# Entanglement and violation of CHSC Bell's inequality

**Edy Alberto Flores Leal**[1]

[1]KTH Royal Institute of Technology, Stockholm, Sweden

**Abstract.** We tested Bell's inequality by generating entangled photon pairs via spontaneous parametric down conversion (SPDC) using the qutools quED Entanglement Demonstrator. First, we characterized the source with nonentangled photon pairs, measuring visibilities in the horizontal and diagonal bases. We then repeated the measurements for entangled photon pairs in the $|\Phi^+\rangle$ state. Setting sixteen polarization combinations, we obtained a correlation parameter of $|S| = 2.34 \pm 0.21$, confirming the violation of Bell's inequality.

## 1. Introduction

Entanglement is one of the most bizarre features of quantum mechanics. Its counterintuitive nature was questioned even by Einstein, who, along with Podolsky and Rosen, doubted the completeness of quantum mechanics because of it. Their concern arose from the the apparent violation of locality, as entanglement seemed to allow information to travel faster than light. This led to their 1935 paper "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," where they proposed the existence of hidden variables to reconcile with the theory [1].

However, it was not until 1964 that John Bell demonstrated entanglement to be an inherently nonlocal phenomenon [2]. Bell's work led to the formulation of Bell's inequality, a crucial test for the incomptatibility of quantum mechanics and local hidden-variable theories. Several versions of Bell's inequality exist, but one of the most widely used to test Bell's theorem is the Clauser–Horne–Shimony–Holt (CHSH) inequality [3]:

$$\begin{aligned} |S(\alpha,\alpha',\beta,\beta')| = |&E(\alpha,\beta) + E(\alpha',\beta) \\ &+ E(\alpha,\beta') + E(\alpha',\beta')| \leq 2. \end{aligned} \tag{1}$$

Here, $\alpha$ ($\alpha'$) and $\beta$ ($\beta'$) denote the orientations of polarizing filters placed along the two arms of the source. Also, $E(\alpha,\beta)$ is the quantum correlation, defined as the expectation value of the product of the experimental "outcomes" and are determined from coincidence counts $C(\alpha,\beta)$ in the following fashion:

$$E(\alpha,\beta) = \frac{C(\alpha,\beta) - C(\alpha,\beta_\perp) + C(\alpha_\perp,\beta) + C(\alpha_\perp,\beta_\perp)}{C(\alpha,\beta) + C(\alpha,\beta_\perp) + C(\alpha_\perp,\beta) + C(\alpha_\perp,\beta_\perp)}. \tag{2}$$

The parameters $\alpha_\perp$ and $\beta_\perp$ correspond to rotations of $\alpha$ and $\beta$ by $+90°$, respectively. In this report, we experimentally tested Bell's inequality and its violation using the qutools quED Entanglement Demonstrator device. Specifically, we measured the visibility of both nonentangled and entangled photon pairs and, finally, determined the experimental value of the $S$ parameter. Our results confirmed the violation of Bell's inequality, proving the nonlocality of quantum mechanics.

## 2. Experiment

### 2.1. Experimental setup

For this experiment, we used qutool's quED Entanglement Demonstrator. The quED setup consists of two main parts: the pump assembly and the down-conversion block [4]. The first part of the setup consists of a blue laser diode, mirrors, a half-wave plate (HWP), birefringent pre- and post-compensation crystals (YVO), and a nonlinear down-conversion crystal (made of barium borate or BBO). This device produces entangled photon pairs via a process known as spontaneous parametric down-conversion (SPDC) that occurs in the BBO.

The system employs two adjacent type-I phase-matching nonlinear crystals with orthogonal optical axes to generate polarization-entangled states. The photon pairs will have the same polarization as we employ type-I crystals. By pumping the crystals with linearly polarized light at $45°$ with respect to the horizontal and vertical direction, there will be an equal probability that the pump photon undergoes down-conversion in either crystal, producing the entangled state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 + \mathrm{e}^{\mathrm{i}\phi}|V\rangle_1|V\rangle_2\right), \tag{3}$$

where $|H\rangle$ denotes the horizontal polarization state, while $|V\rangle$ represents the vertical polarization state. The system ensures spatial and temporal indistinguishability of photon pairs using thin nonlinear crystals and single-mode fiber coupling to preserve the coherence. Additional birefringent crystals precompensate temporal delays caused by group-velocity mismatch and dispersion effects, ensuring high entanglement fidelity. Finally, the system detects the photon pairs employing silicon avalanche photodiodes inside the electronic control unit.

### 2.2. Experimental methods

The experiment began with the source characterization using nonentangled photon pairs. We measured the coincidence counts and visibility in both HV and DA bases. We estimated the visibility using the following expression:

$$V = \frac{C_\parallel - C_\perp}{C_\parallel + C_\perp}. \tag{4}$$

Here, $C_{\|}$ and $C_{\perp}$ represented the extrema of the coincidence count rates. To estimate the error $\Delta V$, we used the Gaussian error propagation rule given by

$$\Delta V = \sqrt{\left(\frac{\partial V}{\partial C_{\|}}\Delta C_{\|}\right)^2 + \left(\frac{\partial V}{\partial C_{\perp}}\Delta C_{\perp}\right)^2}, \tag{5}$$

where $\Delta C_{\|} = \sqrt{C_{\|}}$ and $\Delta C_{\perp} = \sqrt{C_{\perp}}$, and

$$\frac{\partial V}{\partial C_{\|}} = \frac{2C_{\perp}}{(C_{\|}+C_{\perp})^2}, \quad \frac{\partial V}{\partial C_{\perp}} = -\frac{2C_{\|}}{(C_{\|}+C_{\perp})^2}, \tag{6}$$

Using these parameters, we determined the experimental visibility, $V^{\text{exp}}_{\text{basis}} = V \pm \Delta V$. Then, one polarizer was fixed at the H state while the other was rotated in $5°$ steps to examine the behavior of the coincidence counts against the polarization.

After characterizing the source, we generated entangled photon pairs and prepared the Bell state $|\Phi^+\rangle$. This was achieved by inserting a HWP in front of the pump. Next, we repeated the previous coincidence count and visibility measurements. We recorded coincidence counts for sixteen polarization combinations, shown in table 1, and experimentally evaluated the $S$ parameter to test Bell's inequality.

**Table 1:** Different polarization combinations to measure the CHSH inequality

| $\alpha$ | $\beta$ | $\alpha$ | $\beta$ |
|---|---|---|---|
| 0° | 22.5° | 45° | 22.5° |
| 0° | 67.5° | 45° | 67.5° |
| 0° | 112.5° | 45° | 112.5° |
| 0° | 157.5° | 45° | 157.5° |
| 90° | 22.5° | 135° | 22.5° |
| 90° | 67.5° | 135° | 67.5° |
| 90° | 112.5° | 135° | 112.5° |
| 90° | 157.5° | 135° | 157.5° |

The $S$ parameter was obtained using Eq. (1), and its statistical error was computed using, once again, Gaussian error propagation, leading to the expression

$$\Delta E = \frac{2[C(\alpha,\beta)+C(\alpha_\perp,\beta_\perp)][C(\alpha,\beta_\perp)+C(\alpha_\perp,\beta)]}{[C(\alpha,\beta)+C(\alpha,\beta_\perp)+C(\alpha_\perp,\beta)+C(\alpha_\perp,\beta_\perp)]^2} \times \sqrt{\frac{1}{C(\alpha,\beta)+C(\alpha_\perp,\beta_\perp)} + \frac{1}{C(\alpha,\beta_\perp)+C(\alpha_\perp,\beta)}}. \tag{7}$$

Both expressions were automatically computed by the quED system once we set the sixteen polarization combinations.

## 3. Results

We began the experiment by characterizing the source. We measured the parallel and perpendicular coincidence counts without a HWP. The measurements were made in the HV and DA bases, as shown in Table 2:

**Table 2:** Coincidence counts for each $\alpha$ and $\beta$ polarization combinations in the HV and DA bases without HWP

| **Coin. counts** | HV basis | DA basis |
|---|---|---|
| $C_{\|}$ | 2763 | 982 |
| $C_{\perp}$ | 11 | 636 |

For the HV basis, we used the quantities

$$C_{\|} = C_{\text{HH}} + C_{\text{VV}}, \tag{8}$$
$$C_{\perp} = C_{\text{HV}} + C_{\text{VH}} \tag{9}$$

with polarization settings $\alpha = \beta = 90°$ and $\alpha = \beta = 0°$, respectively, while we used the quantities

$$C_{\|} = C_{\text{DD}} + C_{\text{AA}}, \tag{10}$$
$$C_{\perp} = C_{\text{DA}} + C_{\text{AD}} \tag{11}$$

for the DA basis and for the polarization configurations $\alpha = \beta = 45°$ and $\alpha = \beta = 135°$, respectively. Here, $\alpha$ denoted the polarization in the left arm, and $\beta$ for the polarization in right arm. Therefore, the experimental visibilities on the HV and DA bases without HWP were

$$V^{\text{exp}}_{\text{HV}} = 0.99 \pm 0.002, \tag{12}$$
$$V^{\text{exp}}_{\text{DA}} = 0.21 \pm 0.02. \tag{13}$$

The next step was fixing one polarizer to be in the H state and then rotating the second polarizer by steps of $5°$. We shown the observed behavior of the coincidence counts against the polarization in Fig. 1:

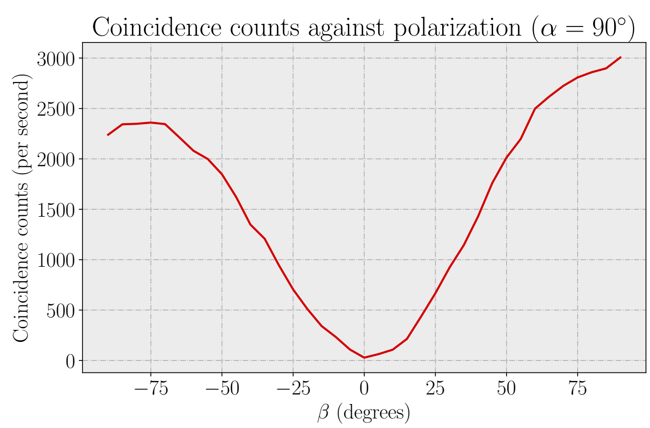


**Figure 1:** Coincidence counts against polarization in the right arm of the array. The plot exhibits a sinusoidal trend.

Once the source has been characterized, we proceeded to create the Bell states. We inserted the HWP in front of the pump and measured the coincidence rates in the HV and DA bases, as shown in Table 3:

**Table 3:** Coincidence counts for each $\alpha$ and $\beta$ polarization combinations in the HV and DA bases with HWP

| **Coin. counts** | HV basis | DA basis |
|---|---|---|
| $C_{\parallel}$ | 2766 | 2458 |
| $C_{\perp}$ | 63 | 356 |

Thus, the visibilities obtained in both bases when using a HWP were the following:

$$V_{\mathrm{HV}}^{\mathrm{exp}} = 0.96 \pm 0.01, \tag{14}$$
$$V_{\mathrm{DA}}^{\mathrm{exp}} = 0.75 \pm 0.01. \tag{15}$$

Finally, we recorded the coincidences for sixteen different polarizer combinations presented in Table 17. By doing so, we tested Bell's inequality and got the $|S|$ parameter

$$|S| = 2.34 \pm 0.21 \tag{16}$$

for a $\Delta t = 60$ s integration time. The interpretation of these results was discussed in the following section.

## 4. Discussion

The first part of the experiment consisted in the generation of unentangled photons. We obtained that the visibility for photons produced in the same polarization settings was higher in contrast to the photons produced in different polarizations, as expected. Next, we plotted the relation between the coincidence counts and the (right arm) polarization angle for a fixed $\alpha = 90°$. As known from the theory [4], the probability that one particle passes a polarizer under the angle $\alpha$ and a second particle passes a polarizer under the angle $\beta$ is given by

$$P(\alpha, \beta) = \frac{1}{2}\cos^2(\alpha - \beta). \tag{17}$$

It is relevant to mention that we usually measure the coincidence rate (as it has been done in this report) instead of probabilities. We notice that the maximum coincidence count was achieved when $\alpha \approx \beta$, as expected from Eq. (17). However, we noticed in Fig. 1 that we did not get a perfect $\cos^2$ shape.

The third and last part of the experiment dealt with the generation of entangled photon pairs. As stated before, we inserted a HWP and determined the visibilities in both bases. This time, we noticed that the measured visibility in the HV basis slightly decreased while the visibility in the DA basis increased significantly. We recall that the Bell state $|\Phi^+\rangle$ exhibits similar correlations between the HV and DA bases since

$$\begin{aligned} |\Phi^+\rangle &= \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle_1|\mathrm{H}\rangle_2 + |\mathrm{V}\rangle_1|\mathrm{V}\rangle_2) \\ &= \frac{1}{\sqrt{2}}(|\mathrm{D}\rangle_1|\mathrm{D}\rangle_2 + |\mathrm{A}\rangle_1|\mathrm{A}\rangle_2). \end{aligned} \tag{18}$$

Therefore, we could have expected similar results for the visibilities in the HV and DA basis. Finally, we obtained a correlation of $|S| = 2.34 \pm 0.21$. Since the statistical error falls in the range of the inequality $S > 2$, we confirmed the violation of the Bell inequality.

It is relevant to mention that there are several (possible) sources of errors that lead to the unsatisfactory experimental results. For instance, the wrong manipulation of the experimental apparatuses, i.e. polarization misalignments, could be one of the main error sources. Also, there are a couple environmental factors like background noise coming from other light sources. Finally, something that was not taken into account was the accidental coincidence rate. These factors could hinder the quality of the experimental measurements and further research is needed to identify their impact in the experimental outcomes.

## 5. Conclusions

In summary, we experimentally reproduced the Bell state $|\Phi^+\rangle$ with an $S$ value of $|S| = 2.34 \pm 0.21$ by recording the coincidence rate for sixteen polarization settings. We also measured the visibilities in the HV and DA bases for entangled and unentangled photon pairs, validating the theoretical expectations for each case. However, there are several sources of error that reduce the quality of the measurements. In our case, we did not consider the accidental count rate or human mistakes during the alignment. Further study is needed to identify theses sources and refine the experimental outcomes.

## References

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