Polynomial Optimization Based Schemes for Solving AC Optimal Power Flow Problems

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Abstract

The Classical Semidefinite Relaxation of the ACOPF problem fails to globally solve several network instances. The Moment-SOS hierarchy of semidefinite relaxations is capable of globally solving a broader class of ACOPF instances through the use of moment and localizing matrices. In this work, we take the idea of localizing matrices in order to build an original tightening of the Classical ACOPF Semidefinite Relaxation. Our tightening introduces localizing-like constraints by breaking the symmetry of the ACOPF model and deriving auxiliary box constraints for each voltage variable. The addition of these new constraints results in a tighter relaxation. The proposed optimization scheme is tested on a variety of cases and is shown to be tighter than the classical semidefinite relaxation.

1 Introduction

The optimal power flow problem was introduced by Carpentier in 1962 [1]. As an optimization model, the general OPF problem may be modeled using linear, nonlinear, or mixed integer nonlinear optimization [7]. The scope of this work will consider only the nonlinear version known as alternating current optimal power flow (ACOPF) problem. In particular, we focus on developing an original convex relaxation as an attempt to globally solve some challenging instances.

The ACOPF problem is nonconvex and NP-hard [5], [10] and, therefore, local solvers may fail even at the task of finding a feasible solution. For such a problem, an adequate solution is to relax the nonconvex constraints. We base our discussion on the well-known Semidefinite (SD) relaxation of the ACOPF.

Previous work on tightening SD relaxations of the ACOPF, including Polynomial Optimization techniques, can be found in [8], [9], [11], [14]–[16]. Since Semidefinite programming is computationally demanding, many efforts to reduce the computational burden are still being made. In particular, in this work we focus on maintaining a reasonable size on the SD matrices associated with the optimization process.

The remainder of the paper is organized as follows. Section 2 covers the basics on sparse semidefinite programming. The nonlinear formulation of the ACOPF problem is given in Section 3. Then, the classical ACOPF relaxation can be described in Section 4. Our contribution is presented in Section 5 and the obtained numerical results are presented in Section 6. Finally, conclusions are made in Section 7.

2 Sparse Semidefinite Programming

In this Section, we review the concepts of exploiting sparsity in semidefinite programming needed for our exposition in Sections 4 and 5.

For given symmetric matrices $C \in \mathbb{S}^n$, $A_i \in \mathbb{S}^n$, $i = 1, ..., m$, and $b \in \mathbb{R}^m$, a semidefinite program (SDP) is defined as

$$\inf_{y \in \mathbb{R}^m, S \in \mathbb{S}^n} y^\top b \quad \text{s.t.} \quad S = C - \sum_{i=1}^{m} y_i A_i \succeq 0, \tag{1}$$

where the condition $S \succeq 0$ does not allow S to have any negative eigenvalues. A semidefinite program can be solve efficiently using interior point methods, however SDP solvers are not as advanced as linear solvers, for example, and the computational burden can increase significantly as n increases. Fortunately, performance of SDP solvers can be improved considerably if matrix S is sparse.

Definition 1 (Sparsity Graph of a Matrix) *An undirected graph $G(V, E)$ with nodes $V = \{1, 2, ..., n\}$ and edge set E is called the sparsity graph of matrix $S \in \mathbb{S}^n$ when $(i, j) \in E$ if and only if $S_{ij} \neq 0$.*

Our interest will lye on the case where G is a chordal graph, that is, every graph cycle of length four or greater has a cycle chord. We next recall that a subgraph of G such that every two distinct vertices are adjacent is called a *clique*. A maximal clique of G is a clique that is not a proper subset of a larger clique.

Following the lead of [9], we consider a Completion Matrix Theorem [2], [3] in order to exploit the sparsity pattern of S.

Theorem 1 *Let $G(V, E)$ the sparsity graph of $S \in \mathbb{S}^n$. Let $C_1, ..., C_p$ be the maximal cliques of G. The matrix S can completed to a positive semidefinite matrix (i.e., unknown entries can be chosen such that $S \succeq 0$) if and only if the submatrices associated with each of the maximal cliques of G are all positive semidefinite, that is, $S[C_k, C_k] \succeq 0$, $k = 1, ..., p$.*

Example When the sparsity graph of a matrix is not chordal, we compute a chordal extension so that Theorem 1 can be applied and maximal cliques can be determined in linear time. Consider the following matrix

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 \\ s_{21} & s_{22} & s_{23} & s_{24} & 0 \\ s_{31} & s_{32} & s_{33} & 0 & s_{35} \\ 0 & s_{42} & 0 & s_{44} & s_{45} \\ 0 & 0 & s_{53} & s_{54} & s_{55} \end{bmatrix} \tag{2}$$

with sparsity graph as in Figure 1.

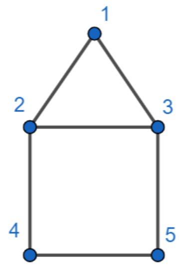

Figure 1: Sparsity graph of matrix (2)

Note that S does not have a chordal sparsity graph since the cycle $(2, 3, 4, 5)$ of length four has no chord. We can readily extend this graph to a chordal graph by adding the edge $(3, 4)$, see Figure 2.

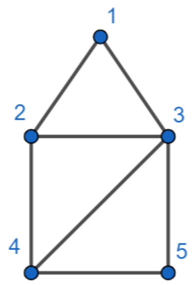

Figure 2: Chordal extension of the graph of matrix (2)

Finally, we have the maximal cliques $C_1 = \{1, 2, 3\}$, $C_2 = \{2, 3, 4\}$ and $C_3 = \{3, 4, 5\}$, and the application of Theorem 1 allows us to replace the condition $S \succeq 0$ by semidefinite constraints on smaller matrices:

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \succeq 0, \quad \begin{bmatrix} s_{22} & s_{23} & s_{24} \\ s_{32} & s_{33} & s_{34} \\ s_{42} & s_{43} & s_{44} \end{bmatrix} \succeq 0, \quad \text{and} \quad \begin{bmatrix} s_{33} & s_{34} & s_{35} \\ s_{43} & s_{44} & s_{45} \\ s_{54} & s_{54} & s_{55} \end{bmatrix} \succeq 0.$$

We will give more details on chordal extension computing in Section 4. We now proceed to present our target optimization model: the ACOPF problem.

3 ACOPF Problem

For our model formulation, we consider notations on [9], [15].

Consider a typical power network $P = (\mathcal{N}, \mathcal{L})$ where $\mathcal{N} = \{1, ..., n\}$ and $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ denote respectively the set of buses and the set of branches (transmission lines, transformers and phase shifters). Each branch $l \in \mathcal{L}$ has a *from* end k (on the *tap side*) and a *to* end m as modeled in [6]. We note $l = (k, m)$.

The ACOPF problem is given as:

$$\min \sum_{g \in \mathcal{G}} c_{g2} p_{Gg}^2 + c_{g1} p_{Gg} + c_{g0} \tag{3a}$$

over the variables $p_G, q_G \in \mathbb{R}^{|\mathcal{G}|}$, $p_f, q_f, p_t, q_t \in \mathbb{R}^{|\mathcal{L}|}$ and $v_d, v_q \in \mathbb{R}^{|\mathcal{N}|}$, subject to

- Power balance equations:

$$\sum_{g\in\mathcal{G}_k} p_{Gg} - p_{Dk} - g'_k(v_{dk}^2 + v_{qk}^2) = \sum_{l=(k,m)\in\mathcal{L}} p_{fl} + \sum_{l=(m,k)\in\mathcal{L}} p_{tl} \quad \forall k \in \mathcal{N}, \tag{3b}$$

$$\sum_{g\in\mathcal{G}_k} q_{Gg} - q_{Dk} + b'_k(v_{dk}^2 + v_{qk}^2) = \sum_{l=(k,m)\in\mathcal{L}} q_{fl} + \sum_{l=(m,k)\in\mathcal{L}} q_{tl} \quad \forall k \in \mathcal{N}, \tag{3c}$$

- Line flow equations:
  For all $l = (k, m) \in \mathcal{L}$:

$$p_{fl} = \frac{g_l}{\tau_l^2}(v_{dk}^2 + v_{qk}^2) - \frac{g_l}{\tau_l}(v_{dk}v_{dm} + v_{qk}v_{qm}) + \frac{b_l}{\tau_l}(v_{dk}v_{qm} - v_{qk}v_{dm}), \tag{3d}$$

$$p_{tl} = g_l(v_{dm}^2 + v_{qm}^2) - \frac{g_l}{\tau_l}(v_{dk}v_{dm} + v_{qk}v_{qm}) - \frac{b_l}{\tau_l}(v_{dk}v_{qm} - v_{qk}v_{dm}), \tag{3e}$$

$$p_{fl} = -\frac{b_l + b_{s,l}/2}{\tau_l^2}(v_{dk}^2 + v_{qk}^2) + \frac{b_l}{\tau_l}(v_{dk}v_{dm} + v_{qk}v_{qm}) + \frac{g_l}{\tau_l}(v_{dk}v_{qm} - v_{qk}v_{dm}), \tag{3f}$$

$$p_{tl} = -(b_l + b_{s,l}/2)(v_{dm}^2 + v_{qm}^2) + \frac{b_l}{\tau_l}(v_{dk}v_{dm} + v_{qk}v_{qm}) + \frac{g_l}{\tau_l}(-v_{dk}v_{qm} + v_{qk}v_{dm}), \tag{3g}$$

- Generator power capacities:

$$\underline{p}_{Gg} \le p_{Gg} \le \overline{p}_{Gg}, \quad \underline{q}_{Gg} \le q_{Gg} \le \overline{q}_{Gg} \quad \forall g \in \mathcal{G}, \tag{3h}$$

- Line thermal limits:

$$p_{fl}^2 + q_{fl}^2 \le \overline{s}_l^2, \quad p_{tl}^2 + q_{tl}^2 \le \overline{s}_l^2 \quad \forall l \in \mathcal{L}, \tag{3i}$$

- Voltage magnitude limits:

$$\underline{v}_k^2 \le v_{dk}^2 + v_{qk}^2 \le \overline{v}_k^2 \tag{3j}$$

- Reference bus constraint:

$$v_{q0} = 0. \tag{3k}$$

Objective function (3a) is commonly known as generation cost minimization. Constraints (3b)-(3g) are derived from Kirchhoff's laws and represent power flows in the network. Constraint (3k) specifies bus $k = 1$ as the reference bus. We assume that $\underline{v}_k > 0$ for all $k \in \mathcal{N}$ in (3j), and the generation cost $c_{g2}p_{Gg}^2 + c_{g1}p_{Gg} + c_{g0}$ is a convex function for all $g \in \mathcal{G}$. The complete list of parameters is shown in Table 3.

Note that nonconvexity of the problem comes from the quadratic terms of v_d and v_q in constraints (3b)-(3g) and (3j). Therefore, using local solvers may lead to a local optimum or even may fail to converge in the absence of a good initial solution. In the next Section we detail our implementation of the classical SDP relaxation of the ACOPF problem, a powerful technique to compute lower bounds or even find global solutions to this problem.

Symbol	Description
$P = (\mathcal{N}, \mathcal{L})$	Power Network
$\mathcal{N}$	Set of buses
$\mathcal{G} = \bigcup_{k\in\mathcal{N}} \mathcal{G}_k$	Set of generators
$\mathcal{G}_k$	Set of generators connected to bus k
$\mathcal{L}$	Set of branches
p_{Dk}/q_{Dk}	Active/reactive power demand at bus k
g'_k/b'_k	Conductance/susceptance of shunt element at bus k
c_{g2}, c_{g1}, c_{g0}	Generation cost coefficients of generator g
$y_l = 1/(r_l + jx_l)$	Series impedance of branch l
τ_l	Turns ratio of branch l
$g_l + jb_l$	Series admittance of branch l
$g_{s,l} + jb_{s,l}$	Total shunt admittance of branch l.

Table 1: Power network parameters

4 Semidefinite Relaxation of the ACOPF Problem

Let

$$v = \begin{bmatrix} v_d \\ v_q \end{bmatrix} \in \mathbb{R}^{2|\mathcal{N}|} \tag{4}$$

and $V = vv^\top$. Then, the ACOPF problem (3) can be restated as

$$\min \sum_{g\in\mathcal{G}} c_{g2}p_{Gg}^2 + c_{g1}p_{Gg} + c_{g0} \tag{5a}$$

subject to (3h), (3i), (3k),

$$\sum_{g\in\mathcal{G}_k} p_{Gg} - p_{Dk} - g'_k(V_{k,k} + V_{k+n,k+n}) = \sum_{l=(k,m)\in\mathcal{L}} p_{fl} + \sum_{l=(m,k)\in\mathcal{L}} p_{tl} \quad \forall k \in \mathcal{N}, \tag{5b}$$

$$\sum_{g\in\mathcal{G}_k} q_{Gg} - q_{Dk} + b'_k(V_{k,k} + V_{k+n,k+n}) = \sum_{l=(k,m)\in\mathcal{L}} q_{fl} + \sum_{l=(m,k)\in\mathcal{L}} q_{tl} \quad \forall k \in \mathcal{N}, \tag{5c}$$

For all $l = (k, m) \in \mathcal{L}$:

$$p_{fl} = \frac{g_l}{\tau_l^2}(V_{k,k} + V_{k+n,k+n}) - \frac{g_l}{\tau_l}(V_{k,m} + V_{k+n,m+n}) + \frac{b_l}{\tau_l}(V_{k,m+n} - V_{k+n,m}), \tag{5d}$$

$$p_{tl} = g_l(V_{m,m} + V_{m+n,m+n}) - \frac{g_l}{\tau_l}(V_{k,m} + V_{k+n,m+n}) - \frac{b_l}{\tau_l}(V_{k,m+n} - V_{k+n,m}), \tag{5e}$$

$$p_{fl} = -\frac{b_l + b_{s,l}/2}{\tau_l^2}(V_{k,k} + V_{k+n,k+n}) + \frac{b_l}{\tau_l}(V_{k,m} + V_{k+n,m+n}) + \frac{g_l}{\tau_l}(V_{k,m+n} - V_{k+n,m}), \tag{5f}$$

$$p_{tl} = -(b_l + b_{s,l}/2)(V_{m,m} + V_{m+n,m+n}) + \frac{b_l}{\tau_l}(V_{k,m} + V_{k+n,m+n}) + \frac{g_l}{\tau_l}(-V_{k,m+n} + V_{k+n,m}), \tag{5g}$$

For all $k \in \mathcal{N}$

$$\underline{v}_k^2 \leq V_{k,k} + V_{k+n,k+n} \leq \overline{v}_k^2 \tag{5h}$$

and

$$V = vv^T. \tag{5i}$$

Note that constraint (5i) can be replaced by $V \succeq 0$ and $\mathrm{rank}(V) = 1$. Then, the semidefinite relaxation of the ACOPF problem is obtained by dropping the rank constraint and replacing (5i) by $V \succeq 0$ in model (5).

4.1 Exploiting Sparsity

Solving the semidefinite relaxation of the ACOPF is computationally very expensive for large-scale power systems. Fortunately, as mentioned in Section 2, the sparsity pattern of V can be exploited in order to reduce the computational burden.

There are two options when it comes to building a sparsity pattern for V: correlative sparsity or term sparsity. For our case and if term sparsity is considered, the sparsity graph contains the edge (i, j) if the product $v_i v_j$, for v defined as in (4), appears in one or more ACOPF constraints. This kind of sparsity pattern yields small maximal cliques in general and therefore is a very suitable option for the semidefinite relaxation (5). However, we will focus on correlative sparsity since it will be more adequate for the tightening proposed in Section 5. In correlative sparsity, the sparsity graph contains the edge (i, j) if variables v_i and v_j appear in the same ACOPF constraint (even in different terms). This subtle difference will produce a sparsity graph with larger maximal cliques and therefore, it will be computationally more expensive than term sparsity.

Assume that M_V is the adjacency matrix of the sparsity graph of V. A chordal extension of the sparsity graph can be computed via the fill-in pattern of a Cholesky decomposition of M_V. In our implementation, we add an adequate scaled identity matrix to M_V in order to perform the Cholesky decomposition numerically.

Once the extended sparsity graph is chordal, maximal cliques can be computed in linear time, and we can replace the constraint $V \succeq 0$ by

$$V[C_t, C_t] \succeq 0, \quad \text{for each maximal clique } t.$$

5 Proposed Tightening of the ACOPF Semidefinite Relaxation

The classical SD relaxation of the ACOPF is capable of solving a variety of ACOPF instances to global optimality. However, there are cases were the SD relaxation only yields a lower bound of the objective function. For this cases, [8], [9], [16] proposed to apply Laserre's Hierarchy of Moment relaxations [4] in order to achieve exactness of semidefinite relaxations. From these works, we take the concept of localizing matrices to develop our own tightening of the SD relaxation by the introduction of localizing-like matrices.

The main feature of our tightening is that localizing matrices are not build with the original ACOPF constraints but by deriving extra auxiliary constrains. Let us proceed to this task.

First, note that constraint (3k) allows to choose the sign of v_{d1} and write the constraint

$$\underline{v}_1 \leq v_{d1} \leq \overline{v}_1 \tag{6}$$

without changing the optimal solution of (3). Similarly, for $k = 2, ..., n$ we can write the constraints

$$-\overline{v}_k \leq v_{dk} \leq \overline{v}_k, \quad -\overline{v}_k \leq v_{qk} \leq \overline{v}_k. \tag{7}$$

For the sake of notation, we group constraints (6) and (7) as

$$h(k, w, z) = \begin{cases} v_{d1} - \underline{v}_1 & \text{if } k = 1, w = 0, z = 1 \\ \overline{v}_1 - v_{d1} & \text{if } k = 1, w = 1, z = 1 \\ v_{dk} + \overline{v}_k & \text{if } 2 \leq k \leq n, w = 0, z = 1 \\ \overline{v}_k - v_{dk} & \text{if } 2 \leq k \leq n, w = 1, z = 1 \\ v_{qk} + \overline{v}_k & \text{if } 2 \leq k \leq n, w = 0, z = 0 \\ \overline{v}_k - v_{qk} & \text{if } 2 \leq k \leq n, w = 1, z = 0 \\ 0 & \text{otherwise} \end{cases}. \tag{8}$$

Therefore, we claim that adding constraints

$$h(k, w, z) \geq 0, \quad \text{for } k \in \mathcal{N}, \quad w, z = 0, 1, \tag{9}$$

to the ACOPF model does not affect the optimal solution since they are redundant.

On the other hand, we define the neighborhood of bus k as

$$Nb(k) := \{1, v_{dk}, v_{qk}\} \bigcup \{v_{dt}, v_{qt} | (k, t) \in \mathcal{L} \text{ or } (t, k) \in \mathcal{L}\}. \tag{10}$$

Let $r_k = |Nb(k)|$ and $u_k \in \mathbb{R}^{r_k}$ the vector that contains the elements of $Nb(k)$, we define

$$M_k := u_k u_k^\top.$$

Since $M_k \succeq 0$, for a feasible solution of the ACOPF we have that

$$h(k', w, z) M_k \succeq 0, \tag{11}$$

for $k' \in \mathcal{N}$ and any $w, z = 0, 1$. We will generate tightening constraints for the ACOPF semidefinite relaxation using the entry-wise linearization of (11) in the style of localizing matrices. For more details on localizing matrices in Polynomial Optimization see [4]. Actually, the ACOPF semidefinite relaxation corresponds to the first order Moment relaxation for this problem [8], [9] and we have already replaced products of the type $v_{dk} v_{qm}$ by linear variables $V_{k,m+n}$, for instance. Note that in order to linearize the entries of (11) we need more variables than those in V, those that correspond to the linearization of third-order monomials like $u \cdot s \cdot t$ and they will be denoted as $L(u \cdot s \cdot t)$, for any s, u, t in vector variable v. In order to keep things tractable, we propose to add this tightening constraints within neighborhoods, that is, for each $k \in \mathcal{N}$ we add the constraints

$$L(h(k', w, z) M_k) \succeq 0, \text{ for all } k' \text{ such that } v_{dk'} \in Nb(k) \text{ or } v_{qk'} \in Nb(k), w, z = 0, 1. \tag{12}$$

A connection between (12) and the original ACOPF constraints still has to de made. Note that $\sum_{g\in\mathcal{G}_k} p_{Gg}$ and $\sum_{g\in\mathcal{G}_k} q_{Gg}$ can be written as polynomial functions on the variable vector v, see [9], and let us denote them by $p_k(v)$ and $q_k(v)$. For both polynomial functions we can derive box constraints $\underline{p}_k \leq p_k(v) \leq \overline{p}_k$ and $\underline{q}_k \leq q_k(v) \leq \overline{q}_k$. Finally, we add tightening constraints for each bus $k \in \mathcal{N}$,

$$L((\overline{p}_k - p_k(v))h(k', w, z)) \geq 0 \tag{13a}$$

$$L((\overline{p}_k - p_k(v))h(k', w, z)) \geq 0 \tag{13b}$$

$$L((q_k(v) - \underline{q}_k)h(k', w, z)) \geq 0 \tag{13c}$$

$$L((\overline{q}_k - q_k(v))h(k', w, z)) \geq 0 \tag{13d}$$

$$L((\overline{v}_k^2 - v_{dk}^2 - v_{qk}^2)h(k', w, z)) \geq 0 \tag{13e}$$

$$L((v_{dk}^2 + v_{qk}^2 - \underline{v}_k^2)h(k', w, z)) \geq 0 \tag{13f}$$

for all k' such that $v_{dk'} \in Nb(k)$ or $v_{qk'} \in Nb(k)$, $w, z = 0, 1$. Then, the main featuring of this work is the convex relaxation given by

Variables	$p_G, q_G \in \mathbb{R}^{\lvert\mathcal{G}\rvert}$ $p_f, q_f, p_t, q_t \in \mathbb{R}^{\lvert\mathcal{L}\rvert}$ $v \in \mathbb{R}^{2n+1}$, $v = \begin{bmatrix} v_d \\ v_q \\ 1 \end{bmatrix}$ $V \in \mathbb{R}^{2n+1,2n+1}$, $V = L(vv^\top)$ $L(u \cdot s \cdot t)$ for all $k \in \mathcal{N}$, for all $u, s, t \in Nb(k)$
Objective	(3a)
Constraints	(3i), (3h), (3k) (5b)-(5h) $V[C, C] \succeq 0$ for each maximal cliques C. (12), (13) $v_{d1} \geq \frac{V_{1,1} + \underline{v}_1 \overline{v}_1}{\underline{v}_1 + \overline{v}_1}$ (taken from [15])

6 Numerical Results

As mentioned before, the Semidefinite Relaxation of the ACOPF problem is not exact for all OPF instances. In order to test our tightening technique, we consider the testcases of [9], cases where the semidefinite relaxation fails to be exact.

Our implementation https://github.com/efuentes41664/ACOPFrelaxations/tree/main/POBS_ACOPF is made in Julia 1.6.7 [12] using JuMP v1.1.1 [13] to build the optimization model. The resulting semidefinite program is solved with state of the art solver Mosek 9.3.20. All experiments are executed in an ASUS laptop with Corei5 8thGen 1.60GHz processor and 8GB of memory.

We say that a convex relaxation is exact if a feasible solution for the original nonlinear ACOPF problem can be extracted from the solution of the relaxation (up to a given tolerance). In the case of the semidefinite relaxations considered in this work, the relaxation is exact if, at optimal solution V^*, all numerical matrices

$$V^*[C, C] \succeq 0$$

have rank equal to 1. Thus, the metric to check the quality of our tightening will be considered as the order of the largest second eigenvalue among all $V^*[C, C]$ matrices.

The next table resumes the result of our numerical experiments.

Testcase	Objective Value	Time (s)	Max. 2nd Eigval
case14Q			
SD Relaxation	3301.34749	0.06	0.030737
Tightening	3301.80121	2.12	1.48236e-5
case14L			
SD Relaxation	9353.58290	0.08	0.03929
Tightening	9358.91756	4.27	0.00231
case39Q			
SD Relaxation	10804.08281	0.32	0.03456
Tightening	10820.59594	9.98	0.01254
case39L			
SD Relaxation	41906.84949	0.36	0.01441
Tightening	41907.47819	7.29	1.02601e-7
case57Q			
SD Relaxation	7350.73666	1.75	0.03034
Tightening	7351.78375	27.92	0.00013
case57L			
SD Relaxation	43909.75058	2.90	0.01621
Tightening	43982.04489	36.11	2.94808e-5
case118Q			
SD Relaxation	81427.88634	3.92	0.07948
Tightening	81508.46427	297.58	0.00016
case118L			
SD Relaxation	133833.69227	6.38	0.06228
Tightening	133887.13503	464.59	0.07427
case300			
SD Relaxation	719710.17936	16.91	0.02338
Tightening	719724.79435	2408.04	0.00058

It is observed that, for all cases, our tightening yields a better lower bound on the objective function when compared to the SD relaxation; clearly, a larger computational time is required to achieve this. Regarding the exactness of our proposed tightening, let us choose a tolerance of 1e-3 for the maximum second eigenvalue among all the solution matrices corresponding to maximal cliques. Then, we globally solved cases 14Q, 39L, 57Q, 57L, 118Q and 300. For the rest of the cases, even if the obtained lower bound is tighter than the one of the SD relaxation, the maximum second eigenvalue metric suggests that no feasible solution of the nonlinear ACOPF model can be extracted from the relaxation solution.

7 Conclusion

We have derived an effective tightening of the classical SD relaxation of the ACOPF problem. The proposed tightening constraints are original and novel but, more importantly, effective. However, the inherent computational burden of semidefinite programming cannot be avoided by our model and an expensive price has to paid regarding the CPU time.

Therefore, possible future work includes the incorporation of these new constraints to other polynomial optimization based schemes to solve ACOPF problems. In particular, one appealing research direction is the application of selective tightening for a subset of buses in the power network, since this idea has been proven to deliver tight lower bounds in the relaxations of [9].

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