AERO70003 - Advanced Propulsion

Group 2 Team Project: Numerical Ramjet Engine

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Abstract

This report assesses the variation of thermal and propulsive efficiency of a ramjet engine across a wide range of different operating conditions. The analysis is based on a code developed in MATLAB, the accuracy of which is also examined. When comparing the code to reference calculations, the results are all within 8.47% error. However, for a certain range of inputs, the code gives efficiencies that are greater than 1, negative or complex. These solutions either correspond to physically impossible solutions or solutions where the engine no longer acts as an engine. The reason for these results is explained along with their physical implications.

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1 Definitions

Table 1 includes the nomenclature for inputs, outputs variables and corresponding terminology used in the code by referring to the schematic ramjet engine with station labels as shown in Figure 1. Table 2 represents a set of reasonable base inputs used to run the function. They are taken from the example in Lecture 3 [1]. The thrust was assumed to be 50 kN, an estimate taken from historical data [2].

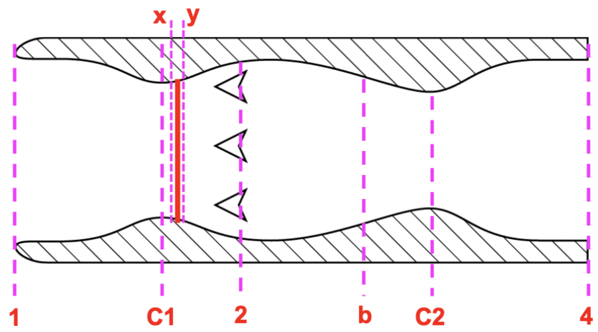

Figure 1: Schematic of a ramjet engine with station labels[3]. (1) Intake (freestream). (C1) Inlet throat. (x) Just before the shock. (y) Just after the shock. (2) Beginning of burner. (b) End of burner (burn complete). (C2) Nozzle throat. (4) Engine exhaust.

Table 1: Nomenclature of inputs and outputs.

	Variable	Report Term	MATLAB Term
Inputs	Free-stream pressure	P_1	`P_FS`
	Free-stream temperature	T_1	`T_FS`
	Flight Mach number	M_1	`M`
	Normal Shock Strength	M_x	`S_shock`
	Burner entry Mach number	M_2	`M_burner`
	Burner temperature	T_2	`T_burner`
	Burner pressure ratio	$\frac{P_b}{P_2}$	`ratio_Pb`
	Exhaust pressure ratio	$\frac{P_4}{P_1}$	`ratio_Pe`
	Required Thrust	T	`T`
Outputs	Inlet area	A_1	`A1`
	Inlet throat area	A_{C1}	`AC1`
	Burner entry area	A_2	`A2`
	Burner exit area	A_b	`Ab`
	Nozzle throat area	A_{C2}	`AC2`
	Exhaust area	A_4	`A4`
	Thermodynamic efficiency	η_{th}	`eta_th`
	Propulsive efficiency	η_p	`eta_p`

Table 2: Values of Inputs

Variable	Report Term	Values
Free-stream pressure	P_1	70 kPa
Free-stream temperature	T_1	210 K
Flight Mach number	M_1	2.8
Normal Shock Strength	M_x	1.1
Burner entry Mach number	M_2	0.2
Burner temperature	T_2	1700 K
Burner pressure ratio	$\frac{P_b}{P_2}$	1
Exhaust pressure ratio	$\frac{P_4}{P_1}$	1
Required Thrust	T	50 kN

1.1 Equations

These are the equations that will be referenced in order to explain certain features in the graphs.

$$\eta_p = \frac{F}{P_1 A_1} \times \frac{2T}{\gamma(T_4 M_4^2 \frac{P4}{P1} - T_1 M_1^2)} \tag{1}$$

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_b - T_2} \tag{2}$$

$$M_b = \frac{1}{2}\sqrt{\frac{T_2}{T_b}}\left(M_2 + \frac{1}{\gamma M_2}\right) \pm \frac{1}{2}\sqrt{\frac{T_2}{T_b}\left(M_2 + \frac{1}{\gamma M_2}\right)^2 - \frac{4}{\gamma}} \tag{3}$$

Isentropic relations:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}M^2 \tag{4}$$

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\gamma/(\gamma-1)} \tag{5}$$

$$\frac{A}{A_t} = \frac{1}{M}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{6}$$

2 Assumptions

2.1 Global assumptions

1. Specific heat ratio $\gamma = 1.4$ and is constant. Values for heat capacities are also constant. This is the standard value for air at low temperatures, but this assumption is flawed once the air is mixed with fuel, especially when this is ignited and brought to high temperatures. Essentially, air is assumed to be an ideal gas when it is not. At 1500K, γ is 1.33, a 5% difference. This may not seem like much, but γ intervenes in almost every formula, so the compound effect makes the answers deviate significantly.
2. The fuel-to-air ratio is low, so the mass of fuel added is negligible. This mainly affects the accuracy of the thrust equation, which uses the momentum of the exhaust. It also goes hand in hand with assumption (1), where it is assumed the fuel does not change the properties of the air.
3. No friction in the boundary layer means expansions and compressions are isentropic (except across the normal shock). This assumption is quite bold, as, for such high Mach numbers, the boundary layer causes a lot of friction, which changes the properties of the flow. The boundary layer reduces the effective area, changing the flow velocity, and also creates heating. More broadly, the flow is assumed to be inviscid, so the development of boundary layers is not possible.
4. All heat is added between stations 2 and b, while the rest of the engine is a perfect insulator. Compared to the heating from the burner, the heating from the rest of the engine is indeed small. However, it is still an important consideration that the code is neglecting, especially for high Mach numbers. Essentially, the entire engine (apart from the burner) is assumed to be adiabatic. The transit time is short as the flow is at such high speed, so the time for heat to diffuse into the flow is very short.
5. No Mach waves nor separation occur. For this, the surfaces must be perfectly smooth. If the pressure gradient is exceedingly high, the flow is likely to separate, though this is not considered in the code.
6. Flow is steady. It is valid to assume the flow is steady, especially as the flow is supersonic. However, if a sudden change in conditions were to occur, this is not accounted for.
7. Assume 1-D Flow. Due to this assumption, this code only gives the areas required at the relevant stations. It does not describe the nozzle shapes which achieve uniform flow downstream and therefore does not account for potentially non-uniform flow. Too large of an angle could also result in the boundary layer inside the engine tripping, which is not accounted for under the 1-D assumption.

2.2 Inlet assumptions

1. The shock is always swallowed (at the throat). The optimal position of shock is right after the inlet throat because it minimises Mach number and thus stagnation pressure loss. Another solution exists in nature, where the shock is in front of the engine, though this is not of interest to the report's analysis, so it will be assumed that the shock is swallowed.
2. The width of the shock is thin and A_x and A_y are equal. This is a valid assumption because steady flow is considered, so the shock is regarded as a discontinuity in flow parameters.

2.3 Burner assumptions

1. The temperature rise from T_2 to T_b is assumed instantaneous because combustion occurs at very high speeds. This is a valid assumption, given the design is for Mach 2.8.
2. The pressure in the burner is assumed to stay constant (isobaric). Typically, the design is fine-tuned to make sure this is true, as it ensures a constant combustion rate. From experimental results, the pressure only drops 1 to 2%, so this is a reasonable assumption [4].
3. The dimensions of the fuel injectors are assumed to be small. Thus, its contribution to reducing the sectional area of the burner can be neglected. With a well-designed engine, this assumption is justifiable.
4. Fuel and Air are always perfectly mixed, leading to a complete and stable combustion. In reality, turbulence is necessary to mix the two, which can impact flow properties.
5. The compression and expansion efficiencies are unity, $\eta_c = 1$ and $\eta_e = 1$.
6. Assume the weak solution for burner Mach number in Eq. 3. This is the one that is naturally taken because the strong solution would lead to flow choking.

2.4 Nozzle assumptions

1. Similar to the other parts, the expansion to P_{exit} is assumed isentropic.

3 Source Code

```
% Ramjet Calculator Code
% Advanced Propulsion Group 2


%Initialising variables - example from Lecture 3
Free_stream_P = 70*10^3;
Free_stream_T = 210;
Flight_M = 2.8;
Shock_strength = 1.1;
Burner_entry_M = 0.2;
Burner_temperature = 1700;
Burner_pressure_ratio = 1;
Exhaust_pressure_ratio = 1;
Required_thrust = 50000;

%Plotting results

%Plotting one graph - efficiency as a function of Mach number - but other graphs are
    plotted in a similar manner
%Flight Mach Number
Flight_M = linspace(0,8,10000);
plotting_array = [];

for i = 1:length(Flight_M)
    %Call function
    [A1, AC1, A2, Ab, AC2, A4, eta_th, eta_p] = ramjet(Free_stream_P, Free_stream_T,
    Flight_M(i), Shock_strength, Burner_entry_M, Burner_temperature,
    Burner_pressure_ratio, Exhaust_pressure_ratio, Required_thrust);
    %Append to array
```

```
    plotting_array = [plotting_array; eta_th, eta_p];
end
hold off
plot(Flight_M, plotting_array(:,1), 'Linewidth',2)
hold on
plot(Flight_M, plotting_array(:,2), 'Linewidth',2)
plot(Flight_M, plotting_array(:,1).*plotting_array(:,2),'Linewidth',2)
grid on
ylabel('Efficiencies')
xlabel('Flight mach Number')
%Left and right red shaded region
f_left = fill([0,1.78,1.78,0],[0,0,1.3,1.3],'r','FaceAlpha',0.1,'LineStyle','none');
f_right = fill([8,6,6,8],[0,0,1.3,1.3],'r','FaceAlpha',0.1,'LineStyle','none');
%eta = 1 line
yline(1,'--');
axis([0 8 0 1.3])
legend('Thermodynamic Efficiency ', 'Propulsive Efficiency', 'Total Efficiency','','','');



%%
function [T0_T, P0_P, A_At] = isentropic_relation(M,gamma)
%Isentropic relations for stagnation values and throat area

   %Stagnation Temperature
   T0_T = 1+ ((gamma-1)/2) * (M^2) ;

   %Stagnation Pressure
   P0_P = (1+ ((gamma-1)/2)*(M^2)) ^(gamma/(gamma-1));

   %Throat area
   A_At = (1/M) * (((2/(gamma + 1)) * (1 + ((gamma-1)/2) * (M^2) ))^((gamma+1)/(2*(gamma-1))));

end




function [A1, AC1, A2, Ab, AC2, A4, eta_th, eta_p] = ramjet(P_FS, T_FS, M, S_shock, M_burner, T_burner, ratio_Pb, ratio_Pe, T)
   %Main function to return the areas at each station and the two
   %efficiencies

    %Global Parameters
    R = 287;
    gamma = 1.4; %Assume constant


    %Inlet - throat
    [T01_T1, P01_P1, A1_AC1] = isentropic_relation(M,gamma);

    %Find M after Shock as a function of shock strength
    M_y = sqrt(( (gamma-1) * S_shock^2 + 2) / (2*gamma*S_shock^2 - (gamma-1) ));

    %Ax over throat area - using shock strength
    [T0x_Tx, P0x_Px, Ax_AC1] = isentropic_relation(S_shock,gamma);

    %Ay over Ay star - using M after shock
    [T0y_Ty, P0y_Py, Ay_Ay_star] = isentropic_relation(M_y,gamma);

    %A2 over Ay star - using M at the entrance to the burner
    [T0y_T2, P02_P2, A2_Ay_star] = isentropic_relation(M_burner,gamma);

    %Combine to find A2 over A1 - where A2 is just before burner
    A2_A1 = A2_Ay_star * Ay_Ay_star^-1 * Ax_AC1 * A1_AC1^-1;


    %Burner Area


    %Find temperature ratio across shock: Ty/Tx using normal shock relation
```

```
    Ty_Tx = ((2*gamma*S_shock^2 - (gamma-1)) * (2+ (gamma-1) * S_shock^2)) / ((gamma+1)
    ^2 * S_shock^2);


    %Relate them all to find T2 / T1
    T2_T1 = T0y_T2^-1 * T0y_Ty *  Ty_Tx * T0x_Tx^-1 *T01_T1;
    T2 = T2_T1 * T_FS;


    %Find Mach number at the burner exit: Mb_exit
    %Note that the formula choses the (-) solution, corrsponding to the
    %weak shock solution, which is the one that natrually occurs.
    %hard-coding in the negative solution aloso makes it a continuous
    %function.
    Mb_exit = 1/2 * sqrt(T2/T_burner) * (M_burner + 1/(gamma * M_burner)) - 1/2 * sqrt(
    T2/T_burner * (M_burner + 1/ (gamma * M_burner))^2 - 4/gamma );


    %Use Mb and M2 and pressure ratio to find area ratio
    %A2 over Ab
    A2_Ab = (gamma*Mb_exit^2 +1) / (gamma * M_burner^2 +1) * ratio_Pb;

    %relate to A1
    Ab_A1 = A2_Ab^-1 * A2_A1;


    %Second throat area - AC2
    [T0b_Tb,P0b_Pb, Ab_AC2] = isentropic_relation(Mb_exit,gamma);
    %relate to A1
    AC2_A1 = Ab_AC2^-1 * Ab_A1;


    %Now, find Exit pressure to find exit mach number
    Py_Px = (2*gamma*S_shock^2 - (gamma-1)) / (gamma+1);


    %Combine for P04 over P4
    P04_P4 = ratio_Pe^-1 * P01_P1 * P0x_Px^-1 * Py_Px * P0y_Py * ratio_Pb * P02_P2^-1 *
    P0b_Pb;
    %This gives us the Mach number by rearranging the isentropic formula
    M_exit = sqrt(2/(gamma-1) * (( P04_P4 ) ^ ((gamma-1)/gamma) - 1));

    %Use mach number to relate to AC2 by isentropic throat formula
    [T04_T4, ~ , A4_AC2] = isentropic_relation(M_exit,gamma);
    %Relate to A1
    A4_A1 = A4_AC2 * AC2_A1;


    %Calculate A1 using thrust equation, accounting for pressure difference (P4/P1)
    A1 =  T / P_FS * ( gamma * M^2 * (M_exit^2 / M^2 * ratio_Pe * A4_A1 -1) + ratio_Pe *
     A4_A1 -1 )^-1;

    %Calculate numerical values of areas
    AC1 = A1_AC1^-1 * A1;
    A2 = A2_A1 * A1;
    Ab = Ab_A1 * A1;
    AC2 = AC2_A1 * A1;
    A4 = A4_A1 * A1;

    %Find Temperatures
    T0b = T0b_Tb * T_burner;
    T4 = T04_T4^-1 * T0b;

    %Efficiencies
    %1. Thermodynamic: can not use cycle formula as ideal Brayton
    %assumption is invalid
    eta_th = 1 - (T4 - T_FS) / (T_burner - T2);


    %2. Propulsive
    %Find T4/T1
    T4_T1 = T04_T4^-1 * T0b_Tb * T_burner / T_FS;
    %Eta_p uses thrust nromalised by inlet area, and accounts for the
```

```
    %pressures acting on A1 and A4
    %Normalised Thrust: F_norm = F/(P_FS*A1)
    F_norm = gamma * M^2 * ( ratio_Pe * A4_A1 * (M_exit / M)^2 -1);
    eta_p = ( F_norm * 2/gamma) * (1/(T4_T1 * ratio_Pe * M_exit^2 - M^2));


end
```

ramjet_final.m

4 Results and Discussions

4.1 Freestream Pressure

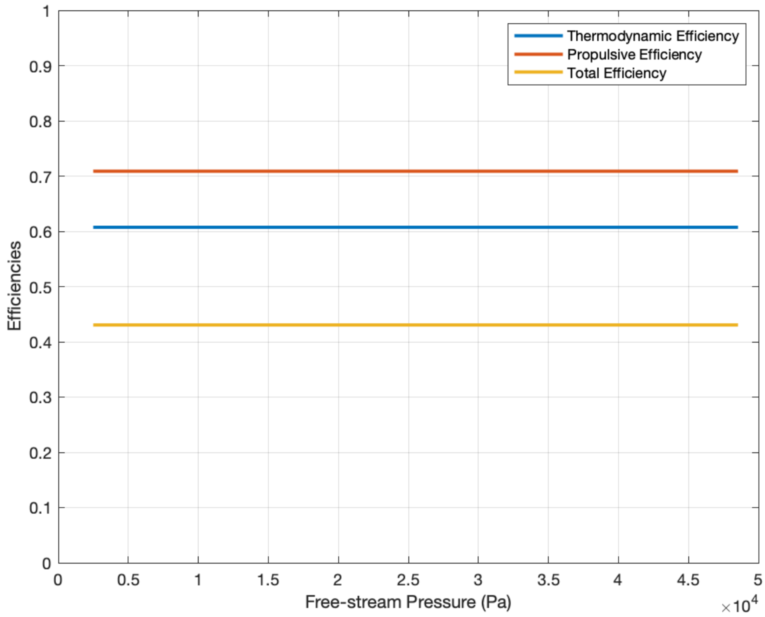

Figure 2: Variation in efficiencies with Freestream Pressure

Fig.2 shows that the free stream pressure does not impact the thermodynamic and propulsive efficiencies. This is expected for η_{th} because the work done on the air during the thermodynamic cycle and the heat added to the air during the combustion process are independent of P_1. Similarly, η_p depends primarily on the velocity of the exhaust gases exiting the engine, which is determined by the pressure ratios across the engine and the geometry of the nozzle rather than P_1.

4.2 Freestream Temperature

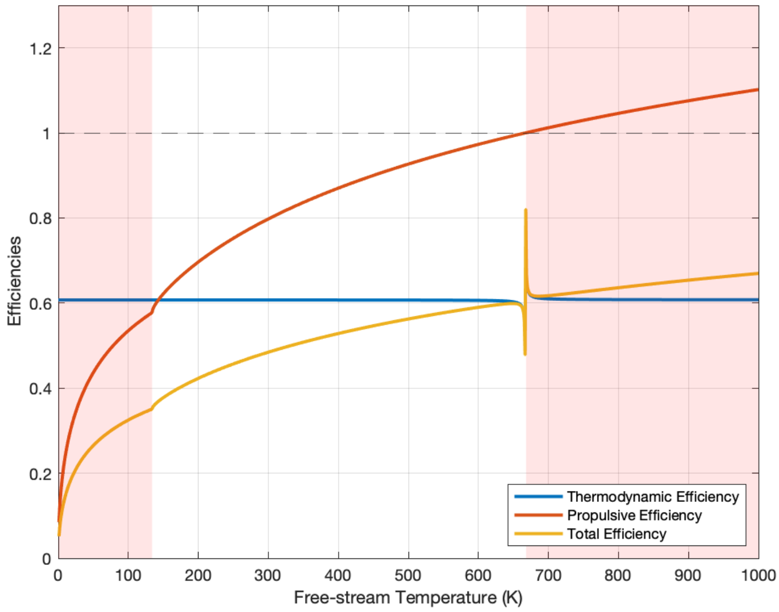

Figure 3: Variation in efficiencies with Freestream Temperature

Since T_4 is independent of T_1 and M_4 decreases as T_1 increases above 134 K, as seen in Fig.4, the denominator in the latter fraction in Eq.1 decreases with increasing T_1. Thus, η_p increases with increasing T_1. This is expected because the higher the T_1, the less heat needs to be added to achieve the desired temperature for combustion. As a result, fuel consumption and heat addition in the burner can be reduced. This decrease in heat addition to the burner results in less expansion of the gases in the combustion chamber, which subsequently leads to a decrease in the velocity of the exhaust gases. Since the jet momentum power is directly proportional to the velocity of exhaust gases, this leads to a decrease in jet momentum power, thus increasing η_p.

However, the code can no longer generate real solutions as T_1 decreases beyond a certain point. This is because the results of the equation within the square root in Eq.3 become negative, meaning that solutions for M_b are complex. The imaginary parts of the solutions are ignored to plot Fig.3 in MATLAB, resulting in a kink around 134 K as MATLAB tries to connect the non-real and real solutions. The reason why solutions for M_b are complex below a certain T_1 is because T_2 decreases linearly with decreasing T_1 because the stagnation temperature is conserved across a shock. Physically, this means that below the T_1 value of 134 K, the engine's inlet would be unable to compress the incoming air sufficiently to achieve the necessary temperature for sustained combustion to occur. Hence, a ramjet engine would not work below a certain T_1.

On the other hand, thermal efficiency (η_{th}) remains essentially constant for most of the range of free stream temperatures. It decreases by a total of 0.1% over the entire range apart from the discontinuity at 668 K, as seen in Fig.3 . At this free stream temperature (T_1), T_2 equals the set T_b value of 1700 K, resulting in zero heat addition in the burner. Thus, the fraction in Eq.2 tends to infinity. As a result, the non-linearity of Eq.2 becomes increasingly significant as T_2 approaches T_b, even though T_2 varies linearly with T_1 as shown in Fig.4. This yields the only relevant change in thermodynamic efficiency.

As T_1 continues to increase beyond this point, T_2 becomes greater than T_b, which means that heat is absorbed in the burner, physically making the engine a refrigerator. It can also be observed from Fig.3 that η_p beyond 668 K is greater than 1, which is not physically possible as it would imply that the engine is generating more thrust than the available momentum in the exhaust gases. Therefore, the red highlighted region beyond 668 K is invalid.

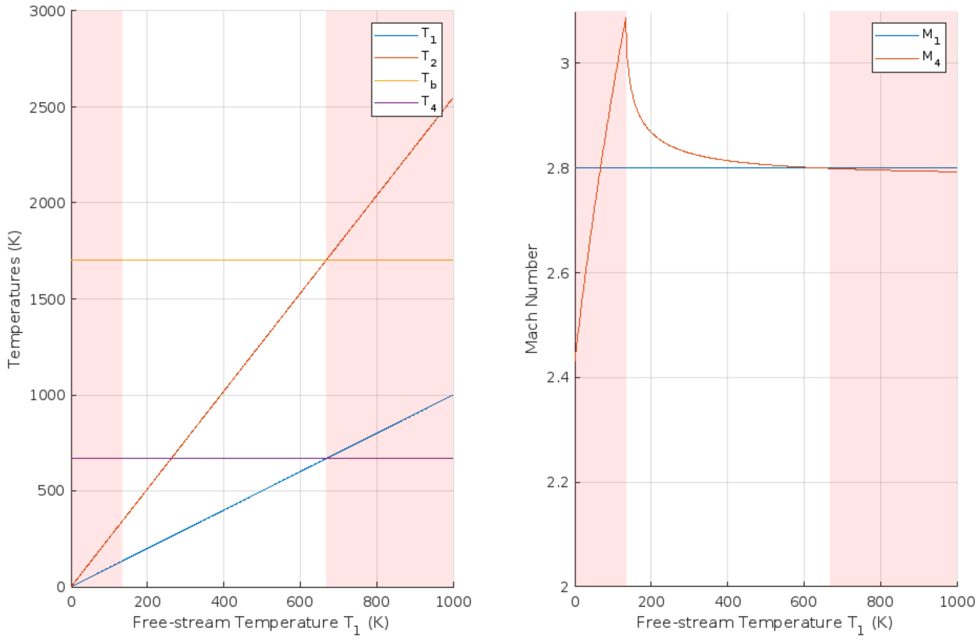

Figure 4: Left:Temperatures at internal stations with varying Freestream Temperature. Right: Mach numbers at the inlet and exhaust with varying Freestream Temperature

4.3 Flight Mach Number

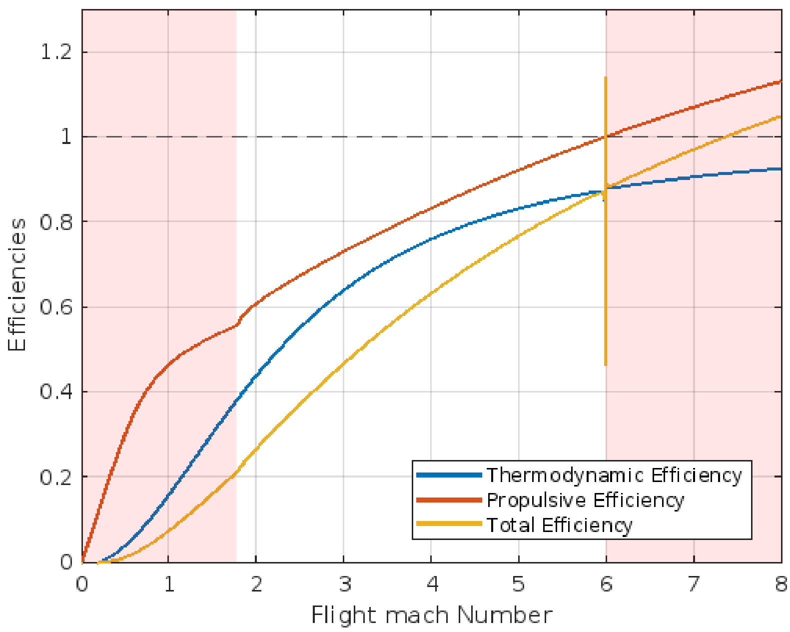

Figure 5: Variation in efficiencies with Flight Mach Number

As the free stream Mach number increases, stagnation temperature increases. Stagnation temperature remains constant over a shock, therefore resulting in an increase in burner entry temperature (T_2). This then results in a decrease in T_4 as seen in Fig.6. This is due to an increase in the ratio $\frac{T_{04}}{T_4}$ due to a higher exit Mach number while T_{04} which equals T_{0b} remains fairly constant. This decrease in T_4 then results in an increase in thermodynamic efficiency (η_{th}) according to Eq.2. This yields the overall relationship where an increase in free stream Mach number increases thermodynamic efficiency as seen in Fig.5.

Propulsive efficiency is defined by Eq.1. The variables which change and affect propulsive efficiency are shown in Fig.6. In Fig.6 Mach number squared is graphed for indications of magnitude in accordance with Eq.1. From Fig. 6 it can be seen that M_1^2 and M_4^2 have very similar values for most of the range of Mach numbers. On the other hand, the left-hand graph shows that T4 varies significantly with flight Mach number, so while M4 and M1 increase together, T4 and T1 are changing drastically. As propulsive efficiency depends inversely on T_4, this significant decrease in T_4 yields the increase in propulsive efficiency seen in fig.5.

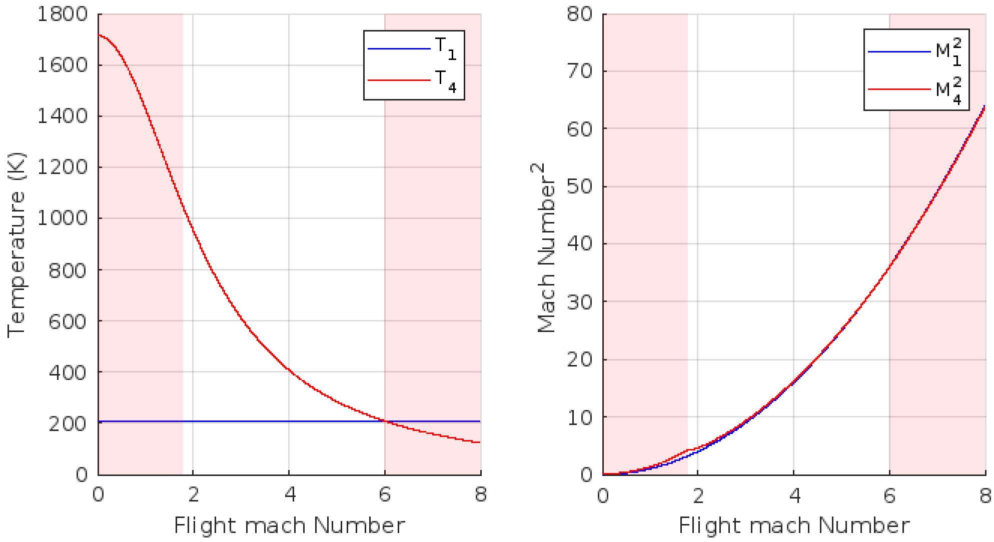

Figure 6: Change in Temperature and Mach number squared at stations 1 and 4 as Free Stream Mach number (M_1) changes.

While propulsive efficiency increases over the entire range of free stream Mach numbers shown in Fig.5, its behaviour changes in certain ranges shown in red. For $M_1 < 1.78$ the calculated values of propulsive efficiency are complex, resulting in the discontinuity seen at $M_1 = 1.78$. These complex roots result from the inside of the square root in Eq.3 being negative. This indicates that the solution is not physically possible. Physically, this is a property of ramjets that can not operate at low Mach numbers as they rely on the stagnation pressure of the free stream flow to complete the required compression. The results for propulsive and, therefore, total efficiency below $M_1 < 1.78$ are non-real and should be disregarded as the ramjet would not be able to operate (They are simply a plot of the real parts of the complex solution). This lines up with other sources suggesting ramjets are ineffective below a free stream velocity of approximately Mach 2 [5].

At $M_1 = 6$ there is a discontinuity in thermodynamic efficiency. This is caused by T_2 equalling T_b, therefore, resulting in the denominator of the fraction in Eq.2 equalling zero causing the discontinuity. Physically, this corresponds to the Mach number being sufficiently high such that the increase in stagnation temperature results in a burner temperature equal to the constant prescribed value of T_b. This means that at such a Mach number, no fuel is added in the engine - Simply slowing the flow down results in the maximum prescribed temperature at the end of the burner. This implies that at higher Mach numbers (region shown in red on the right), energy is removed from the engine to meet the prescribed burner temperature. Therefore, the modeled engine would no longer be an engine but a refrigerator and should not be considered a reasonable operating range. This is also shown by the fact that propulsive efficiency exceeds a value of one beyond this range.

These findings imply that both efficiencies of an engine are at their highest when operating at the highest possible free stream Mach number.

4.4 Normal Shock Strength

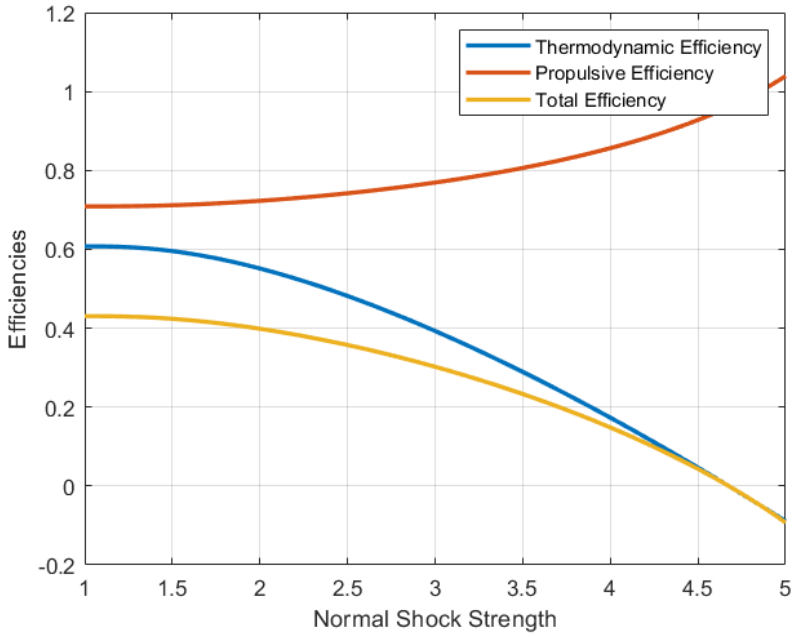

Figure 7: Variation in efficiencies with Normal Shock Strength

The normal shock strength is the Mach number of the shock near the throat of the ramjet. In theory, it is most efficient for the shock strength to equal 1, but in practicality, it is greater than 1, to ensure a stable system. This is so that the shock is not suddenly "spat out" the front, where the shock would now be in front of the engine, leading to high "fist of God" levels of drag. It is usually at around Mach 1.1, but for this graph, shown in Fig.7, it is varied from 1 to 5. In reality, it is likely to not be above 1.2, as then you would simply be wasting space and the design would be cumbersome. The graph shows that the overall efficiency decreases as normal shock strength increases. This negative relationship makes sense, as a higher Mach of the shock means a larger drop in stagnation pressure, meaning losses occur. The overall negative trend comes from a decreasing thermodynamic efficiency, which outweighs the increasing propulsive efficiency. Looking at the terms in the thermal efficiency equation, it is seen that free-stream temperature, burner temperature, and burner inlet temperature are all independent of shock strength (burner inlet temperature is fixed by the prescribed M at burner entry). As such, the only change is in T4. As the shock strength increases, the pressure and temperature jumps increase. This carries through and then is exhausted out at a higher temperature. Because of the negative dependency of thermal efficiency of T4, it goes down. In other words, T_4 is higher so the rejected heat is higher.

That said, the reason for the increase in propulsive efficiency is less clear. As mentioned, a higher shock strength means higher temperature and pressure in the burner. The ratio of $\frac{P04}{P4}$ is increased, which the exit Mach number depends negatively on, as seen by rearranging the isentropic pressure formula in Eq.5. Seen as the propulsive efficiency has an inverse square correlation to the exit Mach number, it goes up. Propulsive efficiency also has an inverse relationship with T4, which, as mentioned earlier, increases. The square relationship is enough to outweigh the linear relationship, and the propulsive efficiency increases.

When the shock strength is exceedingly high, the propulsive efficiency is supposedly above 1. At this strength, all the areas after the throat would have to be exponentially large, as seen in Fig.8. As the area expands and the shock strength increases, the flow is expanded and the pressure drops. Eventually, the pressure goes below the pressure after the shock and the strength of the shock is capped. As such, these incredibly large areas are unrealistic for a practical design, and achieving such shock strength is impossible in reality.

According to the graph, the ideal shock strength is at 1. However, for a stable system, it is desirable for it to be slightly above. That said, the normal shock also serves to mix the fuel and air mixture. If the strength is too low, this mixing will not be as effective, which can cause incomplete combustion and reduce efficiency. This is not considered by the code and is an area for improvement.

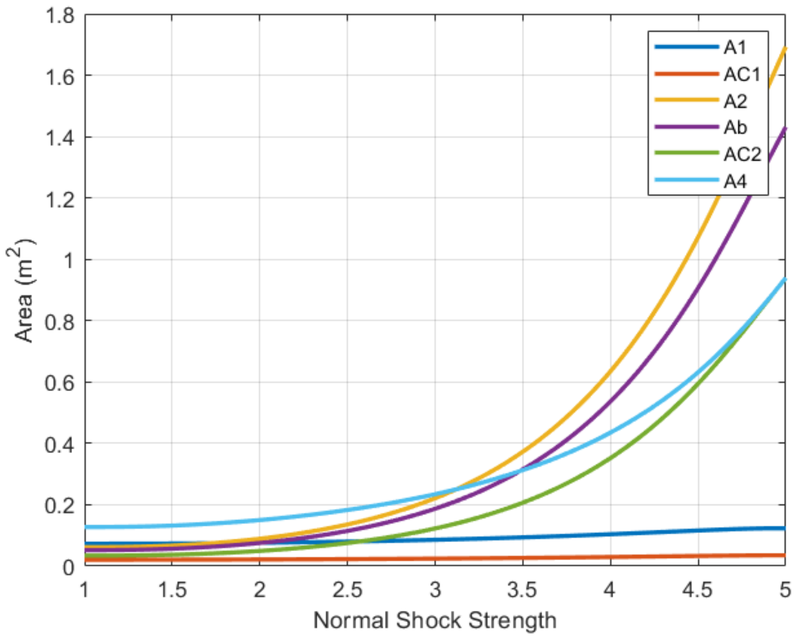

Figure 8: Variation in area with Normal Shock Strength

4.5 Burner Entry Mach Number

According to the Fig.9 depicting the relationship between efficiencies and the burner entry Mach number, it can be observed that the thermodynamic efficiency remains constant at 0.61. However, the propulsive efficiency demonstrates a slight decrease until reaching 0.68, after which it becomes complex.
During the process of obtaining the burner Mach number, it is assumed that there is no fuel mass loss or drag present. This assumption enables the application of the conservation of mass and conservation of momentum principles. By utilizing the ideal gas law and solving the resulting quadratic equation, the burner entry Mach number can be expressed as Eq.3.

From the aforementioned Eq.3, two potential solutions can be deduced. However, only the solution that falls between zero and one is deemed relevant as it corresponds to the weak solution. The weak solution is chosen as it is the one that is taken by nature. It should be noted that any burner entry Mach number above one will cause the flow to become choked. Additionally, in cases where the expression under the square root is less than zero, the Mach number will be non-real, rendering it physically unattainable. Consequently, the shaded area in the plot is disregarded as the corresponding burner entry Mach number is complex and not physically achievable.

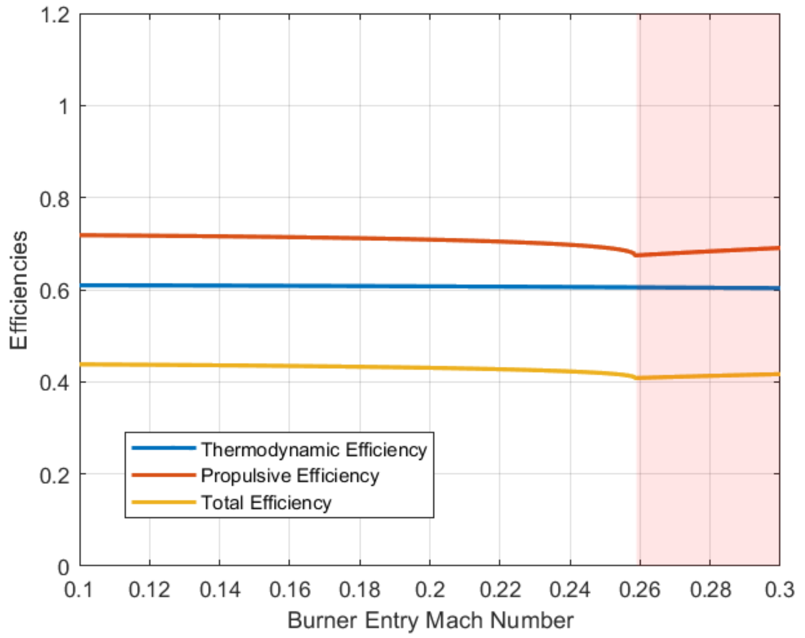

Figure 9: Variation in efficiencies with Burner Entry Mach Number

4.6 Burner Temperature

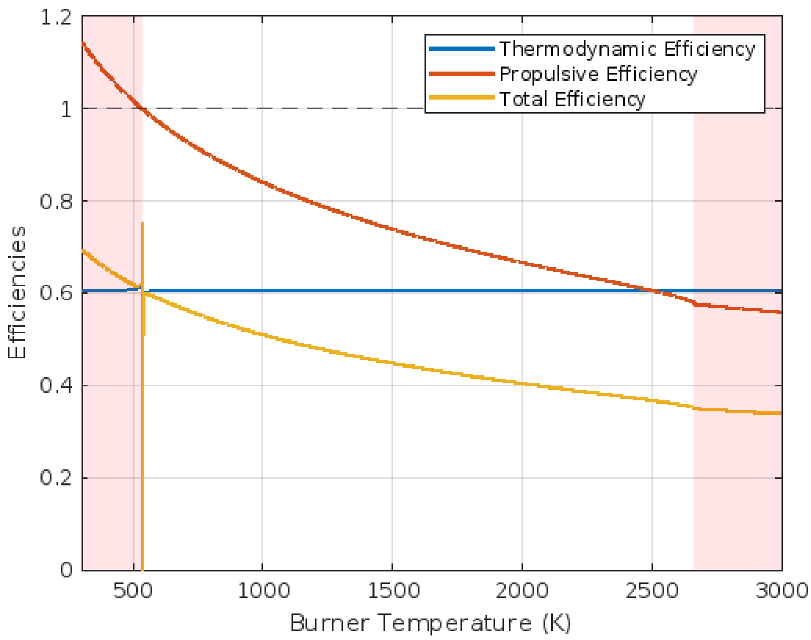

Figure 10: Variation in efficiencies with Burner Temperature

As burner temperature (T_b) increases, T_4 increases linearly with it as seen in Fig.11. This relationship is linear as only isentropic expansion is assumed between stations b and 4 with no shocks or heat addition. This linear relationship between T_b and T_4 therefore results in constant thermodynamic efficiency (η_{th}) according to Eq.2 except at the discontinuity. As before, the discontinuity is caused when the burner temperature (T_b) equals the temperature at station 2 simply due to slowing the flow down. This results in the denominator of Eq.2 equalling zero resulting in a discontinuity. This occurs at a burner temperature of 536K for the other assumed inputs. Lower burner exit temperatures than this physically represent having to remove heat from the flow, making it a refrigerator. This temperature defines the boundary of an impossible operating range. This is also evidenced as after this burner temperature, propulsive efficiency exceeds 1. This lower boundary on burner exit temperature is a function of upstream variables that affect M_2, such as free stream temperature, pressure Mach number and shock strength.

Fig.11 shows the variables which affect propulsive efficiency. It can be seen that T_4 and M_4^2 increase while T_1 and M_1^2 are constant. This increases the denominator of Eq.1, resulting in decreased propulsive efficiency seen in Fig.10.

Above a burner temperature of 2660K (for the assumed inputs), propulsive efficiency becomes complex. This, again, is a result of the inside of the square root of Eq.3 being negative due to a too high value of T_b. Physically this simply represents a condition in which Eq.3's constituent equations of mass, energy and momentum cannot be met.

This limit occurs at temperatures significantly higher than typical burner exit temperatures for the assumed input conditions, which are of reasonable values for a typical ramjet. Practically this upper boundary is set based on the material choice and cooling system used.

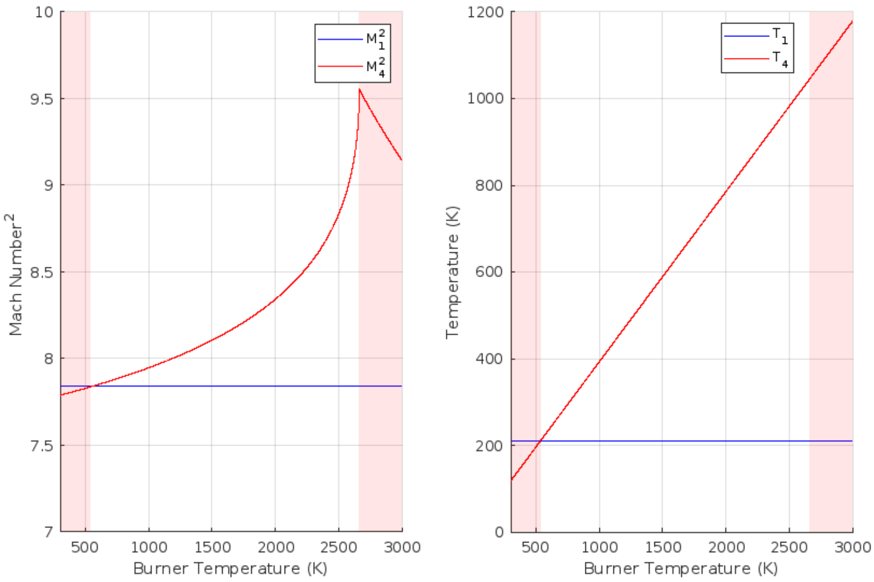

Figure 11: Change in Temperature and Mach number squared at stations 1 and 4 as Burner entry Temperature (T_2) changes

4.7 Burner Pressure Ratio

The graphical representation of the relationship between burner pressure ratio and efficiencies, Fig.12, demonstrates that the thermodynamic efficiency experiences a sharp initial increase, which slows down until it asymptotically approaches unity. It is noteworthy that the propulsive efficiency in Eq.1, which is determined by normalizing the thrust with respect to the inlet area, considers the pressure exerted on A_1 and A_4 as the exhaust may not be ideally expanded.

The graphical illustration reveals that the propulsive efficiency surpasses unity in scenarios where the burner pressure ratio is below 0.086. This phenomenon arises from the marginal difference in jet velocities, where the exhaust velocity is greater than the inlet velocity. In such cases, the engine generates negative thrust, which is fundamentally unfeasible from a physical standpoint.

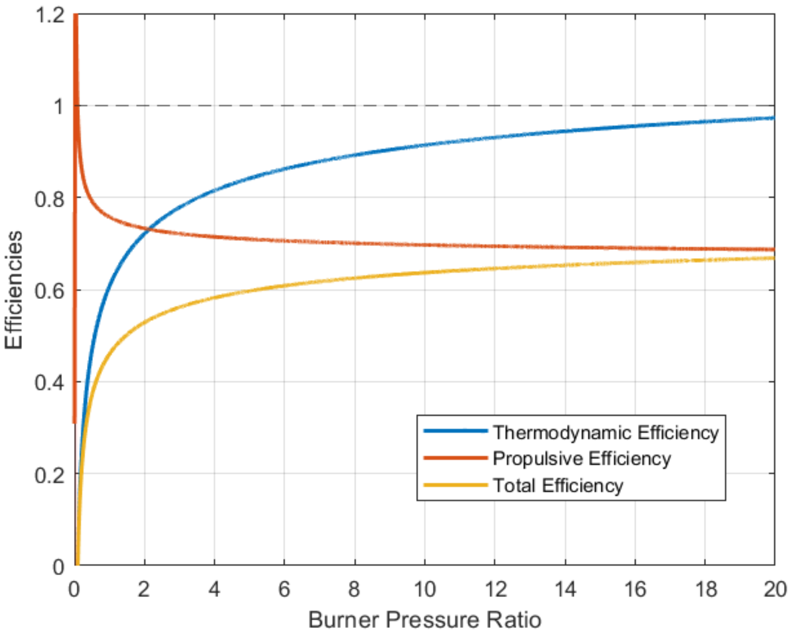

Figure 12: Variation in efficiencies with Burner Pressure Ratio

4.8 Exhaust Pressure Ratio

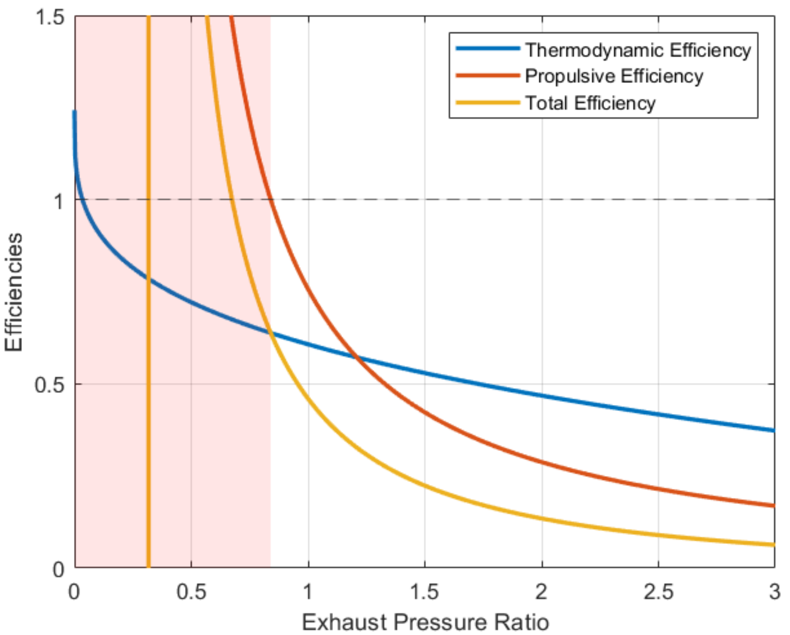

Figure 13: Variation in efficiencies with Exhaust Pressure Ratio

As the exhaust pressure ratio goes to zero, the flow exiting the nozzle must be flowing infinitely fast to have infinitesimally small pressure, meaning M_4 tends to infinity. This explains the asymptotic behaviour of the propulsive efficiency. Similarly, the temperature would tend to 0, explaining the asymptote of the thermal efficiency. Below a ratio of 0.84, the propulsive efficiency goes above 1. This value is slightly incoherent, as most ramjets are designed to be able to operate with a slightly negative exhaust pressure ratio to ensure a robust design. The efficiencies all trend downwards with exhaust pressure ratio. This makes sense as if the flow is further away from being perfectly expanded, there is still kinetic energy in the air that has not been "harvested" by the nozzle. So thrust is left over and propulsive efficiency goes down. Thermal efficiency also decreases as the exit temperature is higher than it should be, and η_{th} depends negatively on T_4. If the graph is scaled back, it can be seen that the propulsive and hence total efficiencies tend to 0 as seen in fig.14. This follows from the explanation given previously, though the behaviour of the thermodynamic efficiency, which dips below 0, is not physically possible. By this point, the flow would have long been reverted, and the ramjet would no longer act as an engine, so calculating an efficiency doesn't make sense.

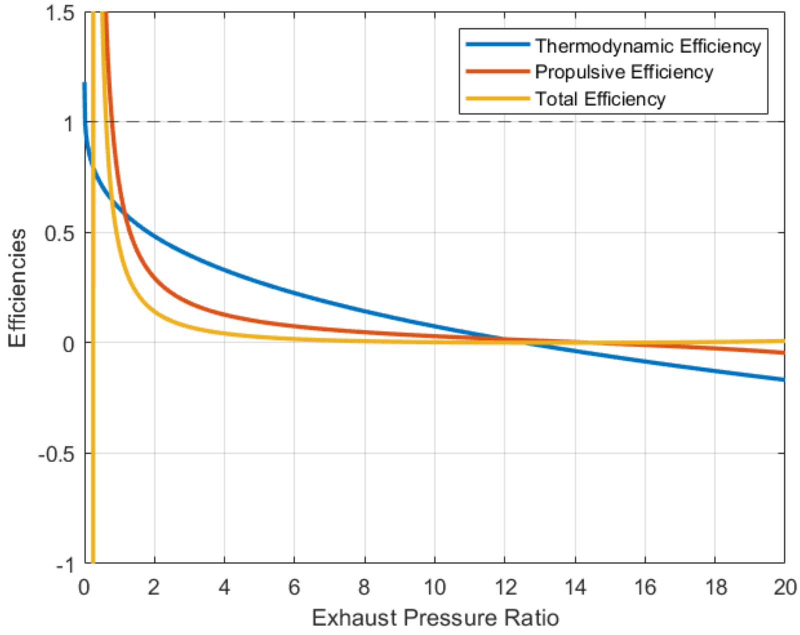

Figure 14: Variation in efficiencies with Exhaust Pressure Ratio

4.9 Required Thrust

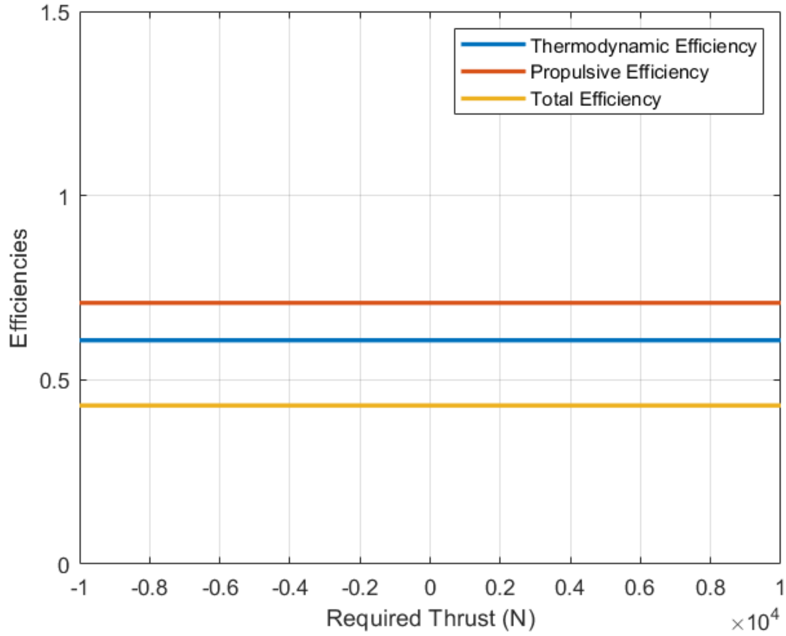

Figure 15: Variation in efficiencies with Required Thrust

As seen, the required thrust has no impact on any of the efficiencies. This is because the thrust linearly determines the size of A_1. It does not change the relative sizes of any of the areas nor any of the temperatures or pressures, it simply scales the whole thing to fit the required thrust. As such, it can be seen that this even works in theory for negative thrust. This doesn't make sense in reality, as it would correspond to a large-scale vacuum cleaner, and the fuel would be injected after the burner, meaning the engine wouldn't actually work. In reality, the efficiency is likely to vary with the required thrust for reasons beyond the scope of this report. Certain components are unable to scale, for example, the fuel injectors will have to be designed differently to make sure all the air and fuel is mixed. Other simplifications, such as the assumption of a normal shock, also start to break down for exceedingly large areas.

5 Conclusion

The output results shown in Table 3 are obtained using the source code in section 3 by inputting values from Table 2. As seen below in Fig.16, the overall dimensions of the engine seem to be coherent. Note that the figure shows the height, which is calculated from the areas assuming a circular cross-section. The area ratios shown in Table 3 were compared with the calculations completed in the reference lecture [1]. From this the largest percentage difference was found to be 8.47%. Such a difference is likely the difference between this code using the exact equations while the reference calculations used nearest neighbour values from reference tables. Another potential cause of this percentage difference could be taking into account the exhaust pressure ratio $\frac{P_4}{P_1}$ when obtaining the propulsive efficiency. However, in this case, we assumed the exhaust pressure ratio is one which should not be a factor causing the percentage difference.

Table 3: Values of Outputs

Variable	Report Term	Values
Inlet area	A_1	$0.0187\,\mathrm{m}^2$
Inlet throat area	A_{C1}	$0.0053\,\mathrm{m}^2$
Burner entry area	A_2	$0.0159\,\mathrm{m}^2$
Burner exit area	A_b	$0.0141\,\mathrm{m}^2$
Nozzle throat area	A_{C2}	$0.0082\,\mathrm{m}^2$
Exhaust area	A_4	$0.0298\,\mathrm{m}^2$
Thermodynamic efficiency	η_{th}	0.6073
Propulsive efficiency	η_p	0.7572
Inlet throat area ratio	$\frac{A_{C1}}{A_1}$	0.283
Burner entry area ratio	$\frac{A_2}{A_1}$	0.850
Burner exit area ratio	$\frac{A_b}{A_1}$	0.754
Nozzle throat area ratio	$\frac{A_{C2}}{A_1}$	0.439
Exhaust area ratio	$\frac{A_4}{A_1}$	1.594

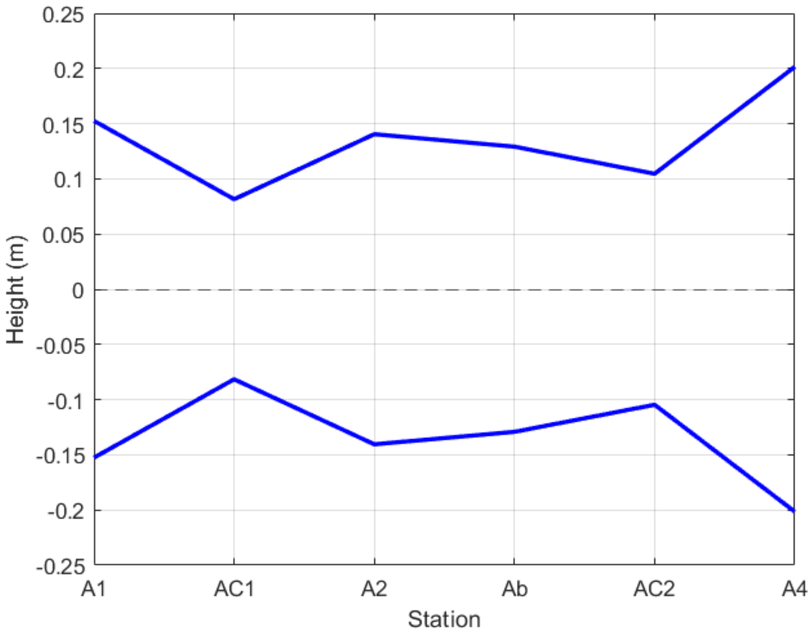

Figure 16: Simplified Engine Cross-section Drawing using Lecture 3 Input Values

Although the code produces results with reasonable values, there are still limitations of applying the code.

As discussed previously, there are many input ranges which were deemed as unreasonable operating conditions. This is partially due to non-physical operating conditions yielding complex results. There are also conditions in which the engine no longer acts as an engine and either removes heat from the flow or produces negative thrust/ acts as a vacuum which were therefore deemed as unreasonable operating conditions. Specifically there are non-physical complex results for sufficiently low free stream temperatures ($T_1 < 134K$), low free stream Mach number ($M_1 < 1.78$), high burner entry Mach number ($M_2 > 2.59$) and high burner exit temperatures ($T_b > 2660K$). There are unreasonable operating conditions for sufficiently high free stream temperature ($T_1 > 668K$), high free stream Mach number ($M_1 > 6$), high normal shock strength ($M_x > 4.88$) low burner exit temperature ($T_b < 536K$) and low exhaust pressure ratio ($\frac{P_4}{P_1} < 0.84$). Additionally there is also a practical limit on the burner exit temperature imposed by the melting point of the material used and the cooling system used. All these conditions would therefore define the envelope over which the proposed ramjet could operate and which the results are valid. The specific numbers are however a function of the input parameters used and are included simply to give reasonable limits.

With regard to the limitations outlined earlier, it should be noted that the code is exclusively applicable within the assumptions specified in section 2. The most crucial of these assumptions are ignoring boundary layer effects and assuming there are no oblique shocks. It is imperative to acknowledge that in real-world scenarios, the impact of boundary layer effects must be taken into account, as they can significantly interfere with the flow dynamics and behavior within the engine. Also, some oblique shock waves may form within the ramjet engine if the air entering the engine is supersonic. The presence of oblique shock waves can have a significant impact on the performance of the engine, causing flow separation and hence reducing the overall efficiency. Furthermore, the thrust equation considers pressure drag but fails to consider the additional parasitic drag that comes with increasing the areas. This would reduce the propulsive efficiency.

Overall, the code produces reasonable results for a ramjet engine over a limited range of reasonable values. The code breaks down once it is asked to compute things that no longer make physical sense. It can be improved by considering more complex phenomena such as boundary layers, heat transfers, flow disruptions from fuel injectors, and more.

References

[1] Birch DM. Advanced Propulsion Lecture 3. Department of Aeronautics, Imperial College London; 2023.

[2] Flack R. Fundamentals of jet propulsion with applications. New York: Cambridge University Press; 2012.

[3] Birch DM. Team project: numerical ramjet engine. Department of Aeronautics, Imperial College London; 2023.

[4] Benson T. Burner Thermodynamics;.

[5] Felicione C. Is it possible for a ramjet to start from 0 velocity?;.