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Gravity disturbance or gravity anomaly?

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SUMMARY
Gravity anomalies have long been used by geophysicists for the purpose of determining density distributions in subsurface.
However, gravity anomalies
In this paper, we discuss the fundamental concepts
URGENTE: (Marussi et al. 1974), (Torge & Müller 2012), section 4.2.1

Key words: potential fields – gravity disturbance – gravity anomaly – gravity modeling.

1 INTRODUCTION

The resultant of gravitational force and centrifugal force acting on a body at rest on the Earth's surface is called *gravity vector* and its intensity is called simply *gravity* (Hofmann-Wellenhof & Moritz 2005). In the case of gravimetry on moving platforms (e.g., airplanes, helicopters, marine vessels), there are additional non-gravitational accelerations due to the vehicle motion, such as Coriolis acceleration and high-frequency vibrations (Glennie et al. 2000; Nabighian et al. 2005; Baumann et al. 2012). Geophysicists use gravity for estimating the Earth's internal density distribution whereas geodesists use gravity to estimate the geoid (Li & Götze 2001). Hence, geophysicists are usually interested in the gravitational component of the observed gravity, which is produced by the Earth's internal density distribution. The first step of the procedure for isolating this gravitational component consists in removing the non-gravitational effects due to the vehicle motion and also the time variations such as Earth tides, instrumental drift and barometric pressure changes, for example. If these effects are properly removed, the resultant gravity data can be considered as the sum of a centrifugal component due to the Earth's rotation and a gravitational component produced by the whole Earth's internal density distribution. The isolation of this particular gravitational component and its subsequent use for estimating density distributions related to geological structures in subsurface are the main goals in applied geophysics (Blakely 1996).

Based on well-established concepts of the literature, we present a discussion aiming at bringing some light to the following question: in geophysical applications, should we use the gravity disturbance or gravity anomaly? It seems that this theoretical issue has been debated within the scientific community from a more geodetic than geophysical point of view (LaFehr 1991; Chapin 1996; Li & Götze 2001; Fairhead et al. 2003; Hackney & Featherstone 2003; Hinze et al. 2005). Our reasoning suggests that the gravity disturbance is more appropriated than gravity anomalies for approximating the gravitational effect produced by the Earth's internal density distribution.

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2 NORMAL EARTH AND NORMAL GRAVITY

Traditionally, the Earth's gravity field is approximated by the gravity field produced by a geocentric and rigid ellipsoid of revolution, which has the minor axis b coincident with the mean rotating axis of the Earth Z, the same total mass (including the atmosphere) and also the same angular velocity of the Earth (Heiskanen & Moritz 1967; Vaníček & Krakiwsky 1987; Hofmann-Wellenhof & Moritz 2005; Torge & Müller 2012). Another characteristic of this model is that its limiting surface coincides with a particular equipotential of its own gravity field. Here, we follow (Torge & Müller 2012) and call this model as *normal Earth*.

Similarly to the gravity vector and gravity, the resultant of the virtual gravitational and centrifugal forces exerted by the normal Earth on a body at rest at a point P is called *normal gravity vector* and its intensity is called simply *normal gravity*. In geodesy, any model used to represent the normal gravity field can be arbitrarily defined for the only purpose of keeping the difference from the actual gravity field as small as possible (Vaníček & Krakiwsky 1987).

It is worth noting that, although the normal Earth has the same total mass (including the atmosphere) of the Earth, its internal density distribution is unknown. The search for physically meaningful mass distributions that generate a required normal gravity field has geophysical rather than geodetic motives (Marussi et al. 1974). The only condition imposed on its internal density distribution is that it produces a gravity field having a particular equipotential which coincides with its limiting surface. For convenience, we denote any density distribution satisfying this condition as a *normal density distribution*.

The normal Earth gives rise to the *geodetic coordinate system* (Heiskanen & Moritz 1967; Soler 1976; Torge & Müller 2012;

Bouman et al. 2013). In this coordinate system, the position of a point P is defined by the *geometric height* h, *geodetic latitude* φ and *longitude* λ (Figure A1). Geodetic coordinates (h, φ, λ) can be easily converted into geocentric Cartesian coordinates (X, Y, Z) (Figure A1). The plane containing the point P, the axis Z and the origin O of this *geocentric Cartesian coordinate system* is called meridian plane (gray plane in Figure A1).

At a given point (h, φ, λ), there are three unit vectors (Figure A1) given by (Soler 1976):

$$\begin{aligned} \hat{\boldsymbol{u}} &= \begin{bmatrix} \cos\varphi\,\cos\lambda \\ \cos\varphi\,\sin\lambda \\ \sin\varphi \end{bmatrix} \\ \hat{\boldsymbol{v}} &= \begin{bmatrix} -\sin\varphi\,\cos\lambda \\ -\sin\varphi\,\sin\lambda \\ \cos\varphi \end{bmatrix} , \\ \hat{\boldsymbol{w}} &= \begin{bmatrix} -\sin\lambda \\ \cos\lambda \\ 0 \end{bmatrix} \end{aligned} \tag{1}$$

Notice that these unit vectors are mutually orthogonal. The normal gravity vector $\boldsymbol{\gamma}_P$ at a point P is opposite to the unit vector $\hat{\boldsymbol{u}}_P$ at P.

3 GRAVITY DISTURBANCE

It is worth noting that, by definition, the centrifugal component of the normal gravity field is equal to the centrifugal component of the Earth's gravity field if they are evaluated at the same point. Then, the differences between the gravity vector (corrected from non-gravitational effects) and the normal gravity vector, at the same point, represents a purely gravitational and consequently harmonic disturbing field. This gravitational disturbance is caused by contrasts between the actual internal density distribution of the Earth and the internal density distribution of the normal Earth. In applied geophysics, these density differences are generally called "anomalous masses" (e.g., Hammer, 1945; LaFehr, 1965), "density anomalies" (e.g., Forsberg, 1984) or "gravity sources" (e.g., Blakely, 1996). Here, we opted for using the last term.

The difference between the observed gravity and the normal gravity, at the same point, is called gravity disturbance (Hofmann-Wellenhof & Moritz 2005). Notice that the gravity disturbance is not equivalent to the magnitude of the difference between the gravity vector and the normal gravity vector at the same point. As properly pointed out by Hackney & Featherstone (2003), the gravity disturbance is a very-well established quantity in geodesy, but appears to be less well known in geophysics.

The gravity anomaly is the commonly used quantity in applied geophysics. It is defined as the difference between the gravity on the geoid and the normal gravity on the ellipsoid, both at the same geodetic latitude and longitude. Notice that, by definition, the gravity anomaly depends on longitude and latitude only and is not a function in space (Barthelmes 2013). Different gravity anomalies can be calculated, depending on the corrections applied to them (Blakely 1996; Hofmann-Wellenhof & Moritz 2005). These corrections are usually called gravity reductions. For example, the Free-air anomaly is an approximation of the gravity disturbance whereas the Bouguer anomaly is an approximation of the terrain corrected gravity disturbance. The last one is commonly used by geophysicists as the gravitational effect produced by the gravity sources. Although this approximation is valid for most practical applications, it is important to bear in mind not only the terminology changes, but also the conceptual assumptions.

There was a certain lack of comprehension regarding the geophysical meaning of gravity anomalies until the mid 90's. As properly pointed out by Chapin (1996) at that time, "although the corrections which bring about a Bouguer gravity anomaly are well established, the reasons for doing them are not well understood. One cause of this common misunderstanding is that the subject has been poorly presented in many of the basic texts". In his seminal book, Blakely (1996) brought some light on the geophysical meaning of gravity anomalies from the perspective of applied geophysics. Blakely (1996) correctly defined gravity sources as density contrasts between the actual internal density distribution of the Earth and the internal density distribution of the normal Earth. However, he did not stress that, by removing the normal gravity evaluated on the ellipsoid from the gravity measured on the Earth's surface, the remaining disturbing field will reflect not only the effect produced by the gravity sources, but also a small combination of gravitational and centrifugal effects. This additional, non-harmonic and undesired effect is simply due to the calculation of the normal gravity at a point other than that were the gravity is measured.

4 MATHEMATICAL DESCRIPTION OF THE GRAVITY DISTURBANCE IN A LOCAL COORDINATE SYSTEM

In a local- or regional-gravity study, geophysicists commonly use a topocentric Cartesian coordinate system with origin at a point P and axes x, y and z defined by the unit vectors $\hat{\boldsymbol{v}}_P$, $\hat{\boldsymbol{w}}_P$ and $-\hat{\boldsymbol{u}}_P$ (equation 1), respectively (Figures A1 and A2). In this coordinate system, the observed gravity vector $\boldsymbol{g}_i$, at a point (x_i, y_i, z_i), $i = 1, ..., N$, can be represented by

$$\boldsymbol{g}_i = \boldsymbol{\gamma}_i + \Delta\boldsymbol{g}_i \ , \tag{2}$$

where $\boldsymbol{\gamma}_i$ and $\Delta\boldsymbol{g}_i$ are, respectively, the normal gravity vector and a disturbing gravitational attraction produced by the anomalous masses at the point (x_i, y_i, z_i).

By definition, the gravity disturbance δg_i, at the point (x_i, y_i, z_i), is given by (Hofmann-Wellenhof & Moritz 2005):

$$\delta g_i = g_i - \gamma_i \ , \tag{3}$$

where $g_i = \|\boldsymbol{g}_i\|$ and $\gamma_i = \|\boldsymbol{\gamma}_i\|$ are, respectively, the observed gravity and the normal gravity at the point (x_i, y_i, z_i). Fortunately, the condition $\gamma_i \gg \|\Delta\mathbf{g}_i\|$ is met at all points located above or on the Earth's surface. By combining this condition and the definition of observed gravity vector (equation 2), we can approximate the observed gravity g_i by a first order Taylor's expansion as follows (Sansò & Sideris 2013):

$$g_i \approx \gamma_i + \hat{\boldsymbol{\gamma}}_i^\top \Delta\boldsymbol{g}_i \ , \tag{4}$$

where $\hat{\boldsymbol{\gamma}}_i = -\hat{\boldsymbol{u}}_i$ is a unit vector with the same direction as the normal gravity vector $\boldsymbol{\gamma}_i$ at the point (x_i, y_i, z_i) and $\hat{\boldsymbol{u}}_i$ is the unit vector $\hat{\boldsymbol{u}}$ (equation 1) evaluated at the point $(h_i, \varphi_i, \lambda_i)$, which is obtained by transforming (x_i, y_i, z_i) from the topocentric coordinate system (Figure A2) to the geodetic coordinate system (Figure A1).

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This approximation, which is known in geodesy (e.g., Sansò & Sideris 2013), is largely used in applied geophysics for representing total-field anomalies (e.g., Blakely 1996). Notice that, local- or regional-gravity studies, the unit vector $\hat{\boldsymbol{\gamma}}_i$ (equation 4) coincides with the z axis of the local Cartesian coordinate system defined at the beginning of this section. Consequently, by using the approximation defined in equation 4, the gravity disturbance (equation 3) can be rewritten as follows

$$\delta g_i^o \approx \hat{\boldsymbol{z}}^\top \Delta \boldsymbol{g}_i^o \, , \tag{5}$$

where $\hat{\boldsymbol{z}}^\top = [\,0\ 0\ 1]$. According to equation 5, the gravity disturbance δg_i^o (equation 3) represents the vertical component of the gravitational attraction exerted by the gravity sources at the point (x_i, y_i, z_i). As a consequence, the gravity disturbance produced by a homogeneous gravity source can be represented by the following harmonic function

$$d_i^o = k_g \, G \, \rho \, \partial_z \phi_i \, , \tag{6}$$

where G is the Newtonian constant of gravitation (in $m^3/(kg\,s^2)$), $k_g = 10^5$ is a constant factor transforming from m/s^2 to milligal (mGal), and $\partial_z \phi_i$ is a harmonic function representing the first derivative, evaluated at the observation point (x_i, y_i, z_i), $i = 1, ..., N$, of the function

$$\phi(x, y, z) = \int \int_v \int \frac{1}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} \, dv \tag{7}$$

with respect to the variable z. The integral is conducted over the coordinates x', y' and z' within the volume v of the gravity source. This equation can be easily generalized for the case of multiple gravity sources.

5 GRAVITY DISTURBANCE VERSUS GRAVITY ANOMALY

Almost all interpretation techniques assume, implicitly or directly, that the gravity data is harmonic (e.g., upward/downward continuation, data processing with equivalent layer, conversions between gravity and magnetic data, computation of vertical derivatives via Fourier and Hilbert transforms).

Consequently, they implicitly or directly assume that the gravity data approximates the gravitational disturbance.

Almost all forward modelling techniques compute the vertical component of the gravitational attraction exerted by the geological bodies at the observation points.

Hence, almost all geophysicists implicitly compute the vertical component of the gravitational disturbance.

The gravity anomaly is defined as the difference between the gravity $\|\boldsymbol{g}_P\|$, at a point P on the Geoid, and the normal gravity $\|\boldsymbol{\gamma}_Q\|$, on the reference elipsoid, where P and Q have the same geodetic latitude and longitude.

Consequently, the gravity anomaly is a function of the geodetic latitude and longitude only and cannot be calculated at arbitrary heights.

Gravity anomalies require the computation of gravity within the topographic masses, where the gravitational disturbance is not harmonic.

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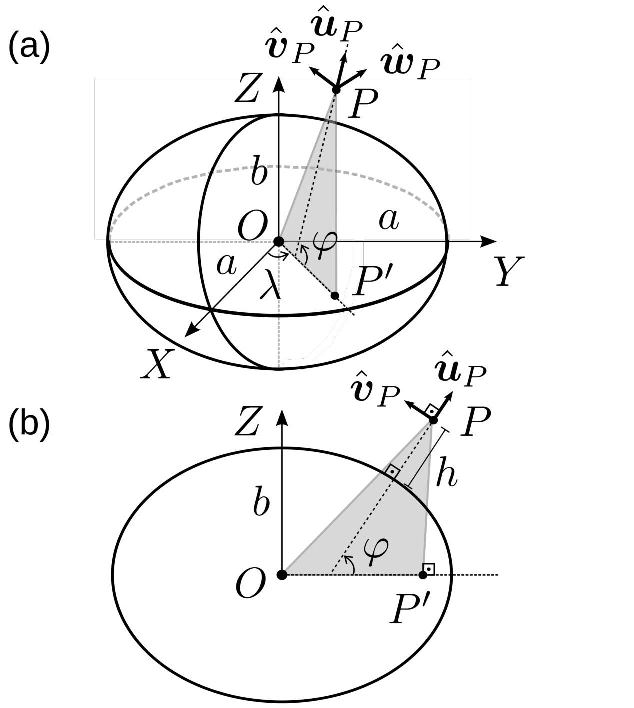

Figure A1. Schematic representation of the geodetic coordinate system defined by an oblate ellipsoid with semi-minor axis b, coincident with the mean Earth's rotation axis, and a semi-major axis a. In this coordinate system, the position of a point is determined by the geometric height h, the geodetic latitude φ and longitude λ. The Earth's center of mass is represented by O, P represents a point (h, φ, λ) and P' represents the projection of P on the plane XY (Equatorial plane). The plane containing O, P and P' is represented by the gray triangle in (a) and (b). The unit vectors $\hat{\boldsymbol{u}}_P$, $\hat{\boldsymbol{v}}_P$ and $\hat{\boldsymbol{w}}_P$ define three mutually orthogonal directions at each point P. In (b), the point Q represents the projection of P on the ellipsoid surface, at the same latitude φ and longitude λ.

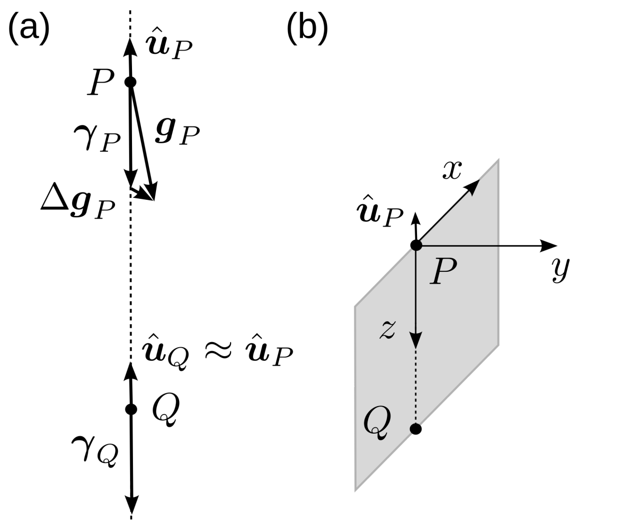

Figure A2. (a) Schematic representation of the gravity vector $\boldsymbol{g}_P$, normal gravity vector $\boldsymbol{\gamma}_P$, disturbing gravitational attraction $\Delta\boldsymbol{g}_P$ (equation 2) and the unit vector $\hat{\boldsymbol{u}}_P$ (equation 1) at the point P and also the normal gravity vector $\boldsymbol{\gamma}_Q$ at the point Q on the reference ellipsoid. At the point P the normal gravity vector $\boldsymbol{\gamma}_P$ is opposite to the unit vector $\hat{\boldsymbol{u}}_P$ and the normal gravity $\boldsymbol{\gamma}_Q$ at the point Q is opposite the another unit vector $\hat{\boldsymbol{u}}_Q$, which is close to $\hat{\boldsymbol{u}}_P$. (b) Schematic representation of a topocentric Cartesian coordinate system with origin at a point P and x, y and z axes defined by the unit vectors $\hat{\boldsymbol{v}}_P$, $\hat{\boldsymbol{w}}_P$ and $-\hat{\boldsymbol{u}}_P$ (Figure A1), respectively.