UNIVERSITY OF ENERGY AND NATURAL RESOURCES, SUNYANI, GHANA

SCHOOL OF SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

LEVEL 300 END OF SEMESTER EXAMINATIONS, 2020/2021

BSc. AGRIC/COMP/CIVIL/EEE /ENV /REN /MECH/ PET ENGINEERING

STAT 309: PROBABILITY AND STATISTICS FOR ENGINEERS

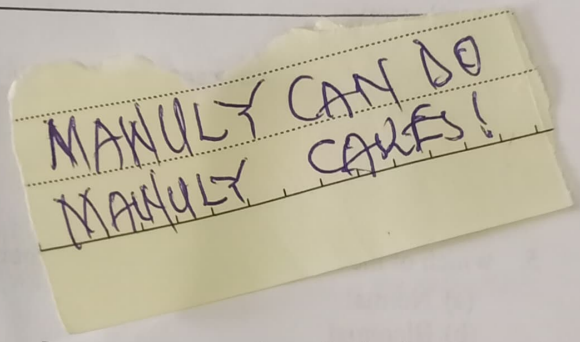
APRIL, 2021 **DURATION: 2 HOURS**

Index Number: _____

Program of Study: _____

Materials Required

1. ***Standard normal table***
2. ***Supplementary Sheets***
3. ***Non-Programmable calculator***



INSTRUCTIONS

- Write down your **index number and programme of study** in the spaces provided on the question booklet before starting the examination.
- The paper consists of only Section A.
- There are **12** printed pages including the back and front pages. Make sure you have all the pages before starting the examination.
- Answer **ALL** questions on the Question paper by circling or ticking the correct answer from options provided
- Submit Question Papers and, supplementary sheet(s) to the invigilator at the end of the examination.

SECTION A: ANSWER ALL QUESTIONS BY CIRCLING OR TICKING THE CORRECT ANSWER FROM THE OPTIONS PROVIDED

1. In a sample of 800 students at UENR, 160, or 20%, are Engineering Students. Based on the above information, the school's paper reported that "20% of all the students at the university are Engineering Students. This report is an example of:
   (a) a sample
   (b) a population
   (c) statistical inference
   (d) descriptive statistics
2. Which the following is NOT an example of a primary data collection
   (a) Personal interviews
   (b) Questionnaires
   (c) Surveys
   (d) Internet source
3. A laboratory assistant measures the weight of a sample of bread. This variable is:
   (a) nominal
   (b) ordinal
   (c) interval
   (d) ratio
4. Which of the following can be termed as observations measured on a numerical scale?
   (a) qualitative data
   (b) quantitative data
   (c) categorical data
   (d) nominal data
5. Which of the following probability distributions has its mean and variance equal?
   (a) Normal
   (b) Binomial
   (c) Poisson
   (d) Gamma
6. How many elements are in the sample space of rolling a die three times?
   (a) 216
   (b) 8
   (c) 36
   (d) 72

Use the information to answer questions 7-8

Suppose that the number of dog bite cases at Sunyani Regional hospital in a year has a Poisson distribution with average 6 bite cases.

7. What is the probability that in a year the number of dog bite cases will be 7?
   (a) 0.1429
   (b) 0.8571
   (c) 0.13768
   (d) 0

8. What is the probability that in a year the number of dog bite cases will be less than 2?

(a) 0.1429
(b) 0.28571
(c) 0.3333
(d) 0.01735

9. A random variable X has the pdf $f(x)$ given by $f(x) = \begin{cases} cxe^{-1}, & x > 0 \\ 0 & x \leq 0 \end{cases}$.

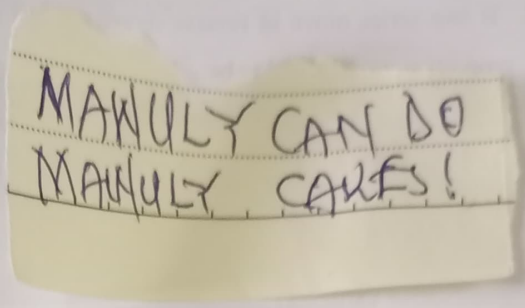
Determine the value of c.

(a) $\frac{1}{6}$

(b) $\frac{1}{9}$

(c) $\frac{1}{3}$

(d) 1



10. The number of telephone calls received by an office averages 4 per minute. Find the probability that no call will arrive in a given one-minute period

(a) 0.01833
(b) 0.9833
(c) 1
(d) 0

11. A large retailer purchases a certain kind of product from a manufacturer. The manufacturer indicates that the defective rate of the product is 3% in a shipment. The inspector of the retailer randomly picks 20 items of the product from a shipment. What is the probability that there will be (3) defective items?

(a) 6.667
(b) 0.150
(c) 0.030
(d) 0.018

12. The weekly amount spent for maintenance and repairs in a certain company was observed, over a long period of time, to be approximately normally distributed with a mean of \$400 and a standard deviation of \$20. If \$450 is budgeted for the week, what is the probability that the actual costs will exceed the budgeted amount?

(a) 0.05
(b) 0.02
(c) 0.0062
(d) 0.9938

13. Using the Normal table, find $P(z \leq -1.95)$.

(a) 0.97441
(b) 0.02619
(c) 0.02559
(d) 0.97381

14. If two series move in reverse directions and the variations in their values are always proportionate, it is said to be:

(a) Negative correlation
(b) Positive correlation
(c) Perfect negative correlation
(d) Perfect positive correlation

15. Which one of the following is NOT appropriate for studying the relationship between two quantitative variables?

(a) Scatterplot
(b) Correlation
(c) Bar chart
(d) Regression

16. Let X be a discrete random variable with probability function $P(X = x) = \frac{2}{3^x}$ for $x = 1,2,3, \ldots$ What is the probability that X is even?

(a) $\frac{1}{4}$
(b) $\frac{2}{7}$
(c) $\frac{1}{3}$
(d) $\frac{2}{3}$

17. For a certain discrete random variable on the non-negative integers, the probability function satisfies the relationships

$P(0) = P(1)$ and $P(k+1) = \frac{1}{k} \cdot P(k)$ for $k = 1,2,3, \ldots$

Find $P(0)$

(a) $\ln e$
(b) e^{-1}
(c) $(e+1)^{-1}$
(d) $e - 1$

18. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} 6x(1-x) & for\ 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Calculate $P\left(|X - 1/2| > \frac{1}{4}\right)$

(a) 0.0521
(b) 0.1563
(c) 0.3125
(d) 0.5000

19. Let X have the density function $f(x) = \begin{cases} {}^{3x^2}/_{\theta^3} & for\ 0 < x < \theta \\ 0 & otherwise \end{cases}$

If $P(X > 1) = {}^{7}/_{8}$, Find the value of θ

(a) ${}^{1}/_{2}$

(b) $\left({}^{7}/_{8}\right)^{1/3}$

(c) $\left({}^{8}/_{7}\right)^{1/3}$

(d) 2

20. If X is a loss random variable with the density function

$f(x) = \begin{cases} 1.4e^{-2x} + 0.9e^{-3x} & for\ x > 0 \\ 0 & otherwise \end{cases}$. Find $E[X]$

(a) ${}^{9}/_{20}$

(b) ${}^{5}/_{6}$

(c) 1

(d) ${}^{23}/_{10}$

21. Let X be a continuous random variable with density function

$f(x) = \begin{cases} \frac{1}{30}x(1+3x) & for\ 1 < x < 3 \\ 0 & otherwise \end{cases}$. Find $E\left[\frac{1}{X}\right]$

(a) ${}^{1}/_{12}$

(b) ${}^{7}/_{15}$

(c) ${}^{45}/_{104}$

(d) ${}^{11}/_{20}$

22. A system made of 7 components with independent, identically distributed lifetimes will operate until any of 1 of the system's component fails. If the lifetime X of each component has density function $f(x) = \begin{cases} \frac{3}{x^4} & for \quad 1 < x \\ 0 & otherwise \end{cases}$

What is the expected lifetime until failure of the system?

(a) 1.02

(b) 1.03

(c) 1.04

(d) 1.05

23. Two players put one dollar into a pot. They decide to throw a pair of dice alternatively. The first one who throws a total of 5 on both dice wins the pot. How much should the player who starts add to the pot to make this a fair game?

(a) ${}^{9}/_{17}$

(b) ${}^{8}/_{17}$

(c) ${}^{1}/_{8}$

(d) ${}^{2}/_{9}$

MAWULY CAN DO MAWULY CAN'T!

24. Let X have a density function $f(x) = \begin{cases} \frac{2x}{k^2} & for \quad 0 < x < k \\ 0 & otherwise \end{cases}$

For what value of k is the variance of X equal to 2?

(a) 2
(b) 6
(c) 9
(d) 18

25. A student received a grade of 80 in French final exam where the mean grade was 72 and the standard deviation was s. In the Statistics final, he received a 90, where the mean grade was 80 and standard deviation was 15. If the standardized scores were the same in each case, find s.

(a) 10
(b) 12
(c) 16
(d) 18

26. If X has a normal distribution with mean 1 and variance 4, find $P(X^2 - 2X \leq 8)$

(a) 0.13
(b) 0.43
(c) 0.75
(d) 0.87

27. A survey of 1000 people determined that 80% like walking and 60% like biking and all like at least one of the two activities. What percentage of people like biking but not walking?

(a) 0
(b) 0.1
(c) 0.2
(d) 0.3

28. Let A, B, C and D be events such that $B = A'$, $C \cap D = \emptyset$, $P(A) = 1/4$, $P(B) = 3/4$, $P(C|A) = 1/2$, $P(C|B) = 3/4$, $P(D|A) = 1/4$ and $P(D|B) = 1/8$. Calculate $P(C \cup D)$

(a) $5/32$
(b) $1/4$
(c) $27/32$
(d) 1

29. Two bowls each contain 5 black and 5 white balls. A ball is chosen at random from bowl 1 and put into bowl 2. A ball is then chosen at random from bowl 2 and put into bowl 1. Find the probability that bowl 1 still has 5 5 black and 5 white balls

(a) $^2/_3$

(b) $^3/_5$

(c) $^6/_{11}$

(d) $^6/_{13}$

30. A fair coin is tossed. If a head occurs, 1 fair die is rolled; if a tail occurs, 2 fair dice are rolled. If Y is the total on the die or dice, Find $P(Y = 6)$

(a) $^1/_9$

(b) $^5/_{36}$

(c) $^{11}/_{72}$

(d) $^1/_6$

Use the following to answer questions 31 - 33

Urn I has 2 white and 3 black balls; Urn **II**, 4 white and 1 black; and Urn **III**, 3 white and 4 black. An urn is selected at random and a ball drawn at random is found to be white.

31. Find the total probability of selecting a white ball at random.

(a) $\frac{1}{3} \times \frac{2}{5}$

(b) $\frac{2}{5} + \frac{4}{5} + \frac{4}{5}$

(c) $\frac{1}{3}\left(\frac{2}{5} + \frac{4}{5} + \frac{3}{7}\right)$

(d) $\frac{9}{17}$

32. Find the total probability of selecting a black ball at random.

(a) $\frac{1}{3} \times \frac{3}{5}$

(b) $\frac{3}{5} + \frac{1}{5} + \frac{4}{5}$

(c) $\frac{1}{3}\left(\frac{2}{5} + \frac{4}{5} + \frac{3}{7}\right)$

(d) $1 - \frac{9}{17}$

33. Find the probability that Urn I was selected.

(a) $\frac{14}{57}$

(b) $\frac{2}{15}$

(c) $\frac{16}{35}$

(d) $\frac{19}{35}$

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34. A sample space consists of 3 sample points with associated probabilities given by $2p$, p^2 and $4p - 1$. Find the value of p.

(a) $-3 \pm \sqrt{11}$
(b) $-3 - \sqrt{11}$
(c) $-3 + \sqrt{11}$
(d) $3 - \sqrt{11}$

Use the following to answer questions 35 and 36

Suppose that $p(x) = c/3^x, x = 1,2, \ldots,$ is the probability function for a random variable X.

35. Determine c.
(a) 2
(b) 2.25
(c) 1.5
(d) 1.8

36. Find $P(2 \leq X < 5)$
(a) $\frac{26}{81}$
(b) $\frac{13}{36}$
(c) $\frac{13}{54}$
(d) $\frac{13}{45}$

37. Which of the following is a **false** property of a standard normal distribution?
I: the mean is zero (0) and the standard deviation is 1.
II: the distribution is symmetric about the mean.
III: the mean, mode and median are the same.
IV: $P(-1 \leq Z \leq 1) \approx 0.68$.
(a) I only
(b) IV only
(c) All the above
(d) None of the above.

38. Let C and D be two events with $P(C) = 0.25, P(D) = 0.45$, and $P(C \cap D) = 0.1$. What is $P(C^c \cap D)$?
(a) 0.35
(b) 0.15
(c) 0.1125
(d) 0.045

39. What percentage of a normal distribution is found within a range of z scores from $P(-2 \leq Z \leq 2)$?
(a) 68%
(b) 80%
(c) 95%
(d) 54%

40. Suppose X is a random variable with $E(X) = 5$ and $\text{Var}(X) = 2$. What is $E(X^2)$
(a) 27
(b) 25
(c) 4
(d) 24

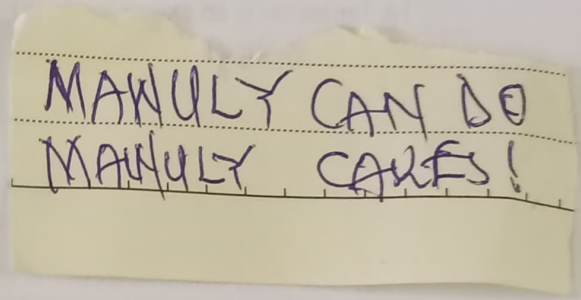
41. Suppose that X takes values between 0 and 1 and has probability density function $2x$. Compute the $E(X)$
    (a) 2/3
    (b) 1/3
    (c) 1
    (d) 2

42. Let x be a binomial random variable with $n = 25$ and $p = 0.6$. Using the normal distribution to approximate the binomial, determine the probability that more than 12 successes will occur.
    (a) 0.0808
    (b) 0.1539
    (c) 0.9192
    (d) 0.8461

Use the following to answer questions 43 – 45

Many utility companies have begun to promote energy conservation by offering discount rates to customers who keep their energy usage below certain established subsidy standards. A recent EPA report notes that 70% of residents in a town have reduced their electricity consumption sufficiently to qualify for discount rates. Suppose 20 residential subscribers are randomly selected from the town.

43. Find the expected number of residents who qualify for the subsidy.
    (a) 6
    (b) 14
    (c) 4.2
    (d) 25



44. What is the variance of this distribution?
    (a) 6
    (b) 4.2
    (c) 12
    (d) 5

45. What is the probability that at least four qualify for the favorable rates?
    (a) 0.8929
    (b) 0.9999
    (c) 0.5436
    (d) 0.7756

46. An instructor grades exam 20%, term paper 30% and final exam 50%. A student had grades of 83,72 and 90, respectively for exams, term paper and final exam. Compute the student's final average using the weighted mean.
    (a) 83.2
    (b) 57.5
    (c) 33.3
    (d) 81.7

47. The variance of 15 observations is 4. If each observation is increased by 9, the variance of the resulting observation is:
    (a) 2
    (b) 3
    (c) 4
    (d) 5

48. Probability that a man will be alive 25 years hence is 0.3 and the probability that his wife will be alive 25 years hence is 0.4. Find the probability that 25 years hence only the man will be alive
(a) 0.12
(b) 0.18
(c) 0.28
(d) 0.42

49. In a certain college, the students engage in sports in the following proportion Football (F): 60% of all students Basketball (B): 50% of all students. Both football and basketball: 30% of all students. If a student is selected at random the probability that he will play neither sports is
(a) 0.80
(b) 0.10
(c) 0.70
(d) 0.20

50. A researcher is interested in the travel time of UENR students to school. A group of 50 students is interviewed. Their mean travel time in 16.7 minutes. For this study the mean of 16.7 minutes is an example of a(n)
(a) Parameter
(b) Statistic
(c) Population
(d) Sample

51. Which of the following statements is the most accurate description for the concept of standard deviation?
(a) The total distance from the smallest score to the highest score.
(b) The square root of the total distance from the smallest score to the highest score.
(c) The squared average distance between all scores and the mean.
(d) The average distance between a score and the mean.

52. A population has a mean of $\mu=35$ and a standard deviation of $\sigma=5$. After 3 points are added to every score in the population, what are the new values for the mean and standard deviation?
(a) $\mu=35$ and $\sigma=5$
(b) $\mu=35$ and $\sigma=8$
(c) $\mu=38$ and $\sigma=5$
(d) $\mu=38$ and $\sigma=8$

53. A sample of 400 Regina households is selected and several variables are recorded. Which of the following statements is correct?
(a) Total household income (in $) is interval level data.
(b) Socioeconomic status (recorded as "low income", "middle income", or "high income" is nominal level data.
(c) The number of people living in a household is a discrete variable.
(d) The primary language spoken in the household is ordinal level data

Use the following information below to questions 54 and 55

The cholesterol content of large chicken eggs is normally distributed with a mean of 200 milligrams and standard deviation 15 milligrams.

54. What proportion of these eggs have cholesterol content above 205 milligrams?
    (a) 0.2004
    (b) 0.6293
    (c) 0.3300
    (d) 0.3707
55. In sixty-seven percent of the eggs, the cholesterol content is less than a certain value "C". Find the value of "C".
    (a) 0.33
    (b) 206.6
    (c) 210
    (d) 193.4
56. The following table lists the number of days that five houses had been up for sale, as well as their selling price.

X (days)	45	12	3	17	32
Y (1000$)	275	401	420	212	365

Calculate the correlation coefficient r between the number of days (X) and the selling price (Y) for this sample.

    (a) 0.465
    (b) 0.219
    (c) 0.219
    (d) 0.512
57. According to a survey, 75% of all customers will return to the same grocery store. Suppose eight customers are selected at random. What is the probability that exactly five of the customers will return?
    (a) 0.0467
    (b) 0.2076
    (c) 0 .2541
    (d) 0.1468
58. Let X be a binomial variable with parameters n and p. If $n = 1$, the distribution of X reduces to
    (a) Poisson
    (b) Binomial
    (c) Bernoulli
    (d) Geometric
59. The number of arrivals per hour at an automatic teller machine is Poisson distributed with a mean of 3.5 arrivals/hour. What is the probability that more than three arrivals occur in an hour?
    (a) 0.3209
    (b) 0.4633
    (c) 0.5367
    (d) 0.6791

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60. A statistics instructor has established that 10 percent of all the students who take his course receive a failing grade. If 10 students have enrolled for his course next semester, the probability that *at most one* of these students will fail is

(a) 0.387.
(b) 0.100.
(c) 0.651.
(d) 0.736

END OF PAPER