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1  """
2  Statistical Risk Model
3  Author: Evan Fotopoulos
4  """
5
6
7  import numpy as np
8  import pandas as pd
9  from sklearn.decomposition import PCA
10
11  class Risk_Model():
12      """
13      Description:
14          class to calculate statistical risk model from historical returns.
15
16      Arguments:
17          returns {pd.DataFrame} -- returns, stocks as columns, dates as index
18          num_factors {int} -- Number of PCs to use
19
20      Keyword Arguments:
21          annualisation_factor {int} -- Number of rows per year in returns (default:
22          {252})
23
24      Returns:
25          component_returns {pd.DataFrame} --
26          component_cov_matrix {} --
27          stock_betas {} --
28
29      Functions:
30          idiosyncratic_variance_matrix
31          idiosyncratic_variance_vector
32          predict_portfolio_vol
33      """
34
35  def __init__(self, returns, num_factors, annualisation_factor = 252):
36      self.returns = returns
37      self.num_factors = num_factors
38      self.annualisation_factor = annualisation_factor
39      self.stocks = returns.columns
40      self.dates = returns.index
41
42      self.pca = PCA(n_components = num_factors).fit(returns)
43
44      #Apply dimensionality reduction to returns
45      self.component_returns = pd.DataFrame(self.pca.transform(returns),
46      self.dates,
47      np.arange(1,num_factors+1))
48
49      self.component_cov_matrix =
50      pd.DataFrame(np.diag(self.component_returns.var(axis = 0,
51      ddof = 1)
52      ),
53      np.arange(1,num_factors+1),
54      np.arange(1,num_factors+1))
55
56      self.stock_betas = pd.DataFrame(self.pca.components_.T,
57      self.stocks,
58      np.arange(1,num_factors+1))
59
60  def idiosyncratic_variance_matrix(self, stocks):
61      try:
62          stock_betas = self.stock_betas.loc[stocks]
63          common = pd.DataFrame(np.dot(self.component_returns, stock_betas.T),
64          self.dates,
65          stocks)
66          residuals = self.returns - common
67          return pd.DataFrame(np.diag(np.var(residuals))*self.annualisation_factor,
68          stocks,
69          stocks)
70
71  except KeyError:
72      print("stock list contains stock not in returns data used in PCA")

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70         return np.nan
71
72
73     def idiosyncratic_variance_vector(self, stocks):
74         return pd.DataFrame(np.diagonal(self.idiosyncratic_variance_matrix(stocks)),
75                                stocks)
76
77     def predict_portfolio_vol(self, weights):
78         try:
79             npstock_betas = self.stock_betas.loc[(weights.index)].values
80             npcommon =
81             npstock_betas.dot(self.component_cov_matrix.values.dot(npstock_betas.T))
82             npidio = self.idiosyncratic_variance_matrix(weights.index).values
83             npweights = weights.values
84             return np.sqrt(npweights.T.dot(npcommon + npidio).dot(npweights))[0][0]
85         except KeyError:
86             print("portfolio contains stock not in returns data used in PCA")
87             return np.nan
88
89     """
90     Get the predicted portfolio risk
91
92     Formula for predicted portfolio risk is  $\sqrt{X.T(BFB.T + S)X}$  where:
93     X is the portfolio weights
94     B is the factor betas
95     F is the factor covariance matrix
96     S is the idiosyncratic variance matrix
97
98     Parameters
99     -----
100     factor_betas : DataFrame
101         Factor betas
102     factor_cov_matrix : 2 dimensional Narray
103         Factor covariance matrix
104     idiosyncratic_var_matrix : DataFrame
105         Idiosyncratic variance matrix
106     weights : DataFrame
107         Portfolio weights
108
109     Returns
110     -----
111     predicted_portfolio_risk : float
112         Predicted portfolio risk
113     """

```