CHAPTER 3

Half-Wave Rectifiers

The Basics of Analysis

3.1 INTRODUCTION

A rectifier converts ac to dc. The purpose of a rectifier may be to produce an output that is purely dc, or the purpose may be to produce a voltage or current waveform that has a specified dc component.

In practice, the half-wave rectifier is used most often in low-power applications because the average current in the supply will not be zero, and nonzero average current may cause problems in transformer performance. While practical applications of this circuit are limited, it is very worthwhile to analyze the half-wave rectifier in detail. A thorough understanding of the half-wave rectifier circuit will enable the student to advance to the analysis of more complicated circuits with a minimum of effort.

The objectives of this chapter are to introduce general analysis techniques for power electronics circuits, to apply the power computation concepts of the previous chapter, and to illustrate PSpice solutions.

3.2 RESISTIVE LOAD

Creating a DC Component Using an Electronic Switch

A basic half-wave rectifier with a resistive load is shown in Fig. 3-1*a*. The source is ac, and the objective is to create a load voltage that has a nonzero dc component. The diode is a basic electronic switch that allows current in one direction only. For the positive half-cycle of the source in this circuit, the diode is on (forward-biased).

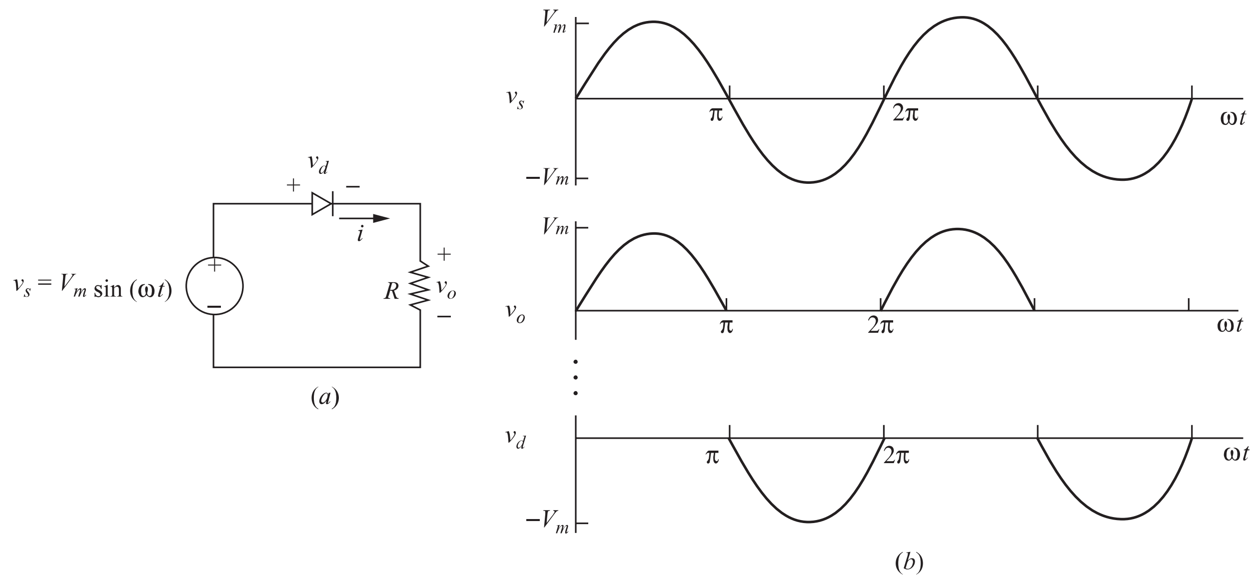


Figure 3-1 (*a*) Half-wave rectifier with resistive load; (*b*) Voltage waveforms.

Considering the diode to be ideal, the voltage across a forward-biased diode is zero and the current is positive.

For the negative half-cycle of the source, the diode is reverse-biased, making the current zero. The voltage across the reverse-biased diode is the source voltage, which has a negative value.

The voltage waveforms across the source, load, and diode are shown in Fig. 3-1*b*. Note that the units on the horizontal axis are in terms of angle (ωt). This representation is useful because the values are independent of frequency. The dc component V_o of the output voltage is the average value of a half-wave rectified sinusoid

$$V_o = V_{\text{avg}} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi} \tag{3-1}$$

The dc component of the current for the purely resistive load is

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} \tag{3-2}$$

Average power absorbed by the resistor in Fig. 3-1*a* can be computed from $P = I^2_{\text{rms}}R = V^2_{\text{rms}}/R$. When the voltage and current are half-wave rectified sine waves,

$$\begin{aligned} V_{\text{rms}} &= \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \sin(\omega t)]^2\, d(\omega t)} = \frac{V_m}{2} \\ I_{\text{rms}} &= \frac{V_m}{2R} \end{aligned} \tag{3-3}$$

In the preceding discussion, the diode was assumed to be ideal. For a real diode, the diode voltage drop will cause the load voltage and current to be reduced, but not appreciably if V_m is large. For circuits that have voltages much larger than the typical diode drop, the improved diode model may have only second-order effects on the load voltage and current computations.

EXAMPLE 3-1

Half-Wave Rectifier with Resistive Load

For the half-wave rectifier of Fig. 3-1*a*, the source is a sinusoid of 120 V rms at a frequency of 60 Hz. The load resistor is 5 Ω. Determine (*a*) the average load current, (*b*) the average power absorbed by the load and (*c*) the power factor of the circuit.

■ **Solution**

(*a*) The voltage across the resistor is a half-wave rectified sine wave with peak value $V_m = 120\sqrt{2} = 169.7$ V. From Eq. (3-2), the average voltage is V_m/π, and average current is

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} = \frac{\sqrt{2}(120)}{5\pi} = 10.8 \text{ A}$$

(*b*) From Eq. (3-3), the rms voltage across the resistor for a half-wave rectified sinusoid is

$$V_{\text{rms}} = \frac{V_m}{2} = \frac{\sqrt{2}(120)}{2} = 84.9 \text{ V}$$

The power absorbed by the resistor is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{84.9^2}{4} = 1440 \text{ W}$$

The rms current in the resistor is $V_m/(2R) = 17.0$ A, and the power could also be calculated from $I_{\text{rms}}^2 R = (17.0)^2(5) = 1440$ W.

(*c*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{1440}{(120)(17)} = 0.707$$

3.3 RESISTIVE-INDUCTIVE LOAD

Industrial loads typically contain inductance as well as resistance. As the source voltage goes through zero, becoming positive in the circuit of Fig. 3-2*a*, the diode becomes forward-biased. The Kirchhoff voltage law equation that describes the current in the circuit for the forward-biased ideal diode is

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} \tag{3-4}$$

The solution can be obtained by expressing the current as the sum of the forced response and the natural response:

$$i(t) = i_f(t) + i_n(t) \tag{3-5}$$

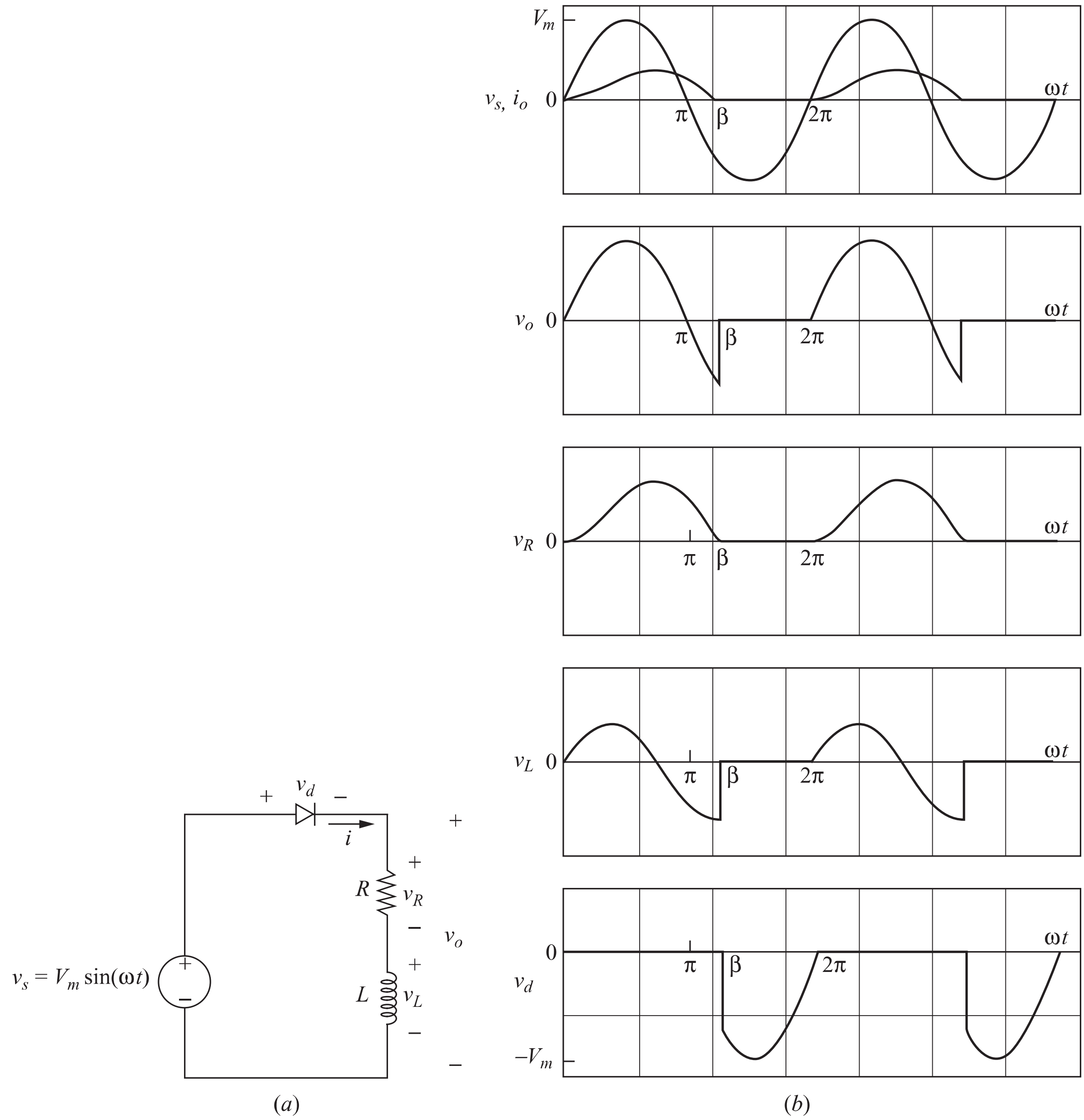


Figure 3-2 (*a*) Half-wave rectifier with an *RL* load; (*b*) Waveforms.

The forced response for this circuit is the current that exists after the natural response has decayed to zero. In this case, the forced response is the steady-state sinusoidal current that would exist in the circuit if the diode were not present. This steady-state current can be found from phasor analysis, resulting in

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta) \tag{3-6}$$

$$\text{where} \qquad Z = \sqrt{R^2 + (\omega L)^2} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The natural response is the transient that occurs when the load is energized. It is the solution to the homogeneous differential equation for the circuit without the source or diode:

$$Ri(t) + L\frac{di(t)}{dt} = 0 \tag{3-7}$$

For this first-order circuit, the natural response has the form

$$i_n(t) = Ae^{-t/\tau} \tag{3-8}$$

where τ is the time constant L/R and A is a constant that is determined from the initial condition. Adding the forced and natural responses gets the complete solution.

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau} \tag{3-9}$$

The constant A is evaluated by using the initial condition for current. The initial condition of current in the inductor is zero because it was zero before the diode started conducting and it cannot change instantaneously.

Using the initial condition and Eq. (3-9) to evaluate A yields

$$\begin{aligned} i(0) &= \frac{V_m}{Z}\sin(0 - \theta) + Ae^0 = 0 \\ A &= -\frac{V_m}{Z}\sin(-\theta) = \frac{V_m}{Z}\sin\theta \end{aligned} \tag{3-10}$$

Substituting for A in Eq. (3-9) gives

$$\begin{aligned} i(t) &= \frac{V_m}{Z}\sin(\omega t - \theta) + \frac{V_m}{Z}\sin(\theta)e^{-t/\tau} \\ &= \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-t/\tau}\right] \end{aligned} \tag{3-11}$$

It is often convenient to write the function in terms of the angle ωt rather than time. This merely requires ωt to be the variable instead of t. To write the above equation in terms of angle, t in the exponential must be written as ωt, which requires τ to be multiplied by ω also. The result is

$$i(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] \tag{3-12}$$

A typical graph of circuit current is shown in Fig. 3-2*b*. Equation (3-12) is valid for positive currents only because of the diode in the circuit, so current is zero when the function in Eq. (3-12) is negative. When the source voltage again becomes positive, the diode turns on, and the positive part of the waveform in Fig. 3-2*b* is repeated. This occurs at every positive half-cycle of the source. The voltage waveforms for each element are shown in Fig. 3-2*b*.

Note that the diode remains forward-biased longer than π rad and that the source is negative for the last part of the conduction interval. This may seem unusual, but an examination of the voltages reveals that Kirchhoff's voltage law is satisfied and there is no contradiction. Also note that the inductor voltage is negative when the current is decreasing ($v_L = L\,di/dt$).

The point when the current reaches zero in Eq. (3-12) occurs when the diode turns off. The first positive value of ωt in Eq. (3-12) that results in zero current is called the extinction angle β.

Substituting $\omega t = \beta$ in Eq. (3-12), the equation that must be solved is

$$i(\beta) = \frac{V_m}{Z}\left[\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau}\right] = 0 \tag{3-13}$$

which reduces to

$$\boxed{\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau} = 0} \tag{3-14}$$

There is no closed-form solution for β, and some numerical method is required. To summarize, the current in the half-wave rectifier circuit with *RL* load (Fig. 3-2) is expressed as

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases} \tag{3-15}$$

$$\text{where} \quad Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{and} \quad \tau = \frac{L}{R}$$

The average power absorbed by the load is $I_{\text{rms}}^2 R$, since the average power absorbed by the inductor is zero. The rms value of the current is determined from the current function of Eq. (3-15).

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\,d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\,d(\omega t)} \tag{3-16}$$

Average current is

$$I_o = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)\,d(\omega t) \tag{3-17}$$

EXAMPLE 3-2

Half-Wave Rectifier with *RL* Load

For the half-wave rectifier of Fig. 3-2*a*, $R = 100\ \Omega$, $L = 0.1$ H, $\omega = 377$ rad/s, and $V_m = 100$ V. Determine (*a*) an expression for the current in this circuit, (*b*) the average current, (*c*) the rms current, (*d*) the power absorbed by the *RL* load, and (*e*) the power factor.

■ **Solution**

For the parameters given,

$$Z = [R^2 + (\omega L)^2]^{0.5} = 106.9\,\Omega$$
$$\theta = \tan^{-1}(\omega L/R) = 20.7° = 0.361 \text{ rad}$$
$$\omega t = \omega L/R = 0.377 \text{ rad}$$

(*a*) Equation (3-15) for current becomes

$$i(\omega t) = 0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377} \quad \text{A} \qquad \text{for } 0 \le \omega t \le \beta$$

Beta is found from Eq. (3-14).

$$\sin(\beta - 0.361) + \sin(0.361)e^{-\beta/0.377} = 0$$

Using a numerical root-finding program, β is found to be 3.50 rad, or 201°

(*b*) Average current is determined from Eq. (3-17).

$$I_o = \frac{1}{2\pi}\int_0^{3.50} \left[0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}\right] d(\omega t) = 0.308 \text{ A}$$

(A numerical integration program is recommended.)

(*c*) The rms current is found from Eq. (3-16) to be

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{3.50} \left[0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}\right]^2 d(\omega t)} = 0.474 \text{ A}$$

(*d*) The power absorbed by the resistor is

$$P = I_{\text{rms}}^2 R = (0.474)^2(100) = 22.4 \text{ W}$$

The average power absorbed by the inductor is zero. Also P can be computed from the definition of average power:

$$P = \frac{1}{2\pi}\int_0^{2\pi} p(\omega t)\,d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t) i(\omega t)\,d(\omega t)$$
$$= \frac{1}{2\pi}\int_0^{3.50} [100\sin(\omega t)]\left[0.936\sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}\right] d(\omega t)$$
$$= 22.4 \text{ W}$$

(*e*) The power factor is computed from the definition pf $= P/S$, and P is power supplied by the source, which must be the same as that absorbed by the load.

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{\text{rms}}} = \frac{22.4}{\left(100/\sqrt{2}\right)0.474} = 0.67$$

Note that the power factor is *not* cos θ.

3.4 PSPICE SIMULATION

Using Simulation Software for Numerical Computations

A computer simulation of the half-wave rectifier can be performed using PSpice. PSpice offers the advantage of having the postprocessor program Probe which can display the voltage and current waveforms in the circuit and perform numerical computations. Quantities such as the rms and average currents, average power absorbed by the load, and power factor can be determined directly with PSpice. Harmonic content can be determined from the PSpice output.

A transient analysis produces the desired voltages and currents. One complete period is a sufficient time interval for the transient analysis.

EXAMPLE 3-3

PSpice Analysis

Use PSpice to analyze the circuit of Example 3-2.

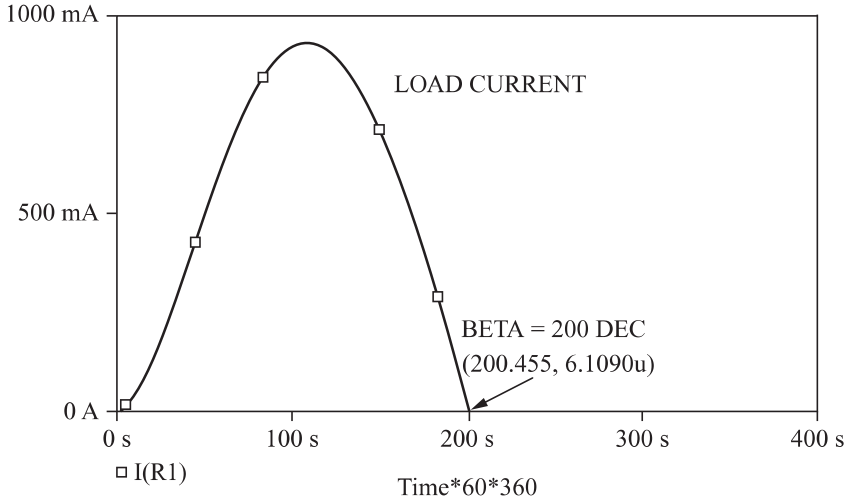
■ Solution

The circuit of Fig. 3-2*a* is created using VSIN for the source and Dbreak for the diode. In the simulation settings, choose Time Domain (transient) for the analysis type, and set the Run Time to 16.67 ms for one period of the source. Set the Maximum Step Size to 10 μs to get adequate sampling of the waveforms. A transient analysis with a run time of 16.67 ms (one period for 60 Hz) and a maximum step size of 10 μs is used for the simulation settings.

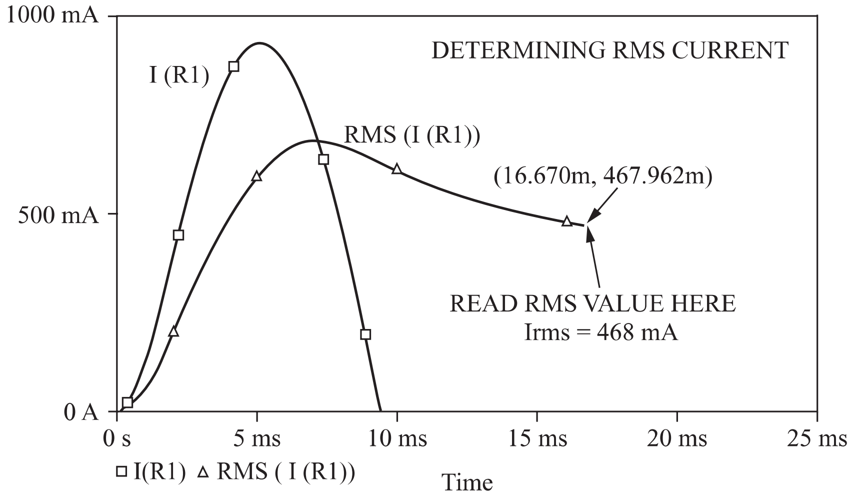
If a diode model that approximates an ideal diode is desired for the purpose of comparing the simulation with analytical results, editing the PSpice model and using $n = 0.001$ will make the voltage drop across the forward-biased diode close to zero. Alternatively, a model for a power diode may be used to obtain a better representation of a real rectifier circuit. For many circuits, voltages and currents will not be affected significantly when different diode models are used. Therefore, it may be convenient to use the Dbreak diode model for a preliminary analysis.

When the transient analysis is performed and the Probe screen appears, display the current waveform by entering the expression I(R1). A method to display angle instead of time on the x axis is to use the x-variable option within the x-axis menu, entering TIME*60*360. The factor of 60 converts the axis to periods ($f = 60$ Hz), and the factor 360 converts the axis to degrees. Entering TIME*60*2*3.14 for the x variable converts the x axis to radians. Figure 3-3*a* shows the result. The extinction angle β is found to be 200° using the cursor option. Note that using the default diode model in PSpice resulted in a value of β very close to the 201° in Example 3-2.

Probe can be used to determine numerically the rms value of a waveform. While in Probe, enter the expression RMS(I(R1)) to obtain the rms value of the resistor current. Probe displays a "running" value of the integration in Eq. (3-16), so the appropriate value is *at the end of one or more complete periods* of the waveform. Figure 3-3*b* shows how to obtain the rms current. The rms current is read as approximately 468 mA. This compares



(*a*)



(*b*)

Figure 3-3 (*a*) Determining the extinction angle β in Probe. The time axis is changed to angle using the x-variable option and entering Time*60*360; (*b*) Determining the rms value of current in Probe.

very well with the 474 mA calculated in Example 3-2. Remember that the default diode model is used in PSpice and an ideal diode was used in Example 3-2. The average current is found by entering AVG(I(R1)), resulting in I_o = 304 mA.

PSpice is also useful in the design process. For example, the objective may be to design a half-wave rectifier circuit to produce a specified value of average current by selecting the proper value of L in an RL load. Since there is no closed-form solution, a trial-and-error iterative method must be used. A PSpice simulation that includes a parametric sweep is used to try several values of L. Example 3-4 illustrates this method.

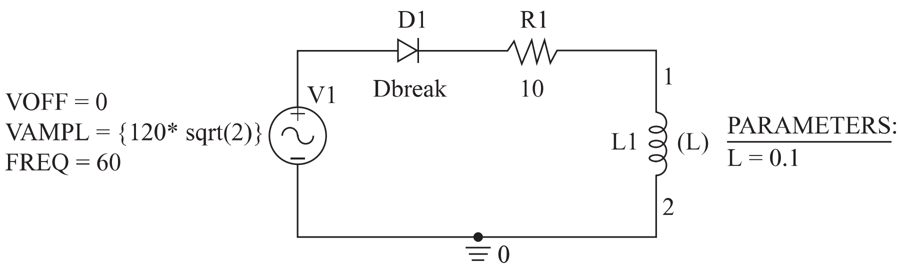
EXAMPLE 3-4

Half-Wave Rectifier Design Using PSpice

Design a circuit to produce an average current of 2.0 A in a 10-Ω resistance. The source is 120 V rms at 60 Hz.

■ Solution

A half-wave rectifier is one circuit that can be used for this application. If a simple half-wave rectifier with the 10-Ω resistance were used, the average current would be $(120\sqrt{2}/\pi)/8 = 6.5$ A. Some means must be found to reduce the average current to the specified 2 A. A series resistance could be added to the load, but resistances absorb power. An added series inductance will reduce the current without adding losses, so an



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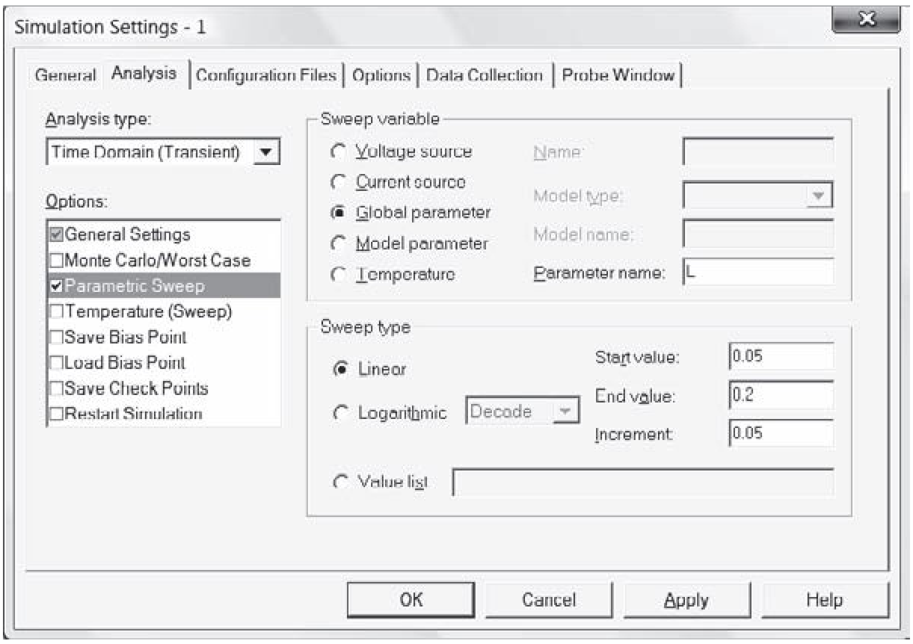
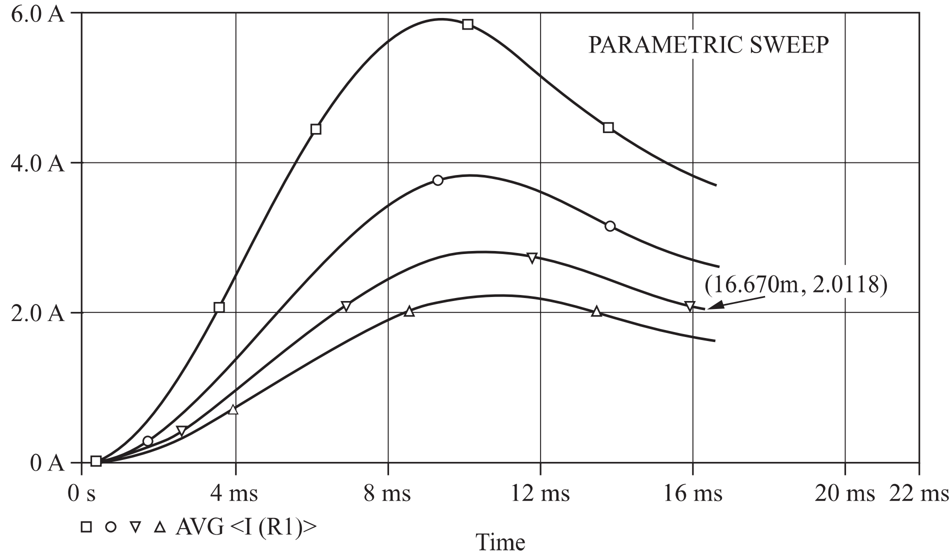


(*b*)

Figure 3-4 (*a*) PSpice circuit for Example 3-4; (*b*) A parametric sweep is established in the Simulation Settings box; (*c*) $L = 0.15$ H for an average current of approximately 2 A.



(*c*)

Figure 3-4 (*continued*)

inductor is chosen. Equations (3-15) and (3-17) describe the current function and its average for *RL* loads. There is no closed-form solution for *L*. A trial-and-error technique in PSpice uses the parameter (PARAM) part and a parametric sweep to try a series of values for *L*. The PSpice circuit and the Simulation Settings box are shown in Fig. 3-4.

Average current in the resistor is found by entering AVG(I(R1)) in Probe, yielding a family of curves for different inductance values (Fig. 3-4*c*). The third inductance in the sweep (0.15 H) results in an average current of 2.0118 A in the resistor, which is very close to the design objective. If further precision is necessary, subsequent simulations can be performed, narrowing the range of *L*.

3.5 *RL*-SOURCE LOAD

Supplying Power to a DC Source from an AC Source

Another variation of the half-wave rectifier is shown in Fig. 3-5*a*. The load consists of a resistance, an inductance, and a dc voltage. Starting the analysis at $\omega t = 0$ and assuming the initial current is zero, recognize that the diode will remain off as long as the voltage of the ac source is less than the dc voltage. Letting α be the value of ωt that causes the source voltage to be equal to V_{dc},

$$V_m \sin \alpha = V_{dc}$$

or

$$\boxed{\alpha = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)} \tag{3-18}$$

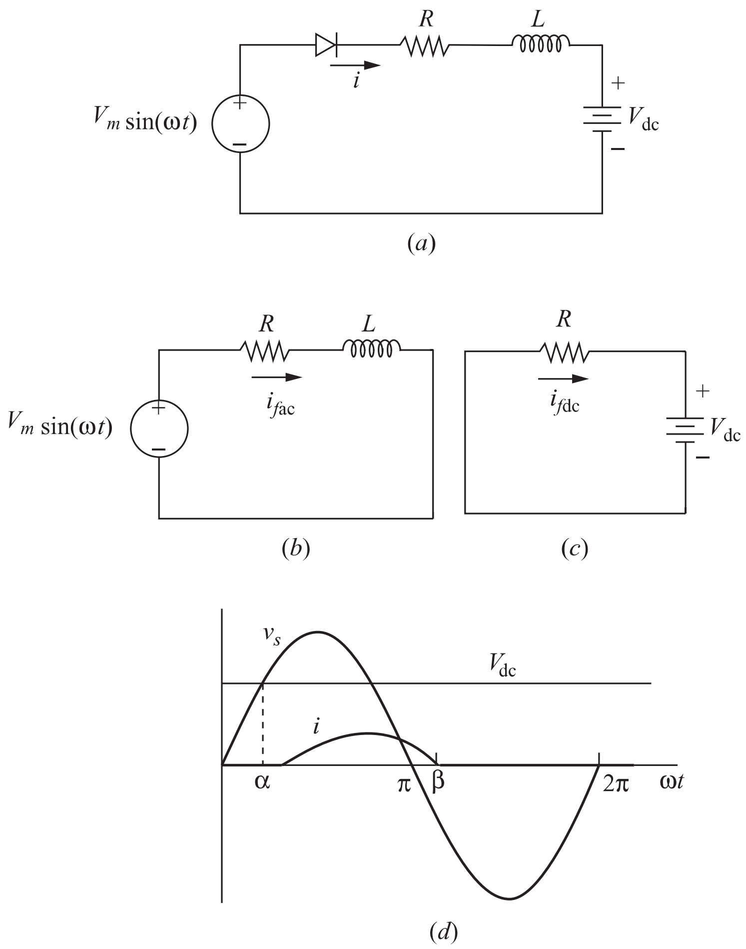


Figure 3-5 (*a*) Half-wave rectifier with *RL* source load; (*b*) Circuit for forced responce from ac source; (*c*) Circuit for forced responce from dc source; (*d*) Waveforms.

The diode starts to conduct at $\omega t = \alpha$. With the diode conducting, Kirchhoff's voltage law for the circuit yields the equation

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} + V_{dc} \tag{3-19}$$

Total current is determined by summing the forced and natural responses:

$$i(t) = i_f(t) + i_n(t)$$

The current $i_f(t)$ is determined using superposition for the two sources. The forced response from the ac source (Fig. 3-5*b*) is $(V_m/Z)\sin(\omega t - \theta)$. The forced response due to the dc source (Fig. 3-5*c*) is $-V_{dc}/R$. The entire forced response is

$$i_f(t) = \frac{V_m}{Z}\sin(\omega t - \theta) - \frac{V_{dc}}{R} \tag{3-20}$$

The natural response is

$$i_n(t) = Ae^{-t/\tau} \tag{3-21}$$

Adding the forced and natural responses gives the complete response.

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}} \tag{3-22}$$

The extinction angle β is defined as the angle at which the current reaches zero, as was done earlier in Eq. (3-15). Using the initial condition of $i(\alpha) = 0$ and solving for A,

$$\boxed{A = \left[-\frac{V_m}{Z}\sin(\alpha - \beta) + \frac{V_{\text{dc}}}{R}\right]e^{\alpha/\omega\tau}} \tag{3-23}$$

Figure 3-5*d* shows voltage and current waveforms for a half-wave rectifier with *RL*-source load.

The average power absorbed by the resistor is $I^2_{\text{rms}}R$, where

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_\alpha^\beta i^2(\omega t)\,d(\omega t)} \tag{3-24}$$

The average power absorbed by the dc source is

$$P_{\text{dc}} = I_o V_{\text{dc}} \tag{3-25}$$

where I_o is the average current, that is,

$$I_o = \frac{1}{2\pi}\int_\alpha^\beta i(\omega t)\,d(\omega t) \tag{3-26}$$

Assuming the diode and the inductor to be ideal, there is no average power absorbed by either. The power supplied by the ac source is equal to the sum of the power absorbed by the resistor and the dc source

$$P_{\text{ac}} = I^2_{\text{rms}}R + I_o V_{\text{dc}} \tag{3-27}$$

or it can be computed from

$$P_{\text{ac}} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t)\,i(\omega t)\,d(\omega t) = \frac{1}{2\pi}\int_\alpha^\beta (V_m \sin\omega t)\,i(\omega t)\,d(\omega t) \tag{3-28}$$

EXAMPLE 3-5

Half-Wave Rectifier with *RL*-Source Load

For the circuit of Fig. 3-5*a*, $R = 2\ \Omega$, $L = 20$ mH, and $V_{dc} = 100$ V. The ac source is 120 V rms at 60 Hz. Determine (*a*) an expression for the current in the circuit, (*b*) the power absorbed by the resistor, (*c*) the power absorbed by the dc source, and (*d*) the power supplied by the ac source and the power factor of the circuit.

■ Solution

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = 7.80\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = 1.31 \text{ rad}$$
$$\alpha = \sin^{-1}(100/169.7) = 36.1° = 0.630 \text{ rad}$$
$$\omega\tau = 377(0.02/2) = 3.77 \text{ rad}$$

(*a*) Using Eq. (3-22),

$$i(\omega t) = 21.8 \sin(\omega t - 1.31) - 50 + 75.3e^{-\omega t/3.77} \quad \text{A}$$

The extinction angle β is found from the solution of

$$i(\beta) = 21.8 \sin(\beta - 1.31) - 50 + 75.3e^{-\beta/3.77} = 0$$

which results in $\beta = 3.37$ rad (193°) using root-finding software.

(*b*) Using the preceding expression for $i(\omega t)$ in Eq. (3-24) and using a numerical integration program, the rms current is

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{0.63}^{3.37} i^2(\omega t)\, d(\omega t)} = 3.98 \text{ A}$$

resulting in

$$P_R = I_{\text{rms}}^2 R = 3.98^2(2) = 31.7 \text{ W}$$

(*c*) The power absorbed by the dc source is $I_o V_{dc}$. Using Eq. (3-26),

$$I_o = \frac{1}{2\pi}\int_{0.63}^{3.37} i(\omega t)\, d(\omega t) = 2.25 \text{ A}$$

yielding

$$P_{dc} = I_o V_{dc} = (2.25)(100) = 225 \text{ W}$$

(*d*) The power supplied by the ac source is the sum of the powers absorbed by the load.

$$P_s = P_R + P_{dc} = 31.2 + 225 = 256 \text{ W}$$

The power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{\text{rms}}} = \frac{256}{(120)(3.98)} = 0.54$$

■ **PSpice Solution**

The power quantities in this example can be determined from a PSpice simulation of this circuit. The circuit of Fig. 3-5*a* is created using VSIN, Dbreak, *R*, and *L*. In the simulation settings, choose Time Domain (transient) for the analysis type, and set the Run Time to 16.67 ms for one period of the source. Set the Maximum Step Size to 10 μs to get adequate sampling of the waveforms. A transient analysis with a run time of 16.67 ms (one period for 60 Hz) and a maximum step size of 10 μs is used for the simulation settings.

Average power absorbed by the 2-Ω resistor can be computed in Probe from the basic definition of the average of $p(t)$ by entering AVG(W(R1)), resulting in 29.7 W, or from I_{rms}^2R by entering RMS(I(R1))*RMS(I(R1))*2. The average power absorbed by the dc source is computed from the Probe expression AVG(W(Vdc)), yielding 217 W.

The PSpice values differ slightly from the values obtained analytically because of the diode model. However, the default diode is more realistic than the ideal diode in predicting actual circuit performance.

3.6 INDUCTOR-SOURCE LOAD

Using Inductance to Limit Current

Another variation of the half-wave rectifier circuit has a load that consists of an inductor and a dc source, as shown in Fig. 3-6. Although a practical implementation of this circuit would contain some resistance, the resistance may be negligible compared to other circuit parameters.

Starting at $\omega t = 0$ and assuming zero initial current in the inductor, the diode remains reverse-biased until the ac source voltage reaches the dc voltage. The value of ωt at which the diode starts to conduct is α, calculated using Eq. (3-18). With the diode conducting, Kirchhoff's voltage law for the circuit is

$$V_m \sin(\omega t) = L\frac{di(t)}{dt} + V_{dc} \tag{3-29}$$

or

$$V_m \sin(\omega t) = \frac{L}{\omega}\frac{di(\omega t)}{dt} + V_{dc} \tag{3-30}$$

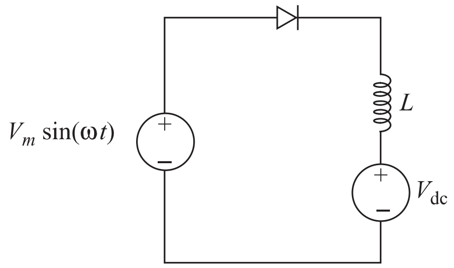


Figure 3-6 Half-wave rectifier with inductor source load.

Rearranging gives

$$\frac{di(\omega t)}{dt} = \frac{V_m \sin(\omega t) - V_{dc}}{\omega L} \tag{3-31}$$

Solving for $i(\omega t)$,

$$i(\omega t) = \frac{1}{\omega L}\int_{\alpha}^{\omega t} V_m \sin\lambda \, d(\lambda) - \frac{1}{\omega L}\int_{\alpha}^{\omega t} V_{dc}\, d(\lambda) \tag{3-32}$$

Performing the integration,

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{\omega L}(\cos\alpha - \cos\omega t) + \dfrac{V_{dc}}{\omega L}(\alpha - \omega t) & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}} \tag{3-33}$$

A distinct feature of this circuit is that the power supplied by the source is the same as that absorbed by the dc source, less any losses associated with a nonideal diode and inductor. If the objective is to transfer power from the ac source to the dc source, losses are kept to a minimum by using this circuit.

EXAMPLE 3-6

Half-Wave Rectifier with Inductor-Source Load

For the circuit of Fig. 3-6, the ac source is 120 V rms at 60 Hz, $L = 50$ mH, and $V_{dc} = 72$ V. Determine (*a*) an expression for the current, (*b*) the power absorbed by the dc source, and (*c*) the power factor.

■ Solution

For the parameters given,

$$\alpha = \sin^{-1}\left(\frac{72}{120\sqrt{2}}\right) = 25.1° = 0.438 \text{ rad}$$

(*a*) The equation for current is found from Eq. (3-33).

$$i(\omega t) = 9.83 - 9.00\cos(\omega t) - 3.82\,\omega t \qquad \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

where β is found to be 4.04 rad from the numerical solution of $9.83 - 9.00\cos\beta - 3.82\beta = 0$.

(*b*) The power absorbed by the dc source is $I_o V_{dc}$, where

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_{0.438}^{4.04} [9.83 - 9.00\cos(\omega t) - 3.82\,\omega t]\, d(\omega t) = 2.46 \text{ A}$$

resulting in

$$P_{dc} = V_{dc} I_o = (2.46)(72) = 177 \text{ W}$$

(*c*) The rms current is found from

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)} = 3.81 \text{ A}$$

Therefore,

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{rms} I_{rms}} = \frac{177}{(120)(3.81)} = 0.388$$

3.7 THE FREEWHEELING DIODE

Creating a DC Current

A freewheeling diode, D_2 in Fig. 3-7*a*, can be connected across an *RL* load as shown. The behavior of this circuit is somewhat different from that of the half-wave rectifier of Fig. 3-2. The key to the analysis of this circuit is to determine when each diode conducts. First, it is observed that both diodes cannot be forward-biased at the same time. Kirchhoff's voltage law around the path containing the source and the two diodes shows that one diode must be reverse-biased. Diode D_1 will be on when the source is positive, and diode D_2 will be on when the source is negative.

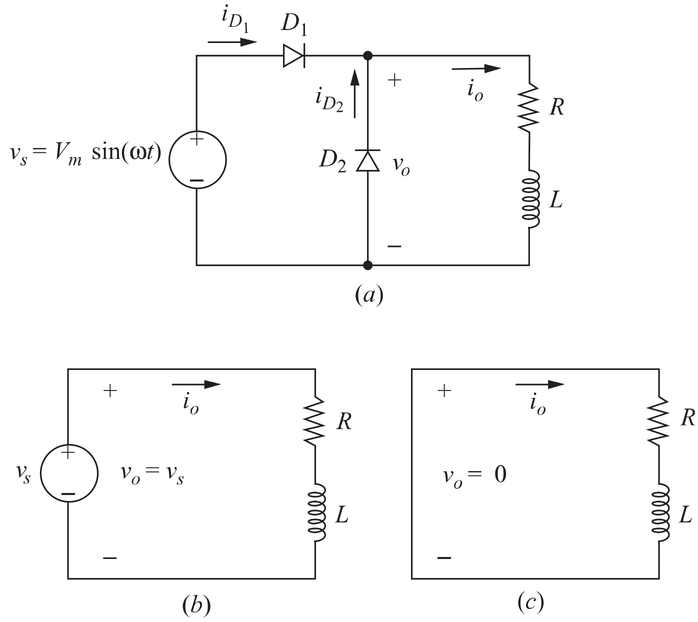


Figure 3-7 (*a*) Half-wave rectifier with freewheeling diode; (*b*) Equivalent circuit for $v_s > 0$; (*c*) Equivalent circuit for $v_s < 0$.

For a positive source voltage,

- D_1 is on.
- D_2 is off.
- The equivalent circuit is the same as that of Fig. 3-2, shown again in Fig. 3-7*b*.
- The voltage across the *RL* load is the same as the source.

For a negative source voltage,

- D_1 is off.
- D_2 is on.
- The equivalent circuit is the same at that of Fig. 3-7*c*.
- The voltage across the *RL* load is zero.

Since the voltage across the *RL* load is the same as the source voltage when the source is positive and is zero when the source is negative, the load voltage is a half-wave rectified sine wave.

When the circuit is first energized, the load current is zero and cannot change instantaneously. The current reaches periodic steady state after a few periods (depending on the *L*/*R* time constant), which means that the current at the end of a period is the same as the current at the beginning of the period, as shown in Fig. 3-8. The steady-state current is usually of greater interest than the transient that occurs when the circuit is first energized. Steady-state load, source, and diode currents are shown in Fig. 3-9.

The Fourier series for the half-wave rectified sine wave for the voltage across the load is

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin(\omega_0 t) - \sum_{n=2,4,6\ldots}^{\infty} \frac{2V_m}{(n^2-1)\pi}\cos(n\omega_0 t) \tag{3-34}$$

The current in the load can be expressed as a Fourier series by using superposition, taking each frequency separately. The Fourier series method is illustrated in Example 3-7.

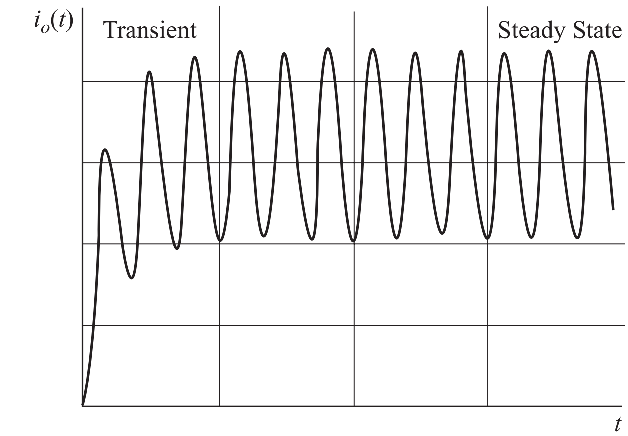


Figure 3-8 Load current reaching steady state after the circuit is energized.

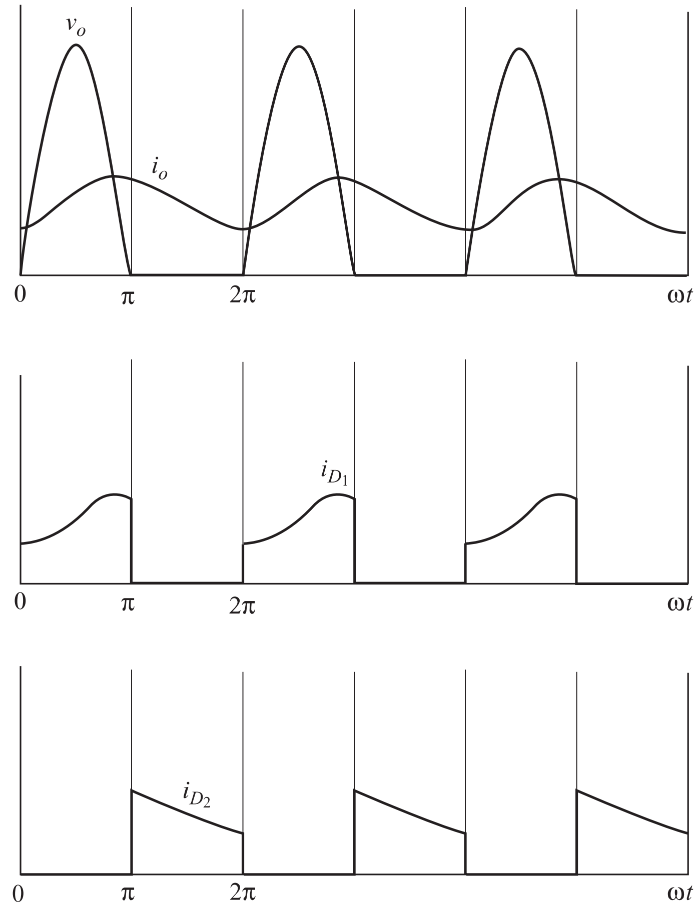


Figure 3-9 Steady-state load voltage and current waveforms with freewheeling diode.

EXAMPLE 3-7

Half-Wave Rectifier with Freewheeling Diode

Determine the average load voltage and current, and determine the power absorbed by the resistor in the circuit of Fig. 3-7*a*, where $R = 2\ \Omega$ and $L = 25$ mH, V_m is 100 V, and the frequency is 60 Hz.

■ Solution

The Fourier series for this half-wave rectified voltage that appears across the load is obtained from Eq. (3-34). The average load voltage is the dc term in the Fourier series:

$$V_o = \frac{V_m}{\pi} = \frac{100}{\pi} = 31.8 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{31.8}{2} = 15.9 \text{ A}$$

Load power can be determined from $I^2_{rms}R$, and rms current is determined from the Fourier components of current. The amplitudes of the ac current components are determined from phasor analysis:

$$I_n = \frac{V_n}{Z_n}$$

where $$Z_n = |R + jn\omega_0 L| = |2 + jn377(0.025)|$$

The ac voltage amplitudes are determined from Eq. (3-34), resulting in

$$V_1 = \frac{V_m}{2} = \frac{100}{2} = 50 \text{ V}$$

$$V_2 = \frac{2V_m}{(2^2 - 1)\pi} = 21.2 \text{ V}$$

$$V_4 = \frac{2V_m}{(4^2 - 1)\pi} = 4.24 \text{ V}$$

$$V_6 = \frac{2V_m}{(6^2 - 1)\pi} = 1.82 \text{ V}$$

The resulting Fourier terms are as follows:

n	V_n (V)	Z_n (Ω)	I_n (A)
0	31.8	2.00	15.9
1	50.0	9.63	5.19
2	21.2	18.96	1.12
4	4.24	37.75	0.11
6	1.82	56.58	0.03

The rms current is obtained using Eq. (2-64).

$$I_{rms} = \sqrt{\sum_{k=0}^{\infty} I_{k,rms}} \approx \sqrt{15.9^2 + \left(\frac{5.19}{\sqrt{2}}\right)^2 + \left(\frac{1.12}{\sqrt{2}}\right)^2 + \left(\frac{0.11}{\sqrt{2}}\right)^2} = 16.34 \text{ A}$$

Notice that the contribution to rms current from the harmonics decreases as n increases, and higher-order terms are not significant. Power in the resistor is $I^2_{rms}R = (16.34)^2 2 = 534$ W.

■ PSpice Solution

The circuit of Fig. 3-7*a* is created using VSIN, Dbreak, *R*, and *L*. The PSpice model for Dbreak is changed to make $n = 0.001$ to approximate an ideal diode. A transient analysis is run with a run time of 150 ms with data saved after 100 ms to eliminate the start-up transient from the data. A maximum step size of 10 μs gives a smooth waveform.

A portion of the output file is as follows:

```
****     FOURIER ANALYSIS       TEMPERATURE =  27.000 DEG C

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(OUT)

DC COMPONENT =      3.183002E+01
```

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	6.000E+01	5.000E+01	1.000E+00	-5.804E-05	0.000E+00
2	1.200E+02	2.122E+01	4.244E-01	-9.000E+01	-9.000E+01
3	1.800E+02	5.651E-05	1.130E-06	-8.831E+01	-8.831E+01
4	2.400E+02	4.244E+00	8.488E-02	-9.000E+01	-9.000E+01
5	3.000E+02	5.699E-05	1.140E-06	-9.064E+01	-9.064E+01
6	3.600E+02	1.819E+00	3.638E-02	-9.000E+01	-9.000E+01
7	4.200E+02	5.691E-05	1.138E-06	-9.111E+01	-9.110E+01
8	4.800E+02	1.011E+00	2.021E-02	-9.000E+01	-9.000E+01
9	5.400E+02	5.687E-05	1.137E-06	-9.080E+01	-9.079E+01

FOURIER COMPONENTS OF TRANSIENT RESPONSE I(R_R1)

DC COMPONENT = 1.591512E+01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	6.000E+01	5.189E+00	1.000E+00	-7.802E+01	0.000E+00
2	1.200E+02	1.120E+00	2.158E-01	-1.739E+02	-1.788E+01
3	1.800E+02	1.963E-04	3.782E-05	-3.719E+01	1.969E+02
4	2.400E+02	1.123E-01	2.164E-02	-1.770E+02	1.351E+02
5	3.000E+02	7.524E-05	1.450E-05	6.226E+01	4.524E+02
6	3.600E+02	3.217E-02	6.200E-03	-1.781E+02	2.900E+02
7	4.200E+02	8.331E-05	1.605E-05	1.693E+02	7.154E+02
8	4.800E+02	1.345E-02	2.592E-03	-1.783E+02	4.458E+02
9	5.400E+02	5.435E-05	1.047E-05	-1.074E+02	5.948E+02

Note the close agreement between the analytically obtained Fourier terms and the PSpice output. Average current can be obtained in Probe by entering AVG(I(R1)), yielding 15.9 A. Average power in the resistor can be obtained by entering AVG(W(R1)), yielding $P = 535$ W. It is important that the simulation represent steady-state periodic current for the results to be valid.

Reducing Load Current Harmonics

The average current in the RL load is a function of the applied voltage and the resistance but not the inductance. The inductance affects only the ac terms in the Fourier series. If the inductance is infinitely large, the impedance of the load to ac terms in the Fourier series is infinite, and the load current is purely dc. The load current is then

$$\boxed{i_o(t) \approx I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} \qquad \frac{L}{R} \to \infty} \tag{3-35}$$

A large inductor ($L/R \gg T$) with a freewheeling diode provides a means of establishing a nearly constant load current. Zero-to-peak fluctuation in load current can be estimated as being equal to the amplitude of the first ac term in the Fourier series. The peak-to-peak ripple is then

$$\Delta I_o \approx 2I_1 \tag{3-36}$$

EXAMPLE 3-8

Half-Wave Rectifier with Freewheeling Diode: $L/R \to \infty$

For the half-wave rectifier with a freewheeling diode and RL load as shown in Fig. 3-7a, the source is 240 V rms at 60 Hz and $R = 8\ \Omega$. (a) Assume L is infinitely large. Determine the power absorbed by the load and the power factor as seen by the source. Sketch v_o, i_{D_1}, and i_{D_2}. (b) Determine the average current in each diode. (c) For a finite inductance, determine L such that the peak-to-peak current is no more than 10 percent of the average current.

■ **Solution**

(a) The voltage across the RL load is a half-wave rectified sine wave, which has an average value of V_m/π. The load current is

$$i(\omega t) = I_o = \frac{V_o}{R} = \frac{V_m/\pi}{R} = \frac{\left(240\sqrt{2}\right)/\pi}{8} = 13.5\text{ A} \approx I_{\text{rms}}$$

Power in the resistor is

$$P = (I_{\text{rms}})^2 R = (13.5)^2 8 = 1459\text{ W}$$

Source rms current is computed from

$$I_{s,\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(13.5)^2\, d(\omega t)} = 9.55\text{ A}$$

The power factor is

$$\text{pf} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{1459}{(240)(9.55)} = 0.637$$

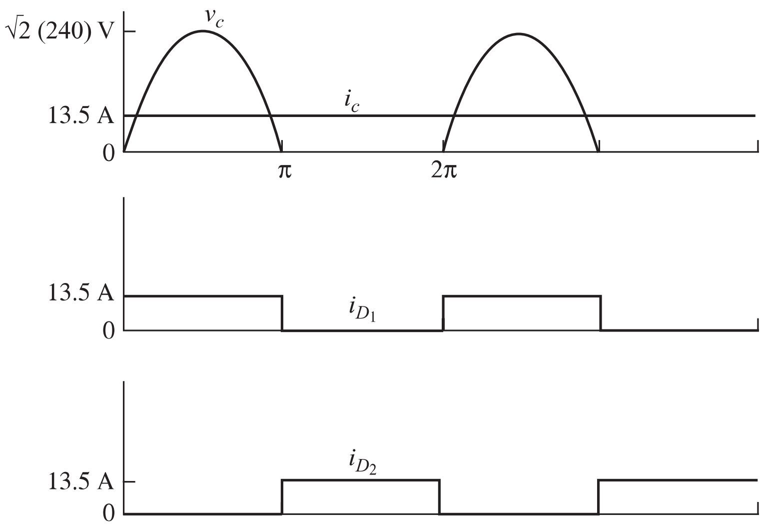


Figure 3-10 Waveforms for the half-wave rectifier with freewheeling diode of Example 3-8 with $L/R \rightarrow \infty$.

Voltage and current waveforms are shown in Fig. 3-10.

(*b*) Each diode conducts for one-half of the time. Average current for each diode is $I_o/2 = 13.5/2 = 6.75$ A.

(*c*) The value of inductance required to limit the variation in load current to 10 percent can be approximated from the fundamental frequency of the Fourier series. The voltage input to the load for $n = 1$ in Eq. (3-34) has amplitude $V_m/2 = \sqrt{2}(240)/2 = 170$ V the peak-to-peak current must be limited to

$$\Delta I_o = (0.10)(I_o) = (0.10)(13.5) = 1.35 \text{ A}$$

which corresponds to an amplitude of $1.35/2 = 0.675$ A. The load impedance at the fundamental frequency must then be

$$Z_1 = \frac{V_1}{I_1} = \frac{170}{0.675} = 251\ \Omega$$

The load impedance is

$$Z_1 = 251 = |R + j\omega L| = |8 + j377L|$$

Since the 8-Ω resistance is negligible compared to the total impedance, the inductance can be approximated as

$$L \approx \frac{Z_1}{\omega} = \frac{251}{377} = 0.67 \text{ H}$$

The inductance will have to be slightly larger than 0.67 H because Fourier terms higher than $n = 1$ were neglected in this estimate.

3.8 HALF-WAVE RECTIFIER WITH A CAPACITOR FILTER

Creating a DC Voltage from an AC Source

A common application of rectifier circuits is to convert an ac voltage input to a dc voltage output. The half-wave rectifier of Fig. 3-11*a* has a parallel *RC* load. The purpose of the capacitor is to reduce the variation in the output voltage, making it more like dc. The resistance may represent an external load, and the capacitor may be a filter which is part of the rectifier circuit.

Assuming the capacitor is initially uncharged and the circuit is energized at $\omega t = 0$, the diode becomes forward-biased as the source becomes positive. With the diode on, the output voltage is the same as the source voltage, and the capacitor charges. The capacitor is charged to V_m when the input voltage reaches its positive peak at $\omega t = \pi/2$.

As the source decreases after $\omega t = \pi/2$, the capacitor discharges into the load resistor. At some point, the voltage of the source becomes less than the output voltage, reverse-biasing the diode and isolating the load from the source. The output voltage is a decaying exponential with time constant *RC* while the diode is off.

The point when the diode turns off is determined by comparing the rates of change of the source and the capacitor voltages. The diode turns off when the

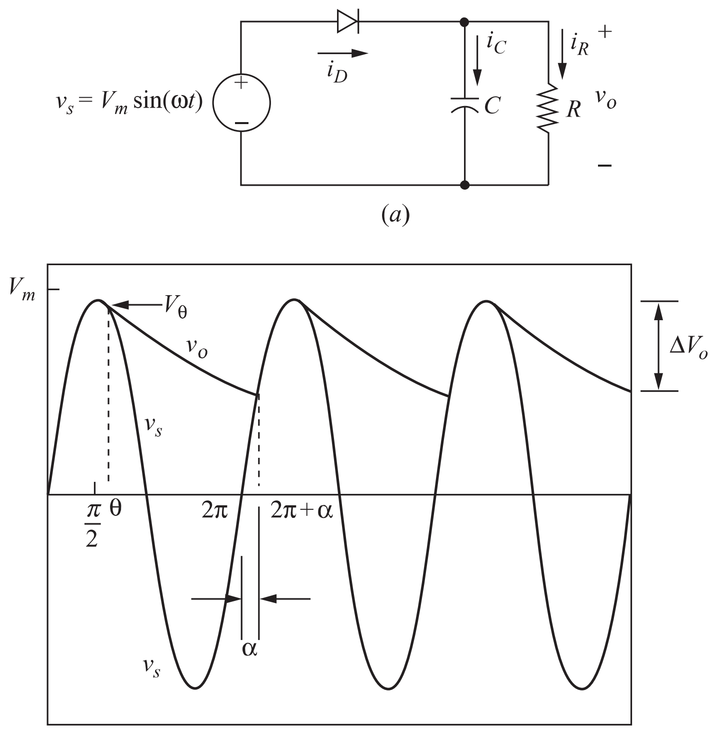


Figure 3-11 (*a*) Half-wave rectifier with *RC* load; (*b*) Input and output voltages.

downward rate of change of the source exceeds that permitted by the time constant of the RC load. The angle $\omega t = \theta$ is the point when the diode turns off in Fig. 3-11b. The output voltage is described by

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases} \tag{3-37}$$

where

$$V_\theta = V_m \sin \theta \tag{3-38}$$

The slopes of these functions are

$$\frac{d}{d(\omega t)}[V_m \sin(\omega t)] = V_m \cos(\omega t) \tag{3-39}$$

and

$$\frac{d}{d(\omega t)}\left(V_m \sin\theta\, e^{-(\omega t - \theta)/\omega RC}\right) = V_m \sin\theta\left(-\frac{1}{\omega RC}\right)e^{-(\omega t - \theta)/\omega RC} \tag{3-40}$$

At $\omega t = \theta$, the slopes of the voltage functions are equal:

$$V_m \cos\theta = \left(\frac{V_m \sin\theta}{-\omega RC}\right)e^{-(\theta - \theta)/\omega RC} = \frac{V_m \sin\theta}{-\omega RC}$$

$$\frac{V_m \cos\theta}{V_m \sin\theta} = \frac{1}{-\omega RC}$$

$$\frac{1}{\tan\theta} = \frac{1}{-\omega RC}$$

Solving for θ and expressing θ so it is in the proper quadrant, we have

$$\boxed{\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi} \tag{3-41}$$

In practical circuits where the time constant is large,

$$\boxed{\theta \approx \frac{\pi}{2} \qquad \text{and} \qquad V_m \sin\theta \approx V_m} \tag{3-42}$$

When the source voltage comes back up to the value of the output voltage in the next period, the diode becomes forward-biased, and the output again is the same as the source voltage. The angle at which the diode turns on in the second period, $\omega t = 2\pi + \alpha$, is the point when the sinusoidal source reaches the same value as the decaying exponential output:

$$V_m \sin(2\pi + \alpha) = (V_m \sin\theta)e^{-(2\pi + \alpha - \theta)/\omega RC}$$

or

$$\boxed{\sin\alpha - (\sin\theta)e^{-(2\pi+\alpha-\theta)/\omega RC} = 0} \tag{3-43}$$

Equation (3-43) must be solved numerically for α.

The current in the resistor is calculated from $i_R = v_o/R$. The current in the capacitor is calculated from

$$i_C(t) = C\frac{dv_o(t)}{dt}$$

which can also be expressed, using ωt as the variable, as

$$i_C(\omega t) = \omega C\frac{dv_o(\omega t)}{d(\omega t)}$$

Using v_o from Eq. (3-37),

$$\boxed{i_C(\omega t) = \begin{cases} -\left(\dfrac{V_m \sin\theta}{R}\right)e^{-(\omega t-\theta)/\omega RC} & \text{for } \theta \le \omega t \le 2\pi + \alpha \quad \text{(diode off)} \\ \omega C V_m \cos(\omega t) & \text{for } 2\pi + \alpha \le \omega t \le 2\pi + \theta \quad \text{(diode on)} \end{cases}} \tag{3-44}$$

The source current, which is the same as the diode current, is

$$i_S = i_D = i_R + i_C \tag{3-45}$$

The average capacitor current is zero, so the average diode current is the same as the average load current. Since the diode is on for a short time in each cycle, the peak diode current is generally much larger than the average diode current. Peak capacitor current occurs when the diode turns on at $\omega t = 2\pi + \alpha$. From Eq. (3-44),

$$I_{C,\text{peak}} = \omega C V_m \cos(2\pi + \alpha) = \omega C V_m \cos\alpha \tag{3-46}$$

Resistor current at $\omega t = 2\pi + \alpha$ is obtained from Eq. (3-37).

$$i_R(2\omega t + \alpha) = \frac{V_m \sin(2\omega t + \alpha)}{R} = \frac{V_m \sin\alpha}{R} \tag{3-47}$$

Peak diode current is

$$\boxed{I_{D,\text{peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C\cos\alpha + \frac{\sin\alpha}{R}\right)} \tag{3-48}$$

The effectiveness of the capacitor filter is determined by the variation in output voltage. This may be expressed as the difference between the maximum and minimum output voltage, which is the peak-to-peak ripple voltage. For the half-wave rectifier of Fig. 3-11*a*, the maximum output voltage is V_m. The minimum output voltage occurs at $\omega t = 2\pi + \alpha$, which can be computed from $V_m \sin \alpha$. The peak-to-peak ripple for the circuit of Fig. 3-11*a* is expressed as

$$\boxed{\Delta V_o = V_m - V_m \sin \alpha = V_m(1 - \sin \alpha)} \tag{3-49}$$

In circuits where the capacitor is selected to provide for a nearly constant dc output voltage, the *RC* time constant is large compared to the period of the sine wave, and Eq. (3-42) applies. Moreover, the diode turns on close to the peak of the sine wave when $\alpha \approx \pi/2$. The change in output voltage when the diode is off is described in Eq. (3-37). In Eq. (3-37), if $V_\theta \approx \mathrm{V_m}$ and $\theta \approx \pi/2$, then Eq. (3-37) evaluated at $\alpha = \pi/2$ is

$$v_o(2\pi + \alpha) = V_m e^{-(2\pi + \pi/2 - \pi/2)\omega RC} = V_m e^{-2\pi/\omega RC}$$

The ripple voltage can then be approximated as

$$\Delta V_o \approx V_m - V_m e^{-2\pi/\omega RC} = V_m\left(1 - e^{-2\pi/\omega RC}\right) \tag{3-50}$$

Furthermore, the exponential in the above equation can be approximated by the series expansion:

$$e^{-2\pi/\omega RC} \approx 1 - \frac{2\pi}{\omega RC}$$

Substituting for the exponential in Eq. (3-50), the peak-to-peak ripple is approximately

$$\boxed{\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}} \tag{3-51}$$

The output voltage ripple is reduced by increasing the filter capacitor *C*. As *C* increases, the conduction interval for the diode decreases. Therefore, increasing the capacitance to reduce the output voltage ripple results in a larger peak diode current.

EXAMPLE 3-9

Half-Wave Rectifier with *RC* Load

The half-wave rectifier of Fig. 3-11*a* has a 120-V rms source at 60 Hz, $R = 500\ \Omega$, and $C = 100\ \mu\mathrm{F}$. Determine (*a*) an expression for output voltage, (*b*) the peak-to-peak voltage variation on the output, (*c*) an expression for capacitor current, (*d*) the peak diode current, and (*e*) the value of *C* such that ΔV_o is 1 percent of V_m.

■ **Solution**

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7\ \mathrm{V}$$

$$\omega RC = (2\pi 60)(500)(10)^{-6} = 18.85\ \mathrm{rad}$$

The angle θ is determined from Eq. (3-41).

$$\theta = -\tan^{-1}(18.85) + \pi = 1.62\,\text{rad} = 93°$$

$$V_m \sin\theta = 169.5 \text{ V}$$

The angle α is determined from the numerical solution of Eq. (3-43).

$$\sin\alpha - \sin(1.62)e^{-(2\pi+\alpha-1.62/18.85)} = 0$$

yielding

$$\alpha = 0.843 \text{ rad} = 48°$$

(*a*) Output voltage is expressed from Eq. (3-37).

$$v_o(\omega t) = \begin{cases} 169.7\sin(\omega t) & 2\pi + \alpha \le \omega t \le 2\pi + \theta \\ 169.5e^{-(\omega t - 1.62)/18.85} & \theta \le \omega t \le 2\pi + \alpha \end{cases}$$

(*b*) Peak-to-peak output voltage is described by Eq. (3-49).

$$\Delta V_o = V_m(1 - \sin\alpha) = 169.7(1 - \sin 0.843) = 43 \text{ V}$$

(*c*) The capacitor current is determined from Eq. (3-44).

$$i_C(\omega t) = \begin{cases} -0.339e^{-(\omega t - 1.62)/18.85} \text{ A} & \theta \le \omega t \le 2\pi + \alpha \\ 6.4\cos(\omega t) \text{ A} & 2\pi + \alpha \le \omega t \le 2\pi + \theta \end{cases}$$

(*d*) Peak diode current is determined from Eq. (3-48).

$$I_{D,\text{peak}} = \sqrt{2}(120)\left[377(10)^{-4}\cos 0.843 + \frac{\sin 8.43}{500}\right]$$

$$= 4.26 + 0.34 = 4.50 \text{ A}$$

(*e*) For $\Delta V_o = 0.01V_m$, Eq. (3-51) can be used.

$$C \approx \frac{V_m}{fR(\Delta V_o)} = \frac{V_m}{(60)(500)(0.01V_m)} = \frac{1}{300}\text{F} = 3333\ \mu\text{F}$$

Note that peak diode current can be determined from Eq. (3-48) using an estimate of α from Eq. (3-49).

$$\alpha \approx \sin^{-1}\left(1 - \frac{\Delta V_o}{V_m}\right) = \sin^{-1}\left(1 - \frac{1}{fRC}\right) = 81.9°$$

From Eq. (3-48), peak diode current is 30.4 A.

■ PSpice Solution

A PSpice circuit is created for Fig. 3-11*a* using VSIN, Dbreak, *R*, and *C*. The diode Dbreak used in this analysis causes the results to differ slightly from the analytic solution based on the ideal diode. The diode drop causes the maximum output voltage to be slightly less than that of the source.

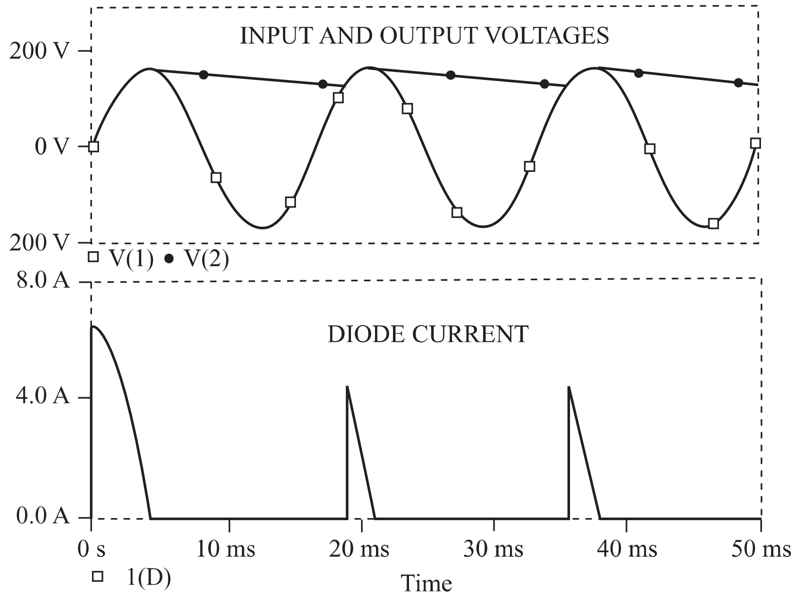


Figure 3-12 Probe output for Example 3-9.

The Probe output is shown in Fig. 3-12. Angles θ and α are determined directly by first modifying the x-variable to indicate degrees (x-variable = time*60*360) and then using the cursor option. The restrict data option is used to compute quantities based on steady-state values (16.67 to 50 ms). Steady state is characterized by waveforms beginning and ending a period at the same values. Note that the peak diode current is largest in the first period because the capacitor is initially uncharged.

■ Results from the Probe Cursor

Quantity	Result
$\alpha + 360°$	408° ($\alpha = 48°$)
θ	98.5°
V_o max	168.9 V
V_o min	126 V
ΔV_o	42.9 V
$I_{D,\text{peak}}$	4.42 A steady state; 6.36 A first period
$I_{C,\text{peak}}$	4.12 A steady state; 6.39 A first period

■ Results after Restricting the Data to Steady State

Quantity	Probe Expression	Result
$I_{D,\text{avg}}$	AVG(I(D1))	0.295 A
$I_{C,\text{rms}}$	RMS(I(C1))	0.905 A
$I_{R,\text{avg}}$	AVG(W(R1))	43.8 W
P_s	AVG(W(Vs))	−44.1 W
P_D	AVG(W(D1))	345 mW

In this example, the ripple, or variation in output voltage, is very large, and the capacitor is not an effective filter. In many applications, it is desirable to produce an output that is closer to dc. This requires the time constant RC to be large compared to the period of the input voltage, resulting in little decay of the output voltage. For an effective filter capacitor, the output voltage is essentially the same as the peak voltage of the input.

3.9 THE CONTROLLED HALF-WAVE RECTIFIER

The half-wave rectifiers analyzed previously in this chapter are classified as uncontrolled rectifiers. Once the source and load parameters are established, the dc level of the output and the power transferred to the load are fixed quantities.

A way to control the output of a half-wave rectifier is to use an SCR[1] instead of a diode. Figure 3-13*a* shows a basic controlled half-wave rectifier with a resistive load. Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased ($v_{SCR} > 0$).
2. A current must be applied to the gate of the SCR.

Unlike the diode, the SCR will not begin to conduct as soon as the source becomes positive. Conduction is delayed until a gate current is applied, which is the basis for using the SCR as a means of control. Once the SCR is conducting, the gate current can be removed and the SCR remains on until the current goes to zero.

Resistive Load

Figure 3-13*b* shows the voltage waveforms for a controlled half-wave rectifier with a resistive load. A gate signal is applied to the SCR at $\omega t = \alpha$, where α is the delay angle. The average (dc) voltage across the load resistor in Fig. 3-13*a* is

$$\boxed{V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)} \tag{3-52}$$

The power absorbed by the resistor is V^2_{rms}/R, where the rms voltage across the resistor is computed from

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} v_o^2(\omega t)\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} [V_m \sin(\omega t)]^2\, d(\omega t)}$$

$$= \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}} \tag{3-53}$$

[1] Switching with other controlled turn-on devices such as transistors or IGBTs can be used to control the output of a converter.

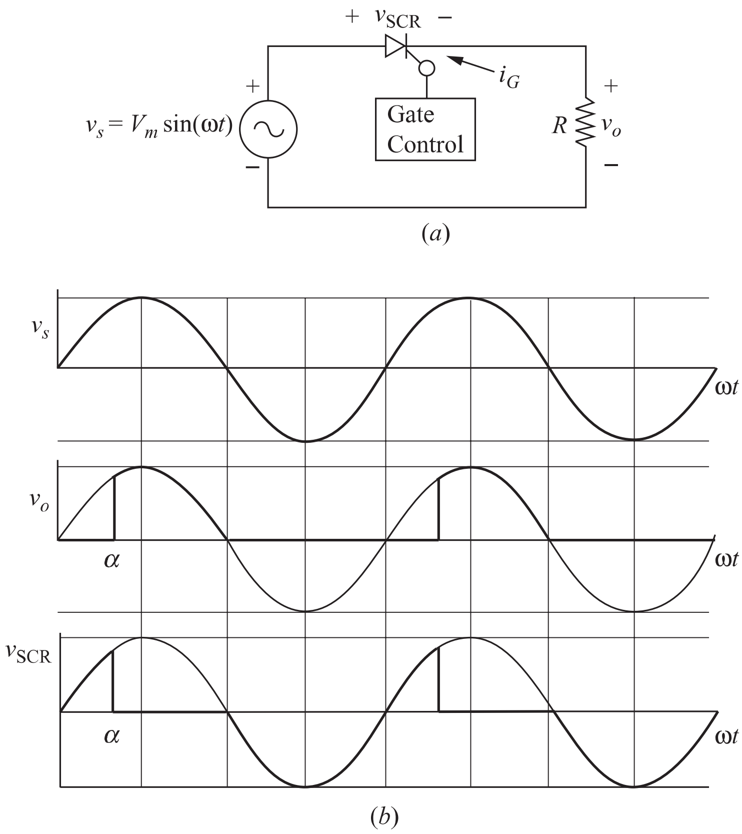


Figure 3-13 (*a*) A basic controlled rectifier; (*b*) Voltage waveforms.

EXAMPLE 3-10

Controlled Half-Wave Rectifier with Resistive Load

Design a circuit to produce an average voltage of 40 V across a 100-Ω load resistor from a 120-V rms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

■ Solution

If an uncontrolled half-wave rectifier is used, the average voltage will be $V_m/\pi = 120\sqrt{2}/\pi = 54$ V. Some means of reducing the average resistor voltage to the design specification of 40 V must be found. A series resistance or inductance could be added to an uncontrolled rectifier, or a controlled rectifier could be used. The controlled rectifier of Fig. 3-13*a* has the advantage of not altering the load or introducing losses, so it is selected for this application.

Equation (3-52) is rearranged to determine the required delay angle:

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right]$$

$$= \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2° = 1.07 \text{ rad}$$

Equation (3-53) gives

$$V_{\text{rms}} = \frac{\sqrt{2}(120)}{2}\sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$

Load power is

$$P_R = \frac{V_{\text{rms}}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

The power factor of the circuit is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S,\text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$

RL Load

A controlled half-wave rectifier with an *RL* load is shown in Fig. 3-14*a*. The analysis of this circuit is similar to that of the uncontrolled rectifier. The current is the sum of the forced and natural responses, and Eq. (3-9) applies:

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

The constant A is determined from the initial condition $i(\alpha) = 0$:

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right] = e^{\alpha/\omega\tau} \tag{3-54}$$

Substituting for A and simplifying,

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}} \tag{3-55}$$

The *extinction angle* β is defined as the angle at which the current returns to zero, as in the case of the uncontrolled rectifier. When $\omega t = \beta$,

$$\boxed{i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)e^{(\alpha - \beta)/\omega\tau}\right]} \tag{3-56}$$

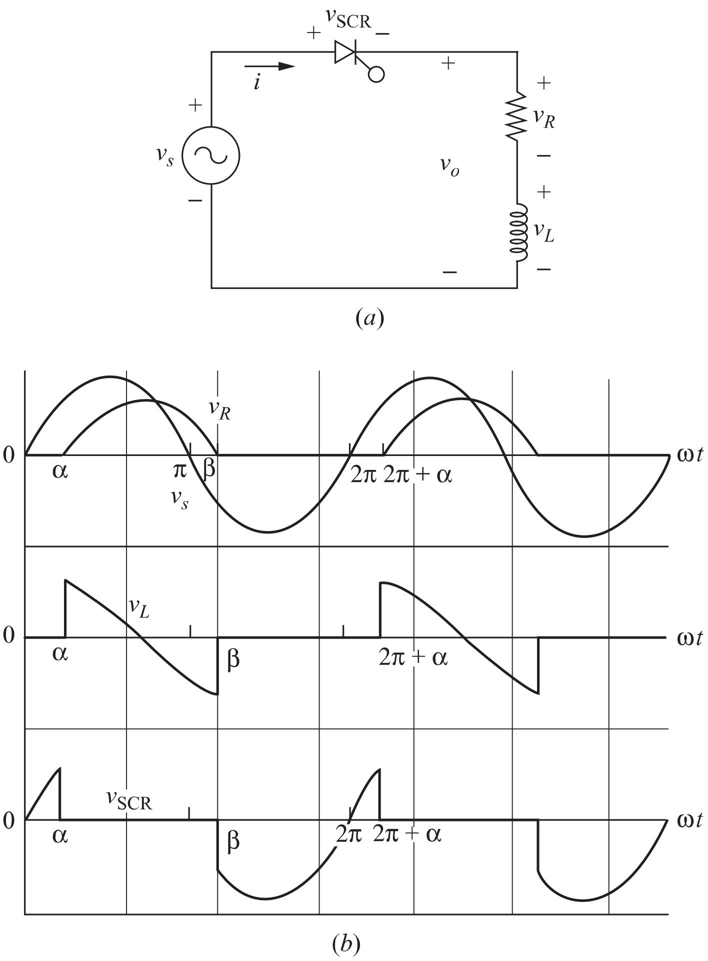


Figure 3-14 (*a*) Controlled half-wave rectifier with *RL* load; (*b*) Voltage waveforms.

which must be solved numerically for β. The angle $\beta - \alpha$ is called the *conduction angle* γ. Figure 3-14*b* shows the voltage waveforms.

The average (dc) output voltage is

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) \tag{3-57}$$

The average current is computed from

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t) \tag{3-58}$$

where $i(\omega t)$ is defined in Eq. (3-55). Power absorbed by the load is $I_{rms}^2 R$, where the rms current is computed from

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)} \tag{3-59}$$

EXAMPLE 3-11

Controlled Half-Wave Rectifier with *RL* Load

For the circuit of Fig. 3-14*a*, the source is 120 V rms at 60 Hz, $R = 20\ \Omega$, $L = 0.04$ H, and the delay angle is 45°. Determine (*a*) an expression for $i(\omega t)$, (*b*) the average current, (*c*) the power absorbed by the load, and (*d*) the power factor.

■ Solution

(*a*) From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = [20^2 + (377*0.04)^2]^{0.5} = 25.0\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.04/20) = 0.646 \text{ rad}$$
$$\omega\tau = \omega L/R = 377*0.04/20 = 0.754$$
$$\alpha = 45° = 0.785 \text{ rad}$$

Substituting the preceding quantities into Eq. (3-55), current is expressed as

$$i(\omega t) = 6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754} \quad \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

The preceding equation is valid from α to β, where β is found numerically by setting the equation to zero and solving for ωt, with the result $\beta = 3.79$ rad (217°). The conduction angle is $\gamma = \beta - \alpha = 3.79 - 0.785 = 3.01$ rad $= 172°$.

(*b*) Average current is determined from Eq. (3-58).

$$I_o = \frac{1}{2\pi}\int_{0.785}^{3.79}\left[6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754}\right]d(\omega t) = 2.19 \text{ A}$$

(*c*) The power absorbed by the load is computed from $I_{\text{rms}}^2 R$, where

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{0.785}^{3.79}\left[6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754}\right]^2 d(\omega t)} = 3.26 \text{ A}$$

yielding

$$P = I_{\text{rms}}^2 R = (3.26)^2(20) = 213 \text{ W}$$

(*d*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{213}{(120)(3.26)} = 0.54$$

RL-Source Load

A controlled rectifier with a series resistance, inductance, and dc source is shown in Fig. 3-15. The analysis of this circuit is very similar to that of the uncontrolled half-wave rectifier discussed earlier in this chapter. The major difference is that for the uncontrolled rectifier, conduction begins as soon as the source voltage

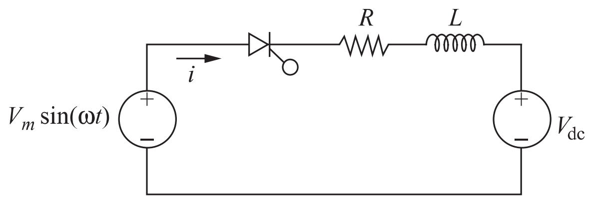


Figure 3-15 Controlled rectifier with *RL*-source load.

reaches the level of the dc voltage. For the controlled rectifier, conduction begins when a gate signal is applied to the SCR, provided the SCR is forward-biased. Thus, the gate signal may be applied at any time that the ac source is larger than the dc source:

$$\alpha_{\min} = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right) \tag{3-60}$$

Current is expressed as in Eq. (3-22), with α specified within the allowable range:

$$i(\omega t) \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases} \tag{3-61}$$

where A is determined from Eq. (3-61):

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta) + \frac{V_{dc}}{R}\right]e^{\alpha/\omega\tau}$$

EXAMPLE 3-12

Controlled Rectifier with *RL*-Source Load

The controlled half-wave rectifier of Fig. 3-15 has an ac input of 120 V rms at 60 Hz, $R = 2\ \Omega$, $L = 20$ mH, and $V_{dc} = 100$ V. The delay angle α is 45°. Determine (*a*) an expression for the current, (*b*) the power absorbed by the resistor, and (*c*) the power absorbed by the dc source in the load.

■ Solution:

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = [2^2 + (377{*}0.02)^2]^{0.5} = 7.80\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377{*}0.02)/2) = 1.312 \text{ rad}$$
$$\omega\tau = \omega L/R = 377{*}0.02/2 = 3.77$$
$$\alpha = 45° = 0.785 \text{ rad}$$

(*a*) First, use Eq. (3-60) to determine if $\alpha = 45°$ is allowable. The minimum delay angle is

$$\alpha_{\min} = \sin^{-1}\left(\frac{100}{120\sqrt{2}}\right) = 36°$$

which indicates that 45° is allowable. Equation (3-61) becomes

$$i(\omega t) = 21.8 \sin(\omega t - 1.312) - 50 + 75.0e^{-\omega t/3.77} \text{ A} \quad \text{for } 0.785 \leq \omega t \leq 3.37 \text{ rad}$$

where the extinction angle β is found numerically to be 3.37 rad from the equation $i(\beta) = 0$.

(*b*) Power absorbed by the resistor is $I^2_{\text{rms}}R$, where I_{rms} is computed from Eq. (3-59) using the preceding expression for $i(\omega t)$.

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\,d(\omega t)} = 3.90 \text{ A}$$

$$P = (3.90)^2(2) = 30.4 \text{ W}$$

(*c*) Power absorbed by the dc source is $I_o V_{\text{dc}}$, where I_o is computed from Eq. (3-58).

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\,d(\omega t) = 2.19 \text{ A}$$

$$P_{\text{dc}} = I_o V_{\text{dc}} = (2.19)(100) = 219 \text{ W}$$

3.10 PSPICE SOLUTIONS FOR CONTROLLED RECTIFIERS

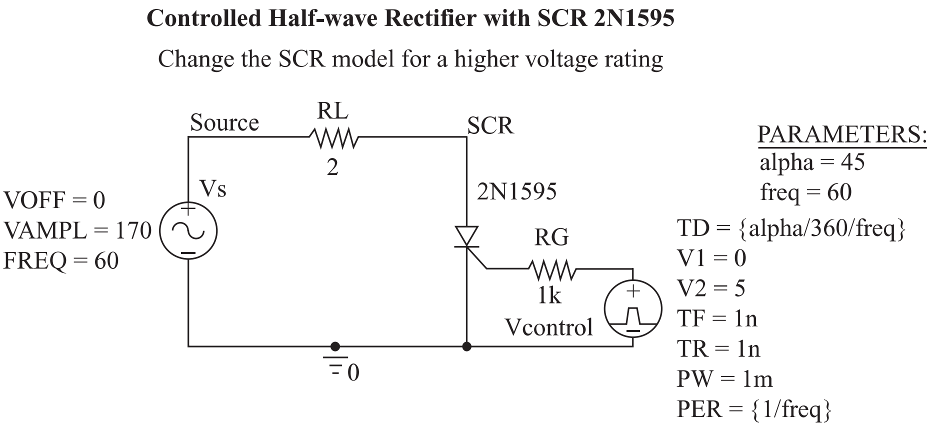
Modeling the SCR in PSpice

To simulate the controlled half-wave rectifier in PSpice, a model for the SCR must be selected. An SCR model available in a device library can be utilized in the simulation of a controlled half-wave rectifier. A circuit for Example 3-10 using the 2N1595 SCR in the PSpice demo version library of devices is shown in Fig. 3-16*a*. An alternative model for the SCR is a voltage-controlled switch and a diode as described in Chap. 1. The switch controls when the SCR begins to conduct, and the diode allows current in only one direction. The switch must be closed for at least the conduction angle of the current. An advantage of using this SCR model is that the device can be made ideal. The major disadvantage of the model is that the switch control must keep the switch closed for the entire conduction period and open the switch before the source becomes positive again. A circuit for the circuit in Example 3-11 is shown in Fig. 3-16*b*.

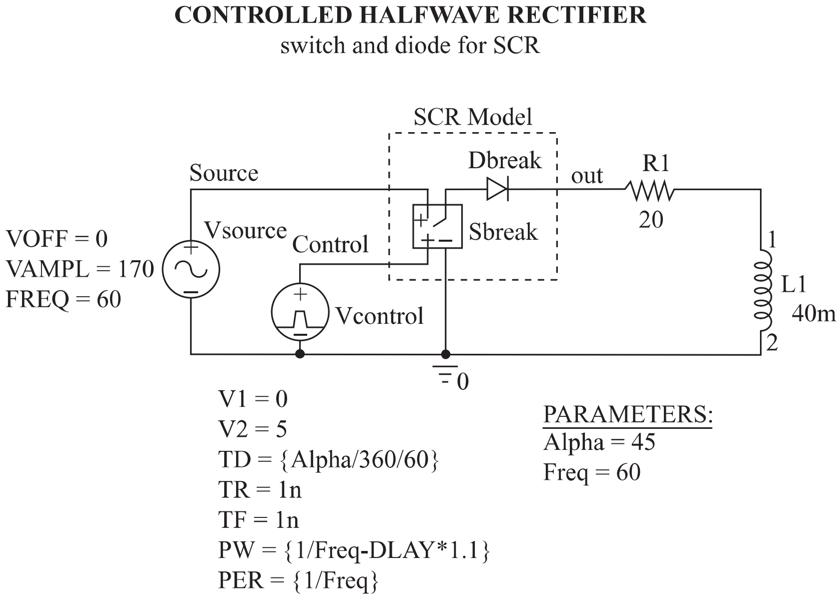
EXAMPLE 3-13

Controlled Half-Wave Rectifier Design Using PSpice

A load consists of a series-connected resistance, inductance, and dc voltage source with $R = 2\ \Omega$, $L = 20$ mH, and $V_{\text{dc}} = 100$ V. Design a circuit that will deliver 150 W to the dc voltage source from a 120-*V* rms 60-Hz ac source.



(*a*)



(*b*)

Figure 3-16 (*a*) A controlled half-wave rectifier using an SCR from the library of devices; (*b*) An SCR model using a voltage-controlled switch and a diode.

■ Solution

Power in the dc source of 150 W requires an average load current of 150 W/100 V = 1.5 A. An uncontrolled rectifier with this source and load will have an average current of 2.25 A and an average power in the dc source of 225 W, as was computed in Example 3-5 previously. A means of limiting the average current to 1.5 A must be found. Options include the addition of series resistance or inductance. Another option that is chosen for this application is the controlled half-wave rectifier of Fig. 3-15. The power delivered to the load components is determined by the delay angle α. Since there is no closed-form solution for α, a trial-and-error iterative method must be used. A PSpice simulation that includes a parametric sweep is used to try several values of alpha. The parametric sweep

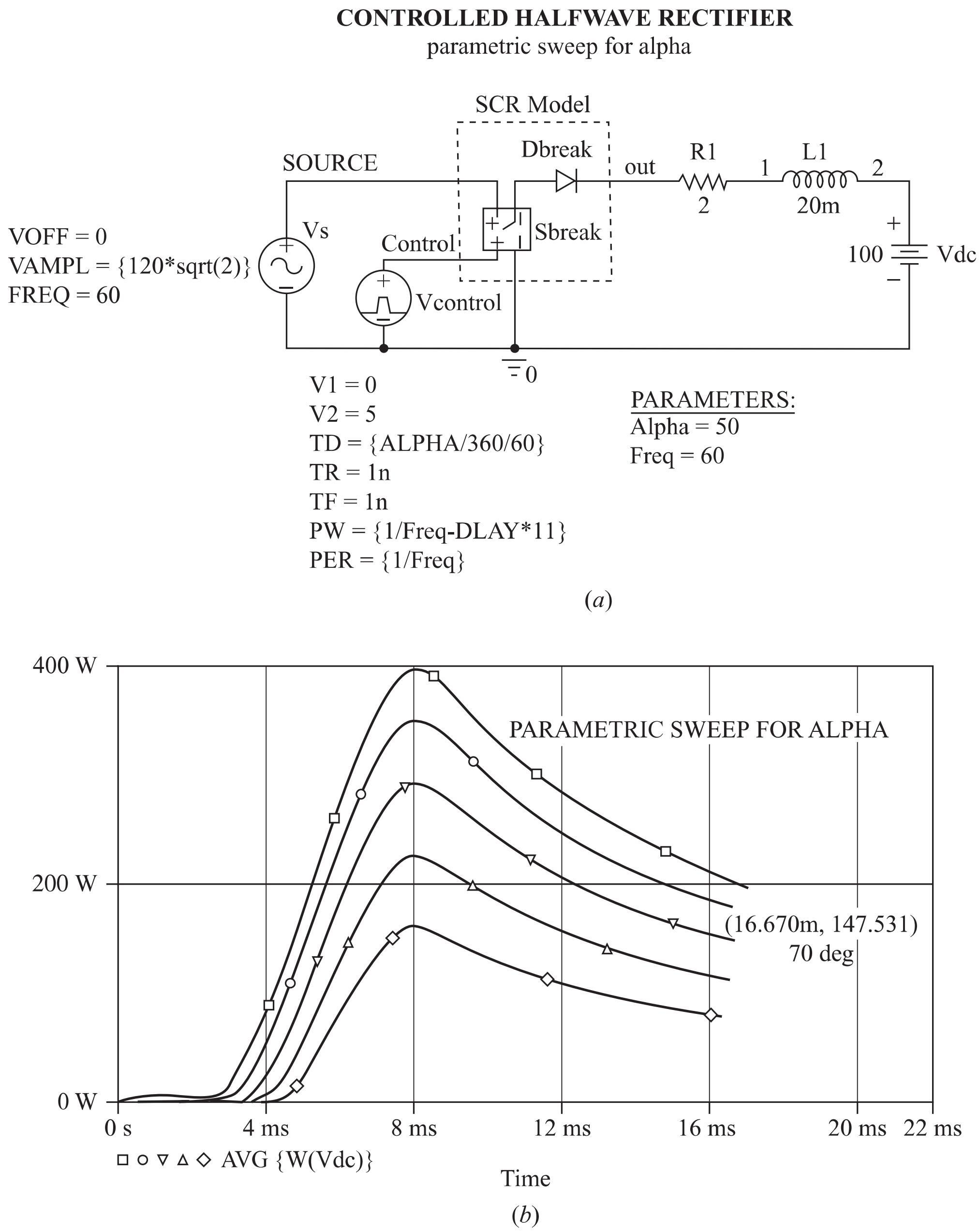


Figure 3-17 (*a*) PSpice circuit for Example 3-13; (*b*) Probe output for showing a family of curves for different delay angles.

is established in the Simulation Setting menu (see Example 3-4). A PSpice circuit is shown in Fig. 3-17*a*.

When the expression AVG(W(Vdc)) is entered, Probe produces a family of curves representing the results for a number of values of α, as shown in Fig. 3-17*b*. An α of 70°, which results in about 148 W delivered to the load, is the approximate solution.

The following results are obtained from Probe for $\alpha = 70°$:

Quantity	Expression	Result
DC source power	AVG(W(Vdc))	148 W (design objective of 150 W)
RMS current	RMS(I(R1))	2.87 A
Resistor power	AVG(W(R1))	16.5 W
Source apparent power	RMS(V(SOURCE))*RMS(I(Vs))	344 VA
Source average power	AVG(W(Vs))	166 W
Power factor (P/S)	166/344	0.48

3.11 COMMUTATION

The Effect of Source Inductance

The preceding discussion on half-wave rectifiers assumed an ideal source. In practical circuits, the source has an equivalent impedance which is predominantly inductive reactance. For the single-diode half-wave rectifiers of Figs. 3-1 and 3-2, the nonideal circuit is analyzed by including the source inductance with the load elements. However, the source inductance causes a fundamental change in circuit behavior for circuits like the half-wave rectifier with a freewheeling diode.

A half-wave rectifier with a freewheeling diode and source inductance L_s is shown in Fig. 3-18*a*. Assume that the load inductance is very large, making the load current constant. At $t = 0^-$, the load current is I_L, D_1 is off, and D_2 is on. As the source voltage becomes positive, D_1 turns on, but the source current does not instantly equal the load current because of L_s. Consequently, D_2 must remain on while the current in L_s and D_1 increases to that of the load. The interval when both D_1 and D_2 are on is called the commutation time or commutation angle. *Commutation is the process of turning off an electronic switch, which usually involves transferring the load current from one switch to another.*[2]

When both D_1 and D_2 are on, the voltage across L_s is

$$v_{Ls} = V_m \sin(\omega t) \tag{3-62}$$

and current in L_s and the source is

$$i_s = \frac{1}{\omega L_s}\int_0^{\omega t} v_{Ls}\, d(\omega t) + i_s(0) = \frac{1}{\omega L_s}\int_0^{\omega t} V_m \sin(\omega t)\, d(\omega t) + 0$$

$$i_s = \frac{V_m}{\omega L_s}(1 - \cos \omega t) \tag{3-63}$$

[2] Commutation in this case is an example of *natural commutation* or *line commutation,* where the change in instantaneous line voltage results in a device turning off. Other applications may use *forced commutation,* where current in a device such as a thyristor is forced to zero by additional circuitry. *Load commutation* makes use of inherent oscillating currents produced by the load to turn a device off.

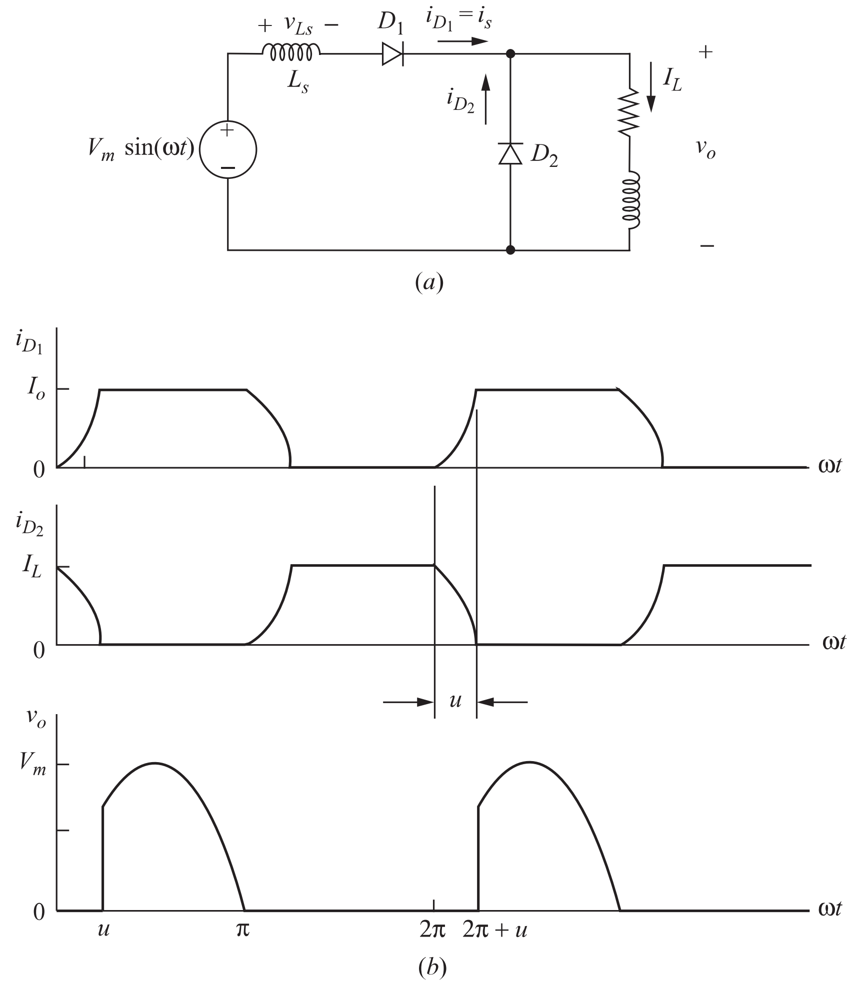


Figure 3-18 (*a*) Half-wave rectifier with freewheeling diode and source inductance; (*b*) Diode currents and load voltage showing the effects of Commutation.

Current in D_2 is

$$i_{D_2} = I_L - i_s = I_L - \frac{V_m}{\omega L_s}(1 - \cos \omega t)$$

The current in D_2 starts at I_L and decreases to zero. Letting the angle at which the current reaches zero be $\omega t = u$,

$$i_{D_2}(u) = I_L - \frac{V_m}{\omega L_s}(1 - \cos u) = 0$$

Solving for u,

$$\boxed{u = \cos^{-1}\left(1 - \frac{I_L \omega L_s}{V_m}\right) = \cos^{-1}\left(1 - \frac{I_L X_s}{V_m}\right)} \tag{3-64}$$

where $X_s = \omega L_s$ is the reactance of the source. Figure 3-18*b* shows the effect of the source reactance on the diode currents. The commutation from D_1 to D_2 is analyzed similarly, yielding an identical result for the commutation angle u.

The commutation angle affects the voltage across the load. Since the voltage across the load is zero when D_2 is conducting, the load voltage remains at zero through the commutation angle, as shown in Fig. 3-17*b*. Recall that the load voltage is a half-wave rectified sinusoid when the source is ideal.

Average load voltage is

$$V_o = \frac{1}{2\pi}\int_u^{\pi} V_m \sin(\omega t)\, d(\omega t)$$

$$= \frac{V_m}{2\pi}[-\cos(\omega t)]\Big|_u^{\pi} = \frac{V_m}{2\pi}(1 + \cos u)$$

Using u from Eq. (3-64),

$$\boxed{V_o = \frac{V_m}{\pi}\left(1 - \frac{I_L X_s}{2V_m}\right)} \tag{3-65}$$

Recall that the average of a half-wave rectified sine wave is V_m/π. Source reactance thus reduces average load voltage.

3.12 Summary

- A rectifier converts ac to dc. Power transfer is from the ac source to the dc load.
- The half-wave rectifier with a resistive load has an average load voltage of V_m/π and an average load current of $V_m/\pi R$.
- The current in a half-wave rectifier with an *RL* load contains a natural and a forced response, resulting in

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases}$$

$$\text{where} \quad Z = \sqrt{R^2 + (\omega L)^2}, \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{and} \quad \tau = \frac{L}{R}$$

The diode remains on as long as the current is positive. Power in the *RL* load is $I_{rms}^2 R$.

- A half-wave rectifier with an *RL*-source load does not begin to conduct until the ac source reaches the dc voltage in the load. Power in the resistance is $I_{rms}^2 R$, and power absorbed by the dc source is $I_o V_{dc}$, where I_o is the average load current. The load current is expressed as

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

where

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \beta) + \frac{V_{dc}}{R}\right]e^{\alpha/\omega\tau}$$

- A freewheeling diode forces the voltage across an *RL* load to be a half-wave rectified sine wave. The load current can be analyzed using Fourier analysis. A large load inductance results in a nearly constant load current.
- A large filter capacitor across a resistive load makes the load voltage nearly constant. Average diode current must be the same as average load current, making the peak diode current large.
- An SCR in place of the diode in a half-wave rectifier provides a means of controlling output current and voltage.
- PSpice simulation is an effective way of analyzing circuit performance. The parametric sweep in PSpice allows several values of a circuit parameter to be tried and is an aid in circuit design.

3.13 Bibliography

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Problems

Half-Wave Rectifier with Resistive Load

3-1. The half-wave rectifier circuit of Fig. 3-1*a* has $v_s(t) = 170 \sin(377t)$ V and a load resistance $R = 15\ \Omega$. Determine (*a*) the average load current, (*b*) the rms load current, (*c*) the power absorbed by the load, (*d*) the apparent power supplied by the source, and (*e*) the power factor of the circuit.

3-2. The half-wave rectifier circuit of Fig. 3-1*a* has a transformer inserted between the source and the remainder of the circuit. The source is 240 V rms at 60 Hz, and the load resistor is 20 Ω. (*a*) Determine the required turns ratio of the transformer such that the average load current is 12 A. (*b*) Determine the average current in the primary winding of the transformer.

3-3. For a half-wave rectifier with a resistive load, (*a*) show that the power factor is $1/\sqrt{2}$ and (*b*) determine the displacement power factor and the distortion factor as defined in Chap. 2. The Fourier series for the half-wave rectified voltage is given in Eq. (3-34).

Half-Wave Rectifier with *RL* Load

3-4. A half-wave rectifier has a source of 120 V rms at 60 Hz and an *RL* load with $R = 12\ \Omega$ and $L = 12$ mH. Determine (*a*) an expression for load current, (*b*) the average current, (*c*) the power absorbed by the resistor, and (*d*) the power factor.

3-5. A half-wave rectifier has a source of 120 V rms at 60 Hz and an *RL* load with $R = 10\ \Omega$ and $L = 15$ mH. Determine (*a*) an expression for load current, (*b*) the average current, (*c*) the power absorbed by the resistor, and (*d*) the power factor.

3-6. A half-wave rectifier has a source of 240 V rms at 60 Hz and an *RL* load with $R = 15\ \Omega$ and $L = 80$ mH. Determine (*a*) an expression for load current, (*b*) the average current, (*c*) the power absorbed by the resistor, and (*d*) the power factor. (*e*) Use PSpice to simulate the circuit. Use the default diode model and compare your PSpice results with analytical results.

3-7. The inductor in Fig. 3-2*a* represents an electromagnet modeled as a 0.1-H inductance. The source is 240 V at 60 Hz. Use PSpice to determine the value of a series resistance such that the average current is 2.0 A.

Half-Wave Rectifier with *RL*-Source Load

3-8. A half-wave rectifier of Fig. 3-5*a* has a 240 V rms, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 75$ mH, $R = 10\ \Omega$, and $V_{dc} = 100$ V. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

3-9. A half-wave rectifier of Fig. 3-5*a* has a 120 V rms, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 120$ mH, $R = 12\ \Omega$, and $V_{dc} = 48$ V. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

3-10. A half-wave rectifier of Fig. 3-6 has a 120 V rms, 60 Hz ac source. The load is a series inductance and dc voltage with $L = 100$ mH and $V_{dc} = 48$ V. Determine the power absorbed by the dc voltage source.

3-11. A half-wave rectifier with a series inductor-source load has an ac source of 240 V rms, 60 Hz. The dc source is 96 V. Use PSpice to determine the value of inductance which results in 150 W absorbed by the dc source. Use the default diode model.

3-12. A half-wave rectifier with a series inductor and dc source has an ac source of 120 V rms, 60 Hz. The dc source is 24 V. Use PSpice to determine the value of inductance which results in 50 W absorbed by the dc source. Use the default diode.

Freewheeling Diode

3-13. The half-wave rectifier with a freewheeling diode (Fig. 3-7*a*) has $R = 12\ \Omega$ and $L = 60$ mH. The source is 120 V rms at 60 Hz. (*a*) From the Fourier series of the half-wave rectified sine wave that appears across the load, determine the dc component of the current. (*b*) Determine the amplitudes of the first four nonzero ac terms in the Fourier series. Comment on the results.

3-14. In Example 3-8, the inductance required to limit the peak-to-peak ripple in load current was estimated by using the first ac term in the Fourier series. Use PSpice to determine the peak-to-peak ripple with this inductance, and compare it to the estimate. Use the ideal diode model ($n = 0.001$).

3-15. The half-wave rectifier with a freewheeling diode (Fig. 3-7*a*) has $R = 4\ \Omega$ and a source with $V_m = 50$ V at 60 Hz. (*a*) Determine a value of L such that the amplitude of the first ac current term in the Fourier series is less than 5 percent of the dc current. (*b*) Verify your results with PSpice, and determine the peak-to-peak current.

3-16. The circuit of Fig. P3-16 is similar to the circuit of Fig. 3-7*a* except that a dc source has been added to the load. The circuit has $v_s(t) = 170 \sin(377t)$ V, $R = 10\ \Omega$, and $V_{dc} = 24$ V. From the Fourier series, (*a*) determine the value of L such that the peak-to-peak variation in load current is no more than 1 A. (*b*) Determine the power absorbed by the dc source. (*c*) Determine the power absorbed by the resistor.

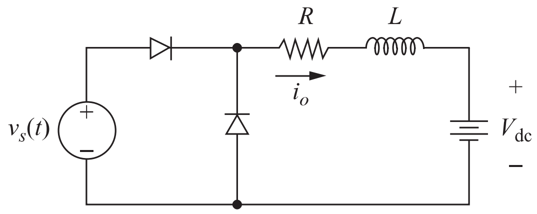

Figure P3-16

Half-Wave Rectifier with a Filter Capacitor

3-17. A half-wave rectifier with a capacitor filter has $V_m = 200$ V, $R = 1$ kω, $C = 1000\ \mu$F, and $\omega = 377$. (*a*) Determine the ratio of the RC time constant to the period of the input sine wave. What is the significance of this ratio? (*b*) Determine the peak-to-peak ripple voltage using the exact equations. (*c*) Determine the ripple using the approximate formula in Eq. (3-51).

3-18. Repeat Prob. 3-17 with (*a*) $R = 100\ \Omega$ and (*b*) $R = 10\ \Omega$. Comment on the results.

3-19. A half-wave rectifier with a 1-kΩ load has a parallel capacitor. The source is 120 V rms, 60 Hz. Determine the peak-to-peak ripple of the output voltage when the capacitor is (*a*) 4000 μF and (*b*) 20 μF. Is the approximation of Eq. (3-51) reasonable in each case?

3-20. Repeat Prob. 3-19 with $R = 500\ \Omega$.

3-21. A half-wave rectifier has a 120 V rms, 60 Hz ac source. The load is 750 Ω. Determine the value of a filter capacitor to keep the peak-to-peak ripple across the load to less than 2 V. Determine the average and peak values of diode current.

3-22. A half-wave rectifier has a 120 V rms 60 Hz ac source. The load is 50 W. (*a*) Determine the value of a filter capacitor to keep the peak-to-peak ripple across the load to less than 1.5 V. (*b*) Determine the average and peak values of diode current.

Controlled Half-Wave Rectifier

3-23. Show that the controlled half-wave rectifier with a resistive load in Fig. 3-13*a* has a power factor of

$$\text{pf} = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

3-24. For the controlled half-wave rectifier with resistive load, the source is 120 V rms at 60 Hz. The resistance is 100 Ω, and the delay angle α is 45°. (*a*) Determine the average voltage across the resistor. (*b*) Determine the power absorbed by the resistor. (*c*) Determine the power factor as seen by the source.

3-25. A controlled half-wave rectifier has an ac source of 240 V rms at 60 Hz. The load is a 30-Ω resistor. (*a*) Determine the delay angle such that the average load current is 2.5 A. (*b*) Determine the power absorbed by the load. (*c*) Determine the power factor.

3-26. A controlled half-wave rectifier has a 120 V rms 60 Hz ac source. The series *RL* load has $R = 25\ \omega$ and $L = 50$ mH. The delay angle is 30°. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

3-27. A controlled half-wave rectifier has a 120 V rms 60 Hz ac source. The series *RL* load has $R = 40\ \Omega$ and $L = 75$ mH. The delay angle is 60°. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

3-28. A controlled half-wave rectifier has an *RL* load with $R = 20\ \Omega$ and $L = 40$ mH. The source is 120 V rms at 60 Hz. Use PSpice to determine the delay angle required to produce an average current of 2.0 A in the load. Use the default diode in the simulation.

3-29. A controlled half-wave rectifier has an *RL* load with $R = 16\ \Omega$ and $L = 60$ mH. The source is 120 V rms at 60 Hz. Use PSpice to determine the delay angle required to produce an average current of 1.8 A in the load. Use the default diode in the simulation.

3-30. A controlled half-wave rectifier has a 120 V, 60 Hz ac source. The load is a series inductance, resistance, and dc source, with $L = 100$ mH, $R = 12\ \Omega$, and $V_{dc} = 48$ V. The delay angle is 50°. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

3-31. A controlled half-wave rectifier has a 240 V rms 60 Hz ac source. The load is a series resistance, inductance, and dc source with $R = 100\ \Omega$, $L = 150$ mH, and $V_{dc} = 96$ V. The delay angle is 60°. Determine (*a*) the power absorbed by the dc voltage source, (*b*) the power absorbed by the resistance, and (*c*) the power factor.

3-32. Use PSpice to determine the delay angle required such that the dc source in Prob. 3-31 absorbs 35 W.

3-33. A controlled half-wave rectifier has a series resistance, inductance, and dc voltage source with $R = 2\ \Omega$, $L = 75$ mH, and $V_{dc} = 48$ V. The source is 120 V rms at 60 Hz. The delay angle is 50°. Determine (*a*) an expression for load current, (*b*) the power absorbed by the dc voltage source, and (*c*) the power absorbed by the resistor.

3-34. Use PSpice to determine the delay angle required such that the dc source in Prob. 3-33 absorbs 50 W.

3-35. Develop an expression for current in a controlled half-wave rectifier circuit that has a load consisting of a series inductance L and dc voltage V_{dc}. The source is $v_s = V_m \sin \omega t$, and the delay angle is α. (*a*) Determine the average current if $V_m = 100$ V, $L = 35$ mH, $V_{dc} = 24$ V, $\omega = 2\pi 60$ rad/s, and $\alpha = 75°$. (*b*) Verify your result with PSpice.

3-36. A controlled half-wave rectifier has an *RL* load. A freewheeling diode is placed in parallel with the load. The inductance is large enough to consider the load current to be constant. Determine the load current as a function of the delay angle alpha. Sketch the current in the SCR and the freewheeling diode. Sketch the voltage across the load.

Commutation

3-37. The half-wave rectifier with freewheeling diode of Fig. 3-18*a* has a 120 V rms ac source that has an inductance of 1.5 mH. The load current is a constant 5 A. Determine the commutation angle and the average output voltage. Use PSpice to verify your results. Use ideal diodes in the simulation. Verify that the commutation angle for D_1 to D_2 is the same as for D_2 to D_1.

3-38. The half-wave rectifier with freewheeling diode of Fig. 3-18*a* has a 120 V rms ac source which has an inductance of 10 mH. The load is a series resistance-inductance with $R = 20\ \Omega$ and $L = 500$ mH. Use PSpice to determine (*a*) the steady-state average load current, (*b*) the average load voltage, and (*c*) the commutation angle. Use the default diode in the simulation. Comment on the results.

3-39. The half-wave rectifier with freewheeling diode of Fig. 3-18*a* has a 120 V rms ac source which has an inductance of 5 mH. The load is a series resistance-inductance with $R = 15\ \Omega$ and $L = 500$ mH. Use PSpice to determine (*a*) the steady-state average load current, (*b*) the average load voltage, and (*c*) the commutation angle. Use the default diode in the simulation.

3-40. The commutation angle given in Eq. (3-64) for the half-wave rectifier with a freewheeling diode was developed for commutation of load current from D_2 to D_1. Show that the commutation angle is the same for commutation from D_1 to D_2.

3-41. Diode D_1 in Fig. 3-18*a* is replaced with an SCR to make a controlled half-wave rectifier. Show that the angle for commutation from the diode to the SCR is

$$u = \cos^{-1}\left(\cos \alpha - \frac{I_L X_s}{V_m}\right) - \alpha$$

where α is the delay angle of the SCR.

Design Problems

3-42. A certain situation requires that either 160 or 75 W be supplied to a 48 V battery from a 120 V rms 60 Hz ac source. There is a two-position switch on a control panel set at either 160 or 75. Design a single circuit to deliver both values of power, and specify what the control switch will do. Specify the values of all the components in your circuit. The internal resistance of the battery is 0.1 Ω.

3-43. Design a circuit to produce an average current of 2 A in an inductance of 100 mH. The ac source available is 120 V rms at 60 Hz. Verify your design with PSpice. Give alternative circuits that could be used to satisfy the design specifications, and give reasons for your selection.

3-44. Design a circuit that will deliver 100 W to a 48 V dc source from a 120 V rms 60 Hz ac source. Verify your design with PSpice. Give alternative circuits that could be used to satisfy the design specifications, and give reasons for your selection.

3-45. Design a circuit which will deliver 150 W to a 100 V dc source from a 120 V rms 60 Hz ac source. Verify your design with PSpice. Give alternative circuits that could be used to satisfy the design specifications, and give reasons for your selection.