How far from equilibrium is active matter?

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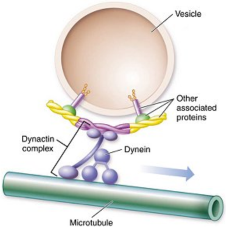
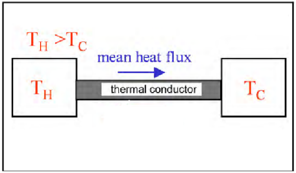
Active Liquids – Leiden

Active matter: intrinsically out-of-equilibrium, but...

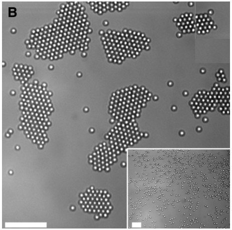
each degree of freedom takes energy from the environment and consume it to self-propel

... but ...





Macroscopic heat flux



No net macroscopic current

suspension of synthetic photoactivated colloidal particles

Palacci et al., Science, 2013

how active matter is far from equilibrium?

- Is there an effective equilibrium regime (time symmetry respected)?
- How can we prove that we are close/far from equilibrium?
  - good observables to look at?
  - in which parameter is close/far from equilibrium?

A simple framework: active Ornstein-Uhlenbeck

$$\dot{q}_i = -\nabla_i \phi + v_i$$

$$\langle v_i(t) v_j(0) \rangle = \delta_{ij} \frac{D}{\tau} \exp(-t/\tau)$$

- $D \propto u_0^2 \tau$
- u_0 : magnitude of self-propulsion
- τ : persistence time-scale
- ϕ : interaction potential

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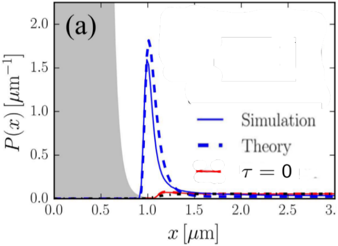
- $\tau = 0, D$ fixed : equilibrium
  - $\dot{q}_i = -\nabla_i \phi + \sqrt{2D} \eta_i$
  - $\rho_{ss} \propto \exp(-\phi/D)$

- $\tau \neq 0$: velocities correlated in time
  - non-Boltzmann-Gibbs P_{ss}

Mapping to equilibrium

Accumulation at boundaries



Maggi et al., Scientific Reports, 2015

Approximating the dynamics (for small τ)

[Fox, 1986; Hanggi, 1987; Maggi et al. 2014-2015; Farage et al. 2015]

$$\dot{q}_i \simeq -\nabla_i \phi + \sqrt{2D}\,\mathbb{N}_{ik}^{-1}\eta_k \qquad \mathbb{N}^2 = \mathbf{1} + \tau\nabla\nabla\phi$$

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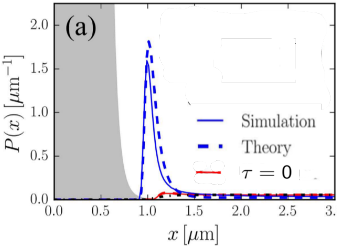

Maggi et al., Scientific Reports, 2015

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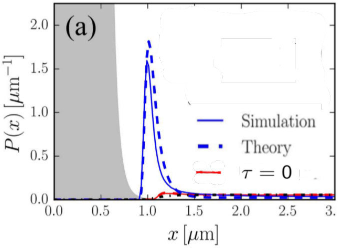
$$\dot{q}_i \simeq -\nabla_i \phi + \sqrt{2D}\,\mathbb{N}_{ik}^{-1}\eta_k \qquad \mathbb{N}^2 = \mathbf{1} + \tau\nabla\nabla\phi$$

Time reversible!

$$\rho_{ss} = Z \exp\left(-\frac{\phi}{D} - \frac{\tau}{2D}|\nabla\phi|^2\right) \det(\mathbf{1} + \tau\nabla\nabla\phi)$$

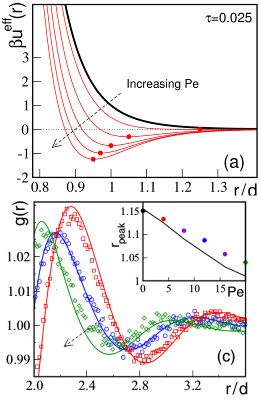
Accumulation at boundaries



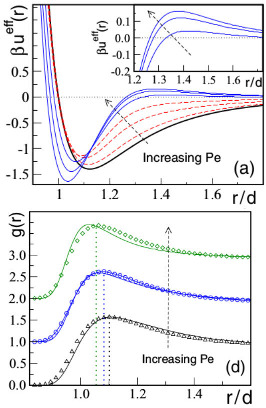
Maggi et al., Scientific Reports, 2015

Mapping to equilibrium

Effective attraction from repulsive ϕ



Effective repulsion from attractive ϕ



Farage et al., PRE, 2015

Excellent predictions of the steady density profile and two-body correlations


SUMMERTIME IN SCOTLAND. IT IS NOT ALWAYS EASY TO FEEL AT HOME.

For any $\rho_{ss}(q_1, ..., q_N)$, I can find φ such that $\rho_{ss} = \exp(-\varphi)$

Of course, φ is horrible in general ...

Time symmetry

Nonequilibrium: Breakdown of time reversal invariance

$$\langle A(t_1)B(t_2)\rangle \neq \langle B(t_1)A(t_2)\rangle$$

Entropy production rate $\quad \sigma = \lim_{T\to\infty} \frac{1}{T} \ln \frac{\mathcal{P}_{\mathrm{F}}}{\mathcal{P}_{\mathrm{B}}}$

- $\mathcal{P}_{\mathrm{F}}$ probability of the trajectory $\{q_t, t \in [0, T]\}$
- $\mathcal{P}_{\mathrm{B}}$ probability of time reversed trajectory $\{q_t^{\mathrm{R}} = q_{T-t}\}$

Equilibrium $\quad \sigma = 0$

Time symmetry

Approximated dynamics (Fox/UCNA) $\sigma = 0$

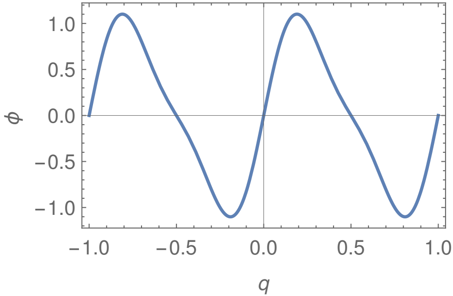
Active Ornstein-Uhlenbeck $\sigma = \mathcal{O}\left(\tau^2\right)$

$$\rho_{\mathsf{ss}}(q) \propto \exp\left[-\frac{\phi}{D} - \frac{\tau}{2D}\left|\nabla\phi\right|^2 + \tau\nabla^2\phi + \mathcal{O}\left(\tau^2\right)\right]$$

Small τ regime $\begin{cases} \text{Time symmetry} \\ \text{Non-Boltzmann statistics} \end{cases}$

How to measure time symmetry?

Ratchet experiment



Time symmetry breakdown $\rightarrow$ Particle current $= \mathcal{O}\left(\tau^2\right)$

How to measure time symmetry?

Fluctuation-dissipation relation

$$\dot{q} = -\nabla\phi + v + f \quad \rightarrow \quad R(t) = \left.\frac{\delta \langle q(t) \rangle}{\delta f(0)}\right|_{f=0}$$

Thermal limit $\quad R(t) \underset{\tau=0}{=} -\frac{1}{D}\frac{\mathrm{d}}{\mathrm{d}t}\langle q(t)q(0)\rangle$

Small τ regime: time symmetry assumption

$$R(t) = -\frac{1}{D}\frac{\mathrm{d}}{\mathrm{d}t}\Big[\langle q(t)q(0)\rangle + \tau^2\langle \dot{q}(t)\dot{q}(0)\rangle\Big]$$

Conclusion

Is there a regime where time symmetry holds
and
we observe the standard phenomenology of active matter?

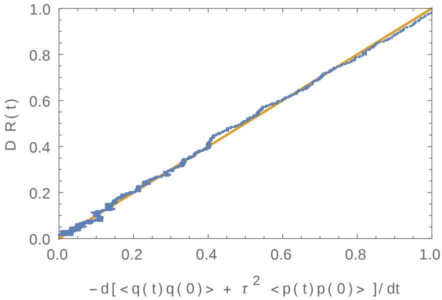
$$\text{Effective equilibrium} \quad \begin{cases} \text{Fluctuation-dissipation relation} \\ \text{Accumulation at the boundaries, MIPS} \end{cases}$$

What about other theoretical models/experimental systems?

Fluctuation-dissipation relation

External potential $\quad \phi = -k\frac{q^2}{2} + \beta\frac{q^4}{4}$

$DR(t) = -\frac{\mathrm{d}}{\mathrm{d}t}\left[\langle q(t)q(0)\rangle + \tau^2\langle \dot{q}(t)\dot{q}(0)\rangle\right]$



$DR(t) \neq -\frac{\mathrm{d}}{\mathrm{d}t}\langle q(t)q(0)\rangle$