Self-propelled particles as an active matter system

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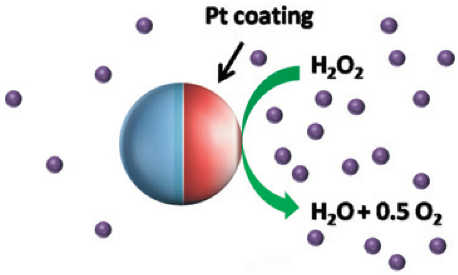
Introduction

Active matter: interacting self-propelled particles

Janus particle



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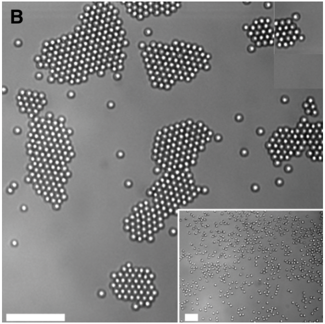
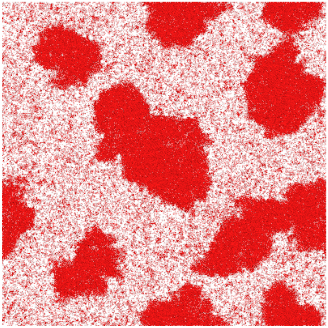
Introduction

Light-induced clusters of colloids



J. Palacci, S. Sacanna, A. P. Steinberg, D. J. Pine, and P. M. Chaikin, Science **339**, 936 (2013)

Simulated interacting active colloids



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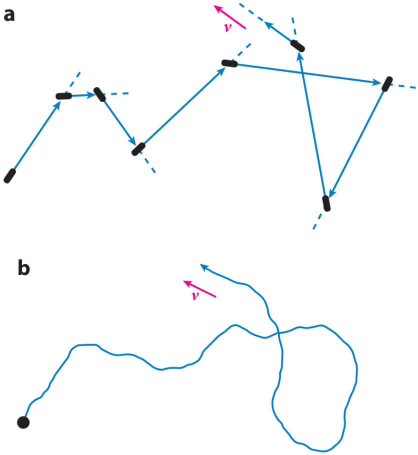
Introduction

Run-and-tumble and active Brownian particles



M. E . Cates and J. Tailleur, Ann. Rev. CMP **6**, 219 (2015)

Introduction

Overdamped dynamics

$$0 = -\nabla_i U - \gamma \dot{r}_i + \xi_i$$

Equilibrium fluctuations ξ_i

No memory and Gaussian

Boltzmann distribution

$$P_\mathrm{s} \sim \mathrm{e}^{-U/T}$$

Introduction

Overdamped dynamics

$$\dot{r}_i = -\mu \nabla_i U + v_i$$

Nonequilibrium self-propulsion v_i

Memory and non-Gaussian

Non-Boltzmann distribution

Introduction

Main questions of interest

- Equation of state: temperature, pressure
- Phase diagram: steady state
- Nonequilibrium properties

Outline

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1. Searching for a minimal model of self-propulsion
2. Quantifying nonequilibrium
3. Effective equilibrium regime

Outline

Searching for a minimal model of self-propulsion

Overdamped dynamics

$$\dot{r}_i = -\mu \nabla_i U + v_i$$

Nonequilibrium self-propulsion v_i

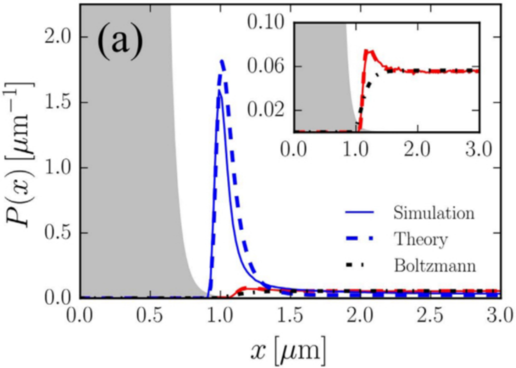
Memory and Gaussian

Persistence time τ

Equilibrium reference $\tau = 0$

Searching for a minimal model of self-propulsion

Non-interacting particles in external potential



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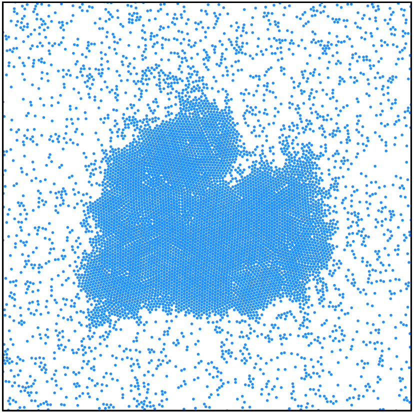
Searching for a minimal model of self-propulsion

Interacting repulsive particles



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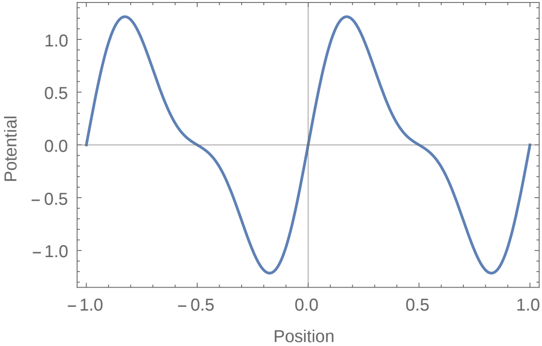
Outline

Quantifying nonequilibrium

Breakdown of time reversal



Current in a ratchet

Quantifying nonequilibrium

Quantifying irreversibility

$$\sigma = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\mathcal{P}}{\mathcal{P}^{\mathsf{R}}}$$

Entropy production rate

Equilibrium $\quad \sigma = 0$

Quantifying nonequilibrium

Brownian particles + non-conservative force F_i

$$\sigma = \frac{\langle \dot{r}_i F_i \rangle}{T} = \frac{\text{power of } F_i}{\text{temperature}}$$

Self-propelled particles

$$\sigma = \frac{\mu \tau^2}{2T} \langle \dot{r}_i^3 \nabla_i^3 U \rangle$$

No simple energetic interpretation

Quantifying nonequilibrium

Péclet number

$$\mathrm{Pe} = \frac{\text{persistence length}}{\text{interaction range}}$$

$$\mathrm{Pe} = \sqrt{\frac{\text{persistence time}}{\text{relaxation time}}}$$

Competition self-propulsion/interaction

Quantifying nonequilibrium

Perturbative treatment in Pe

- $P_\mathrm{S} \sim \mathrm{e}^{-U/T}\left[1+\mathcal{O}\left(\mathrm{Pe}^2\right)\right]$
- $\left\langle \dot{r}_i^3 \nabla_i^3 U \right\rangle = \mathcal{O}\left(\mathrm{Pe}^3\right)$

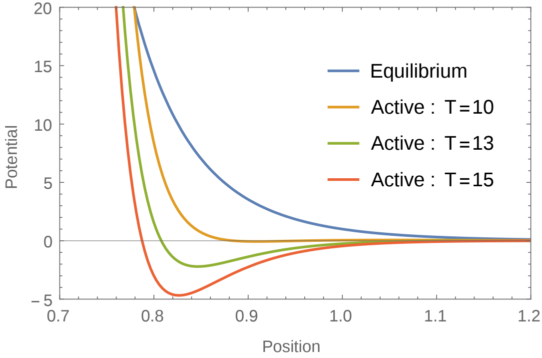
$$\text{Small Pe regime} \quad \begin{cases} \text{Non-Boltzmann statistics} \\ \text{Time reversal} \end{cases}$$

Outline

Effective equilibrium regime

Effective potential

$$U_{\text{eff}} = U + \tau \left[\frac{(\nabla_i U)^2}{2} - T \nabla_i^2 U \right]$$



Pair-wise potential

$$U = \sum_{\{i,j\}=1}^{N} \phi(r_i - r_j)$$

$$\phi \sim \frac{1}{r^{12}}$$

Effective equilibrium regime

External perturbation $f_i(t)$

Response function $R(t)$

$$\langle \delta r_i(t) \rangle = \int_0^t R(t-s) f_i(s) \mathrm{d}s$$

Brownian particles

$$R(t) = -\frac{1}{T} \frac{\mathrm{d}}{\mathrm{d}t} \langle r_i(t) r_i(0) \rangle$$

Fluctuation-dissipation theorem

Effective equilibrium regime

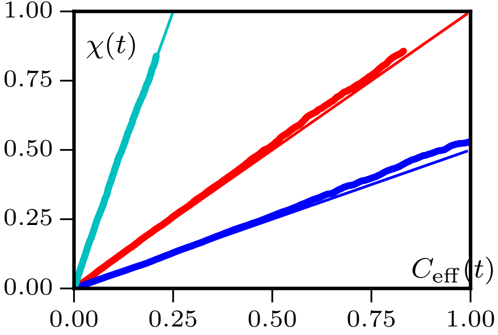
Self-propelled particles

$$R(t) = -\frac{1}{T}\frac{\mathrm{d}}{\mathrm{d}t}\left\langle r_i(t) r_i(0) + \tau^2 \dot{r}_i(t)\dot{r}_i(0)\right\rangle$$

Effective equilibrium regime

Effective equilibrium regime

Numerical simulations: 3 values of T



Susceptibility

$\chi(t) = \int_0^t R(s)\mathrm{d}s$

Correlation

$$C_{\rm eff}(t) = \left\langle r_i^2(t) + \mathrm{Pe}^2 \dot{r}_i^2(t) \right\rangle - \left\langle r_i(t) r_i(0) + \mathrm{Pe}^2 \dot{r}_i(t) \dot{r}_i(0) \right\rangle$$

Conclusion

Minimal model of self-propulsion

Gaussian with memory

- Phase separation for repulsive particles
- Effective equilibrium
- Fluctuation-dissipation relation

Outlook

Future directions

- Collective modes

  Hydrodynamic equations, passive tracer

- Other models of self-propulsion

  Non-Gaussian without memory