Statistics & Causal Inference:

Neyman & Fisher

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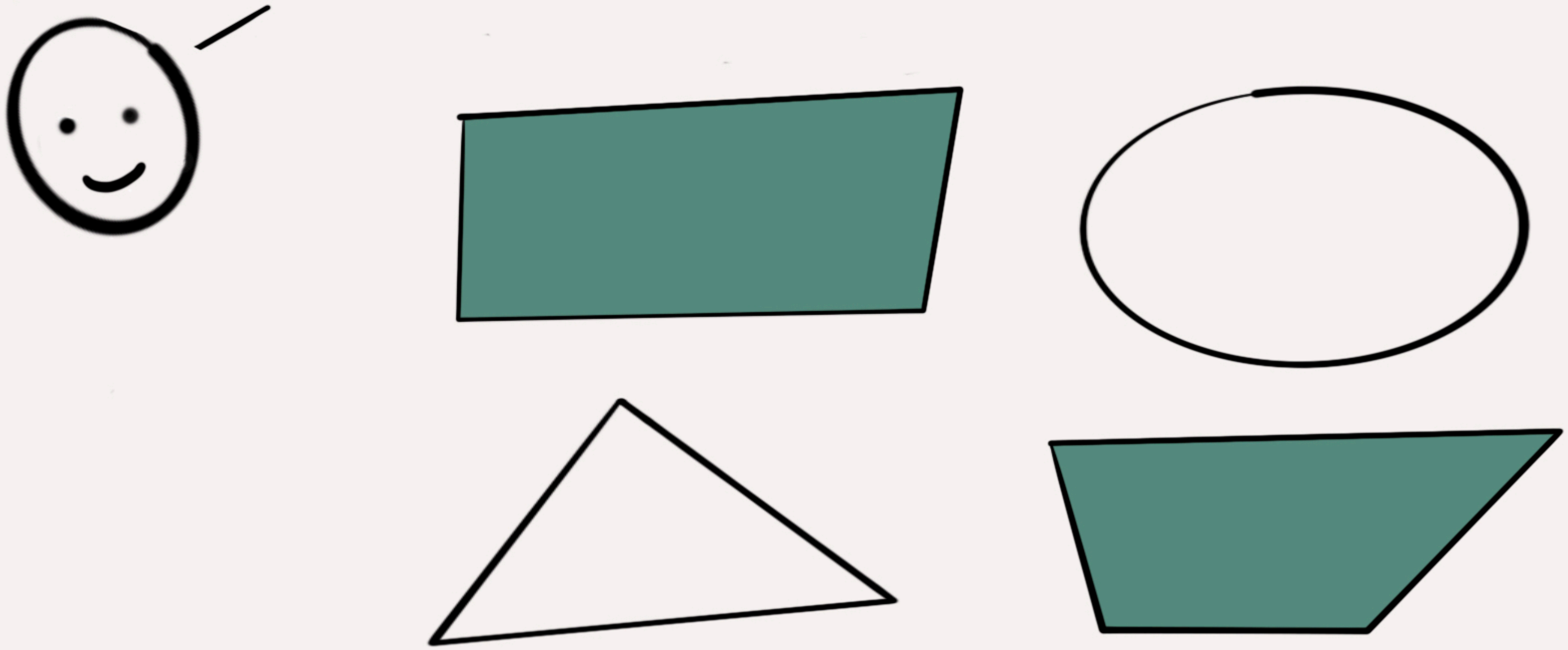
http://jakebowers.org

EGAP 2021
19 July 2021

What is the causal effect?



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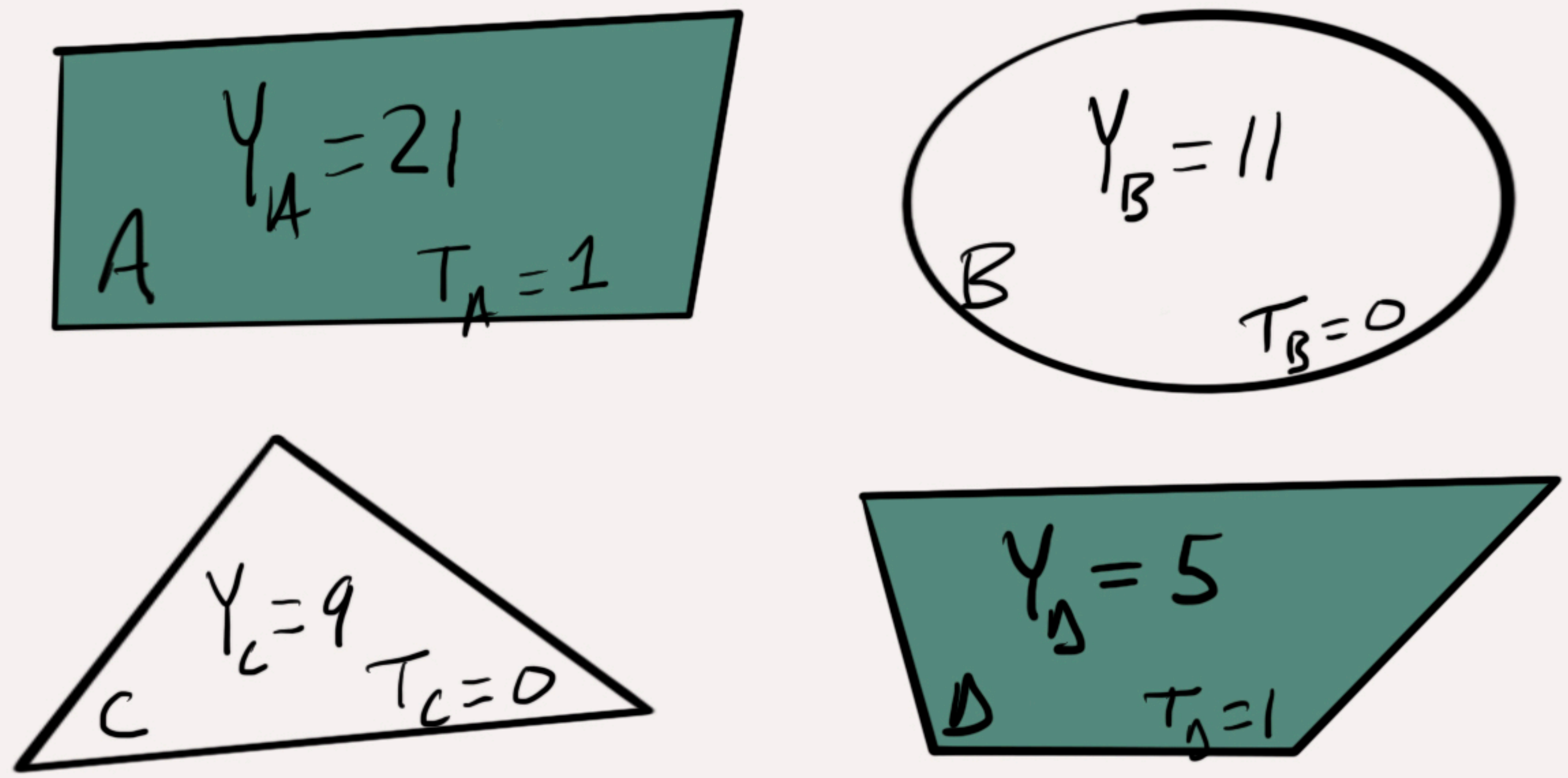
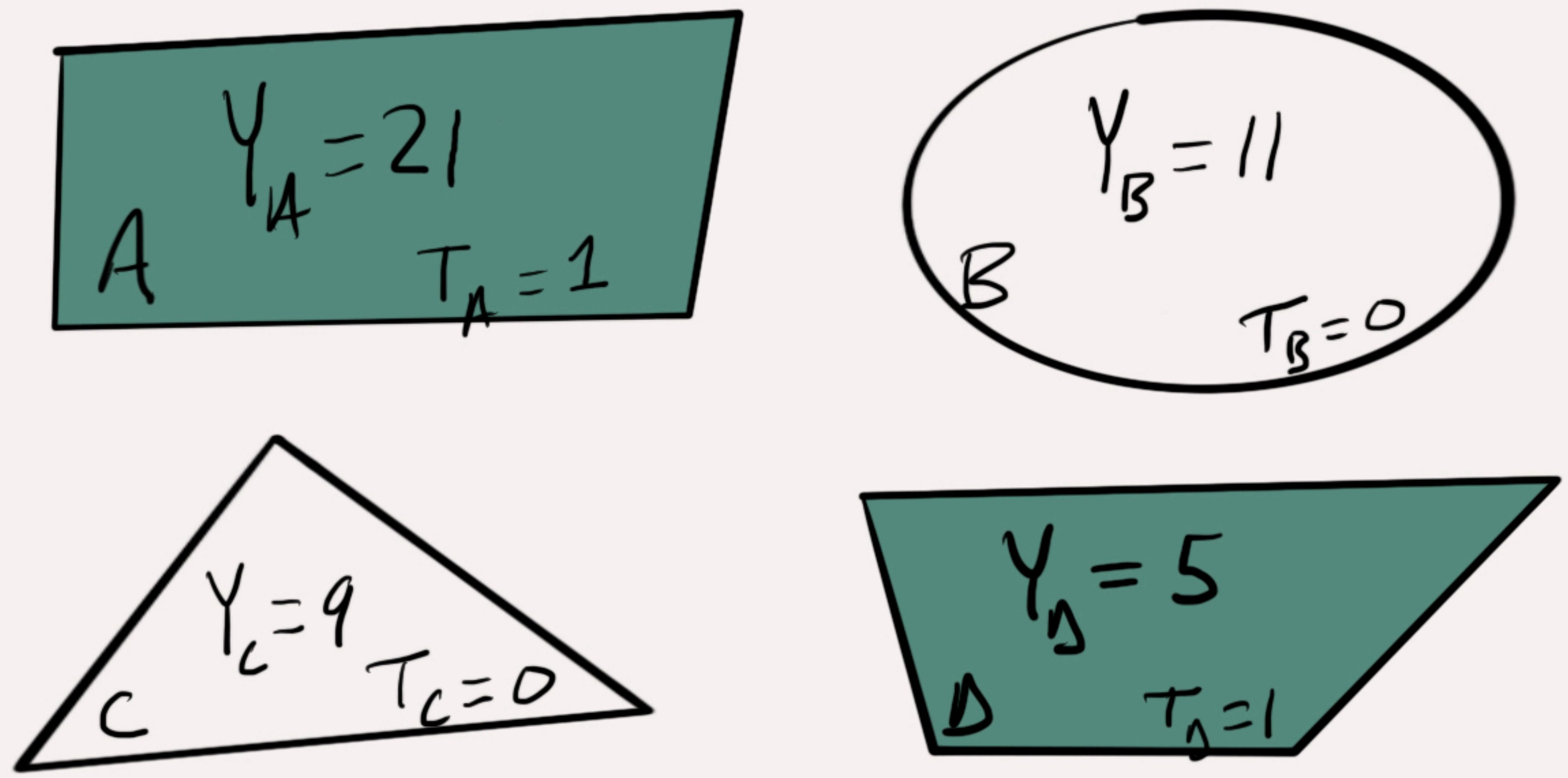


i Unit	T Treatment	Y Observed Outcome
A	1	21
B	0	11
C	0	9
D	1	5

What is the causal effect?



i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
A	1	21	21	?	$21 - ? = \tau_A$
B	0	11	?	11	$? - 11 = \tau_B$
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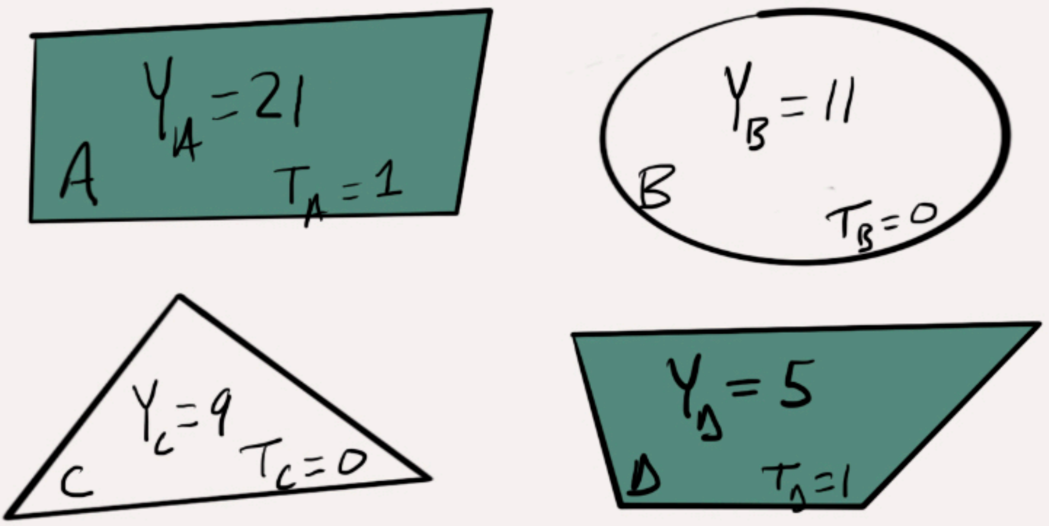
D	1	5	5	?	$5 - ? = \tau_D$

We don't know!



I don't know about the individual causal effects. BUT I can help you ESTIMATE the AVERAGE causal effect!

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
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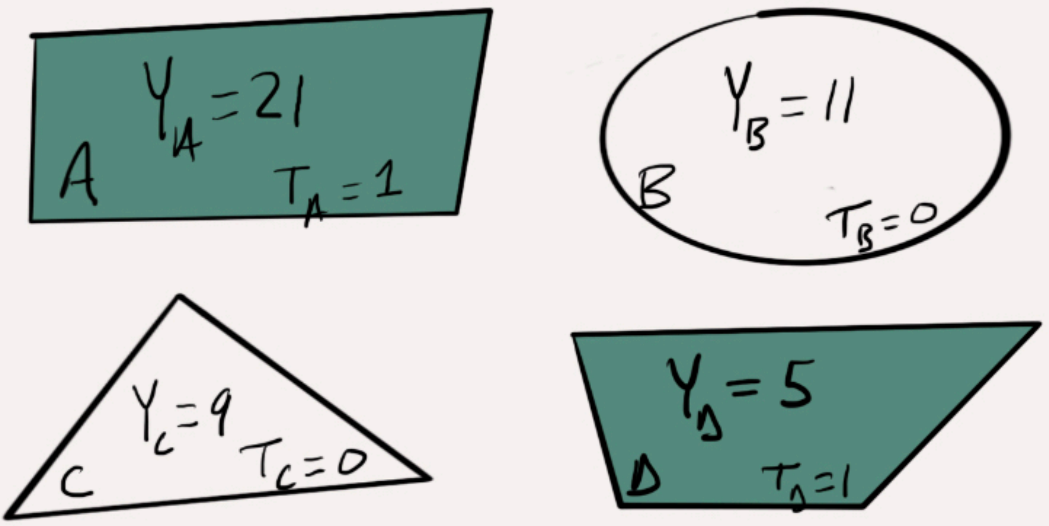
C	0	9	?	9	$? - 9 = \tau_C$
D	1	5	5	?	$5 - ? = \tau_D$

Define: $\bar{\tau} = (\tau_A + \tau_B + \tau_C + \tau_D)/4$ (The avg. causal effect)

Notice: $\tau_A = Y_A(1) - Y_A(0)$, $\tau_B = Y_B(1) - Y_B(0)$, ...

$$\bar{\tau} = \big((Y_A(1) - Y_A(0)) + \dots + (Y_D(1) - Y_D(0))\big)/4$$

$$= (Y_A(1) + \dots + Y_D(1))/4 - (Y_A(0) + \dots + Y_D(0))/4$$



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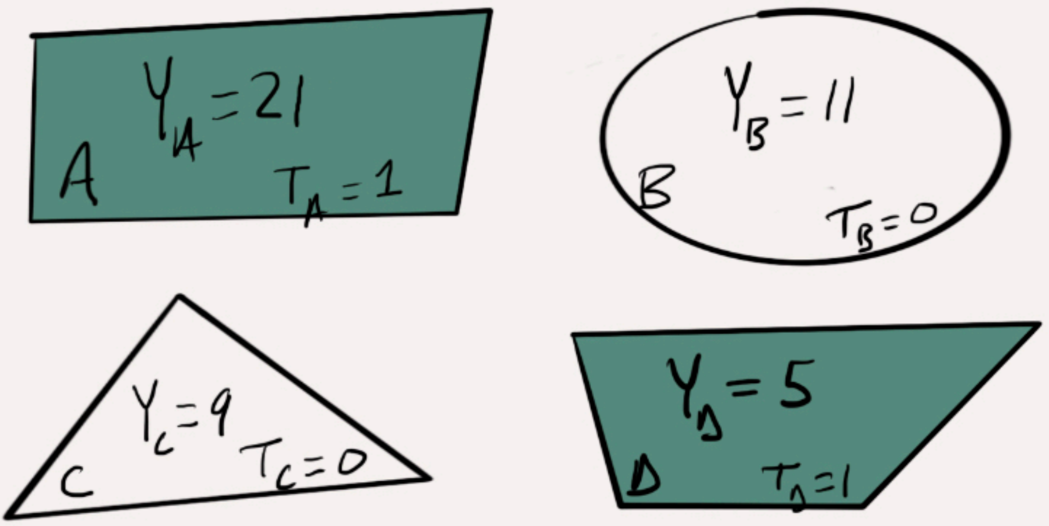
Define: $\bar{\tau} = (\tau_A + \tau_B + \tau_C + \tau_D)/4$ (The avg. causal effect)

$$= (Y_A(1) + \dots + Y_D(1))/4 - (Y_A(0) + \dots + Y_D(0))/4$$

an Unobserved Estimand

$$\bar{\tau} = \bar{Y}(1) - \bar{Y}(0)$$



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Define an Unobserved Estimand

$$\bar{\tau}=(\tau_A+\tau_B+\tau_C+\tau_D)/4$$

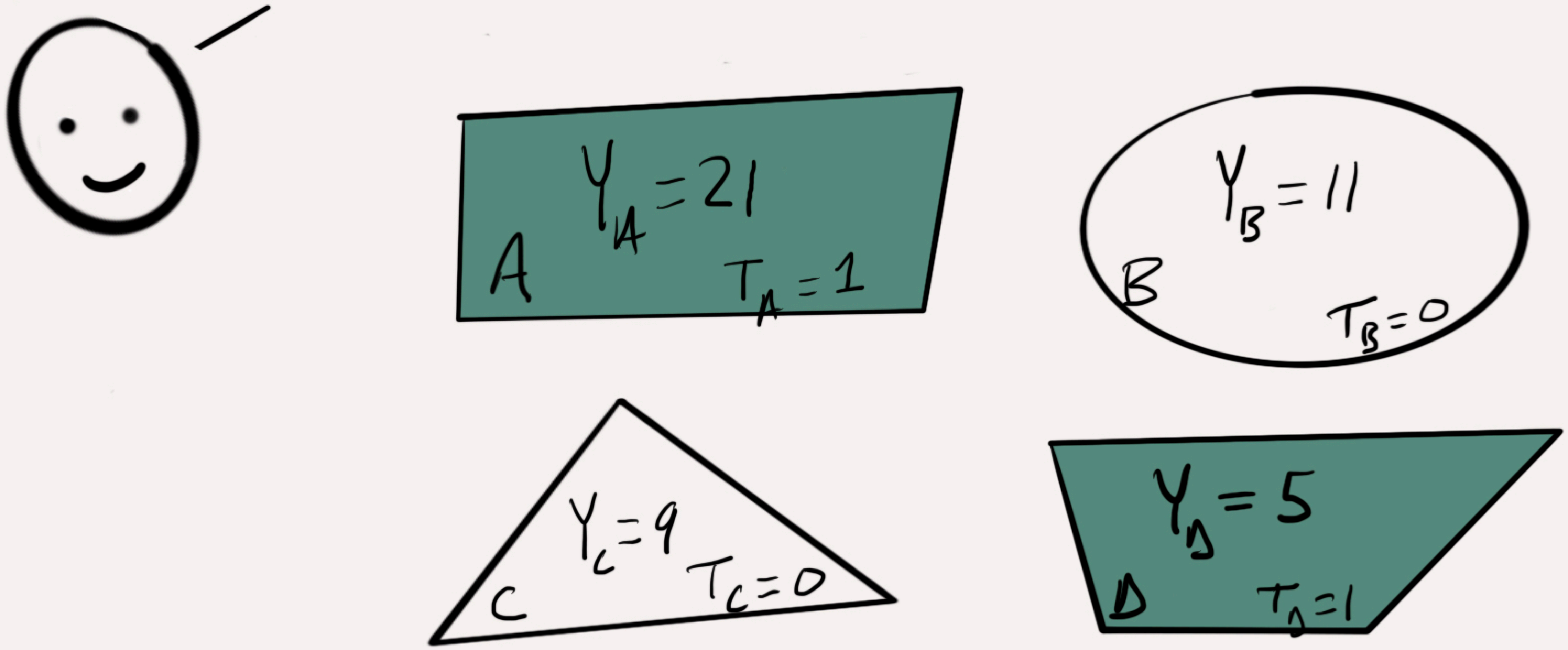
$$\bar{\tau}=\bar{Y}(1)-\bar{Y}(0)$$

Choose an Estimator

$$\hat{\bar{\tau}}=\hat{\bar{Y}}(1)-\hat{\bar{Y}}(0)=\frac{(21+5)}{2}-\frac{(11+9)}{2}=13-10=3$$

(Show (Later) that $\hat{\bar{\tau}}$ is a good estimator of $\bar{\tau}$)

What is the causal effect?



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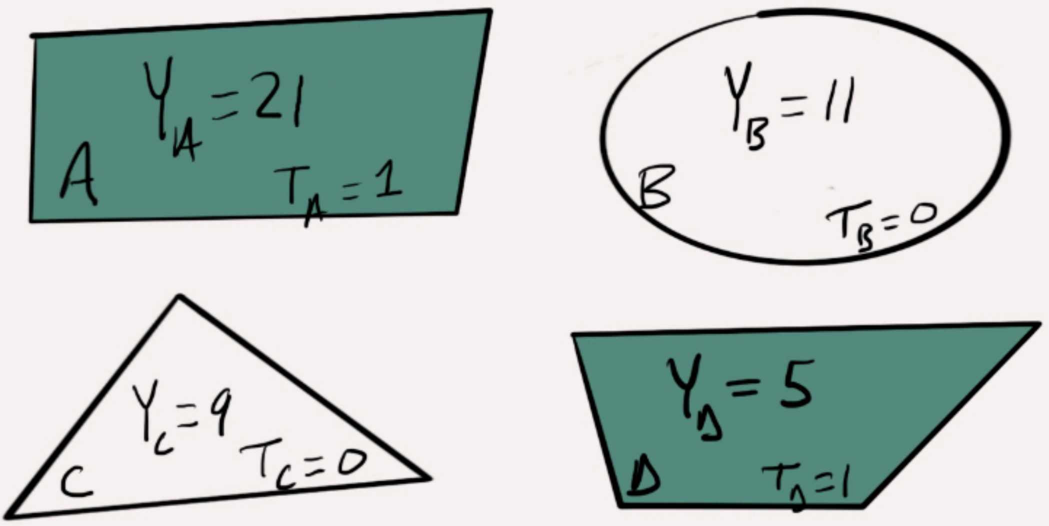

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An idea: "No effects"

"$H_0: Y_i(1) = Y_i(0)$"

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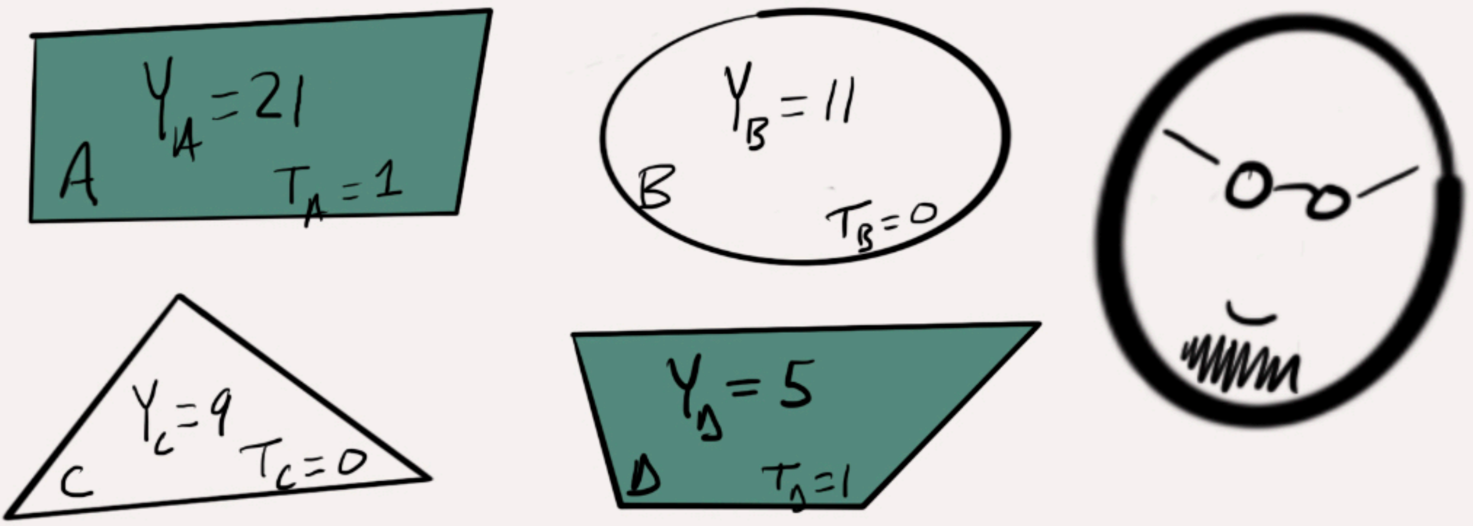

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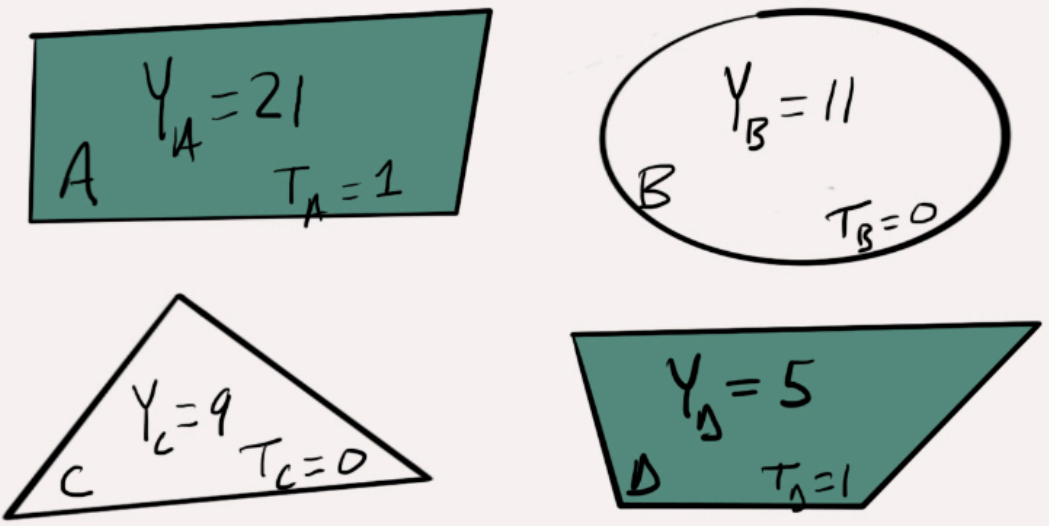
B	0	11	?	11	$? - 11 = \tau_B$
C	0	9	?	9	$? - 9 = \tau_C$
D	1	5	5	5	$5 - ? = \tau_D$

What does this idea imply?

Notice: $Y_i = T_i Y_i(1) + (1 - T_i) Y_i(0)$

So H_0 implies: $Y_i = T_i Y_i(0) + (1 - T_i) Y_i(0)$

$= Y_i(0)$



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An idea: "No effects"

"$H_0: Y_i(1) = Y_i(0)$"

$H_0 \Rightarrow Y_i = Y_i(0)$

observed

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
A	1	21	21	21	$21 - ? = \tau_A$
B	0	11	?	11	$? - 11 = \tau_B$
C	0	9	?	9	$? - 9 = \tau_C$
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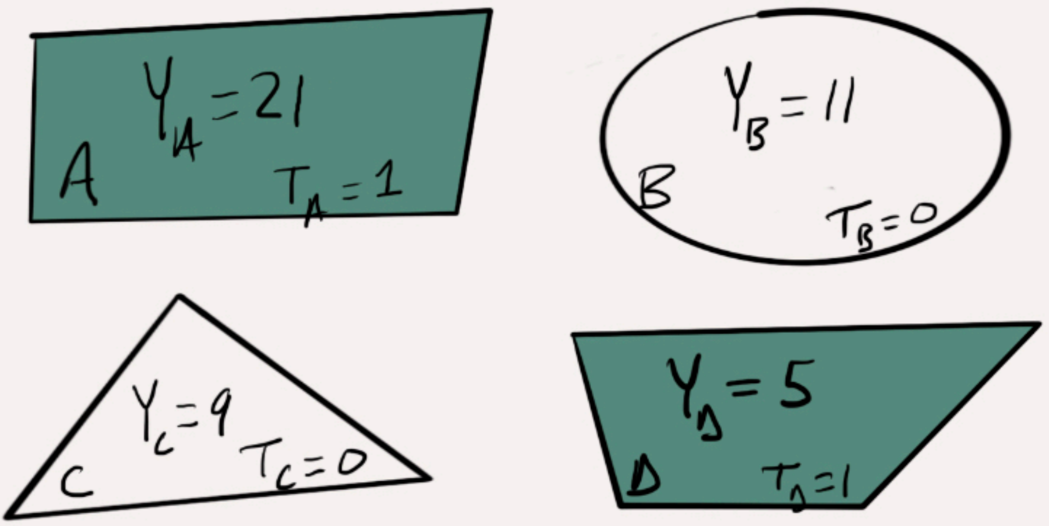
So: $\frac{(21+5)}{2} - \frac{(11+9)}{2} = 3$ is compatible with H_0 even if not $\emptyset$.

But, A & D are only Treated by chance.

If A & B were treated, $H_0 \Rightarrow \frac{(21+11)}{2} - \frac{(9+5)}{2} = 9$

If C & D were treated, $H_0 \Rightarrow \frac{(9+5)}{2} - \frac{(21+11)}{2} = -9$



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1. Idea/H_0: $Y_i(1)=Y_i(0)$
2. H_0 implies $Y_i=Y_i(0)$
3. H_0 implies that a test statistic summarizing $T\to Y$,

$$t(T,Y)=\left(\frac{21+5}{2}\right)-\left(\frac{11+9}{2}\right)$$
$$=3$$

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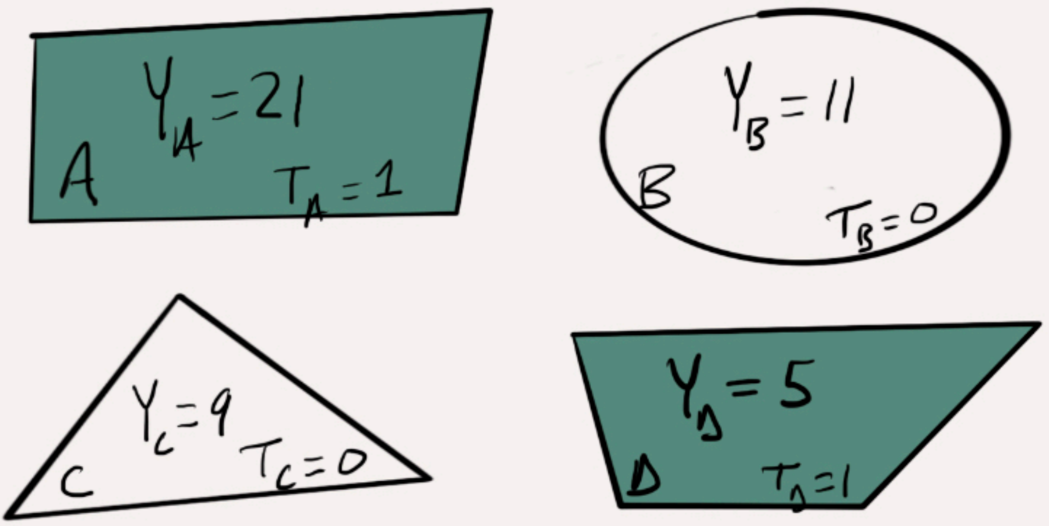
But also these other possible $t(T,Y)$s :

i	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$	$\tilde{T}_4$	$\tilde{T}_5$	$\tilde{T}_6$
A	1	1	1	0	0	0
B	0	1	0	1	1	0
C	0	0	1	1	0	1
D	1	0	0	0	1	1
	3	9	7	−3	−7	−9



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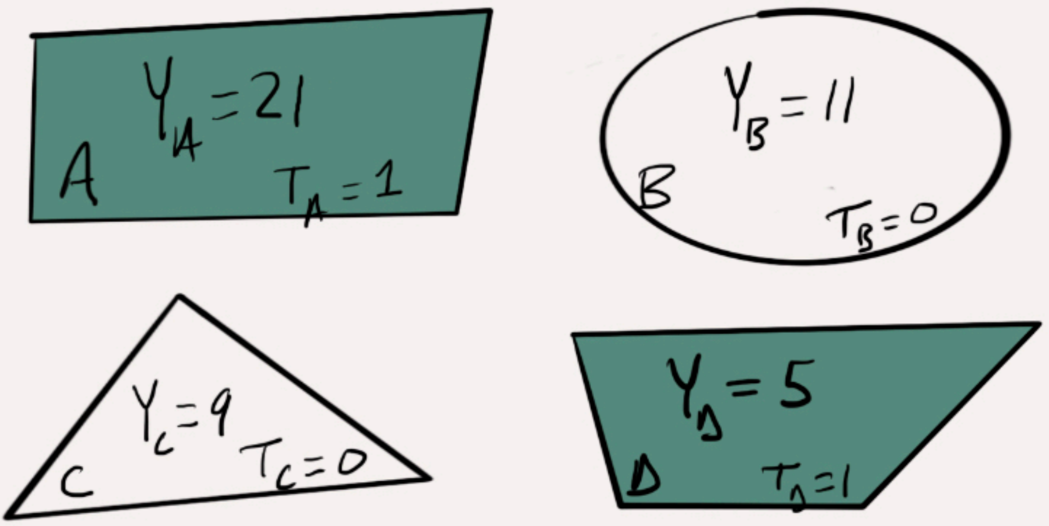
1. Idea/H_0: $Y_i(1) = Y_i(0)$
2. H_0 implies $Y_i = Y_i(0)$
3. H_0 and Randomization imply 6 equally likely $t(T, Y)$ values.

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
A	1	21	21	21	$21 - ? = \tau_A$
B	0	11	?	11	$? - 11 = \tau_B$
C	0	9	?	9	$? - 9 = \tau_C$
D	1	5	5	5	$5 - ? = \tau_D$

In this experiment, "No effects" means

i	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$	$\tilde{T}_4$	$\tilde{T}_5$	$\tilde{T}_6$
A	1	1	1	0	0	0
B	0	1	0	1	1	0
C	0	0	1	1	0	1
D	1	0	0	0	1	1
	3	9	7	−3	−7	−9

$t(T, Y)$ No effects



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② H_0 implies $Y_i = Y_i(0)$

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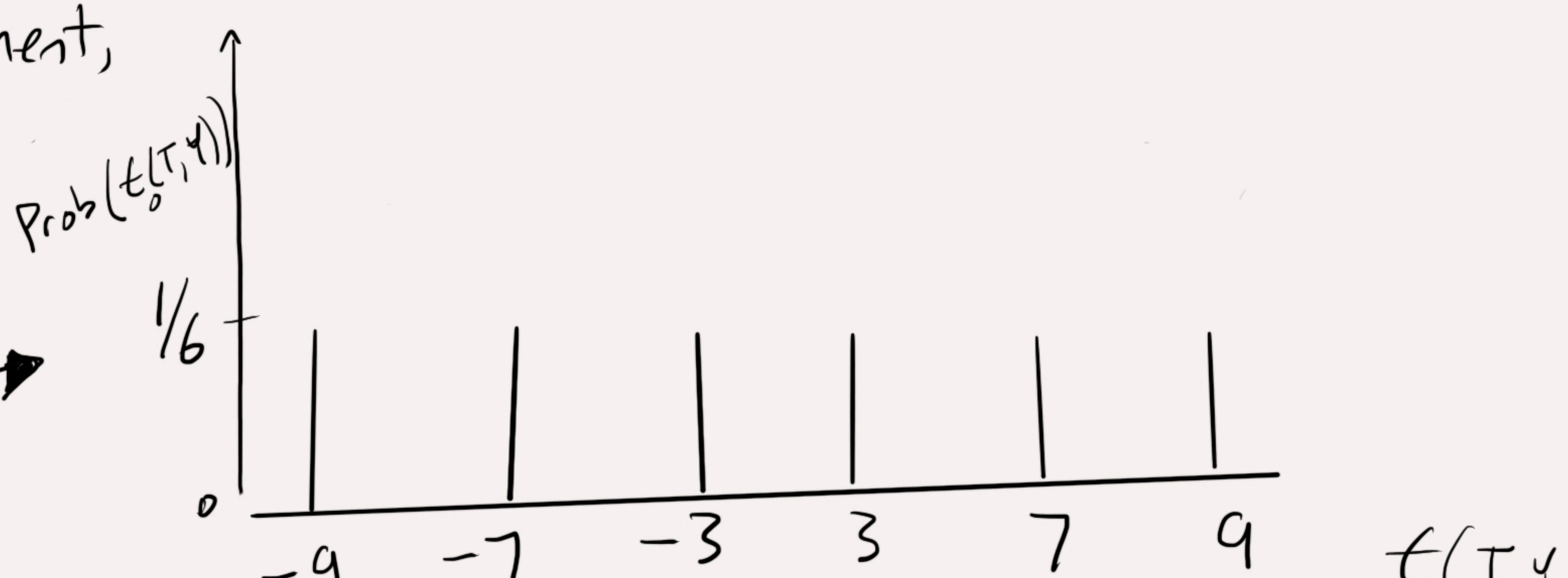

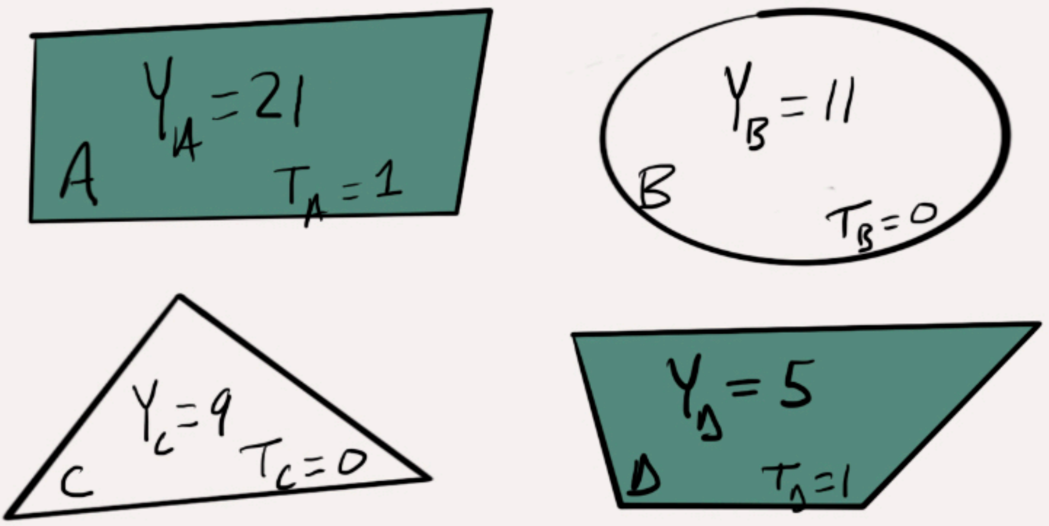

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① Idea / H_0: $Y_i(1) = Y_i(0)$

② H_0 implies $Y_i = Y_i(0)$

③ H_0 and Randomization imply $\underline{\underline{6}}$ equally likely $t_0(T, Y)$ values.

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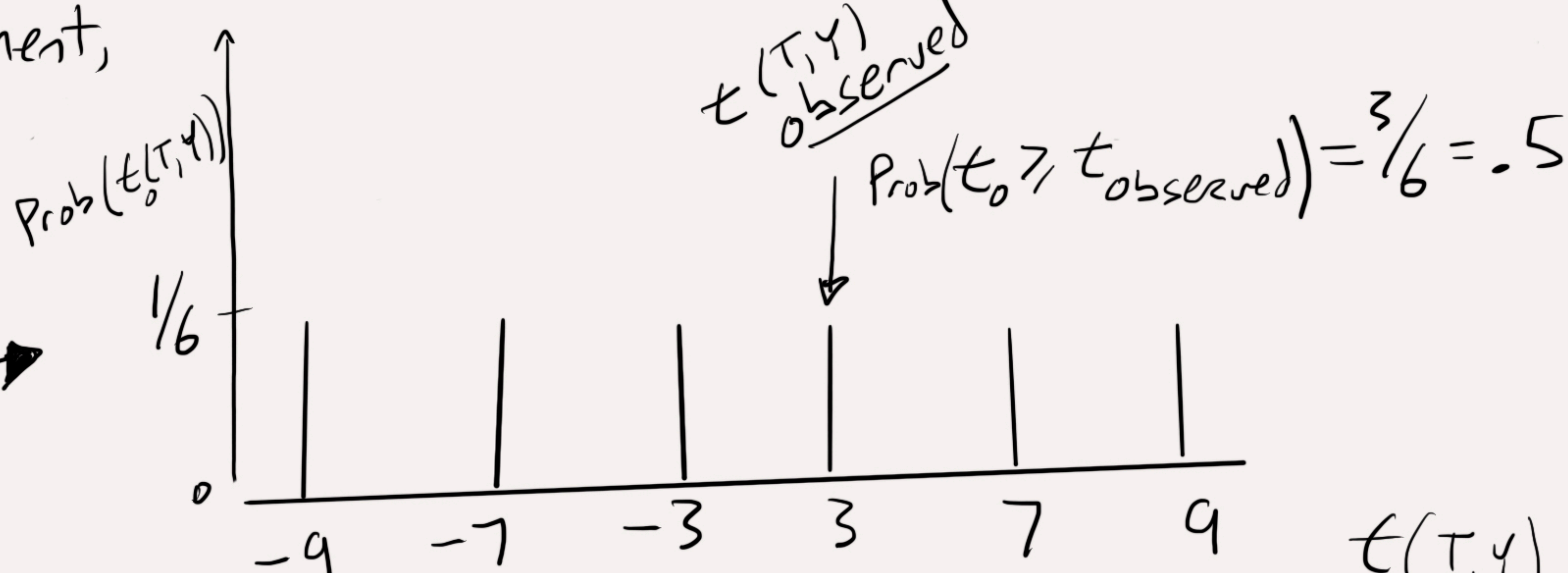
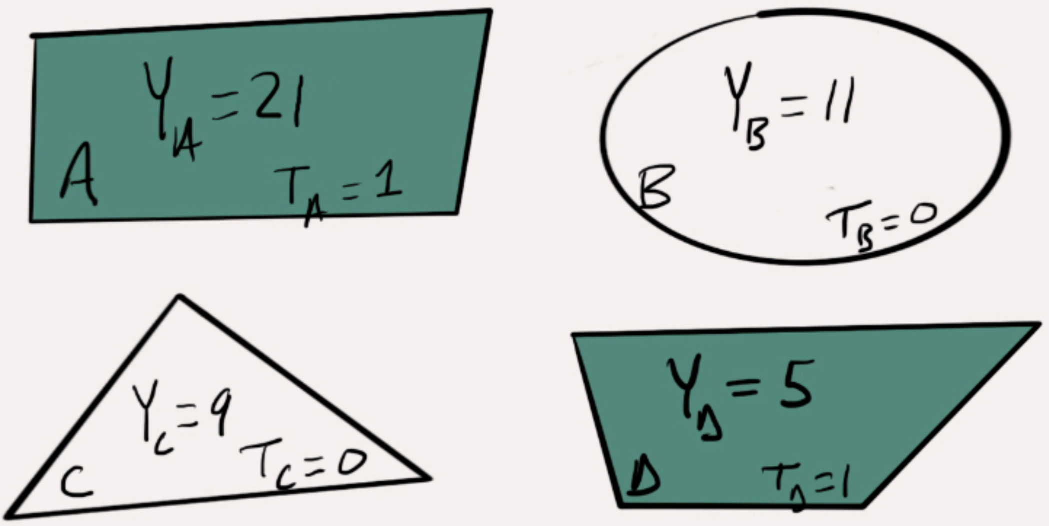
In this experiment, "No effects" means





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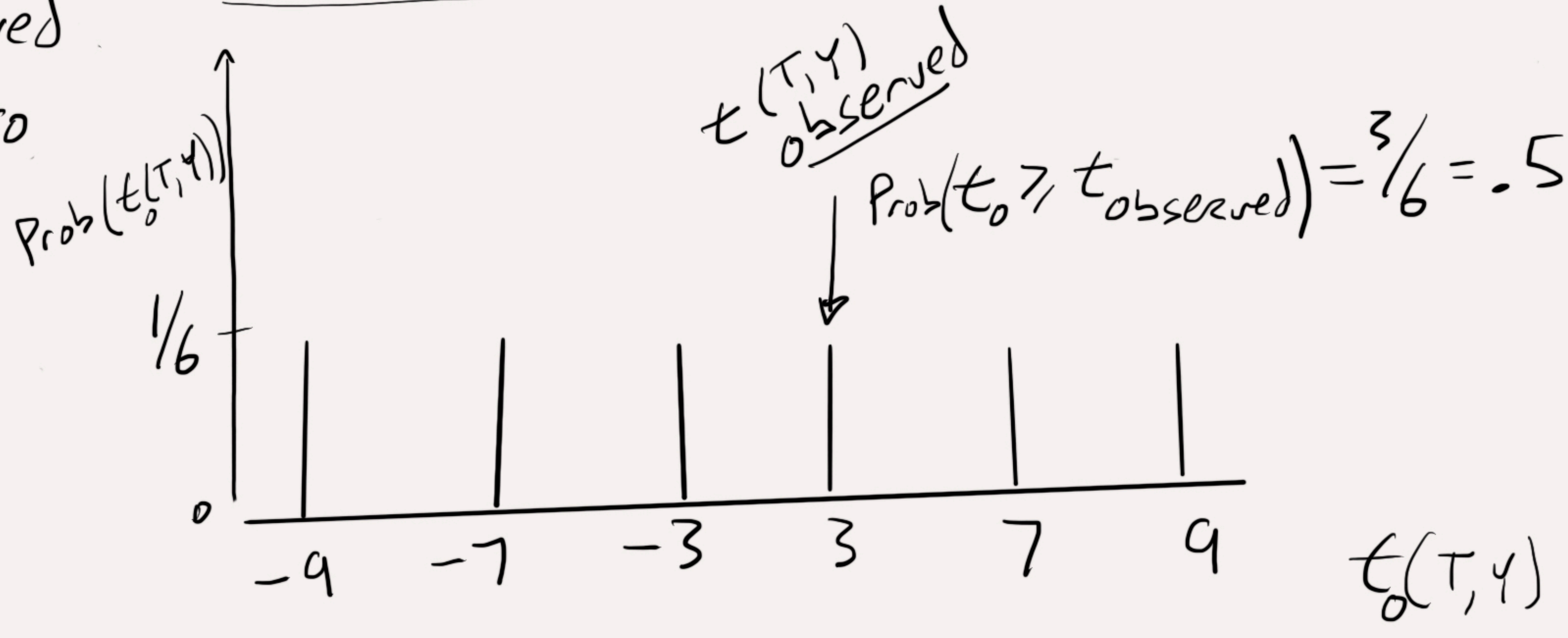
④ Compare observed to implied by H_0 using a p-value

⑤ (Later) Show that this is a good test



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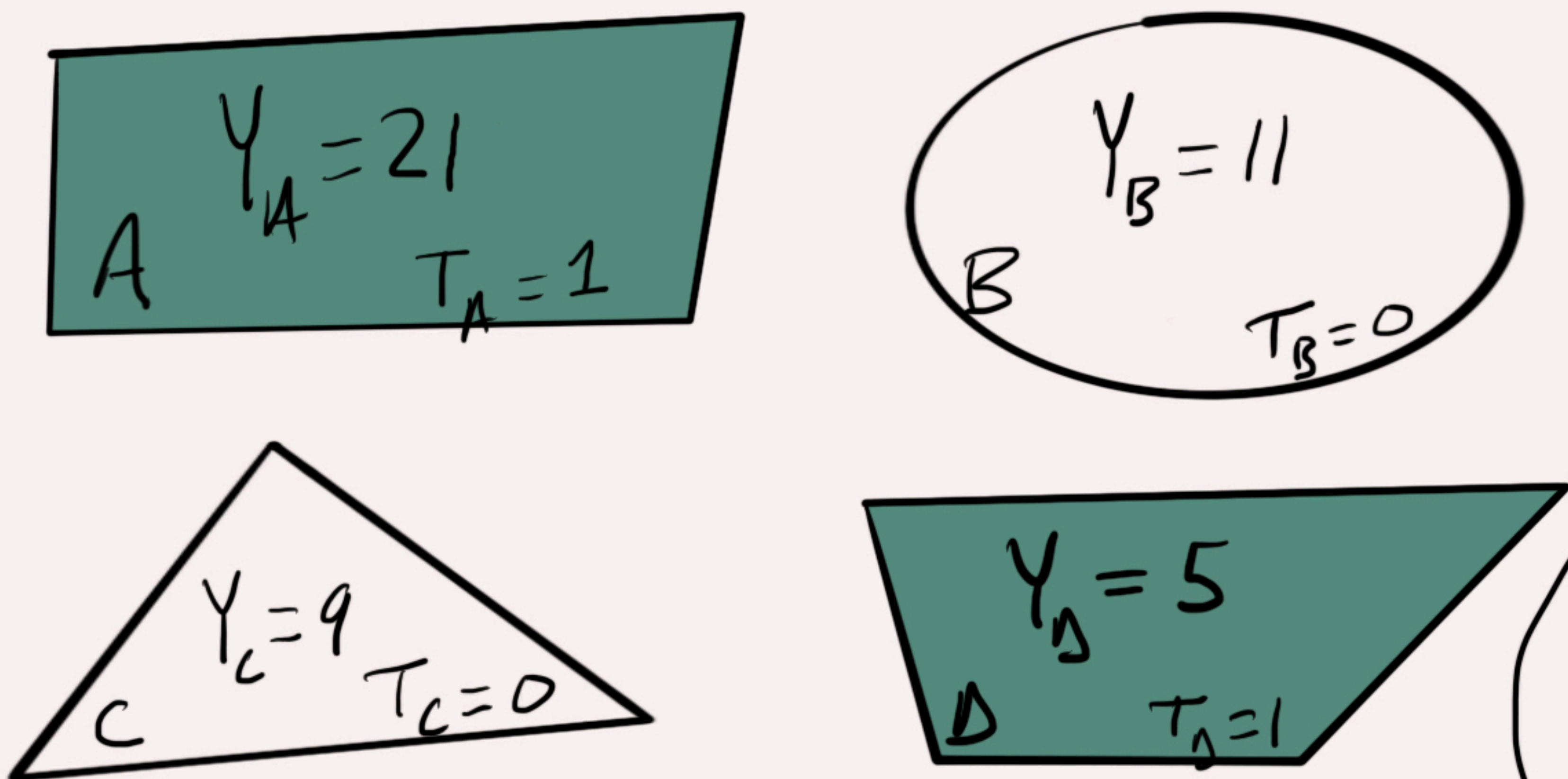

We don't know!

But...

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You can estimate the average of the τ_i.

You can test ideas about the τ_i.