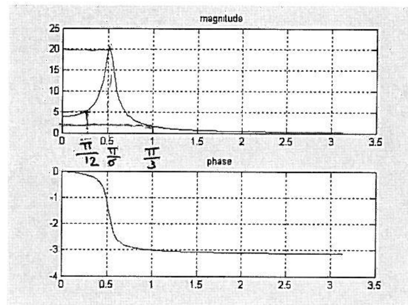
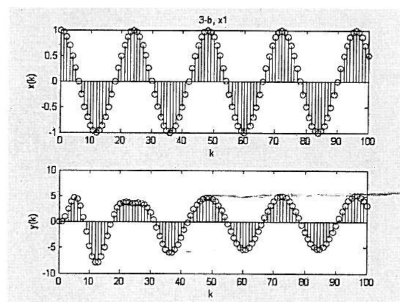
Problem One(1)

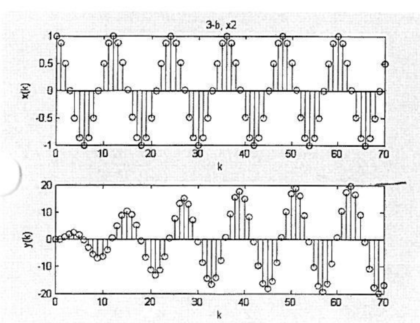
```
% problem (3-a)

w = 0:pi/120:pi;                    % create 120 points to evaluate H(ejw).
h = exp(j*w)./((exp(j*w)).^2 - 2*0.95*cos(pi/6)*(exp(j*w)) + 0.95^2);
subplot(2,1,1)
plot(w,abs(h))
title (' magnitude')
grid on
subplot(2,1,2)
plot(w,angle(h))
title (' phase')
grid on
```



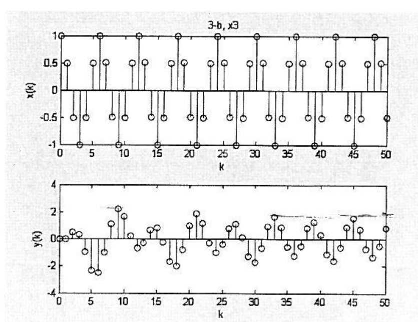

```
% matlab code for part (b) problem 3

N = 50;
k = [0:N];
a= pi/3;  % for x2 a=pi/6 , for x1 a = pi/12
x = cos(a.*k);
y=0*k;
for i = 1:N-1
   y(i+2)=2*0.95*cos(pi/6)*y(i+1)-(0.95^2)*y(i) + x(i+1);
end
subplot(2,1,1)
stem(k,x)
xlabel('k')
ylabel('x(k)')
title('3-b, x3')
subplot(2,1,2)
stem(k,y)
xlabel('k')
ylabel('y(k)'
```



amplitude ≈ 5 also the same as in the freq. response.



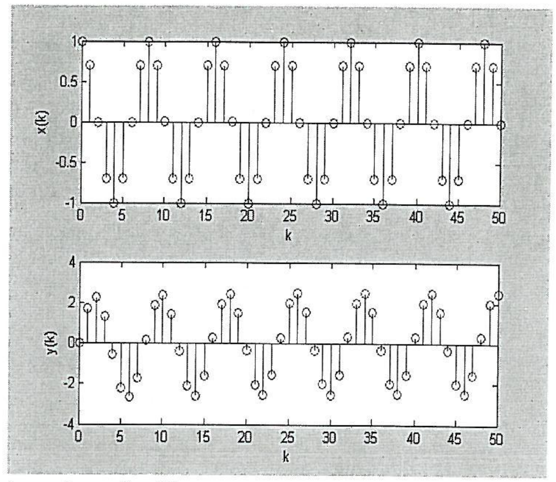
amplitude≈20 same as freq. response



amplitude ≈ 2 same as freq. response.

Problem 2 (Continue from Last page)

```
% matlab code for part (b)

N = 50;
k = [0:N];
a= pi/4;
x = cos(a.*k);
y=0*k;
for i = 1:N
    y(i+1)=0.9*y(i) + x(i+1) + x(i);
end
subplot(2,1,1)
stem(k,x)
xlabel('k')
ylabel('x(k)')
subplot(2,1,2)
stem(k,y)
xlabel('k')
ylabel('y(k)')
```

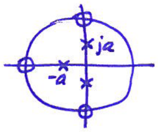


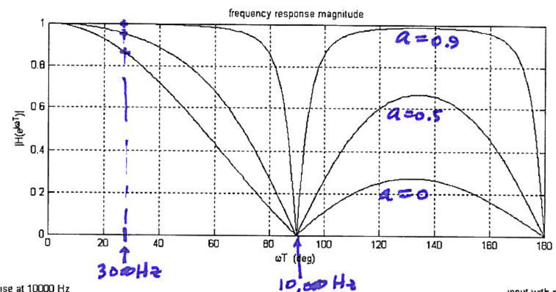
input is cos(kπ/4)

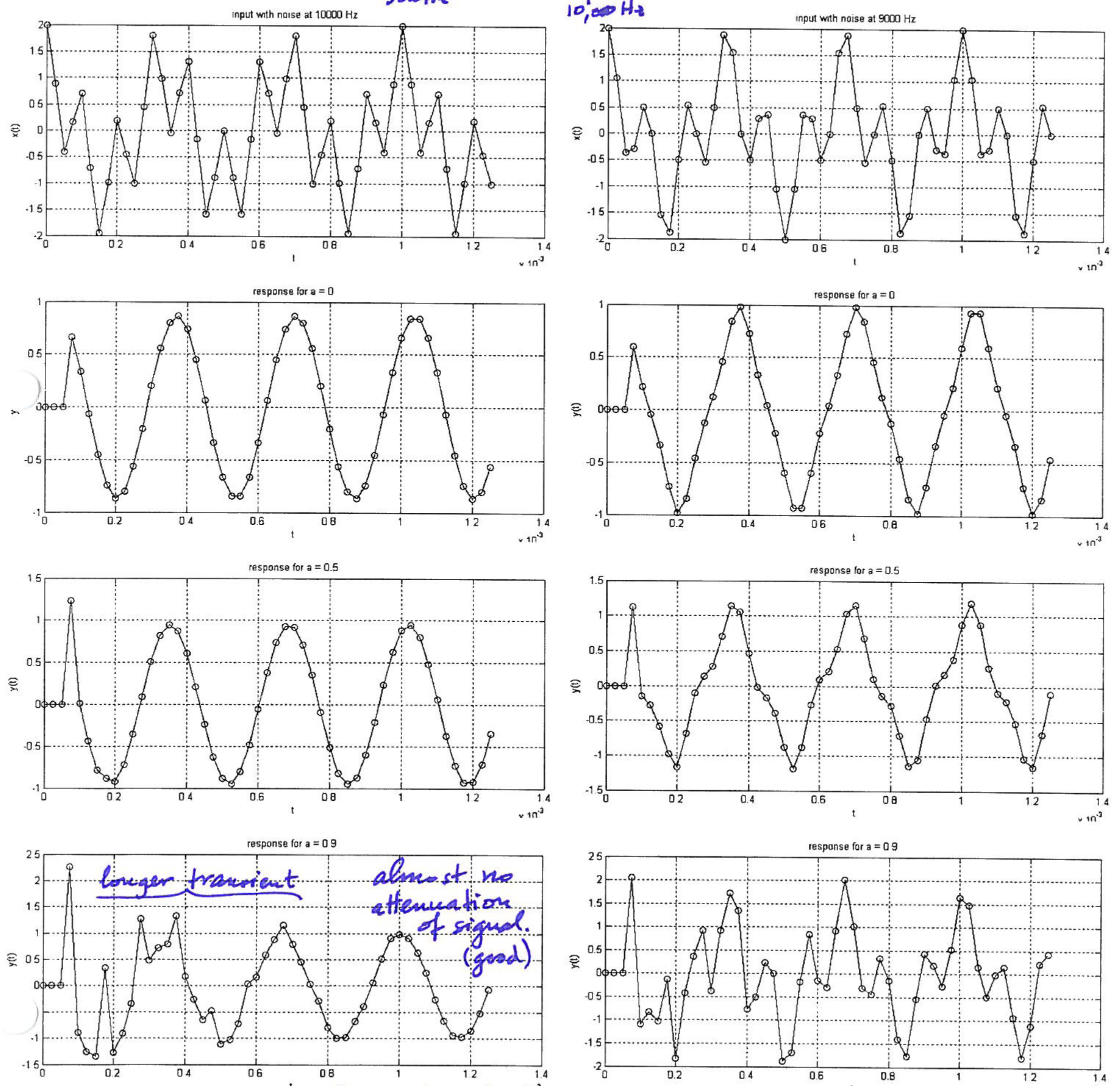
The Maximum amplitude after steady state is ≈ 2.5 and This can be seen from Freq. response.

Problem Two(2)

$$H(z) = \frac{(1+a^2)(1+a)}{4} \cdot \frac{(z^2+1)(z+1)}{(z^2+a^2)(z+a)}$$







All three filters eliminate the noise at 10,000 Hz.

The narrower the "notch," the more the 9000-Hz noise passes through the filter.

TO FILTER 10,000 Hz & 20,000 Hz The ZEROS ARE ON the UNIT CIRCLE

NOW DETERMINE ωT:

$$\omega = 2\pi f$$

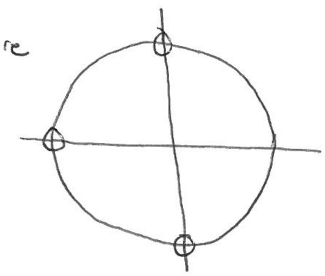
so

$$2\pi f_1 T = 2\pi\, 10{,}000\,(.000025) = 1.57 = \pi/2$$

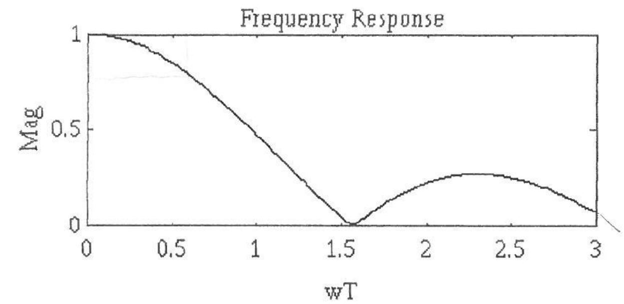
$$2\pi f_2 T = 2\pi\, 20{,}000\,(.000025) = 3.14 = \pi$$

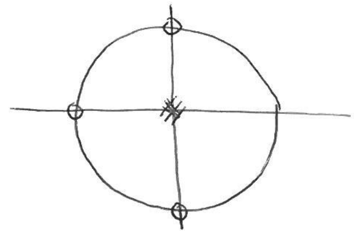
Therefore

THE ZERO ARE AS SHOWN

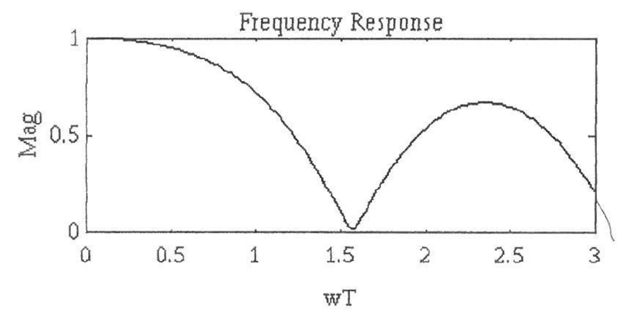


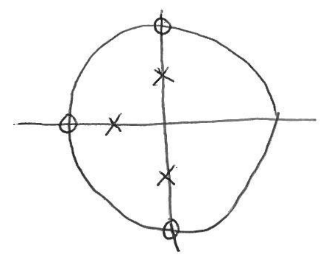
Homework #12 problem 1
Part a) Frequency response curves
For a = 0



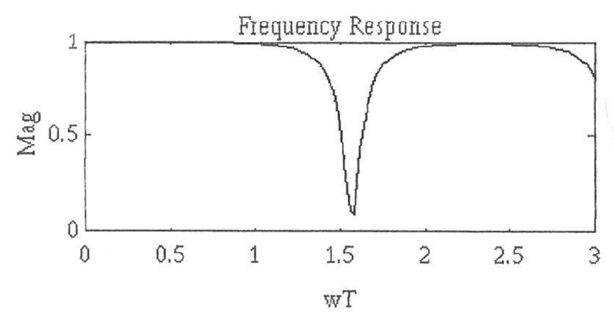


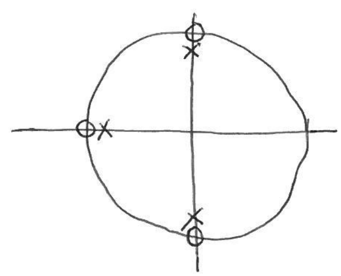
For a = .5



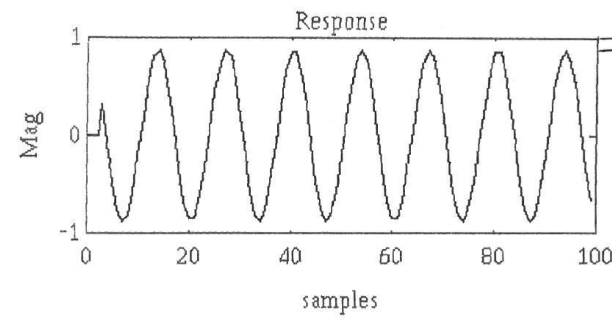


For a = .9



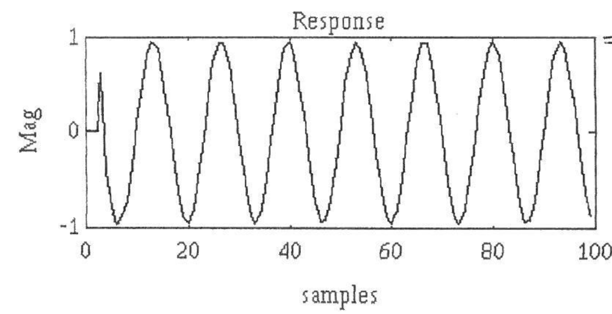


For part b) response to x(k)=cos(2*pi*3000*k*T)+cos(2*pi*10000*k*T);
For a = 0



ATTENUATES SIGNAL

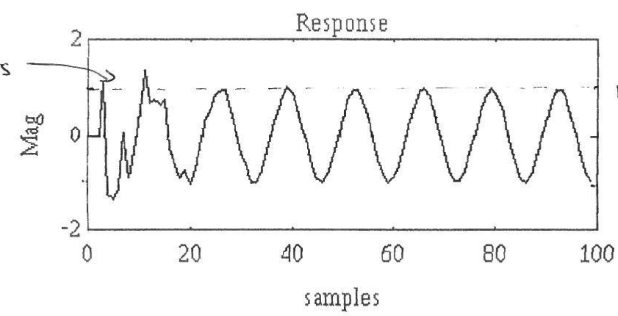
For a = .5



VERY SMALL ATTENUATION

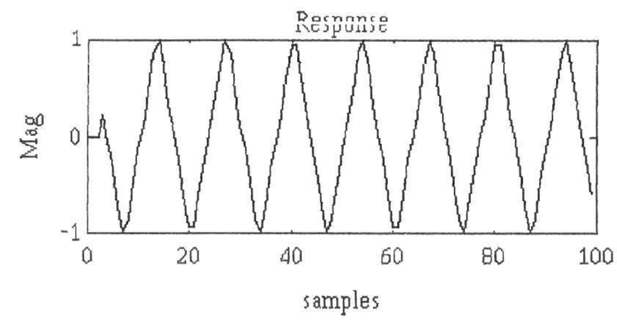
For a = .9

START UP TRANSIENTS



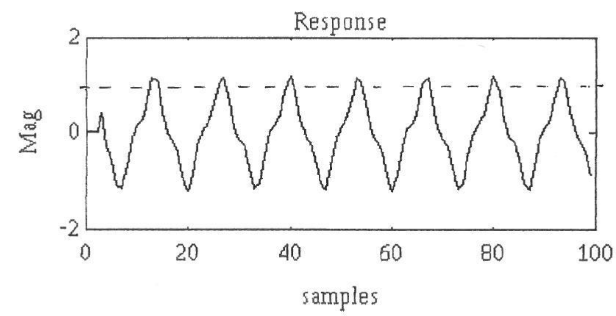
NEAR NO ATTENUATION

For part c) response to x(k)=cos(2*pi*3000*k*T)+cos(2*pi*9000*k*T);
For a = 0



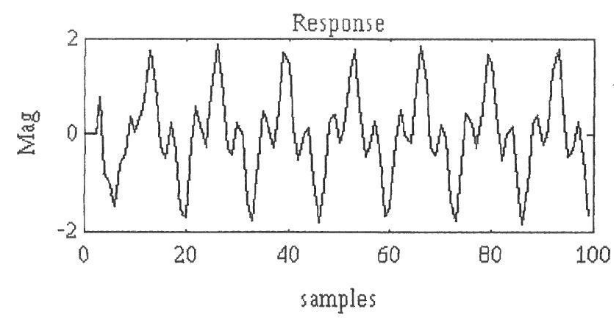
SOME DISTORTION

For a = .5



DISTORTION WITH
NOISE AMPLIFICATION

For a = .9



THIS IS A MESS.

Problem Three(3)

1. (a) $A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$ $\quad \det(\lambda I - A) = \det\begin{pmatrix} \lambda-1 & -1 \\ 0 & \lambda-2 \end{pmatrix} = (\lambda-1)(\lambda-2)$

eigenvalues are $\boxed{\lambda_1 = 1,\ \lambda_2 = 2}$

eigenvectors: $(\lambda_1 I - A)v_1 = 0$

$\begin{pmatrix} 0 & -1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow b = 0 \Rightarrow \boxed{v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}}$

$(\lambda_2 I - A)v_2 = 0$

$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow a = b \Rightarrow \boxed{v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

$A^{-1} = \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix}$ eigenvalues are $\boxed{\lambda_1 = 1,\ \lambda_2 = \frac{1}{2}}$ (reciprocals of the eigenvalues of A)

eigenvectors: $(\lambda_1 I - A)v_1 = 0 \Rightarrow \boxed{v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}}$

$(\lambda_2 I - A)v_2 = 0 \Rightarrow \boxed{v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$ (same as the eigenvectors of A)

(b) $A = \begin{pmatrix} 2 & 1 \\ 6 & 3 \end{pmatrix}$ $\quad \det(\lambda I - A) = \det\begin{pmatrix} \lambda-2 & -1 \\ -6 & \lambda-3 \end{pmatrix} = \lambda^2 - 5\lambda = \lambda(\lambda-5)$

eigenvalues are $\boxed{\lambda_1 = 0,\ \lambda_2 = 5}$

eigenvectors: $\begin{pmatrix} -2 & -1 \\ -6 & -3 \end{pmatrix} v_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \boxed{v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}}$

$\begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} v_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \boxed{v_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}}$

A^{-1} does not exist. $\boxed{(\text{rank}(A) = 1.)}$

(c) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{pmatrix}$ $\quad \det(\lambda I - A) = \det\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 0 & 2 & \lambda+3 \end{pmatrix} = \lambda(\lambda^2 + 3\lambda + 2)$

$= \lambda(\lambda+1)(\lambda+2)$

eigenvalues are $\boxed{\lambda_1 = 0,\ \lambda_2 = -1,\ \lambda_3 = -2}$

eigenvectors: $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 2 & 3 \end{pmatrix} v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{pmatrix} v_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow v_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} -2 & -1 & 0 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{pmatrix} v_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow v_3 = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$

A^{-1} does not exist.

$\boxed{\text{rank}(A) = 2}$

Problem Four(4)

1/3

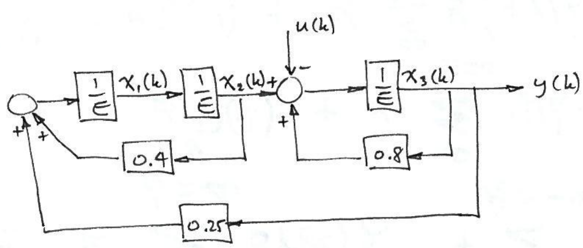
3. (a) $x_1(k)$ = "young" cows

$x_2(k)$ = "medium" cows

$x_3(k)$ = "old" cows $\qquad u(k)$ = # sold.

$$\boxed{\begin{aligned} x_1(k+1) &= \tfrac{1}{2}\left[0.8\, x_2(k) + 0.5\, x_3(k)\right] \\ x_2(k+1) &= x_1(k) \\ x_3(k+1) &= x_2(k) + 0.8\, x_3(k) - u(k) \end{aligned}}$$

$$\left\{ \begin{array}{ll} A = \begin{bmatrix} 0 & 0.4 & 0.25 \\ 1 & 0 & 0 \\ 0 & 1 & 0.8 \end{bmatrix} & B = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} & D = 0. \end{array} \right.$$

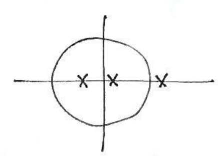


(b) $H(E) = C(EI - A)^{-1}B = \dfrac{-(E^2 - 0.4)}{E^3 - 0.8E^2 - 0.4E + 0.07}$

Eigenvalues of A: $\lambda_1 = 0.1419$

$\lambda_2 = -0.4466$

$\lambda_3 = 1.1047$



The system is not stable. The farmer would not want a stable system. If it were stable, its state variables would all decay to zero.