Signals and Systems
Fall 2012

Solution to Homework Assignment #15

1.

(a) $X(s)=\dfrac{3s+2}{s^2+6s+25}=3\times\dfrac{(s+3)}{(s+3)^2+4^2}-\dfrac{7}{4}\times\dfrac{4}{(s+3)^2+4^2}$

We know that:

$$\frac{b}{(s+a)^2+b^2}\xleftrightarrow{L}e^{-at}\sin bt\cdot u_s(t)$$

$$\frac{s+a}{(s+a)^2+b^2}\xleftrightarrow{L}e^{-at}\cos bt\cdot u_s(t)$$

Therefore, we obtain the following signal in time domain:

$$x(t)=3\times e^{-4t}\cos 3t\cdot u_s(t)-\frac{7}{4}\times e^{-4t}\sin 3t\cdot u_s(t)$$

(b) $X(s)=\dfrac{1-e^{-4s}}{s+2}=\dfrac{1}{s+2}-e^{-4s}\times\dfrac{1}{s+2}$

We know that

$$e^{-at}u_s(t)\xleftrightarrow{L}\frac{1}{1+a}$$

and

$$x(t-a)u_s(t-a)\xleftrightarrow{L}e^{-as}X(s),a>0$$

Therefore, we obtain the following signal in time domain:

$$x(t)=e^{-2t}u_s(t)-e^{-2(t-4)}u_s(t-4)$$

2.

$$\dddot{y}(t)+0.2\ddot{y}(t)+\dot{y}(t)=\dot{x}(t)+2x(t-4)$$

Taking the Laplace transform of the above equation:

$$Y(s)\left(s^3+0.2s^2+s\right)=X(s)\left(s+2e^{-4s}\right)$$

We can write the input as below:

$$x(t)=\begin{cases}0 & t<0\\ e^{-t} & 0\le t<2\\ e^{-2} & t\ge 2\end{cases}.$$

$$
\begin{aligned}
x(t) &= e^{-t}\left(u_s(t)-u_s(t-2)\right)+e^{-2}u_s(t-2) \\
&= e^{-t}u_s(t)-e^{-2}e^{-(t-2)}u_s(t-2)+e^{-2}u_s(t-2)
\end{aligned}
$$

Taking the Laplace transform of $x(t)$:

$$X(s)=\frac{1}{s+1}-e^{-2}\frac{e^{-2s}}{s+1}+e^{-2}\frac{e^{-2s}}{s}=\frac{s+e^{-2(s+1)}}{s(s+1)}$$

$Y(s)$ can then be obtained as:

$$Y(s)=\frac{\left(s+2e^{-4s}\right)\left(s+e^{-2(s+1)}\right)}{s^2\left(s^2+0.2s+1\right)(s+1)}$$

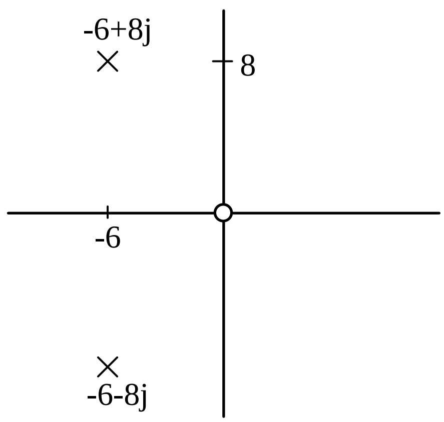
3. A system has the transfer function

$$H(s)=\frac{100s}{(s^2+12s+100)}=\frac{s}{\frac{1}{100}s^2+\frac{12}{100}s+1}.$$

(a) There is one zero at the origin. In addition, we have two complex poles, which are:

$$\left[-\frac{12}{100}\pm\sqrt{\left(\frac{12}{100}\right)^2-\frac{4}{100}}\right]\Bigg/\frac{2}{100}=-6\pm 8j$$

The pole-zero plot of $H(s)$ is depicted below:



(b) The d.c. gain of the system.

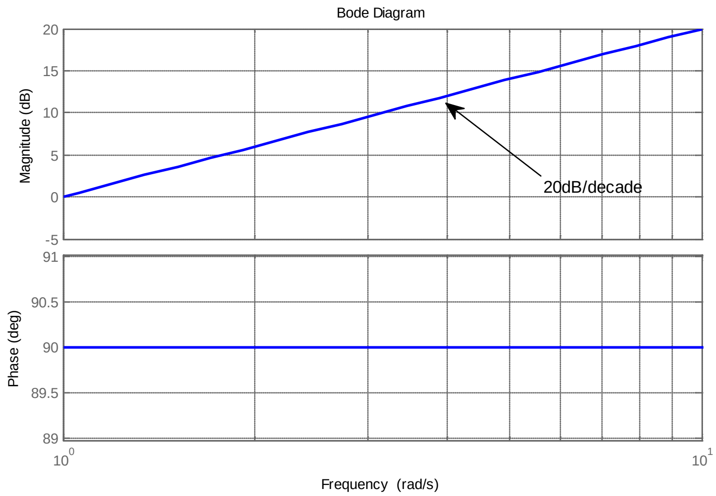
$$H(0)=\frac{0}{(0+6-8j)(0+6-8j)}=0$$

(c) Sketch the Bode approximations of the magnitude and angle of $H(j\omega)$:

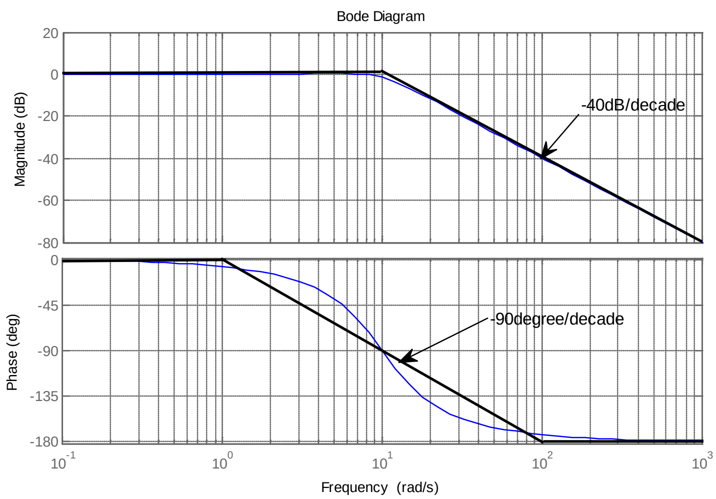
$$H(j\omega)=\frac{j\omega}{\frac{1}{100}(j\omega)^2+\frac{12}{100}(j\omega)+1}$$

$$\Rightarrow\ 20\log_{10}|H(j\omega)|=20\log_{10}|j\omega|-20\log_{10}\left|\tfrac{1}{100}(j\omega)^2+\tfrac{12}{100}(j\omega)+1\right|$$

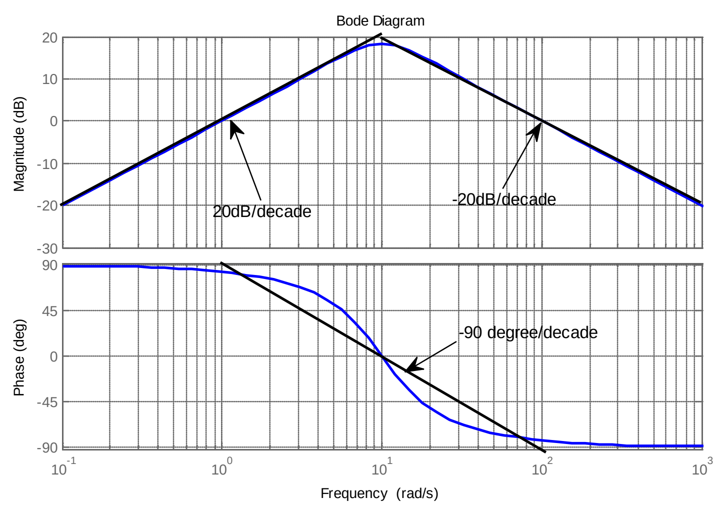
-For the numerator: $20\log_{10}\left|j\omega\right|$,



-For the denominator: $20\log_{10}\left|\frac{1}{100}(j\omega)^2+\frac{12}{100}(j\omega)+1\right|$,



- For the whole transfer function:



(d) Observe from the Bode plot at *w*=10 rad/s, we achieve a magnitude gain of 20dB (a gain factor of 10) and a phase of 0 degrees. Hence, the forced response is

$$y(t) = 10 \cdot 0.25 \cdot \cos(10t + 0) = 2.5\cos(10t).$$

(e) At *w*=10 rad/s, we have $s = 10j$. It then follows that:

$$H(10j) = \frac{100 \times 10j}{(10j)^2 + 12(10j) + 100} = \frac{25}{3}\angle 0.$$

The forced response can then be expressed as:

$$y(t) = \frac{25}{3} \cdot 0.25 \cdot \cos(10t + 0) = 2.08\cos(10t).$$