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# Theoretical aspects in gravity modeling

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**SUMMARY**
Gravity anomalies have long been used by geophysicists for the purpose of determining density distributions in subsurface.
However, gravity anomalies
In this paper, we discuss the fundamental concepts
URGENTE: (Marussi et al. 1974), (Torge & Müller 2012), section 4.2.1

**Key words:** potential fields – gravity disturbance – gravity anomaly – gravity modeling.

## 1 INTRODUCTION

The resultant of gravitational force and centrifugal force acting on a body at rest on the Earth's surface is called *gravity vector* and its intensity is called *gravity* (Heiskanen & Moritz 1967; Hofmann-Wellenhof & Moritz 2005). In the case of gravimetry on moving platforms (e.g., airplanes, helicopters, marine vessels), there are additional non-gravitational accelerations due to the vehicle motion, such as Coriolis acceleration and high-frequency vibrations (Glennie et al. 2000; Nabighian et al. 2005; Baumann et al. 2012). Geophysicists use gravity for estimating the Earth's internal density distribution whereas geodesists use gravity to estimate the geoid (Li & Götze 2001). Hence, geophysicists are usually interested in the gravitational component of the observed gravity, which is produced by the Earth's internal density distribution.

The first step of the procedure for isolating this gravitational component consists in removing the non-gravitational effects due to the vehicle motion and also the time variations such as Earth tides, instrumental drift and barometric pressure changes, for example. If these effects are properly removed, the resultant gravity data can be considered as the sum of a centrifugal component due to the Earth's rotation and a gravitational component produced by the whole Earth's internal density distribution. The isolation of this particular gravitational component and its subsequent use for estimating density distributions related to geological structures in subsurface are the main goals in applied geophysics (Blakely 1996). The computation of the gravitational effect produced by the geological structures is commonly known in geophysics as *gravity modeling*. This term is different from *gravity field modeling*, which is commonly used in geodesy to denote the characterization of the gravity field in a local, regional or global scale.

Based on well-established concepts of the literature, we present a discussion aiming at bringing some light to the following question: in geophysical applications, should we use the gravity disturbance or gravity anomaly? It seems that this theoretical issue has been debated within the scientific community from a more geodetic than geophysical point of view (LaFehr 1991; Chapin 1996; Li & Götze 2001; Fairhead et al. 2003; Hackney & Featherstone 2003; Hinze et al. 2005). Our reasoning suggests that the gravity disturbance is more appropriated than the gravity anomaly for geophysical purposes.

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## 2 NORMAL EARTH AND NORMAL GRAVITY

Traditionally, the Earth's gravity field is approximated by the gravity field produced by a reference ellipsoid (or level ellipsoid), which is rigid and geocentric. This reference ellipsoid has the minor axis $b$ coincident with the mean rotating axis of the Earth $Z$, the same total mass (including the atmosphere) and also the same angular velocity of the Earth (Heiskanen & Moritz 1967; Vaníček & Krakiwsky 1987; Hofmann-Wellenhof & Moritz 2005; Torge & Müller 2012). Another characteristic of this model is that its limiting surface coincides with a particular equipotential of its own gravity field. Here, we follow (Torge & Müller 2012) and call this model as *normal Earth*.

Similarly to the gravity vector and gravity, the resultant of the virtual gravitational and centrifugal forces exerted by the normal Earth on a body at rest at a point $P$ is called *normal gravity vector* and its intensity is called *normal gravity*. In geodesy, any model used to represent the normal gravity field can be arbitrarily defined for the only purpose of keeping the difference from the actual gravity field as small as possible (Vaníček & Krakiwsky 1987).

It is worth noting that, although the normal Earth has the same total mass (including the atmosphere) of the Earth, its internal density distribution is unknown. The search for physically meaningful mass distributions that generate a required normal gravity field has geophysical rather than geodetic motives (Marussi et al. 1974). The only condition imposed on its internal density distribution is that it produces a gravity field having a particular equipotential which coincides with its limiting surface. For convenience, we denote any density distribution satisfying this condition as a *normal density distribution*.

## 3 TERRESTRIAL REFERENCE SYSTEMS USED IN GRAVITY MODELING

For geophysical purposes, there are three important Terrestrial Reference Systems used in gravity modeling. They rotate with the Earth and are used for describing positions and movements of objects on and close to the Earths surface (Torge & Müller 2012).

The first one is a geocentric system of Cartesian coordinates having the $Z$-axis coincident with the mean Earth's rotational axis, the $X$-axis pointing to the Greenwich meridian and the $Y$-axis directed so as to obtain a right-handed system (Figure A1). This reference system can be found in the literature with different names: Mean Terrestrial System (e.g., Soler 1976), Earth-fixed geocentric Cartesian system (e.g., Torge & Müller 2012) or Earth-centered Earth-fixed system (e.g., Bouman et al. 2013), for example. Here, we opted for simply using the term Geocentric Cartesian System (GCS).

Other important reference system is a geocentric system of geodetic coordinates, which is defined by the reference ellipsoid used in the Normal Earth model (Heiskanen & Moritz 1967; Soler 1976; Torge & Müller 2012; Bouman et al. 2013). In this coordinate system, the position of a point is defined by the *geometric height* $h$, *geodetic latitude* $\varphi$ and *longitude* $\lambda$ (Figure A1). For convenience, we call this system Geocentric Geodetic System (GGS). At a given point $(h, \varphi, \lambda)$, there are three mutually-orthogonal unit vectors (Figure A1) given by (Soler 1976):

$$\hat{\boldsymbol{u}} = \begin{bmatrix} \cos\varphi \, \cos\lambda \\ \cos\varphi \, \sin\lambda \\ \sin\varphi \end{bmatrix}$$
$$\hat{\boldsymbol{v}} = \begin{bmatrix} -\sin\varphi \, \cos\lambda \\ -\sin\varphi \, \sin\lambda \\ \cos\varphi \end{bmatrix} . \tag{1}$$
$$\hat{\boldsymbol{w}} = \begin{bmatrix} -\sin\lambda \\ \cos\lambda \\ 0 \end{bmatrix}$$

The required equations to convert coordinates $(h, \varphi, \lambda)$ referred to the GGS into coordinates $(X, Y, Z)$ referred to the GCS (Figure A1) and vice versa can be easily found in the literature (e.g., Heiskanen & Moritz 1967; Torge & Müller 2012; Bouman et al. 2013).

In a local or regional study, geophysicists commonly use a topocentric Cartesian coordinate system (TCS) with origin at a point $P$ on or close to the Earth's surface and axes $x$, $y$ and $z$ (Figure A2). In the TCS, the axes $x$ and $y$ are parallel to the unit vectors $\hat{\boldsymbol{v}}_P$ $\hat{\boldsymbol{w}}_P$, respectively, whereas the $z$-axis is opposite to the unit vector $\hat{\boldsymbol{u}}_P$ (equation 1) and points downward. Consider, for example, a TCS (Figure A2) with origin at a point $P$ with coordinates $(X_P, Y_P, Z_P)$ referred to the GCS (Figure A1). In this case, the relationship between the coordinates $(X, Y, Z)$ of a point in the GCS and the coordinates $(x, y, z)$ of the same point in the TCS is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \boldsymbol{R}^\top \begin{bmatrix} X - X_P \\ Y - Y_P \\ Z - Z_P \end{bmatrix} , \tag{2}$$

where $\boldsymbol{R}$ is an orthogonal matrix whose columns are defined by the unit vectors $\hat{\boldsymbol{u}}$, $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{w}}$ (equation 1) as follows:

$$\boldsymbol{R} = \begin{bmatrix} \hat{\boldsymbol{v}} & \hat{\boldsymbol{w}} & -\hat{\boldsymbol{u}} \end{bmatrix} . \tag{3}$$

## 4 ANOMALOUS QUANTITIES IN GRAVITY MODELING

Let $\boldsymbol{g}_P$ and $\boldsymbol{\gamma}_P$ be, respectively, the gravity vector (corrected from non-gravitational effects due to vehicle motion and time variations such as Earth tides, instrumental drift and barometric pressure changes, for example) and the normal gravity vector at a point $P$. In this case, the gravity vector $\boldsymbol{g}_P$ represents the gradient of a scalar potential called *gravity potential*, which is the sum of a scalar gravitational potential and a scalar centrifugal potential. Similarly, the normal gravity vector $\boldsymbol{\gamma}_P$ represents the gradient of a scalar potential called *normal potential*, which is also the sum of a scalar gravitational potential and a scalar centrifugal potential. By definition, the centrifugal part of the normal potential is equal to that of the gravity potential. Then, the both gravity vector and normal gravity vectors are represented by the sum of two vectors: the gradient of a gravitational potential and the gradient of a centrifugal potential. Notice that, as the centrifugal part of the gravity potential and normal potential are the same, the centrifugal part of the gravity vector and normal gravity vector are also equal to each other.

The difference between $\boldsymbol{g}_P$ and $\boldsymbol{\gamma}_P$, at the same point $P$, defines a quantity called *gravity disturbance vector*, which is given by:

$$\delta\boldsymbol{g}_P = \boldsymbol{g}_P - \boldsymbol{\gamma}_P . \tag{4}$$

As the centrifugal part of the gravity vector and the normal gravity vector is equal to the centrifugal component of the gravity vector, the gravity disturbance vector $\delta\boldsymbol{g}_P$ represents a purely gravitational (and consequently harmonic) quantity, which is caused by contrasts between the actual internal density distribution of the Earth and the (unknown) internal density distribution of the normal Earth. In applied geophysics, these density differences are generally called *anomalous masses* (e.g., Hammer, 1945; LaFehr, 1965), *density anomalies* (e.g., Forsberg, 1984) or *gravity sources* (e.g., Blakely, 1996). Here, we opted for using the last term. Figure A3 illustrates the gravity vector $\boldsymbol{g}_P$, normal gravity vector $\boldsymbol{\gamma}_P$ and the gravity disturbance vector $\delta\boldsymbol{g}_P$ at a point $P$ located on the Earth's surface.

The difference between gravity $g_P$ (magnitude of the gravity vector $\boldsymbol{g}_P$) and normal gravity $\gamma_P$ (magnitude of the normal gravity vector $\boldsymbol{\gamma}_P$), at the same point $P$, is called *gravity disturbance* (Heiskanen & Moritz 1967; Hofmann-Wellenhof & Moritz 2005) and can be represented as follows:

$$\delta g_P = g_P - \gamma_P . \tag{5}$$

Notice that the gravity disturbance $\delta g_P$ is not equivalent to the magnitude of the gravity disturbance vector $\delta\boldsymbol{g}_P$ (Barthelmes 2013; Sansò & Sideris 2013). As properly pointed out by Hackney & Featherstone (2003), the gravity disturbance is a very-well established quantity in geodesy, but appears to be less well known in geophysics.

The gravity anomaly is the commonly used quantity in applied geophysics. It is defined as the difference between the gravity at a point $Q'$ on the geoid (a particular equipotential surface of the gravity potential) and the normal gravity at a point $Q$ on the ellipsoid surface, both at the same geodetic latitude and longitude (Figure A3). Notice that, by definition, the gravity anomaly depends on longitude and latitude only and is not a function of height (Barthelmes 2013). As a consequence, it is not possible to compute, for example, the upward continuation of a gravity anomaly. However, many authors in the literature have computed the upward continuation of gravity anomalies. All these authors have implicitly considered that the gravity anomaly is an approximation of the gravity disturbance, which in turn can be represented by a harmonic function that can be continued upward.

Different gravity anomalies can be calculated (e.g., Free-air and Bouguer anomalies), depending on the corrections applied to them (Blakely 1996; Hofmann-Wellenhof & Moritz 2005). These corrections are usually called gravity reductions. It can be shown that the Free-air anomaly is an approximation of the gravity disturbance whereas the Bouguer anomaly is an approximation of the terrain corrected gravity disturbance. The Bouguer anomaly is commonly used by geophysicists as the gravitational effect produced by the gravity sources. Although this approximation is valid for most practical applications, it is important to bear in mind not only the terminology changes, but also the conceptual assumptions.

There was a certain lack of comprehension regarding the geophysical meaning of gravity anomalies until the mid 90's. As properly pointed out by Chapin (1996) at that time, "although the corrections which bring about a Bouguer gravity anomaly are well established, the reasons for doing them are not well understood. One cause of this common misunderstanding is that the subject has been poorly presented in many of the basic texts". In his seminal book, Blakely (1996) brought some light on the geophysical meaning of gravity anomalies from the perspective of applied geophysics. Blakely (1996) correctly defined gravity sources as density contrasts between the actual internal density distribution of the Earth and the internal density distribution of the normal Earth. However, he did not stress that, by removing the normal gravity evaluated on the ellipsoid from the gravity measured on the Earth's surface, the remaining disturbing field will reflect not only the effect produced by the gravity sources, but also a small combination of gravitational and centrifugal effects. This additional, non-harmonic and undesired effect is simply due to the calculation of the normal gravity at a point other than that were the gravity is measured.

## 5 MATHEMATICAL DESCRIPTION OF THE GRAVITY DISTURBANCE IN A LOCAL COORDINATE SYSTEM

In this coordinate system, the observed gravity vector $\boldsymbol{g}_i$, at a point $(x_i, y_i, z_i)$, $i = 1, ..., N$, can be represented by

$$\boldsymbol{g}_i = \boldsymbol{\gamma}_i + \Delta\boldsymbol{g}_i \; , \tag{6}$$

where $\boldsymbol{\gamma}_i$ and $\Delta\boldsymbol{g}_i$ are, respectively, the normal gravity vector and a disturbing gravitational attraction produced by the anomalous masses at the point $(x_i, y_i, z_i)$.

For each point $(x_i, y_i, z_i)$ in the topocentric Cartesian coordinate system (Figure A2b), there is a corresponding point $(h_i, \varphi_i, \lambda_i)$ in the geodetic coordinate system (Figure A1).

By definition, the gravity disturbance $\delta g_i$, at the point $(x_i, y_i, z_i)$, is given by (Heiskanen & Moritz 1967; Hofmann-Wellenhof & Moritz 2005):

$$\delta g_i = g_i - \gamma_i \; , \tag{7}$$

where $g_i = \|\boldsymbol{g}_i\|$ and $\gamma_i = \|\boldsymbol{\gamma}_i\|$ are, respectively, the observed gravity and the normal gravity at the point $(x_i, y_i, z_i)$. Fortunately, the condition $\gamma_i \gg \|\Delta\mathbf{g}_i\|$ is met at all points located above or on the Earth's surface. By combining this condition and the definition of observed gravity vector (equation 6), we can approximate the observed gravity $g_i$ by a first order Taylor's expansion as follows (Sansò & Sideris 2013):

$$g_i \approx \gamma_i + \hat{\boldsymbol{\gamma}}_i^{\top} \Delta\boldsymbol{g}_i \; , \tag{8}$$

where $^{\top}$ denotes transposition, $\hat{\boldsymbol{\gamma}}_i = -\hat{\boldsymbol{u}}_i$ is a unit vector with the same direction as the normal gravity vector $\boldsymbol{\gamma}_i$ at the point $(x_i, y_i, z_i)$, in the topocentric Cartesian coordinate system (Figure A2b), and $\hat{\boldsymbol{u}}_i$ is the unit vector $\hat{\boldsymbol{u}}$ (equation 1) evaluated at the corresponding point $(h_i, \varphi_i, \lambda_i)$ in the geodetic coordinate system (Figure A1).

This approximation, which is known in geodesy (e.g., Sansò & Sideris 2013), is largely used in applied geophysics for representing total-field anomalies (e.g., Blakely 1996). Notice that, in local- or regional-gravity studies, the unit vector $\hat{\boldsymbol{\gamma}}_i$ (equation 8) may be considered constant throughout the study area and parallel to the $z$ axis of the topocentric Cartesian coordinate system (Figure A2b). Consequently, by using the approximation defined in equation 8, the gravity disturbance (equation 7) can be rewritten as follows

$$\delta g_i \approx \hat{\boldsymbol{\gamma}}_P^{\top} \Delta\boldsymbol{g}_i \; , \tag{9}$$

where $\hat{\boldsymbol{\gamma}}_P$ represents the unit vector with the same direction as the normal gravity vector $\boldsymbol{\gamma}_P$ at the origin $P$ of the topocentric Cartesian coordinate system (Figure A2). This equation shows that the gravity disturbance $\delta g_i$ (equation 7) is different from the magnitude of the disturbing gravitational attraction $\Delta\boldsymbol{g}_i$ produced by the gravity sources. Rather, it represents the component of the $\Delta\boldsymbol{g}_i$ on the direction of the normal gravity vector (Sansò & Sideris 2013). In the topocentric Cartesian coordinate system, the gravity disturbance $\delta g_i$ (equation 9) can be defined as the vertical component of the gravitational attraction exerted by the gravity sources at the point $(x_i, y_i, z_i)$. As a consequence, the gravity disturbance produced by a homogeneous gravity source with density contrast $\Delta\rho$ (in $kg/m^3$) can be represented by the following harmonic function:

$$d_i = k_g \, G \, \Delta\rho \, \partial_z \phi_i \; , \tag{10}$$

where $G$ is the Newtonian constant of gravitation (in $m^3/(kg\, s^2)$), $k_g = 10^5$ is a constant factor transforming from $m/s^2$ to milligal (mGal), and $\partial_z \phi_i$ is a harmonic function representing the first derivative, evaluated at the observation point $(x_i, y_i, z_i)$, $i = 1, ..., N$, of the function

$$\phi(x, y, z) = \iiint\limits_v \frac{1}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} \, dv \tag{11}$$

with respect to the variable $z$. The integral is conducted over the coordinates $x'$, $y'$ and $z'$ within the volume $v$ of the gravity source. This equation can be easily generalized for the case of multiple gravity sources.

Practically all the literature about gravity modeling use the quantity $d_i$ (equation 10) to represent the gravity anomaly produced by gravity sources (e.g., Blakely 1996). Consequently, almost all the geophysicists use the gravity anomaly as an approximation of the gravity disturbance produced by the gravity sources. Notice that $d_i$ (equation 10) does not depend on the geoidal surface. Rather, it depends on the relative position of the observation points $(x_i, y_i, z_i)$ with respect to the gravity sources in a topocentric Cartesian coordinate system (Figure A2b).

## 6 CONCLUSIONS

We debate the conceptual differences between the gravity disturbance and the gravity anomaly. Our reasoning suggest that the gravity disturbance is the more appropriated quantity for representing the gravity effect produced by the gravity sources. In summary, we point out that:

(i) Almost all interpretation techniques assume, implicitly or directly, that the gravity data is harmonic (e.g., upward/downward continuation, data processing with equivalent layer, conversions between gravity and magnetic data, computation of vertical derivatives via Fourier and Hilbert transforms). As a consequence, they implicitly or directly assume that the gravity data approximates the gravitational disturbance.

(ii) Almost all forward modeling techniques compute the vertical component of the gravitational attraction exerted by the geological bodies at the observation points. Notice that the gravity anomaly requires the computation of gravity on the Geoid, which is generally within the topographic masses. Hence, almost all geophysicists implicitly compute the gravity disturbance.

(iii) The gravity anomaly is defined as the difference between the gravity on the Geoid and the normal gravity on the reference ellipsoid, both at the same geodetic latitude and longitude. Consequently, the gravity anomaly is a function of the geodetic latitude and longitude only and cannot be calculated at arbitrary heights. On the other hand, the gravity disturbance can be computed at arbitrary points outside the sources.

The principal theoretical implication of this study is that is that, although the gravity anomaly may be used as a good approximation of the gravity effect produced by the sources for most practical applications, the more appropriated quantity for gravity modeling is the gravity disturbance. We stress that, more important than the the terminology changes, the geophysicist must bear in mind the conceptual assumptions used in gravity modeling.

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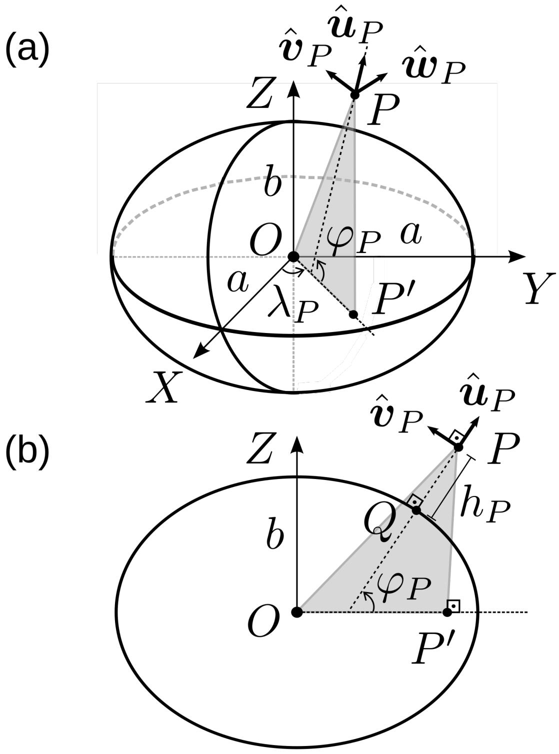


**Figure A1.** Schematic representation of the Geocentric Cartesian System (GCS) and the Geocentric Geodetic System (GGS). The GCS has the $Z$-axis coincident with the mean Earth's rotational axis, the $X$-axis pointing to the Greenwich meridian and the $Y$-axis directed so as to obtain a right-handed system. The GGS is defined by an oblate ellipsoid with semi-minor axis $b$, coincident with the $Z$-axis of GCS, and a semi-major axis $a$. In this coordinate system, the position of a point is determined by the geometric height $h$, geodetic latitude $\varphi$ and longitude $\lambda$. The Earth's center of mass is represented by $O$, $P$ represents a point $(h_P, \varphi_P, \lambda_P)$ and $P'$ its projection onto the plane $XY$ (Equatorial plane). The plane containing $O$, $P$ and $P'$ is represented in gray in (a) and (b). The unit vectors $\hat{\boldsymbol{u}}_P$, $\hat{\boldsymbol{v}}_P$ and $\hat{\boldsymbol{w}}_P$ define three mutually orthogonal directions at $P$ (equation 1). In (b), $Q$ represents a point $(h_Q, \varphi_Q, \lambda_Q)$, which is the projection of $P$ onto the reference ellipsoid, at the same latitude and longitude ($\varphi_Q = \varphi_P$ and $\lambda_Q = \lambda_P$).

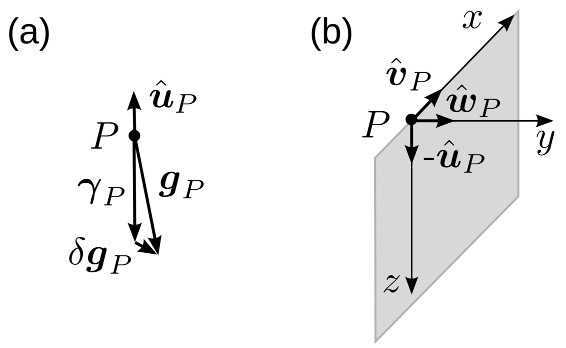


**Figure A2.** (a) Schematic representation of the gravity vector $\boldsymbol{g}_P$, normal gravity vector $\boldsymbol{\gamma}_P$, gravity disturbance vector $\delta\boldsymbol{g}_P$ (equation 6) and unit vector $\hat{\boldsymbol{u}}_P$ (equation 1) at a point $P$. (b) Schematic representation of a topocentric Cartesian coordinate system with origin at a point $P$. The axes $x$ and $y$ are parallel to the unit vectors $\hat{\boldsymbol{v}}_P$ and $\hat{\boldsymbol{w}}_P$ (equation 1 and Figure A1), respectively. On the other hand, the $z$ axis is opposite to the unit vector $\hat{\boldsymbol{u}}_P$ (equation 1 and Figure A1) and points downward. The gray plane shown in (b) is the same gray plane shown in Figure A1.

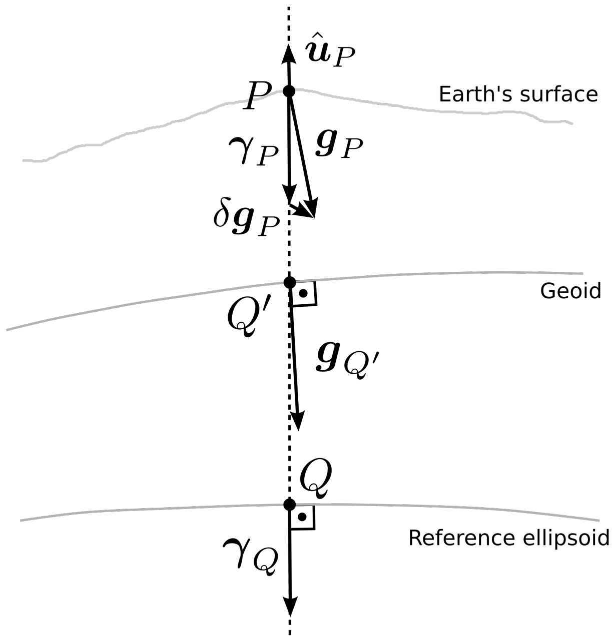


**Figure A3.** Schematic representation of the gravity vector $\boldsymbol{g}_P$, normal gravity vector $\boldsymbol{\gamma}_P$, gravity disturbance vector $\delta\boldsymbol{g}_P$ (equation 6) and unit vector $\hat{\boldsymbol{u}}_P$ (equation 1) at a point $P$ on the surface of the Earth; gravity vector $\boldsymbol{g}_{Q'}$ at a point $Q'$ on the Geoid; normal gravity vector $\boldsymbol{\gamma}_Q$ at a point $Q$ on the reference ellipsoid. The dashed line is normal to the surface of the reference ellipsoid at $Q$.