How far from equilibrium is active matter?

Étienne Fodor,[1,2] Cesare Nardini,[1] Elsen Tjhung,[1] Michael E. Cates,[1]
Julien Tailleur,[2] Paolo Visco,[2] Frédéric van Wijland[2]

1. Department of Applied Mathematics and Theoretical Physics, University of Cambridge
2. Laboratoire Matière et Systèmes Complexes, Université Paris Diderot

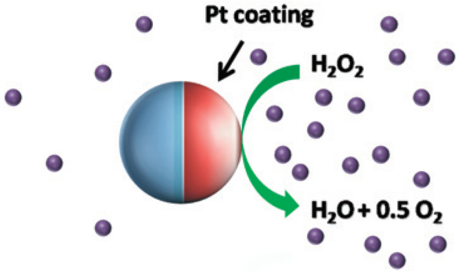
Edwards Centre Mini-Conference

University of Cambridge

Introduction

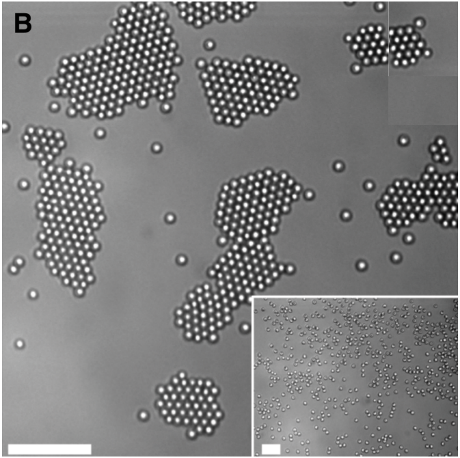
Active matter: interacting self-propelled particles

Janus particle



A. Walther and A. H. E. Müller, Soft Matter **4**, 663 (2008)

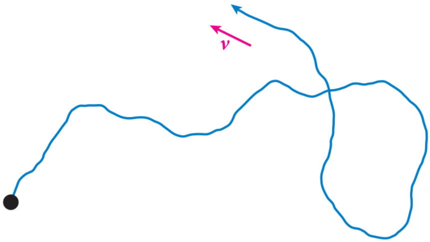
Light-induced clusters of colloids



J. Palacci *et al.*, Science **339**, 936 (2013)

Many-body dynamics

$$0 = -\nabla_i U - \frac{\mathrm{d}x_i}{\mathrm{d}t} + v_i$$



M. E. Cates and J. Tailleur,

Ann. Rev. CMP **6**, 2119 (2015)

Nonequilibrium self-propulsion v_i

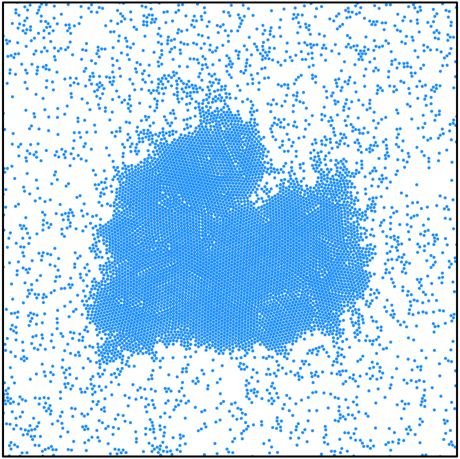
$$\langle v_i(t) v_j(0) \rangle \equiv T \delta_{ij} \frac{\mathrm{e}^{-|t|/\tau}}{\tau}$$

Persistence time τ

Equilibrium $\tau \to 0$

Introduction

Interacting repulsive particles



ÉF *et al.*, Phys. Rev. Lett. **117**, 038103 (2016)

Introduction

Main questions of interest

- Phase diagram: equilibrium mapping

  J. Tailleur *et al.*, Phys. Rev. Lett. **100**, 218103 (2008)

- Equation of state: pressure vs. density

  A. P. Solon *at al.*, Phys. Rev. Lett. **114**, 198301 (2015)

How far from equilibrium is active matter?

Interacting active particles

Comparing forward and backward dynamics

$$\mathcal{S} \equiv \lim_{t\to\infty} \frac{1}{t} \ln \frac{\mathcal{P}}{\mathcal{P}^{\mathsf{R}}}$$

Entropy production rate

Equilibrium limit $\quad \mathcal{S} \underset{\tau=0}{=} 0$

Self-propelled particles

$$\mathcal{S} = \frac{\tau^2}{2T} \left\langle \left(\frac{\mathrm{d}x_i}{\mathrm{d}t} \nabla_i \right)^3 U \right\rangle$$

Interacting active particles

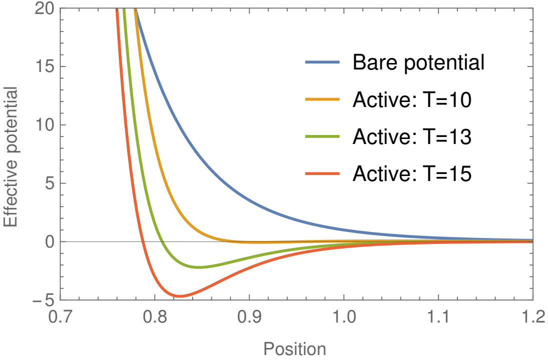
Perturbative treatment in $\mathrm{Pe} \equiv \sqrt{\frac{\text{persistence time}}{\text{relaxation time}}}$

- $P_\mathrm{S} \sim \mathrm{e}^{-U/T}\left[1 + \mathcal{O}\left(\mathrm{Pe}^2\right)\right]$
- $\left\langle \left(\frac{\mathrm{d}x_i}{\mathrm{d}t}\nabla_i\right)^3 U \right\rangle = \mathcal{O}\left(\mathrm{Pe}^3\right)$

Effective equilibrium regime $\begin{cases} \text{Non-Boltzmann statistics} \\ \text{Time reversal symmetry} \end{cases}$

Effective potential

$$U_{\text{eff}} \equiv U + \tau \left[\frac{(\nabla_i U)^2}{2} - T \nabla_i^2 U \right]$$



Original pair-wise potential

$$U \equiv \frac{1}{2} \sum_{\{i,j\}=1}^{N} V(r_i - r_j)$$

$$V \sim \frac{1}{x^{12}}$$

Interacting active particles

External perturbation $f_i(t)$

$$\langle \delta x_i(t) \rangle \equiv \int_0^t R(t-s) f_i(s) \mathrm{d}s \ \rightarrow \ \text{Response function } R(t)$$

Fluctuation-dissipation relation

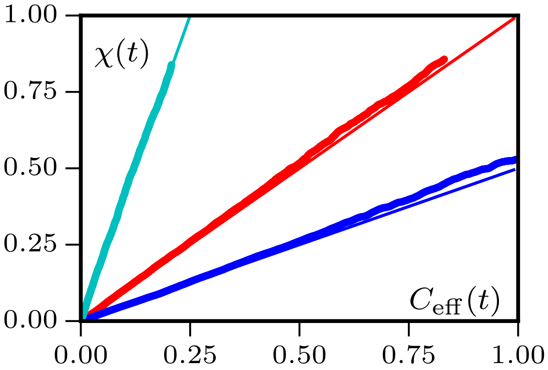
$$R(t) \underset{\tau=0}{=} -\frac{1}{T}\frac{\mathrm{d}}{\mathrm{d}t}\langle x_i(t) x_i(0) \rangle$$

$$R(t) = -\frac{1}{T}\frac{\mathrm{d}}{\mathrm{d}t}\left\langle x_i(t) x_i(0) + \tau^2 \frac{\mathrm{d}x_i(t)}{\mathrm{d}t}\frac{\mathrm{d}x_i(0)}{\mathrm{d}t} \right\rangle$$

Probing effective equilibrium regime

Interacting active particles

Numerical simulations: 3 values of T



Susceptibility

$$\chi(t) \equiv \frac{1}{N}\int_0^t R(t-s)\mathrm{d}s$$

Correlation

$$C_{\rm eff}(t) \equiv \frac{1}{N}\left\langle x_i^2(t) + \tau^2 \left[\frac{\mathrm{d}x_i(t)}{\mathrm{d}t}\right]^2 \right\rangle - \frac{1}{N}\left\langle x_i(t)x_i(0) + \tau^2 \frac{\mathrm{d}x_i(t)}{\mathrm{d}t}\frac{\mathrm{d}x_i(0)}{\mathrm{d}t} \right\rangle$$

Conclusion

Nonequilibrium properties of active systems

- Interacting active particles

  Effective equilibrium regime

- Coarse-grained dynamics

  Spatial structure of nonequilibrium

References: Phys. Rev. Lett. **117**, 038103 (2016) | `arXiv:1604.00953`

Phys. Rev. X **7**, 021007 (2017) | `arXiv:1610.06112`