Signals and Systems
Fall 2012

Solution to Homework Assignment #12

1.

(a)

$$x(t) = 2u_s(t-1) - u_s(t-2) - 2u_s(t-3.5)$$

(b)

$$x(t) = 2t \cdot u_s(t) - 2(t-1)u_s(t-1) - 2u_s(t-2) + (t-2)u_s(t-2)$$
$$-2(t-3)u_s(t-3) + (t-4)u_s(t-4)$$

(c)

$$x(t) = u_s(t) - u_s(t-1) + 2(t-1)u_s(t-1) - 2(t-2)u_s(t-2)$$
$$-2u_s(t-2) - u_s(t-3) + (t-3)u_s(t-3)$$

2.

(a)

Let $\mathbf{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$,

$$\left\langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\rangle = a + 2b + 3c = 0 \qquad (1)$$

$$\left\langle \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\rangle = 3a + 2b + c = 0 \qquad (2)$$

$$(1) - (2) = 0 \Rightarrow a = c \qquad (3)$$

Given that a=c, from (3), we obtain $b = -2a$.

Therefore, any vector $\mathbf{v} = a\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ with $a \neq 0$ is orthogonal to both $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$.

(b)

Let $\mathbf{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$\left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\rangle = 2a + b + 3c = 0$$

Let $c = 0$, then $b = -2a$, we have

$$\therefore \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ -2a \\ 0 \end{pmatrix} = a \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \text{ for } a \neq 0$$

We then need to find another vector that is linearly independent with $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$. Certainly, there are infinite numbers of these vectors. For example, let $a = 2, b = 1$. It then follows that $c = -\frac{5}{3}$. It is not hard to see that $\begin{pmatrix} 2 \\ 1 \\ -5/3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ are linearly independent.

(c)

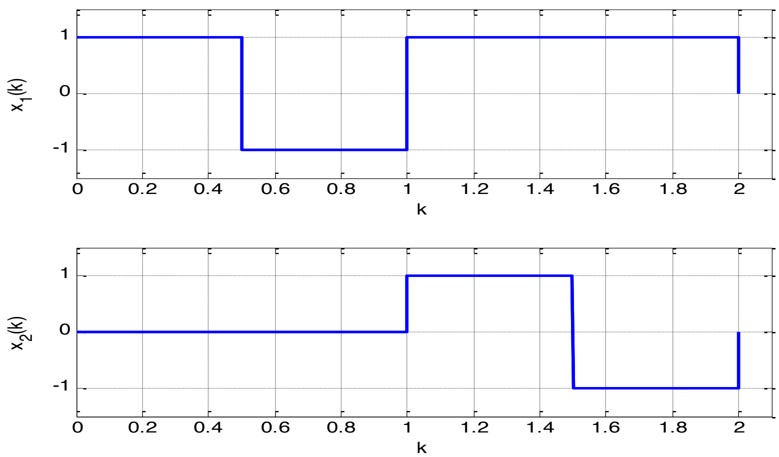
$\langle x, y \rangle = |x| \cdot |y|$. We know that $\langle x, x \rangle = |x| \cdot |x|$. Therefore, $\langle x, kx \rangle = |x| \cdot |kx|$ for $k > 0$.

As a result, the non-zero vector y can be expressed as

: $$y = k \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \text{ where } k > 0.$$

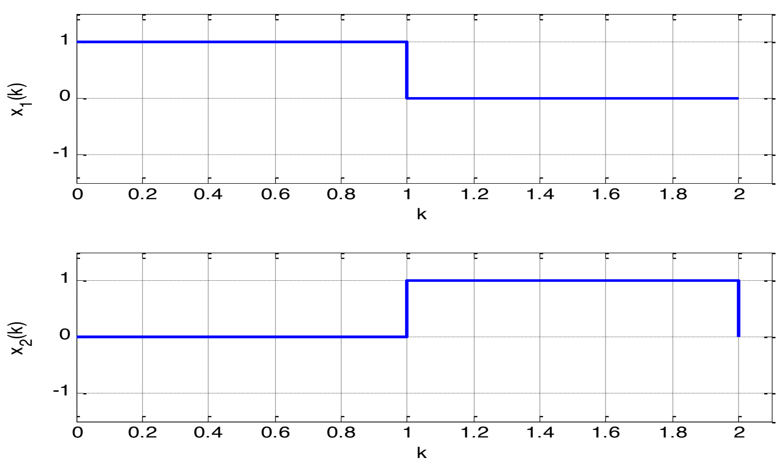
3.

(a)



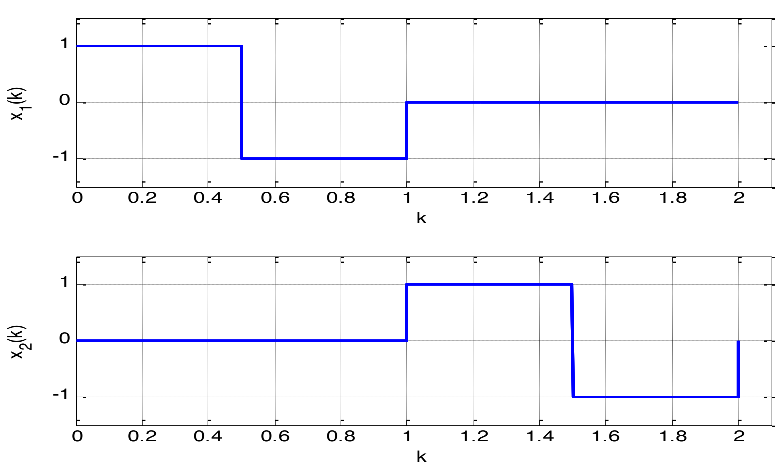
x1(k)
1
0
-1
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
k
x2(k)
1
0
-1
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
k

(b)



x1(k)
1
0
-1
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
k
x2(k)
1
0
-1
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
k

(c)



x1(k)
1
0
-1
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
k
x2(k)
1
0
-1
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
k