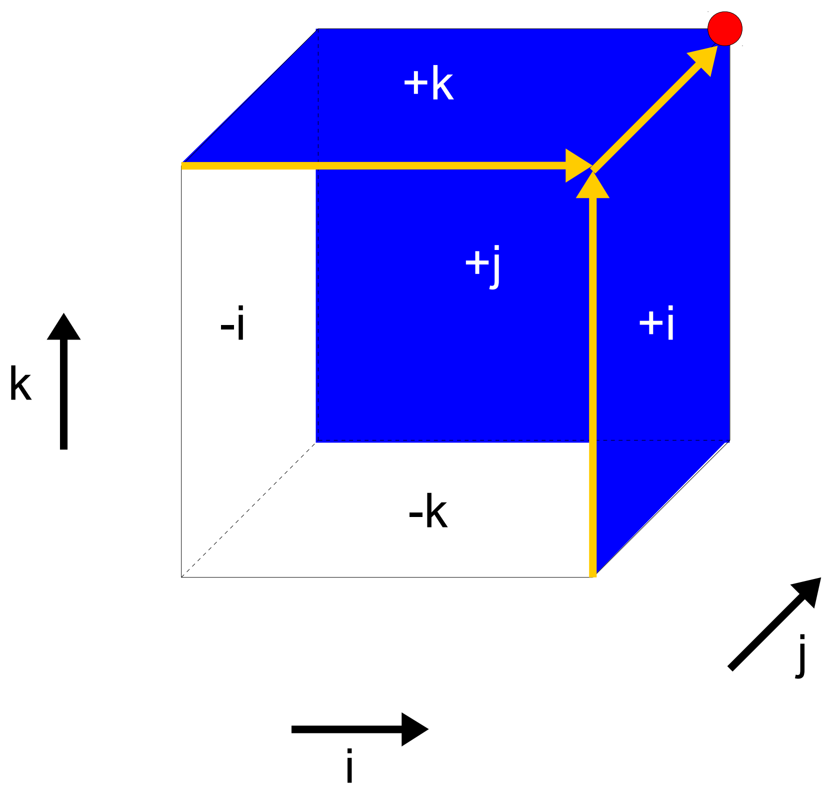
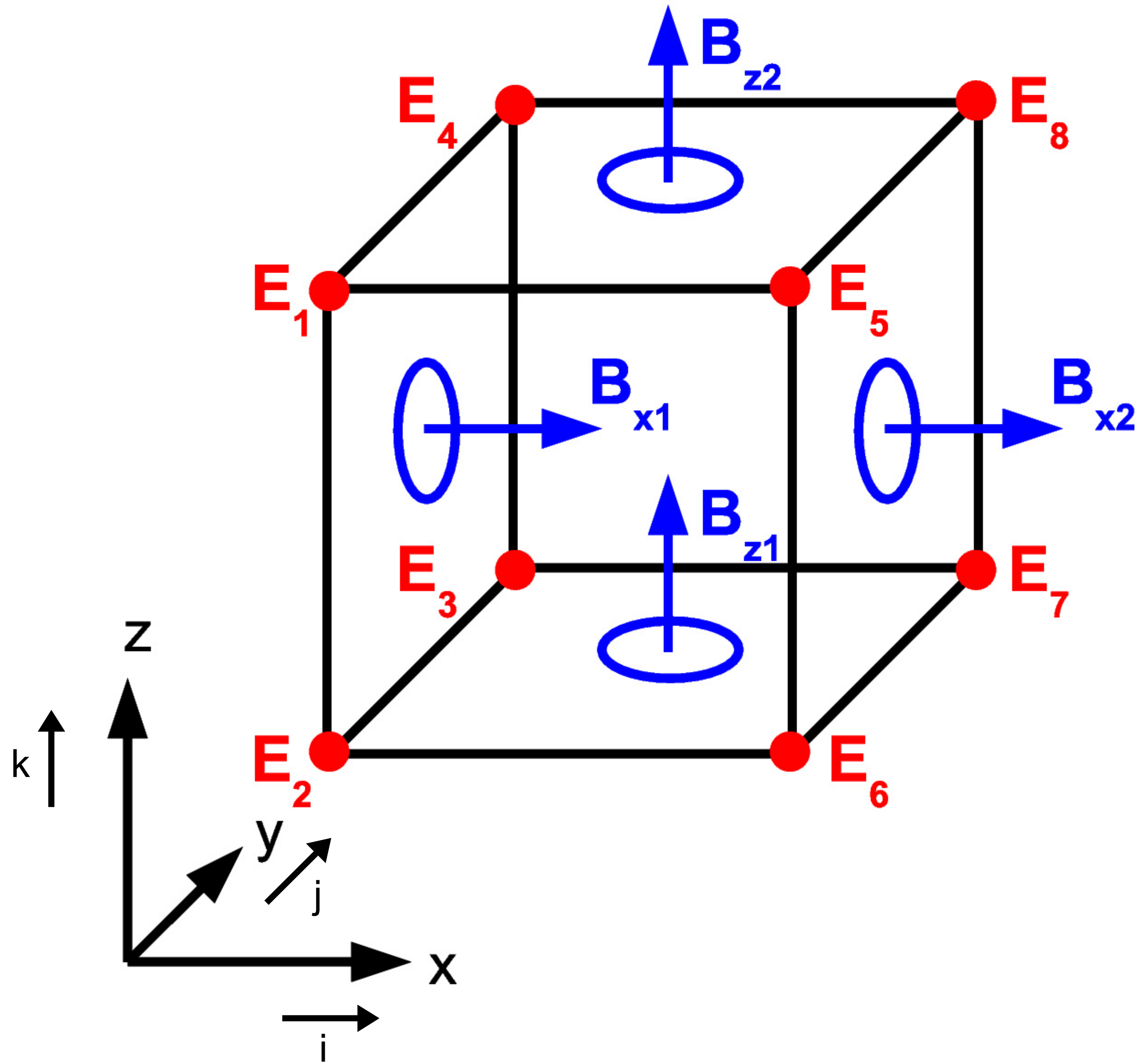
RHybrid cell indexing

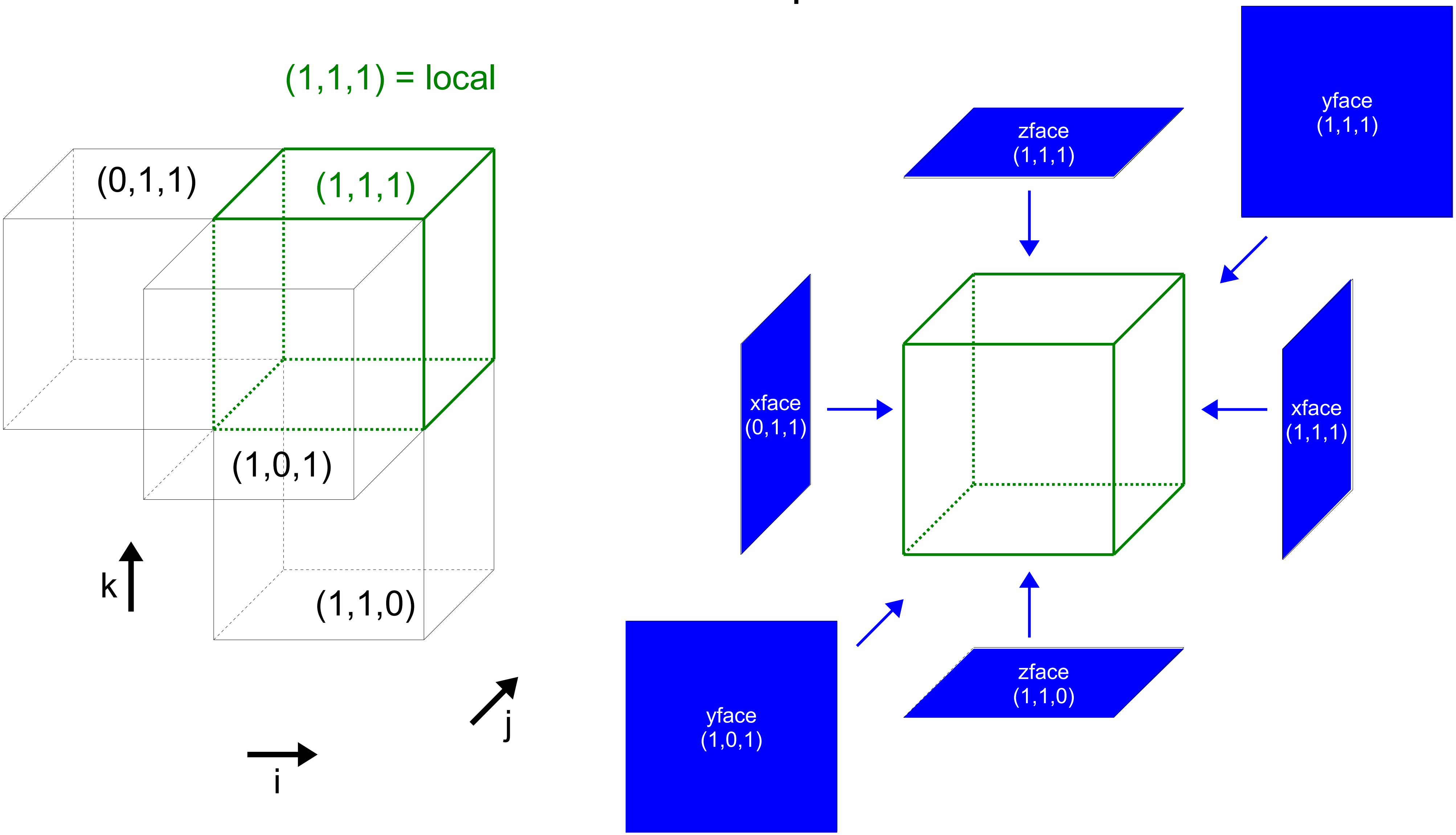
- cell volume average
  - Bx, By, Bz
- upper front corner node (+i,+j,+k)
  - Ex, Ey, Ez
  - Bx, By, Bz
  - Jx, Jy, Jz
- three face surface averages
  - Φx = dA × Bx (+i face)
  - Φx = dA × By (+j face)
  - Φz = dA × Bz (+k face)
- edges
  - Jx, Jy, Jz





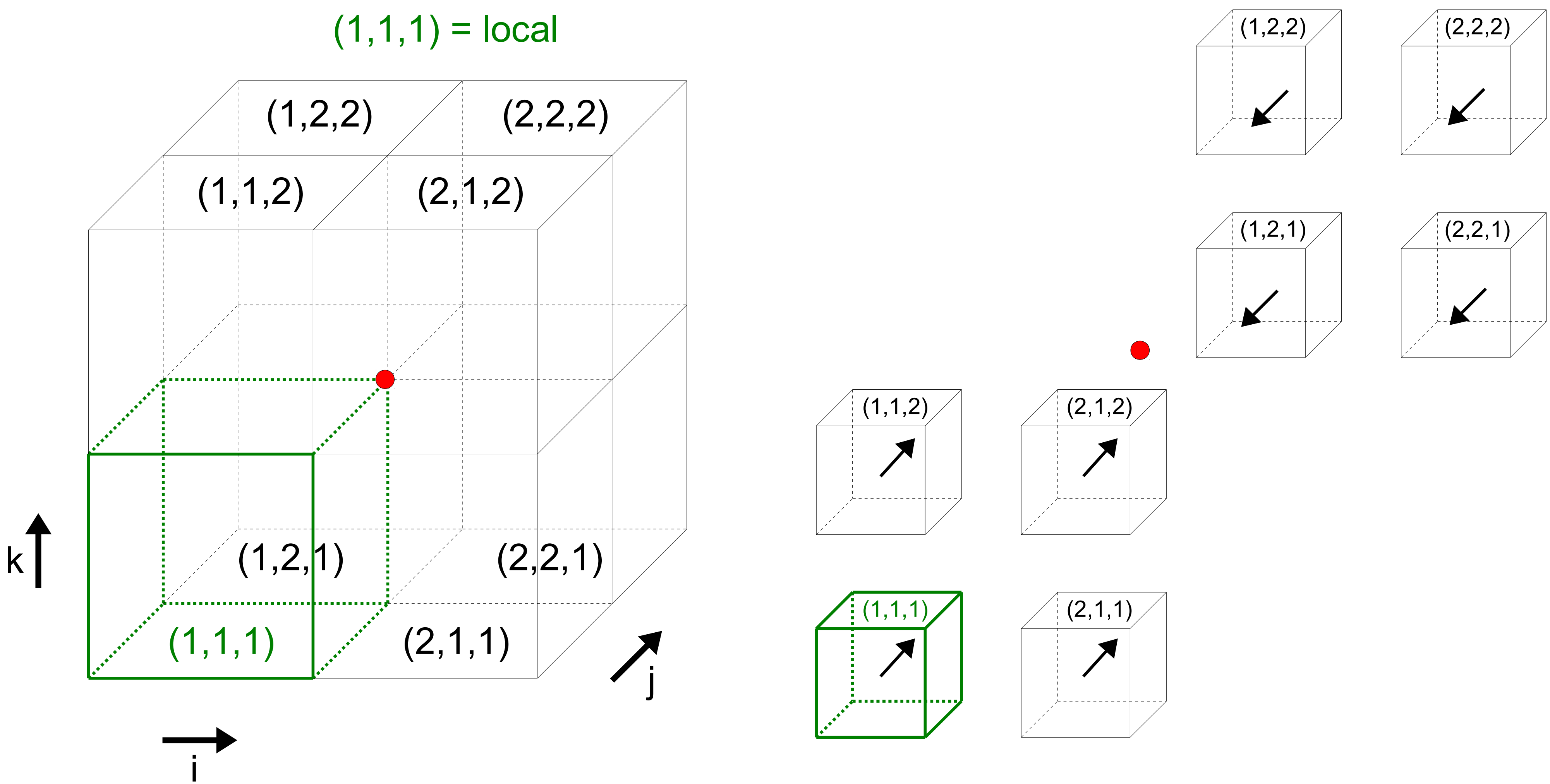
B z2
E 4
E 8
E 1
E 5
B x1
B x2
B z1
E 3
E 7
z
k
E 2
E 6
y
j
x
i

face to cell interpolation



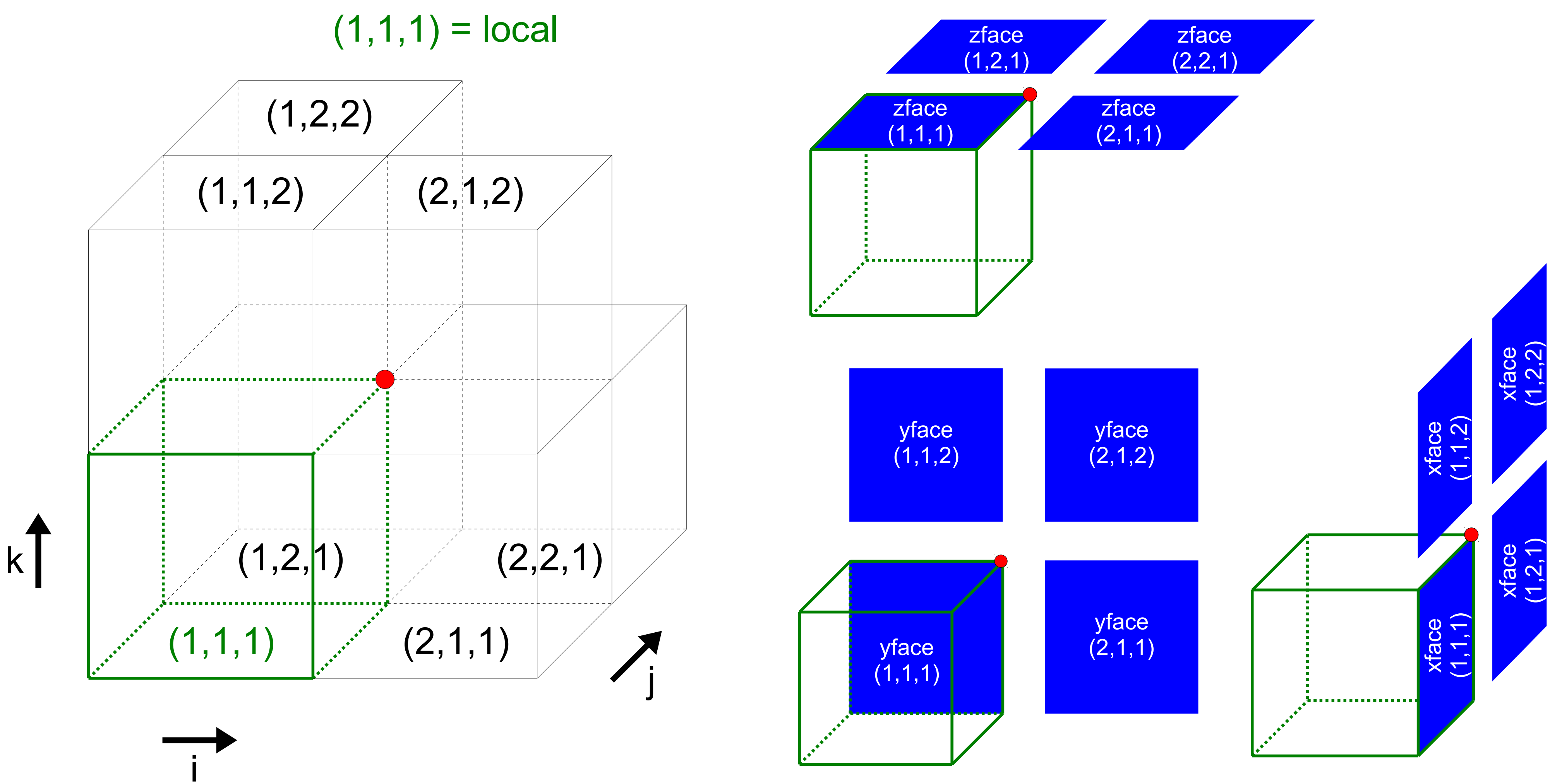

```
celldata_x =   0.5 × ( facedata_x(1,1,1) + facedata_x(0,1,1) )
celldata_y =   0.5 × ( facedata_y(1,1,1) + facedata_y(1,0,1) )
celldata_z =   0.5 × ( facedata_z(1,1,1) + facedata_z(1,1,0) )
```

cell to node interpolation



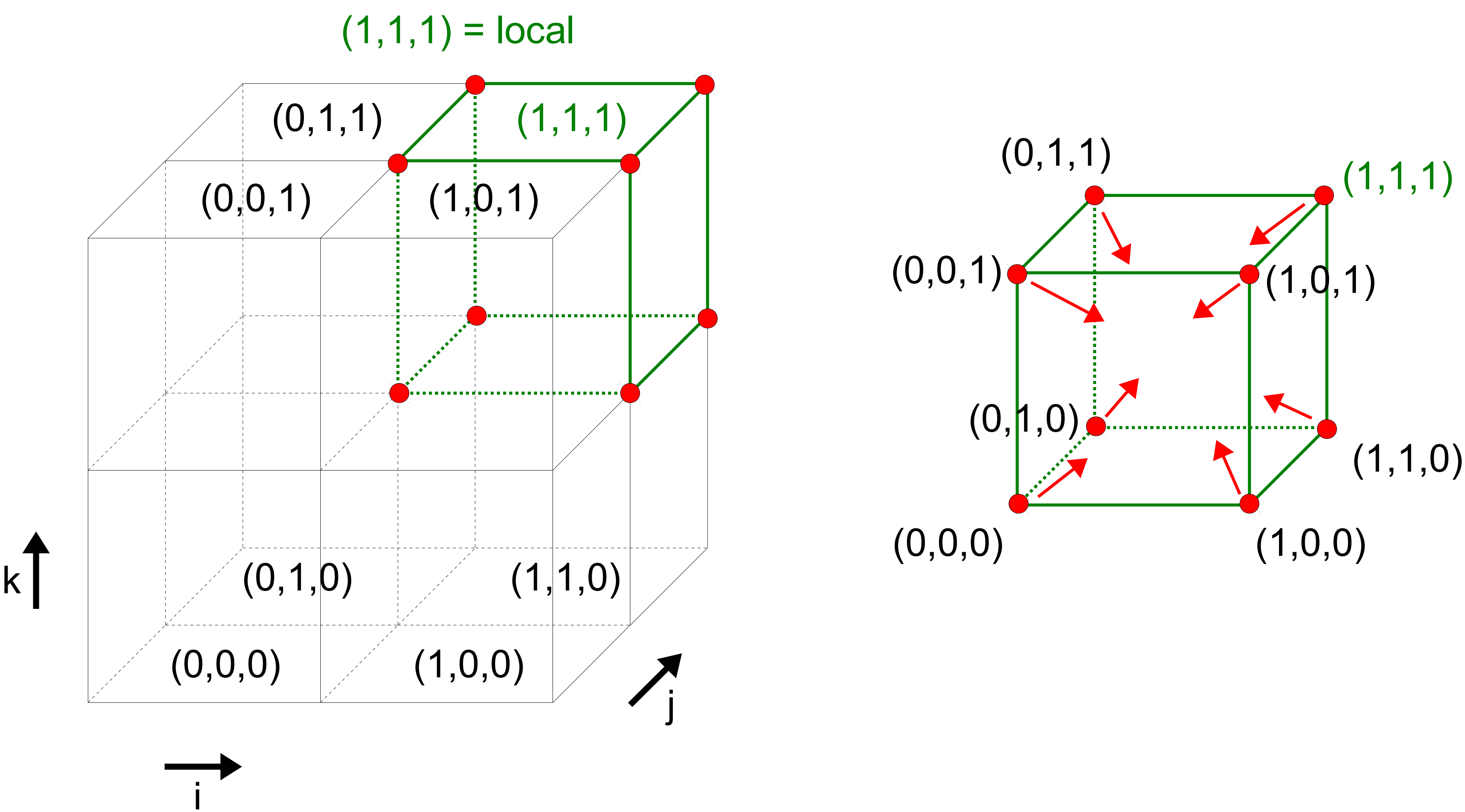

```
nodedata_i   = 0.125 × (  celldata_i(1,1,1) + celldata_i(2,1,1)
                         +celldata_i(1,1,2) + celldata_i(2,1,2)
                         +celldata_i(1,2,1) + celldata_i(2,2,1)
                         +celldata_i(1,2,2) + celldata_i(2,2,2)  )
```

face to node interpolation



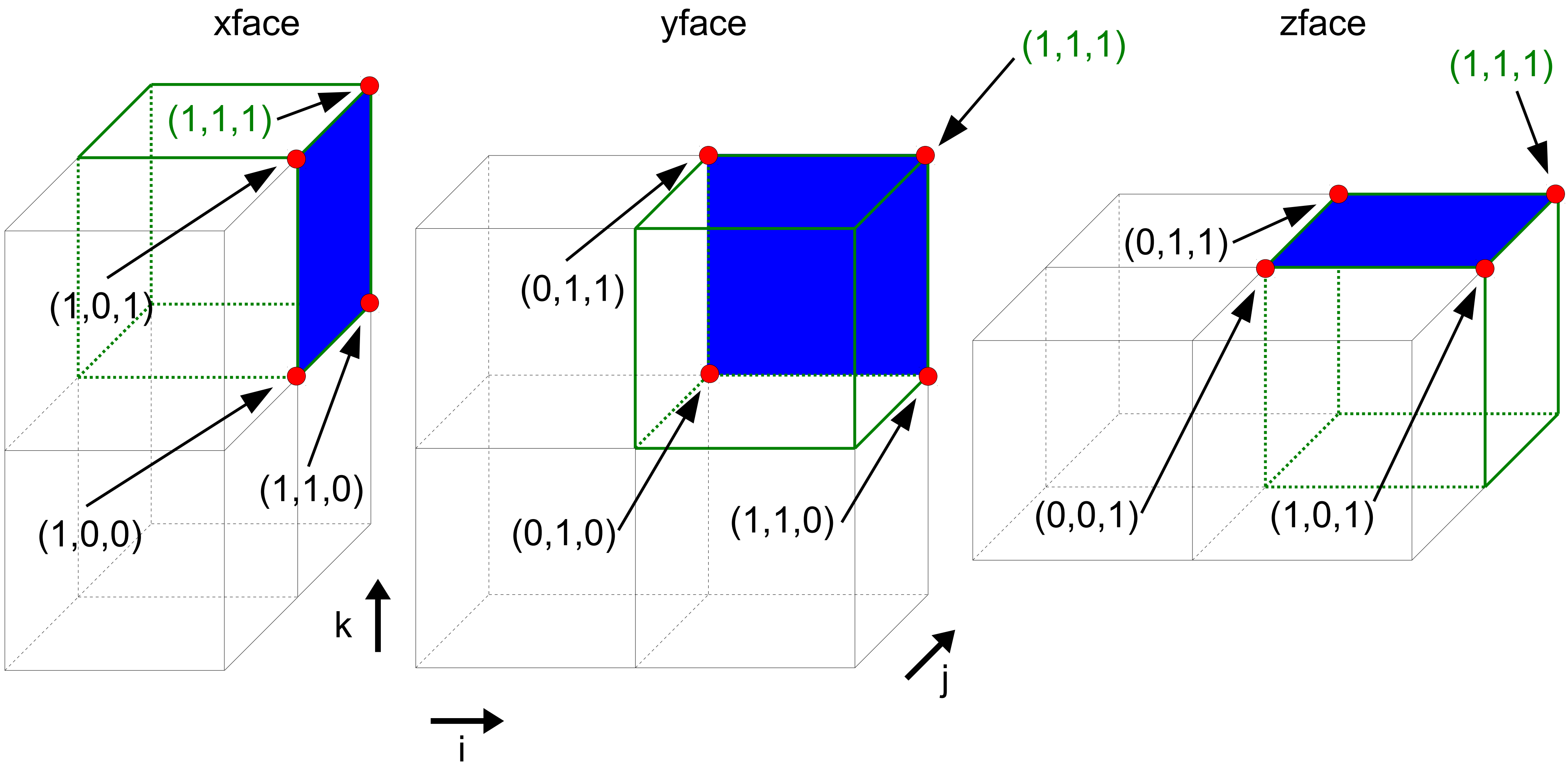

```
nodedata_x = 0.25 × ( facedata_x(1,1,1) + facedata_x(1,1,2) + facedata_x(1,2,2) + facedata_x(1,2,1) )
nodedata_y = 0.25 × ( facedata_y(1,1,1) + facedata_y(1,1,2) + facedata_y(2,1,2) + facedata_y(2,1,1) )
nodedata_z = 0.25 × ( facedata_z(1,1,1) + facedata_z(1,2,1) + facedata_z(2,2,1) + facedata_z(2,1,1) )
```

node to cell interpolation



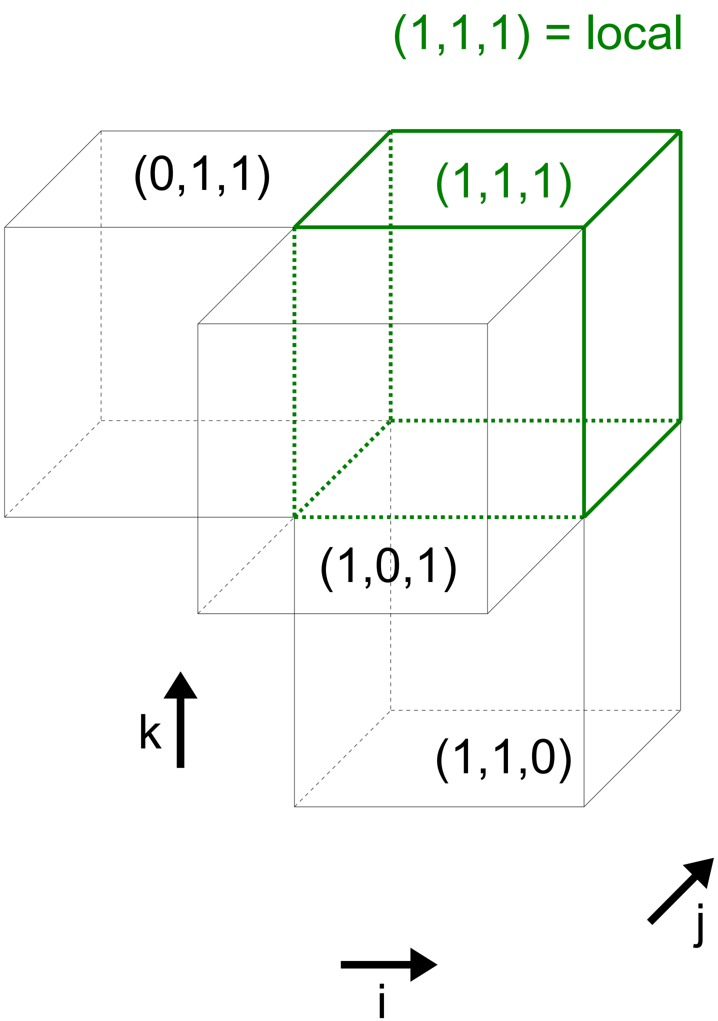

```
celldata_i = 0.125 × (nodedata_i(1,1,1) + nodedata_i(0,1,1)
                    + nodedata_i(0,0,1) + nodedata_i(1,0,1)
                    + nodedata_i(1,1,0) + nodedata_i(0,1,0)
                    + nodedata_i(0,0,0) + nodedata_i(1,0,0) )
```

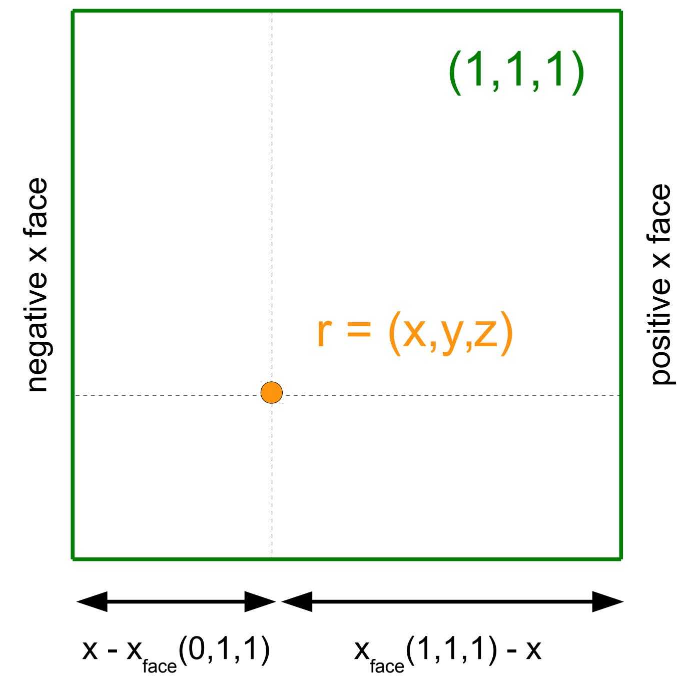
node to face interpolation




```
facedata_x = 0.25 × ( nodedata_x(1,1,1) + nodedata_x(1,0,1) + nodedata_x(1,0,0) + nodedata_x(1,1,0) )
facedata_y = 0.25 × ( nodedata_y(1,1,1) + nodedata_y(1,1,0) + nodedata_y(0,1,0) + nodedata_y(0,1,1) )
facedata_z = 0.25 × ( nodedata_z(1,1,1) + nodedata_z(0,1,1) + nodedata_z(0,0,1) + nodedata_z(1,0,1) )
```

face to r interpolation

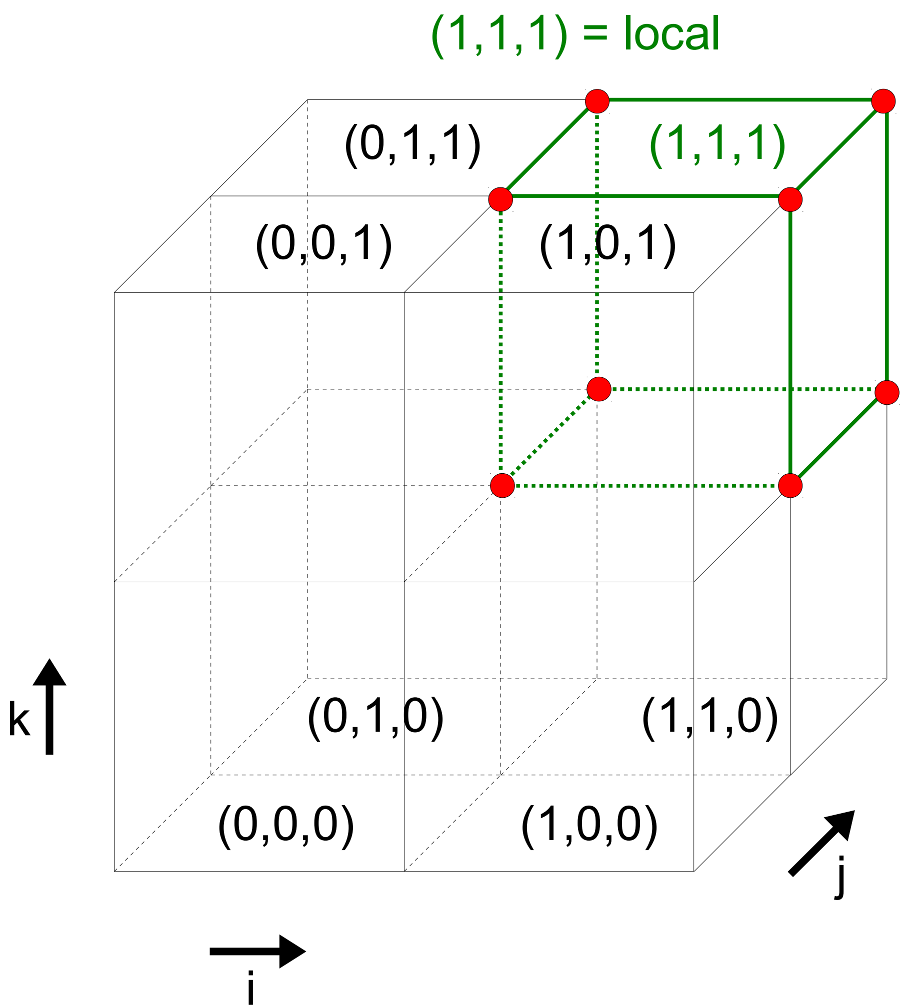


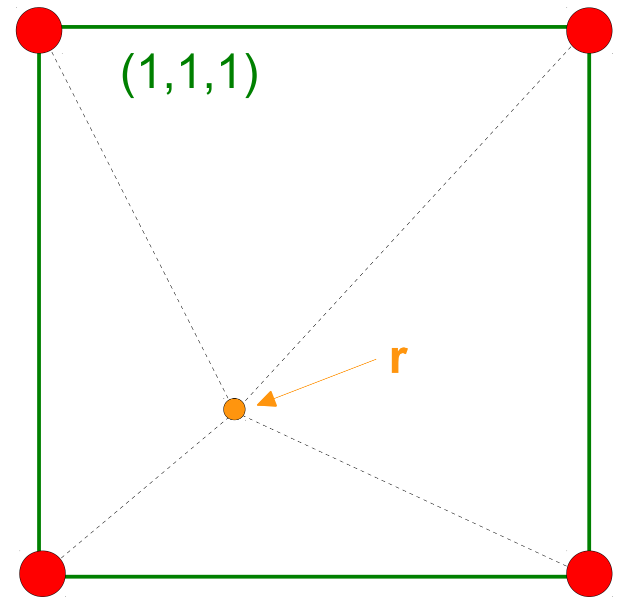


$$w111x = +(x - x_{face}(0,1,1))/dx$$
$$w111y = +(y - y_{face}(1,0,1))/dx$$
$$w111z = +(z - z_{face}(1,1,0))/dx$$
$$w011x = -(x - x_{face}(1,1,1))/dx$$
$$w101y = -(y - y_{face}(1,1,1))/dx$$
$$w110z = -(z - z_{face}(1,1,1))/dx$$

```
data_x(r) = w111x × facedata_x(1,1,1) + w011x × facedata_x(0,1,1)
data_y(r) = w111y × facedata_y(1,1,1) + w101y × facedata_y(1,0,1)
data_z(r) = w111z × facedata_z(1,1,1) + w110z × facedata_z(1,1,0)
```

node to r interpolation



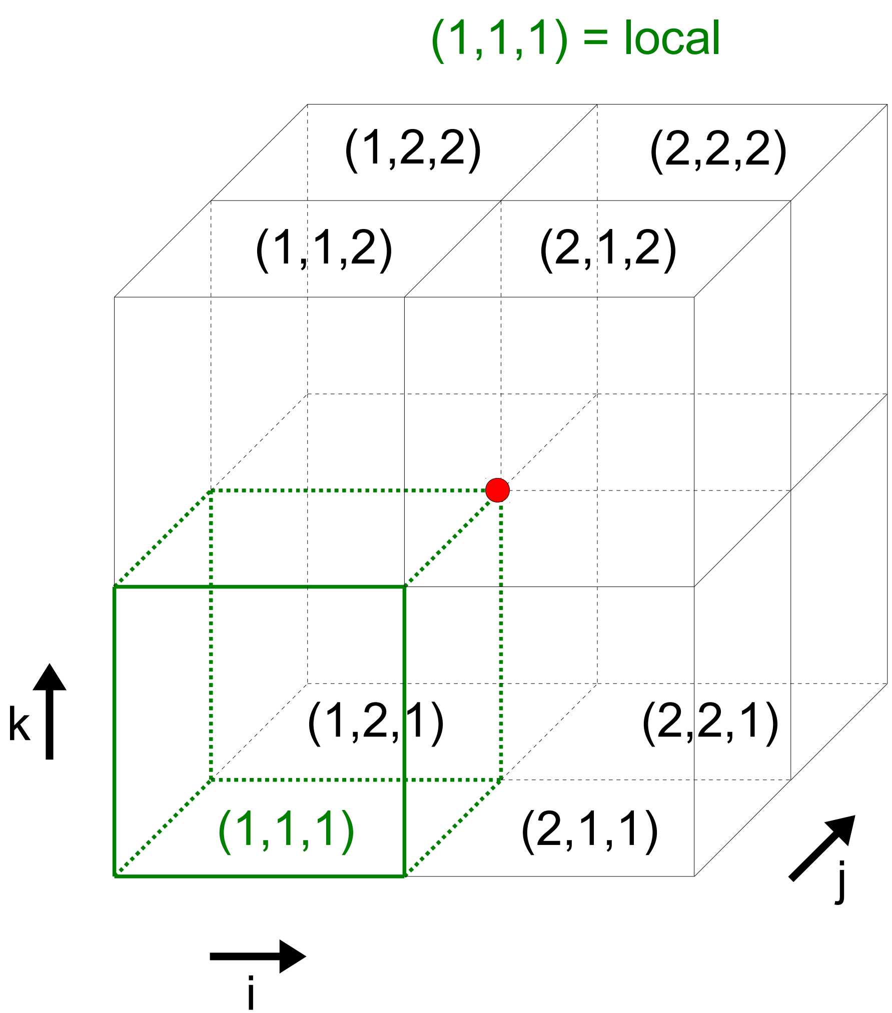


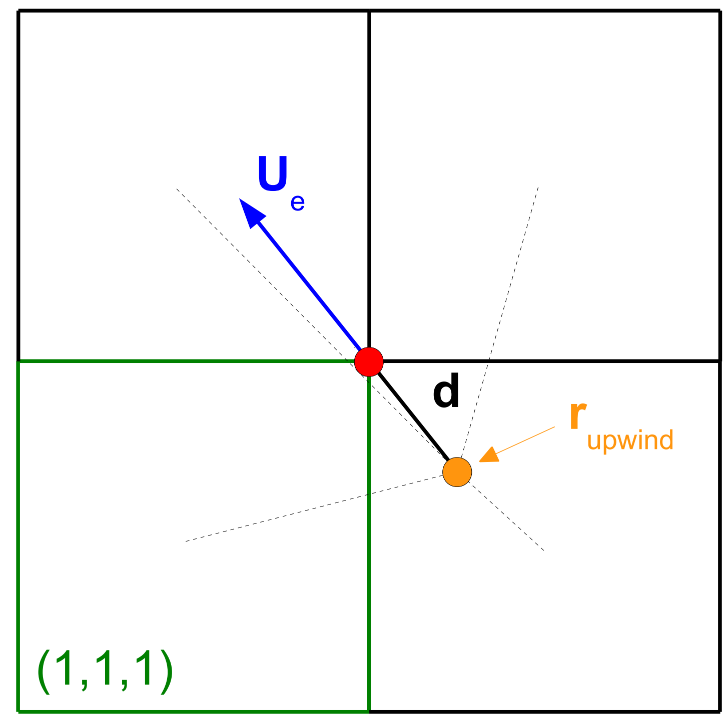
Weight factors: $w\_i = 1/|\mathbf{r}_{node\_i} - \mathbf{r}|$

Sum of weights: wsum = sum_i(w_i)

```
data(r) = (  w(1,1,1) × nodedata(1,1,1)   + w(0,1,1) × nodedata(0,1,1) +
             w(1,0,1) × nodedata(1,0,1)   + w(0,0,1) × nodedata(0,0,1) +
             w(1,1,0) × nodedata(1,1,0)   + w(0,1,0) × nodedata(0,1,0) +
             w(1,0,0) × nodedata(1,0,0)   + w(0,0,0) × nodedata(0,0,0) )/wsum
```

upwind node data





Displacement vector: $\mathbf{d} = 0.5 \times \mathbf{U}_e/|U_e|$

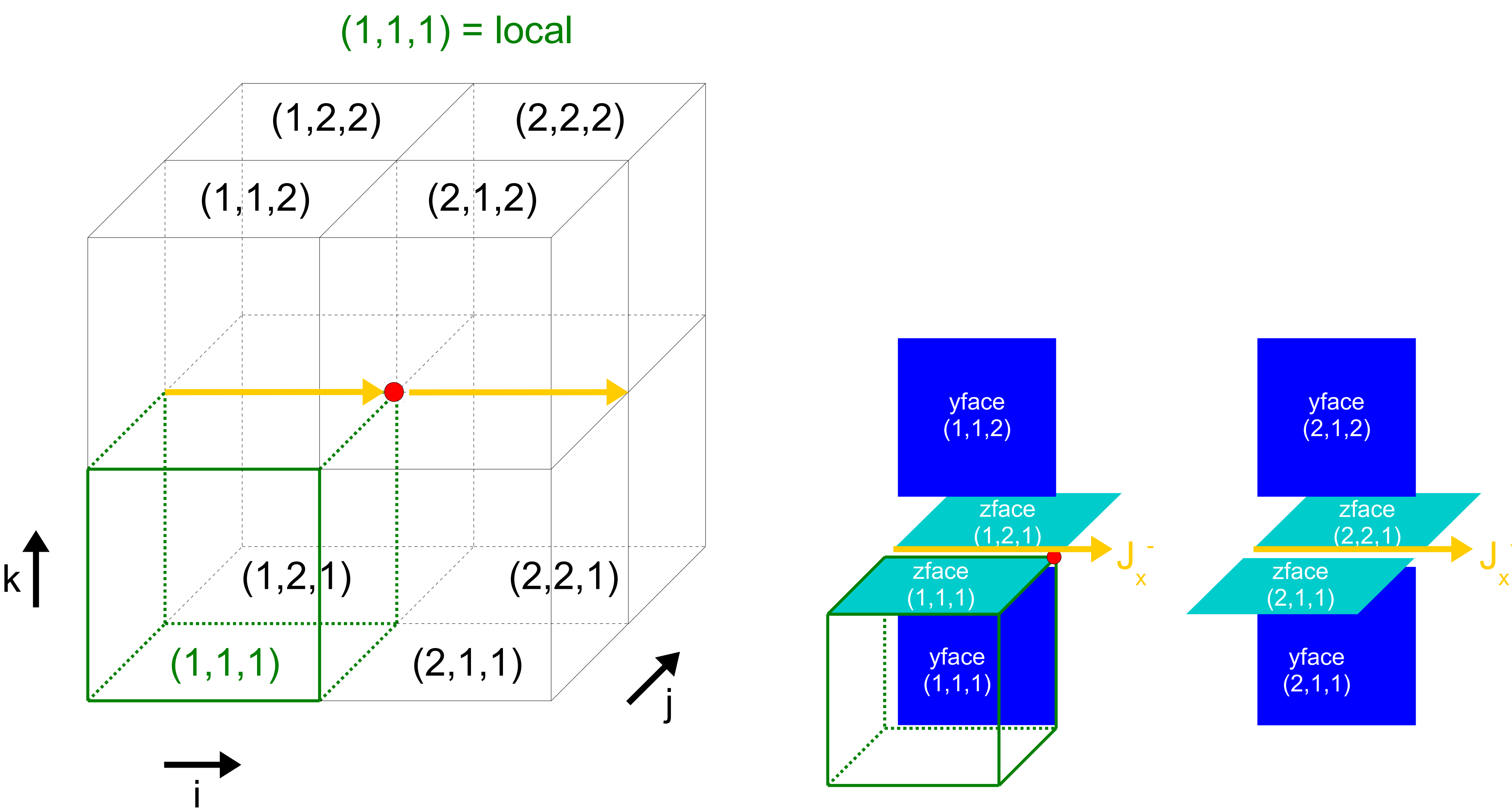
Upwind position: $\mathbf{r}_{upwind} = \mathbf{r}_{node} - \mathbf{d}$

Weight factors: $w\_i = 1/|\mathbf{r}_{cell\_i} - \mathbf{r}_{upwind}|$

Sum of weights: wsum = sum_i(w_i)

```
Nodedata(1,1,1) → nodedata(1,1,1) =
        (   w(1,1,1) × celldata(1,1,1) + w(1,1,2) × celldata(1,1,2) +
            w(1,2,1) × celldata(1,2,1) + w(2,1,1) × celldata(2,1,1) +
            w(1,2,2) × celldata(1,2,2) + w(2,2,1) × celldata(2,2,1) +
            w(2,1,2) × celldata(2,1,2) + w(2,2,2) × celldata(2,2,2) )/wsum
```

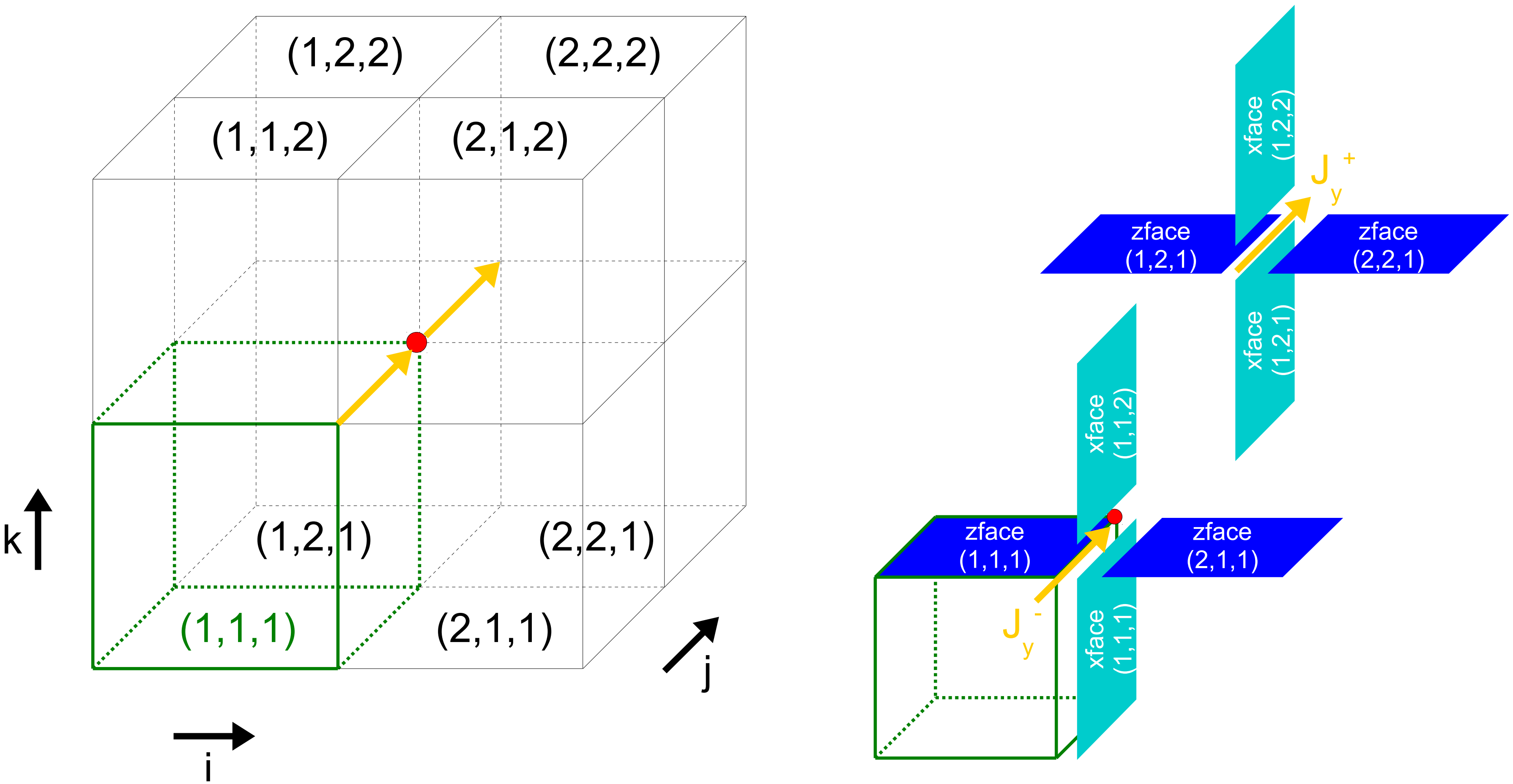
Calculation of Node Jx



$$\mathrm{edgeJx}^- = (-\mathrm{faceBz}(1,1,1) + \mathrm{faceBy}(1,1,1) + \mathrm{faceBz}(1,2,1) - \mathrm{faceBy}(1,1,2))/(\mathrm{dx} \times \mathrm{mu0})$$
$$\mathrm{edgeJx}^+ = (-\mathrm{faceBz}(2,1,1) + \mathrm{faceBy}(2,1,1) + \mathrm{faceBz}(2,2,1) - \mathrm{faceBy}(2,1,2))/(\mathrm{dx} \times \mathrm{mu0})$$
$$\mathrm{nodeJx} = 0.5 \times (\ \mathrm{edgeJx}^+ + \mathrm{edgeJx}^-\ )$$

Calculation of Node Jy

(1,1,1) = local

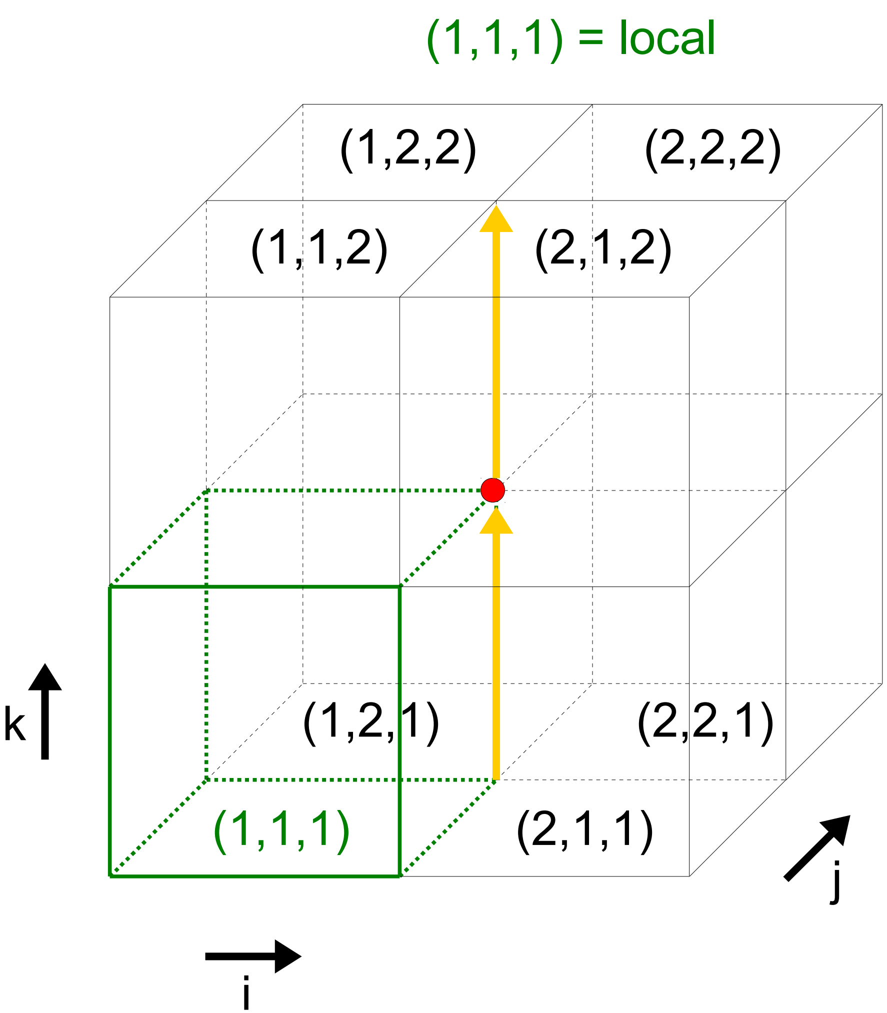


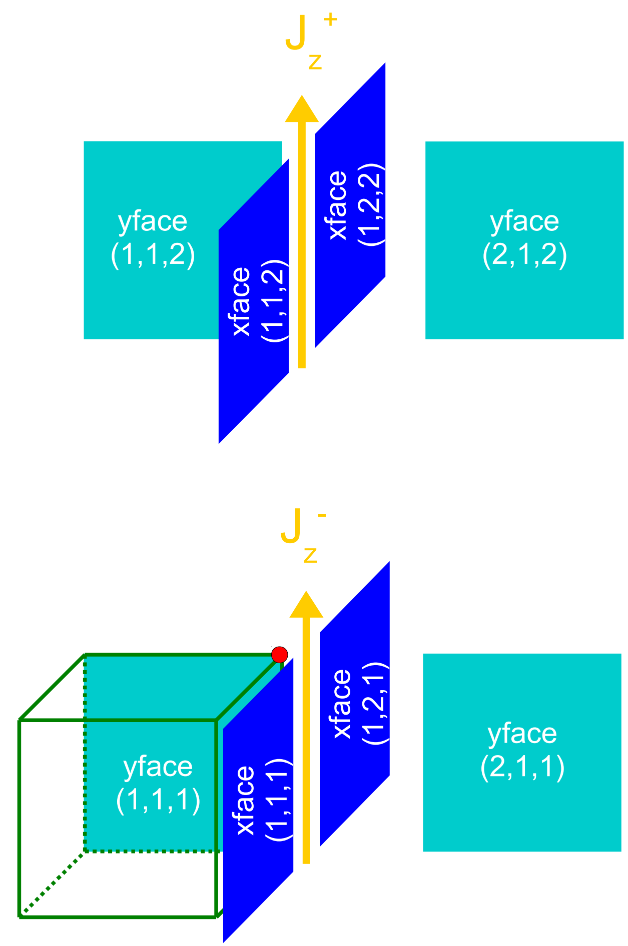
$$edgeJy^- = (+faceBz(1,1,1) + faceBx(1,1,2) - faceBz(2,1,1) - faceBx(1,1,1))/(dx \times mu0)$$

$$edgeJy^+ = (+faceBz(1,2,1) + faceBx(1,2,2) - faceBz(2,2,1) - faceBx(1,2,1))/(dx \times mu0)$$

$$nodeJy = 0.5 \times (\ edgeJy^+ + edgeJy^-\ )$$

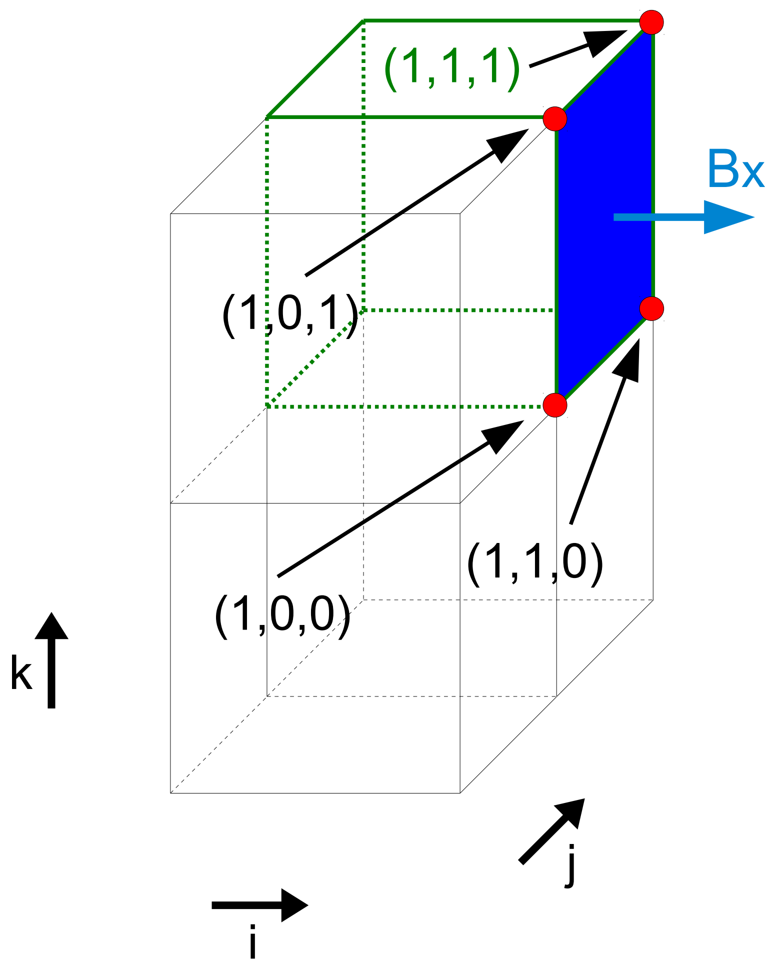
Calculation of Node Jz



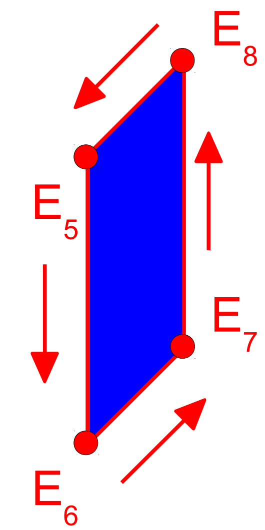


$$\mathrm{edgeJz}^{-} = (-\mathrm{faceBy}(1,1,1) + \mathrm{faceBx}(1,1,1) + \mathrm{faceBy}(2,1,1) - \mathrm{faceBx}(1,2,1))/(\mathrm{dx} \times \mathrm{mu0})$$
$$\mathrm{edgeJz}^{+} = (-\mathrm{faceBy}(1,1,2) + \mathrm{faceBx}(1,1,2) + \mathrm{faceBy}(2,1,2) - \mathrm{faceBx}(1,2,2))/(\mathrm{dx} \times \mathrm{mu0})$$
$$\mathrm{nodeJz} = 0.5 \times (\ \mathrm{edgeJz}^{+} + \mathrm{edgeJz}^{-}\ )$$

propagation of B on xface / xface curl



face curl

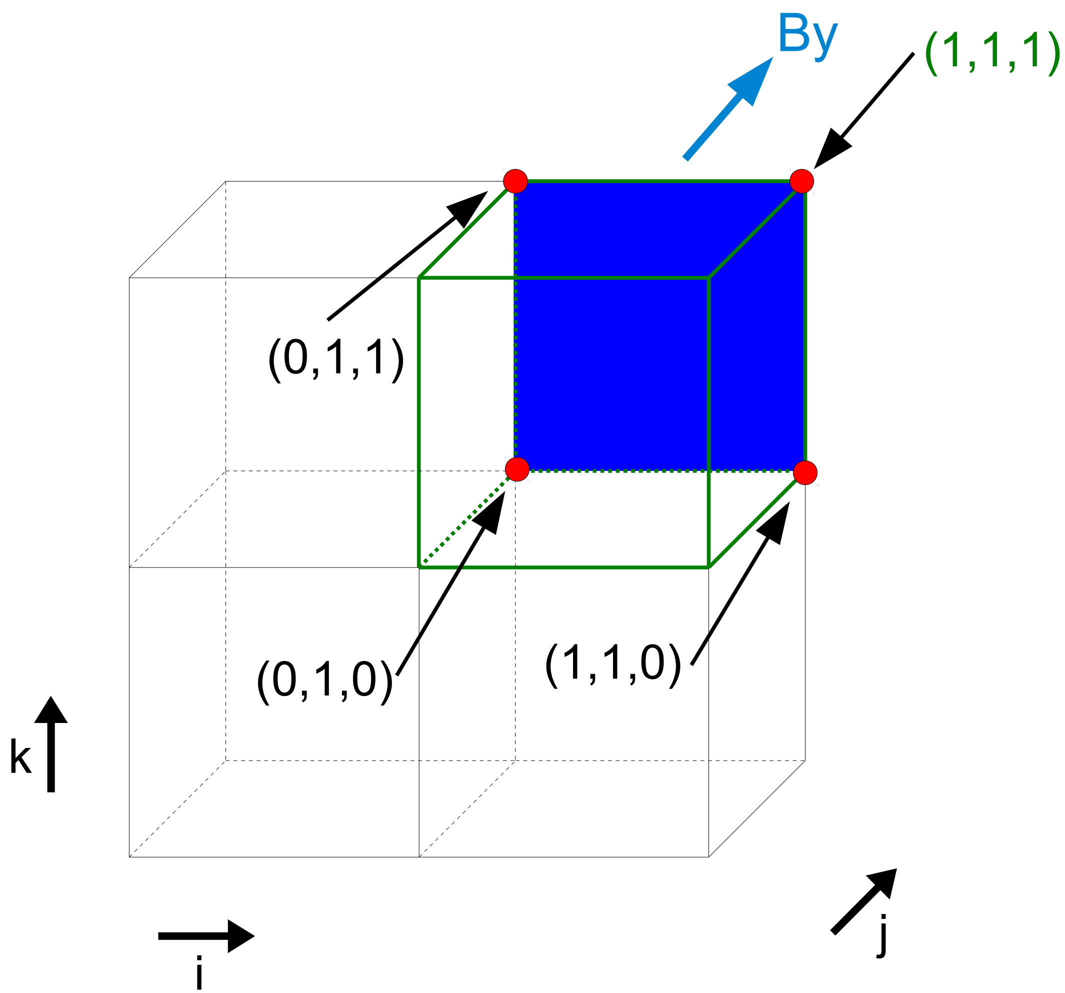


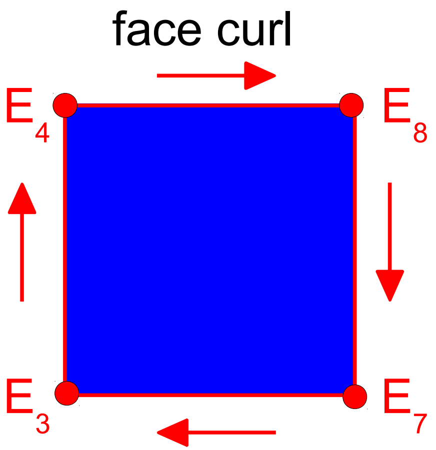
$E_5 = E(1,0,1)$
$E_6 = E(1,0,0)$
$E_7 = E(1,1,0)$
$E_8 = E(1,1,1) = \text{local}$

$$\partial(\int d\mathbf{A}_x \cdot \mathbf{B})/\partial t \quad = -\int d\mathbf{A}_x \cdot (\nabla \times \mathbf{E}) \quad = 0.5 \times dx \times (E_{5z} + E_{6z} - E_{6y} - E_{7y} - E_{7z} - E_{8z} + E_{8y} + E_{5y})$$

$$\mathbf{J}_x \quad = (\nabla \times \mathbf{B})_x / \mu_0 \quad = -0.5 \times dx \times (B_{5z} + B_{6z} - B_{6y} - B_{7y} - B_{7z} - B_{8z} + B_{8y} + B_{5y})$$

propagation of B on yface / yface curl





$E_4 = E(0,1,1)$

$E_8 = E(1,1,1) = \text{local}$

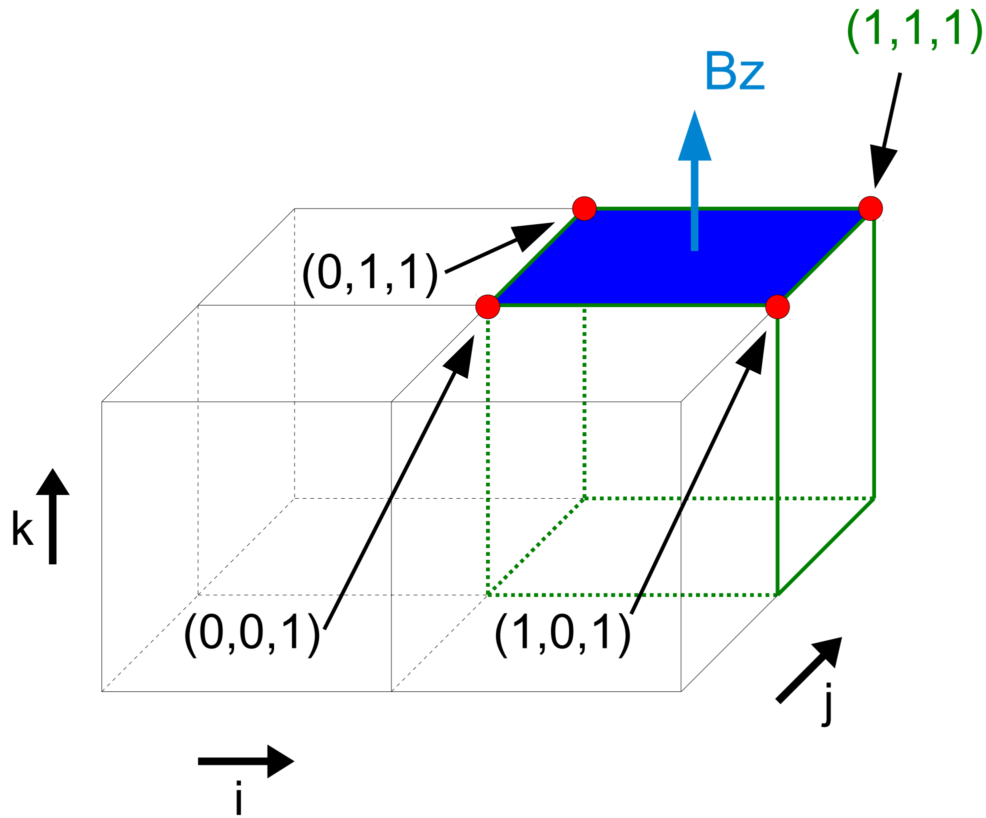
$E_7 = E(1,1,0)$

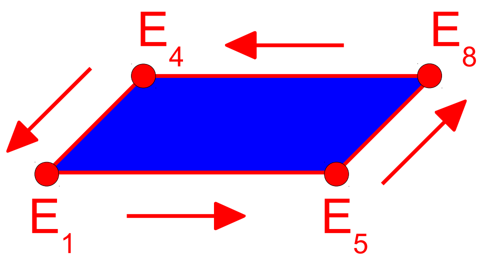
$E_3 = E(0,1,0)$

$$\partial\left(\int d\mathbf{A}_y \cdot \mathbf{B}\right)/\partial t \quad = -\int d\mathbf{A}_y \cdot (\nabla \times \mathbf{E}) \quad = 0.5 \times dx \times (-E_{4x} - E_{8x} + E_{8z} + E_{7z} + E_{7x} + E_{3x} - E_{3z} - E_{4z})$$

$$\mathbf{J}_y \quad = (\nabla \times \mathbf{B})_y / \mu_0 \quad = -0.5 \times dx \times (-B_{4x} - B_{8x} + B_{8z} + B_{7z} + B_{7x} + B_{3x} - B_{3z} - B_{4z})$$

propagation of B on zface / zface curl





$E_1 = E(0,0,1)$
$E_5 = E(1,0,1)$
$E_8 = E(1,1,1)$ = local
$E_4 = E(0,1,1)$

$$\partial(\int d\mathbf{A}_z\cdot\mathbf{B})/\partial t \quad = -\int d\mathbf{A}_z\cdot(\nabla\times\mathbf{E}) \quad = 0.5 \times dx \times (-E_{1x} - E_{5x} - E_{5y} - E_{8y} + E_{8x} + E_{4x} + E_{4y} + E_{1y})$$

$$\mathbf{J}_z \quad = (\nabla\times\mathbf{B})_z/\mu_0 \quad = -0.5 \times dx \times (-B_{1x} - B_{5x} - B_{5y} - B_{8y} + B_{8x} + B_{4x} + B_{4y} + B_{1y})$$