Statistics of Causal Inference: Estimands, Estimation, Hypotheses, Tests

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05 June, 2022

EVIDENCE IN GOVERNANCE AND POLITICS

Key points for this lecture

- Randomization helps us learn about the unobservable using the observed
- We can assess the evidence pertaining to provisional ideas arising from theories explaining the treatment mechanism (like the hypothesis that despite our theoretical expectations, the treatment had no effect on any unit).
  - We can evaluate tests: We'd like them to rarely cast doubt on truth claims (low and controlled false positive rate) and often cast doubt on false claims (high statistical power).

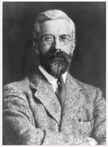
Key points for this lecture

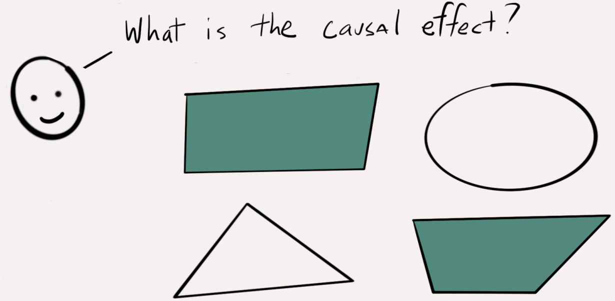
- Randomization helps us learn about the unobservable using the observed
- We can calculate guesses (estimates) of certain unobserved aggregated quantities (like the average of the causal effects).
  - The recipe for calculating estimates is called an "estimator"
  - The recipe for summarizing how the guesses might vary depending on different specific manifestations of the design (different randomizations) is called the "variance" of the estimator: $\sqrt{\text{variance}}$ is the "standard error".
  - We can evaluate estimators (we'd like them to be "unbiased" or at least "consistent" or "precise" or "efficient").

Statistics and Causal Inference: Review of Hypothesis Testing

Statistics & Causal Inference:

Neyman & Fisher

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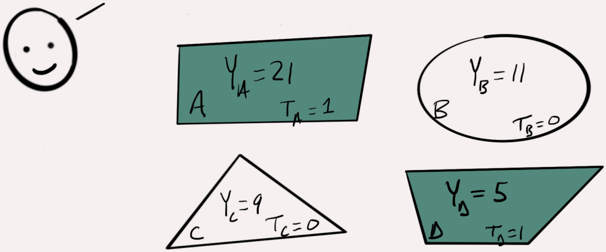
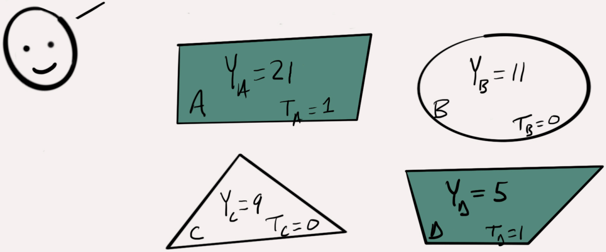


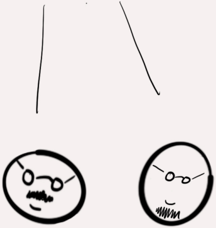
i Unit	T Treatment	Y Observed Outcome
A	1	21
B	0	11
C	0	9
D	1	5

What is the causal effect?



i	T	Y	Y(T=1)	Y(T=0)	Effect (τ)
A	1	21	21	?	$21-?=\tau_A$
B	0	11	?	11	$?-11=\tau_B$
C	0	9	?	9	$?-9=\tau_C$
D	1	5	5	?	$5-?=\tau_D$

We don't know!



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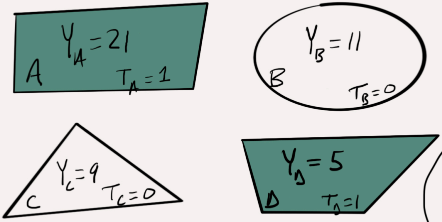

We don't know!

But...

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
A	1	21	21	?	$21 - ? = \tau_A$
B	0	11	?	11	$? - 11 = \tau_B$
C	0	9	?	9	$? - 9 = \tau_C$
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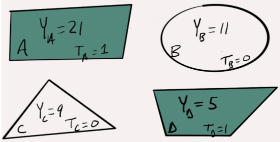
You can estimate the average of the τ_i.

You can test ideas about the τ_i.



I don't Know about the individual causal effects. BUT I can help you TEST your ideas / HYPOTHESES about them.

An idea: "No effects"

"$H_0: Y_i(1) = Y_i(0)$"

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A	1	21	21	?	$21 - ? = \tau_A$
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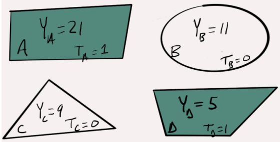

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C	0	9	?	9	$? - 9 = \tau_C$
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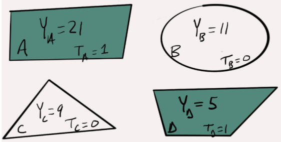
What does this idea imply?

Notice: $Y_i = T_i \, Y_i(1) + (1 - T_i) \, Y_i(0)$

So H_0 implies: $Y_i = T_i \, Y_i(0) + (1 - T_i) \, Y_i(0)$

$= Y_i(0)$



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An idea: "No effects"

"$H_0: Y_i(1) = Y_i(0)$"

$H_0 \Rightarrow Y_i = Y_i(0)$

observed

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
A	1	21	21	21	$21 - ? = \tau_A$
B	0	11	?	11	$? - 11 = \tau_B$
C	0	9	?	9	$? - 9 = \tau_C$
D	1	5	5	5	$5 - ? = \tau_D$

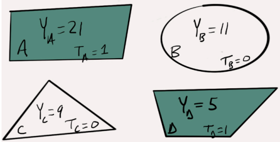
So: $\frac{(21+5)}{2} - \frac{(11+9)}{2} = 3$ is compatible with H_0 even if not $\emptyset$.

But, A & D are only Treated by chance.

If A & B were treated, $H_0 \Rightarrow \frac{(21+11)}{2} - \frac{(9+5)}{2} = 9$

If C & D were treated, $H_0 \Rightarrow \frac{(9+5)}{2} - \frac{(21+11)}{2} = -9$

EVIDENCE IN GOVERNANCE AND POLITICS



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① Idea/H_0: $Y_i(1) = Y_i(0)$

② H_0 implies $Y_i = Y_i(0)$

③ H_0 implies that a test statistic summarizing $T \rightarrow Y$,

$$t(T, Y) = \left(\frac{21+5}{2}\right) - \left(\frac{11+9}{2}\right) = 3$$

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
A	1	21	21	21	$21 - ? = \tau_A$
B	0	11	?	11	$? - 11 = \tau_B$
C	0	9	?	9	$? - 9 = \tau_C$
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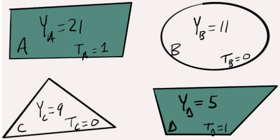
But also these other possible $t(T, Y)$s:

i	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$	$\tilde{T}_4$	$\tilde{T}_5$	$\tilde{T}_6$
A	1	1	1	0	0	0
B	0	1	0	1	1	0
C	0	0	1	1	0	1
D	1	0	0	0	1	1
	3	9	7	−3	−7	−9



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① Idea/H_0: $Y_i(1) = Y_i(0)$

② H_0 implies $Y_i = Y_i(0)$

③ H_0 and Randomization imply 6 equally likely $t(T, Y)$ values.

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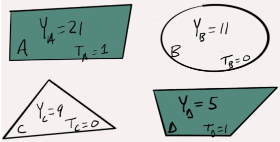
A	1	21	21	21	$21 - ? = \tau_A$
B	0	11	?	11	$? - 11 = \tau_B$
C	0	9	?	9	$? - 9 = \tau_C$
D	1	5	5	5	$5 - ? = \tau_D$

In this experiment, "No effects" means

i	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$	$\tilde{T}_4$	$\tilde{T}_5$	$\tilde{T}_6$
A	1	1	1	0	0	0
B	0	1	0	1	1	0
C	0	0	1	1	0	1
D	1	0	0	0	1	1
	3	9	7	−3	−7	−9

$t(T, Y)$ no effects



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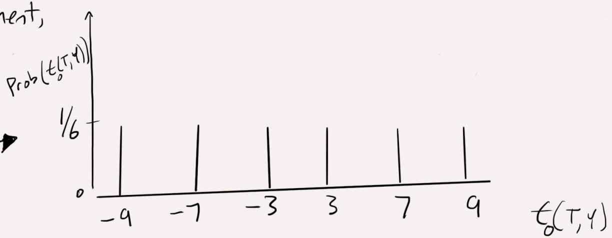

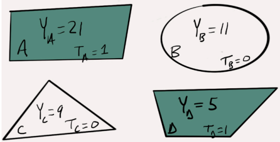

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1. Idea/H_0: $Y_i(1)=Y_i(0)$
2. H_0 implies $Y_i=Y_i(0)$
3. H_0 and Randomization imply $\underline{6}$ equally likely $t_0(T,Y)$ values.

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
A	1	21	21	21	$21-?=\tau_A$
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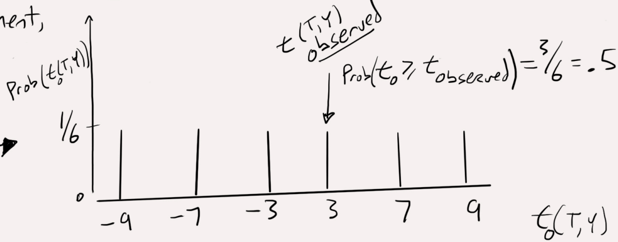

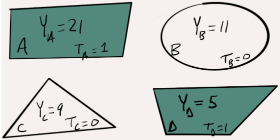

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3. H_0 and Randomization imply $\underline{6}$ equally likely $t_0(T, Y)$ values.
4. Compare observed to implied by H_0 using a p-value
5. (Later) Show that this is a good test

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
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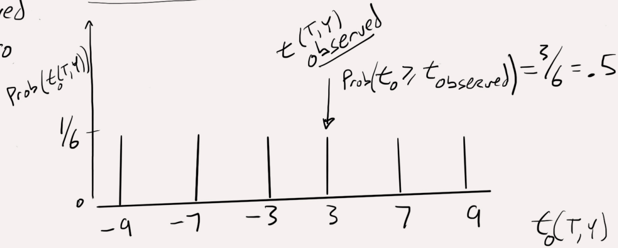


What else to know about hypothesis testing?

Things to learn about later. Things to remember.

- An estimator can be used as a test-statistic. (Regression tables are confusing! Bowers and Leavitt (2020) explain how the difference of means produced as an estimator of the average causal effect in a regression table can also be a test statistic for the weak null of no effects if it is an unbiased estimator.)

Things to learn later:

- A good testing procedure controls the false positive rate and keeps it low (if I'm willing to reject the null up to 5% of the time, then my test should not encourage me to reject the null 10% of the time).
- A good testing procedure should be sensitive to departures from the null (it should detect signal from noise; it has high statistical power; it has low false negative rate — rarely says "not enough information to seriously doubt the null of no effects" if we should doubt the null (because the treatment had an effect).

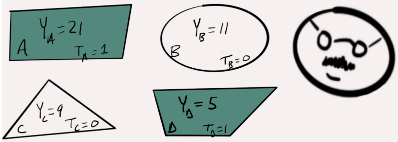
Quiz:

What are the key ingredients of the hypothesis testing approach to statistical inference for counterfactual causal effects? (Roughly 5 or 6 ingredients).

Some quiz answers

- A hypothesis stated in terms of potential outcomes
- A connection between potential outcomes and observed outcomes. (The identity function above, for example).
- A way to reflect the hypothesis in terms of observed data — an answer to to "What would we see granting the hypothesis for the sake of argument?" (A test statistic and a connection between potential outcomes and observed outcomes.)
- The distribution the test statistic would take on if the hypothesis were true. An explanation for why we would use **this** over **that** distribution. In an experiment we use the design of the study to construct this explanation.
- An observed value of the test statistic.
- A *p*-value to summarize the information that the design and test statistic contain that pertains to the hypothesis: a large *p*-value suggests that the design and test statistic has little to say about the hypothesis, a small *p*-value says that the observed value of the test statistic would be surprising from the perspective of the hypothesis — it encourages us to doubt the hypothesis.

Statistics and Causal Inference: Estimation



I don't know about the individual causal effects. BUT I can help you ESTIMATE the AVERAGE causal effect!

i	T	Y	$Y(T=1)$	$Y(T=0)$	Effect (τ)
A	1	21	21	?	$21-?=\tau_A$
B	0	11	?	11	$?-11=\tau_B$
C	0	9	?	9	$?-9=\tau_C$
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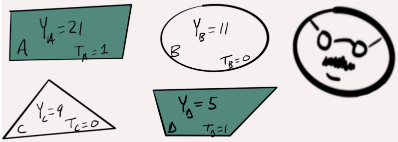
Define: $\bar{\tau}=(\tau_A+\tau_B+\tau_C+\tau_D)/4$ (The avg. causal effect)

Notice: $\tau_A=Y_A(1)-Y_A(0),\ \tau_B=Y_B(1)-Y_B(0), \ldots$

$$\bar{\tau}=\left((Y_A(1)-Y_A(0))+\ldots+(Y_D(1)-Y_D(0))\right)/4$$

$$=(Y_A(1)+\ldots+Y_D(1))/4-(Y_A(0)+\ldots+Y_D(0))/4$$



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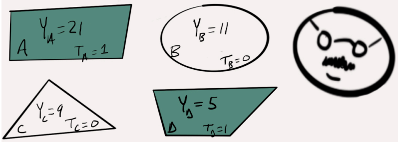
D	1	5	5	?	$5-?=\tau_D$

Define: $\bar{\tau} = (\tau_A + \tau_B + \tau_C + \tau_D)/4$ (The avg. causal effect)

an Unobserved Estimand

$$= (Y_A(1) + \dots + Y_D(1))/4 - (Y_A(0) + \dots + Y_D(0))/4$$

$$\bar{\tau} = \overline{Y(1)} - \overline{Y(0)}$$



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Define an Unobserved Estimand

$$\bar{\tau}=(\tau_A+\tau_B+\tau_C+\tau_D)/4$$

$$\bar{\tau}=\overline{Y(1)}-\overline{Y(0)}$$

Choose an Estimator

$$\hat{\bar{\tau}}=\hat{\bar{Y}}(1)-\hat{\bar{Y}}(0)=\frac{(21+5)}{2}-\frac{(11+9)}{2}=13-10=3$$

(Show (Later) that $\hat{\bar{\tau}}$ is a good estimator of $\bar{\tau}$)

References

References

Bowers, Jake, and Thomas Leavitt. 2020. "Causality & Design-Based Inference." In *The SAGE Handbook of Research Methods in Political Science and International Relations*, edited by Luigi Curini and Robert Franzese. Sage Publications Ltd.