Report on « Proof of uniform convergence for a cell-centered AP discretization of the hyperbolic heat equation on general meshes »

This article is devoted to the analysis of semi-discrete schemes for a linear hyperbolic system and its asymptotic behavior towards the solution to the linear heat equation. The main motivation is to propose a complete analysis of Asymptotic Preserving schemes of general meshes.

The motivation in itself is interesting but the problem under study is relatively simple and smooth solutions can be considered and it is what the authors are actually doing.

First the authors propose an analysis of a semi-discrete scheme in 1d for non uniform meshes but under a regularity assumption of the mesh. The analysis is classical for AP scheme and consists in getting error estimates using two different paths and to take the minimum. It allows to get uniform accuracy. This first part is only technical but not difficult. Furthermore, the authors consider semi-discrete scheme while the main difficulty in practice is to construct schemes which are not restricted to a CFL condition and to get uniform accuracy. The fact that the authors neglect the time discretization is the major weakness of the present paper.

On the other hand, the second part is devoted to the 2D case on unstructured meshes, but under a regularity condition. Unfortunately this section suffers from the same problem as before. The authors consider

- a linear hyperbolic problem and the relaxation to the linear diffusion equation
- the scheme is semi-discrete and the time discretization is neglected
- the initial data is well prepared and the issue of the initial layer is not adressed

In conclusion, this work is certainly interesting, but the issue concerning the time discretization must be addressed correctly.

Some comments should be given on the extension of the scheme (not necessarily the analysis) to nonlinear problem where the limit corresponds to a nonlinear heat equation.

Additional references on similar topic should be quoted and discussed

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