How far from equilibrium is active matter?

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Can we understand active matter with equilibrium physics?

Active Ornstein-Uhlenbeck Particles (AOUPs)

Self-propelled interacting colloids

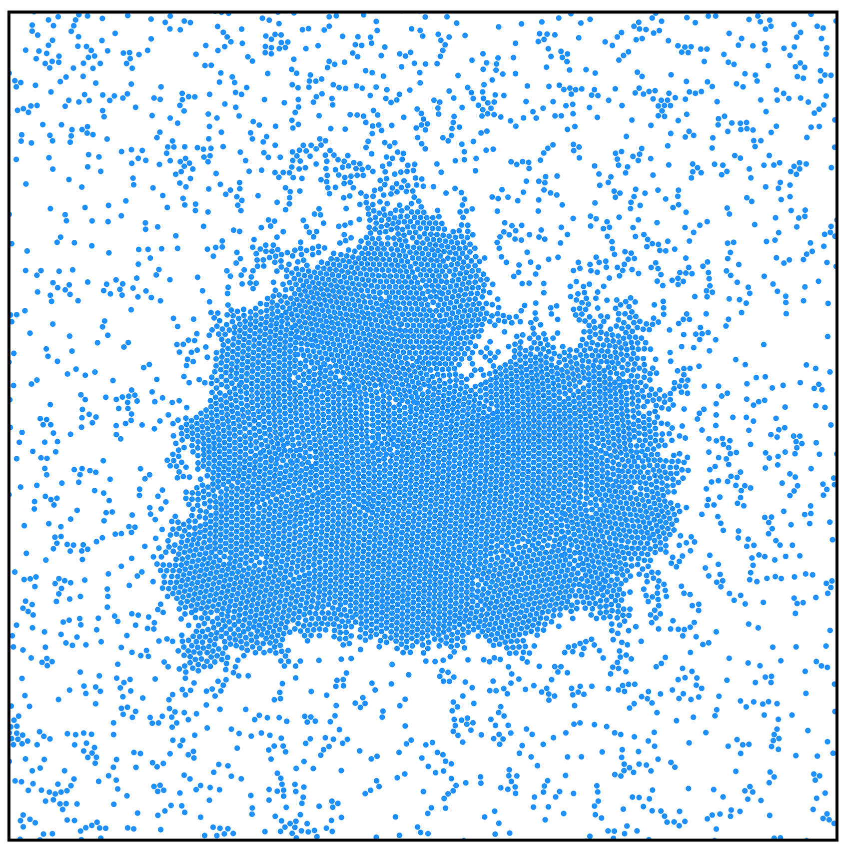
$$\dot{\mathbf{r}}_i = -\nabla_i U + \mathbf{v}_i$$

$$\langle v_{i\alpha}(t) v_{j\beta}(0)\rangle = \delta_{ij}\delta_{\alpha\beta}\frac{T}{\tau}\mathrm{e}^{-|t|/\tau}$$

Nonequilibrium dynamics:
persistent fluctuations with
instantaneous damping

Equilibrium dynamics for
vanishing persistence

$$\langle v_{i\alpha}(t) v_{j\beta}(0)\rangle \xrightarrow[\tau\to 0]{} \delta_{ij}\delta_{\alpha\beta} 2T\delta(t)$$



Phase separation under purely repulsive interactions

Steady state: perturbative treatment

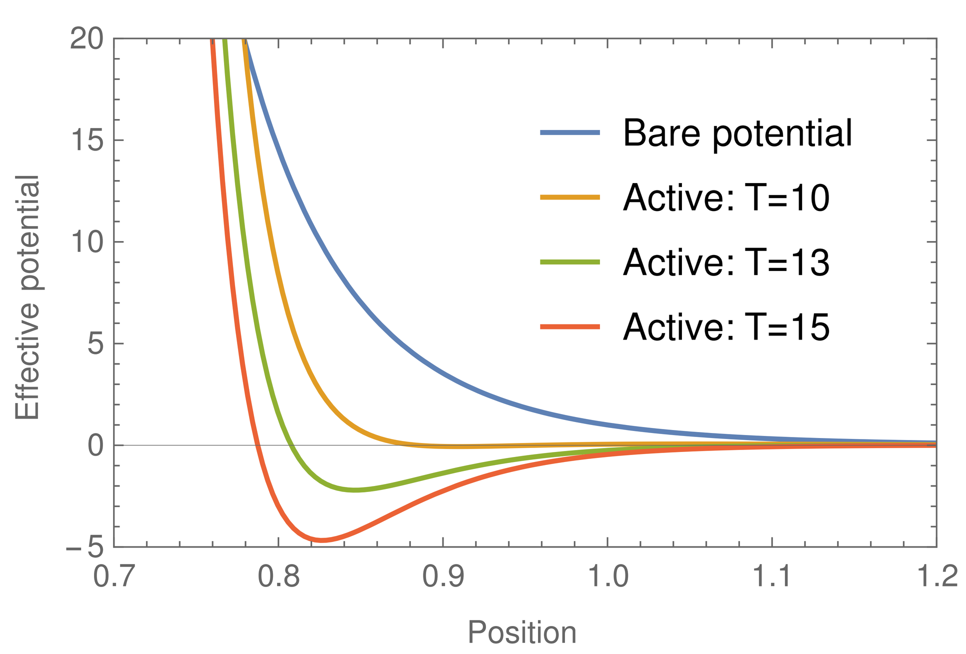
Small persistence expansion of stationary distribution

$$P = P(\{\mathbf{r}_i, \tilde{\mathbf{p}}_i = \sqrt{\tau}\mathbf{p}_i\}), \quad \rho = \rho(\{\mathbf{r}_i\})$$

$$P = \exp\left(-\frac{U}{T} - \frac{\tilde{\mathbf{p}}_i^2}{2T}\right)\left[1 - \frac{\tau}{2T}\left[(\nabla_i U)^2 + (\tilde{\mathbf{p}}_i\cdot\nabla_i)^2 U - 3T\nabla_i^2 U\right] + \frac{\tau^{3/2}}{6T}\left[(\tilde{\mathbf{p}}_i\cdot\nabla_i)^3 U - 3T(\tilde{\mathbf{p}}_i\cdot\nabla_i)\nabla_j^2 U\right] + \mathcal{O}(\tau^2)\right]$$

Modified equipartition $\quad \langle\tilde{\mathbf{p}}_i^2\rangle = T - \tau\left\langle(\nabla_i U)^2\right\rangle_{\mathrm{B}} + \mathcal{O}(\tau^2)$

where $\langle\cdot\rangle_{\mathrm{B}}$ denotes an average with respect to the Boltzmann distribution



Effective potential $\quad U_{\mathrm{eff}} = -T\ln\rho$

$$U_{\mathrm{eff}} = U + \tau\left[\frac{(\nabla_i U)^2}{2} - T\nabla_i^2 U\right] + \mathcal{O}(\tau^2)$$

Effective attraction emerges
from bare repulsion

Quantifying irreversibility

Likelihood between forward and time-reversed dynamics

$$\mathbf{r}_i^{\mathrm{R}}(t) = \mathbf{r}_i(-t), \quad \mathbf{p}_i^{\mathrm{R}}(t) = -\mathbf{p}_i(-t)$$

Entropy production rate $\quad \mathcal{S} = \lim_{t\to\infty}\frac{1}{t}\ln\frac{\mathcal{P}\left[\{\mathbf{r}_i, \mathbf{p}_i\}\right]}{\mathcal{P}\left[\{\mathbf{r}_i^{\mathrm{R}}, \mathbf{p}_i^{\mathrm{R}}\}\right]}$

$$\mathcal{S} = \frac{\tau^2}{2T}\left\langle(\mathbf{p}_i\cdot\nabla_i)^3 U\right\rangle = \frac{T\tau^2}{2}\left\langle(\nabla_i\nabla_j\nabla_k)^2 U\right\rangle_{\mathrm{B}} + \mathcal{O}(\tau^3)$$

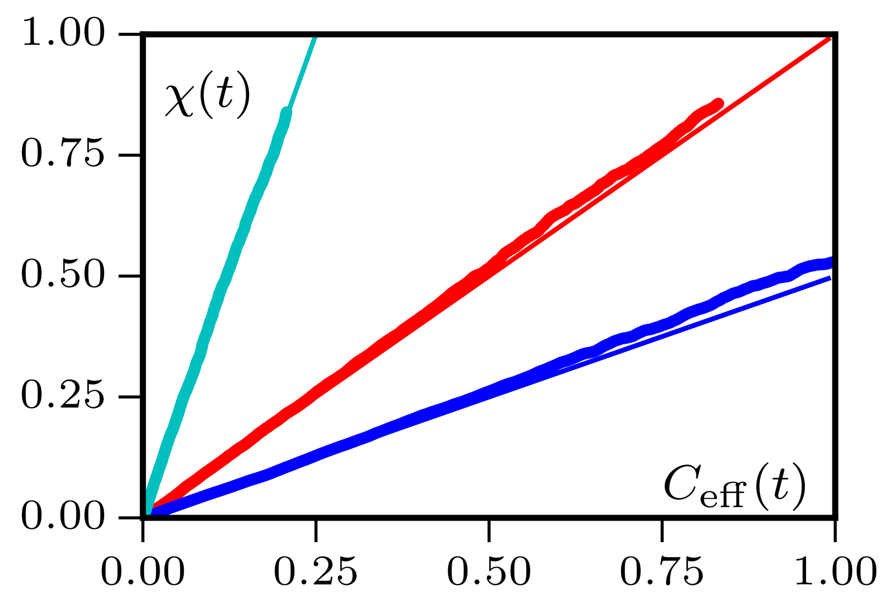
Effective equilibrium regime $\begin{cases}\text{Time-reversal symmetry}\\ \text{Non-Boltzmann statistics}\end{cases}$

Fluctuation-dissipation relation

Protocol to probe the effective equilibrium regime:
compare response with correlations

Response $\quad R(t) = \left.\frac{\delta\langle r_{i\alpha}(t)\rangle}{\delta f_{i\alpha}(0)}\right|_{f=0}$, $\quad$ Susceptibility $\quad \chi(t) = \int_0^t R(t-s)\,\mathrm{d}s$

Correlations $\quad C_r(t) = \langle r_{i\alpha}(t) r_{i\alpha}(0)\rangle$, $\quad C_p(t) = \langle p_{i\alpha}(t) p_{i\alpha}(0)\rangle$



Fluctuation-dissipation relation valid
in the effective equilibrium regime

$$R = -\frac{1}{T}\frac{\mathrm{d}}{\mathrm{d}t}\left(C_r + \tau^2 C_p\right)$$

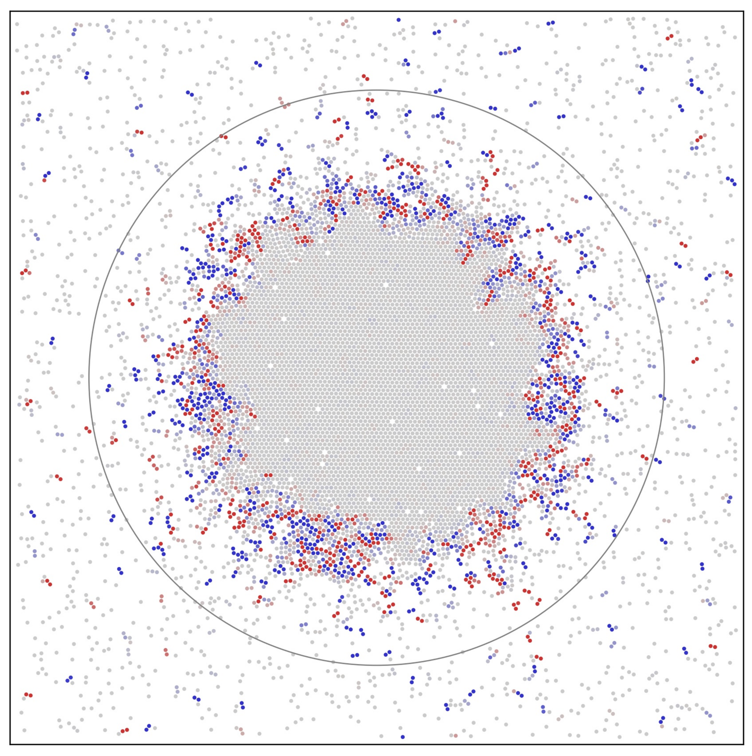
Connection with entropy production

$$\mathcal{S} = \frac{1}{T}\int\omega\left[2T\tilde{R}''(\omega) + \omega\left(\tilde{C}_r(\omega) + \tau^2\tilde{C}_p(\omega)\right)\right]\frac{\mathrm{d}\omega}{2\pi}$$

Violation of fluctuation-dissipation relation

Local entropy production

Relate position density profile with entropy production

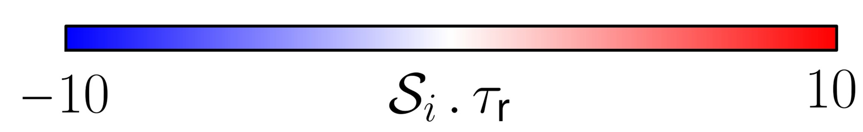


Pair-wise interactions

$$U(\{\mathbf{r}_i\}) = \frac{1}{2}\sum_{i,j} V(\mathbf{r}_i - \mathbf{r}_j)$$

$$\mathcal{S} = \frac{\tau^2}{4T}\sum_{i,j}\left\langle(\mathbf{p}_i - \mathbf{p}_j)^3 : \nabla_i^3 V(\mathbf{r}_i - \mathbf{r}_j)\right\rangle$$

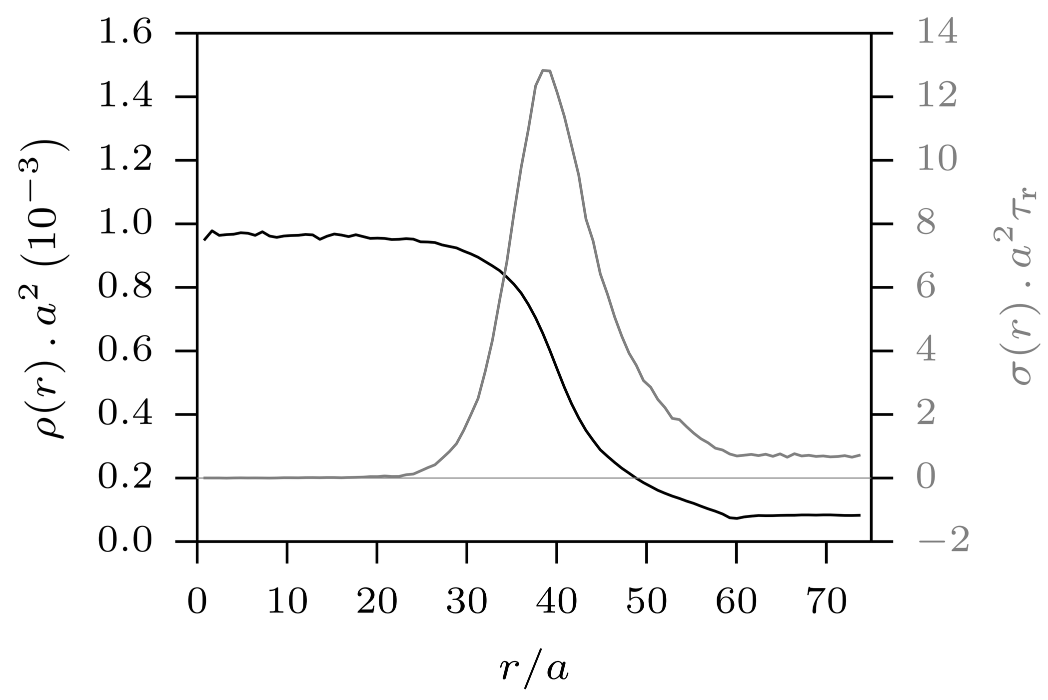
Particle-based measurement of entropy



Spatial decomposition

$$\mathcal{S} = \int\sigma(\mathbf{r})\,\mathrm{d}\mathbf{r}$$

The main contribution to
entropy production is located
at phase interface [1]



Conclusion and perspectives

- **Effective equilibrium regime** controlled by microscopic persistence
  - $\rightarrow$ Fluctuating hydrodynamics of coarse-grained density fields
- Stochastic thermodynamics: defining extracted work and dissipated heat
  - $\rightarrow$ Heat engine operating with active particles

[1] C. Nardini, É. Fodor, E. Tjhung, F. van Wijland, J. Tailleur, and M. E. Cates, Phys. Rev. X **7**, 021007 (2017)

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