EEE3080F

Communication Network and System Fundamentals

http://web.uct.ac.za/depts/commnetwork/eee3080

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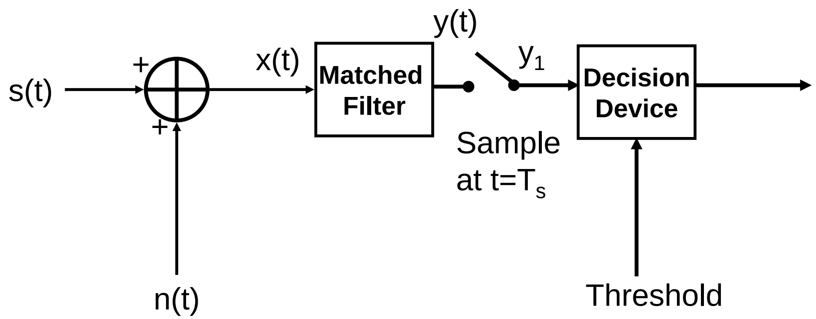
Probability of error in transmission and Eb/n

Study Sklar 3.1.4 – 5

Do and understand exercises

Receiver for baseband transmission of binary-encoded PCM wave.



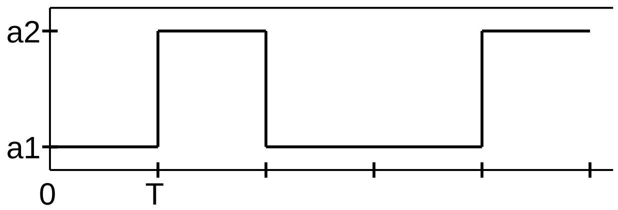
- Disregarding the matched filter for the time being.

Binary signals

- In a binary PCM system, binary digits may be represented by two pulse levels.
- If these levels are chosen to be 0 and A, the signal is termed an on-off (or unipolar) binary signal.
- If the level switches between –A/2 and A/2 it is called a polar binary signal.
- In general, the signals are at levels a1 and a2.
- In what follows we do not consider modulating the signal—it is transmitted at baseband.

Transmitted signals

- Suppose we are transmitting digital information, and decide to do this using two-level pulses each with period T :



- The binary digit 0 is represented by a signal of level a1 for the duration T of the transmission, and the digit 1 is represented by the signal level a2 .

Channel noise

- Channel noise is modeled as additive white Gaussian noise (AWGN), n(t), and is added to the transmitted signal.

- The probability density function of n(t) has a Gaussian distribution with a mean of 0 and a variance of σ^2:

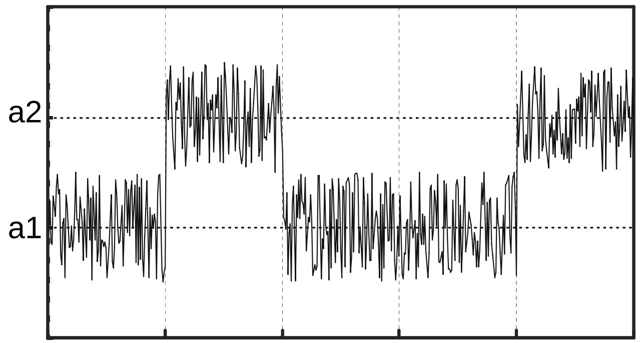
$$p_N(n) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{n^2}{2\sigma^2}\right)$$

Baseband received signals

- Owing to the presence of noise, the received signal waveform y(t) at the receiver for the bit transmitted between time 0 and time T is
- y(t) = s(t) + n(t)
- where the ideal noise-free signal is
- s(t) = a1 (0 if unipolar): symbol 0 transmitted.
- s(t) = a2 (A if unipolar) : symbol 1 transmitted.

Baseband received signals

- In the event of a noisy Gaussian channel (with high bandwidth) y(t) may look as follows:



Probability of error in transmission

- In what follows, it is assumed that the transmitter and the receiver are synchronized, so the receiver has perfect knowledge of the arrival times of sequences of pulses. The means of achieving this synchronization is not considered here. This means that without loss of generality we can always assume that the bit to be received lies in the interval (0, T).

Baseband detection

- The detector samples the received signal at some time instant Ts in the range (0, T), and uses that value to make a decision. The value obtained would be one of the following:

- y(Ts) = a1 + n(Ts) if 0 was sent
- y(Ts) = a2 + n(Ts) if 1 was sent

- For unipolar signals, for example,
- y(Ts) = n(Ts) signal absent.
- y(Ts) = A + n(Ts) signal present.

Probability of error in transmission

- The function of a receiver is to distinguish the digit 0 from the digit 1. The most important performance characteristic of the receiver is the probability that an error will be made in such a determination.

Gaussian Noise Distribution

- Suppose now that n(Ts) has a Gaussian distribution with a mean of 0 and a variance of σ^2:

$$p_N(n) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{n^2}{2\sigma^2} \right)$$

- When a1 was sent the conditional probability density of y is $$p_{Y|a1}(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y-a1)^2}{2\sigma^2} \right)$$

- When a2 was sent the conditional probability density of y is

$$p_{Y|a2}(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y-a2)^2}{2\sigma^2} \right)$$

Gaussian Noise Distribution

- The conditional probability functions

$$p_{Y|a1}(y) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(y-a1)^2}{2\sigma^2}\right)$$

$$p_{Y|a2}(y) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(y-a2)^2}{2\sigma^2}\right)$$

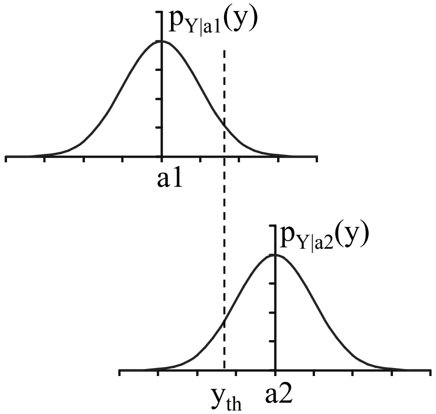
- are also called the likelihood functions.

Baseband detection

- Since the value n(T) is random, we cannot decide with certainty whether the signal was a1 or a2 at the time of the sample. However, a reasonable rule for the decision of whether a 0 or a 1 was received is the following:
- $y(T) \le y_{th}$ signal absent — a1 received
- $y(T) > y_{th}$ signal present— a2 received:
- The quantity y_{th} is a threshold which we would usually choose somewhere between a1 and a2. For convenience we denote y(Ts) by y.

Gaussian Noise Distribution

- Using the decision rule described, it is evident that we sometimes decide that a signal is a2 even when it is in fact a1, and vice versa.



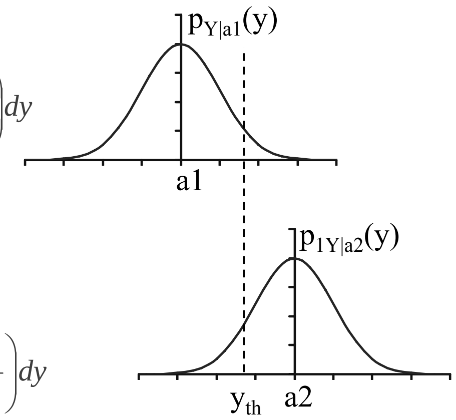
Gaussian Noise Distribution

- The probability of a false alarm occurring (mistaking a1 for a a2) is

$$p_{a2|a1} = \frac{1}{\sqrt{2\pi}\sigma}\int_{y_{th}}^{\infty}\exp\left(-\frac{(y-a1)^2}{2\sigma^2}\right)dy$$

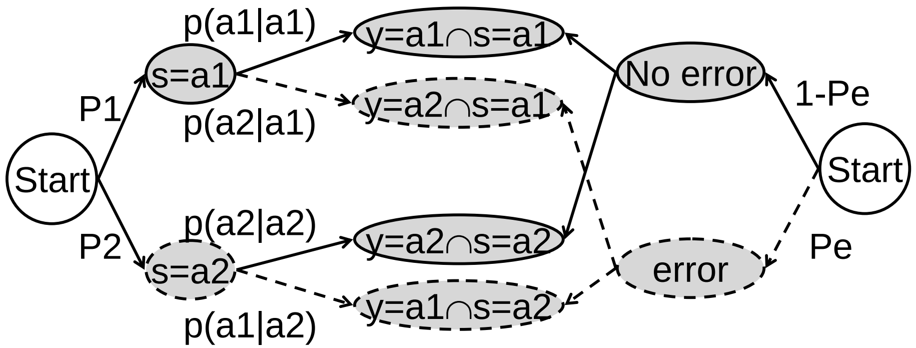
- Similarly, the probability of a missed detection (mistaking a2 for a1) is

$$p_{a1|a2} = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{y_{th}}\exp\left(-\frac{(y-a2)^2}{2\sigma^2}\right)dy$$



Baseband detection

- Letting P1 and P2 be the source symbol probabilities of a1 and a2 respectively, we can define the overall probability of error to be
- Pe = P1 $p_{a2|a1}$ + P2 $p_{a1|a2}$.



Baseband detection

- Pe = P1 $p_{a2|a1}$ + P2 $p_{a1|a2}$.
- $= P1 \times \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{y_{th}} \exp\left(-\frac{(y-a2)^2}{2\sigma^2}\right)dy + P2 \times \frac{1}{\sqrt{2\pi}\sigma}\int_{y_{th}}^{\infty} \exp\left(-\frac{(y-a1)^2}{2\sigma^2}\right)dy$
- = P1 Q{(a2- y_{th})/ σ } + P2 Q{(y_{th} -a1)/ σ }
- In the equally probable case (maximize information) Pe becomes
- Pe = (1/2) [Q{(a2- y_{th})/ σ } + Q{(y_{th} -a1)/ σ }]
- The sum of these two errors will be minimized for y_{th} = (a1+a2)/2.
- Pe = Q{(a2-a1)/(2 σ)}

Baseband detection with unipolar signals

- For equally probable unipolar signals, the sum of the two errors
- Pe = (1/2) (P_{e0} + P_{e1}).
- will be minimized for y_{th} = A/2. This sets the decision threshold for a minimum probability of error for P0 = P1 = 1/2 .
- The selection of voltages 0 and A may be difficult for baseband transmission, since an overall DC current flow is implied.

Baseband detection with unipolar signals

- In that case the probabilities of each type of error are equal, so the overall probability of error is

$$P_e = \frac{1}{\sqrt{2\pi}\sigma}\int_{A/2}^{\infty} \exp\left(-\frac{y^2}{2\sigma^2}\right)dy$$

- Making the change of variables z $\equiv$ y/ σ this integral becomes

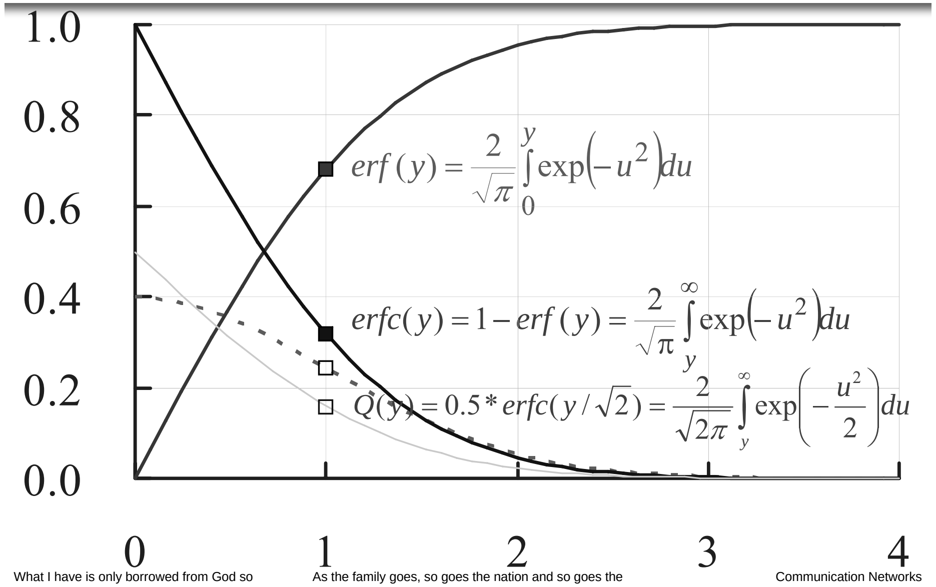
$$P_e = \frac{1}{\sqrt{2\pi}}\int_{A/(2\sigma)}^{\infty} \exp\left(-\frac{z^2}{2}\right)dz$$

- Pe = Q{A/(2 σ)}

Baseband detection with unipolar signals

- The function Pe may be written in a more useful form by noting that the average signal power is S = $A^2/2$, and the noise power is N = σ^2. The probability of error for on-off binary is therefore
- Pe = $Q\{A/(2\sigma)\} = Q\{(S/2N)^{1/2}\}$

Q function versus Complementary error function



Baseband detection with polar signals

- Consider polar signals with a1 = –A/2 and a2 = A/2, then
- Pe = $Q\{(a2-a1)/(2\sigma)\} = Q\{A/(2\sigma)\} = Q\{[A^2/(4\sigma^2)]^{1/2}\}$
- $= Q\{(S/N)^{1/2}\}$
- and S = $A^2/4$
- The on-off binary signal therefore requires twice the signal power of the polar binary signal to achieve the same error rate.

Decision

- Detector needs to figure out which waveform was sent, by looking at differences in amplitude, phase, and/or frequency
- Use a “Decision Statistic” such as a single sample or a sum of samples to decide
- Probability of error, Pe, is the probability of making a mistake in identifying a symbol given the signal strength, noise, channel, interference, etc
- Probability of bit error, Pb, is the probability of making a mistake on the overall bit stream over time.

SNR for Digital Communication Systems

- SNR, average signal power to average noise power is important for measuring performance in analog systems
- In digital communication, the ratio is the bit energy (E_b) divided by noise spectrum density (η), a normalized version of SNR
- Allows comparison when M-ary systems are used

E_b/η

- Bit energy E_b = Signal Power S times the bit time T_b ,
- Bit time T_b = 1 over bit rate R_b

- Noise power spectral density $\eta/2$ = noise power N divided by bandwidth 2B

$$\frac{E_b}{\eta} = \frac{S * T_b}{N / B} = \frac{S / R_b}{N / B} = \left(\frac{S}{N}\right)\left(\frac{B}{R_b}\right)$$

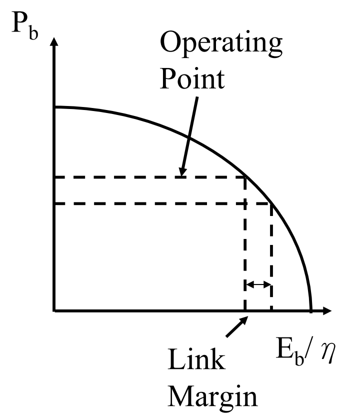
Why E_b/η ?

- Why not SNR?
  - Power Signal: finite average power, infinite energy, good model for analog signal
  - Energy Signal: zero average power, finite energy

- Power signals are good for analog signals since they can be thought of as existing for a long time
- Digital symbols exist over one symbol or bit interval, T_b, so this allows comparison between different M-ary signals

How performance is measured?

- Waterfall curve shows how bit error and energy/bit can be compared
- Ideally, want low error and low energy/bit



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As the family goes, so goes the nation and so goes the whole world in which we live. (Pope John Paul II)