$$h[n] = \left(\frac{1}{3}\right)^n u[n]$$

$$x[n] = \cos\left(\frac{\pi n}{50}\right) + 0{,}2\cos\left(\frac{\pi}{10}n\right)$$

transformando cos en exponenciais complexas

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$x[n] = \frac{1}{2}\cdot\left(e^{j\frac{\pi}{50}n} + e^{-j\frac{\pi}{50}n}\right) + \frac{1}{10}\cdot\left(e^{j\frac{\pi}{10}n} + e^{-j\frac{\pi}{10}n}\right)$$

$$x[n] = \frac{1}{2}e^{j\frac{\pi}{50}n} + \frac{1}{2}e^{-j\frac{\pi}{50}n} + \frac{1}{10}e^{j\frac{\pi}{10}n} + \frac{1}{10}e^{-j\frac{\pi}{10}n}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]\,x[n-k]$$

$$x[n-k] = \frac{1}{2}e^{j\frac{\pi}{50}(n-k)} + \frac{1}{2}e^{j\left(-\frac{\pi}{50}\right)(n-k)} + \frac{1}{10}e^{j\frac{\pi}{10}(n-k)} + \frac{1}{10}e^{-j\left(-\frac{\pi}{10}\right)(n-k)}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]\cdot\frac{1}{2}e^{j\frac{\pi}{50}(n-k)} + h[k]\cdot\frac{1}{2}e^{j\left(\frac{-\pi}{50}\right)(n-k)} + h[k]\cdot\frac{1}{10}e^{j\frac{\pi}{10}(n-k)} + h[k]\cdot\frac{1}{10}e^{j\left(\frac{-\pi}{10}\right)(n-k)}$$

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]\cdot\frac{1}{2}e^{j\frac{\pi}{50}(n-k)} + \sum_{k=-\infty}^{+\infty} h[k]\cdot\frac{1}{2}e^{j\left(-\frac{\pi}{50}\right)(n-k)} + \sum_{k=-\infty}^{+\infty} h[k]\cdot\frac{1}{10}e^{j\frac{\pi}{10}(n-k)} + \sum_{k=-\infty}^{+\infty} h[k]\cdot\frac{1}{10}e^{j\left(\frac{-\pi}{10}\right)(n-k)}$$

Sabendo que: $\sum_{k=-\infty}^{+\infty} h[k] \cdot A e^{jw_0(n-k)} = H(e^{jw_0}) \cdot A e^{jw_0 n}$

então

buscando $H(e^{jw_0})$ com $h[n] = \left(\frac{1}{3}\right)^n u[n]$

$$H(e^{jw_0}) = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{3}\right)^n u[n] e^{-jwn} = \sum_{n=0}^{+\infty} \left(\frac{1}{3}\right)^n e^{-jwn}$$

$$H(e^{jw}) = \sum_{n=0}^{+\infty} \left(\frac{1}{3} \cdot e^{-jw}\right)^n$$

$$H(e^{jw}) = \frac{1}{1 - \frac{1}{3} e^{-jw}}$$

Aplicando em $Y[n]$

$$Y[n] = \frac{\frac{1}{2} \cdot e^{j\frac{\pi}{50}n}}{1 - \frac{1}{3} e^{-j\frac{\pi}{50}}} + \frac{\frac{1}{2} e^{-j\frac{\pi}{50}n}}{1 - \frac{1}{3} e^{j\frac{\pi}{50}}} + \frac{\frac{1}{10} e^{j\frac{\pi}{10}n}}{1 - \frac{1}{3} e^{-j\frac{\pi}{10}}} + \frac{\frac{1}{10} e^{-j\frac{\pi}{10}n}}{1 - \frac{1}{3} e^{j\frac{\pi}{10}}}$$

transformando as exponenciais complexas em cossenos

$$Y[n] = \frac{\cos\left(\frac{\pi}{50}n\right) - \frac{1}{3}\cos\left(\frac{\pi n}{25}\right)}{1 - \frac{2}{3}\cos\left(\frac{\pi}{50}\right) + \frac{1}{9}} + \frac{\frac{1}{5}\cos\left(\frac{\pi n}{10}\right) - \frac{1}{15}\cos\left(\frac{\pi n}{5}\right)}{1 - \frac{2}{3}\cos\left(\frac{\pi}{10}\right) + 1/9}$$