Signals and Systems
Fall 2012

Solution to Homework Assignment #6

1. Find the z transform of each of the following sequences.

(a) $f(k) = 2\delta(k) - \delta(k-2) = \begin{cases} 2, & k=0 \\ -1, & k=2 \\ 0 & \text{otherwise} \end{cases}$

We then obtain the z-transform as follows

$$\boxed{F(z) = 2 - \frac{1}{z^2} = \frac{2z^2-1}{z^2} \text{ for } |z| \neq 0}$$

(b) $f(k) = \begin{cases} 0, & k<0 \\ 1, & 0 \le k \le 5 \\ 2^{k-5}, & k \ge 5 \end{cases}.$

$\Rightarrow f(k) = u_s(k) - u_s(k-5) + 2^{k-5}u_s(k-5)$

The z-transform can then be obtained as,

$$\boxed{F(z) = \frac{z}{z-1} - \frac{1}{z^5}\times\frac{z}{z-1} + \frac{1}{z^5}\times\frac{z}{z-2} = \frac{z^6-2z^5+1}{z^4(z-1)(z-2)} \text{ for } |z|>2.}$$

(c) $f(k) = \{2,2,0,-1,-1,-1,\underline{2,2,0,-1,-1,-1,2,2,0,-1,-1,-1},\cdots\}$.

[Hint: Express $f(k)$ as the sum of shifted, identical, finite sequences.]

Let $g(k) = \{2,2,0,-1,-1,-1,\ \underline{0,0,0,0,0,0,0,0,0,0,0,0}\cdots\}$.

Then,

$$f(k) = g(k) + \frac{1}{E^6}\{g(k)\} + \frac{1}{E^{12}}\{g(k)\} + \frac{1}{E^{18}}\{g(k)\} + \cdots$$

It then follows that,

$$F(z) = G(z) + \frac{1}{z^6}G(z) + \frac{1}{z^{12}}G(z) + \frac{1}{z^{18}}G(z) + \cdots$$
$$= G(z) = G(z)\sum_{m=0}^{\infty}\frac{1}{(z^6)^m} = G(z)\sum_{m=0}^{\infty}\left(\frac{1}{z^6}\right)^m = G(z)\frac{1}{1-\frac{1}{z^6}}$$

$$\Rightarrow F(z) = G(z)\frac{z^6}{z^6-1}$$

where $G(z) = 2 + \frac{2}{z} - \frac{1}{z^3} - \frac{1}{z^4} - \frac{1}{z^5} = \frac{2z^5+2z^4-z^2-z-1}{z^5}$ for $|z| \neq 0$

Finally, $\boxed{F(z)=\dfrac{z\left(2z^5+2z^4-z^2-z-1\right)}{z^6-1}\ \text{ for } |z|>1}$.

(d) $f(k)=\begin{cases}2, & k \text{ odd}\\ 0, & k \text{ even}\\ 0, & k\le 0\end{cases}$.

$\Rightarrow\ f(k)=\{0,2,0,2,0,2,\ldots\}=u_s(k)-(-1)^k u_s(k)$

It is easy to see that:

$$\boxed{F(z)=\frac{1}{z-1}-\frac{1}{z+1}=\frac{(z+1)-(z-1)}{(z+1)(z-1)}=\frac{2}{(z+1)(z-1)}\ \text{ for } |z|>1}$$

(e) $f(k)=a^{k-1}u_s(k-1)+\dfrac{1}{a}\delta(k)$

We have:

$$F(z)=\frac{1}{z}\times\frac{z}{z-a}+\frac{1}{a}=\frac{1}{z-a}+\frac{1}{a}=\frac{1}{z-a}+\frac{(1/a)(z-a)}{(z-a)}\ \text{ for } |z|>|a|.$$

$$\boxed{\Rightarrow F(z)=\frac{1}{a}\times\frac{z}{z-a}\ \text{ for } |z|>|a|}$$

2. Find the z transform of each of the following sequences.

(a) $f(k)=(0.5)^k\cos(k\frac{\pi}{4})u_s(k)$.

Using the z-transform table, we have

$$\boxed{F(z)=\frac{z\left(z-\dfrac{\sqrt{2}}{4}\right)}{z^2-\dfrac{\sqrt{2}}{2}z+\dfrac{1}{4}}}$$

(b) $f(k)=(0.5)^{(k-2)}\cos((k-2)\frac{\pi}{4})u_s(k-2)$.

$$\boxed{F(z)=\frac{1}{z^2}\times\frac{\left(z-\dfrac{\sqrt{2}}{4}\right)}{z^2-\dfrac{\sqrt{2}}{2}z+\dfrac{1}{4}}=\frac{\left(z-\dfrac{\sqrt{2}}{4}\right)}{z\left(z^2-\dfrac{\sqrt{2}}{2}z+\dfrac{1}{4}\right)}}$$

(c) $f(k) = (0.5)^k \cos(k\frac{\pi}{4})u_s(k-2)$.

We have: $\cos\left(k\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2} - k\frac{\pi}{4}\right) = -\sin\left(\left(k-2\right)k\frac{\pi}{4}\right)$

It follows then:

$\Rightarrow f(k) = -(0.5)^2(0.5)^{k-2}\sin((k-2)\frac{\pi}{4})u_s(k-2)$

Taking the z-transform, yields

$$\boxed{F(z) = -\frac{1}{4}\times\frac{1}{z^2}\times\frac{z\times 0.5\times\sin\left(\pi/4\right)}{z^2 - \left(2\times 0.5\times\cos\left(\pi/4\right)\right)z + 0.5^2} = \frac{-\sqrt{2}}{16}\times\frac{1}{z\left(z^2 - \frac{\sqrt{2}}{2}z + \frac{1}{4}\right)}}$$

(d) $f(k) = [(0.2)^k + (-2)^{k-1}]u_s(k)$.

$$\begin{aligned} f(k) &= (0.2)^k u_s(k) + (-2)^{k-1}u_s(k) \\ &= (0.2)^k u_s(k) - (0.5)(-2)^k u_s(k) \end{aligned}$$

$$\boxed{F(z) = \frac{z}{z-0.2} - 0.5\frac{z}{z+2} = \frac{z(0.5z+2.1)}{(z-0.2)(z+2)} \text{ for } |z| > 2}$$

(e) $f(k) = (0.2)^k u_s(k) + (-2)^{k-1}u_s(k-1)$.

$$\boxed{F(z) = \frac{z}{z-0.2} + \frac{1}{z+2} = \frac{z^2 + 3z - 0.2}{(z-0.2)(z+2)} \text{ for } |z| > 2}$$