Phenomenology of caging in glassy dynamics

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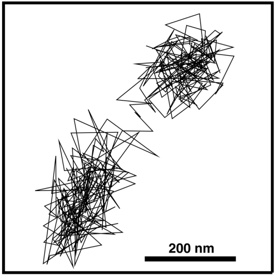
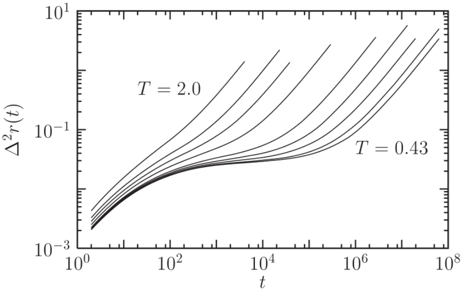
Workshop on non-Gaussian fluctuations
YITP – University of Kyoto

Dense suspension of colloids



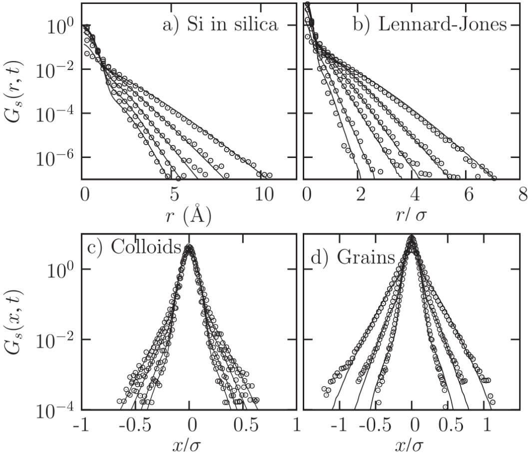
E. R. Weeks, J. C. Crocker, A. C. Levitt, A. Schofield, and D. A. Weitz, Science **287**, 627 (2000)

Lennard-Jones binary mixture



L. Berthier and W. Kob, J. Phys.: Condens. Matter **19**, 205130 (2007)

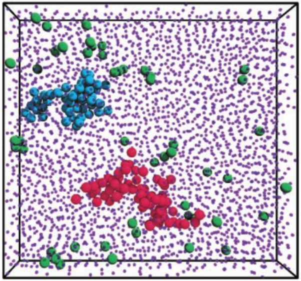
Distribution of displacement



P. Chaudhuri, L. Berthier and W. Kob, Phys. Rev. Lett. **99**, 060604 (2007)

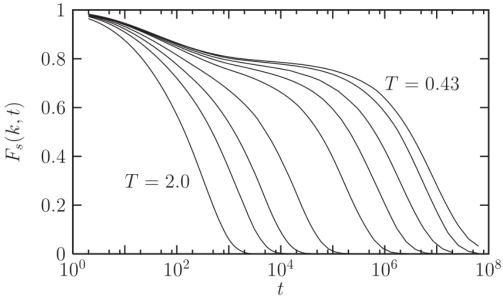
Introduction

Cooperative effects



E. R. Weeks, J. C. Crocker, A. C. Levitt, A. Schofield, and D. A. Weitz, Science **287**, 627 (2000)

Structural relaxation



L. Berthier and W. Kob, J. Phys.: Condens. Matter **19**, 205130 (2007)

Outline

1. Caging model

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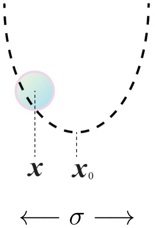
2. Statistics of displacement
3. Structural relaxation
4. Relation with many-body physics

Outline

Caging model

Harmonic cage



Overdamped dynamics

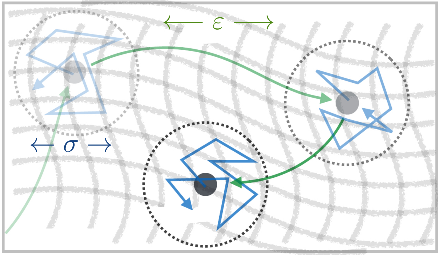
$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{x - x_0}{\tau_\mathrm{R}} + \xi_\mathrm{G}$$

$$\langle \xi_\mathrm{G}(t)\xi_\mathrm{G}(0)\rangle = 2D\delta(t)$$

Diffusion coefficient

$$D = \sigma^2/\tau_\mathrm{R}$$

Active displacement of the cage



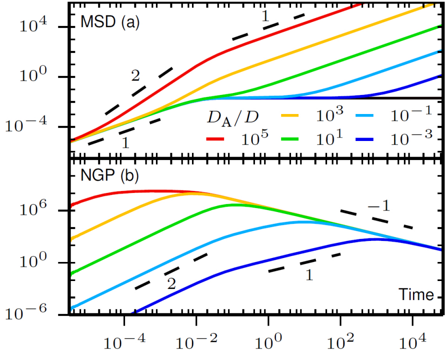
τ_0 time between cage hops

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{x - x_0}{\tau_\mathrm{R}} + \xi_\mathrm{G}, \quad \frac{\mathrm{d}x_0}{\mathrm{d}t} = \xi_\mathrm{NG}$$

$$\langle \xi_\mathrm{NG}(t_1) \cdots \xi_\mathrm{NG}(t_{2n}) \rangle_\mathrm{c} = (2n)! \frac{\varepsilon^{2n}}{\tau_0} \delta(t_1 - t_2) \cdots \delta(t_{2n} - t_{2n-1})$$

Outline

Statistics of displacement



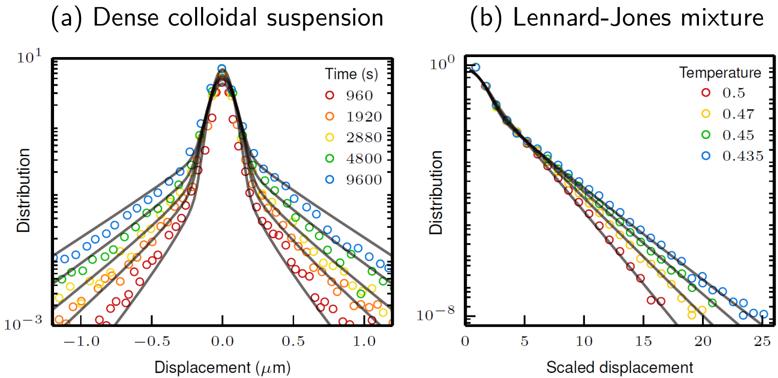
Gaussian regimes

- Short time: MSD $\sim 2Dt$
- Large time: MSD $\sim 2D_A t$

$$D_A = \varepsilon^2/\tau_0$$

$$\text{NGP} = \frac{\langle \Delta x^4 \rangle}{3\langle \Delta x^2 \rangle^2} - 1$$

Statistics of displacement



Data taken from:

(a) Y. Gao and M. L. Kilfoil, Phys. Rev. E **99**, 051406 (2009)

(b) L. Berthier and W. Kob, J. Phys.: Condens. Matter **19**, 205130 (2007)

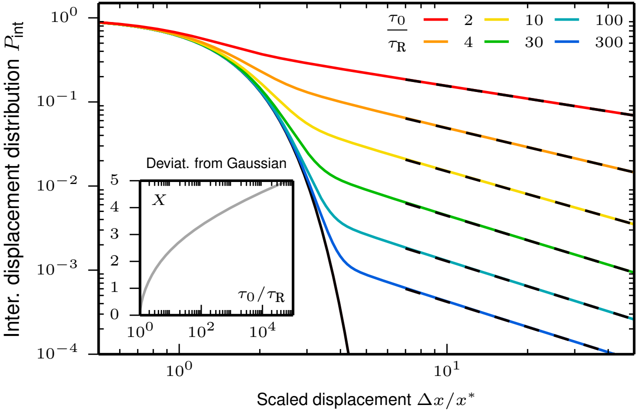
Statistics of displacement

Distribution of intermediate displacement

$$P_{\text{int}}(\Delta x, t) = {}_1F_1\left[\frac{\tau_0 - \tau_{\text{R}}}{2\tau_0}, \frac{1}{2}; -\frac{(\Delta x/\sigma)^2}{4\text{e}^{-t/\tau_{\text{R}}}}\right]$$

- Deviation from Gaussian with power-law
- Scale invariance: variance of central Gaussian

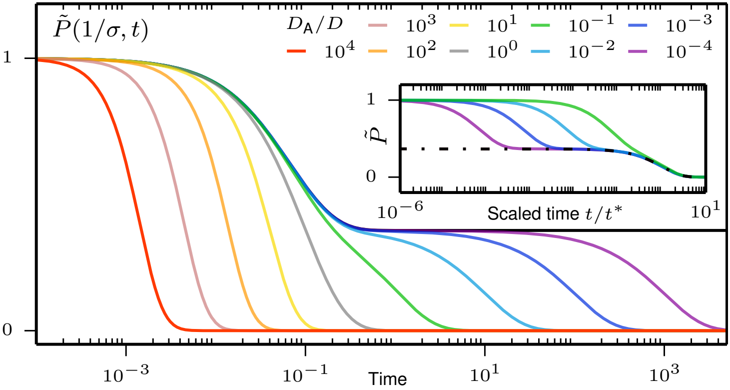
Statistics of displacement



Outline

Structural relaxation

Displacement distribution Fourier transform



Outline

Relation with many-body physics

Displacement distribution

$$\partial_t \tilde{P}(k,t) = -k^2 \left[D_{\text{eff}}(t) + \mu\rho_0 \tilde{V}_{\text{eff}}(k,t) \right] \tilde{P}(k,t)$$

- Diffusion coefficient $\quad D_{\text{eff}}(t) = D \mathrm{e}^{-t/\tau_\mathrm{R}}$
- Potential energy $\quad V_{\text{eff}}(x,t) \propto \exp\left[-\frac{|x|}{\varepsilon\left(1-\mathrm{e}^{-t/\tau_\mathrm{R}}\right)} \right]$

Relation with many-body physics

Interacting particles

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = -\mu \sum_j \frac{\partial}{\partial x_j} V_{\text{eff}}\left(x_i - x_j, t\right) + \sqrt{2D_{\text{eff}}(t)}\xi_{\text{G},i}$$

Density correlation $\quad F(k,t) = \langle \tilde{\rho}(k,t)\tilde{\rho}(k,0)\rangle$

Mean-field equation = one-body caging

Conclusion

Phenomenological model of caging dynamics

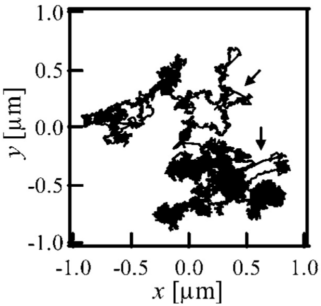
Active cage: non-Gaussian statistics

- Reproduces displacement distribution
- Scale invariance of intermediate displacement
- Relation with many-body physics

ÉF, H. Hayakawa, P. Visco, and F. van Wijland, `arXiv:1601.06613`

Conclusion

Tracer particle in living systems



T. Toyota, D. A. Head, C. F. Schmidt, and D. Mizuno,
Soft Matter **7**, 3234 (2011)