

Matrix Tile Extension

Portable ISA for Vector-Integrated Matrix Units

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Scope and Principles

Provide a simple and portable way to use tensor units linked to vector units for performing the operations $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ and $\mathbf{C} = \mathbf{C} + \mathbf{A} \cdot \mathbf{B}$ where \mathbf{A} , \mathbf{B} , \mathbf{C} are dense matrices of the dimensions, respectively, $m \times k$, $k \times n$, $m \times n$.

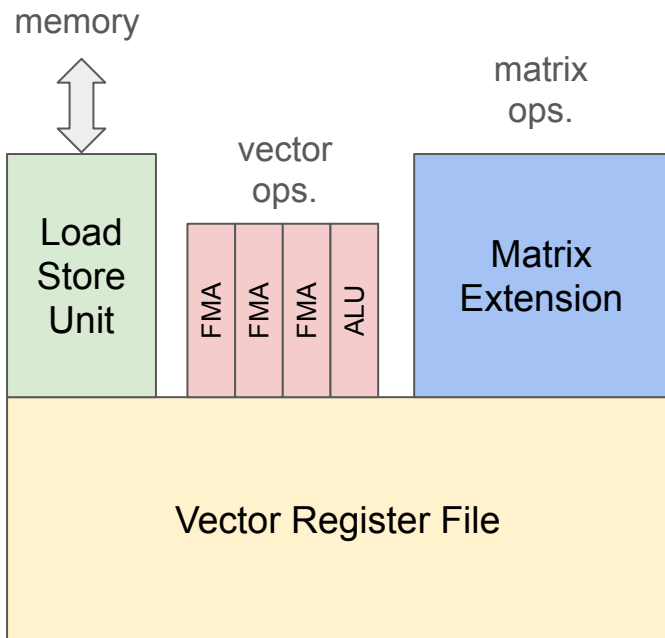
Portability

Few Commands

Simple Usage

The names of the instructions have been chosen to describe their function and are by no means “final”.

Accommodate Diversity



Short Vectors (e.g. VLEN=256)

- fp64: **2x2** or **4x1** matrix per VREG
- fp16: **4x4** or **8x2** matrix per VREG
- VRF: 4-8 read ports (?)
- 4 lanes (or less)

Long Vectors (e.g. VLEN=16384)

- fp64: **16x16**, **32x8**, **64x4**, **128x2**, **256x1** matrix
- fp16: **32x32**, **64x16**, **128x8**, **256x4**, **512x2**, **1024x1** matrix
- VRF: 5-16 read ports per lane, eg. 512 on SX-Aurora VE3
- 8 to 32 lanes
- many possible matrix multiplier geometries!

Square Tiles

fp64

$C = A \cdot B$

16×16



16×16



B



C

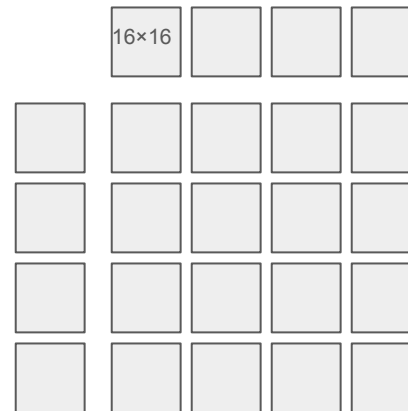
Arithmetic Intensity:

Load: $2 \cdot 256 = 512$

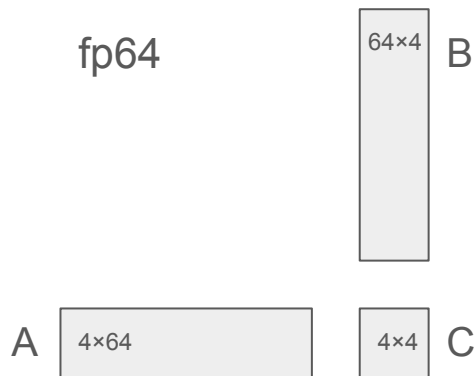
Matmult: $2 \cdot 16 \cdot 16 \cdot 16 = 8192$ ops

Arithmetic intensity = $8192 / 512 = 16$

With 4x4 accumulators: a.i. = 64



Narrow Tiles



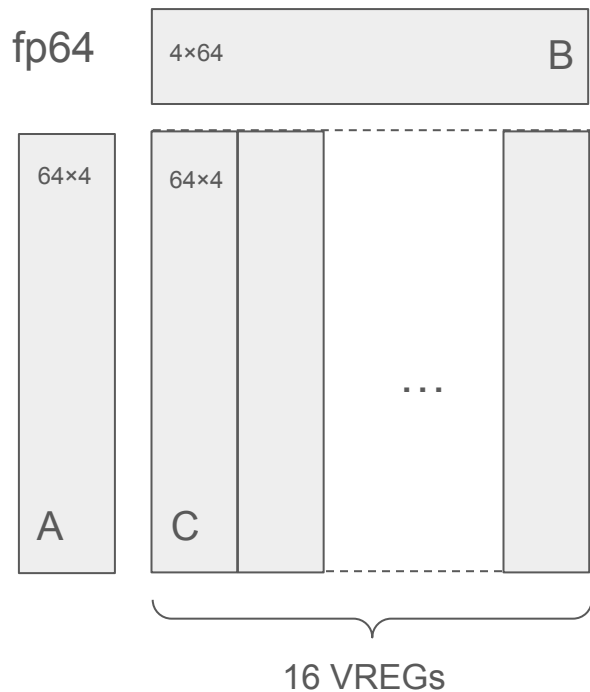
Arithmetic Intensity:

Load: $2 \cdot 4 \cdot 64 = 512$

Matmult: $2 \cdot 4 \cdot 4 \cdot 64 = 2048$ ops

Arithmetic intensity = $2048 / 512 = 4$

Narrow Tiles



Arithmetic Intensity:

Load: $64 \cdot 4 \cdot 2 = 512$

Matmult: $2 \cdot 64 \cdot 64 \cdot 4 = 32768$ ops

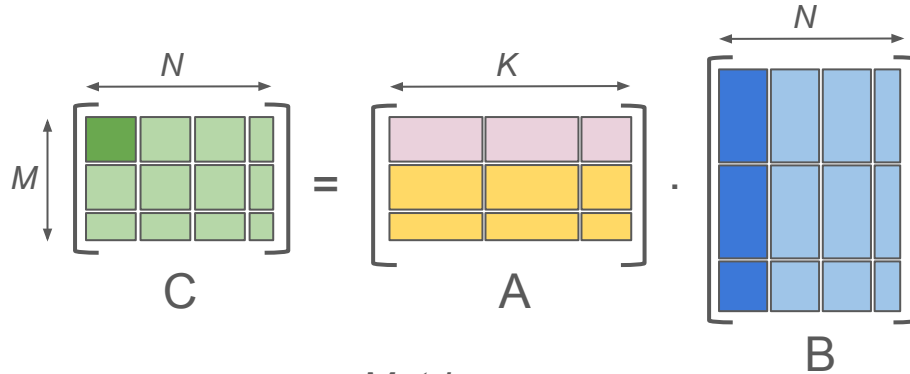
Arithmetic intensity = 64

A, B: 1 VREG each

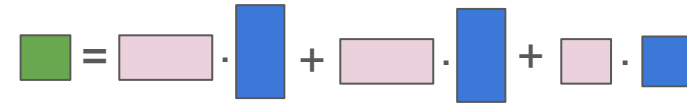
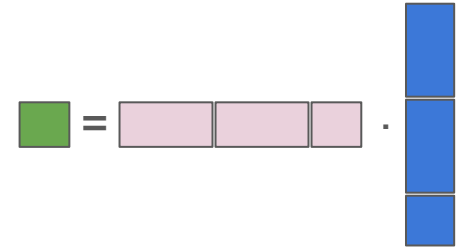
C: 16 VREGs

Fundamentals

Matrix - Tile



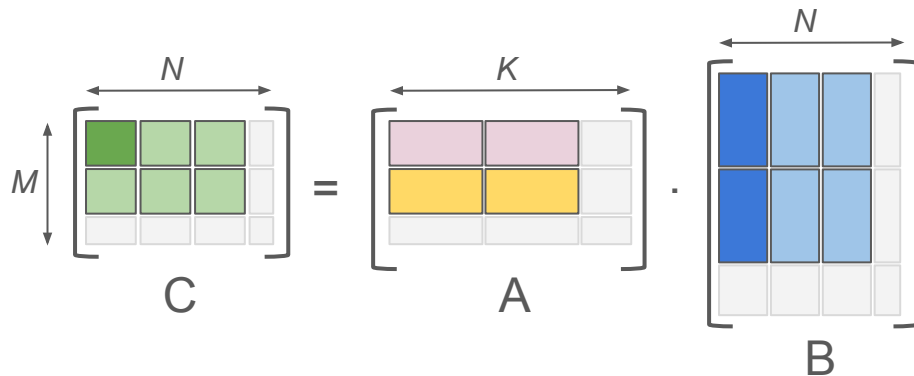
Matrices



Tiles

Multiplication of the large matrices $C = A \cdot B$ is decomposed into sums of their multiplied tiles. Tiles are multiplied and added by the tensor unit.

Matrix - Tile

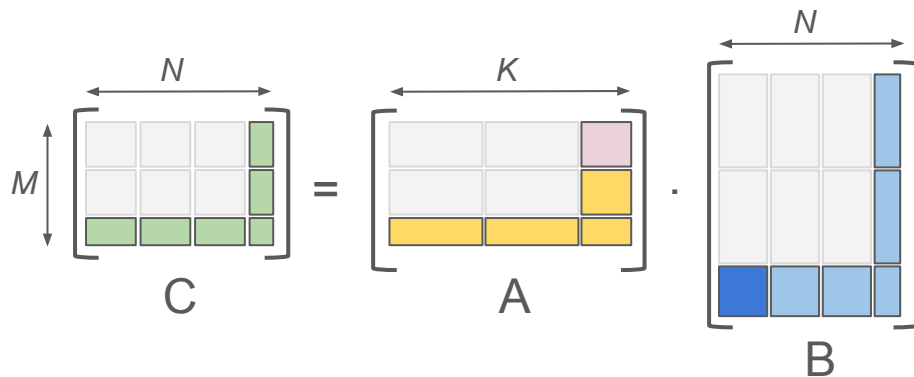


Maximum tile sizes, as supported by the tensor unit. Dimensions:

C-tiles: $m_{\max} \times n_{\max}$

A-tiles: $m_{\max} \times k_{\max}$

B-tiles: $k_{\max} \times n_{\max}$



Border tiles have smaller dimensions. Tensor units may or may not be able to operate on smaller tiles.

Tile - Matrix Unit - Vector Register

Matrix / tensor unit operates on tiles.

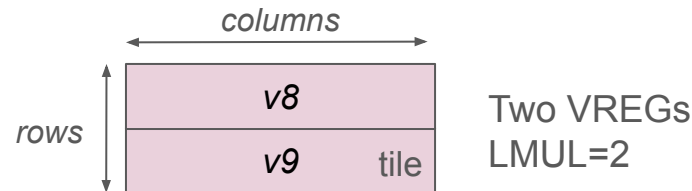
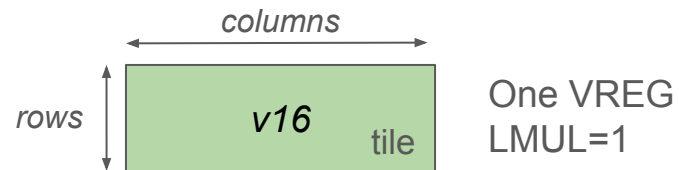
Tile dimensions are constrained by

- Tensor unit geometry: m_{\max} , k_{\max} , n_{\max}
 - May depend on SEW, data type
- Vector register size (VLEN, SEW)

Tiles are stored in vector registers.

- One or several (LMUL)
- Different tiles can have different LMUL

Matrix tile data layout in vector register is **not** specified and left to the implementation!

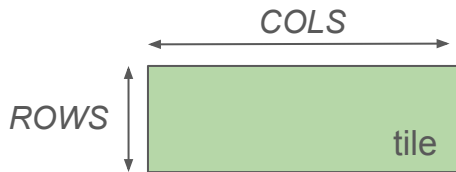


Tile Shape

A matrix tile is characterized by:

- VREG vector register number
- ROWS number of rows
- COLS number of columns
- SEW single element width
- MTYPE matrix type (A, B, C)
- (LMUL)
- (DTYPE data type (signed/unsigned/float/alt float))

***tile
shape***



MAXROWS, MAXCOLS depends on tensor unit geometry and tile type (A, B, C).

Tile Shape Data

The Tile Shape is a 32 bit entity that contains all information needed by the tensor unit to interpret correctly the tile data stored in a vector register or a vector register group.

- **rows/cols**: number of rows, columns of the tile
 - Values can vary widely: eg. 1 to 4096
 - Maximum VLEN=32768: 4096 elements in int8
 - Odd tail / border values should be handled as well
- **sew**: single element width
- **mtype**: tile type enum A, B, C
 - Load unit must recognize whether internal tile representation is in column-major-order or row-major-order.
- (**dtype**: Data type of tile: unsigned/signed/float/alt float)

Example:

```
typedef struct {  
    unsigned int rows : 12;  
    unsigned int cols : 12;  
    unsigned int sew : 3;  
    unsigned int mtype : 2;  
} TileShape_32b;
```

```
typedef enum {  
    MTYPE_A, MTYPE_B, MTYPE_C,  
} mtype_t;
```

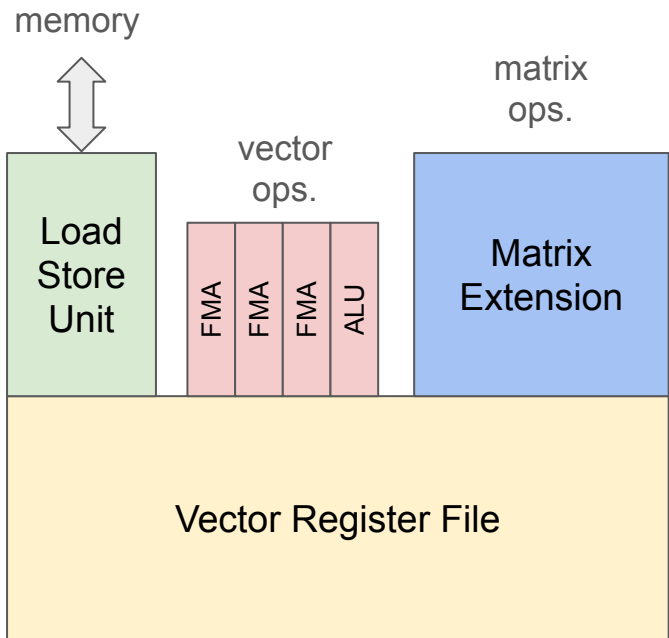
Tensor Unit Configuration Needs

(1) Load matrix tile into VREG

Store matrix tile to memory

- Tile dimensions
- SEW
- Matrix type (A, B, C) for possible internal conversion

=> tile shape



(2) Multiply matrix tiles

- $m \times k \times n$,
- For each tile A, B, C:
 - SEW & DTYPE

or

- The 3 tile shapes of A, B, C
- (DTYPE, if not in tile shape)

(1) and (2) should be able to run simultaneously

Element-wise Matrix Operations

With matrix tiles stored in vector registers element-wise matrix operations can be done by using vector instructions.

- The effective or granted vector length depends on the size and shape of the tile.
- The tensor unit implementation can use masking to represent tiles smaller than the maximum tile size.
- Element-wise operations may need to use the corresponding, implementation-specific, vector mask.

Examples:

- Add two tiles of same shape
- Multiply each element of a tile with scalar value
- Initialize tile
- Apply activation function
- ...

Intrinsics/ISA

Core Logic

1. Get Tile Shape(s) for A, B, C

Iterate, strip-mine.

Like ***vl = vsetvl(avl)*** but for 2D tiles

ask tensor unit for the
largest tile shape needed

tensor unit replies with
largest tile shape it supports

2. Load A, B, C from memory

Like ***vleNN.v*** but for 2D tiles

Use tile shape data

3. Multiply $C=C+A \cdot B$

Like ***vfmac.vv*** but for 2D tiles

Use tile shape data

4. Store C to memory

Like ***vseNN.v*** but for 2D tiles

Use tile shape data

Get Tile Shape

```
uint32_t tsget_T(int8_t mtype, uint32_t max_rows, uint32_t max_columns)
```

Retrieve the largest possible Tile Shape for a particular tile type A, B or C and the data type T. T is one of {i8, i16, i32, i64, u8, u16, u32, u64, **f16**, f32, f64}.

Equivalent to vsetvl intrinsic.

Arguments:

- **mtype**: matrix type enum value for A, B, C type.
- **max_rows**: maximum number of rows needed in the tile.
- **max_columns**: maximum number of columns needed in the tile.

Returns:

- A Tile Shape value that is supported by the tensor unit implementation.

Get Tile Shape

Remarks:

The tile shape value removes the need to have state stored either in the tensor unit or along the vector register containing a tile.

It eases optimization in the compiler like unrolling and reordering tile operations.

Requesting an unsupported data type T could either raise an exception or return a Zero tile shape value.

Letting the tensor implementation return the appropriate tile shape relieves us of the burden to manage various $m_{\max} \times k_{\max} \times n_{\max}$ combinations for different **data types** and **SEWs**.

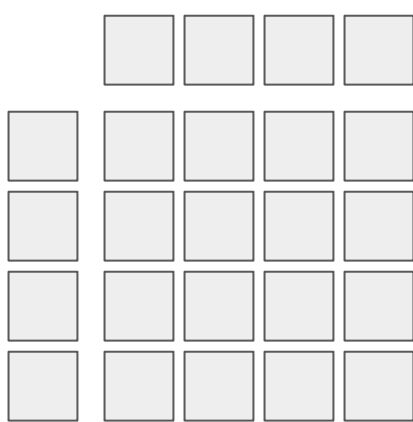
Querying tile shapes simplifies iterating over the tiles when multiplying large matrices.

Add LMUL to type T? Eg. *tsget_f16m4()*

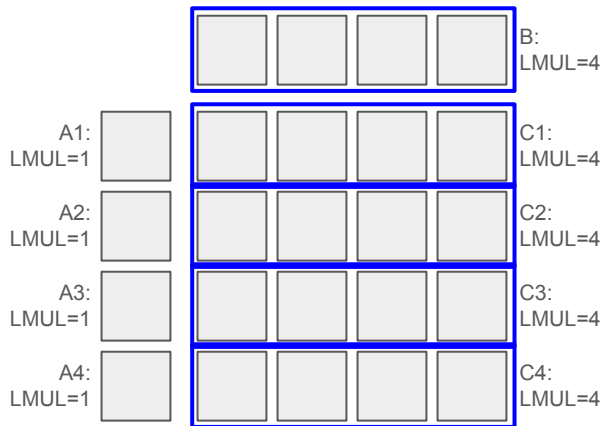
Get Tile Shape

Remarks:

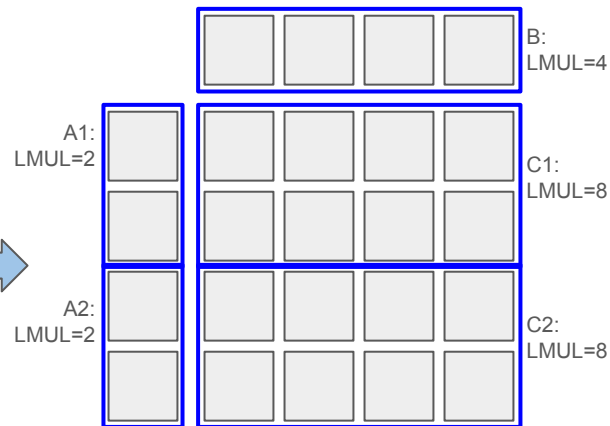
Add LMUL to type T? Eg. `tsget_f16m4()` : Looks useful, needs further discussion.



LMUL=1 : 16 tile multiplies



4 tile multiplies



2 tile multiplies
(but less regular pattern in C)

Load Tile From Memory

```
vT_t tload_T(vT_t vd, uint64_t addr, uint32_t str_col, uint32_t str_row, uint32_t ts)
```

Load a tile shaped according to the tile shape ***ts*** from memory address ***addr*** into the vector register ***vd*** considering the column and row strides ***str_col***, ***str_row***.

T is the tile element data type (eg. f16 or f16m1).

Arguments:

- ***vd***: target vector register
- ***addr***: start address of the tile in memory
- ***str_col***: column stride of the tile data in memory
- ***str_row***: row stride of the tile data in memory
- ***ts***: tile shape formerly retrieved through call to ***tile_get_shape()***

Store Tile To Memory

```
tstore_T(vT_t vs, uint64_t addr, uint32_t str_col, uint32_t str_row, uint32_t ts)
```

Store a tile shaped according to the tile shape ***ts*** from the vector register ***vs*** to memory address ***addr*** considering the column and row strides ***str_col***, ***str_row***.

T is the tile element data type(eg. f32 or f32m1).

Arguments:

- ***vs***: source vector register
- ***addr***: start address of the tile in memory
- ***str_col***: column stride of the tile data
- ***str_row***: row stride of the tile data
- ***ts***: tile shape retrieved through ***tile_get_shape()***

On NEC SX-Aurora VE:

VST2D	vst2d [.nc][.ot] {%vx %vix}, {%sy I}, {%sz	Vector Store 2D
D1 / RVM	Z} [, %vm]	

Load / Store Tile

Remarks:

- The way how tile data is laid out in the vector register(s) is not specified and should remain implementation specific. It can be column-major-order or row-major-order or depend on the A, B, C matrix type.
- Passing both strides (row/column) helps the tensor unit transpose the tile when loading it, eg. if the internal representation requires column-major-order but data is row-major-order in memory. Order in memory doesn't matter.
- 2D slices of higher dimensional tensors can be loaded/stored.
- Strides in bytes or bytes in SEW?
- The intrinsics get transformed into (at least) 2 instructions: a ***vsetvli*** and the actual tile load instruction.

Multiply And Accumulate Tiles

`vTCm1_t tXmacc_TC(vTCm1_t vc, vTABm1_t va, vTABm1_t vb, uint32_t tc, uint32_t ta, uint32_t tb)`

Perform the multiply and accumulate operation $\mathbf{C} = \mathbf{C} + \mathbf{A} \cdot \mathbf{B}$ with tile \mathbf{A} in vector register \mathbf{va} , having shape \mathbf{ta} , \mathbf{B} stored in \mathbf{vb} , shape \mathbf{tb} and the in/out tile \mathbf{C} in \mathbf{vc} with shape \mathbf{tc} .

\mathbf{X} specifies operation type and widening. Eg. tfmacc, twmacc, tfwmacc, tqmacc, etc.

Arguments:

- \mathbf{vc} : vector register that contains tile \mathbf{C} .
- \mathbf{va} : vector register containing the tile \mathbf{A} .
- \mathbf{vb} : vector register containing the tile \mathbf{B} .
- \mathbf{tc} : tile shape of \mathbf{C} .
- \mathbf{ta} : tile shape of \mathbf{A} .
- \mathbf{tb} : tile shape of \mathbf{B} .

Multiply Tiles

Remarks:

- The intrinsics have 6 arguments: 3 x VREG + 3 x TileShape
 - Not feasible for assembler instructions
- Intrinsics get converted into at least two assembler instructions
 - Setup Tensor Unit: takes the three tile shapes as arguments
 - New instruction **tmulset**, similar to **vsetvli**
 - Matrix Multiplication instruction: takes the three VREGs as arguments
- Handling LMUL is more complicated than in the vector case. LMUL in T_C with narrower T_A, T_B does not imply that $LMUL_{A,B} < LMUL_C$

Tile Shape Helpers

Tile Shape Rows And Columns

```
uint16_t tsrows(uint32_t ts)
```

```
uint16_t tscols(uint32_t ts)
```

Extract **rows** and **columns** values from tile shape **ts**. This function can be a macro or compiler builtin, as it only does an AND and a SHIFT operation on the tile shape value. Expands to existing instructions.

Arguments:

- ts: tile shape.

Returns:

- Rows, columns values encoded in the tile shape variable.

Tile Shape Vector Length

```
size_t tsvl(uint32_t ts)
```

Return implementation specific vector length in elements that needs to be used for element-wise vector operations on the tile with the tile shape ***ts***.

Arguments:

- *ts*: tile shape

Returns:

- Vector length for element-wise operations on tile elements.

Tile Shape Vector Mask

```
vboolX_t tsmask(vboolX_t vd, uint32_t ts)
```

The implementation specific vector mask that needs to be used for element-wise vector operations on the tile with the tile shape **ts** is stored in the destination vector register **vd**. Typically masks are needed when the tile shape doesn't have the maximum size.

Arguments:

- **vd**: destination vector register for vector mask.
- **ts**: tile shape.

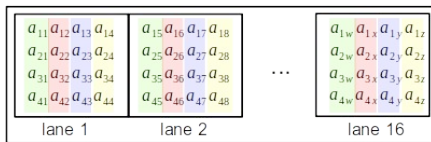
Returns:

- Vector mask for element-wise operations on tile elements.

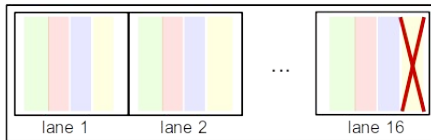
Example:

4 x 64

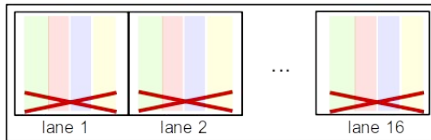
FP64



4 x 63



3 x 64



Assembler

tsget	Tile shape get	
tload	Tile load	
tstore	Tile store	
tmac	Tile multiply-accumulate	tfmacc, tfwmacc, twmacc, tqmacc, tfqmacc, tmacu,...
tmulset	Tile multiplier set	
tsvl	Tile shape vector length	
tmask	Tile shape mask	

NOTE: Adding data type information to Tile Shape reduces number of *tmac* instruction variants.

Tensor Unit CSR (example)
(Could fit into 64 bits)

```
struct TensorUnitCSR {  
    // - tile shapes -  
    unsigned int m : 12;  
    unsigned int k : 12;  
    unsigned int n : 12;  
    unsigned int sew_a : 3;  
    unsigned int sew_b : 3;  
    unsigned int sew_c : 3;  
    // - data type -  
    unsigned int float : 1;  
    unsigned int alt_float : 1;  
    unsigned int unsig : 1;  
    // - further config -  
    unsigned int sparse : 1;  
    // - status -  
    unsigned int status : X;  
    // - implementation specific -  
    unsigned int impl : Y;  
};
```

Example

GEMM $C = A \cdot B$

Matrix data types: C: FP32 A,B: FP32

```
01: for m = 0, M, m += tm do
02:     for n = 0, N, n += tn do
03:         tc = tsget_f32(MTYPE_C, M - m, N - n)
04:         tm = tsrows(tc)
05:         tn = tscols(tc)
06:         gvl = tsvl(tc)
07:         c = __riscv_vfmv_v_f_f32m1(0.0f, gvl)
08:         for k = 0, K, k += tscols(ta) do
09:             ta = tsget_f32(MTYPE_A, tm, K - k)
10:             tb = tsget_f32(MTYPE_B, K - k, tn)
11:             a = tload_f32(&A[m * LDA + k], LDA, 1, ta)
12:             b = tload_f32(&B[k * LDB + n], LDB, 1, tb)
13:             c = tfmacc_f32(c, a, b, tc, ta, tb)
14:         tstore_f32(c, &C[m * LDC + n], LDC, 1, tc)
```

GEMM $C = \alpha \cdot A \cdot B + \beta \cdot C$

Matrix data types: C: FP32 A,B: FP16

```
01: for m = 0, M, m += tm do
02:     for n = 0, N, n += tn do
03:         tc = tsget_f32(MTYPE_C, M - m, N - n)
04:         tm = tsrows(tc)
05:         tn = tscols(tc)
06:         gvl = tsvl(tc)
07:         cmask = tsmask(tc)
09:         c = __riscv_vfmv_v_f_f32m1(0.0f, gvl)
10:         for k = 0, K, k += tscols(ta) do
11:             ta = tsget_f16(MTYPE_A, tm, K - k)
12:             tb = tsget_f16(MTYPE_B, K - k, tn)
13:             a = tload_f16(&A[m * LDA + k], LDA, 1, ta)
14:             b = tload_f16(&B[k * LDB + n], LDB, 1, tb)
15:             c = tfwmacc_f32(c, a, b, tc, ta, tb)
16:         t = tload_f32(&C[m * LDC + n], LDC, 1, tc)
17:         c = __riscv_vfmul_vf_f32m1(c, alpha, gvl, cmask)
18:         c = __riscv_vfadd_vf_f32m1(t, beta, gvl, cmask)
19:         tile_store_fp32(c, &C[m * LDC + n], LDC, 1, tc)
```


TODO

- Discover supported data formats (!)
- Multiple VREG Tiles
 - Code for square matrices pattern (A) with 4x4 C-type + 4 A-type + 4 B-type = 24 VREGs
 - Not optimal for narrow matrices (B): 16 C-type VREGs + 1 A-type + 1 B-type = 18 VREGs
 - Pattern (A) probably still good enough to reach peak performance/throughput!
 - Add new Multi-Tile-Shape to describe optimal topology?
 - Eg. add rows, columns of tiles for optimal performance
 - Generate code dynamically at runtime for presented topology?
 - Or let compiler generate alternative code for extreme cases and chose path according to retrieved shape?
 - LMUL in tile shape: not sufficient for 2D tile patterns
 - If LMUL is hidden in an argument (ts): not possible to do register allocation
 - Helps reduce number of instructions.
- Sparse matrix operations
- Other matrix operations

THE END