Signals and Systems 28 November 2012
Fall 2012 Quiz: Wednesday 5 December 2012

Homework Assignment #15

1. Find the inverse Laplace transform of each of the following functions.

   (a) $X(s) = \dfrac{3s+2}{s^2+6s+25}$

   (b) $X(s) = \dfrac{1-e^{-4s}}{s+2}$

2. A system governed by the differential equation

$$\dddot{y}(t) + 0.2\ddot{y}(t) + \dot{y}(t) = \dot{x}(t) + 2x(t-4)$$

   is initially at rest. The input of the system is

$$x(t) = \begin{cases} 0 & t < 0 \\ e^{-t} & 0 \le t < 2 \\ e^{-2} & t \ge 2 \end{cases}.$$

   Find the Laplace transform of the input $x(t)$ and of the output $y(t)$; *i.e.*, find $X(s)$ and $Y(s)$.

3. A system has the transfer function

$$H(s) = \frac{100s}{(s^2+12s+100)}.$$

   (a) Sketch the pole-zero plot of $H(s)$.

   (b) Determine the d.c. gain of the system.

   (c) Sketch the Bode approximations of the magnitude and angle of $H(j\omega)$. Show your work in plotting the contribution of each factor.

   (d) Use the Bode plots from part (b) to find the (approximate) forced response if the input to the system is

$$x(t) = 0.25 \cdot \cos(10t)$$

   (e) Use the exact value of the frequency response (not the Bode approximation) to determine the exact value of the forced response if the input is the same as in part (d).