

## Problem 5.1.4

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### 1 Problem

Given an image that is 4 pixels by 4 pixels, so that the grayscale representation of the image is a 4x4 matrix called  $\mathbf{A}$ , what matrix  $\mathbf{E}$  would you multiply  $\mathbf{A}$  by in order to switch the leftmost column of pixels with the rightmost column of pixels? Should you multiply  $\mathbf{A}$  by  $\mathbf{E}$  on the right or the left (*i.e.*, would  $\mathbf{AE}$ ,  $\mathbf{EA}$ , or both give the desired result)? It is probably beneficial to experiment with this on paper before providing your answer in your report.

### 2 Solution

We will first use the identity matrix to help us solve this problem as we know that anything multiplied by the identity is itself. We will refer to the identity matrix as  $\mathbf{I}$  from now on. We will take the standard identity matrix:

$$\mathbf{I}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We will now modify the columns in  $\mathbf{A}$  and the changes made here will affect  $\mathbf{A}$  when we compute  $\mathbf{AE}$ . By swapping columns 1 & 4 of  $\mathbf{I}$  we then have:

$$\mathbf{E}_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

We can test our work by using the sample matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$\mathbf{AE} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 3 & 1 \\ 8 & 6 & 7 & 5 \\ 12 & 10 & 11 & 9 \\ 16 & 14 & 15 & 13 \end{bmatrix}$$

This yields us with same matrix values but columns 1 & 4 have now been swapped. If we were to compute  $\mathbf{EA}$  the product would be slightly different, it would be A but with rows 1 & 4 swapped instead of the columns.

### 3 MatLab Code

```

1 A = [1 2 3 4; 5 6 7 8; 9 10 11 12; 13 14 15 16]; %% Sample Matrix
2 E = eye(4); %% Identity 4x4
3
4 E(:, [1 4]) = E(:, [4 1]); %% Swap columns 1 & 4 with each other,
   this logic also applies to swapping rows
5
6 AE = A*E; %% We left multiply E on the right hand side of A which
   swaps A's columns, we left hand multiplication would swap rows
7 disp(AE) %% Display A*E's product

```