Signals and Systems
Fall 2012

Solution to Homework Assignment #1:

1.

$$\frac{Y(s)}{X(s)} = \frac{2.5s}{3s^3 + 2s^2 - 4s + 1}$$

$$\Rightarrow 3s^3 Y(s) + 2s^2 Y(s) - 4sY(s) + Y(s) = 2.5sX(s)$$

By taking the inverse Laplace transform of the above equation, we obtain:

$$3\dddot{y}(t) + 2\ddot{y}(t) - 4\dot{y}(t) + y(t) = 2.5\dot{x}(t)$$

2. The input and output of the considered system are related by:

$$\dot{y}(t) + 3y(t) = x(t) - \int_0^t x(\tau)d\tau$$

By taking the Laplace transform, we have

$$sY(s) - \underbrace{y(0)}_{=0} + 3Y(s) = X(s) - \frac{X(s)}{s}$$

$$\Rightarrow (s+3)Y(s) = \frac{(s-1)}{s} X(s)$$

It then follows that:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{(s-1)}{s^2 + 3s}$$

3.

$$\ddot{y}(t) = 2\dot{y}(t) - y(t) + 2u(t)$$

$$\Rightarrow \ddot{y}(t) - 2\dot{y}(t) + y(t) = 2u(t) \quad (*)$$

We first consider the following problem:

$$\ddot{y}(t) - 2\dot{y}(t) + y(t) = 0 \quad (**)$$

Observe that the corresponding characteristic equation is $k^2 - 2k + 1 = 0$, giving the double root $k = 1$. Hence, the solution of (**) has the following form:

$$y_H = C_1 e^t + C_2 t e^t$$

In addition, it can be seen that the right-hand side (RHS) of (*) gives a particular solution as follows:

$$y_P = 2,\ t \geq 0$$

Therefore, the complete solution to (*) can be expressed as:

$$y = y_H + y_P = C_1 e^t + C_2 t e^t + 2,\ t \geq 0$$

Now, by examining the initial conditions, we have

- $y(0) = 0 \ \Leftrightarrow\ C_1 + 2 = 0 \ \Rightarrow\ C_1 = -2.$

Furthermore, we have:

- $\dot{y}(t) = C_1 e^t + C_2 \left(e^t + t e^t \right)$

$$\Rightarrow \dot{y}(0) = 0 \ \Leftrightarrow\ C_1 + C_2 = 0 \ \Rightarrow C_2 = 2$$

It then follows that $\quad y = -2e^t + 2te^t + 2,\ t \geq 0.$

<u>Different approach:</u>

The above solution can also be obtained using Laplace transform as follows:

By taking the Laplace transform of (*), we have

$$s^2 Y(s) - \underbrace{sy(0)}_{=0} - \underbrace{\dot{y}(0)}_{=0} - 2\left[sY(s) - \underbrace{y(0)}_{=0} \right] + 2s^2 Y(s) + Y(s) = 2U(s)$$

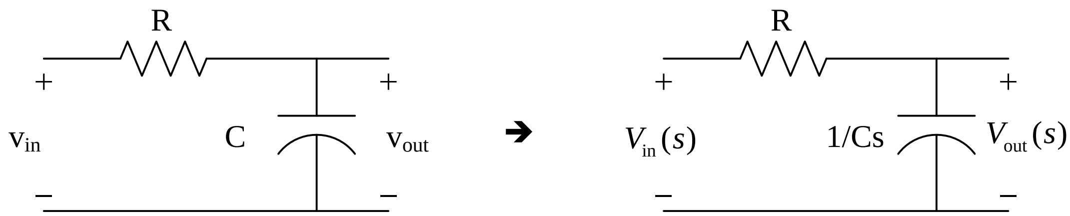
$$\Rightarrow \left(s^2 - 2s + 1 \right) Y(s) = 2 \times \frac{1}{s}$$

$$\Rightarrow Y(s) = \frac{2}{s\left(s-1\right)^2} = \frac{2}{s} - \frac{2}{s-1} + \frac{2}{\left(s-1\right)^2}$$

Now, applying the inverse Laplace transform yields:

$$y = -2e^t + 2te^t + 2,\ t \geq 0.$$

4.



$$\Rightarrow \frac{V_{out}(s)}{V_{in}(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{RCs + 1}$$

Let $s \to j\omega$. The frequency response transfer function can be expressed as :

$$H(j\omega) = \frac{1}{j\omega RC + 1}$$

The corresponding magnitude can then be calculated as:

$$\left|H(j\omega)\right| = \left|\frac{1}{j\omega RC + 1}\right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}}$$

Bode Plot with RC=0.5: