Roulette math

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Roulette is a game with 38 numbers – 18 red numbers, 18 black, 0 and 00 which are considered neither black, red, even, nor odd. The game has players place a bet, and that bet determines the payoff. The probability of success is p, the number of games is n, and the number of hits is m. This gives us a Binomial probability distribution, with probability mass function (PMF):

$$\binom{n}{m} p^m (1-p)^{n-m}$$

So if we're playing straight up – selecting a single number, our probability p is $\frac{1}{38}$ and over single trial X, our expected payout, where we define $q = 1 - p$ is:

$$E[X] = \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k} = np$$

so for roulette, our best strategy over multiple trials is to model it as a Martingale (cite: Gallagher).

Then we know that for trials up to n:

$$E[X_{n+1} | X_1, ..., X_n] \geq X_n$$

which holds due to the best inequality in probability theory, Jensen's inequality:

$$\varphi(E[X]) \leq E[\varphi(X)]$$

and we can show this by considering the convex function φ to be the payout function, each x_i to be our observed payout, and positive weights a_i to be our probability of winning.

$$\varphi\left(\frac{\sum a_i x_i}{a_i}\right) \leq \frac{\sum a_i \varphi(x_i)}{\sum a_i}$$

Thus, each trial of the roulette spin is a fully independent value and the expected value of the $n+1$th spin is identical the the expected value of the nth spin. This is also a rebuttal of the gambler's fallacy that tables can run "hot" or "cold".

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