

$$h[n] = \left(\frac{1}{3}\right)^n u[n] \quad \text{com } u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$X[n] = \cos\left(\frac{\pi}{50}n\right) + 0,2 \cos\left(\frac{\pi}{10}n\right)$$

Sabendo que $\sum_{k=-\infty}^{+\infty} h[k] \cdot A e^{j\omega_0(m-k)} = H(e^{j\omega_0}) \cdot A e^{j\omega_0 m}$

então busco a $H(e^{j\omega_0})$ de $\left(\frac{1}{3}\right)^n u[n]$ termo:

$$H(e^{j\omega_0}) = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{3}\right)^n u[n] e^{-j\omega_0 n} = \sum_{n=0}^{+\infty} \left(\frac{1}{3}\right)^n e^{-j\omega_0 n}$$

$$H(e^{j\omega_0}) = \sum_{n=0}^{+\infty} \left(\frac{1}{3} \cdot e^{-j\omega_0}\right)^n$$

$$H(e^{j\omega_0}) = \frac{1}{1 - \frac{1}{3} e^{-j\omega_0}}$$

Queremos $Y[n] = h[n] * X[n]$ então pela soma de somatórios

$$Y[n] = H(e^{j\frac{\pi}{50}}) \cdot \cos\left(\frac{\pi}{50}n\right) + H(e^{j\frac{\pi}{10}}) \cdot 0,2 \cos\left(\frac{\pi}{10}n\right)$$

$$Y[n] = \frac{\cos\left(\frac{\pi}{50}n\right)}{1 - \frac{1}{3} e^{-j\frac{\pi}{50}}} + \frac{1}{5} \cdot \frac{\cos\left(\frac{\pi}{10}n\right)}{1 - \frac{1}{3} e^{-j\frac{\pi}{10}}}$$