

Introducing Counterfactual Causal Inference

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Some of my recent causal questions

Did a new Hausa television station in northern Nigeria change attitudes about violence, the role of women in society, or the role of youth in society?

Did a short video message from Michelle Obama increase college attendance among low income youth in the USA?

Does information about whether an election is contested increase mail-in voting among US farmers?



What does “cause” mean?

“X causes Y” might mean:

Persistent association “We always/mostly see $Y = 1$ when $X = 1$ and $Y = 0$ when $X = 0$.”

Counterfactual Difference “If X had not been 1, then Y would not have been 1.”

Difference after manipulation “When we change X from 0 to 1, then Y changes from 0 to 1.”
(establishes causal priority of X over Y).

Difference after operation of a mechanism “ X causes Y because once upon a time ...until one day ...and because of that, until finally.”

other...

Extra: If you want to dig into this see Brady (2008).

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Most randomized experiments combine the manipulation and counterfactual definitions.

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Observational studies vs. Randomized studies

Discuss in small groups: Help me design the next project to answer this question (or one of your own causal questions):

Did a new Hausa television station in northern Nigeria change attitudes about violence, the role of women in society, or the role of youth in society?

- ① What would be an ideal observational study design? (no randomization) What questions would people ask when you claim that your results reflect a causal relationship?
- ② What would be an ideal experimental study design? (including randomization) What questions would people ask when you claim that your results reflect a causal relationship?

Counterfactual Causal Inference

How can we use what we see to learn about what we want to know?

The effect of treatment can be defined in different ways using potential outcomes (leaving it vague for now).

City	Pair	Treatment	Baseline	Outcome	Turnout		y_1	y_0
					Newspaper			
Saginaw	1	0	17	16			?	16
Sioux City	1	1	21	22	Sioux City Journal		22	?
Battle Creek	2	0	13	14			?	14
Midland	2	1	12	7	Midland Daily News		7	?
Oxford	3	0	26	23			?	23
Lowell	3	1	25	27	Lowell Sun		27	?
Yakima	4	0	48	58			?	58
Richland	4	1	41	61	Tri-City Herald		61	?

Design and outcomes in the Newspapers Experiment. The Treatment column shows treatment randomized within pair with the newspaper ads as 1 and lack of treatment as 0. The potential outcomes are y_1 for treatment and y_0 for control. Panagopoulos (2006) provides more detail on the design of the experiment.

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Counterfactual Causal Inference

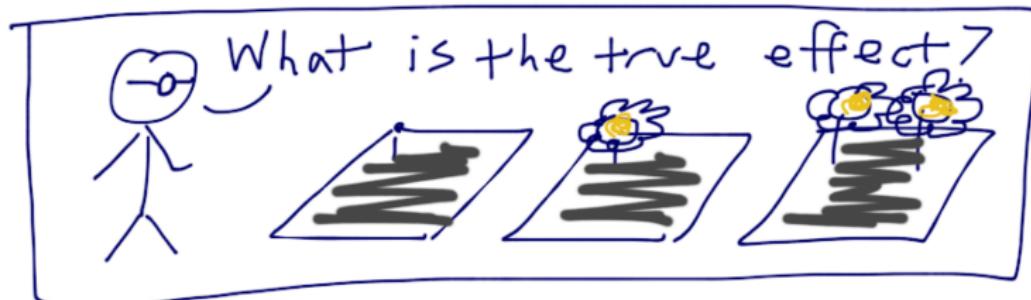
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What is the true effect of the treatment assignment?



We don't know.



What is the true effect of the treatment assignment?



I don't know the truth, but I can provide a good guess of the average causal effect.

i	z_i	y_i	y_{i1}	y_{i0}
A	0	16	?	16
B	1	22	22	?
C	0	7	?	7
D	1	14	14	?

$$\widehat{ATE} = \bar{Y}_1 | z_i=1 - \bar{Y}_0 | z_i=0$$
$$= \frac{22+14}{2} - \frac{16+7}{2} = 6.5$$

What is the true effect of the treatment assignment?

I dew nut knew thee truth,
but, given pryers, I cane
predikte itf
probabeeleetee.



i	Z_i	y_i	y_{i1}	y_{i0}
A	0	16	16	16
B	1	22	22	22
C	0	7	7	7
D	1	14	14	14

$$P(\text{[speech bubble]} \rightarrow f(y_1 - y_0)) = \text{[wavy line]}$$

What is the true effect of the treatment assignment?

I don't know the truth,
but I can assess specific
claims about the truth.


$$H_0: y_{i1} = y_{i0}$$

i	z_i	y_i	y_{i1}	y_{i0}
A	0	16	?	16
B	1	22	22	22
C	0	7	?	7
D	1	14	14	14

$$P(t(y, z))$$

$$\frac{1}{6}$$

$$-8.5$$

$$-6.5$$

$$-.5$$

$$.5$$

$$P = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$6.5$$

$$8.5$$

$$t(y, z)$$

What is the true effect of the treatment assignment?

See also Pearl (2000) and also Richardson and Robins (2013). For more on the potential outcomes approach see Imbens and Rubin (2015).

Estimating an Average Treatment Effect

```
Z <- c(0,1,0,1)
Y <- c(16,22,7,14)
estate <- mean(Y[Z==1]) - mean(Y[Z==0]) ## same as coef(lm(Y~Z))["Z"]
estate

[1] 6.5
```

Testing the Sharp Null of No Effects

```
Om <- matrix(0, ncol=choose(4,2), nrow=length(Z)) ## All possible experiments
whotrtd <- combn(1:4,2)
for(i in 1:choose(4,2)){ Om[cbind(whotrtd[,i],i)]<-1 }
meandifftz <- function(y,z){ mean(y[z==1]) - mean(y[z==0]) }
thedist<-apply(Om, 2, function(z){ meandifftz(Y,z) })
rbind(Om, thedist)
```

```
[,1] [,2] [,3] [,4] [,5] [,6]
1.0 1.0 1.0 0.0 0.0 0.0
1.0 0.0 0.0 1.0 1.0 0.0
0.0 1.0 0.0 1.0 0.0 1.0
0.0 0.0 1.0 0.0 1.0 1.0
```

```
thedist 8.5 -6.5 0.5 -0.5 6.5 -8.5
```

```
table(thedist)
```

```
thedist
-8.5 -6.5 -0.5 0.5 6.5 8.5
 1     1     1     1     1     1
```

```
theobs <- meandifftz(Y,Z)
mean(thedist) >= theobs
```

```
[1] 0.3333333
```

What do we need to interpret our calculations as teaching about causal quantities?

For the sharp null test: Randomization occurred as reported.

What do we need to interpret our calculations as teaching about causal quantities?

For the sharp null test: Randomization occurred as reported.

For the average treatment effect: Randomization occurred as reported plus no interference between units.

- Brady, Henry E. 2008. "Causation and explanation in social science." *Oxford handbook of political methodology* pp. 217–270.
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