Gravitational disturbance

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ABSTRACT

Gravity anomalies have long been used by geophysicists for the purpose of determining density distributions in subsurface.

However, gravity anomalies

In this paper, we discuss the fundamental concepts

URGENTE: (Marussi et al., 1974), (Torge and Müller, 2012), section 4.2.1

INTRODUCTION

The resultant of gravitational force and centrifugal force acting on a body at rest on the Earth's surface is called gravity vector and its intensity is called simply gravity (Hofmann-Wellenhof and Moritz, 2005).

In the case of gravimetry on moving platforms (e.g., airplanes, helicopters, marine vessels), there are additional non-gravitational accelerations due to the vehicle motion, such as Coriolis acceleration and high-frequency vibrations (Glennie et al., 2000; Nabighian et al., 2005; Baumann et al., 2012).

Gravity is the most commonly measured quantity in geophysical surveying. Geophysicists use gravity for estimating the Earth's internal density distribution whereas geodesists use gravity to estimate the geoid (Li and Götze, 2001).

Hence, geophysicists are usually interested in the gravitational component of the observed gravity which is produced by the Earth's internal density distribution.

The first step of the procedure for isolating this gravitational component consists in removing the non-gravitational effects due to the vehicle motion and also the time variations such as Earth tides, instrumental drift and barometric pressure changes, for example.

If these effects are properly removed, the resultant gravity data can be considered as the sum of a centrifugal component due to the Earth's rotation and a gravitational component produced by the whole Earth's internal density distribution.

In applied geophysics, gravity surveys are usually conducted over small areas on the Earth's surface for the purpose of characterizing geological structures located within the crust and upper mantle.

Consequently, geophysicists are only interested in the particular gravitational component of gravity which is produced by these target geological structures.

The isolation of this particular gravitational component and its subsequent use for estimating density distributions related to geological structures in subsurface are the main goals in applied geophysics (Blakely, 1996).

In this paper, we present a discussion based on the well-established concepts of normal Earth, normal gravity, gravity disturbance and gravity anomalies aiming at bringing some light to the following question: in geophysical applications, should we use gravity disturbance or gravity anomaly?

It seems that this theoretical issue has been debated within the scientific community from a more geodetic than geophysical point of view (LaFehr, 1991; Chapin, 1996; Li and Götze, 2001; Fairhead et al., 2003; Hackney and Featherstone, 2003; Hinze et al., 2005).

Our reasoning suggests that the gravity disturbance is conceptually more appropriated than gravity anomalies for approximating the gravitational disturbance and use it for estimating density distributions in subsurface.

NORMAL EARTH AND NORMAL GRAVITY

Traditionally, the Earth's gravity field is approximated by the gravity field produced by a geocentric and rigid ellipsoid of revolution which has the minor axis b coincident with the mean rotating axis of the Earth Z, the same total mass (including the atmosphere) and also the same angular velocity of the Earth (Figure 1). Another characteristic of this model is that its limiting surface coincides with a particular equipotential of its own gravity field. This model is called as normal Earth and its gravity field is called normal gravity field

(Vaníček and Krakiwsky, 1987; Hofmann-Wellenhof and Moritz, 2005; Torge and Müller, 2012).

It is worth noting that, although the normal Earth has the same total mass (including the atmosphere) of the Earth, its internal density distribution is unknown.

The only condition imposed on its internal density distribution is that it produces a normal gravity field having a particular equipotential which coincides with its limiting surface.

For convenience, we denote any density distribution satisfying this condition as a normal density distribution.

The normal Earth gives rise to the geodetic coordinate system. In this coordinate system, the position of a point P is defined by the geometric height h, geodetic latitude φ and longitude λ (Figure 1). Geodetic coordinates (h, φ, λ) can be easily converted into geocentric Cartesian coordinates (X, Y, Z) (Figure 1). The plane containing the point P, the axis Z and the origin O of this geocentric Cartesian coordinate system is called meridian plane (gray plane in Figure 1).

Similarly to the gravity vector and gravity, the resultant of the virtual gravitational and centrifugal forces exerted by the normal Earth on a body at rest at a point P is called normal gravity vector and its intensity is called simply normal gravity.

GRAVITY DISTURBANCE

It is worth noting that, by definition, the centrifugal component of the normal gravity field is equal to the centrifugal component of the Earth's gravity field if they are evaluated at the same point.

Then, the differences between the gravity vector (corrected from non-gravitational effects due to the vehicle motion) and the normal gravity vector, at the same point, represents a purely gravitational and consequently harmonic disturbing field. For convenience, we denote this disturbing field as gravitational disturbance.

It seems logical to expected that the gravitational disturbance is caused by contrasts between the actual internal density distribution of the Earth and the internal density distribution of the normal Earth. In applied geophysics, these density differences are generally called “anomalous masses” (e.g., Hammer, 1945; LaFehr, 1965), “density anomalies” (e.g., Forsberg, 1984) or “gravity sources” (e.g., Blakely, 1996). Here, we opted for using the last term.

The difference between the observed gravity and the normal gravity, at the same point, is called gravity disturbance (Hofmann-Wellenhof and Moritz, 2005). Notice that the gravity disturbance is not equivalent to the magnitude of the difference between the gravity vector and the normal gravity vector at the same point. As properly pointed out by Hackney and Featherstone (2003), the gravity disturbance is a very-well established quantity in geodesy, but appears to be less well known in geophysics.

The gravity anomaly is defined as the difference between the gravity at the geoid and the normal gravity at the ellipsoid and is the commonly used quantity in applied geophysics. Different gravity anomalies can be calculated, depending on the corrections applied to them (Blakely, 1996; Hofmann-Wellenhof and Moritz, 2005). These corrections are usually called gravity reductions. For example, the Free-air anomaly is an approximation of the gravity disturbance whereas the Bouguer anomaly is an approximation of the terrain corrected gravity disturbance. The last one is commonly used by geophysicists as the gravitational effect produced by the gravity sources. Although this approximation is valid for most practical applications, it is important to bear in mind not only the terminology changes, but also the conceptual assumptions.

There was a certain lack of comprehension regarding the geophysical meaning of gravity anomalies until the mid 90's. As properly pointed out by Chapin (1996) at that time, "although the corrections which bring about a Bouguer gravity anomaly are well established, the reasons for doing them are not well understood. One cause of this common misunderstanding is that the subject has been poorly presented in many of the basic texts". In his seminal book, Blakely (1996) brought some light on the geophysical meaning of gravity anomalies from the perspective of applied geophysics. Blakely (1996) correctly defined gravity sources as density contrasts between the actual internal density distribution of the Earth and the internal density distribution of the normal Earth. However, he did not stress that, by removing the normal gravity evaluated on the ellipsoid from the gravity measured on the Earth's surface, the remaining disturbing field will reflect not only the effect produced by the gravity sources, but also a small combination of gravitational and centrifugal effects. This additional, non-harmonic and undesired effect is due to the calculation of the normal gravity at the surface of the ellipsoid instead of at the same points where the gravity is measured.

- Mencionar tambem que anomalias de gravidade requerem o valor da gravidade dentro das fontes

MATHEMATICAL DESCRIPTION OF THE GRAVITY DISTURBANCE IN A LOCAL COORDINATE SYSTEM

In a local- or regional-gravity study, geophysicists commonly use a topocentric Cartesian coordinate system with the x axis pointing to North, the y axis pointing to East and the z axis with the same direction as the normal $\hat{\mathbf{n}}_i$ to the ellipsoid, but pointing downward (Figure 1). In this coordinate system, the observed gravity vector $\mathbf{g}_i^o$, at the point (x_i, y_i, z_i), $i = 1, ..., N$, can be represented by

$$\mathbf{g}_i^o = \boldsymbol{\gamma}_i + \Delta\mathbf{g}_i^o \, , \tag{1}$$

where $\boldsymbol{\gamma}_i$ and $\Delta\mathbf{g}_i^o$ are, respectively, the normal gravity vector and a disturbing gravitational attraction produced by the anomalous masses at the point (x_i, y_i, z_i).

By definition, the gravity disturbance δg_i^o, at the point (x_i, y_i, z_i), is given by

$$\delta g_i^o = \|\delta\mathbf{g}_i^o\| - \|\boldsymbol{\gamma}_i\| \, , \tag{2}$$

where $\|\delta\mathbf{g}_i^o\|$ and $\|\boldsymbol{\gamma}_i\|$ are, respectively, the observed gravity and the normal gravity at the point (x_i, y_i, z_i). Fortunately, the condition $\|\boldsymbol{\gamma}_i\| \gg \|\Delta\mathbf{g}_i^o\|$ is met at all points located above or on the Earth's surface. By combining this condition and the definition of observed gravity vector given in equation 1, we can approximate the observed gravity $\|\delta\mathbf{g}_i^o\|$ (equation 2) by a first order Taylor's expansion as follows:

$$\|\delta\mathbf{g}_i^o\| \approx \|\boldsymbol{\gamma}_i\| + \hat{\boldsymbol{\gamma}}_i^\top \Delta\mathbf{g}_i^o \, , \tag{3}$$

where $\hat{\boldsymbol{\gamma}}_i$ is a unit vector with the same direction as the normal gravity vector $\boldsymbol{\gamma}_i$ at the point (x_i, y_i, z_i). This approximation is largely used in applied geophysics for representing total-field anomalies (e.g., Blakely, 1996). Notice that, local- or regional-gravity studies, the unit vector $\hat{\boldsymbol{\gamma}}_i$ (equation 3) coincides with the z axis of the local Cartesian coordinate system defined at the beginning of this section. Consequently, by using the approximation defined in equation 3, the gravity disturbance (equation 2) can be rewritten as follows

$$\delta g_i^o \approx \hat{\mathbf{z}}^\top \Delta \mathbf{g}_i^o \, , \tag{4}$$

where $\hat{\mathbf{z}}^\top = [0\;0\;1]$. According to equation 4, the gravity disturbance δg_i^o (equation 2) represents the vertical component of the gravitational attraction exerted by the gravity sources at the point (x_i, y_i, z_i). As a consequence, the gravity disturbance produced by a homogeneous gravity source can be represented by the following harmonic function

$$d_i^o = k_g \, G \, \rho \, \partial_z \phi_i \, , \tag{5}$$

where G is the Newtonian constant of gravitation (in $m^3/(kg\, s^2)$), $k_g = 10^5$ is a constant factor transforming from m/s^2 to milligal (mGal), and $\partial_z \phi_i$ is a harmonic function representing the first derivative, evaluated at the observation point (x_i, y_i, z_i), $i = 1, ..., N$, of the function

$$\phi(x, y, z) = \int \underset{v}{\int} \int \frac{1}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} \, dv \tag{6}$$

with respect to the variable z. The integral is conducted over the coordinates x', y' and z' within the volume v of the gravity source. This equation can be easily generalized for the case of multiple gravity sources.

GEOPHYSICAL ARGUMENTS FOR USING THE GRAVITY DISTURBANCE

Almost all interpretation techniques assume, implicitly or directly, that the gravity data is harmonic (e.g., upward/downward continuation, data processing with equivalent layer, conversions between gravity and magnetic data, computation of vertical derivatives via Fourier and Hilbert transforms).

Consequently, they implicitly or directly assume that the gravity data approximates the gravitational disturbance.

Almost all forward modelling techniques compute the vertical component of the gravitational attraction exerted by the geological bodies at the observation points.

Hence, almost all geophysicists implicitly compute the vertical component of the gravitational disturbance.

The gravity anomaly is defined as the difference between the gravity $\|\mathbf{g}_P\|$, at a point P on the Geoid, and the normal gravity $\|\boldsymbol{\gamma}_Q\|$, on the reference elipsoid, where P and Q have the same geodetic latitude and longitude.

Consequently, the gravity anomaly is a function of the geodetic latitude and longitude only and cannot be calculated at arbitrary heights.

Gravity anomalies require the computation of gravity within the topographic masses, where the gravitational disturbance is not harmonic.

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LIST OF FIGURES

1 Schematic representation of the geodetic coordinate system.
2 Schematic representation of the geodetic coordinate system.

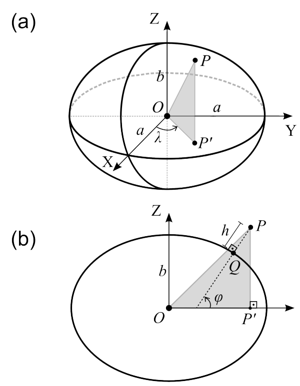

Figure 1: Schematic representation of the geodetic coordinate system.

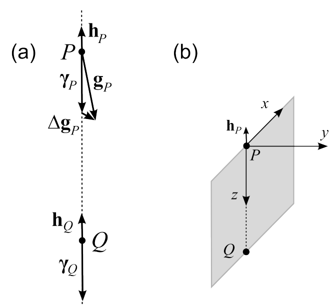

Figure 2: Schematic representation of the geodetic coordinate system.