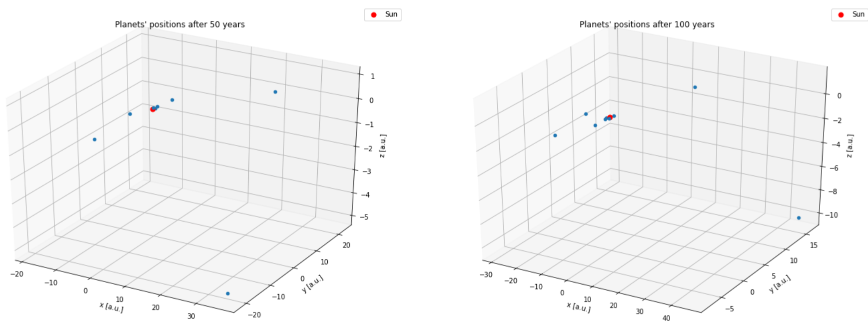
1

2

2.1

The following plots show the location of planets after 50 years and 100 years, using the Velocity Verlet algorithm.

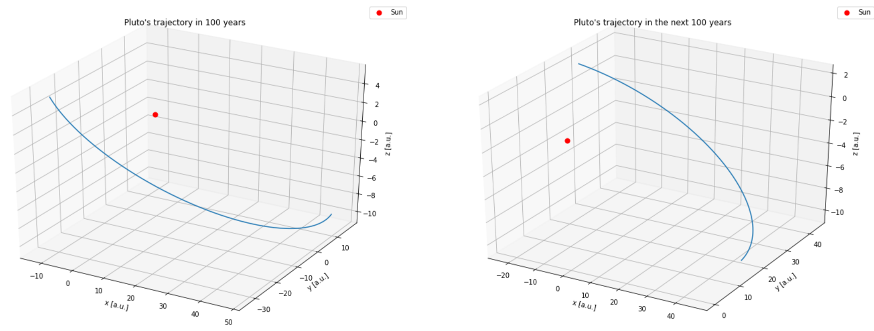


(a) 50 years (b) 100 years

Figure 1: On the left, positions of planets after 50 years obtained with the Velocity Verlet algorithm in steps of half a day. Same on the right, after 100 years.

2.2

The simulation was run over the first 100 years, saving Pluto's coordinates every 100 steps. Then the code was run once more, using the final positions and velocities from the previous run as initial conditions. Results are shown in figure 2.
The orbital period was estimated from Kepler's third law: $\frac{a^3}{T^2} = \frac{GM}{4\pi^2}$, where T is the revolution period, M the Sun mass, G the gravitational constant and a the major semi-axis (estimated from the simulated position data. The calculated orbital period is 276 years (correct value 249 years).



(a) First 100 years

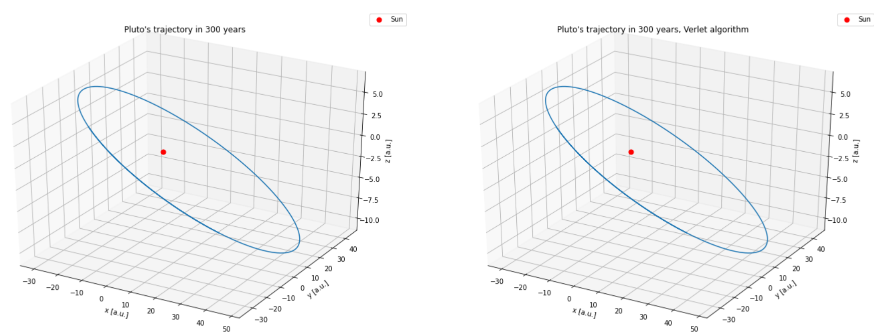
(b) Next 100 years

Figure 2: On the left, trajectory of Pluto over the first 100 years obtained with the Velocity Verlet algorithm in steps of half a day. On the right, trajectory of Pluto over the next 100 years obtained with the Velocity Verlet algorithm in steps of half a day.

3

3.1

The code was modified to implement the Verlet algorithm scheme. Plots in figure 3 compare the trajectories of Pluto obtained with the Velocitiy Verlet algorithm and the Verlet algorithm. As can be seen from the figures, the two trajectories are similar.



(a) Velocity Verlet

(b) Verlet

Figure 3: Trajectory of Pluto over 300 years obtained with the Velocity Verlet algorithm in steps of half a day (left) and with the Verlet algorithm in steps of half a day (right).

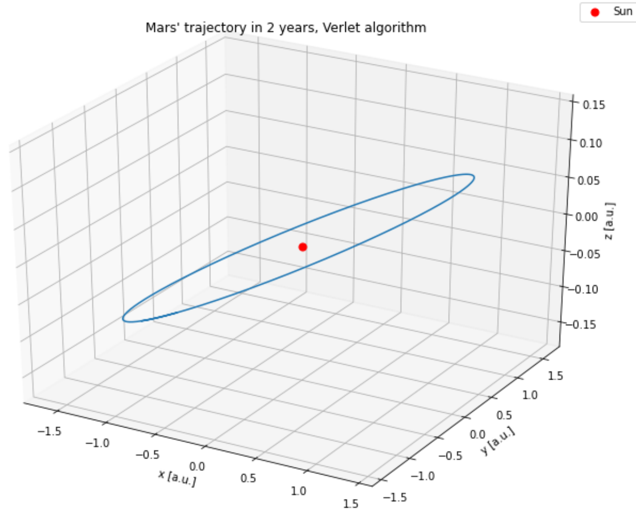


Figure 4: Mars' trajectory over 2 years, obtained with the Verlet algorithm in steps of half a day.

3.2

The orbital period of Mars was estimated from Kepler's third law, too: $\frac{a^3}{T^2} = \frac{GM}{4\pi^2}$, where T is the revolution period, M the Sun mass, G the gravitational constant and a the major semi-axis (estimated from the simulated position data). The calculated orbital period is ≈ 1.77 years (correct value ≈ 1.88 years). The plot 4 shows the trajectory of Mars in 2 years (approximately Mars' orbital period).

4

The code was then modified again to implement the Euler algorithm scheme, and the simulation was run over 300 years with the same initial conditions. The total energy of the system $E = U + K$ was evaluated every 100 steps. As can be noticed from graph 5, E varies throughout the simulation run, but Pluto's trajectory appear to be quite similar to the one obtained with the previous algorithms (figure 6).

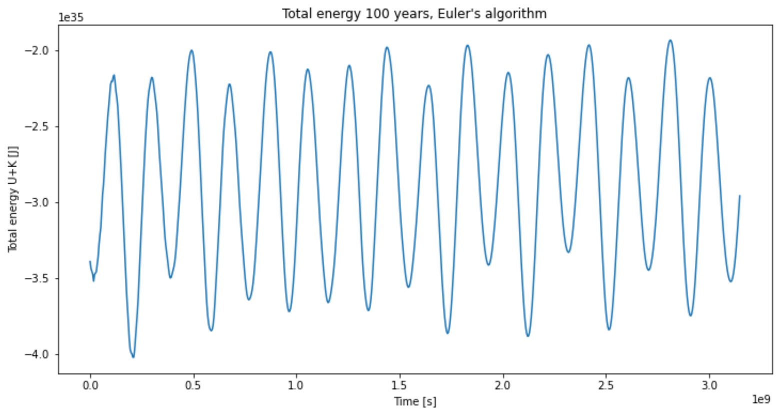


Figure 5: Total energy of the Solar System with Euler algorithm (1 measurement every 100 simulation steps).

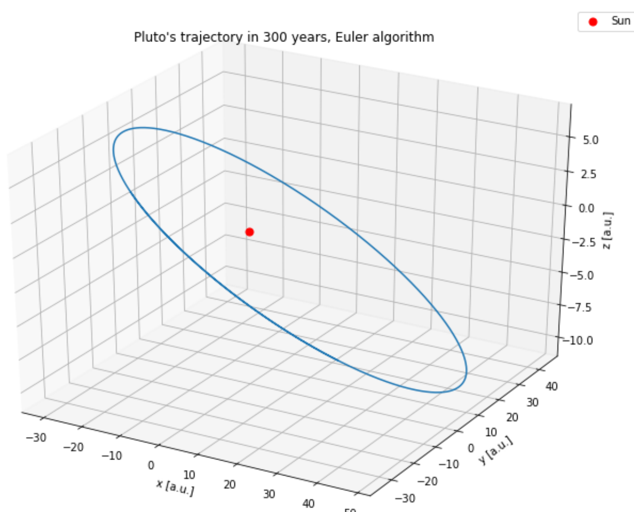


Figure 6: Pluto's trajectory over 300 years, obtained with the Euler algorithm in steps of half a day.