Addendum to Reply from Tuesday, Aug. 11, 2020

Impact of efficiency ratio on the neutron / proton yields ratio

We want to evaluate the impact of the ratio of hadron efficiencies, $R_{\eta_{n/p}} = \eta_n/\eta_p$ on the ratio of neutrons to proton yields $R_{n/p} = N_{en}/N_{ep}$.

To obtain the "true" ratio of neutrons to proton yields $R_{n/p}$ from the recorded "raw" ratio of neutrons to proton yields $R_{n/p,raw}$, this ratio needs to be corrected by $R_{n/p}$.

$$R_{n/p} = R_{n/p,raw}/R_{\eta_{n/p}} \tag{1}$$

Hence, the uncertainty, $\Delta R_{n/p}$ of $R_{n/p}$ could be expressed as:

$$\Delta R_{n/p}/R_{n/p} = \Delta R_{\eta_{n/p}}/R_{\eta_{n/p}}. \tag{2}$$

To evaluate the uncertainty of $R_{\eta_{n/p}} = \eta_p/\eta_n$, we need to account for the strong correlation between η_p and η_n, $\rho_{\eta_{n/p}}$. We can define the covariance between the uncertainties $\Delta\eta_p$ and $\Delta\eta_n$, $\sigma_{\eta_{n/p}} = \rho_{\eta_{n/p}}\Delta\eta_n\Delta\eta_p$ [1].

According to[2], we can write the uncertainty on the ratio efficiencies:

$$\frac{\Delta R_{\eta_{n/p}}}{R_{\eta_{n/p}}} = \sqrt{\left(\frac{\Delta\eta_n}{\eta_n}\right)^2 + \left(\frac{\Delta\eta_p}{\eta_p}\right)^2 - 2\frac{\sigma_{\eta_{n/p}}}{\eta_n\eta_p}} \tag{3}$$

$$= \sqrt{\left(\frac{\Delta\eta_n}{\eta_n}\right)^2 + \left(\frac{\Delta\eta_p}{\eta_p}\right)^2 - 2\frac{\rho_{\eta_{n/p}}\Delta\eta_n\Delta\eta_p}{\eta_n\eta_p}} \tag{4}$$

$\Delta\eta_p$ and $\Delta\eta_n$ are the uncertainties from the calibration measurements described on the Monday write up. The correlation between the variations of the proton and neutron efficiencies $\rho_{\eta_{n/p}}$, depends on the cause of the variations. In the case of the efficiency calibrations runs, discussed in the Monday write-up, the uncertainties of the efficiencies are due to the statistic of the collected events. In such a case, $\rho_{\eta_{n/p}} = 0$. However, in the case of the detector instability, $\rho_{\eta_{n/p}} \sim 1$ (see again Fig. 1).

With the statistics projected for the calibration runs: $\eta_p \pm \Delta\eta_p = 0.915 \pm 0.003$ and $\eta_n \pm \Delta\eta_n = 0.924 \pm 0.014$. Applying these values to Eq. 4, the relative uncertainty on the absolute value of the hadron efficiency ratio becomes:

$$\Delta R_{\eta_{n/p}}/R_{\eta_{n/p}} = 1.6\%. \tag{5}$$

[1] https://en.wikipedia.org/wiki/Covariance_and_correlation

[2] https://www.sagepub.com/upm-data/6427_Chapter_4__Lee_(Analyzing)_I_PDF_6.pdf

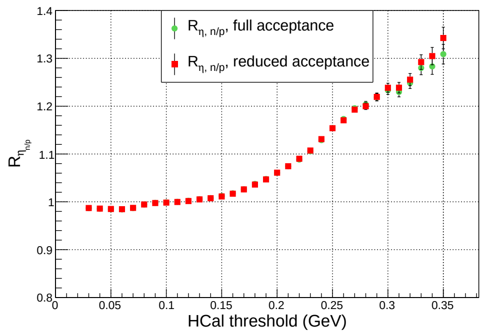

Figure 1: Neutron/proton efficiency ratio $R_{\eta_{n/p}}$ as a function of the calorimeter threshold, for our quasi-elastic sample. The error bars represent the uncertainty from the calibration measurements discussed earlier. The green represents $R_{\eta_{n/p}}$ on the full acceptance, the red represents $R_{\eta_{n/p}}$ on a reduced acceptance.

Considering $R_{\eta_{n/p}} = 0.991$ from Fig. 1, $\Delta R_{n/p}/R_{n/p} = 0.016$. This would add up to the total systematic uncertainty of $R_{n/p}$ to 1.9% for the low ϵ kinematic and to 1.6% for the high ϵ kinematic.

However, the measurement of the Rosenbluth slope, $S^n = \sigma_L/\sigma_T$, is unaffected by the uncertainty of the absolute value of the efficiency ratio $R_{\eta_{n/p}}$. $R_{\eta_{n/p}}$ will be cancelled in the the determination of the neutron Rosenbluth slope S^n, *as long as we control the stability of $R_{\eta_{n/p}}$ over the few days of the measurement.*

Efficiency stability due to the detector parameters drift

As we know, the stability of the hadron detector efficiency is critical for a successful measurement. The instability of the efficiency η over time is mainly due to the drift of the PMTs signals and the related electronics. Such an amplitude drift is typically of 1-2% over a few days. It will be better for our detector thanks to the LED calibration system.

We expect to use a threshold $A_{thr} = 100$ MeV (see plots of amplitude spectrum in the Monday write-up). Using the graph of the nucleon detection efficiency as a function of the threshold A_{thr} on Fig. 2, we find that, in the region $A_{thr} = 90 - 110$ MeV, the efficiency is:

$$\eta = 0.92 - 0.18 \times \frac{A[\mathrm{MeV}] - 100}{100} \tag{6}$$

A one percent variation of A_{thr} leads to 0.2% variation of efficiency.

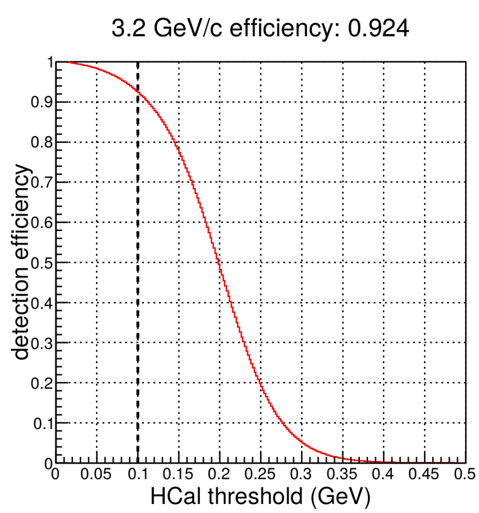

Figure 2: Nucleon detection efficiency vs the calorimeter threshold A_{thr}.

Turning now to the ratio of efficiencies, we plot it as a function of the threshold on Fig. 1. We find that, in the region $A_{thr} = 90 - 110$ MeV, the ratio of efficiencies is:

$$R_{\eta_{n/p}} = \frac{\eta_n}{\eta_p} = 0.998 + 0.011 \times \frac{A[\mathrm{MeV}] - 100}{100}. \tag{7}$$

Using the estimate for a PMT-based system instability of 1-2%, which means that A_{thr} is stable to 1-2 MeV, the ratio of efficiencies $R_{\eta_{n/p}} = \frac{\eta_n}{\eta_p}$ is found to be stable to 0.022%.

The run plan of the GMn experiment (E12-09-019), which will run next summer, has a provision of running on the hydrogen target multiple times with different fields in the SBS dipole, allowing the study of the stability of individual modules of HCal.