_T}} = 10^4$$

$$V_1 - V_2 = nV_T \ln 10^4$$

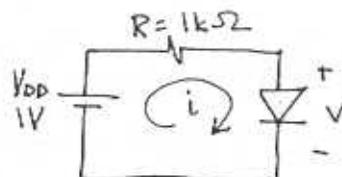
$$= 26 \times 10^{-3} \ln 10^4$$

$$= +0.23V$$

i.e. THE VOLTAGE DIFFERENCE BETWEEN THE TWO DIODES IS +0.23V INDEPENDENT OF THE CURRENT.

HOWEVER, SINCE THE TWO CURRENTS CAN VARY BY A FACTOR OF 3 (0.5 mA TO 1.5 mA) THE DIFFERENCE VOLTAGE WILL BE:
 $0.23V \pm nV_T \ln 3 = 0.23V \pm 2.75\text{ mV}$
 SINCE TEMPERATURE CHANGE AFFECTS BOTH DIODES SIMILARLY THE DIFFERENCE VOLTAGE REMAINS CONSTANT.

3.32



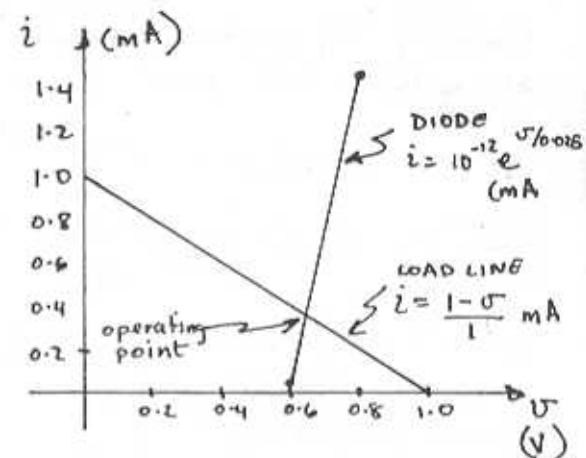
$$i = 10^{-15} e^{V/nV_T}$$

where $n=1$

$$V = 0.7V \quad i = 1.45\text{ mA}$$

$$V = 0.6V \quad i = 0.026\text{ mA}$$

A sketch of the graphical construction to determine the operating point is shown below.



From the above sketch we see that the operating point must lie between $V = 0.6$ and 0.7 V and $i \times 0.3$ to 0.4 mA . To find the point more accurately an enlarged graph is plotted.

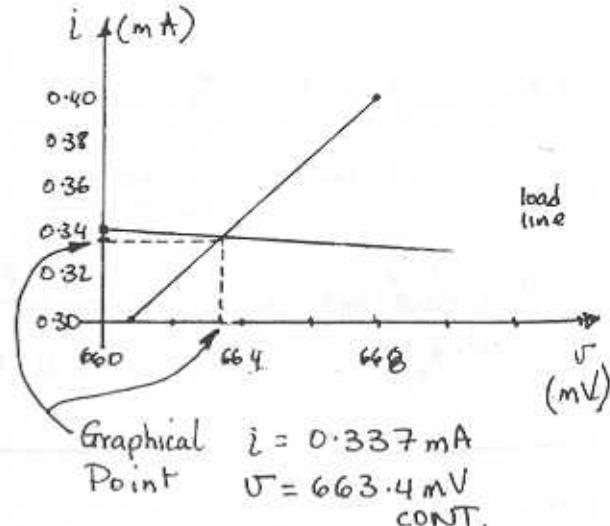
$$\text{For } i = 0.3\text{ mA} = 10^{-12} e^{V/0.025} \Rightarrow V = 660.7\text{ mV}$$

$$\text{For } i = 0.4\text{ mA} = 10^{-12} e^{V/0.025} \Rightarrow V = 667.9\text{ mV}$$

For the load line:

$$V = 660\text{ mV} \Rightarrow i = 0.34\text{ mA}$$

$$V = 670\text{ mV} \Rightarrow i = 0.33\text{ mA}$$



Comparing the graphical results to the exponential model gives:

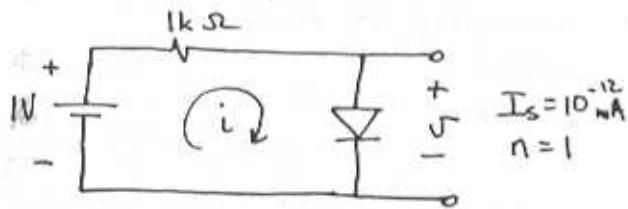
$$\text{At } i = 0.337 \text{ mA} = 10^{-12} e^{U/0.025}$$

$$\Rightarrow U = 663.6 \text{ mV}$$

which is only $(663.6 - 663.4) = \underline{\underline{0.2 \text{ mV}}}$
greater than the value
found graphically!

3.33

Iterative Analysis:



$$\#1 \quad U = 0.7 \text{ V} \quad i = \frac{1 - 0.7}{1} = 0.3 \text{ mA}$$

$$\#2 \quad U = 0.25 \ln\left(\frac{0.3}{10^{-12}}\right) = 0.6607 \text{ V}$$

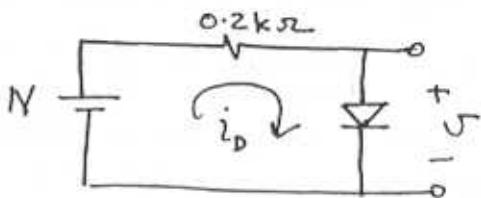
$$i = \frac{1 - 0.6607}{1} = 0.3393 \text{ mA}$$

$$\#3 \quad U = 0.25 \ln\left(\frac{0.3393}{10^{-12}}\right) = \underline{\underline{0.6638 \text{ V}}}$$

$$i = \frac{1 - 0.6638}{1} = \underline{\underline{0.3362}}$$

∴ i did not change by much
stop here.

3.34



$$(a) \quad i_D = \frac{1 - 0.7}{0.2} = \underline{\underline{1.5 \text{ mA}}}$$

(b) Iterative Analysis given $U_D = 0.7 \text{ V}$
at $i_D = 1 \text{ mA}$

$$\#1 \quad U = 0.7 \text{ V} \quad i_D = \frac{1 - 0.7}{0.2} = 1.5 \text{ mA}$$

$$\#2 \quad \therefore i = I_s e^{\frac{U}{nVt}} \quad n=2 \\ \frac{i_2}{i_1} = e^{\frac{U_2 - U_1}{0.05}}$$

$$\text{thus } U_2 = U_1 + 0.05 \ln \frac{i_2}{i_1}$$

$$\therefore \text{for } i = 1.5 \text{ mA}$$

$$U = 0.7 + 0.05 \ln \frac{1.5}{1} \quad \therefore i_D = \frac{1 - 0.720}{0.2} \\ = 0.720 \text{ V} \quad = 1.4 \text{ mA}$$

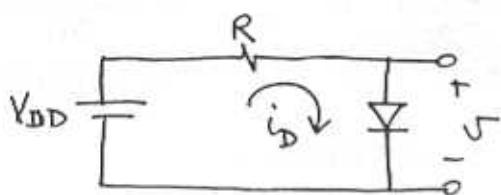
#3

$$U = 0.720 + 0.05 \ln \left(\frac{1.4}{1.5} \right) \quad \therefore i_D = \frac{1 - 0.716}{0.2} \\ = 0.716 \text{ V} \quad = 1.42 \text{ mA}$$

#4

$$U = 0.716 + 0.05 \ln \left(\frac{1.42}{1.4} \right) \quad \therefore i_D = \underline{\underline{1.42 \text{ mA}}} \\ = 0.716 \text{ V}$$

3.35



Derivation of iterative equation

$$i_D = I_s e^{\frac{V}{V_T}}$$

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{V_2 - V_1}{nV_T}}$$

$$\begin{aligned} V_2 &= V_1 + nV_T \ln\left(\frac{i_{D2}}{i_{D1}}\right) \\ &= V_1 + \Delta V \log\left(\frac{i_{D2}}{i_{D1}}\right) \end{aligned}$$

$$(a) V = 0.7 \text{ V} \quad i_D = \frac{10 - 0.7}{9.3} = 1 \text{ mA}$$

$$(b) V = 0.7 \text{ V} \quad i_D = \frac{3 - 0.7}{2.3} = 1 \text{ mA}$$

~ for both these cases the diode is rated at 1mA for 0.7V so stop.

$$(c) V_{DD} = 2 \text{ V} \quad R = 2k\Omega$$

$$\#1 \quad V = 0.7 \text{ V} \quad i_D = \frac{2 - 0.7}{2} = 0.65 \text{ mA}$$

$$\#2 \quad V = 0.7 + 0.1 \log\left(\frac{0.650}{10}\right) = 0.581 \text{ V}$$

$$i_D = \frac{2 - 0.581}{2} = 0.709 \text{ mA}$$

$$\#3 \quad V = 0.581 + 0.1 \log\left(\frac{0.709}{0.650}\right)$$

$$i_D = \frac{0.584}{2} = 0.708 \text{ mA}$$

$$\begin{aligned} (d) \quad V_{DD} &= 2 \text{ V} \quad R = 2k\Omega \\ \#1 \quad V &= 0.7 \text{ V} \quad i_D = \frac{2 - 0.7}{2} \\ &= 0.650 \text{ mA} \end{aligned}$$

#2.

$$\begin{aligned} V &= 0.7 + 0.1 \log\left(\frac{0.650}{1}\right) \quad i_D = \frac{2 - 0.681}{2} \\ &= 0.681 \text{ V} \quad = 0.659 \text{ mA} \end{aligned}$$

#3

$$\begin{aligned} V &= 0.681 + 0.1 \log\left(\frac{0.659}{0.650}\right) \quad i_D = \frac{2 - 0.682}{2} \\ &= \underline{\underline{0.682 \text{ V}}} \quad = \underline{\underline{0.659 \text{ mA}}} \end{aligned}$$

$$(e) \quad V_{DD} = 1 \text{ V} \quad R = 0.3k\Omega$$

$$\#1 \quad V = 0.7 \text{ V} \quad i_D = \frac{1 - 0.7}{0.3} = 1 \text{ mA}$$

#2

$$\begin{aligned} V &= 0.7 + 0.1 \log\frac{1}{10} \quad i_D = \frac{1 - 0.6}{0.3} = 1.333 \text{ mA} \\ &= 0.6 \text{ V} \end{aligned}$$

#3

$$\begin{aligned} V &= 0.6 + 0.1 \log\frac{1.333}{1} \quad i_D = \frac{1 - 0.612}{0.3} = 1.293 \text{ mA} \\ &= 0.612 \text{ V} \end{aligned}$$

#4

$$\begin{aligned} V &= 0.612 + 0.1 \log\frac{1.293}{1.333} \quad i_D = \frac{1 - 0.611}{0.3} = 1.297 \text{ mA} \\ &= 0.611 \text{ V} \end{aligned}$$

#5

$$\begin{aligned} V &= 0.611 + 0.1 \log\frac{1.297}{1.293} \quad i_D = \underline{\underline{1.297 \text{ mA}}} \\ &= \underline{\underline{0.611 \text{ V}}} \end{aligned}$$

$$(f) \quad V_{DD} = 1 \text{ V} \quad R = 0.3k\Omega$$

$$\#1 \quad V = 0.7 \text{ V} \quad i_D = \frac{1 - 0.7}{0.3} = 1 \text{ mA}$$

CONT.

$$\begin{aligned} \text{#2} \\ V &= 0.7 + 0.06 \log \frac{1}{10} \quad i_D = \frac{1 - 0.640}{0.3} \\ &= 0.640 \text{ V} \quad = 1.2 \text{ mA} \end{aligned}$$

$$V = 0.7 + 0.1 \log \frac{3.333 \times 10^{-3}}{10} \quad i_D = \frac{0.5 - 0.352}{30}$$

$$= 0.352 \text{ V} \quad = 4.933 \mu\text{A}$$

$$\#3 \\ V = +0.66 \log \frac{1.2}{1} \quad i_D = \frac{1 - 0.645}{0.3} \\ = 0.645V \quad = 1.183mA$$

$$V = 0.352 + 0.1 \log \frac{4933}{333} \quad i_D = \frac{0.5 - 0.369}{30}$$

$$= 0.369 V \quad = 4.367 \mu A$$

$$V = +0.06 \log \frac{1.183}{1.2} \\ = \underline{\underline{0.645V}} \quad i_D = \underline{\underline{1.183mA}}$$

$$\begin{aligned} \#4 \\ V &= 0.369 + 0.1 \log \frac{4.367}{4.933} & i_D &= \frac{0.5 - 0.364}{30} \\ &= 0.364V & &= 4.533 \mu A \end{aligned}$$

$$\#1 \quad V = 0.7 \quad I_D = \frac{1-0.7}{0.3} = 1 \text{ mA}$$

$$\begin{aligned} \text{#5} \\ U &= 0.364 + 0.1 \log \left(\frac{4.533}{4.367} \right) \quad U_0 = \frac{0.5 - 0.364}{30} \\ &= 0.366V \quad = 4.467\mu A \end{aligned}$$

$$U = 0.7 + 0.12 \log_{10} i_D \quad i_D = \frac{1 - 0.580}{0.3} \\ = 0.580V \quad = 1.381mA$$

$$\#6 \quad V = 0.366 + 0.1 \log \frac{4.467}{4.533} \quad i_D = \frac{0.5 - 0.365}{30}$$

$$= 0.365V \quad = 4.5 \mu A$$

$$\begin{aligned} \#3 \\ U &= 0.680 + 0.12 \log \frac{1.381}{1} \quad I_D = \frac{1 - 0.697}{0.3} \\ &= 0.697 \text{ V} \quad = 1.343 \text{ mA} \end{aligned}$$

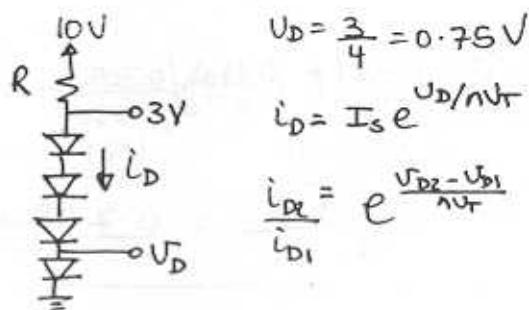
$$\#7 \\ U = 0.365 + 0.1 \log \frac{4.5}{4.467} \\ = \underline{\underline{0.365 \text{ V}}} \quad i_D = \underline{\underline{4.5 \mu A}}$$

$$\begin{aligned} \#5 \\ U &= 0.596 + 0.12 \log \frac{1.347}{1.343} \\ &= \underline{0.596 \text{ V}} \quad I_D = \underline{1.347 \text{ mA}} \end{aligned}$$

3.36

$$(h) \quad V_{DD} = 0.5V \quad R = 30k\Omega$$

$$\#1 \text{ let } V = 0.4V \quad i_D = \frac{0.5 - 0.4}{30} \\ = 3.333 \mu A$$



CONT.

$$\therefore i_D = i_{D2} = i_{D1} e^{\frac{U_{D2}-U_{D1}}{nV_T}}$$

$$= 1 \times e^{\frac{0.76 - 0.7}{1 \times 0.025}}$$

$$= 7.389 \text{ mA}$$

$$\therefore R = \frac{10 - 3}{I_D} = \frac{10 - 3}{7.389} = \underline{0.947 \text{ k}\Omega}$$

$$I_D = \frac{0.815 - V_{DD}}{R_D} = 10 \text{ mA} \quad \textcircled{2}$$

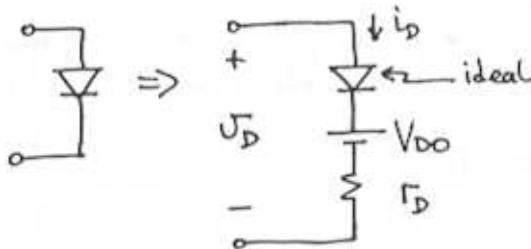
$$\textcircled{2}/\textcircled{1} \Rightarrow \frac{0.815 - V_{DD}}{0.7 - V_{DD}} = 10$$

$$V_{DD} = \underline{0.687 \text{ V}}$$

$$R_D = \frac{0.7 - V_{DD}}{1} = \underline{12.8 \text{ }\Omega}$$

3.37

Piecewise linear model:



Given $n=2$, $V_0=0.7 \text{ V}$ $I_D=1 \text{ mA}$

The current through the diode is given by:

$$i_D = \frac{U_D - V_{DD}}{R_D} \quad \text{need to find the parameters } V_{DD} \text{ and } R_D$$

Using the exponential model to find the diode voltage at 10mA

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{U_{D2}-U_{D1}}{nV_T}}$$

$$U_{D2} = nV_T \ln \left(\frac{i_{D2}}{i_{D1}} \right) = 0.05 \ln \left(\frac{10}{1} \right)$$

$$= 0.815 \text{ V}$$

FINDING V_{DD} & R_D using the given facts:

$$i_D = \frac{0.7 - V_{DD}}{R_D} = 1 \text{ mA} \quad \textcircled{1}$$

Using the piecewise linear model

$$i_D = \frac{U_D - 0.687}{12.8} \Rightarrow U_D = 0.687 + 12.8 i_D$$

Using the exponential model

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{U_{D2}-U_{D1}}{0.05}} \Rightarrow U_D = 0.7 + 0.05 \ln \left(\frac{i_{D2}}{i_{D1}} \right)$$

i_D (mA)	PIECEWISE LINEAR U_D (V)	EXPONENTIAL U_D (V)	ERROR (mV)
0.5	0.693	0.655	28
5	0.751	0.780	-29.5
14	0.866	0.832	34

3.38

Looking at the copy of Fig 3.12 below, we see at

$$i_D = 1 \text{ mA} \rightarrow U_D = 0.7 \text{ V}$$

$$i_D = 10 \text{ mA} \rightarrow U_D = 0.8 \text{ V}$$

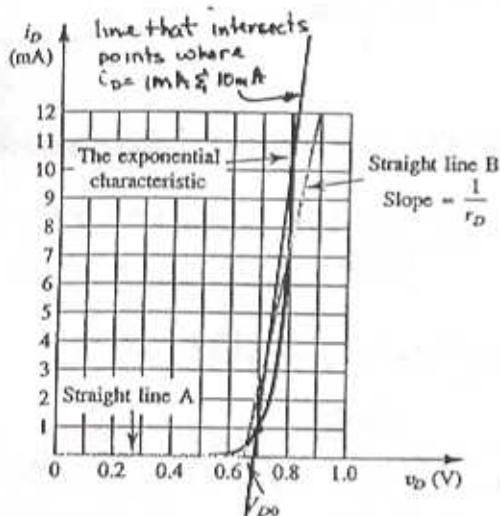
$$\therefore \text{slope} = \frac{1}{R_D} = \frac{0.8 - 0.7}{0.8 - 0.7} = 90 \frac{\text{mA}}{\text{V}}$$

$$\therefore R_D = \frac{1}{90 \times 10^{-3}} = \underline{11.1 \Omega}$$

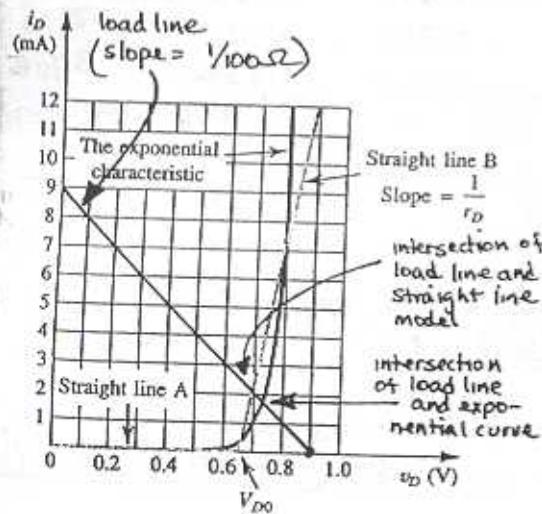
To find V_{DD} :

$$i_D = \frac{V_D - V_{DD}}{r_D}$$

$$10^{-3} = \frac{0.7 - V_{DD}}{11.1} \Rightarrow V_{DD} = 0.689V$$



3.39



(a) The load line intersects the exponential model at:

$$V_D = 0.7V \quad i_D = 1.7mA$$

(b) The load line intersects the straight-line model at

$$V_D = 0.7V \quad i_D = 2mA$$

3.40

Calculating the parameters r_D & V_{DD} for the battery plus resistor model

$$i_D = I_s e^{\frac{V_D}{nV_T}} \quad n=1$$

$$\text{For } i_D = 0.1 I_s$$

$$V_{D2} = 0.7 + 0.025 \ln(0.1) = 0.642V$$

$$\text{For } i_D = 10 I_s$$

$$V_{D3} = 0.7 + 0.025 \ln(10) = 0.758V$$

Note that since the specifications for all of the diodes are given for 0.7V, the end voltages are the same as the voltage change for relative currents are independent to I_D & I_s .

$$\therefore V_{D2} = V_{DD} + i_{D2} r_D \quad ①$$

$$V_{D3} = V_{DD} + i_{D3} r_D \quad ②$$

$$② \rightarrow ①$$

$$V_{D3} - V_{D2} = (i_{D3} - i_{D2}) r_D$$

CONT.

$$0.758 - 0.642 = (0.1I_D - 0.1I_D) R_D$$

$$0.116 = 0.1 I_D R_D$$

$$R_D = \frac{0.0117}{I_D} \quad (3)$$

for (a) $I_D = 1\text{mA}$ $R_D = \frac{0.0117}{1} = \underline{\underline{11.7\Omega}}$

(b) $I_D = 1\text{A}$ $R_D = \frac{0.0117}{1} = \underline{\underline{0.0117\Omega}}$

(c) $I_D = 10\mu\text{A}$ $R_D = \frac{0.0117}{10\mu\text{A}} = \underline{\underline{1.17k\Omega}}$

(3) \rightarrow (1)

$$0.642 = V_{DD} + 0.1 I_D \times \frac{0.0117}{I_D}$$

$$= V_{DD} + 0.00117$$

$$V_{DD} = \underline{\underline{0.641\text{V}}} \quad \leftarrow \text{same for all diodes}$$

3.41

Since for a current of 10mA , the diode voltage is $\underline{\underline{0.8\text{V}}}$, this would be a suitable choice for the constant-voltage-drop model.

3.42

Constant Voltage drop Model:

$$\text{Using } V_D = 0.7\text{V} \Rightarrow I_{D1} = \frac{V - 0.7}{R}$$

$$\text{Using } V_D = 0.6\text{V} \Rightarrow I_{D2} = \frac{V - 0.6}{R}$$

For the difference in currents to vary by only 1% \Rightarrow

$$I_{D2} = 1.01 I_{D1}$$

$$V - 0.6 = 1.01 (V - 0.7)$$

$$V = \underline{\underline{10\text{V}}}$$

For $V = 2\text{V}$ $\frac{1}{R} = 1\text{k}\Omega$
At $V_D = 0.7\text{V}$ $I_{D1} = \frac{2-0.7}{1} = 1.3\text{mA}$

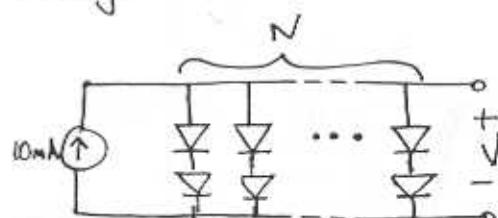
$$V_D = 0.6\text{V} \quad I_{D2} = \frac{2-0.6}{1} = 1.4\text{mA}$$

$$\frac{I_{D2}}{I_{D1}} = \frac{1.4}{1.3} = 1.08$$

Thus the percentage difference is
8%

3.43

Since $2V_D = 1.4\text{V}$ is close to the required 1.25V , use N parallel pairs of diodes to split the current evenly.



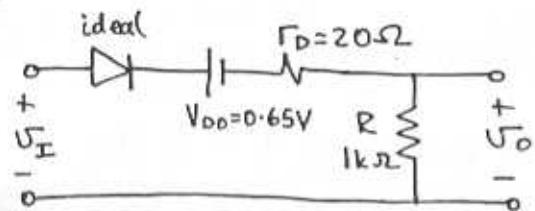
$$\therefore V = 2 \left[0.7 + 0.1 \log \frac{10/N}{20} \right] = 1.25\text{V}$$

$$N = 2.8 \Rightarrow \text{Use } \underline{\underline{3 \text{ sets of diodes}}}$$

$$V = 2 \left(0.7 + 0.1 \log \frac{10/3}{20} \right) = \underline{\underline{1.244\text{V}}}$$

3.44

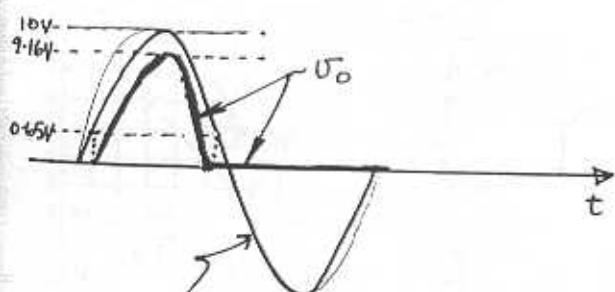
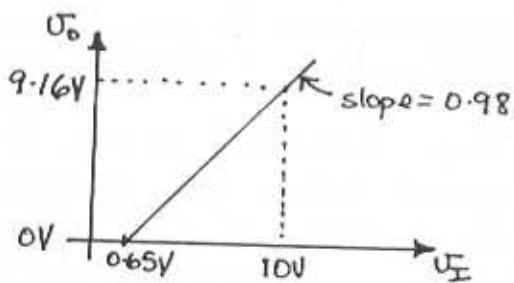
Piecewise linear model in a half-wave rectifier.



$$U_O = \frac{U_I - V_{D0}}{R + r_D} R, \text{ for } U_I \geq 0.65V$$

$$U_O = 0 \quad \text{for } U_I < 0.65V$$

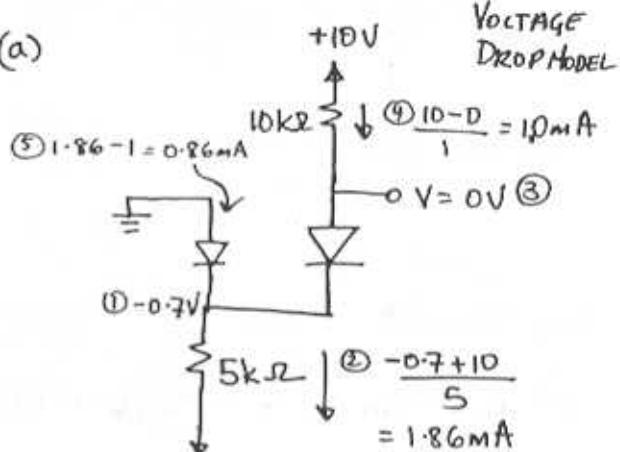
Sketch of transfer characteristic :-



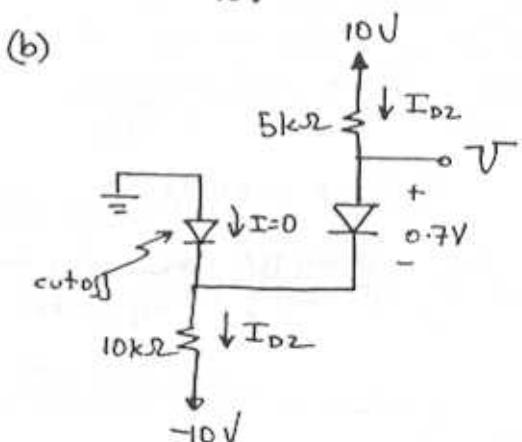
3.45

Refer to example 3-2 ~ CONSTANT VOLTAGE DROP MODEL

(a)



(b)

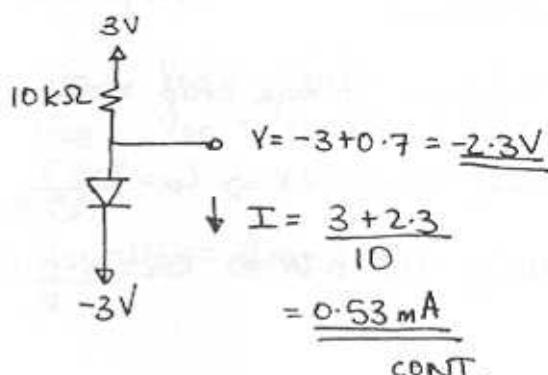


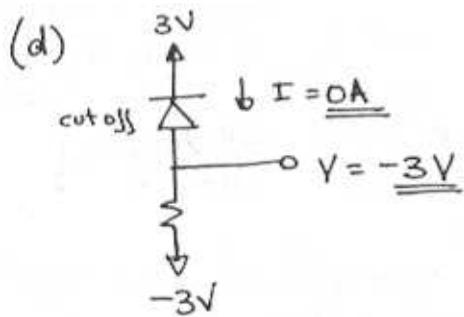
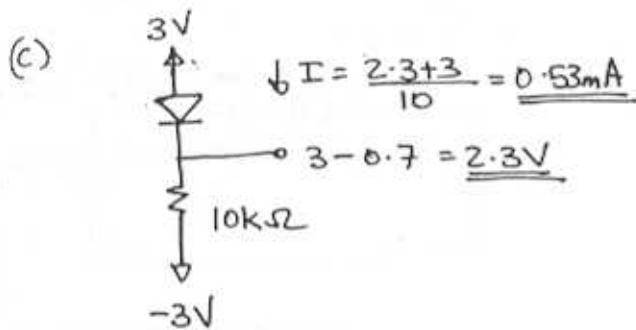
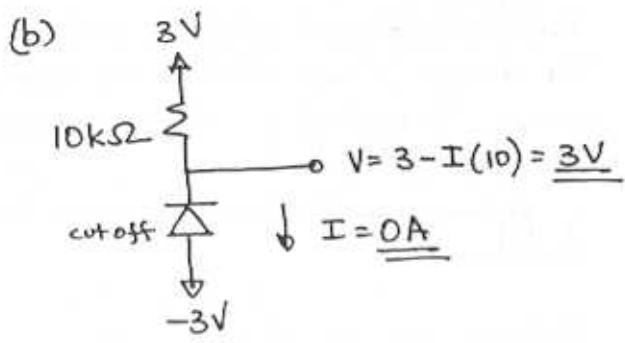
$$I_{D2} = \frac{10 - (-10) - 0.7}{15} = 1.29mA$$

$$U = -10 + 1.29(10) + 0.7 = \underline{\underline{3.6V}}$$

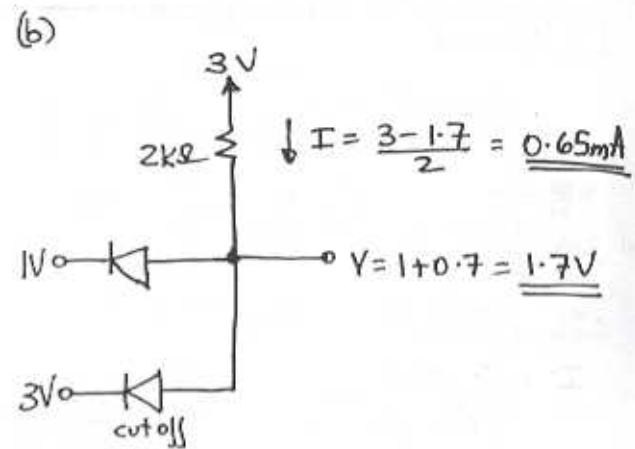
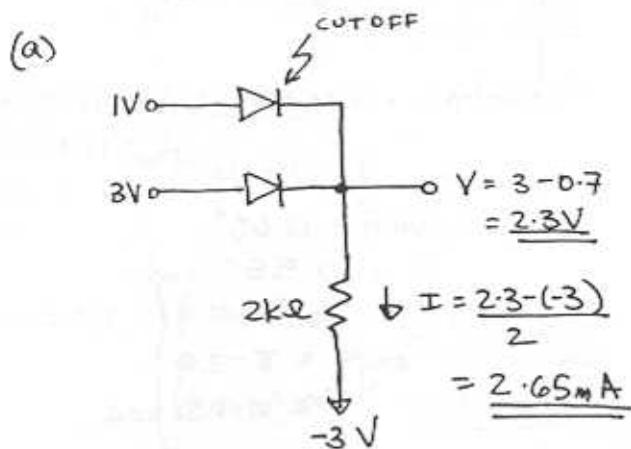
3.46

(a)

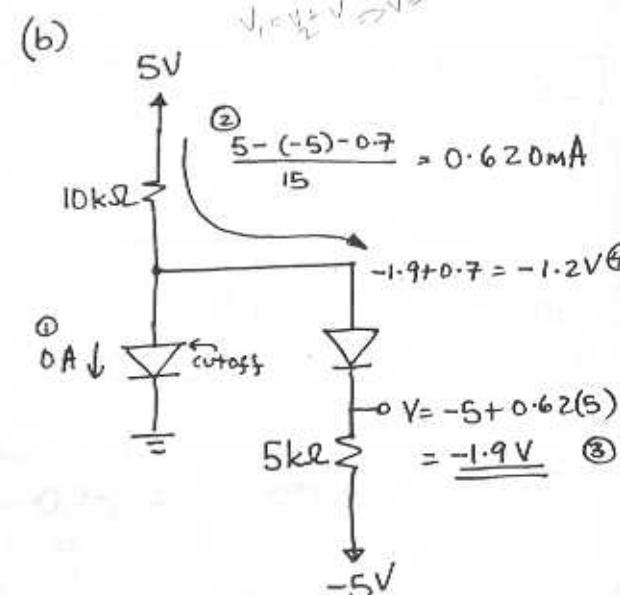
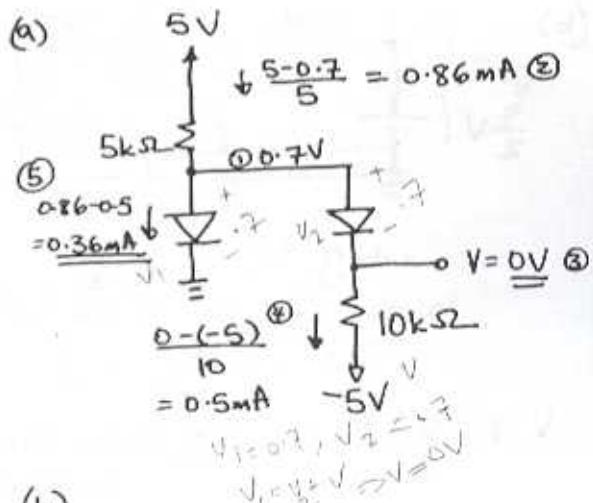




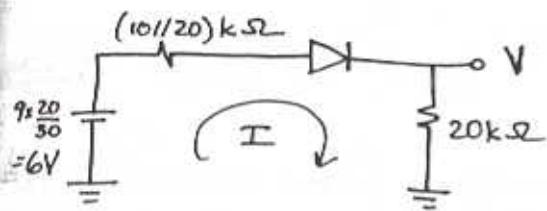
3.47



3.48

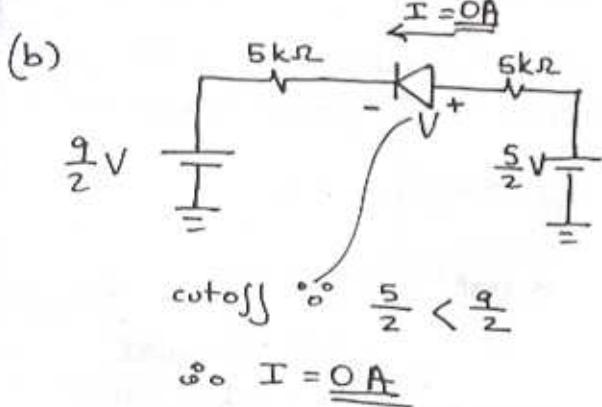


3.49



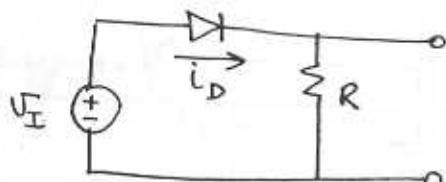
$$I = \frac{6 - 0.7}{(20 + 10)} = 0.199 \text{ mA}$$

$$V = 20I = 3.98 \text{ V}$$



$$V = 9/2 - 9/2 = 0 \text{ V}$$

3.50



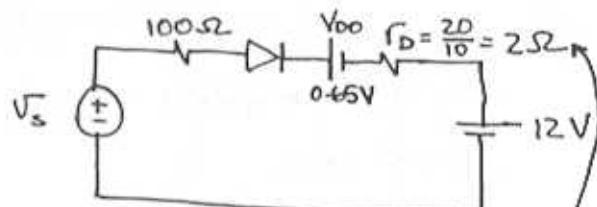
$$i_{D,\text{peak}} = \frac{v_{I,\text{peak}} - 0.7}{R} \leq 50$$

$$R \geq \frac{120\sqrt{2} - 0.7}{50} = 3.38 \text{ k}\Omega$$

Reverse voltage = $120\sqrt{2} = 169.7 \text{ V}$.
The design is essentially the same
since the supply voltage $\gg 0.7 \text{ V}$

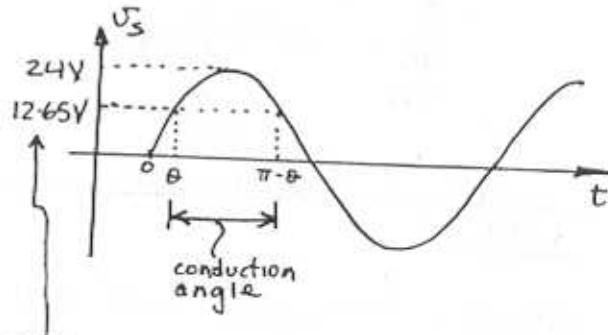
3.51

Battery plus resistance model



since the diode has 10x the area r_D is 1/10 as big.

$$i_D = I_s e^{\frac{V}{nV_T}} = \frac{V_s - V_{DD} - 12}{100 + 2}$$



Conduction starts when $V_s > 12 + V_{DD}$
 $V_s > 12.65 \text{ V}$

$$\therefore 24 \sin \theta = 12.65^\circ$$

$$\theta = 0.555 \text{ rad}$$

$$\text{Conduction angle} = \pi - 2\theta$$

$$= 2.031 \text{ rad.}$$

$$\text{Fraction of cycle for conduction} = \frac{2.031}{2\pi} = 0.323$$

CONT.

$$i_{D, \text{peak}} = \frac{24 - 12.65}{100 + 2} = \underline{\underline{0.111 \text{ A}}}$$

Maximum reverse voltage occurs across the diode when V_S is at its negative peak and is equal to:

$$24 + 12 = \underline{\underline{36 \text{ V}}}$$

3.52

Using the exponential model

$$i_D = I_S e^{\frac{DV}{nV_T}}$$

FOR A +10mV CHANGE

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{0.01}{n(0.025)}} = e^{0.01/n}$$

$$= \begin{cases} 1.492 & \sim n=1 \\ 1.221 & \sim n=2 \end{cases}$$

$$\% \text{ CHANGE} = \frac{i_{D2} - i_{D1}}{i_{D1}} \times 100$$

$$= \begin{cases} (1.492 - 1) \times 100 = +49.2\% & n=1 \\ (1.221 - 1) \times 100 = 22.1\% & n=2 \end{cases}$$

FOR A -10mV CHANGE

$$\frac{i_{D2}}{i_{D1}} = 10^{-0.01/n(0.025)} = \begin{cases} 0.670 & n=1 \\ 0.819 & n=2 \end{cases}$$

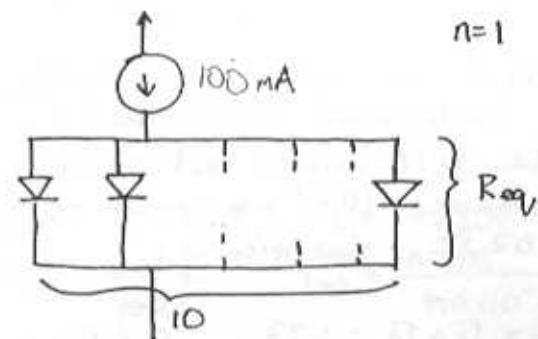
$$\% \text{ CHANGE} = \begin{cases} (0.670 - 1) \times 100 = -33\% & n=1 \\ (0.819 - 1) \times 100 = -18\% & n=2 \end{cases}$$

For a current change limited to $\pm 10\%$

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{\Delta V}{n \times 0.025}} = 0.9 \text{ to } 1.1$$

$$\Delta V = \begin{cases} -2.634 \text{ mV to } 2.383 \text{ mV} & n=1 \\ -5.268 \text{ mV to } 4.766 \text{ mV} & n=2 \end{cases}$$

3.53



Each diode has the current

$$i_D = \frac{0.1}{10} = 0.01 \text{ A}$$

Each diode has a small-signal resistance

$$r_d = \frac{nV_T}{I_D} = \frac{0.025}{0.01} = \underline{\underline{2.5 \Omega}}$$

$$R_{\text{req}} = r_d / 10 = \underline{\underline{0.25 \Omega}}$$

For one diode conducting 0.1 A

$$r_d = nV_T / 0.1 = \frac{0.025}{0.1} = \underline{\underline{0.25 \Omega}}$$

This is the same as R_{req} . We can think of the parallel connection as equivalent to a single diode having 10 times the junction area of each diode. This large diode

is fed with $10\times$ the current (or $0.1A$) and this exhibits the same small-signal resistance as 10 parallel smaller diodes.

Now consider the series resistance of 0.2Ω to connect a diode. For the parallel combination above:

$$R_{eq} = \frac{1}{10} (0.2 + 2.5) = 0.27\Omega$$

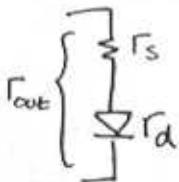
To have an equivalent resistance, the single diode conducting all of the $0.1A$ would need a series resistance $10\times$ as small or 0.025Ω . Specifically:

$$r_{out} = r_s + r_d = 0.27$$

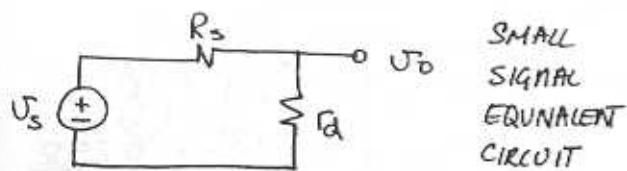
$$= r_s + \frac{nV_T}{I_D} = 0.27$$

$$= r_s + 0.25 = 0.27$$

$$r_s = 0.27 - 0.25 = \underline{\underline{0.025\Omega}}$$



3.54



To find the small-signal response, v_o , open the dc current source I , and short the capacitors C_1 and C_2 . Also replace the diode with its small signal resistance:

$$r_d = \frac{nV_T}{I} \quad n=2$$

Now:

$$v_o = v_s \frac{r_d}{r_d + R_s}$$

$$= v_s \frac{nV_T/I}{nV_T/I + R_s} = v_s \frac{nV_T}{nV_T + IR_s}$$

Q.E.D.

$$v_o = 10mV \frac{0.05}{0.05 + 10^3 I}$$

$$= \begin{cases} 0.476 \text{ mV} & \sim I = 1mA \\ 3.333 \text{ mV} & \sim I = 0.1mA \\ 9.804 \text{ mV} & \sim I = 1\mu A \end{cases}$$

$$\text{For } v_o = \frac{1}{2} v_s = v_s \times \frac{0.05}{0.05 + 10^3 I}$$

$$I = \underline{\underline{50\mu A}}$$

3.55

$R_s = 10k\Omega$, $n=1$, a $1mA$ diode

$$v_o/v_i = \frac{0.025}{0.025 + R_s I}$$

$$= \frac{0.025}{0.025 + 10^4 I} \quad ①$$

For the current change limited to $\pm 10\%$ of I & using the exponential model we get

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/nV_T} = 0.9 \text{ to } 1.1$$

CONT.

$$\Delta V = -2.63 \text{ mV} \text{ to } +38 \text{ mV}$$

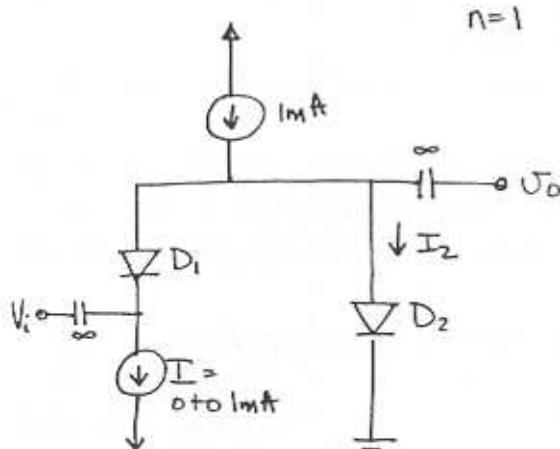
This is the amount the output will vary for a 10% change in diode current. Divide this by the specific gains given in the problem to find the limit on the input signal.

$$\Delta V_s = \frac{\Delta V_o}{V_o/V_I}$$

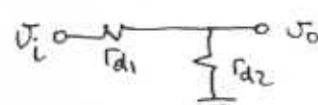
$$= -\frac{2.63 \text{ mV}}{\Delta V_o/V_I} \quad \text{to} \quad \frac{2.38 \text{ mV}}{V_o/V_I} \quad (2)$$

$\frac{V_o}{V_I}$	$I_{use}^{(1)}$ (mA)	V_s (using ②) (mV)
0.5	0.0025	5.26 to 4.76
0.1	0.0225	26.3 to 23.8
0.01	0.25	263 to 238
0.001	2.5	2630 to 2380

3.56



small signal model when D_1 & D_2 are conducting



$$(a) \quad I = 0_{MA}$$

$$D_1 - \text{cutoff} \quad \Rightarrow \quad \frac{U_D}{U_T} = 0 \quad \underline{\underline{V/V}}$$

$$(b) I = 1 \mu A \quad I_2 = 999 \mu A$$

$$r_{d1} = \frac{nV_T}{I}$$

$$= \frac{0.025}{I}$$

$$= 25 \cdot 0.025 \Omega$$

$$= 25 \Omega$$

$$\frac{J_0}{J_I} = \frac{f_{d2}}{f_{a1} + f_{d2}} = 0.001 \frac{V}{V}$$

$$(C) \quad I = 10 \mu A \quad I_2 = 990 \mu A$$

$$r_{d1} = \frac{0.025}{10 \times 10^{-6}} \quad r_{d2} = \frac{0.025}{990 \times 10^{-6}}$$

$$= 2.5 k\Omega \quad = 25.25 \Omega$$

$$(d) \quad I = 100 \mu A \quad I_2 = 900 \mu A$$

$$r_{a1} = \frac{0.025}{100 \times 10^{-6}} \quad r_{a2} = \frac{0.025}{990 \times 10^{-6}}$$

$$= 250 \Omega \quad = 27.78 \Omega$$

$$\frac{V_o}{V_i} = \underline{\underline{0.1 V/V}}$$

$$(e) I = 500 \mu A \quad I_2 = 500 \mu A$$

$$R_1 = R_{d2} = \frac{0.025}{500 \times 10^{-6}} = 50 \Omega$$

$$\frac{5^{\circ}}{5^{\text{h}}} = \underline{\underline{\frac{1}{2} \sqrt{N}}}$$

$$(f) I = 600 \mu A \quad I_2 = 400 \mu A$$

$$r_{d1} = \frac{0.025}{600 \times 10^{-6}} = 41.67 \Omega$$

$$r_{d2} = \frac{0.025}{400 \times 10^{-6}} = 62.5 \Omega$$

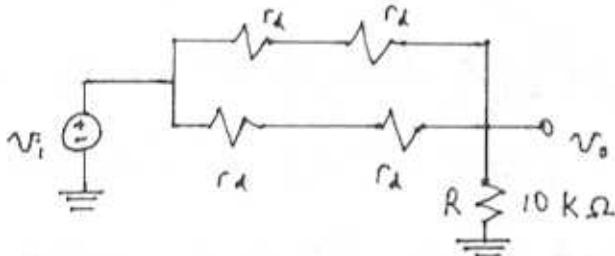
EN THE BIAS CURRENT IN EACH
DE IS $\geq 10\mu A$, THE DIODE RESISTANCE
BE $\leq 2.5 k\Omega$. TO LIMIT THE
CURRENT SIGNAL TO A MAXIMUM OF 10%
BIAS, THE CURRENT SIGNAL MUST BE
 $10\mu A$. THUS, THE SIGNAL VOLTAGE ACROSS
"STARVED" DIODE WILL BE
mV WHICH IS APPROXIMATELY THE
VOLTAGE TO WHICH THE INPUT SIGNAL
SHOULD BE LIMITED.

3.57

$$(a) \frac{V_o}{V_i} = \frac{R}{R + (2r_d // 2r_d)}$$

$$= \frac{R}{R + r_d}$$

WHERE $r_d = \frac{V_T}{I/2} = \frac{2V_T}{I}$
 $= \frac{0.05V}{I}$



I (mA)	V_o/V_i (V/V)
0	0
10^{-3}	0.167
0.01	0.667
0.1	0.952
1.0	0.995
10	0.9995

(b) IF THE SIGNAL CURRENT IS TO BE LIMITED TO $\pm 10I$, THE CHANGE IN DIODE VOLTAGE ΔV_D CAN BE FOUND FROM

$$\frac{i_D}{I} = e^{\Delta V_D / nV_T} = 0.9 \text{ to } 1.1$$

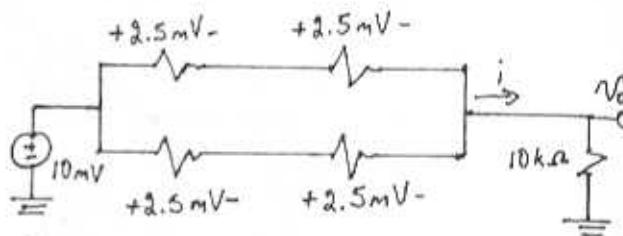
THUS, FOR $n = 1$

$$\Delta V_D = -2.63 \text{ mV to } +2.38 \text{ mV}$$

OR APPROXIMATELY $\pm 2.5 \text{ mV}$

3.57 cont.

(b cont.) FOR THE DIODE CURRENT TO REMAIN WITHIN $\pm 10\%$ OF THEIR DC BIAS CURRENTS, THE SIGNAL VOLTAGE ACROSS EACH DIODE MUST BE LIMITED TO 2.5 mV . NOW, IF $V_{i\text{PEAK}} = 10 \text{ mV}$ WE CAN OBTAIN THE FOLLOWING SITUATION



WE SEE THAT $V_o = 5 \text{ mV}$ AND $i = \frac{5 \text{ mV}}{10 \text{ k}\Omega} = 0.5 \mu\text{A}$.

THUS, EACH DIODE IS CARRYING A CURRENT SIGNAL OF 0.25 mA . FOR THIS TO BE AT MOST 10% OF THE DC CURRENT, THE DC CURRENT IN EACH DIODE MUST BE AT LEAST $2.5 \mu\text{A}$. IT FOLLOWS THAT THE MINIMUM VALUE OF I MUST BE $5 \mu\text{A}$.

(c) FOR $I = 1 \text{ mA}$, $I_D = 0.5 \text{ mA}$, AND FOR MAXIMUM SIGNAL OF 10% , $I_D = 0.05 \text{ mA}$. THUS $i_D = 2i_d = 0.1 \text{ mA}$ AND THE CORRESPONDING MAXIMUM V_o IS $0.1 \text{ mA} \times 10 \text{ k}\Omega = 1 \text{ V}$.

THE CORRESPONDING PEAK INPUT CAN BE FOUND BY DIVIDING V_o BY THE TRANSMISSION FACTOR OF 0.995, THUS

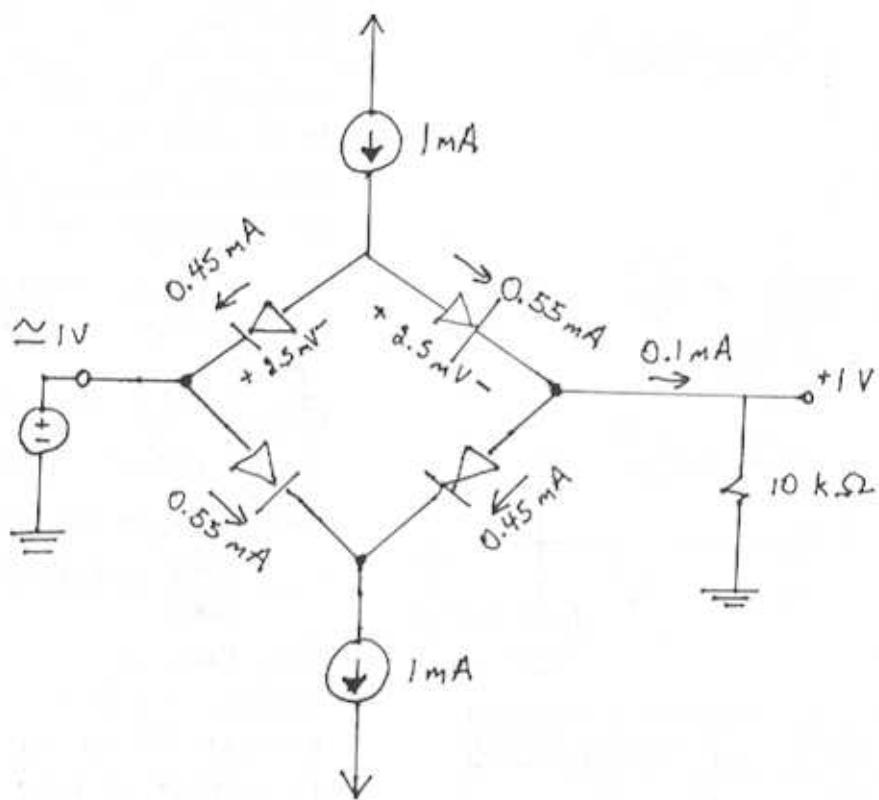
$$V_{i\text{MAX}} = \frac{1 \text{ V}}{0.995 \text{ V}} = \underline{\underline{1.005 \text{ V}}}$$

CONT.

SEE FIGURE.

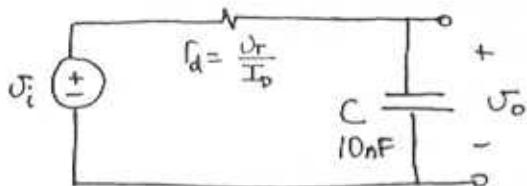
EACH DIODE HAS $r_d = 50\Omega$

3.57 CONT.



3.58

Opening the current source we get
the following small-signal circuit :
(n=1)



$$\frac{V_o}{V_i} = \frac{V_s c}{V_s c + r_d} = \frac{1}{1 + s C r_d}$$

$$\begin{aligned}\text{Phase Shift} &= -\tan^{-1}\left(\frac{\omega C r_d}{1}\right) \\ &= -\tan^{-1}\left(2\pi 10^5 \times 10 \times 10^{-9} \times 0.025/\pi\right)\end{aligned}$$

For a phase shift of -45° we have

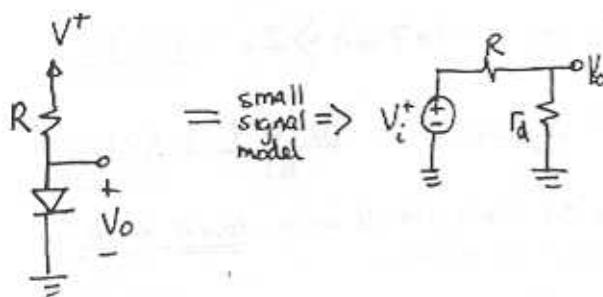
$$2\pi 10^5 \times 10(10^{-9}) \times \frac{0.025}{\pi} = 1$$

$$I = \underline{157 \mu A}$$

Range of phase shift for $I = 15.7 \mu A$
to $1570 \mu A$ is :

$$\underline{-84.3^\circ \text{ to } -5.71^\circ}$$

3.59



CONT.

$$(a) \frac{\Delta V_o}{\Delta V^+} = \frac{r_d}{r_d + R} = \frac{nV_T/I}{nV_T/I + R}$$

$$= \frac{nV_T}{nV_T + IR} \quad \text{where at no load}$$

$$I = \frac{V^+ - 0.7}{R}$$

$$= \frac{nV_T}{nV_T + V^+ - 0.7} \quad \text{Q.E.D.}$$

$$\Delta V_o = - I_L (R \parallel r_d)$$

$$\frac{\Delta V_o}{I_L} = \frac{-(R \parallel r_d)}{R} \quad \text{Q.E.D.}$$

$$(b) \text{ Given at DC } I_D = \frac{V^+ - 0.7}{R}$$

$$\text{Also } r_d = \frac{nV_T}{I_D}$$

(b) For m diodes in series use

$$I = \frac{V^+ - m \times 0.7}{R}$$

Thus:

$$\frac{\Delta V_o}{\Delta V^+} = \frac{m r_d}{m r_d + R} = \frac{m(nV_T)}{m(nV_T) + IR}$$

$$= \frac{m(nV_T)}{m(nV_T) + V^+ - 0.7m}$$

$$\frac{\Delta V_o}{I_L} = - \frac{1}{\frac{1}{R} + \frac{1}{r_d}}$$

$$= - \frac{1}{\frac{I_D}{V^+ - 0.7} + \frac{I_D}{nV_T}}$$

$$= - \frac{nV_T}{I_D} \frac{1}{1 + \frac{nV_T}{V^+ - 0.7}}$$

$$= - \frac{nV_T}{I_D} \frac{V^+ - 0.7}{V^+ - 0.7 + nV_T} \quad \text{Q.E.D.}$$

(c) Line Regulation for $V^+ = 10V$, $n=2$

$$i) m=1 \quad \frac{\Delta V_o}{\Delta V^+} = \underline{\underline{5.35 \text{ mV/V}}}$$

$$\text{For } \frac{\Delta V_o}{I_L} \leq 5 \frac{\text{mV}}{\text{mA}}$$

$$ii) m=3 \quad \frac{\Delta V_o}{\Delta V^+} = \underline{\underline{18.63 \text{ mV/V}}}$$

$$-\frac{2 \times 0.025}{I_D} \times \frac{10 - 0.7}{10 - 0.7 + 0.05} \leq \frac{5 \times 10^{-3}}{10^{-3}}$$

$$I_D > 9.947 \text{ mA} \Rightarrow I_D = \underline{\underline{10 \text{ mA}}}$$

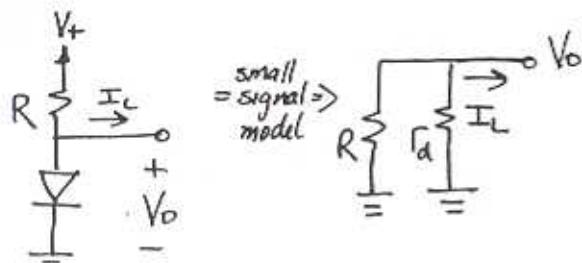
$$R = \frac{V^+ - 0.7}{I_D} = \frac{10 - 0.7}{10} = \underline{\underline{930 \Omega}}$$

Thus the diode should be a 10mA diode.

(c) For m diodes

$$I_D = \frac{V^+ - 0.7m}{R} \quad \text{and} \quad r_d = \frac{m(nV_T)}{I_D}$$

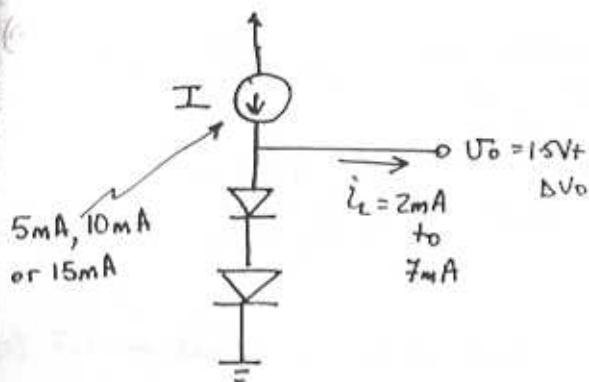
CONT.



$$\begin{aligned}
 \frac{\Delta V_o}{I_L} &= \frac{-1}{\frac{1}{R} + \frac{1}{L}} \\
 &= \frac{-1}{\frac{I_D}{V^+ - 0.7m} + \frac{I_D}{mnV_T}} \\
 &= -\frac{mnV_T}{I_D} \frac{1}{\frac{mnV_T}{V^+ - 0.7m} + 1} \\
 &= -\frac{mnV_T}{I_D} \frac{V^+ - 0.7m}{V^+ - 0.7m + mnV_T}
 \end{aligned}$$

3.61

3.62



For a load current of 2 to 7 mA, I must be greater than 7 mA. Thus the 5 mA source would not do.

We are left to choose between the 10 and 15 mA sources. The 15 mA source provides lower load regulation because the diodes will have more current flowing through them at all times. This is shown below:

Load Regulation if $I = 10\text{mA}$

$$\text{use } \frac{I_{D2}}{I_D} = e^{\frac{\Delta V}{2nV_T}}$$

↑
2 diodes

$$\therefore e^{\frac{\Delta V}{0.05 \times 2}} = \frac{3}{10} \text{ to } \frac{8}{10}$$

$$\Delta U_o = -120\text{mV} \text{ to } -22.3\text{mV}$$

∴ The peak to peak ripple is
 $-120 - (-22.3) \approx -100\text{mV}$

$$\text{Load Regulation} = \frac{\Delta U_o}{I_L} = \frac{-100}{5} = -20$$

$$= -20 \frac{\text{mV}}{\text{mA}}$$

Load Regulation for $I = 15\text{mA}$.
 Here the current through the diodes change from 8 to 13 mA corresponding to

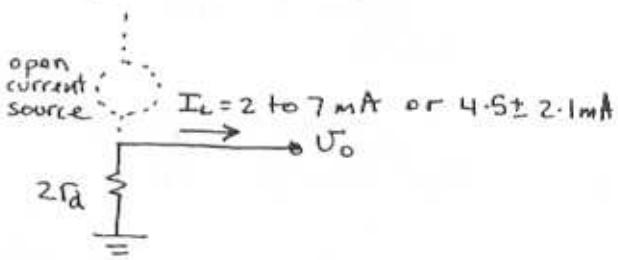
$$\Delta U_o = 0.1 \ln \left(\frac{8}{13} \right)$$

$$= -49\text{mV}$$

$$\text{Load Regulation} = \frac{-49}{5} \approx -10 \frac{\text{mV}}{\text{mA}}$$

The obvious disadvantage of using the 15 mA supply is the requirement of higher current and higher power dissipation.

Alternate solution of Line Regulation using the small signal model



$$\text{Load Regulation} = \frac{\Delta U_o}{I_L} = -2R_d = -\frac{2nV_T}{I_D}$$

Where the bias current $I_D = 10 - 4.5$ for the 10 mA source.

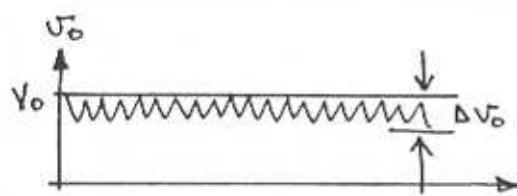
$$\Rightarrow \frac{\Delta U_o}{I_L} = -\frac{2 \times 2 \times 0.025}{10 - 4.5} = -18.2 \frac{\text{mV}}{\text{mA}}$$

For 15 mA source $I_D = 15 - 4.5$

$$\frac{\Delta U_o}{I_L} = -\frac{0.1}{15 - 4.5} = -9.5 \frac{\text{mV}}{\text{mA}}$$

CONT.

Sketch of output:-



3.63

3.64

$$(a) V_Z = V_{Z0} + r_Z I_{ZT}$$

$$10 = 9.6 + r_Z \times 50 \times 10^{-3}$$

$$r_Z = \underline{8\Omega}$$

Power rating:

$$V_Z = V_{Z0} + r_Z \times 2I_{ZT}$$

$$= 9.6 + 8 \times 100 \times 10^{-3}$$

$$= 10.4V$$

$$P = 10.4 \times 100 \times 10^{-3} = \underline{1.04W}$$

$$(b) V_Z = V_{Z0} + r_Z I_{ZT}$$

$$9.1 = V_{Z0} + 30 \times 10 \times 10^{-3}$$

$$V_{Z0} = \underline{8.8V}$$

$$V_Z = 8.8 + 30 \times 20 \times 10^{-3} = 9.4V$$

$$P = 9.4 \times 20 \times 10^{-3} = \underline{188mW}$$

$$(c) 6.8 = 6.6 + 2 \times I_{ZT}$$

$$I_{ZT} = \underline{100mA}$$

$$V_Z = 6.6 + 2 \times 200 \times 10^{-3} = 7V$$

$$P = 7 \times 200 \times 10^{-3} = \underline{1.4W}$$

$$(d) 18 = 17.2 + r_Z \times 5 \times 10^{-3}$$

$$r_Z = \underline{160\Omega}$$

$$V_Z = 17.2 + 160 \times 10 \times 10^{-3} = 18.8V$$

$$P = 18.8 \times 10 \times 10^{-3} = \underline{188mW}$$

$$(e) 7.6 = V_{Z0} + 1.5 \times 200 \times 10^{-3}$$

$$V_{Z0} = \underline{7.2V}$$

$$V_Z = 7.2 + 1.5 \times 400 \times 10^{-3} = 7.8V$$

$$P = 7.8 \times 400 \times 10^{-3} = \underline{3.12W}$$

3.65

(a) Three 6.8V zeners provide $3 \times 6.8 = 20.4V$ with $3 \times 10 = 30\Omega$ resistance. Neglecting R , we have

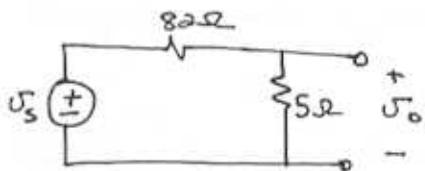
$$\text{Load Regulation} = -\underline{30mV/mA}$$

(b) For 5.1V zeners we use 4 diodes to provide 20.4V with $4 \times 30 = 120\Omega$ resistance.

$$\text{Load regulation} = -\underline{120mV/mA}$$

3.66

Small signal model for line regulation:



$$\frac{\Delta V_o}{\Delta V_s} = \frac{5}{5+82}$$

$$\Delta V_o = \frac{5}{87} \times \Delta V_s$$

$$= \frac{5}{87} \times 1.3$$

$$= \underline{74.7 mV}$$

3.67

$$V_Z = V_{Z0} + r_Z I_{ZT}$$

$$9.1 = V_{Z0} + 5 \times 28 \times 10^{-3}$$

$$V_{Z0} = 8.96V$$

$$V_Z = V_{Z0} + 5I_Z = 8.96 + 5I_Z$$

$$\text{FOR } I_Z = 10mA \quad V_Z = \underline{\underline{9.01V}}$$

$$\text{FOR } I_Z = 100mA \quad V_Z = \underline{\underline{9.46V}}$$

3.68

$$r_e = 30 \Omega$$

$$I_{ZK} = 0.5 \text{ mA}$$

$$V_Z = 7.5 \text{ V}$$

$$I_Z = 12 \text{ mA}$$

$$7.5 = V_{Z0} + 12 \times 30 \times 10^{-3}$$

$$\Rightarrow V_{Z0} = 7.14 \text{ V}$$

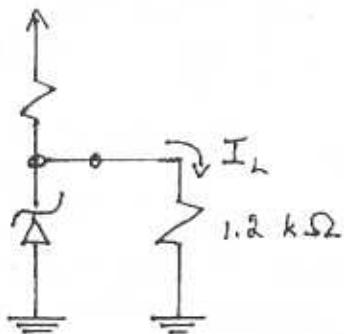
$$I_Z = \frac{7.5}{1.2} = 6.25 \text{ mA}$$

SELECT $I = 10 \text{ mA}$

SO THAT $I_Z = 3.7 \text{ mA}$

WHICH IS $> I_{ZK}$

$$R = \frac{10 - 7.5}{10} = \underline{\underline{250 \Omega}}$$

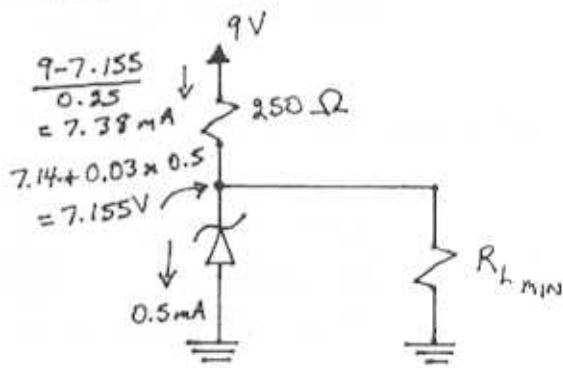
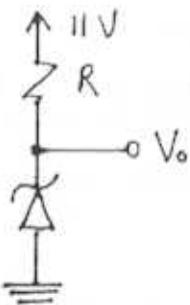


FOR $\Delta V^+ = \pm 1 \text{ V}$

$$\begin{aligned}\Delta V_o &= \pm 1 \times \frac{1.2 // 0.03}{0.250 + (1.2 // 0.03)} \\ &= \pm 0.1 \text{ V}\end{aligned}$$

THUS $V_o = +7.4 \text{ V}$ TO $+7.6 \text{ V}$
WITH $V^+ = 11 \text{ V}$ AND $I_L = 0$

$$\begin{aligned}V_o &= V_{Z0} + \frac{11 - V_o}{0.25} \times 0.03 \\ \Rightarrow V_o &= \underline{\underline{7.55 \text{ V}}}\end{aligned}$$



$$\begin{aligned}R_{L_{\min}} &= \frac{7.155}{7.38 - 0.5} \\ &= \underline{\underline{1.04 \text{ k} \Omega}}\end{aligned}$$

$$\therefore R = \frac{9 - 0.68}{20} = \underline{\underline{110.5\Omega}}$$

$$\begin{aligned}\text{Line Regulation} &= \frac{\Delta V_o}{\Delta V_s} = \frac{r_2}{r_2 + R} \\ &= \frac{5}{5 + 110} \\ &= \underline{\underline{43.5 \frac{mV}{V}}}\end{aligned}$$

SECOND DESIGN ~ limited current from 9V supply

$$I_{zK} = 0.25 \text{ mA}$$

$$V_z = V_{zK} \approx V_{z0} - \text{calculate } V_{z0} \text{ from}$$

$$V_z = V_{z0} + r_2 I_{zT}$$

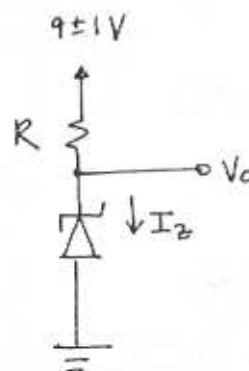
$$6.8 = V_{z0} + 5 \times 0.02$$

$$V_{z0} = 6.7 \text{ V}$$

$$\therefore R = \frac{9 - 6.7}{0.25} = \underline{\underline{9.2 \text{ k}\Omega}}$$

$$\begin{aligned}\text{LINE REGULATION} &= \frac{\Delta V_o}{\Delta V_s} = \frac{7.50}{7.50 + 9.2 \times 10^3} \\ &= \underline{\underline{75.4 \frac{mV}{V}}}\end{aligned}$$

3.69



GIVEN PARAMETERS

$$V_z = 6.8 \text{ V}, r_2 = 5 \Omega, I_{zK} = 20 \text{ mA}$$

By knee

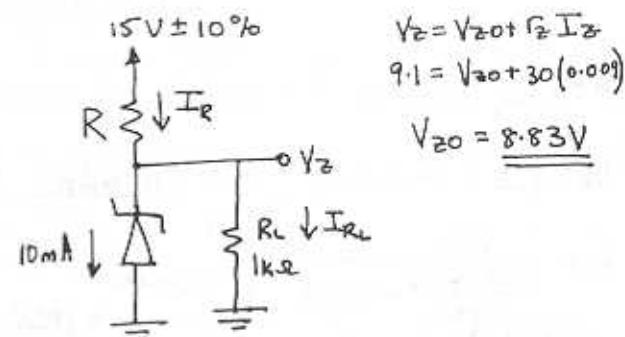
$$I_{zK} = 0.25 \text{ mA}$$

$$r_2 = 7.50 \Omega$$

FIRST DESIGN ~ 9V supply can easily supply current

Let $I_z = 20 \text{ mA}$ ~ well above knee

3.70



$$V_z = V_{z0} + r_2 I_{zT}$$

$$9.1 = V_{z0} + 30(0.001)$$

$$V_{z0} = \underline{\underline{8.83 \text{ V}}}$$

CONT.

$$V_2 = 8.83 + 30(0.01) = 9.13 \text{ V}$$

$$I_{RL} = 9.13/1k\Omega = 9.13 \text{ mA}$$

$$I_R = 10 + 9.13 = \underline{19.13 \text{ mA}}$$

$$\therefore R = \frac{15 - 9.13}{19.13} = 306.8 \Omega \approx \underline{\underline{300 \Omega}}$$

$$V_2 = 8.83 + 30 \left(\frac{15 - V_2}{300} - \frac{V_2}{1000} \right)$$

$$= 10.33 - \frac{V_2}{10} - \frac{3}{100} V_2$$

$$V_2 = 9.14 \text{ V}$$

$$V_2 = 8.83 + 30 \left(\frac{15 \pm 1.5 - V_2}{300} - \frac{V_2}{1000} \right) \\ = \frac{1}{1.13} \left[8.83 + 15 \pm 0.15 \right] = 9.14 \pm 0.13 \text{ V}$$

$\therefore \pm 0.13 \text{ V}$ variation in output voltage
Halving the load current $\equiv R_L$ doubling

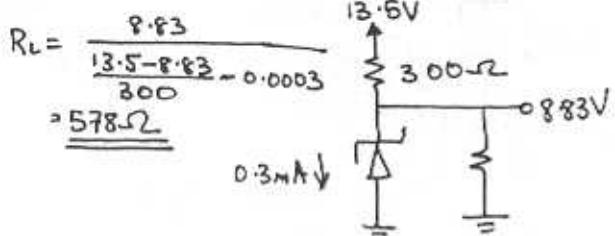
$$V_2 = 8.83 + 30 \left(\frac{15 - V_2}{300} - \frac{V_2}{2000} \right)$$

$$= \frac{10.33}{1.115} = 9.26 \text{ V}$$

$\therefore 9.26 - 9.14 = 0.12 \text{ V}$ increase in output voltage.

At the edge of the breakdown region

$$V_2 \approx V_{20} = 8.83 \text{ V} \quad I_{ZK} = 0.3 \text{ mA}$$



lowest output voltage = 8.83 V

$$\text{Line Regulation} = \frac{V_2}{R + R_2} = \frac{30}{300 + 30} = 90 \frac{\text{mV}}{\text{V}}$$

$$\text{Load Regulation} = -(R_2 || R) = -29.1 \frac{\text{mV}}{\text{mA}}$$

3.71

$$(a) V_{2T} = V_{20} + R_2 I_{2T} \\ 10 = V_{20} + 7(0.025) \\ \Rightarrow V_{20} = \underline{\underline{9.825 \text{ V}}}$$

(b) The minimum zener current of 5mA occurs when $I_L = 20 \text{ mA}$ and V_s is at its minimum of $20(1-0.25) = 15 \text{ V}$. See the circuit below:

$$R \leq \frac{15 - (V_{20} + R_2 I_2)}{20 + 5} \\ \leq \frac{15 - (9.825 + 7(0.005))}{25} \\ \leq 205.6 \Omega$$

\therefore Use $R = \underline{\underline{205 \Omega}}$

$$(c) \text{ Line Regulation} = \frac{7}{205 + 7} = 33 \frac{\text{mV}}{\text{V}}$$

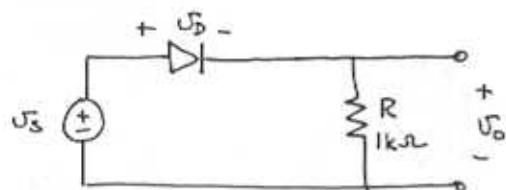
$\pm 25\%$ change in $V_s \equiv \pm 5 \text{ V}$

V_2 changes by $\pm 5 \times 33 = \pm \underline{\underline{165 \text{ mV}}}$

corresponding to $\frac{\pm 165}{10} \times 100 = \pm \underline{\underline{1.65\%}}$

CONT.

3.75



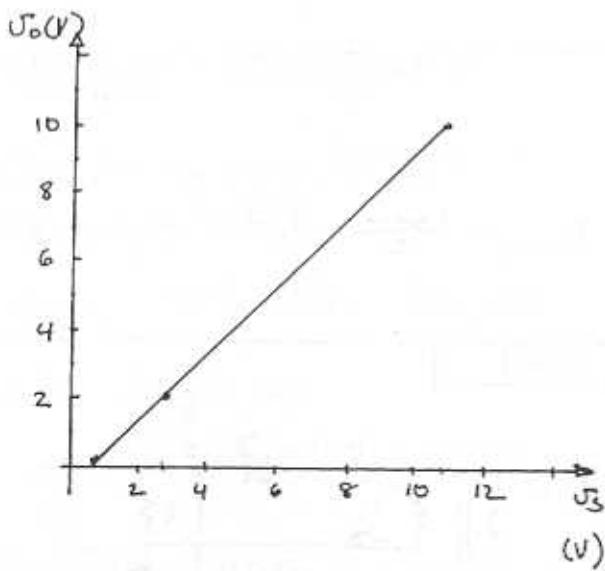
$$i_D = I_S e^{\frac{U_D}{nV_T}}$$

$$\frac{i_D}{1mA} = e^{\frac{U_D - 0.7}{nV_T}}$$

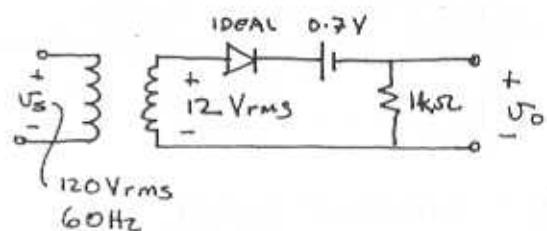
$$U_D - 0.7 = nV_T \ln \left(\frac{i_D}{10^{-3}} \right) = 0.1 \log \left(\frac{i_D}{10^{-3}} \right)$$

$$\begin{aligned} U_D &= 0.7 + 0.1 \log \left(\frac{U_D}{R} \right) \quad R = 1k\Omega \\ &= 0.7 + 0.1 \log \left(\frac{U_o}{1} \right) \end{aligned}$$

U_o (V)	U_D (V)	$U_s = U_D + U_o$ (V)
0.10	0.6	0.7
0.5	0.67	1.17
1	0.7	1.7
2	0.73	2.73
5	0.77	5.77
10	0.8	10.8



3.76



$$V_o = 12\sqrt{2} - 0.7 = 16.27V$$

Conduction begins at

$$\begin{aligned} U_s &= 12\sqrt{2} \sin \theta = 0.7 \\ \theta &= \sin^{-1} \left(\frac{0.7}{12\sqrt{2}} \right) \\ &= 0.0412 \text{ rad} \end{aligned}$$

Conduction ends at $\pi - \theta$

$$\therefore \text{Conduction angle} = \pi - 2\theta = 3.06 \text{ rad}$$

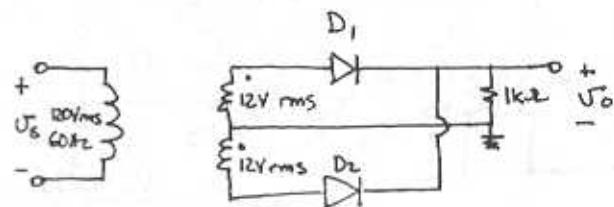
The diode conducts for

$$\frac{3.06}{2\pi} \times 100 = 48.7\% \text{ of the cycle}$$

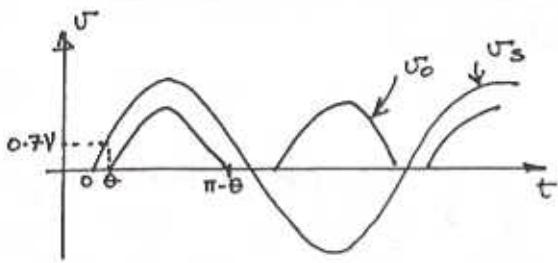
$$\begin{aligned} V_{o, \text{avg}} &= \frac{1}{2\pi} \int_{\theta}^{\pi-\theta} 12\sqrt{2} \sin \phi - 0.7 d\phi \\ &= 5.06V \end{aligned}$$

$$i_{D, \text{avg}} = \frac{V_{o, \text{avg}}}{R} = 5.06 \text{ mA}$$

3.77



CONT.



$$\hat{U}_o = 12\sqrt{2} - V_{D0} = \underline{16.27V}$$

$$\text{Conduction starts at } \theta = \sin^{-1} \frac{0.7}{12\sqrt{2}} = 0.0412 \text{ rad}$$

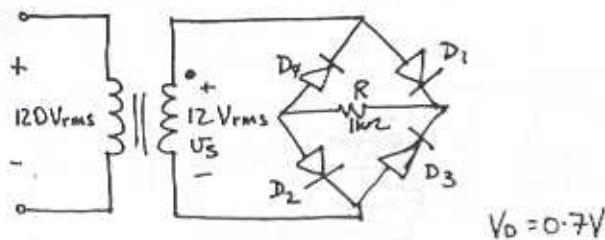
and ends at $\pi - \theta$. Conduction angle $= \pi - 2\theta = 3.06$ rad in each half cycle. Thus the fraction of a cycle for which one of the two diodes conduct $= \frac{2(3.06)}{2\pi} \times 100 = \underline{97.4\%}$

Note that during 97.4% of the cycle there will be conduction. However each of the two diodes conducts for only half the time, i.e. for 48.7% of the cycle.

$$U_{o,\text{avg}} = \frac{1}{\pi} \int_0^{\pi-\theta} 12\sqrt{2} \sin \phi - 0.7 \, d\phi = \underline{10.12V}$$

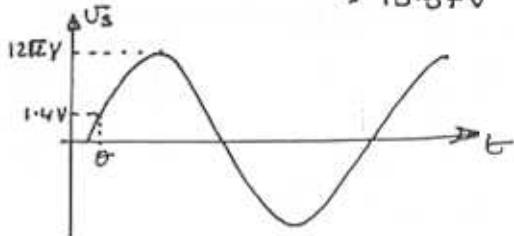
$$i_{D,\text{avg}} = \frac{10.12}{1k\Omega} = \underline{10.12mA}$$

3.78



Peak voltage across $R = 12\sqrt{2} - 2V_D$

$$= 12\sqrt{2} - 1.4 \\ > 16.57V$$



$$\theta = \sin^{-1} \frac{1.4}{12\sqrt{2}} = 0.0826 \text{ rad}$$

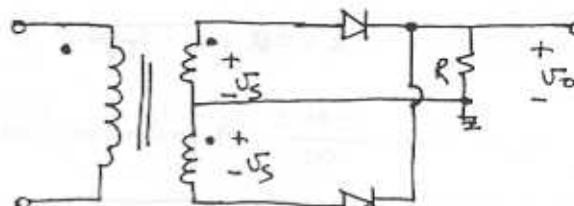
Fraction of cycle that $D_1 \& D_2$ conduct is $\frac{\pi - 2\theta}{2\pi} \times 100 = \underline{47.4\%}$

Note $D_3 \& D_4$ conduct in the other half cycle \Rightarrow that there is $2(47.4) = 94.8\%$ conduction interval.

$$U_{o,\text{avg}} = \frac{1}{2\pi} \int_0^{\pi-\theta} 12\sqrt{2} \sin \phi - 2V_D \, d\phi = \frac{1}{\pi} \left[-12\sqrt{2} \cos \phi - 1.4 \phi \right]_0^{\pi-\theta} = \frac{2(12\sqrt{2} \cos \theta)}{\pi} - \frac{1.4(\pi - 2\theta)}{\pi} = \underline{9.44V}$$

$$i_{R,\text{avg}} = \frac{U_{o,\text{avg}}}{R} = \frac{9.44}{1} = 9.44 \text{ mA}$$

3.79



CONT.

For $V_{DD} \ll V_s$,

$$V_{o,\text{avg}} \approx \frac{2}{\pi} V_s - V_{DD}$$

(a) For $V_{o,\text{avg}} = 10V$

$$10 = \frac{2}{\pi} V_s - 0.7$$

$$\Rightarrow V_s = 16.81V$$

$$\text{Line peak} = 120\sqrt{2}$$

Thus,

$$\text{turns ratio} = \frac{120\sqrt{2}}{16.81} = \underline{\underline{10.1:1}}$$

to each half
of the secondary

OR 5.05:1 centre tapped

(b) For $V_{o,\text{avg}} = 100V$

$$V_s = \frac{\pi}{2} (100.7) = \underline{\underline{158.2V}}$$

$$\text{Turns Ratio} = \frac{120\sqrt{2}}{158.2} = \underline{\underline{1.07:1}} \text{ to each half}$$

OR 0.535:1 centre tapped

3.80

Refer to Fig 3.27.

For $2V_{DD} \ll V_s$,

$$V_{o,\text{avg}} = \frac{2}{\pi} V_s - 2V_{DD} = \frac{2}{\pi} V_s - 1.4$$

(a) For $V_{o,\text{avg}} = 10V$

$$V_s = \frac{\pi}{2} \times 11.4 = 17.91V$$

$$\text{Turns Ratio} = \frac{120\sqrt{2}}{17.91} = \underline{\underline{9.477 \text{ to } 1}}$$

(b) For $V_{o,\text{avg}} = 100V$

$$V_s = \frac{\pi}{2} \times 101.4 = 159.3V$$

$$\text{Turns Ratio} = \frac{120\sqrt{2}}{159.3} = \underline{\underline{1.065 \text{ to } 1}}$$

3.81

$$120\sqrt{2} \pm 10\% : 24\sqrt{2} \pm 10\%$$

\Rightarrow turns Ratio = 5 : 1

$$V_s = \frac{24\sqrt{2}}{2} \pm 10\%$$

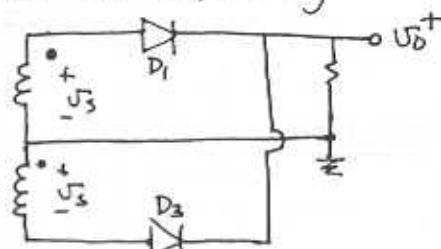
$$\begin{aligned} \text{PIV} &= 2V_s|_{\max} - V_{DD} \\ &= 2 \times \frac{24\sqrt{2}}{2} \times 1.1 - 0.7 \end{aligned}$$

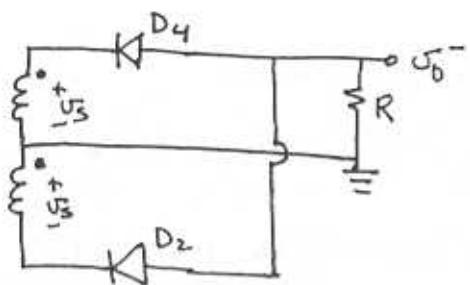
$$= \underline{\underline{36.6V}}$$

Using a factor of 1.5 for safety we select a diode having a PIV rating of 55V.

3.82

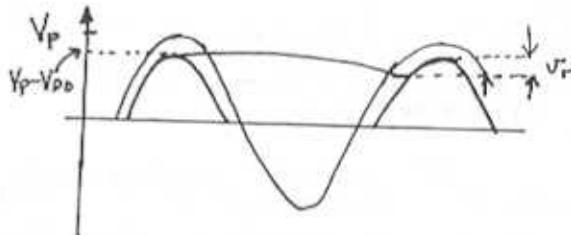
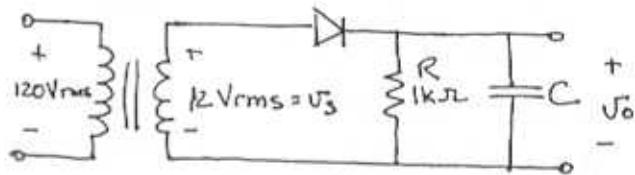
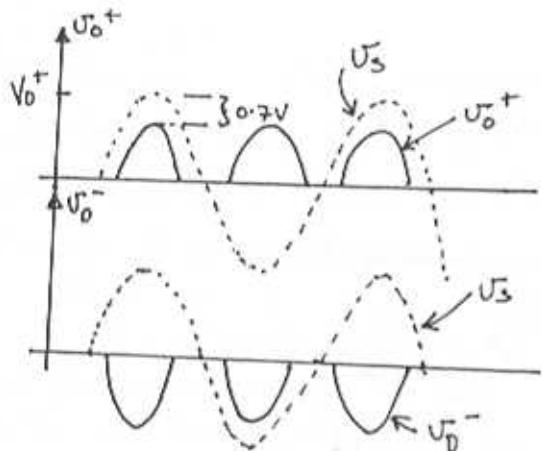
The circuit is a full wave rectifier with centre tapped secondary winding. The circuit can be analyzed by looking at V_o^+ and V_o^- separately.





If choosing a diode, allow a safety margin of $1.5 \text{ PIV} = \underline{\underline{73V}}$

3.83



$$\begin{aligned} V_{D,\text{avg}} &= \frac{1}{2\pi} \int V_s \sin \phi - 0.7 d\phi = 15 \\ &= \frac{2V_s}{\pi} - 0.7 = 15 \quad \text{assumed } V_s > 0.7V \\ V_s &= \frac{15+0.7}{2}\pi = 24.66V \end{aligned}$$

Thus voltage across secondary winding
= $2V_s = \underline{\underline{49.32V}}$

Looking at D_4

$$\begin{aligned} \text{PIV} &= V_s - V_o^- \\ &= V_s + (V_s - 0.7) \\ &= 2V_s - 0.7 \\ &= \underline{\underline{48.6V}} \end{aligned}$$

$$(i) \bar{V}_r \approx (V_p - V_{DD}) \frac{T}{CR} \quad \text{Eq. (3.28)}$$

$$0.1(V_p - V_{DD}) = (V_p - V_{DD}) \frac{T}{CR}$$

$$C = \frac{1}{0.1 \times 60 \times 10^3} = \underline{\underline{166.7 \mu F}}$$

$$(ii) \text{ For } \bar{V}_r = 0.01(V_p - V_{DD}) = (V_p - V_{DD}) \frac{T}{CR}$$

$$C = \underline{\underline{1667 \mu F}}$$

(a)

$$\begin{aligned} (i) \bar{V}_o, \text{avg} &= V_p - V_{DD} - \frac{1}{2} V_r \\ &= 12\sqrt{2} - 0.7 - \frac{1}{2}(2\sqrt{2} - 0.7) 0.1 \\ &= (12\sqrt{2} - 0.7)(1 - \frac{0.1}{2}) \\ &= \underline{\underline{15.5V}} \end{aligned}$$

$$\begin{aligned} (ii) \bar{V}_o, \text{avg} &= (12\sqrt{2} - 0.7)(1 - \frac{0.01}{2}) \\ &= \underline{\underline{16.19V}} \end{aligned}$$

CONT.

(b)

i) Using eq (3.30) we have the conduction angle =

$$\begin{aligned} \omega \Delta t &\approx \sqrt{\frac{2V_r}{(V_p - V_{DD})}} \\ &= \sqrt{\frac{2 \times 0.1 (V_p - 0.7)}{(V_p - 0.7)}} \\ &= \sqrt{0.2} \\ &= 0.447 \text{ rad} \end{aligned}$$

∴ Fraction of cycle for conduction = $\frac{0.447}{2\pi} \times 100$
 $= \underline{7.1\%}$

ii) $\omega \Delta t \approx \sqrt{\frac{2\pi 0.01 (V_p - 0.7)}{V_p - 0.7}} = 0.141 \text{ rad}$

Fraction of cycle = $\frac{0.141}{2\pi} \times 100 = \underline{2.25\%}$

(c) (i) Use eq (3.31)

$$\begin{aligned} i_{D,\text{avg}} &= I_L \left(1 + \pi \sqrt{\frac{2(V_p - V_{DD})}{V_r}} \right) \\ &= \frac{V_{o,\text{avg}}}{R} \left(1 + \pi \sqrt{\frac{2(V_p - V_{DD})}{0.1(V_p - V_{DD})}} \right) \\ &= \frac{15.5}{10^3} \left(1 + \pi \sqrt{\frac{2}{0.1}} \right) \\ &= \underline{2.33 \text{ mA}} \end{aligned}$$

ii) $i_{D,\text{avg}} = \frac{16.19}{10^3} \left(1 + \pi \sqrt{200} \right)$
 $= \underline{7.35 \text{ mA}}$

NB Text uses $I_L \approx V_p/R = \frac{V_p - V_{DD}}{R}$
 but here we used $i_{D,\text{avg}} = \frac{V_p - V_{DD} - \frac{1}{2}V_C}{R}$

which is more accurate.

(d) i) $i_{D,\text{peak}} = I_L \left(1 + 2\pi \sqrt{\frac{2(V_p - V_{DD})}{V_r}} \right)$

$$\begin{aligned} &= \frac{15.42}{10^3} \left(1 + 2\pi \sqrt{\frac{2}{0.1}} \right) \\ &= \underline{449 \text{ mA}} \end{aligned}$$

ii) $i_{D,\text{peak}} = \frac{16.19}{10^3} \left(1 + 2\pi \sqrt{\frac{2}{0.01}} \right)$
 $= \underline{1455 \text{ mA}}$

3.84

i) $V_r = 0.1(V_p - V_{DD}) = \frac{(V_p - V_{DD})}{2fCR}$

the factor of 2 accounts for discharge occurring only half of the period $\frac{1}{2} = \frac{1}{2f}$

$$C = \frac{1}{(2fR)0.1} = \frac{1}{2(60)10^3 \times 0.1} = \underline{833 \mu F}$$

ii) $C = \frac{1}{2(60)10^3 0.01} = \underline{833 \mu F}$

(a) i) $V_o = V_p - V_{DD} - \frac{1}{2}V_r$
 $= (V_p - V_{DD})(1 - \frac{0.1}{2})$
 $= (16.27)(1 - \frac{0.1}{2})$
 $= \underline{15.5V}$

ii) $V_o = (16.27)(1 - \frac{0.01}{2}) = \underline{16.19V}$

(b)

(i) Fraction of cycle = $\frac{2\omega \Delta t}{2\pi} \times 100$
 $= \frac{\sqrt{2V_r/(V_p - V_{DD})}}{\pi} \times 100$
 $= \frac{1}{\pi} \sqrt{2(0.1)} \times 100 = \underline{14.2\%}$

CONT.