Appendix

Proof of Lemma 1. Given the firm knows consumers' type perfectly, it only needs to satisfy the participation constraints for two types of consumers, i.e., the constraints ($\mathrm{IR_H}$) and ($\mathrm{IR_L}$) are blinding. Thus, the prices of the two products can be derived as

$$p_H = q_H - \beta_H(1-\lambda), p_L = \theta q_L + \beta_L \lambda. \tag{A.1}$$

Plugging p_H, p_L into the firm's profit, i.e., $\pi = \lambda(p_H - \frac{1}{2}q_H^2) + (1-\lambda)(p_L - \frac{1}{2}q_L^2)$. By the first-order conditions, we have the optimal quality of two products as $q_H^* = 1, q_L^* = \theta$. Therefore, the product prices, the firm's profit, consumer surplus, and social welfare in equilibrium can be derived as

$$\begin{aligned} &p_H^* = 1 - \beta_H(1-\lambda), p_L^* = \theta^2 + \beta_L\lambda, \pi^* = \frac{1}{2}(\lambda + (1-\lambda)\theta^2) - \lambda(1-\lambda)(\beta_H - \beta_L), \\ &cs_H^* = cs_L^* = 0, sw^* = \frac{1}{2}(\lambda + (1-\lambda)\theta^2) - \lambda(1-\lambda)(\beta_H - \beta_L). \end{aligned} \tag{A.2}$$

By taking the partial derivatives of the price and quality of the two types of products and the firm's profit with respect to β_H and β_L, we can get

$$\frac{\partial p_H^*}{\partial \beta_H} = -(1-\lambda) < 0, \frac{\partial q_H^*}{\partial \beta_H} = 0, \frac{\partial \pi^*}{\partial \beta_H} = -\lambda(1-\lambda) < 0, \frac{\partial p_L^*}{\partial \beta_L} = \lambda > 0, \frac{\partial q_L^*}{\partial \beta_L} = 0, \frac{\partial \pi^*}{\partial \beta_L} = \lambda(1-\lambda) > 0. \tag{A.3}$$

□

Proof of Lemma 2. Since consumer types are private information, the firm's problem is maximizing the total profit by developing two capable offerings, as shown in Equation 3. Depending on which constraints are binding for each type of consumer, there are four possible equilibrium outcomes. Specifically, the four scenarios are as follows. Scenario 1: ($\mathrm{IC_H}$) and ($\mathrm{IR_L}$) are binding; Scenario 2: ($\mathrm{IR_H}$) and ($\mathrm{IC_L}$) are binding; Scenario 3: ($\mathrm{IR_H}$) and ($\mathrm{IR_L}$) are binding; and Scenario 4: ($\mathrm{IC_H}$) and ($\mathrm{IC_L}$) are binding.

Figure A.1 presents the feasible region of the optimization problem. It turns out that there does not exist (p_H, p_L) which satisfies the scenario where both ($\mathrm{IC_H}$) and ($\mathrm{IC_L}$) are binding simultaneously. Hence, Scenario 4 will not happen in the equilibrium. Points 1, 2, and 3 in Figure A.1 represent the three remaining scenarios, which we discuss as follows.

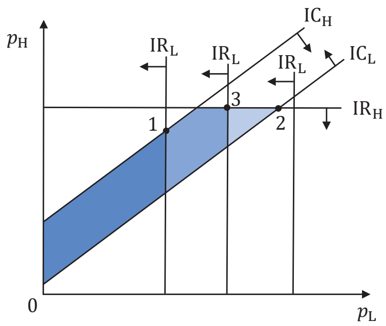

Figure A.1: Self-Selection Constraints

Scenario 1: ($\mathrm{IC_H}$) and ($\mathrm{IR_L}$) are binding.
In this scenario, the optimal prices are

$$p_H = q_H - q_L + \theta q_L + \beta_L\lambda - \beta_H(1-\lambda), p_L = \theta q_L + \beta_L\lambda. \tag{A.4}$$

For Scenario 1 to hold, we need to ensure that ($\mathrm{IR_H}$) and ($\mathrm{IC_L}$) are satisfied in equilibrium. Plugging (p_H, p_L) into ($\mathrm{IR_H}$) and ($\mathrm{IC_L}$), one can show that they are true if and only if $q_L \geq \frac{\beta_L\lambda}{1-\theta}$ and $q_H - q_L \geq \frac{\beta_H(1-\lambda)}{1-\theta}$. Therefore, after substituting (p_H, p_L) into the firm's profit, the problem of the firm can formulated as follows:

$$\begin{aligned} \max_{q_H,q_L} \pi &= -\frac{1}{2}\lambda q_H^2 - \frac{1}{2}(1-\lambda)q_L^2 + \lambda q_H + q_L(\theta-\lambda) + \lambda\left(\beta_L - \beta_H(1-\lambda)\right) \\ \text{s.t.} \qquad & q_L \geq \frac{\beta_L\lambda}{1-\theta}, \\ & q_H - q_L \geq \frac{\beta_H(1-\lambda)}{1-\theta}. \end{aligned} \tag{A.5}$$

It is easy to see that the optimization problem (A.5) is a concave programming problem. We utilize the K-T condition to solve this optimization problem, which can be written as

$$\begin{aligned} \max \pi(q_H,q_L) &= -\frac{1}{2}\lambda q_H^2 - \frac{1}{2}(1-\lambda)q_L^2 + \lambda q_H + q_L(\theta-\lambda) + \lambda\left(\beta_L - \beta_H(1-\lambda)\right) \\ \text{s.t.} \qquad & g_1(q_H,q_L) = q_L - \frac{\beta_L\lambda}{1-\theta} \geq 0, \\ & g_2(q_H,q_L) = q_H - q_L - \frac{\beta_H(1-\lambda)}{1-\theta} \geq 0. \end{aligned} \tag{A.6}$$

We formulate the Lagrange function as:

$$L(q_H,q_L) = \pi(q_H,q_L) + mg_1(q_H,q_L) + ng_2(q_H,q_L). \tag{A.7}$$

Therefore, we generate the following equations:

$$\begin{cases} \frac{\partial L(q_H,q_L)}{\partial q_H} = n + \lambda - \lambda q_H, \\ \frac{\partial L(q_H,q_L)}{\partial q_L} = m - n + \theta - \lambda - q_L(1-\lambda), \\ mg_1(q_H,q_L) = 0, ng_2(q_H,q_L) = 0, m \geq 0, n \geq 0, \end{cases} \tag{A.8}$$

where m, n are the K-T multipliers of $g_1(q_H,q_L)$ and $g_2(q_H,q_L)$. There arise four possible scenarios, depending on the quality constraints $g_1(q_H,q_L) \geq 0$ and $g_2(q_H,q_L) \geq 0$.

Scenario 1.1 When $m=0, n=0$, the quality constraints are loose (i.e., $g_1(q_H,q_L)>0$, $g_2(q_H,q_L)>0$). In this scenario, the first-order conditions $\frac{\partial L}{\partial q_H}=0$ and $\frac{\partial L}{\partial q_L}=0$ render $q_H=1, q_L=\frac{\theta-\lambda}{1-\lambda}$. Substituting p_H, p_L into the quality constraints, we have $\beta_L \leq \beta_{L1} \triangleq \frac{(\theta-\lambda)(1-\theta)}{\lambda(1-\lambda)}$, $\beta_H \leq \beta_{H1} \triangleq \frac{(1-\theta)^2}{(1-\lambda)^2}$, which can satisfy $g_1(q_H,q_L) > 0$ and $g_2(q_H,q_L) > 0$. Correspondingly, the prices and the profit of the firm can be computed as

$$p_H = \frac{1-\theta+\theta(\theta-\lambda)}{1-\lambda} + \beta_L\lambda - \beta_H(1-\lambda), p_L = \frac{\theta(\theta-\lambda)}{1-\lambda} + \beta_L\lambda, \pi = \frac{\theta^2-2\theta\lambda+\lambda}{2(1-\lambda)} + \beta_L\lambda - \beta_H\lambda(1-\lambda). \tag{A.9}$$

Scenario 1.2 When $m>0, n>0$, the quality constraints are binding (i.e., $g_1(q_H,q_L)=0$, $g_2(q_H,q_L)=0$), which gives $q_H = \frac{\beta_L\lambda+\beta_H(1-\lambda)}{1-\theta}$ and $q_L = \frac{\beta_L\lambda}{1-\theta}$. In this scenario, the first-order conditions $\frac{\partial L}{\partial q_H}=0$ and $\frac{\partial L}{\partial q_L}=0$ render $m = \frac{\theta^2-\theta-\beta_H\lambda^2+\beta_H\lambda+\beta_L\lambda}{1-\theta}$, $n = \frac{\lambda(\beta_L\lambda+\beta_H(1-\lambda)-(1-\theta))}{1-\theta}$. Solving $m>0, n>0$, we have ($\beta_L \leq \beta_{L1}, \beta_H \geq \beta_{H2} \triangleq \frac{\theta(1-\theta)-\beta_L\lambda}{\lambda(1-\lambda)}$) or ($\beta_{L1} < \beta_L < \beta_{L3} \triangleq \frac{1-\theta}{\lambda}, \beta_H \geq \beta_{H3} \triangleq \frac{1-\theta-\beta_L\lambda}{1-\lambda}$) or $\beta_L \geq \beta_{L3}$. Correspondingly, the prices and the profit of the firm can be computed as

$$\begin{aligned} p_H &= \frac{\beta_L\lambda+\theta\beta_H(1-\lambda)}{1-\theta}, p_L = \frac{\beta_L\lambda}{1-\theta}, \\ \pi &= \frac{\lambda(\beta_L(2-2\theta-\beta_L\lambda)-\beta_H^2(1-\lambda)^2-2\beta_H(1-\lambda)(\beta_L\lambda-\theta(1-\theta)))}{2(1-\theta)^2}. \end{aligned} \tag{A.10}$$

Scenario 1.3 When $m=0, n>0$, the first quality constraint is loose (i.e., $g_1(q_H,q_L)>0$), while the other is binding (i.e., $g_2(q_H,q_L)=0$). In this scenario, the first-order conditions $\frac{\partial L}{\partial q_H}=0$, $\frac{\partial L}{\partial q_L}=0$ and $m=0$ render $q_H = \theta + \frac{\beta_H(1-\lambda)^2}{1-\theta}$, $q_L = \theta - \frac{\beta_H\lambda(1-\lambda)}{1-\theta}$, $n = \frac{\lambda\left(\beta_H(1-\lambda)^2-(1-\theta)^2\right)}{1-\theta}$. Substituting q_L into $g_1(q_H,q_L)$ and Solving $g_1(q_H,q_L)>0$, we have $\beta_L < \beta_{L1}, \beta_{H1} \leq \beta_H \leq \beta_{H2}$. Correspondingly, the prices and the profit of the firm can be computed as

$$p_H = \theta^2 + \beta_L\lambda + \frac{\theta\beta_H(1-\lambda)^2}{1-\theta}, p_L = \theta^2 + \beta_L\lambda - \frac{\theta\beta_H\lambda(1-\lambda)}{1-\theta}, \pi = \frac{1}{2}\left(\theta^2 + 2\beta_L\lambda - \frac{\lambda\beta_H^2(1-\lambda)^3}{(1-\theta)^2}\right). \tag{A.11}$$

Scenario 1.4 When $m > 0, n = 0$, the first quality constraint is binding (i.e., $g_1(q_H, q_L) = 0$), while the other is loose (i.e., $g_2(q_H, q_L) > 0$). In this scenario, the first-order conditions $\frac{\partial L}{\partial q_H} = 0$, $\frac{\partial L}{\partial q_L} = 0$ and $n = 0$ render $q_H = 1$, $q_L = \frac{\beta_L\lambda}{1-\theta}$, $m = \frac{\theta(\theta-\lambda 1)+\beta_L\lambda(1-\lambda)}{1-\theta}$. Substituting q_H and q_L into $g_2(q_H, q_L)$ and Solving $g_2(q_H, q_L) > 0$, we have $\beta_{L1} \leq \beta_L < \beta_{L3}, \beta_H \leq \beta_{H3}$. Correspondingly, the prices and the profit of the firm can be computed as

$$p_H = 1 - \beta_H(1-\lambda), p_L = \frac{\beta_L\lambda}{1-\theta}, \pi = \frac{\beta_L\lambda(1-\lambda)(2-2\theta-\beta_L\lambda)}{2(1-\theta)^2} + \frac{\lambda}{2} - \beta_H\lambda(1-\lambda). \tag{A.12}$$

Thus we obtain four equilibrium outcomes in Scenario 1.

Scenario 2: ($\mathrm{IR_H}$) and ($\mathrm{IC_L}$) are binding.

In this scenario, the optimal prices are

$$p_H = q_H - \beta_H(1-\lambda), p_L = q_H + \theta(q_L - q_H) - \beta_H(1-\lambda). \tag{A.13}$$

For Scenario 2 to hold, we need to ensure that ($\mathrm{IC_H}$) and ($\mathrm{IR_L}$) are satisfied in equilibrium. Plugging (p_H, p_L) into ($\mathrm{IC_H}$) and ($\mathrm{IR_L}$), one can show that they are true if and only if $q_H \leq \frac{\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}$ and $q_H - q_L \geq \frac{\beta_H(1-\lambda)}{1-\theta}$. Therefore, after substituting (p_H, p_L) into the firm's profit, the problem of the firm can formulated as follows:

$$\begin{aligned} \max_{q_H, q_L} \pi = & -\frac{1}{2}\lambda q_H^2 - \frac{1}{2}(1-\lambda)q_L^2 + q_H(1-\theta(1-\lambda)) + q_L\theta(1-\lambda) - \beta_H(1-\lambda) \\ \text{s.t.} \quad & q_H \leq \frac{\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, \\ & q_H - q_L \geq \frac{\beta_H(1-\lambda)}{1-\theta}. \end{aligned} \tag{A.14}$$

It is easy to see that the optimization problem (A.14) is a concave programming problem. We also utilize the K-T condition to solve this optimization problem, the solving process is similar to Scenario 1, thus we omit it here.

The four equilibrium outcomes in Scenario 2 are as follows:

Scenario 2.1 When ($\beta_{L2} \triangleq \frac{\theta(1-\theta)}{\lambda} \leq \beta_L \leq \beta_{L4} \triangleq \frac{(1-\theta)(1-\theta+\theta\lambda)}{\lambda^2}, \beta_{H4} \triangleq \frac{(1-\theta)(1-\theta+\theta\lambda)-\beta_L\lambda^2}{\lambda(1-\lambda)} \leq \beta_H \leq \beta_{H5} \triangleq \frac{(1-\theta)^2}{\lambda(1-\lambda)}$) or ($\beta_L > \beta_{L4}, \beta_H \leq \beta_{H5}$), the equilibrium quality, price, and the profit of the firm can be derived as

$$\begin{aligned} q_H &= \frac{1-\theta+\theta\lambda}{\lambda}, \ q_L = \theta, p_H = \frac{1-\theta(1-\lambda)}{\lambda} - \beta_H(1-\lambda), \\ p_L &= \frac{1-\theta(2-\theta-\lambda)}{\lambda} - \beta_H(1-\lambda), \pi = \frac{1-\theta(2-\theta)(1-\lambda)}{2\lambda} - \beta_H(1-\lambda). \end{aligned} \tag{A.15}$$

Scenario 2.2 When $\beta_L \leq \beta_{L2}, 0 < \beta_H \leq \beta_{H6} \triangleq \frac{1-\theta-\beta_L\lambda}{\lambda(1-\lambda)}$, the equilibrium quality, price, and the profit of the firm can be derived as

$$\begin{aligned} q_H &= \frac{\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, \ q_L = \frac{\beta_L\lambda}{1-\theta}, p_H = \frac{\theta\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, \ p_L = \frac{\beta_L\lambda}{1-\theta} \\ \pi &= \frac{\lambda(\beta_L(2-2\theta-\beta_L\lambda) - \beta_H^2(1-\lambda)^2 - 2\beta_H(1-\lambda)(\beta_L\lambda - \theta(1-\theta)))}{2(1-\theta)^2}. \end{aligned} \tag{A.16}$$

Scenario 2.3 When ($\beta_L \leq \beta_{L2}, \beta_H \geq \beta_{H6}$) or ($\beta_L > \beta_{L2}, \beta_H \geq \beta_{H5}$), the equilibrium quality, price, and the profit of the firm can be derived as

$$
\begin{aligned}
&q_H = 1 + \frac{\beta_H(1-\lambda)^2}{1-\theta}, q_L = 1 - \frac{\beta_H\lambda(1-\lambda)}{1-\theta}, p_H = 1 + \frac{\beta_H(1-\lambda)(\theta-\lambda)}{1-\theta},\\
&p_L = 1 - \frac{\beta_H\lambda(1-\lambda)}{1-\theta}, \pi = \frac{1}{2}(1 - \beta_H\lambda(1-\lambda)(2 + \frac{\beta_H(1-\lambda)^2}{(1-\theta)^2})).
\end{aligned}
\tag{A.17}
$$

Scenario 2.4 When $\beta_{L2} \le \beta_L < \beta_{L4}, \beta_H \le \beta_{H4}$, the equilibrium quality, price, and the profit of the firm can be derived as

$$
\begin{aligned}
&q_H = \frac{\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, q_L = \theta, p_H = \frac{\theta\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, p_L = \theta^2 + \beta_L\lambda, \\
&\pi = \frac{(1-\lambda)\theta^2(1-\theta)^2 - \lambda\beta_H^2(1-\lambda)^2 - 2\beta_H\lambda(1-\lambda)(\beta_L\lambda - \theta(1-\theta)) + 2\beta_L\lambda(1-\theta)(1-\theta(1-\lambda)) - \beta_L^2\lambda^3}{2(1-\theta)^2}.
\end{aligned}
\tag{A.18}
$$

Scenario 3: ($\mathrm{IR_H}$) and ($\mathrm{IR_L}$) are binding.

In this scenario, the optimal prices are

$$
p_H = q_H - \beta_H(1-\lambda), p_L = \beta_L\lambda + \theta q_L. \tag{A.19}
$$

For Scenario 3 to hold, we need to ensure that ($\mathrm{IC_H}$) and ($\mathrm{IC_L}$) are satisfied in equilibrium. Plugging (p_H, p_L) into ($\mathrm{IC_H}$) and ($\mathrm{IC_L}$), one can show that they are true if and only if $q_H \ge \frac{\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, q_L \le \frac{\beta_L\lambda}{1-\theta}$. Therefore, after substituting (p_H, p_L) into the firm's profit, the problem of the firm can formulated as follows:

$$
\begin{aligned}
\max_{q_H, q_L} \pi = -\frac{1}{2}\lambda q_H^2 - \frac{1}{2}(1-\lambda)q_L^2 + \lambda q_H + \theta q_L(1-\lambda) + \lambda(1-\lambda)(\beta_L - \beta_H)& \\
\text{s.t.} \qquad q_H \ge \frac{\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta},& \\
q_L \le \frac{\beta_L\lambda}{1-\theta}.&
\end{aligned}
\tag{A.20}
$$

It is easy to see that the optimization problem (A.20) is a concave programming problem. We also utilize the K-T condition to solve this optimization problem, the solving process is similar to Scenario 1, thus we omit it here.

The four equilibrium outcomes in Scenario 3 are as follows:

Scenario 3.1 When $\beta_{L2} \le \beta_L < \beta_{L3}, \beta_H \le \beta_{H3}$, the equilibrium quality, price, and the profit of the firm can be derived as

$$
q_H = 1, q_L = \theta, p_H = 1 - \beta_H(1-\lambda), p_L = \theta^2 + \beta_L\lambda, \pi = \frac{1}{2}(\lambda + (1-\lambda)\theta^2) + \lambda(1-\lambda)(\beta_L - \beta_H). \tag{A.21}
$$

Scenario 3.2 When $\beta_L \le \beta_{L2}, \beta_H \ge \beta_{H3}$, the equilibrium quality, price, and the profit of the firm can be derived as

$$
\begin{aligned}
&q_H = \frac{\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, q_L = \frac{\beta_L\lambda}{1-\theta}, p_H = \frac{\theta\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, p_L = \frac{\beta_L\lambda}{1-\theta},\\
&\pi = \frac{\lambda(\beta_L(2-2\theta-\beta_L\lambda) - \beta_H^2(1-\lambda)^2 - 2\beta_H(1-\lambda)(\beta_L\lambda - \theta(1-\theta)))}{2(1-\theta)^2}.
\end{aligned}
\tag{A.22}
$$

Scenario 3.3 When $\beta_L \le \beta_{L2}, \beta_H \le \beta_{H3}$, the equilibrium quality, price, and the profit of the firm can be derived as

$$
q_H = 1, q_L = \frac{\beta_L\lambda}{1-\theta}, p_H = 1 - \beta_H(1-\lambda), p_L = \frac{\beta_L\lambda}{1-\theta}, \pi = \frac{\beta_L\lambda(1-\lambda)(2-2\theta-\beta_L\lambda)}{2(1-\theta)^2} + \frac{\lambda}{2} - \beta_H\lambda(1-\lambda). \tag{A.23}
$$

Scenario 3.4 When ($\beta_{L2} \le \beta_L < \beta_{L3}, \beta_H \le \beta_{H3}$) or ($\beta_L > \beta_{L3}$), the equilibrium quality, price, and the profit of the firm can be derived as

$$q_H=\frac{\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, q_L=\theta, p_H=\frac{\theta\beta_H(1-\lambda)+\beta_L\lambda}{1-\theta}, p_L=\theta^2+\beta_L\lambda, \tag{A.24}$$
$$\pi=\frac{(1-\lambda)\theta^2(1-\theta)^2-\lambda\beta_H^2(1-\lambda)^2-2\beta_H\lambda(1-\lambda)(\beta_L\lambda-\theta(1-\theta))+2\beta_L\lambda(1-\theta)(1-\theta(1-\lambda))-\beta_L^2\lambda^3}{2(1-\theta)^2}.$$

We now refine the equilibria for the twelve scenarios discussed above. Notice that Scenarios 1.2, 2.2, and 3.2 share the same equilibrium outcome, as do Scenarios 1.4 and 3.3, as well as Scenarios 2.4 and 3.4. Thus, we can consolidate the twelve equilibrium outcomes into eight equilibria, as detailed in Table 2. Notably, we denote the case where $\beta_L \leq \beta_{L1}, \beta_H \leq \beta_{H1}$ as Case 1, the case where $\beta_L \leq \beta_{L1}$, $\beta_{H1} < \beta_H < \beta_{H2}$ as Case 2, the case where $\beta_{L1} < \beta_L \leq \beta_{L2}$, $\beta_H \leq \beta_{H3}$ as Case 3, the case where $\beta_L < \beta_{L2}$, $\max\{\beta_{H2}, \beta_{H3}\} + \leq \beta_H \leq \beta_{H6}$ as Case 4, the case where $\beta_{L2} \leq \beta_L < \beta_{L3}$, $\beta_H \leq \beta_{H3}$ as Case 5, the case where $\beta_{L2} < \beta_L < \beta_{L4}$, $\beta_{H3} < \beta_H < \beta_{H4}$ as Case 6, the case where $\beta_{H4} \leq \beta_H \leq \beta_{H5}$ as Case 7, and the case where $\beta_H > \max\{\beta_{H5}, \beta_{H6}\}$ as Case 8. □

Proof of Proposition 1. With social comparison, we compute the quality deviation compared to the first-best quality in 8 cases, where the quality distortion of the high-end product is denoted by $\Delta q_{Hk} \triangleq q_{Hk} - q_H^*$ (the subscript i denotes the Case i), and the quality distortion of the low-end product is denoted by $\Delta q_{Lk} \triangleq q_L^* - q_{Lk}$.

$$\begin{aligned}
&\Delta q_{H1}=0, \Delta q_{L1}=\frac{\lambda(1-\theta)}{1-\lambda}>0, \Delta q_{H2}=\frac{\beta_H(1-\lambda)^2}{1-\theta}-1+\theta>0, \Delta q_{L2}=\frac{\beta_H\lambda(1-\lambda)}{1-\theta}>0,\\
&\Delta q_{H3}=0, \Delta q_{L3}=\theta-\frac{\beta_L\lambda}{1-\theta}>0, \Delta q_{H4}=\frac{\beta_L\lambda+\beta_H(1-\lambda)-1+\theta}{1-\theta}>0, \Delta q_{L4}=\theta-\frac{\beta_L\lambda}{1-\theta}>0,\\
&\Delta q_{H5}=0, \Delta q_{L5}=0, \Delta q_{H6}=\frac{\beta_L\lambda+\beta_H(1-\lambda)-1+\theta}{1-\theta}>0, \Delta q_{L6}=0, \Delta q_{H7}=\frac{(1-\theta)(1-\lambda)}{\lambda}>0,\\
&\Delta q_{L7}=0, \Delta q_{H8}=\frac{\beta_H(1-\lambda)^2}{1-\theta}>0, \Delta q_{L8}=\frac{\beta_H\lambda(1-\lambda)}{1-\theta}-1+\theta>0.
\end{aligned} \tag{A.25}$$

By taking the partial derivatives of the quality distortions of the two types of products with respect to β_L and β_H, we can get

$$\begin{aligned}
&\frac{\partial\Delta q_{L1}}{\partial\beta_L}=0, \frac{\partial\Delta q_{L1}}{\partial\beta_H}=0, \frac{\partial\Delta q_{H1}}{\partial\beta_L}=0, \frac{\partial\Delta q_{H1}}{\partial\beta_H}=0,\\
&\frac{\partial\Delta q_{L2}}{\partial\beta_L}=0, \frac{\partial\Delta q_{L2}}{\partial\beta_H}=\frac{\lambda(1-\lambda)}{1-\theta}>0, \frac{\partial\Delta q_{H2}}{\partial\beta_L}=0, \frac{\partial\Delta q_{H2}}{\partial\beta_H}=\frac{(1-\lambda)^2}{1-\theta}>0,\\
&\frac{\partial\Delta q_{L3}}{\partial\beta_L}=-\frac{\lambda}{1-\theta}<0, \frac{\partial\Delta q_{L3}}{\partial\beta_H}=0, \frac{\partial\Delta q_{H3}}{\partial\beta_L}=0, \frac{\partial\Delta q_{H3}}{\partial\beta_H}=0,\\
&\frac{\partial\Delta q_{L4}}{\partial\beta_L}=-\frac{\lambda}{1-\theta}<0, \frac{\partial\Delta q_{L4}}{\partial\beta_H}=0, \frac{\partial\Delta q_{H4}}{\partial\beta_L}=\frac{\lambda}{1-\theta}>0, \frac{\partial\Delta q_{H4}}{\partial\beta_H}=\frac{1-\lambda}{1-\theta}>0,\\
&\frac{\partial\Delta q_{L5}}{\partial\beta_L}=0, \frac{\partial\Delta q_{L5}}{\partial\beta_H}=0, \frac{\partial\Delta q_{H5}}{\partial\beta_L}=0, \frac{\partial\Delta q_{H5}}{\partial\beta_H}=0,\\
&\frac{\partial\Delta q_{L6}}{\partial\beta_L}=0, \frac{\partial\Delta q_{L6}}{\partial\beta_H}=0, \frac{\partial\Delta q_{H6}}{\partial\beta_L}=\frac{\lambda}{1-\theta}>0, \frac{\partial\Delta q_{H6}}{\partial\beta_H}=\frac{1-\lambda}{1-\theta}>0,\\
&\frac{\partial\Delta q_{L7}}{\partial\beta_L}=0, \frac{\partial\Delta q_{L7}}{\partial\beta_H}=0, \frac{\partial\Delta q_{H7}}{\partial\beta_L}=0, \frac{\partial\Delta q_{H7}}{\partial\beta_H}=0,\\
&\frac{\partial\Delta q_{L8}}{\partial\beta_L}=0, \frac{\partial\Delta q_{L8}}{\partial\beta_H}=\frac{\lambda(1-\lambda)}{1-\theta}>0, \frac{\partial\Delta q_{H8}}{\partial\beta_L}=0, \frac{\partial\Delta q_{H8}}{\partial\beta_H}=\frac{(1-\lambda)^2}{1-\theta}>0.
\end{aligned} \tag{A.26}$$

□

Proof of Corollary 1. The subsequent tables depict the distortion of product quality concerning levels of conformity and exclusivity. From these tables, the following insights can be obtained: (1) When the level of exclusivity is low (i.e., $\beta_H \leq \frac{(1-\theta)^2}{1-\lambda}$), there are five optimal offerings: Cases 1, 3, 5, 6, 7. Among these cases, low levels of conformity result in a downward distortion of the quality of low-end items (i.e., Cases 1, 3), while high conformity levels lead to an upward distortion in the quality of high-end products (i.e., Cases 6, 7). Otherwise, product quality remains unchanged (i.e., Case 5). (2) When the level of exclusivity is high (i.e., $\beta_H \geq \max\{\beta_{H1}, \beta_{H5}\}$), there exists the remaining three optimal offerings: Cases 2, 4, 8. In these instances, the quality of high-end products is distorted upward, and that of low-end products is distorted downward. (3) When the level of exclusivity is moderate (i.e., $\frac{(1-\theta)^2}{1-\lambda} < \beta_H < \max\{\beta_{H1}, \beta_{H5}\}$), the optimal offerings and quality distortions represent a mixture of the aforementioned cases: (Cases 1, 3, 4, 6, 7 when $\frac{(1-\theta)^2}{1-\lambda} < \beta_H \leq$

$\min\{\beta_{H1}, \beta_{H5}\}$), (Cases 2, 4, 6, 7 when $\beta_{H1} < \beta_H < \beta_{H5}$), (Cases 1, 3, 4, 8 when $\beta_{H5} < \beta_H < \beta_{H1}$).

Table A.1: Distortion of quality when β_H is small or large

β_H	$\beta_H \le \frac{(1-\theta)^2}{1-\lambda}$			$\beta_H \ge \max\{\beta_{H1}, \beta_{H5}\}$
β_L	$\beta_L \le \beta_{L2}$	$\beta_{L2} < \beta_L < \beta_{L3}$	$\beta_L \ge \beta_{L3}$	$0 < \beta_L < 1$
q_H	$-$	$-$	$\uparrow$	$\uparrow$
q_L	$\downarrow$	$-$	$-$	$\downarrow$

Note: "$\uparrow$" Upward distortion; "$\downarrow$" Downward distortion; "$-$" No distortion.

Table A.2: Distortion of quality when β_H is moderate

β_H	$\frac{(1-\theta)^2}{1-\lambda} < \beta_H \le \min\{\beta_{H1}, \beta_{H5}\}$			$\min\{\beta_{H1}, \beta_{H5}\} < \beta_H < \max\{\beta_{H1}, \beta_{H5}\}$			
β_L	$\beta_L \le \beta_{L5}$	$\beta_{L5} < \beta_L < \beta_{L2}$	$\beta_L \ge \beta_{L2}$	$\beta_L \le \beta_{L2}$	$\beta_L > \beta_{L2}$	$\beta_L \le \beta_{L5}$	$\beta_L > \beta_{L5}$
q_H	$-$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$-$	$\uparrow$
q_L	$\downarrow$	$\downarrow$	$-$	$\downarrow$	$-$	$\downarrow$	$\downarrow$

Note: $\beta_{L5} \triangleq \frac{1-\theta-(1-\lambda)\beta_H}{\lambda}$.

$\square$

Proof of Corollary 2. The equilibrium outcome observed in Case 5 aligns consistently with the outcome established under complete information as demonstrated in Lemma 1. $\square$

Proof of Proposition 2. Compared to the first-best outcome outlined in Lemma 1, we compute the price deviation in 8 cases, where the price distortion of the high-end product is denoted by $\Delta p_{Hk} \triangleq p_{Hk} - p_H^*$, and the price distortion of the low-end product is denoted by $\Delta p_{Lk} \triangleq p_{Lk} - p_L^*$.

$$\begin{aligned}
&\Delta p_{H1} = \beta_L\lambda - \frac{(1-\theta)(\theta-\lambda)}{1-\lambda} < 0, \Delta p_{L1} = -\frac{\lambda\theta(1-\theta)}{1-\lambda} < 0, \Delta p_{H2} = \theta^2 + \frac{\beta_H(1-\lambda)(1-\theta\lambda)}{1-\theta} + \beta_L\lambda - 1 > 0,\\
&\Delta p_{L2} = -\frac{\beta_H\theta\lambda(1-\lambda)}{1-\theta} < 0, \Delta p_{H3} = 0, \Delta p_{L3} = -\theta\left(\theta - \frac{\beta_L\lambda}{1-\theta}\right) < 0, \Delta p_{H4} = \frac{\beta_L\lambda + \beta_H(1-\lambda)-(1-\theta)}{1-\theta} > 0,\\
&\Delta p_{L4} = -\theta(\theta - \frac{\beta_L\lambda}{1-\theta}) < 0, \Delta p_{H5} = 0, \Delta p_{L5} = 0, \Delta p_{H6} = \frac{\beta_L\lambda + \beta_H(1-\lambda)-(1-\theta)}{1-\theta} > 0, \Delta p_{L6} = 0,\\
&\Delta p_{H7} = \frac{(1-\theta)(1-\lambda)}{\lambda} > 0, \Delta p_{L7} = \frac{(1-\theta)(1-\theta(1-\lambda))}{\lambda} - \beta_H(1-\lambda) - \beta_L\lambda < 0,\\
&\Delta p_{H8} = \frac{\beta_H(1-\lambda)^2}{1-\theta} > 0, \Delta p_{L8} = 1-\theta^2 - \frac{\beta_H\lambda(1-\lambda)}{1-\theta} - \beta_L\lambda < 0.
\end{aligned} \tag{A.27}$$

$\square$

Proof of Lemma 3. By substituting the equilibrium quality and price into the firm's profit, we can obtain the equilibrium profit as shown in Table 3. $\square$

Proof of Proposition 3. By taking the partial derivatives of the profit of the firm with respect to θ, we can get

$$\begin{aligned}
&\frac{\partial\pi_1}{\partial\theta} = \frac{\theta-\lambda}{1-\lambda} > 0, \frac{\partial\pi_3}{\partial\theta} = \frac{\beta_L\lambda(1-\lambda)(1-\theta-\beta_L\lambda)}{(1-\theta)^3} > 0, \frac{\partial\pi_5}{\partial\theta} = \theta(1-\lambda) > 0,\\
&\frac{\partial\pi_2}{\partial\theta} = \theta - \frac{\lambda\beta_H^2(1-\lambda)^3}{(1-\theta)^3} \begin{cases} > 0, & \theta < \theta_1 \\ < 0, & \theta > \theta_1 \end{cases},\\
&\frac{\partial\pi_4}{\partial\theta} = \frac{\lambda\left(\beta_H(1-\lambda)(1-\theta-2\beta_L\lambda) + \beta_L(1-\theta-\beta_L\lambda) - \beta_H^2(1-\lambda)^2\right)}{(1-\theta)^3} \begin{cases} > 0, & \text{if } \theta < \theta_2 \\ < 0, & \text{if } \theta > \theta_2 \end{cases},\\
&\frac{\partial\pi_6}{\partial\theta} = \theta(1-\lambda) + \frac{\lambda(\beta_L\lambda + \beta_H(1-\lambda))(1-\theta-\beta_H(1-\lambda)-\beta_L\lambda)}{(1-\theta)^3} \begin{cases} > 0, & \text{if } \theta < \theta_3 \\ < 0, & \text{if } \theta > \theta_3 \end{cases},\\
&\frac{\partial\pi_7}{\partial\theta} = -\frac{(1-\theta)(1-\lambda)}{\lambda} < 0, \frac{\partial\pi_8}{\partial\theta} = -\frac{\lambda\beta_H^2(1-\lambda)^3}{(1-\theta)^3} < 0.
\end{aligned} \tag{A.28}$$

Where $\theta_1 \triangleq \text{Root}[\theta^4 - 3\theta^3 + 3\theta^2 - \theta + \lambda\beta_H^2(1-\lambda)^3, 2]$, $\theta_2 \triangleq \frac{\beta_H^2(1-\lambda)^2 - \beta_H(1-\lambda)(1-2\beta_L\lambda) - \beta_L(1-\lambda\beta_L)}{-\beta_H(1-\lambda)-\beta_L}$, $\theta_3 \triangleq$

Root$[(1-\lambda)(-\theta^4+3\theta^3-3\theta^2)+\theta\left(1-\beta_H\lambda(1-\lambda)-\beta_L\lambda^2\right)+\lambda\left(\beta_H(1-\lambda)+\beta_L\lambda\right)\left(1-\beta_H(1-\lambda)-\beta_L\lambda\right),2]$.[9] Therefore,

$$\hat{\theta}=\begin{cases}\theta_1, & \text{if } \beta_L\leq\beta_{L1},\ \beta_{H1}<\beta_H<\beta_{H2},\ \text{i.e., Case 2;}\\ \theta_2, & \text{if } \beta_L<\beta_{L2},\ \max\{\beta_{H2},\beta_{H3}\}\leq\beta_H\leq\beta_{H6},\ \text{i.e., Case 4;}\\ \theta_3, & \text{if } \beta_{L2}<\beta_L<\beta_{L4},\ \beta_{H3}<\beta_H<\beta_{H4},\ \text{i.e., Case 6.}\end{cases}$$

□

Proof of Proposition 4. In equilibrium, the surplus for high-type consumers can be expressed as $cs_{Hk}=\lambda(q_{Hk}-p_{Hk}-\beta_H(1-\lambda))$, the surplus for low-type consumers is represented by $cs_{Lk}=(1-\lambda)(\theta q_{Lk}-p_{Lk}+\beta_L\lambda)$. By computing the total surplus $cs_k=cs_{Hk}+cs_{Lk}$, we derive the following outcomes:

$$\begin{aligned}&cs_1=\frac{\lambda(1-\theta)(\theta-\lambda)}{1-\lambda}-\beta_L\lambda^2, cs_2=\lambda\theta(1-\theta)+\lambda^2(\beta_L-\beta_H(1-\lambda)), cs_{3\sim6}=0,\\&cs_7=(1-\lambda)(\beta_L\lambda+\beta_H(1-\lambda))-\frac{(1-\theta)(1-\lambda)(1-\theta(1-\lambda))}{\lambda},\\&cs_8=\lambda(1-\lambda)(\beta_L+\beta_H(1-\lambda))-(1-\theta)(1-\lambda).\end{aligned}\tag{A.29}$$

Given that $sw_k=\pi_k+cs_k$, we can obtain the social welfare as detailed in Table 4. □

Proof of Proposition 5. By taking the partial derivatives of consumer surplus cs_k with respect to β_H, β_L, we can get

$$\begin{aligned}&\frac{\partial cs_1}{\partial\beta_H}=0,\frac{\partial cs_1}{\partial\beta_L}=-\lambda^2<0,\frac{\partial cs_2}{\partial\beta_H}=-(1-\lambda)\lambda^2<0,\frac{\partial cs_2}{\partial\beta_L}=-\lambda^2<0,\\&\frac{\partial cs_3}{\partial\beta_H}=0,\frac{\partial cs_3}{\partial\beta_L}=0,\frac{\partial cs_4}{\partial\beta_H}=0,\frac{\partial cs_4}{\partial\beta_L}=0,\frac{\partial cs_5}{\partial\beta_H}=0,\frac{\partial cs_5}{\partial\beta_L}=0,\frac{\partial cs_6}{\partial\beta_H}=0,\frac{\partial cs_6}{\partial\beta_L}=0,\\&\frac{\partial cs_7}{\partial\beta_H}=(1-\lambda)^2>0,\frac{\partial cs_7}{\partial\beta_L}=\lambda(1-\lambda)^2>0,\frac{\partial cs_8}{\partial\beta_H}=\lambda(1-\lambda)>0,\frac{\partial cs_8}{\partial\beta_L}=\lambda(1-\lambda)>0.\end{aligned}\tag{A.30}$$

□

Proof of Corollary 3. By taking the partial derivatives of consumer surplus cs_k with respect to θ, we have

$$\begin{aligned}&\frac{\partial cs_1}{\partial\theta}=\frac{\lambda(1+\lambda-2\theta)}{1-\lambda}\begin{cases}>0, & \text{if } \theta<\frac{1+\lambda}{2},\\<0, & \text{if } \theta>\frac{1+\lambda}{2}.\end{cases},\frac{\partial cs_2}{\partial\theta}=\lambda(1-2\theta)\begin{cases}>0, & \text{if } \theta<\frac{1}{2},\\<0, & \text{if } \theta>\frac{1}{2}.\end{cases},\\&\frac{\partial cs_7}{\partial\theta}=\frac{(1-\lambda)(2-\lambda-2\theta(1-\lambda))}{\lambda}>0,\frac{\partial cs_8}{\partial\theta}=1-\lambda>0.\end{aligned}\tag{A.31}$$

□

Proof of Corollary 4. By taking the partial derivatives of consumer surplus sw_k with respect to β_H, β_L, we obtain

$$\begin{aligned}&\frac{\partial sw_1}{\partial\beta_H}=-\lambda(1-\lambda)<0,\frac{\partial sw_1}{\partial\beta_L}=\lambda(1-\lambda)>0,\frac{\partial sw_2}{\partial\beta_H}=-\lambda(1-\lambda)(\lambda+\frac{(1-\lambda)^2\beta_H}{(1-\theta)^2})<0,\frac{\partial sw_2}{\partial\beta_L}=\lambda(1-\lambda)>0,\\&\frac{\partial sw_3}{\partial\beta_H}=-\lambda(1-\lambda)<0,\frac{\partial sw_3}{\partial\beta_L}=\frac{\lambda(1-\lambda)\left(1-\theta-\beta_L\lambda\right)}{(1-\theta)^2}>0,\frac{\partial sw_4}{\partial\beta_H}=-\frac{\lambda(1-\lambda)\left(\beta_H(1-\lambda)+\beta_L\lambda-\theta(1-\theta)\right)}{(1-\theta)^2}<0,\\&\frac{\partial sw_4}{\partial\beta_L}=\frac{\lambda\left(1-\theta-\beta_H\lambda(1-\lambda)-\beta_L\lambda\right)}{(1-\theta)^2}>0,\frac{\partial sw_5}{\partial\beta_H}=-\lambda(1-\lambda)<0,\frac{\partial sw_5}{\partial\beta_L}=\lambda(1-\lambda)>0,\\&\frac{\partial sw_6}{\partial\beta_H}=-\frac{\lambda(1-\lambda)\left(\beta_H(1-\lambda)+\beta_L\lambda-\theta(1-\theta)\right)}{(1-\theta)^2}<0,\frac{\partial sw_6}{\partial\beta_L}=\frac{\lambda\left((1-\theta)(1-\theta(1-\lambda))-\beta_H\lambda(1-\lambda)-\beta_L\lambda^2\right)}{(1-\theta)^2}>0,\\&\frac{\partial sw_7}{\partial\beta_H}=-\lambda(1-\lambda)<0,\frac{\partial sw_7}{\partial\beta_L}=\lambda(1-\lambda)>0,\frac{\partial sw_8}{\partial\beta_H}=-\lambda(1-\lambda)(\lambda+\frac{(1-\lambda)^2\beta_H}{(1-\theta)^2})<0,\frac{\partial sw_8}{\partial\beta_L}=\lambda(1-\lambda)>0.\end{aligned}\tag{A.32}$$

□

Proof of Corollary 5. Compared to the first-best outcome under complete information, we can prove that $\pi_5=\pi^*$, $sw_5=sw^*$, $\pi_{1,2,3,4,6,7,8}<\pi^*$, and $sw_{1,2,3,4,6,7,8}<sw^*$. □

Proof of Proposition 6. Through a comparative analysis of a firm's profit derived from producing a single product π_{H0}, with the optimal profits resulting from producing two different products $\pi_i, i\in\{1,8\}$, we establish that $\pi_{H0}<\pi_i$ when $\beta_H<\hat{\beta}_H$. Conversely, when $\beta_H>\hat{\beta}_H$, $\pi_{H0}>\pi_i$. The specific value of $\hat{\beta}_H$ is detailed in the following table.

[9] Root$[f,k]$ represents the exact k^{th} root of the polynomial equation $f[x]=0$.

Table A.3: The specific value of $\hat{\beta}_H$

Case	$\hat{\beta}_H$
1	$\frac{(\theta-\lambda)^2+2\beta_L\lambda(1-\lambda)}{2\lambda(1-\lambda)^2}$
2	$\frac{(1-\theta)\sqrt{\lambda(1-\lambda)(\theta^2-\lambda+2\beta_L\lambda)}}{\lambda(1-\lambda)^2}$
3	$\frac{\beta_L(2-2\theta-\beta_L\lambda)}{2(1-\theta)^2}$
4	$\frac{\theta(1-\theta)-\lambda\beta_L+\sqrt{2\beta_L(1-\theta)(1-\theta\lambda)-\lambda(1-\lambda)\beta_L^2-((1-\theta^2)(1-\theta)^2)}}{1-\lambda}$
5	$\frac{1-\theta-\beta_L\lambda}{1-\lambda}$
6	$\frac{\lambda\theta(1-\theta)+(1-\theta)\sqrt{\lambda(\theta^2-\lambda+2\beta_L\lambda)}-\beta_L\lambda^2}{\lambda(1-\lambda)}$
7	$\frac{\lambda+(1-\theta)^2}{2\lambda}$
8	$\frac{(1-\theta)\sqrt{\lambda(1-\lambda(1-\lambda+\theta(2-\theta)))}-\lambda(1-\theta)^2}{\lambda(1-\lambda)^2}$

□

Proof of Corollary 6. By taking the partial derivatives of product quality with respect to β_H and β_L in both single-product and dual-product scenarios, we get the effects of social comparison on product's quality, as shown in Table 5. □