HW6

Manny Fomundam

2023-11-08

```r
mu <- 5
sigma <- sqrt(100/12)

# Chebyshev's Inequality
k <- 3 / sigma
chebyshev_bound <- 1 - 1 / k^2

# Actual probability
actual_prob <- (8 - 2) / (10 - 0)

chebyshev_bound
```

```
## [1] 0.07407407
```

```r
actual_prob
```

```
## [1] 0.6
```

```r
# Calculate standard deviation numerically
sigma <- sqrt(integrate(function(x) ((x - 5/4)^2 * 5/x^6), lower = 1, upper = Inf)$value)

# Chebyshev's Inequality for P(X >= 2.5)
k_2.5 <- abs(2.5 - 5/4) / sigma
prob_bound_2.5 <- 1 / k_2.5^2
print(paste("Chebyshev's bound for P(X >= 2.5):", prob_bound_2.5))
```

```
## [1] "Chebyshev's bound for P(X >= 2.5): 0.0666666666666667"
```

```r
# Find a such that P(X >= a) <= 0.15
k_a <- sqrt(1 / 0.15)
a <- abs(k_a * sigma - 5/4)
print(paste("Value of a for P(X >= a) <= 0.15:", a))
```

```
## [1] "Value of a for P(X >= a) <= 0.15: 0.416666666666667"
```

```r
# Define the possible outcomes of a die roll
die <- 1:6

# Generate all possible outcomes of rolling two dice
outcomes <- expand.grid(die, die)

# Define the random variables X and Y
X <- outcomes$Var1 + outcomes$Var2
Y <- abs(outcomes$Var1 - outcomes$Var2)

# (a) Find the mean of X
```

```r
mean_X <- mean(X)

# (b) Find the variance of X
var_X <- var(X)

# (c) Find the skewness of X
library(moments)
skew_X <- skewness(X)

# (d) Find the mean of Y
mean_Y <- mean(Y)

# (e) Find the variance of Y
var_Y <- var(Y)

# (f) Find the skewness of Y
skew_Y <- skewness(Y)

# Print the results
print(paste("Mean of X: ", mean_X))
```

```
## [1] "Mean of X:  7"
```

```r
print(paste("Variance of X: ", var_X))
```

```
## [1] "Variance of X:  6"
```

```r
print(paste("Skewness of X: ", skew_X))
```

```
## [1] "Skewness of X:  0"
```

```r
print(paste("Mean of Y: ", mean_Y))
```

```
## [1] "Mean of Y:  1.94444444444444"
```

```r
print(paste("Variance of Y: ", var_Y))
```

```
## [1] "Variance of Y:  2.11111111111111"
```

```r
print(paste("Skewness of Y: ", skew_Y))
```

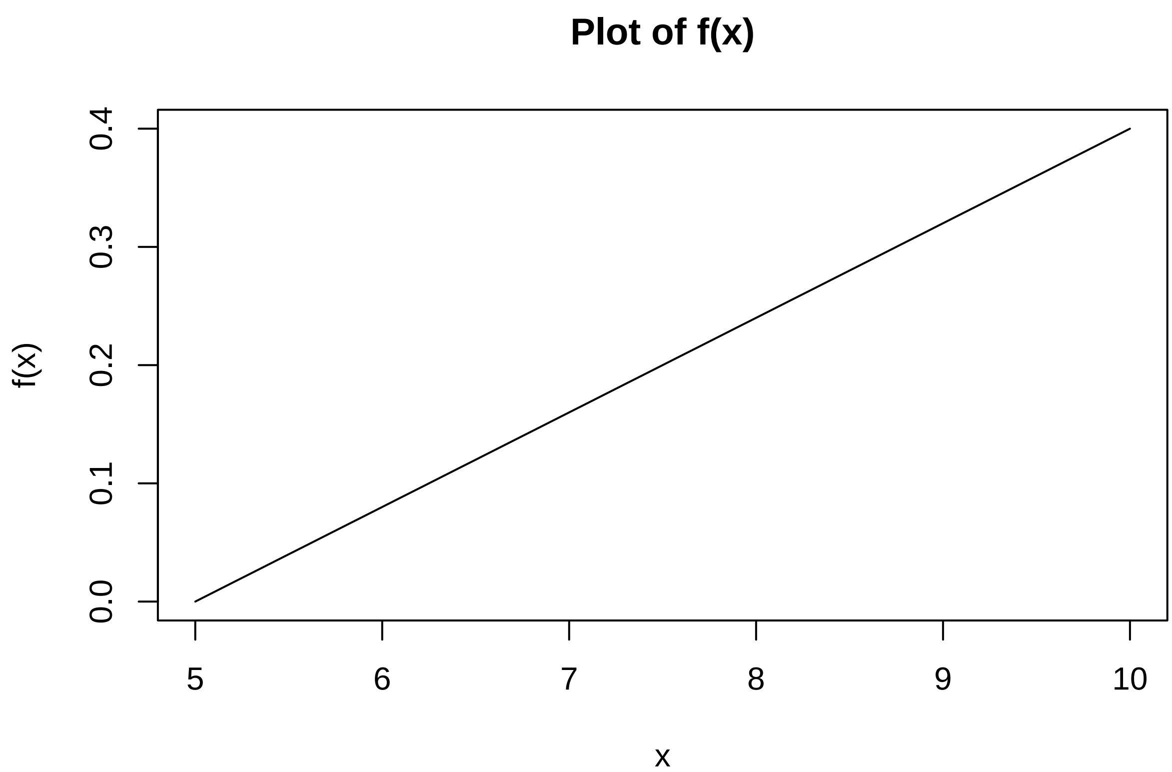
```
## [1] "Skewness of Y:  0.437582915369055"
```

```r
#(b)
# Define the function f(x)
f <- function(x) {
  if (x >= 5 & x <= 10) {
    return(2/25 * (x - 5))
  } else {
    return(0)
  }
}

# Create a sequence of x values
x_values <- seq(5, 10, by = 0.1)

# Compute f(x) for each x
y_values <- sapply(x_values, f)
```

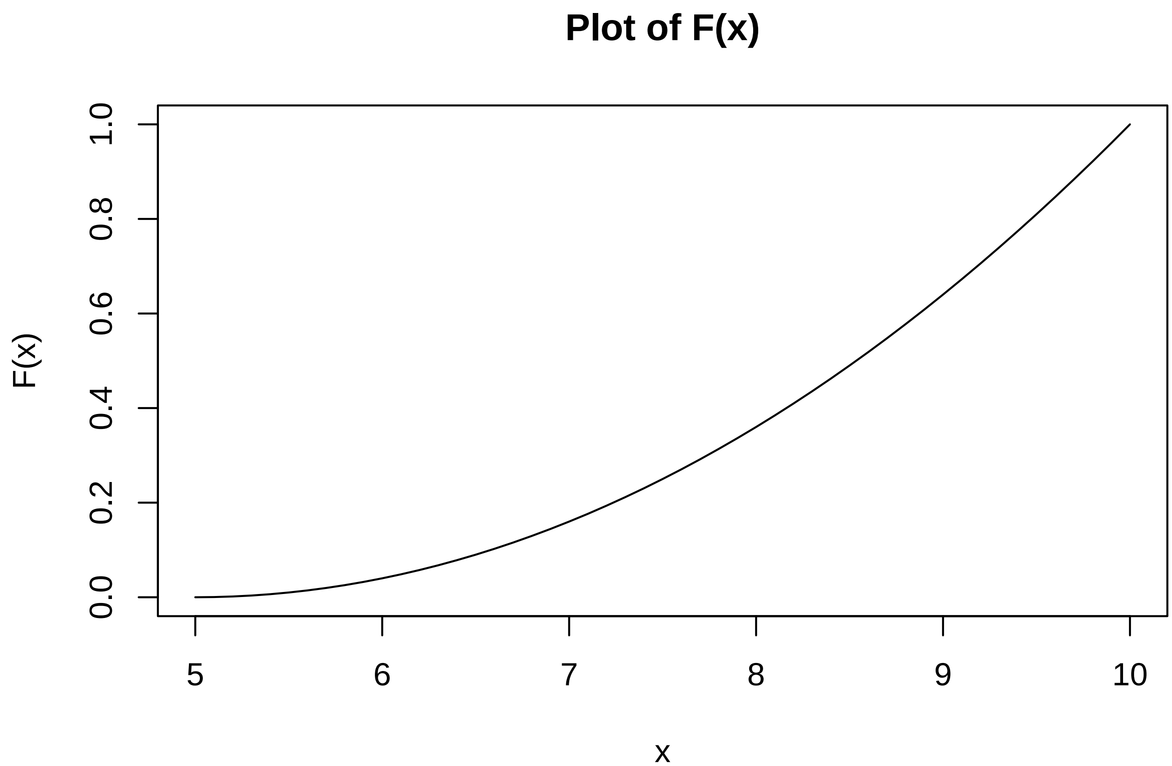
```r
# Plot f(x)
plot(x_values, y_values, type = "l", xlab = "x", ylab = "f(x)", main = "Plot of f(x)")
```




```r
#(c)
# Define the cumulative probability function F(x)
F <- function(x) {
  if (x >= 5 & x <= 10) {
    return(integrate(function(t) 2/25 * (t - 5), lower = 5, upper = x)$value)
  } else {
    return(0)
  }
}

# Compute F(x) for each x
F_values <- sapply(x_values, F)

# Plot F(x)
plot(x_values, F_values, type = "l", xlab = "x", ylab = "F(x)", main = "Plot of F(x)")
```




```r
#(a)
# Define the Poisson probability function
poisson_prob <- function(lambda, k) {
  return(exp(-lambda) * (lambda^k) / factorial(k))
}

# Given lambda (average rate of occurrence)
lambda <- 5

# Probability of receiving more than 4 calls in an hour
prob_more_than_4 <- 1 - sum(sapply(0:4, function(k) poisson_prob(lambda, k)))
cat("Probability of receiving more than 4 calls per hour:", prob_more_than_4, "\n")
```

```
## Probability of receiving more than 4 calls per hour: 0.5595067
```

```r
#(b)
# Probability the first occurrence of 3 calls happens within less than 1 hour
prob_less_than_1_hour_3_calls <- ppois(3, lambda, lower.tail = TRUE)
cat("Probability of waiting less than 1 hour before 3 calls:", prob_less_than_1_hour_3_calls, "\n")
```

```
## Probability of waiting less than 1 hour before 3 calls: 0.2650259
```