

Lecture 4: Convolutional Neural Networks for Computer Vision

Deep Learning @ UvA

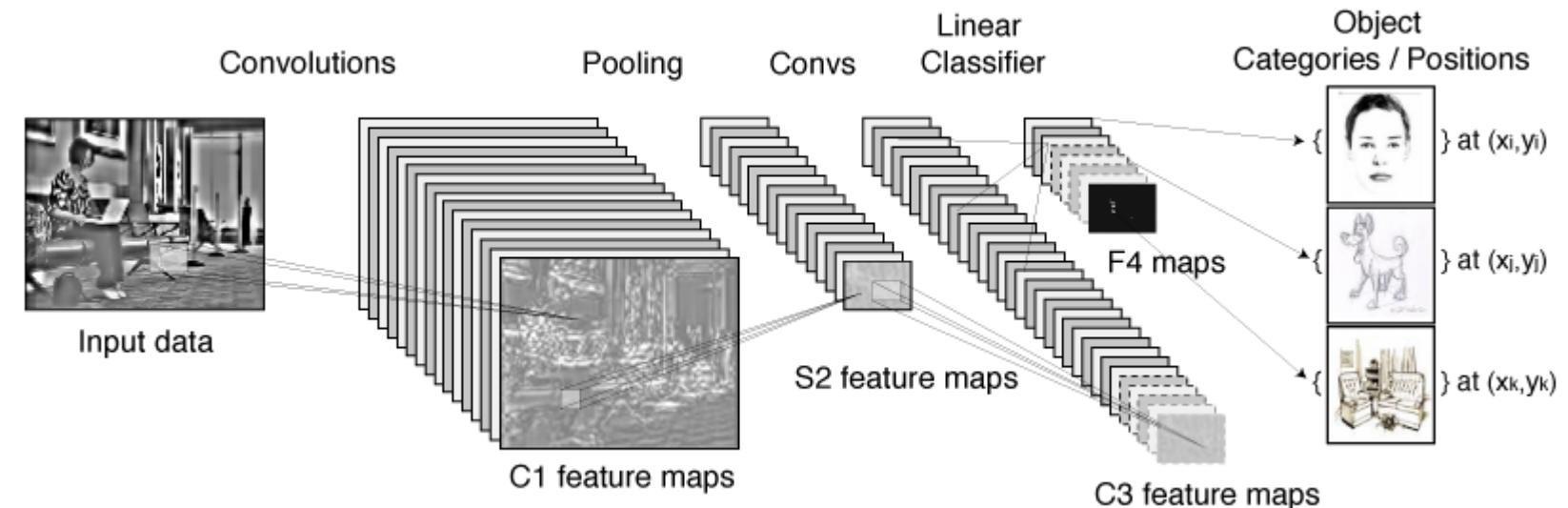
Previous lecture

- How to define your neural network model and optimize it in practice
- Data preprocessing and normalization
- Optimization methods
- Regularizations
- Architectures and architectural hyper-parameters
- Learning rate
- Weight initializations
- Good practices

Lecture overview

- What are the Convolutional Neural Networks?
- Why are they so important for Computer Vision?
- How do they differ from standard Neural Networks?
- How can we train a Convolutional Neural Network?

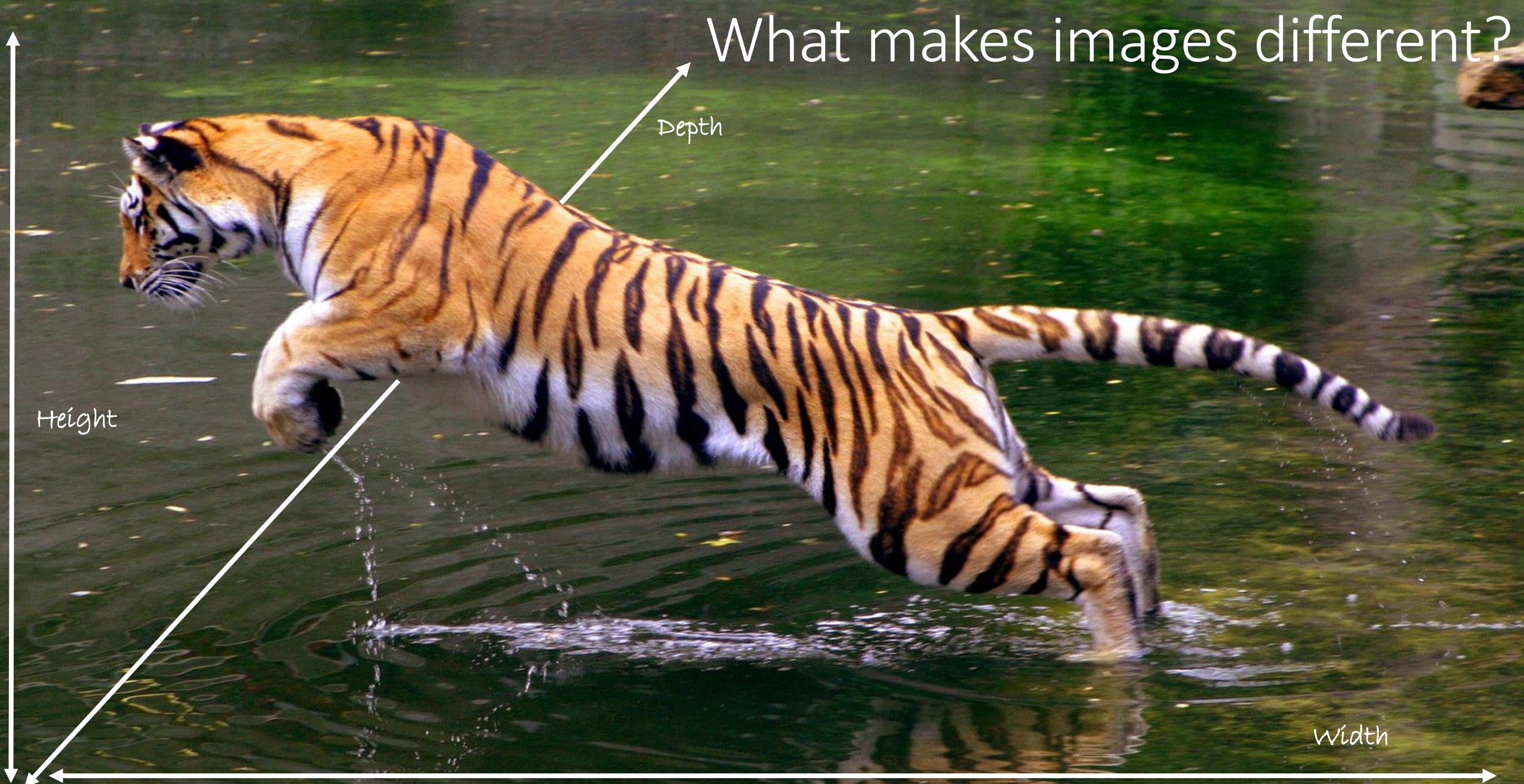
Convolutional Neural Networks



What makes images different?



What makes images different?



What makes images different?



$1920 \times 1080 \times 3 = 6,220,800$ input variables

What makes images different?



What makes images different?



What makes images different?



Image has shifted a bit to the up and the left!

What makes images different?

- An image has spatial structure
- Huge dimensionality
 - A 256x256 RGB image amounts to ~200K input variables
 - 1-layered NN with 1,000 neurons → 200 million parameters
- Images are stationary signals → they share features
 - After variances images are still meaningful
 - Small visual changes (often invisible to naked eye) → big changes to input vector
 - Still, semantics remain
 - Basic natural image statistics are the same

Input dimensions are correlated

Traditional task: Predict my salary!

Shift 1 dimension

Level of education	Age	Years of experience	Previous job	Nationality
"Higher"	28	6	Researcher	Spain

Level of education	Age	Years of experience	Previous job	Nationality
Spain	"Higher"	28	6	Researcher

Vision task: Predict the picture!



First 5x5 values

```
array([[51, 49, 51, 56, 55],  
       [53, 53, 57, 61, 62],  
       [67, 68, 71, 74, 75],  
       [76, 77, 79, 82, 80],  
       [71, 73, 76, 75, 75]], dtype=uint8)
```



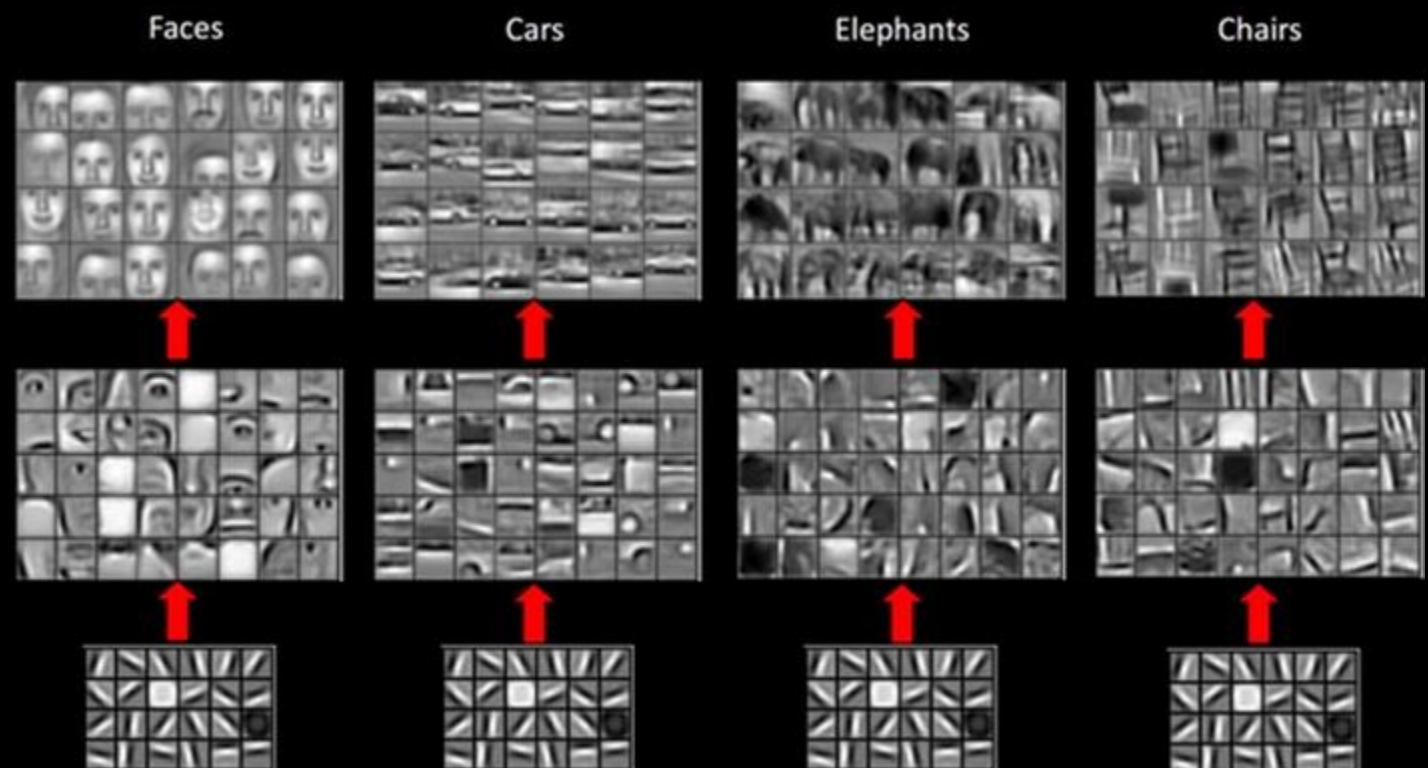
First 5x5 values

```
array([[58, 57, 57, 59, 59],  
       [58, 57, 57, 58, 59],  
       [59, 58, 58, 58, 58],  
       [61, 61, 60, 60, 59],  
       [64, 63, 62, 61, 60]], dtype=uint8)
```

Convolutional Neural Networks

- Question: Spatial structure?
 - Answer: Convolutional filters
- Question: Huge input dimensionalities?
 - Answer: Parameters are shared between filters
- Question: Local variances?
 - Answer: Pooling

Preserving spatial structure

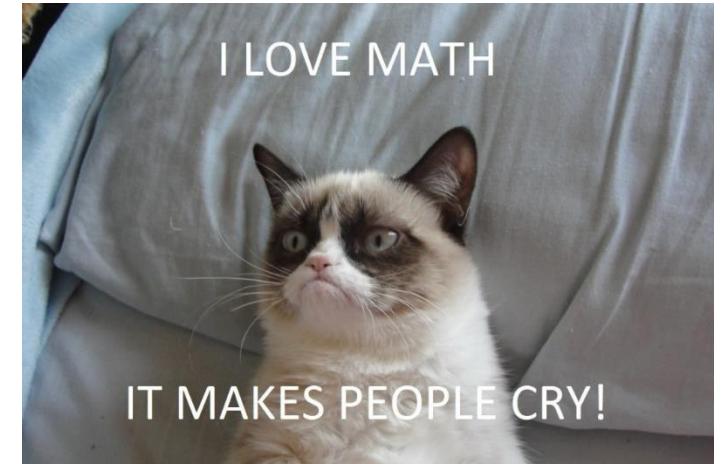


Why spatial?

- Images are 2-D
 - k-D if you also count the extra channels
 - RGB, hyperspectral, etc.

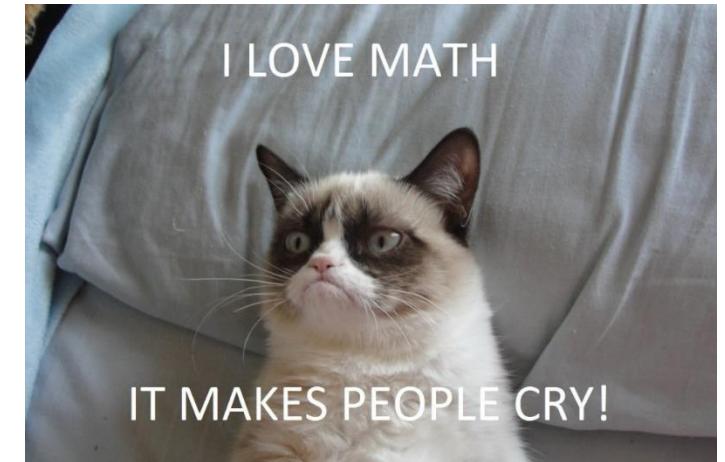
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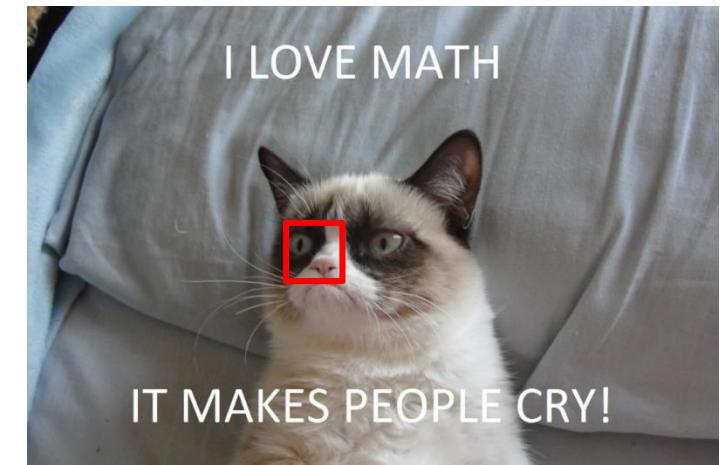
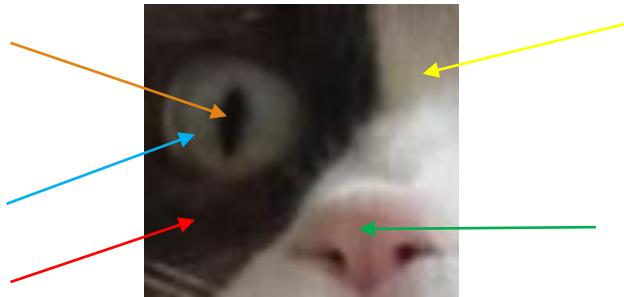
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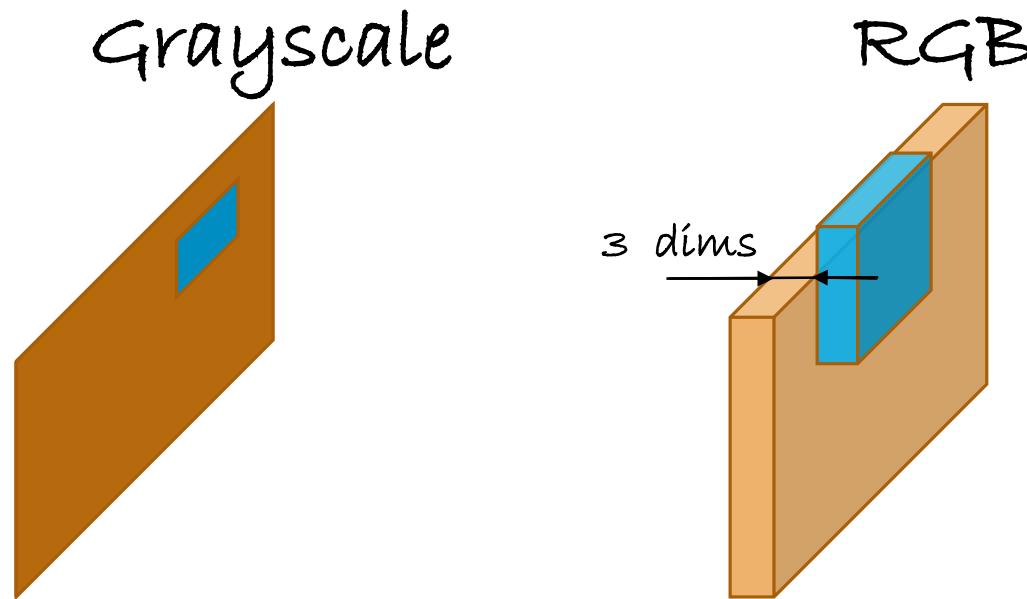
Why spatial?

- Images are 2-D
 - k-D if you also count the extra channels
 - RGB, hyperspectral, etc.
- What does a 2-D input really mean?
 - Neighboring variables are locally correlated



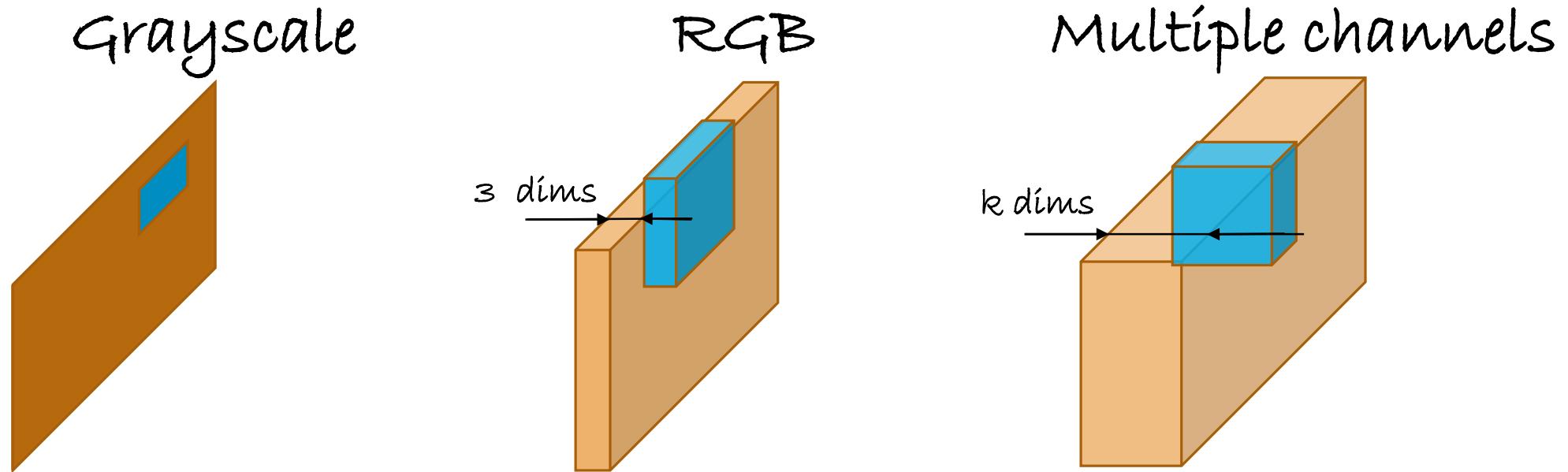
Parameters in k-D == Filters

- If images are 2-D, parameters should also be organized in 2-D
 - That way they can learn the local correlations between input variables
 - That way they can “exploit” the spatial nature of images
- Similarly, if images are k-D, parameters should also be k-D



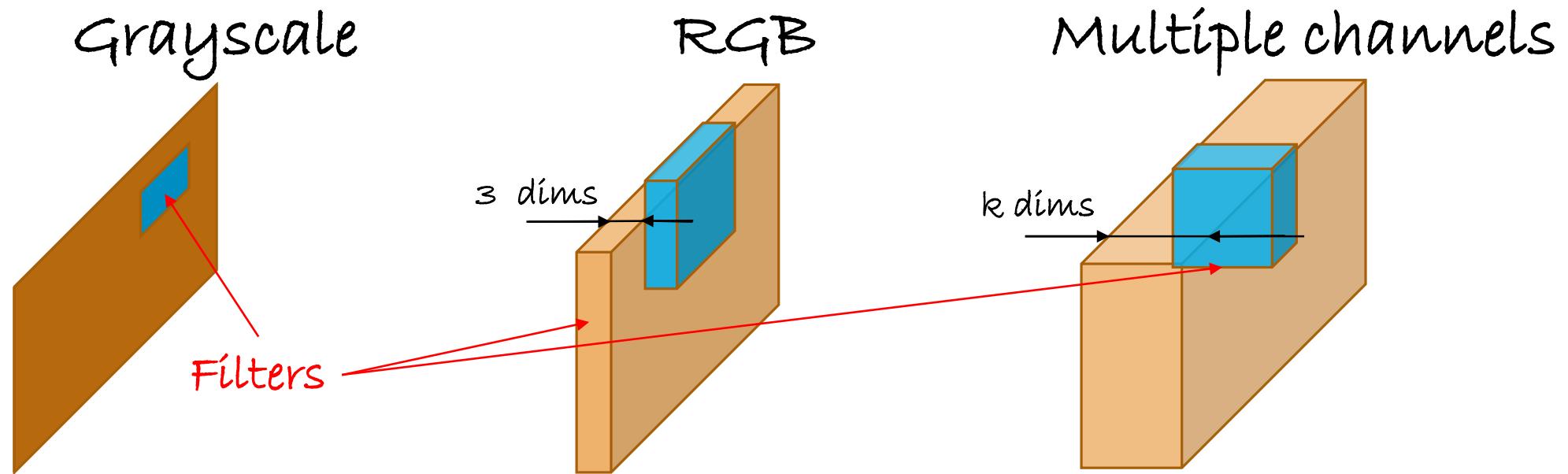
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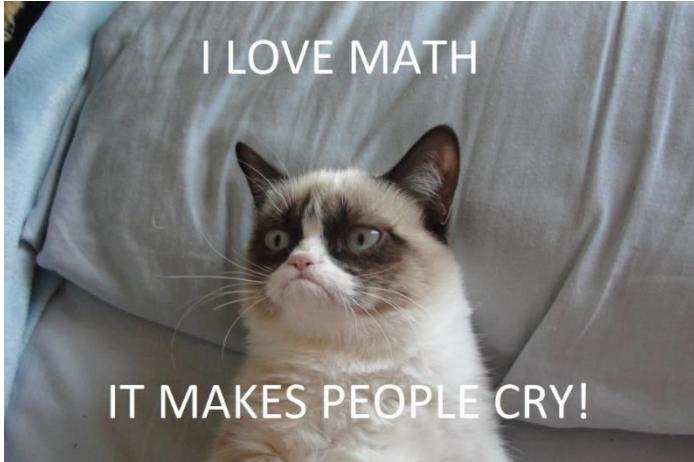


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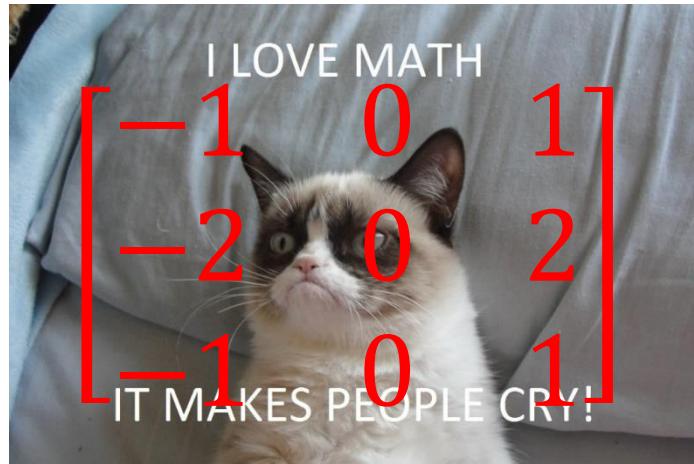


What would a k-D filter look like?



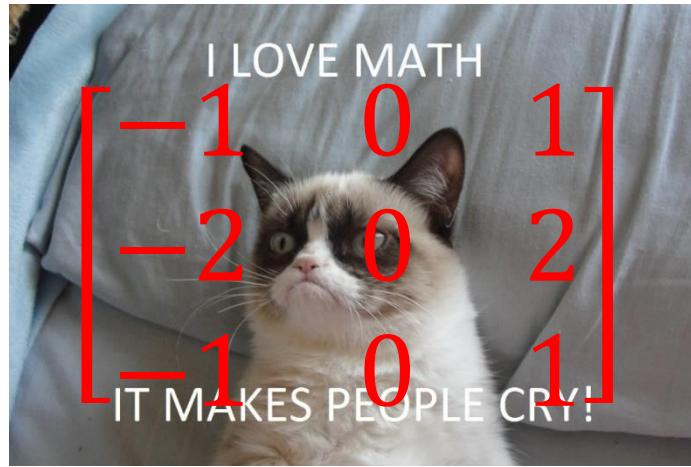
What would a k-D filter look like?

e.g. Sobel 2-D filter



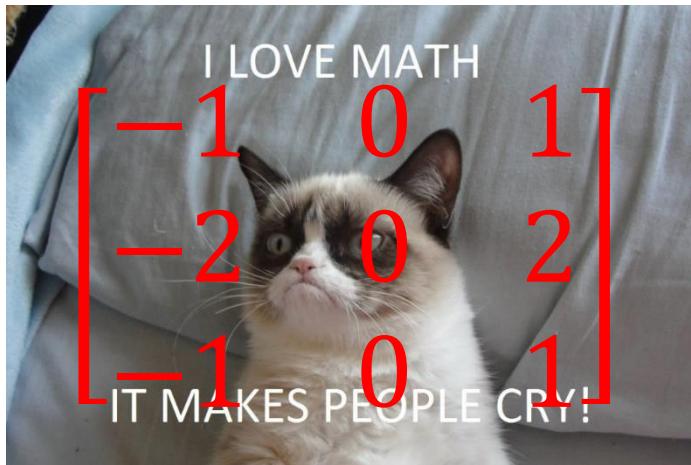
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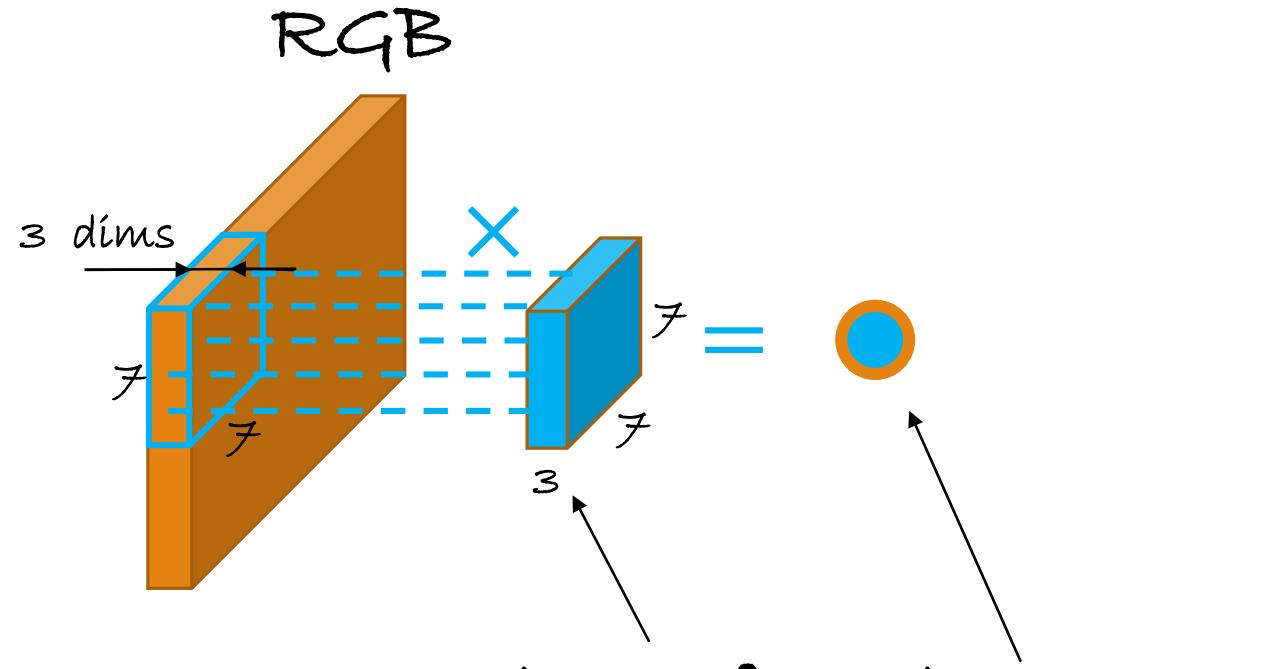
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e.g. Sobel 2-D filter



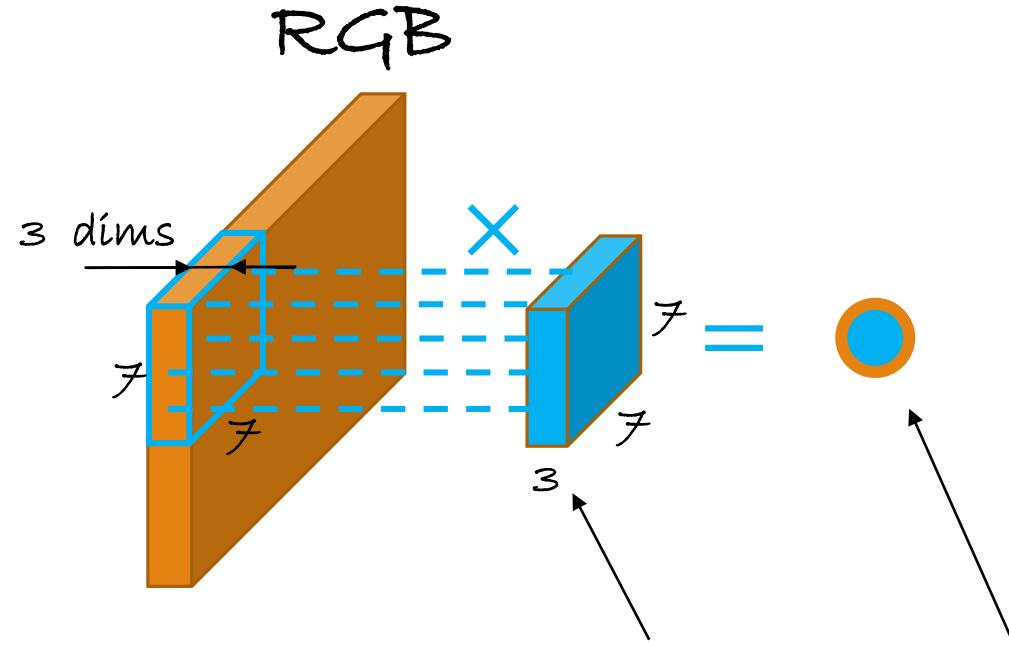
1	5	2	3	9	7
9	7	1	3	9	5
2	4	8	6	1	4
5	3	6	8	2	4

Think in space



How many weights for this neuron?

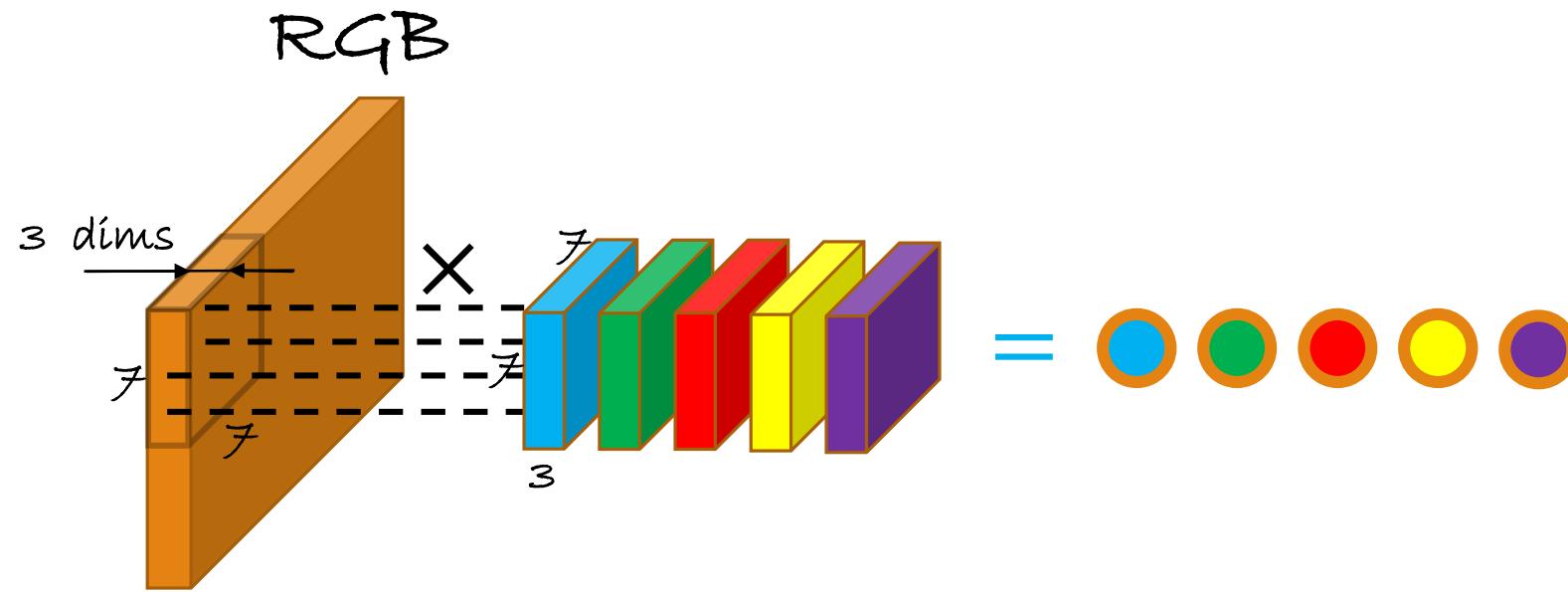
Think in space



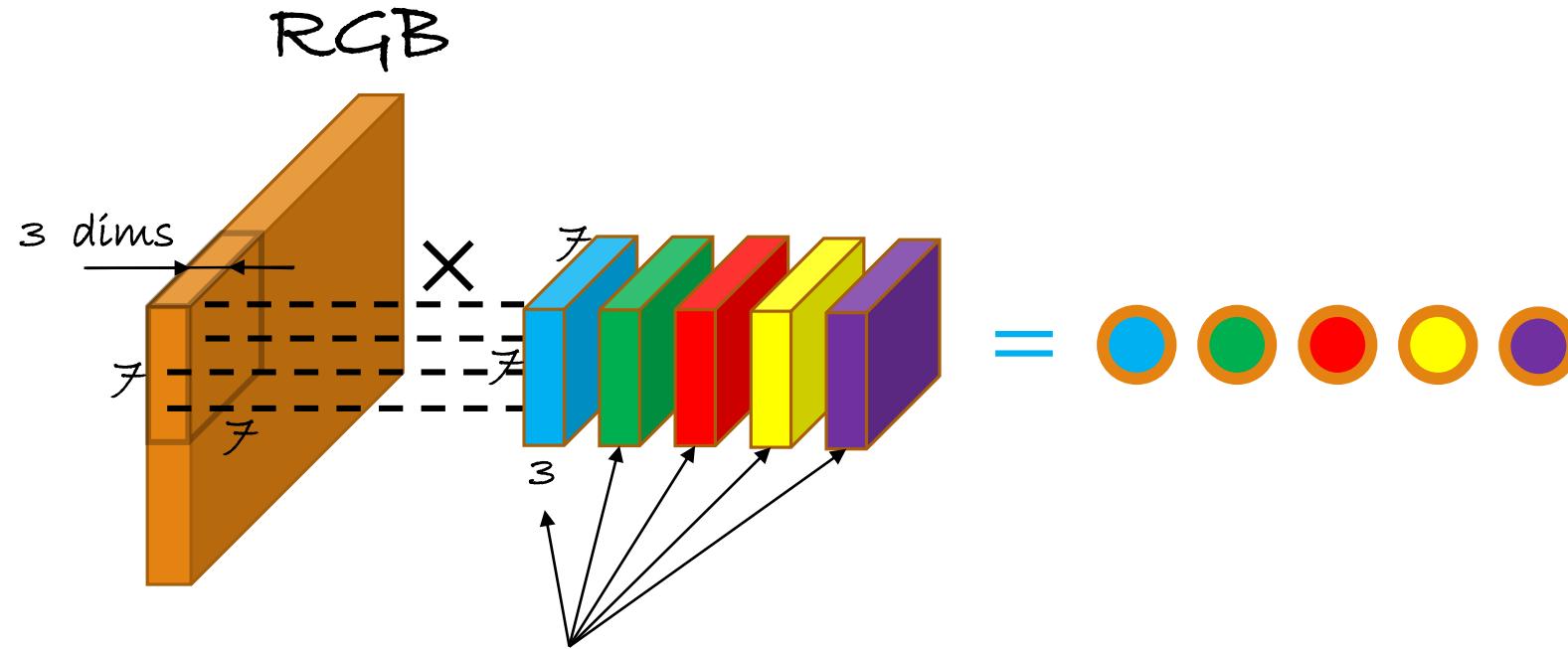
How many weights for this neuron?

$$7 \cdot 7 \cdot 3 = 147$$

Think in space

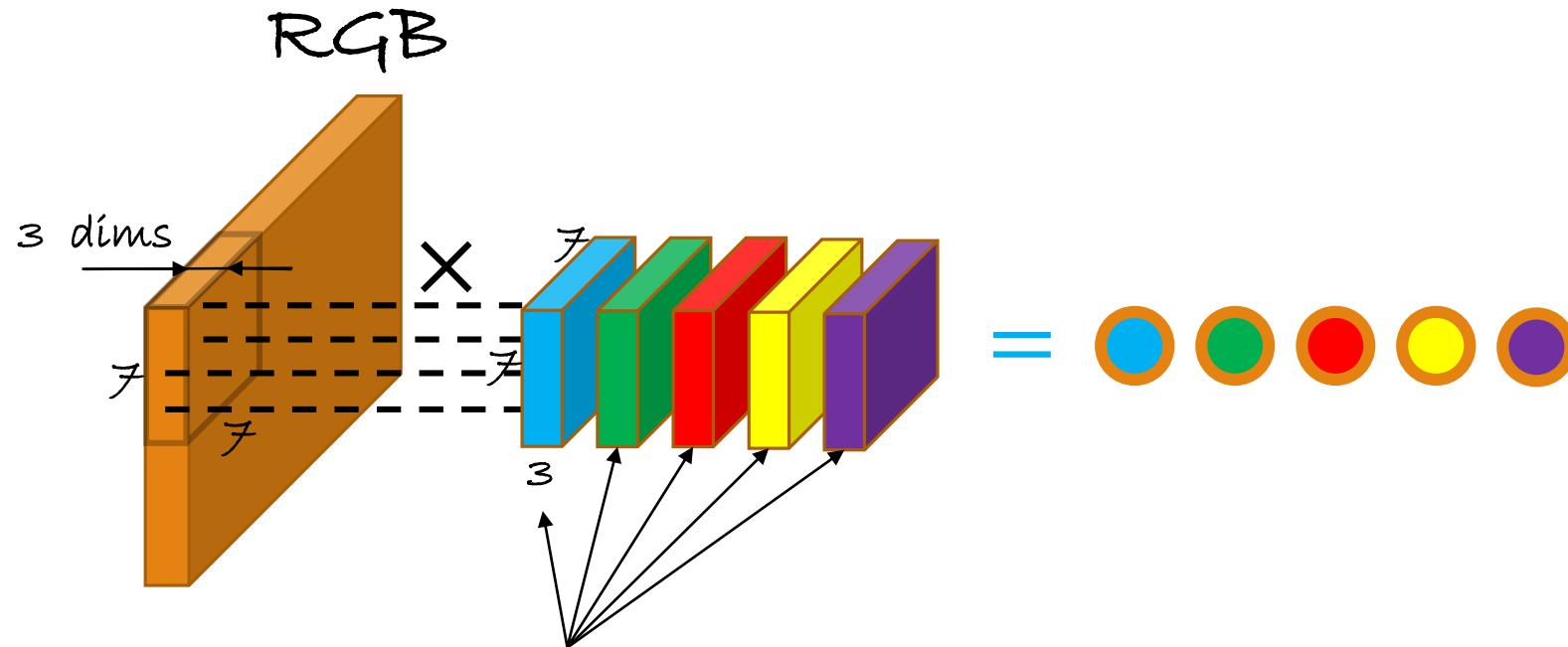


Think in space



How many weights for these 5 neurons?

Think in space

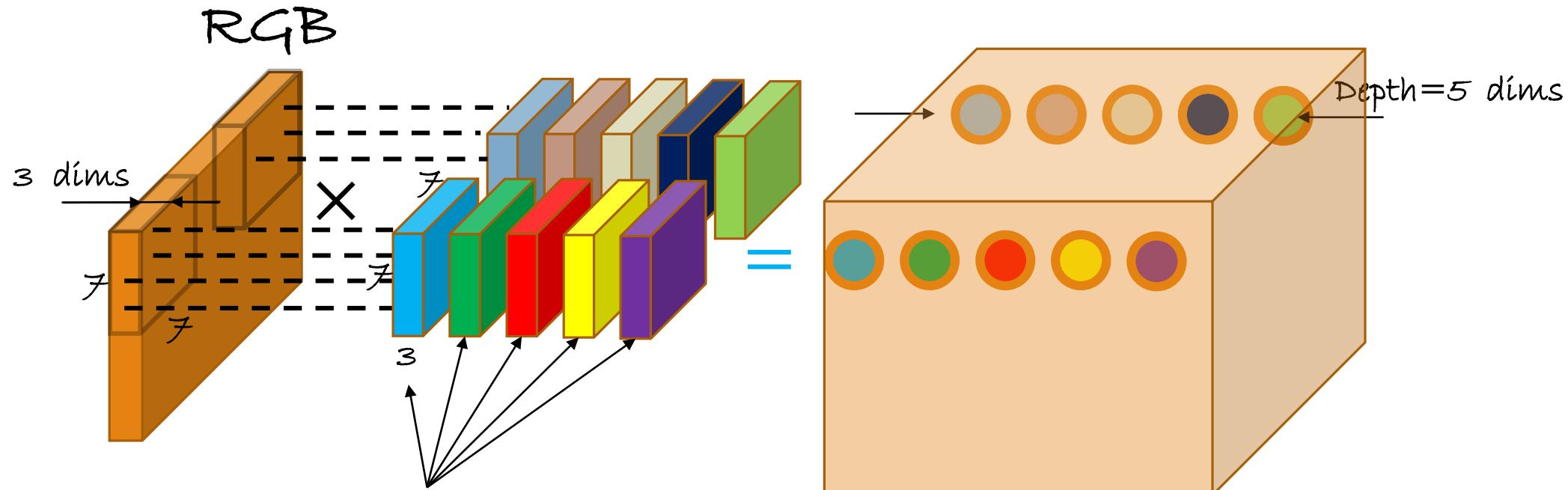


How many weights for these 5 neurons?

$$5 \cdot 7 \cdot 7 \cdot 3 = 735$$

Think in space

The activations of a hidden layer form a volume of neurons, not a 1-d "chain"

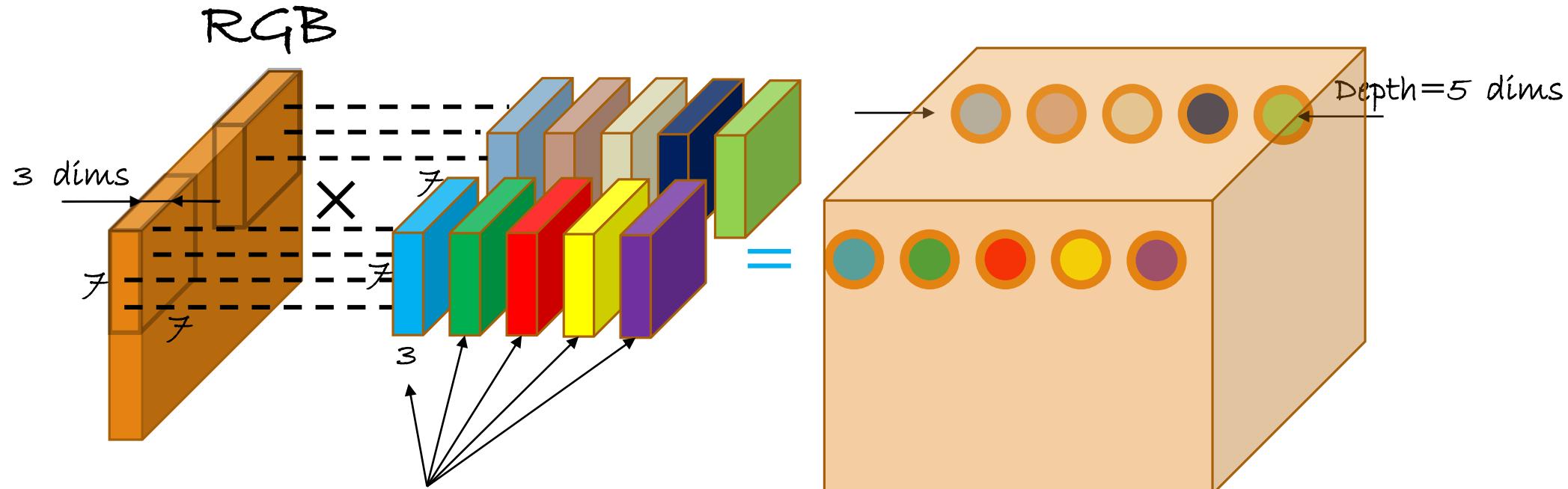


How many weights for these 5 neurons?

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Think in space

The activations of a hidden layer form a volume of neurons, not a 1-d "chain"
This volume has a depth 5, as we have 5 filters

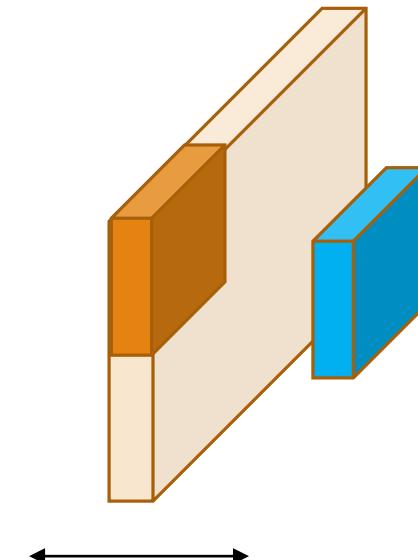
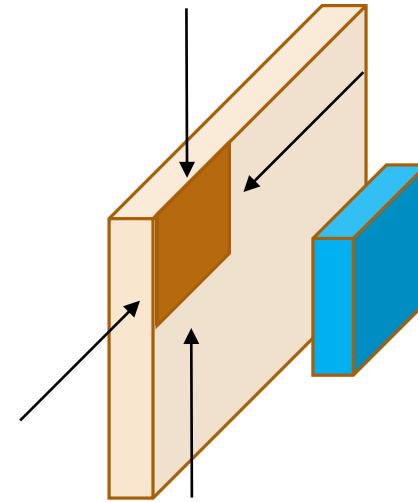


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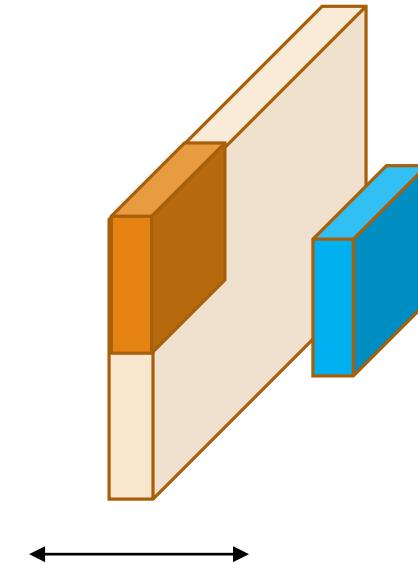
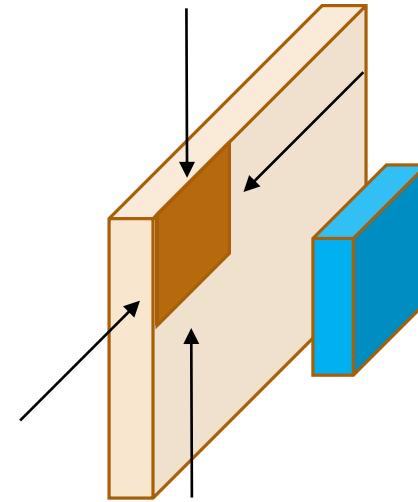
Local connectivity

- The weight connections are surface-wise local!
 - Local connectivity
- The weights connections are depth-wise global



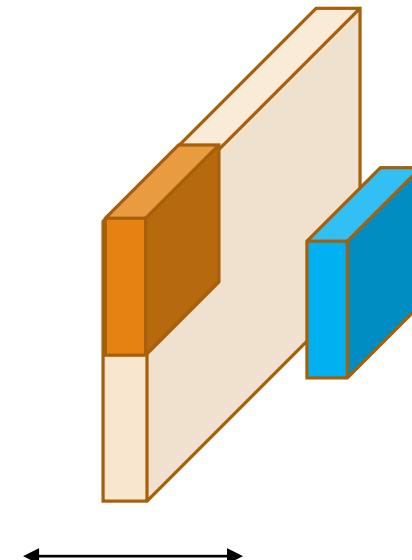
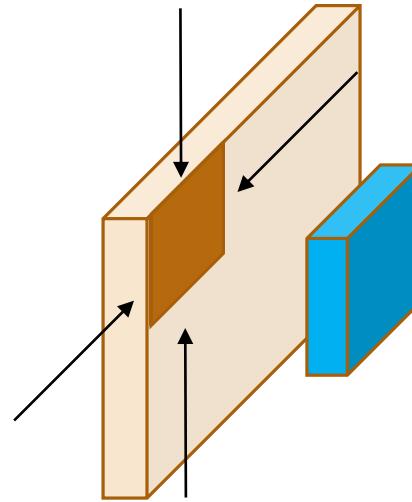
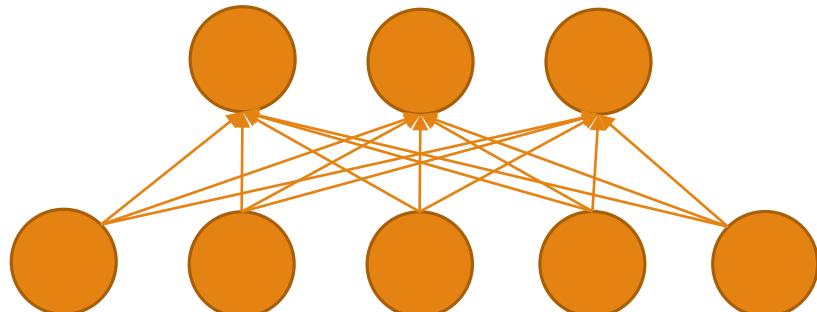
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- For standard neurons no local connectivity

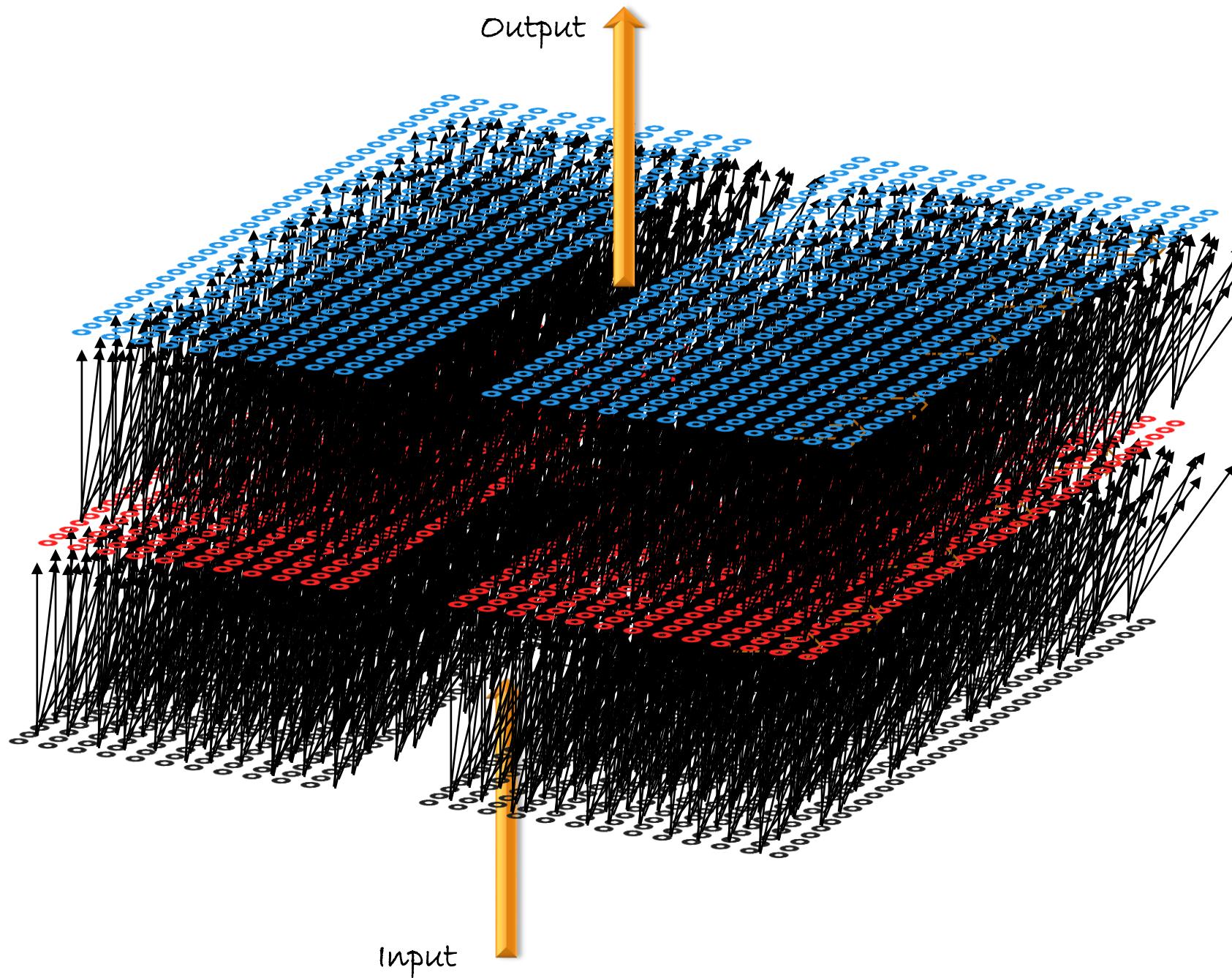


Local connectivity

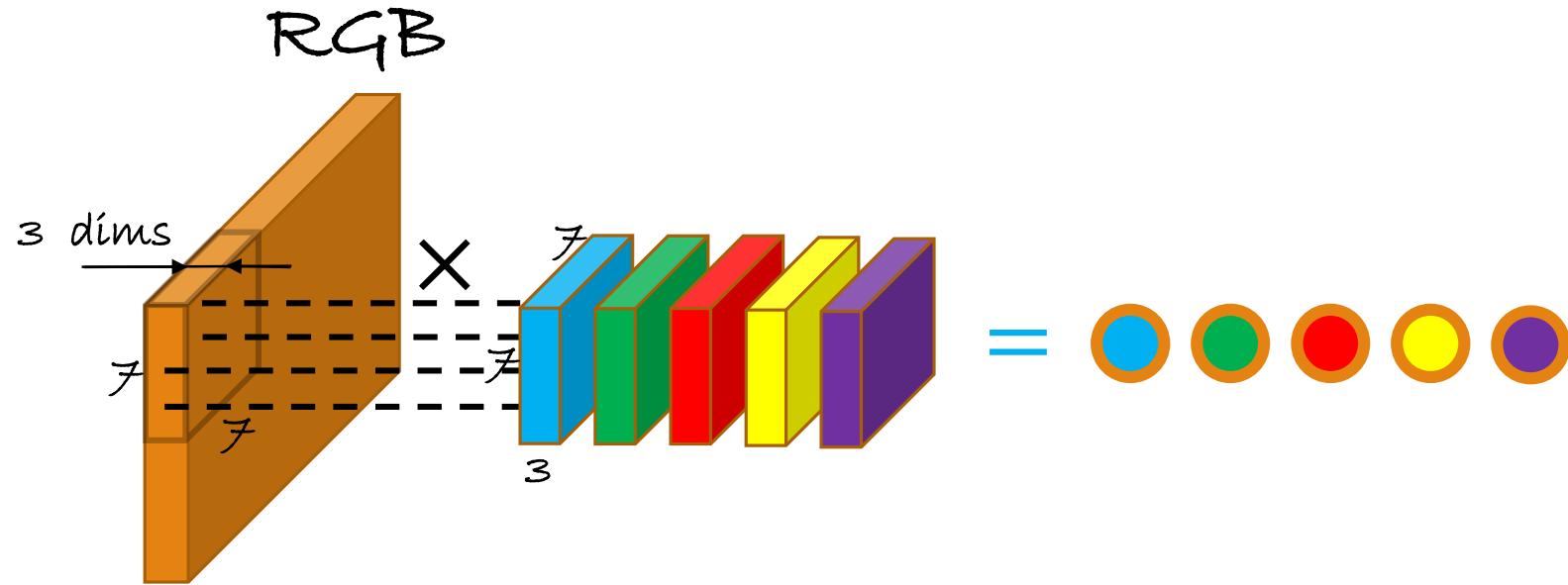
- The weight connections are surface-wise local!
 - Local connectivity
- The weights connections are depth-wise global
- For standard neurons no local connectivity
 - Everything is connected to everything



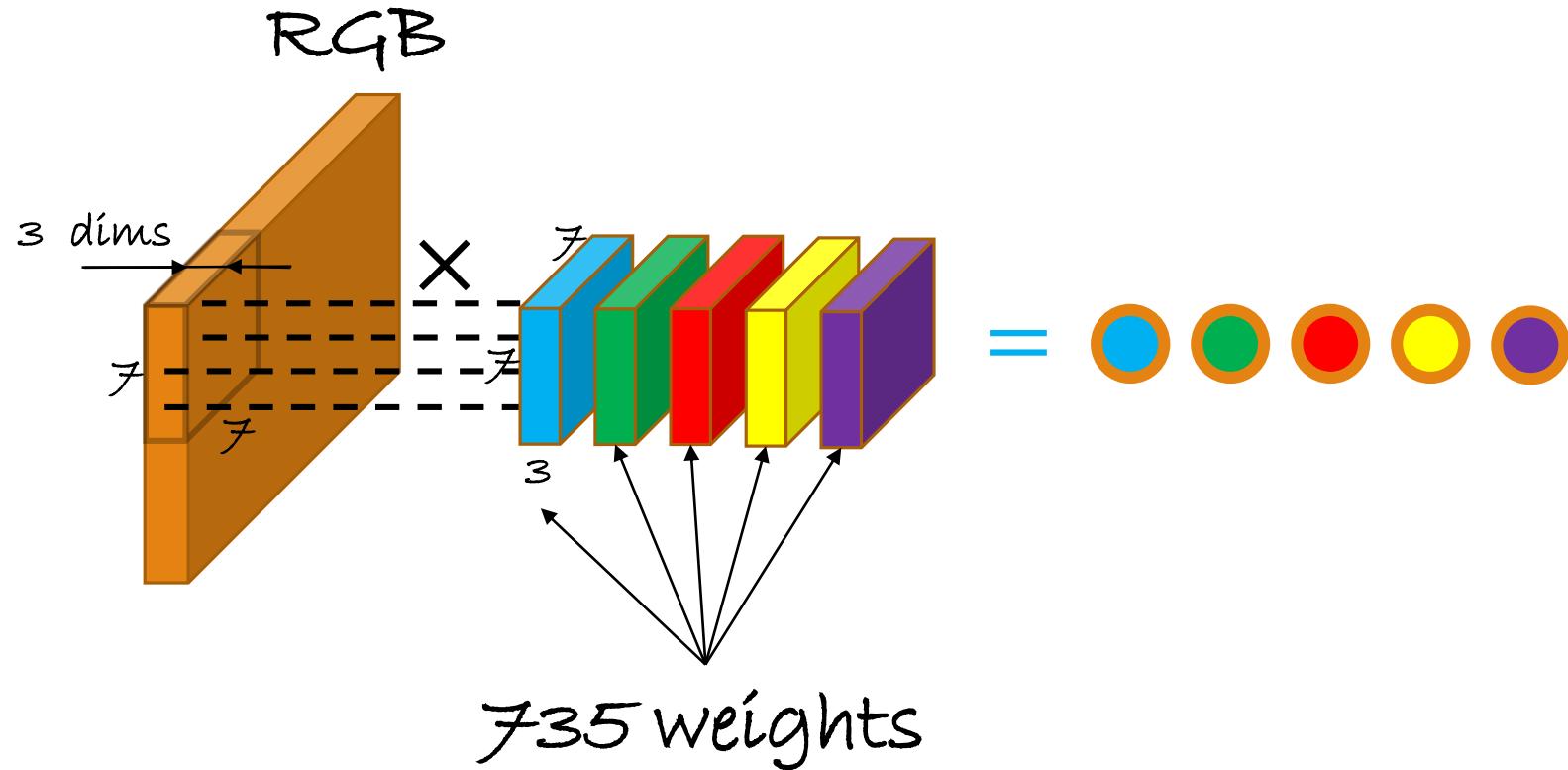
Filters vs Convolutional k-d filters



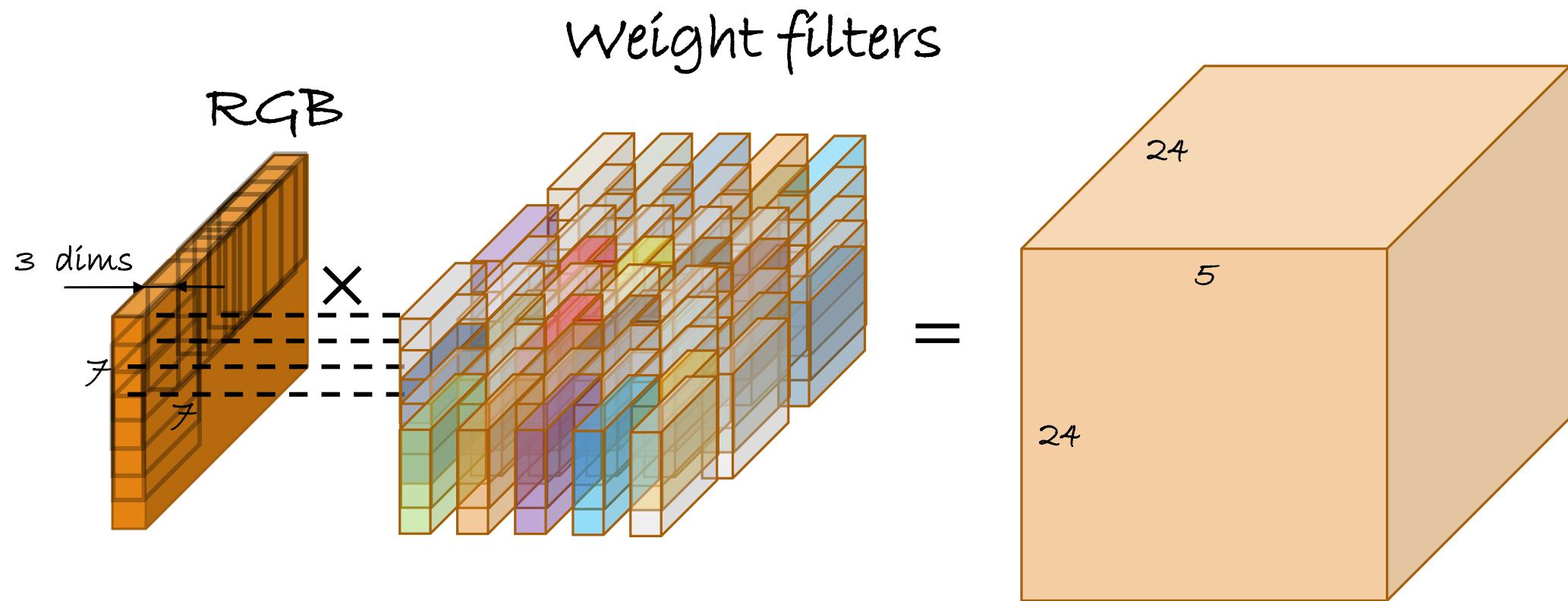
Again, think in space



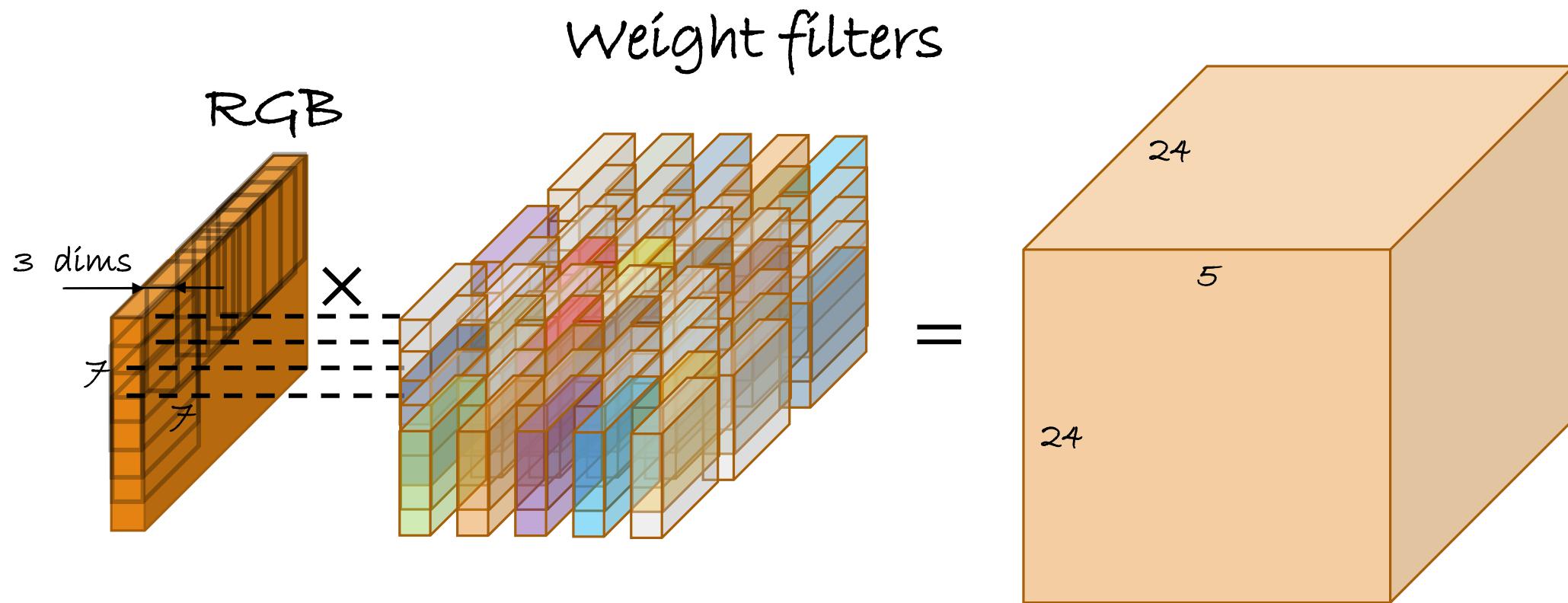
Again, think in space



What about cover the full image with filters?



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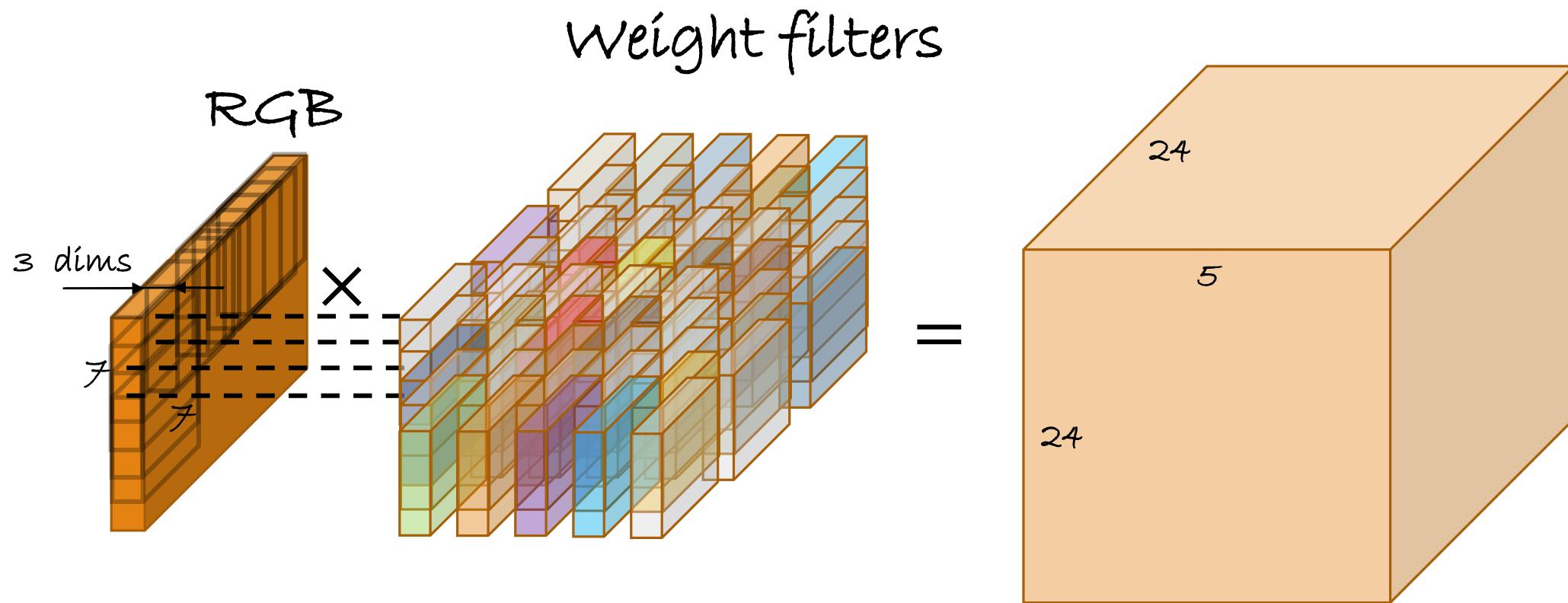


Assume the image is $30 \times 30 \times 3$.

1 filter every pixel (stride = 1)

How many parameters in total?

What about cover the full image with filters?



Assume the image is $30 \times 30 \times 3$.

1 filter every pixel (stride = 1)

How many parameters in total?

24 filters along the x axis
24 filters along the y axis
Depth of 5
 $\times 7 * 7 * 3$ parameters per filter

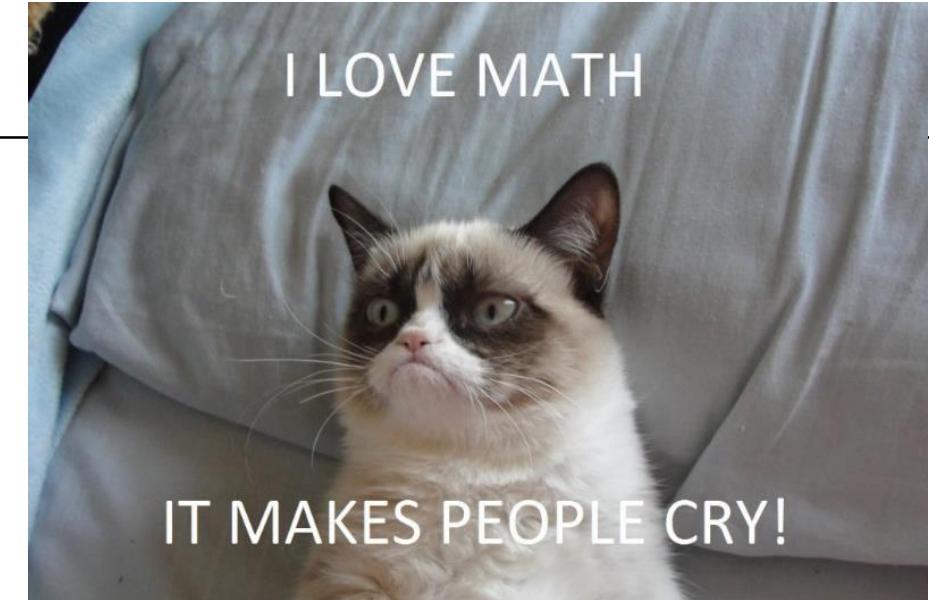
423K parameters in total

Problem!

- Clearly, too many parameters
- With a only 30×30 pixels image and a single hidden layer of depth 5 we would need 85K parameters
 - With a 256×256 image we would need $46 \cdot 10^6$ parameters
- *Problem 1: Fitting a model with that many parameters is not easy*
- *Problem 2: Finding the data for such a model is not easy*
- *Problem 3: Are all these weights necessary?*

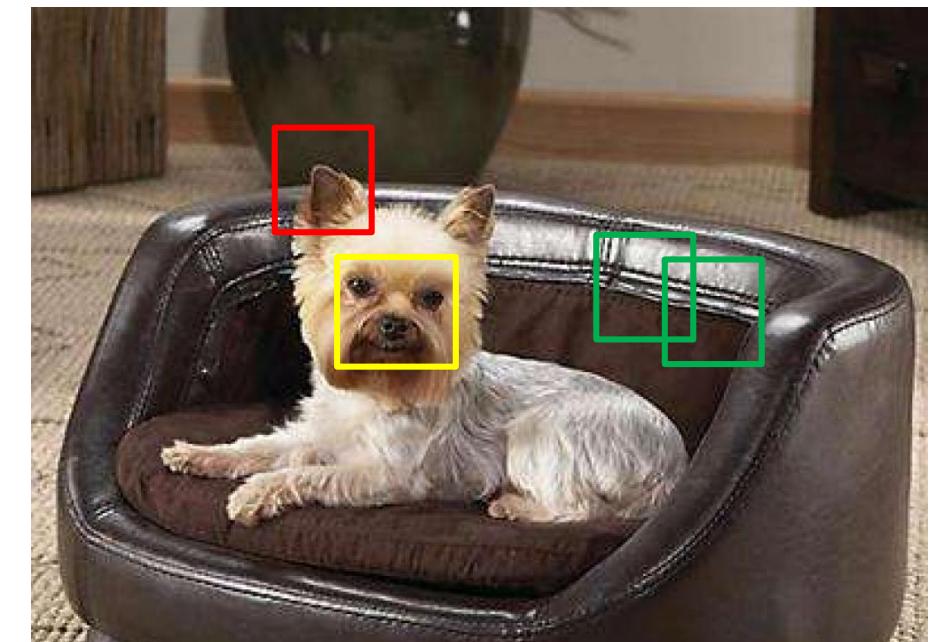
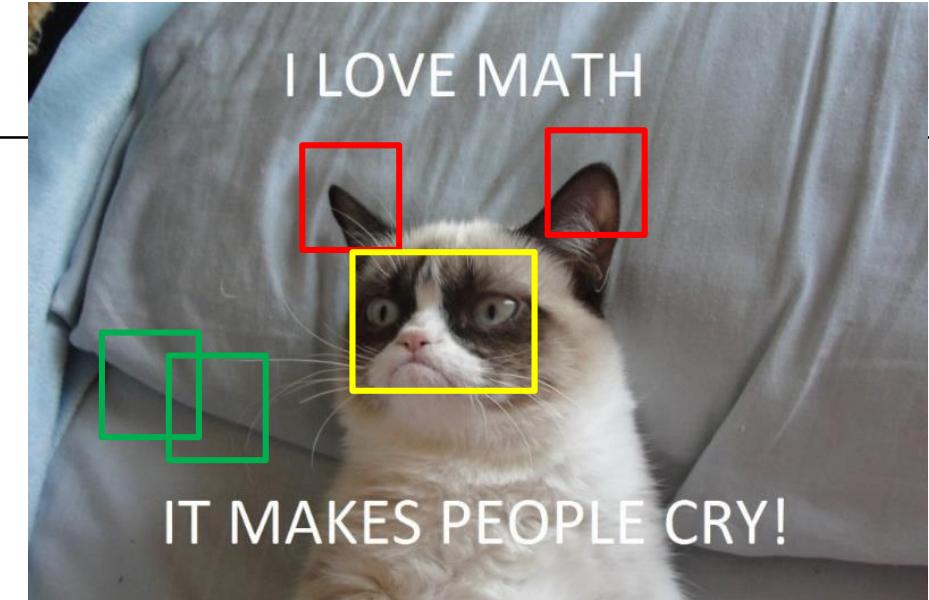
Hypothesis

- Imagine
 - With the right amount of data ...
 - ... and assuming we would connect all input neurons of layer l with all output neurons of layer $l + 1$, ...
 - ... if we would visualize the filters (remember they are 2d) ...
 - ... we would see very similar plots no matter their location



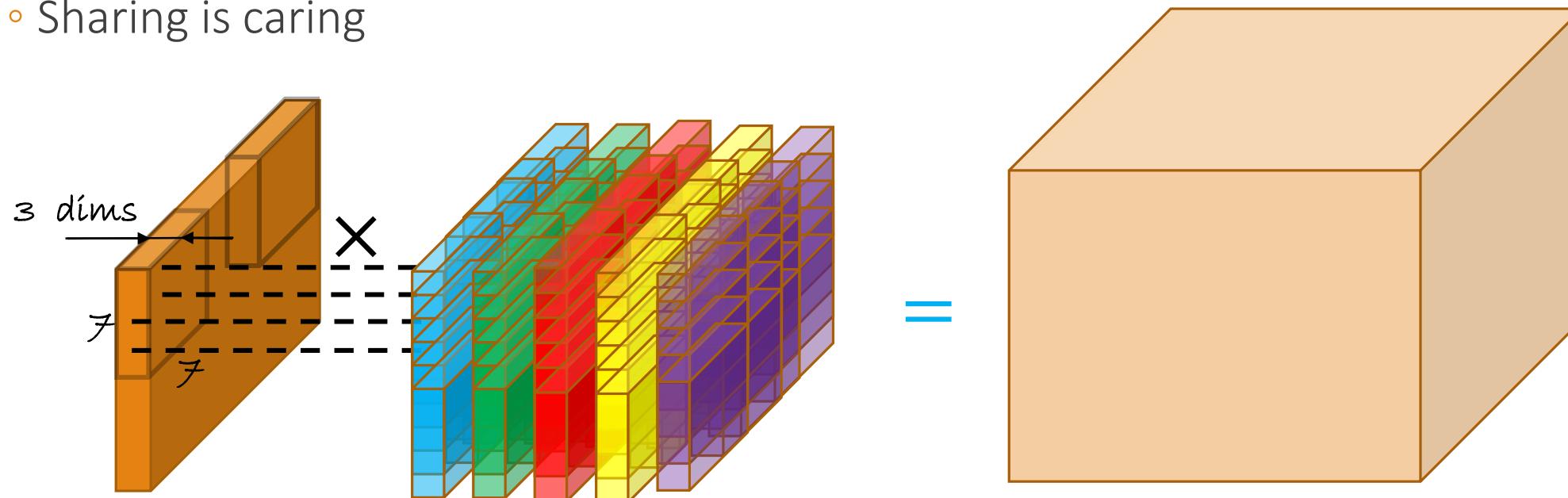
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 - ... and assuming we would connect all input neurons of layer l with all output neurons of layer $l + 1$, ...
 - ... if we would visualize the filters (remember they are 2d) ...
 - ... we would see very similar plots no matter their location
- Why?
 - Natural images are stationary
 - Visual features are common for different parts of one or multiple image



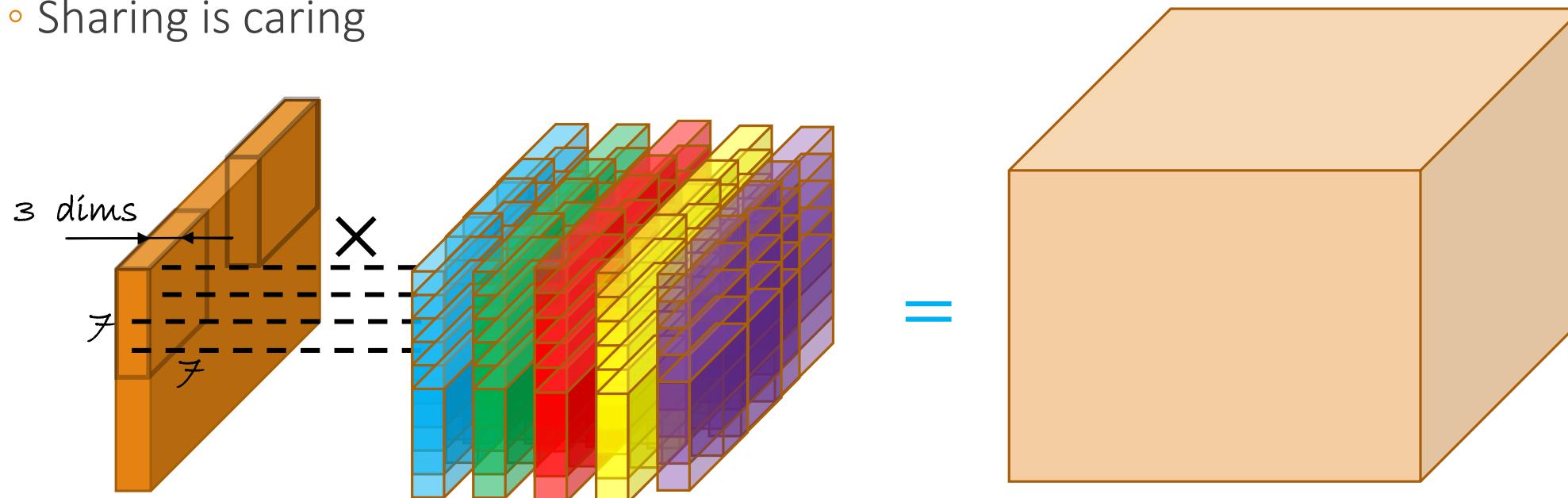
Solution? Share!

- So, if we are anyways going to compute the same filters, why not share?
 - Sharing is caring



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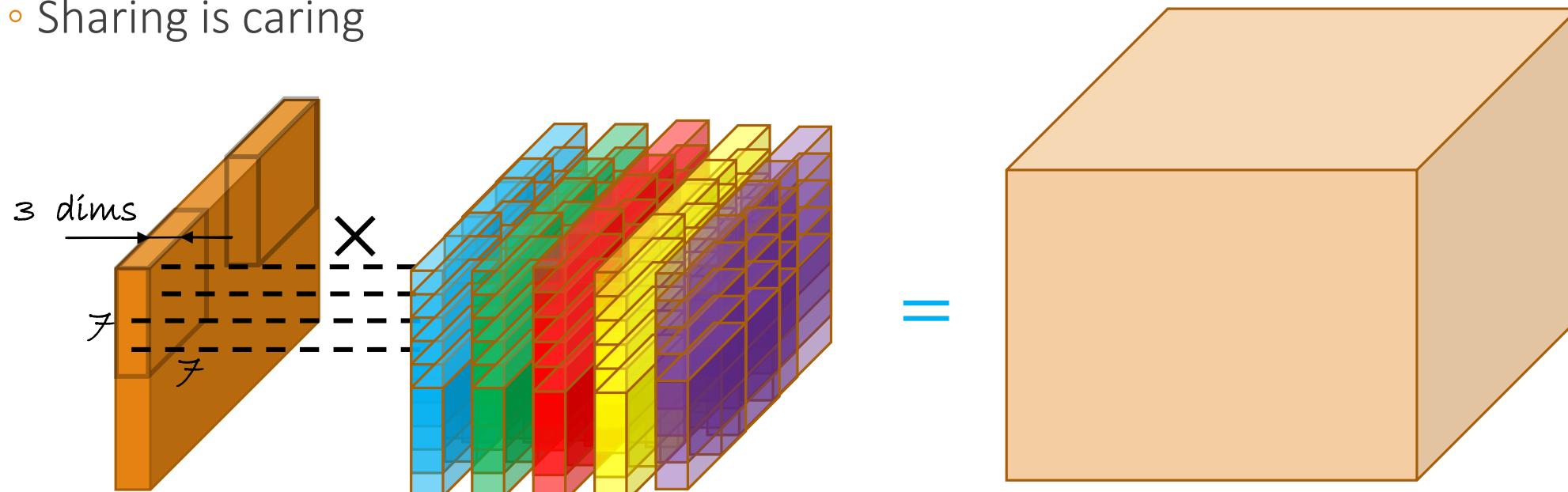
Assume the image is $30 \times 30 \times 3$.

1 column of filters common across the image.

How many parameters in total?

Solution? Share!

- So, if we are anyways going to compute the same filters, why not share?
 - Sharing is caring



Assume the image is $30 \times 30 \times 3$.

1 column of filters common across the image.

How many parameters in total?

$$\frac{\text{Depth of 5} \times 7 \times 7 \times 3 \text{ parameters per filter}}{735 \text{ parameters in total}}$$

Shared 2-D filters → Convolutions

Original image



Shared 2-D filters → Convolutions

Original image



Shared 2-D filters → Convolutions

Original image



Shared 2-D filters → Convolutions

Original image

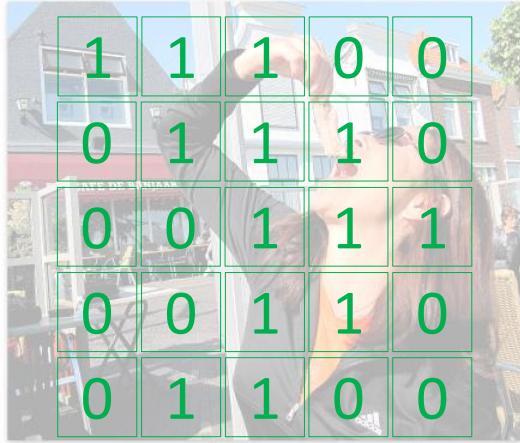


Convolutional filter 1

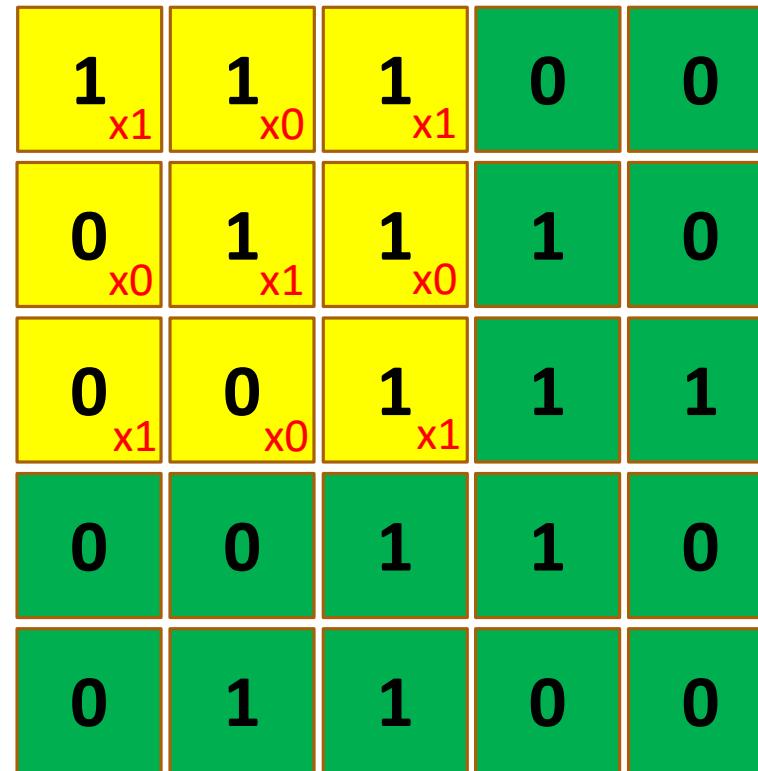
0	0	1
0	1	1
1	1	1

Shared 2-D filters → Convolutions

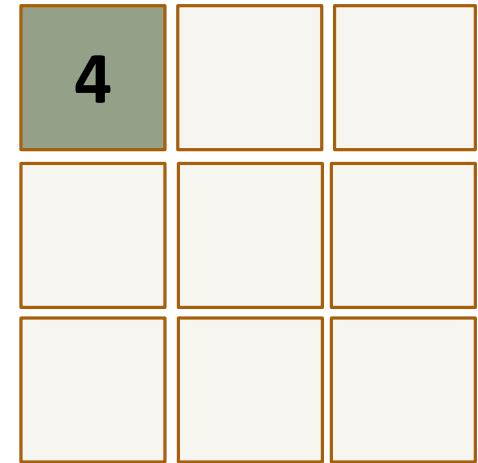
Original image



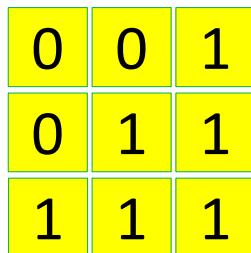
Convolving the image



Result



Convolutional filter 1



$$I(x, y) * h = \sum_{i=-a}^a \sum_{j=-b}^b I(x - i, y - j) \cdot h(i, j)$$

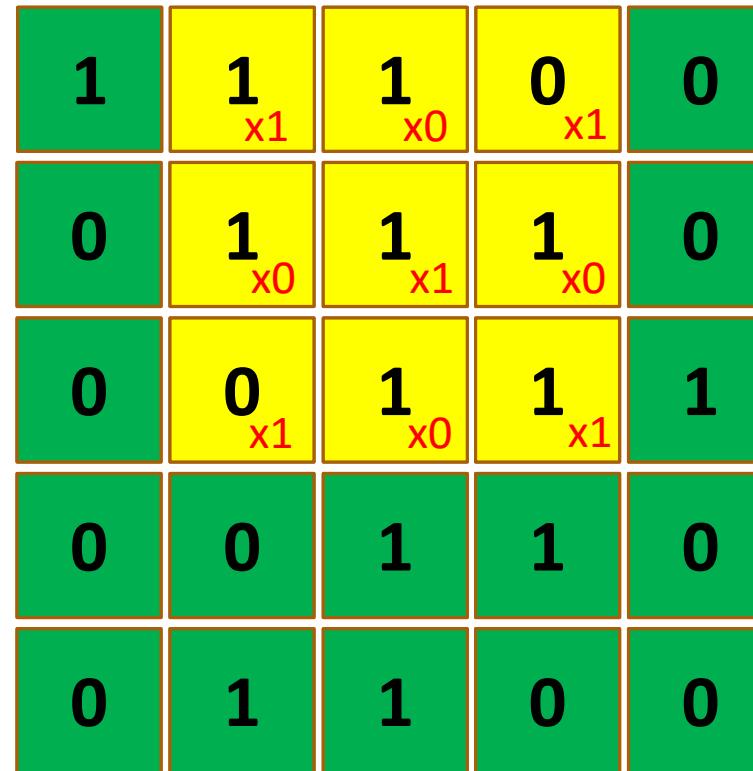
inner product

Shared 2-D filters → Convolutions

Original image



Convolving the image



Result

Convolutional filter 1

0	0	1
0	1	1
1	1	1

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Shared 2-D filters → Convolutions

Original image



Convolving the image



Result

Convolutional filter 1

0	0	1
0	1	1
1	1	1

$$I(x, y) * h = \sum_{i=-a}^a \sum_{j=-b}^b I(x - i, y - j) \cdot h(i, j)$$

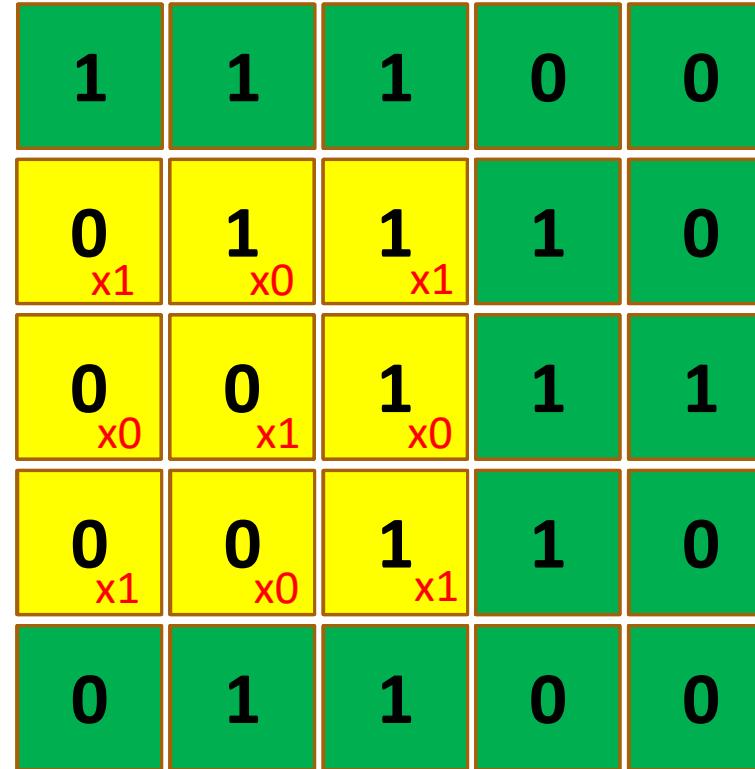
inner product

Shared 2-D filters → Convolutions

Original image



Convolving the image



Result

Convolutional filter 1

0	0	1
0	1	1
1	1	1

$$I(x, y) * h = \sum_{i=-a}^a \sum_{j=-b}^b I(x - i, y - j) \cdot h(i, j)$$

Inner product

Shared 2-D filters → Convolutions

Original image



Convolving the image



Result

4	3	4
2	4	3
2	3	4

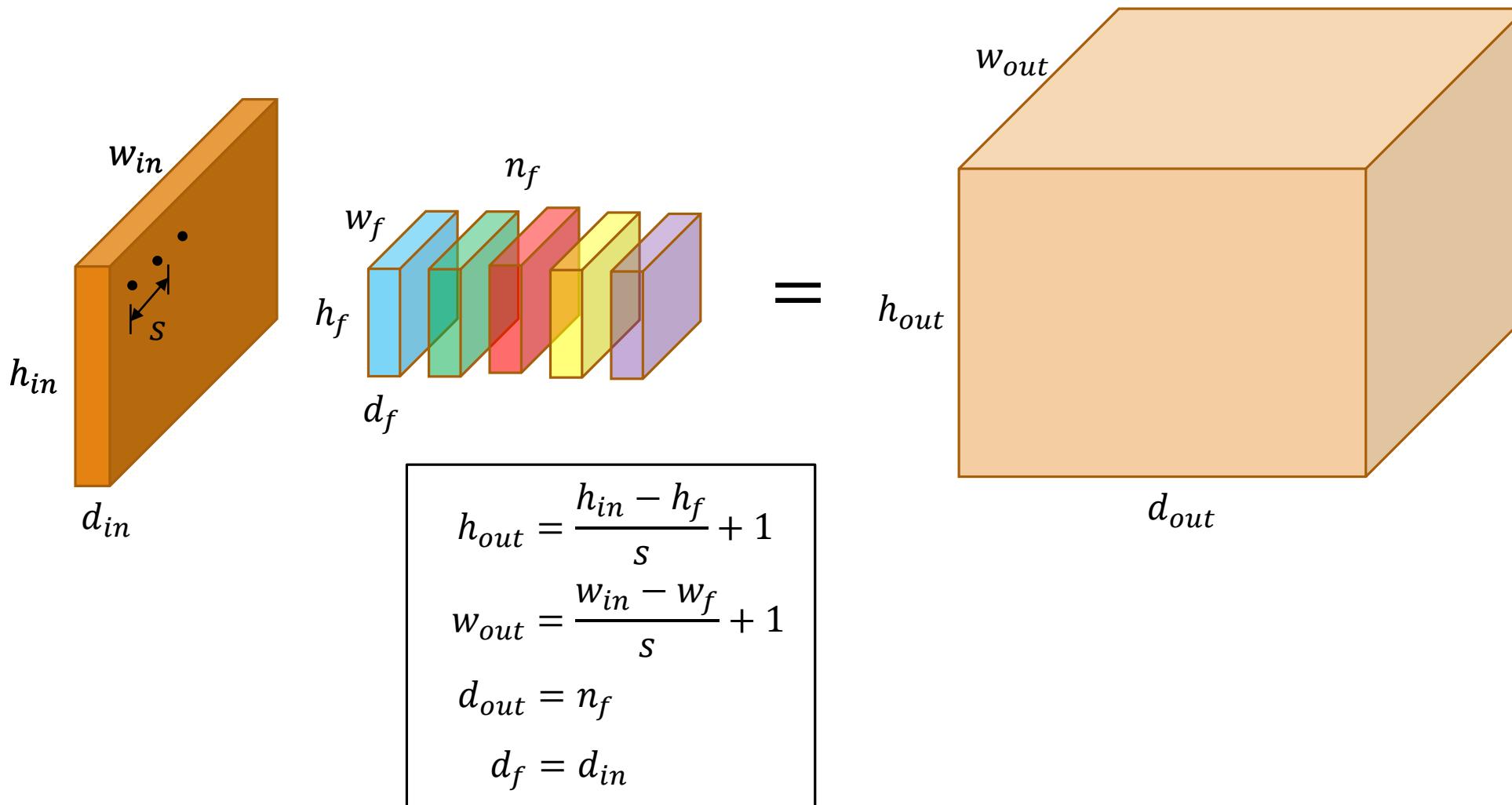
Convolutional filter 1

0	0	1
0	1	1
1	1	1

Inner product

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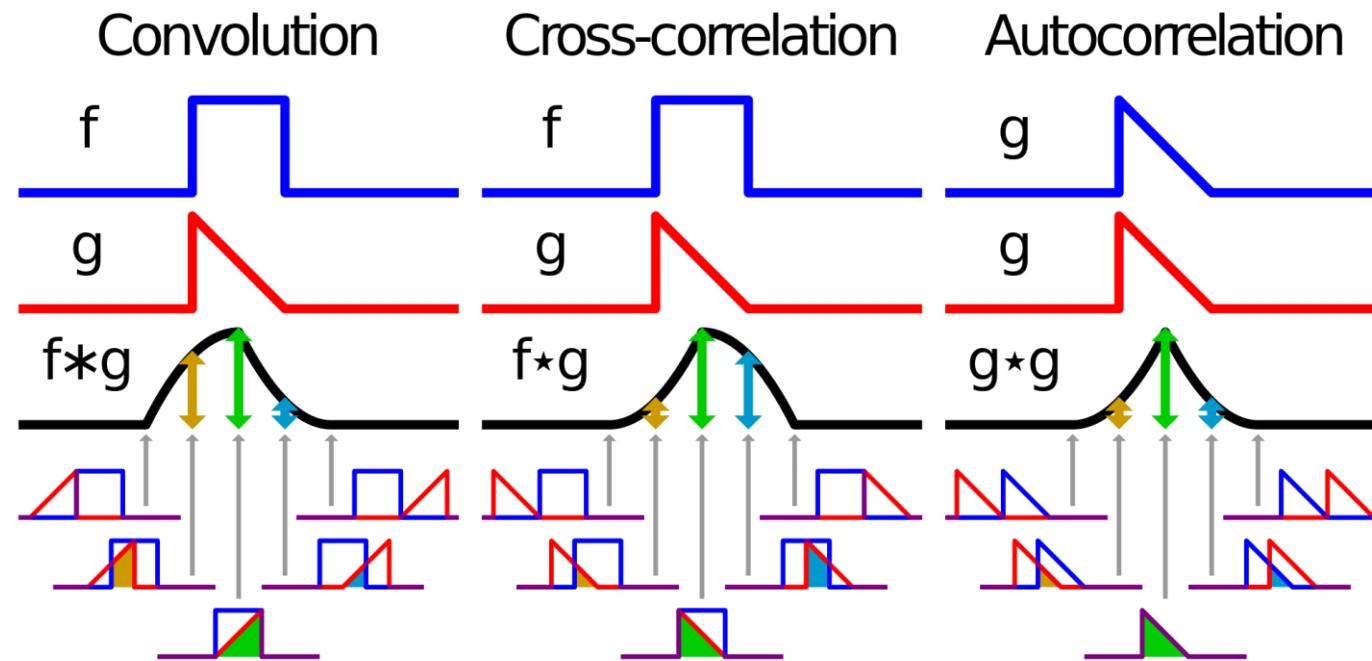
Output dimensions?



Why call them convolutions?

Definition The convolution of two functions f and g is denoted by $*$ as the integral of the product of the two functions after one is reversed and shifted

$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau = \int_{-\infty}^{\infty} f(t - \tau)g(\tau) d\tau$$

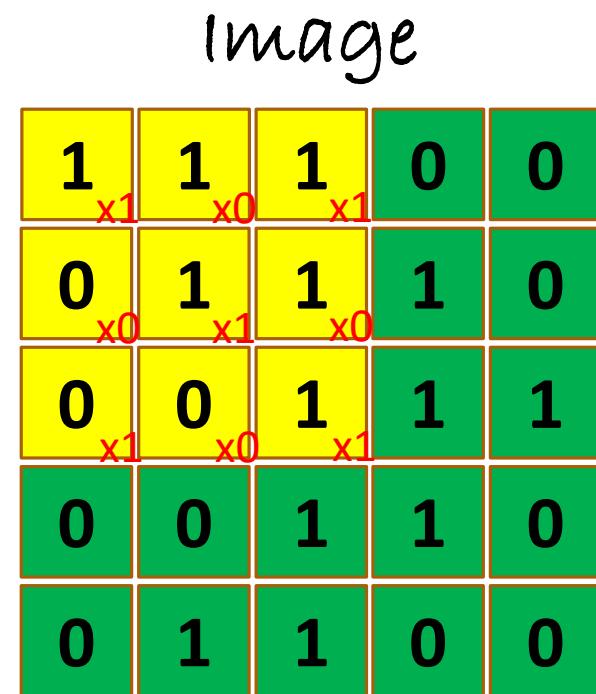


Problem, again :S

- Our images get smaller and smaller
- Not too deep architectures
- Details are lost

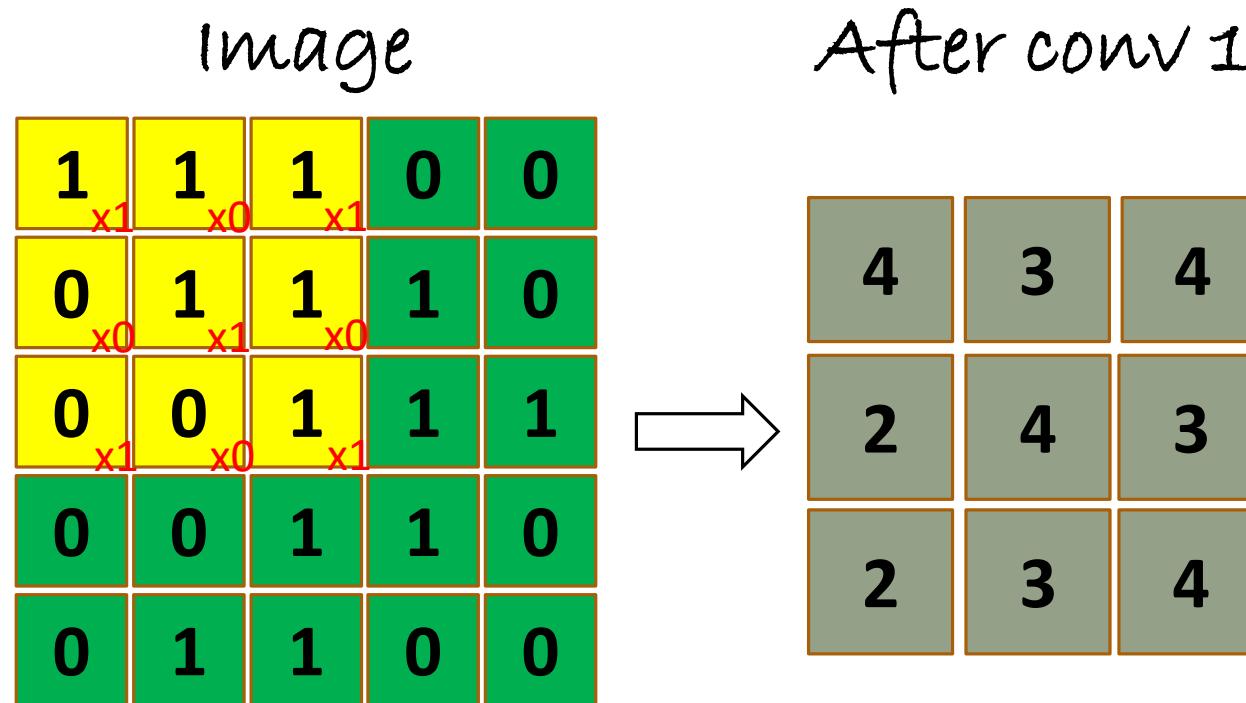
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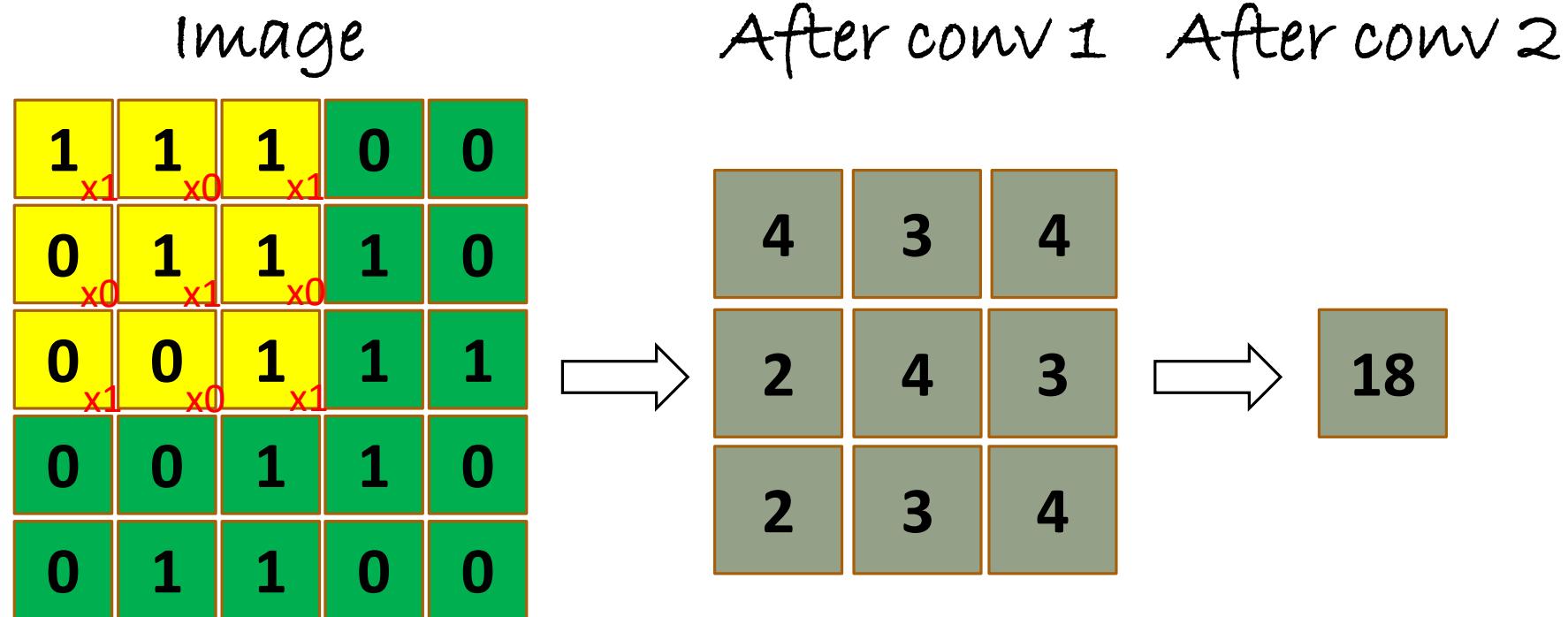
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- Our images get smaller and smaller
- Not too deep architectures
- Details are lost



Solution? Zero-padding!

- For $s = 1$, surround the image with $(h_f - 1)/2$ and $(w_f - 1)/2$ layers of 0

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

1	1	2	0	0
0	1	1	1	0
0	0	1	2	1
1	0	2	1	0
0	1	1	3	0

Solution? Zero-padding!

- For $s = 1$, surround the image with $(h_f - 1)/2$ and $(w_f - 1)/2$ layers of 0

0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0
0	0	0	1	1	1	0	0
0	0	0	1	1	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

1	1	2	0	0
0	1	1	1	0
0	0	1	2	1
1	0	2	1	0
0	1	1	3	0

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0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0
0	0	0	1	1	1	0	0
0	0	0	1	1	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

$$\begin{matrix} * & \begin{matrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \end{matrix}$$

1	1	2	0	0
0	1	1	1	0
0	0	1	2	1
1	0	2	1	0
0	1	1	3	0

Solution? Zero-padding!

- For $s = 1$, surround the image with $(h_f - 1)/2$ and $(w_f - 1)/2$ layers of 0

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{matrix} * \begin{matrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} = \begin{matrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & 1 & 3 & 0 \end{matrix}$$

Convolutional module (New module!!!)

- Activation function

$$a_{rc} = \sum_{i=-a}^a \sum_{j=-b}^b x_{r-i,c-j} \cdot \theta_{ij}$$

- Gradient w.r.t. the parameters

$$\frac{\partial a_{rc}}{\partial \theta_{ij}} = \sum_{r=0}^{N-2a} \sum_{c=0}^{N-2b} x_{r-i,c-j}$$

- Module and variants already implemented in Torch

Convolutional module in Torch

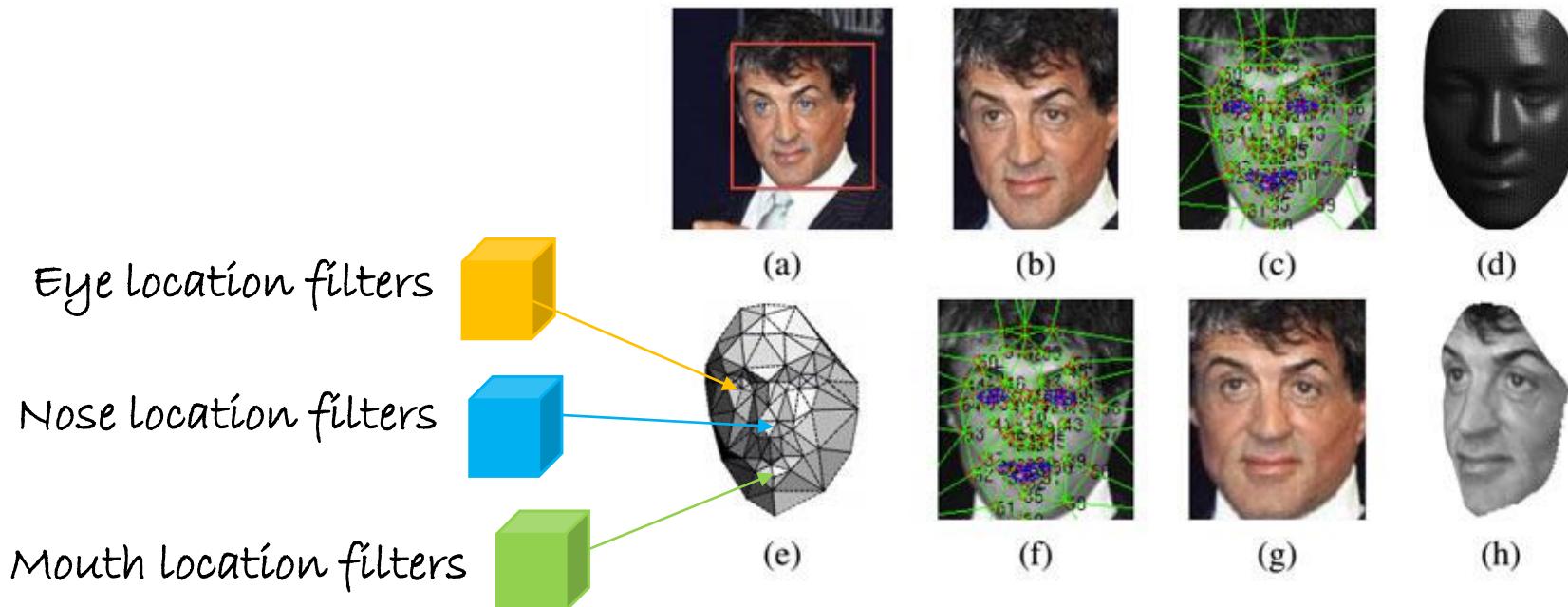
```
require 'nn'  
nn.SpatialConvolution(d_in, d_out, w_f, h_f, s_w, s_h)
```

Good practice

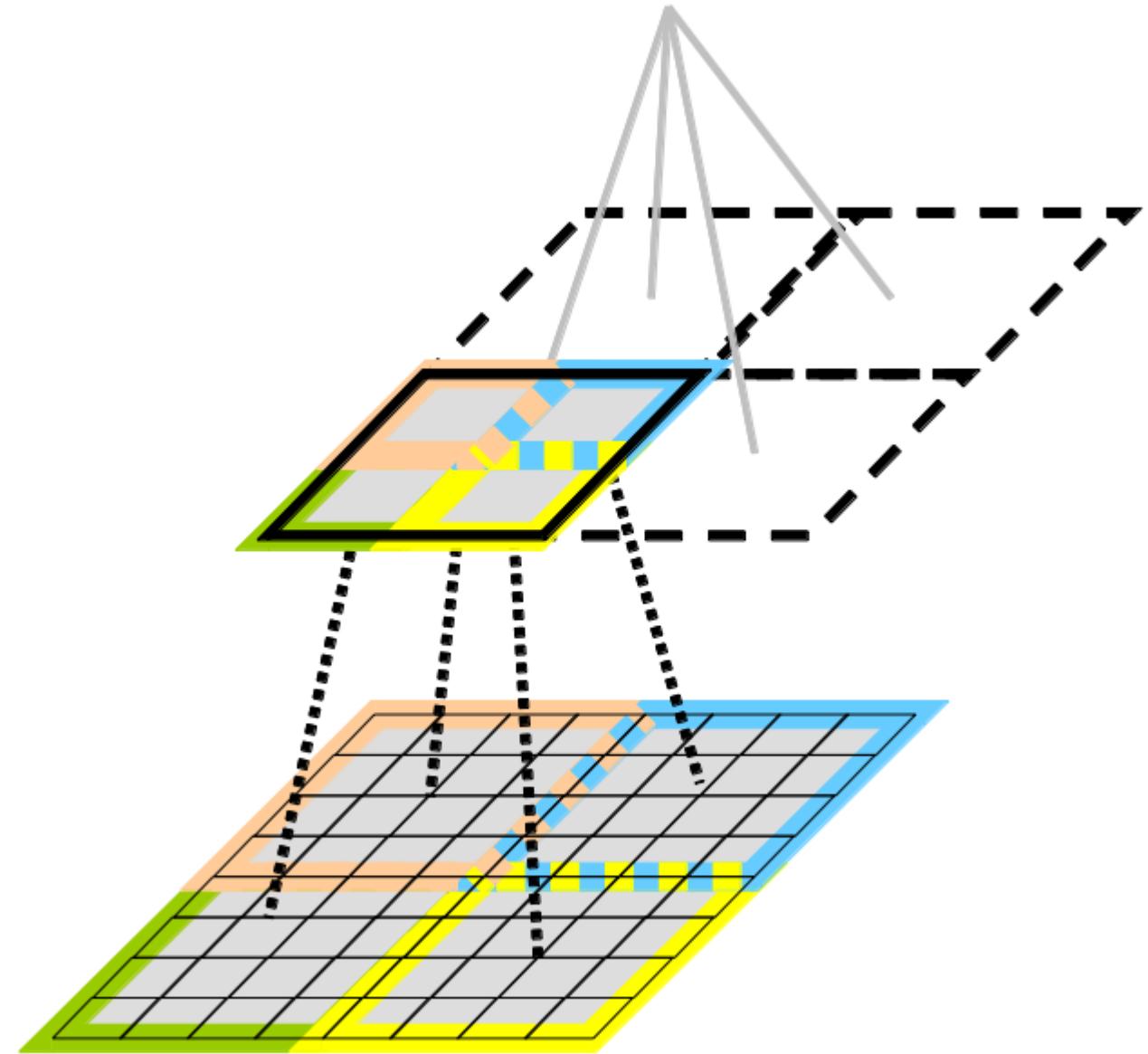
- Resize the image to have a size in the power of 2
- Use stride $s = 1$
- A filter of $(h_f, w_f) = [3 \times 3]$ works quite alright with deep architectures
- Add 1 layer of zero padding
- In general avoid combinations of hyper-parameters that do not click
 - E.g. $s = 1$
 - $[h_f \times w_f] = [3 \times 3]$ and
 - image size $[h_{in} \times w_{in}] = [6 \times 6]$
 - $[h_{out} \times w_{out}] = [2.5 \times 2.5]$
 - Programmatically worse, and worse accuracy because borders are ignored

P.S. Sometimes convolutional filters are not preferred

- When images are registered and each pixel has a particular significance
 - E.g. after face alignment specific pixels hold specific types of inputs, like eyes, nose, etc.
- In these cases maybe better every spatial filter to have different parameters
 - Network learns particular weights for particular image locations [Taigman2014]

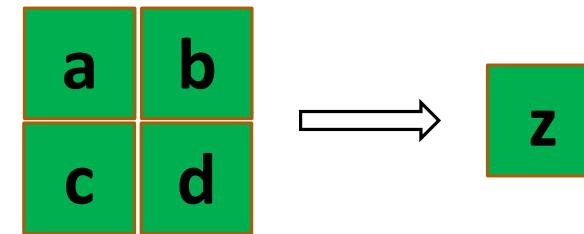


Pooling



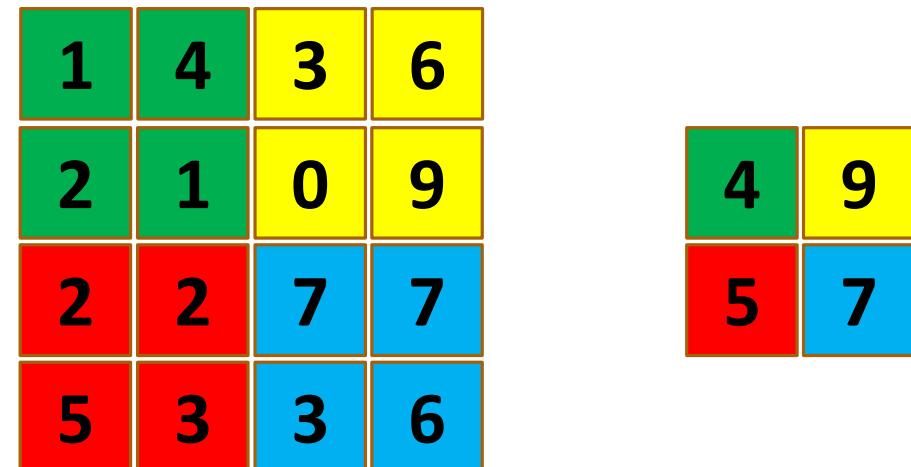
Pooling

- A function that aggregates multiple inputs into a single value
- Reduces the size of the layer output
 - Reduces the input for the next layer
 - Faster computations
 - Keeps the most important information for the next layer
- Max pooling
- Average pooling



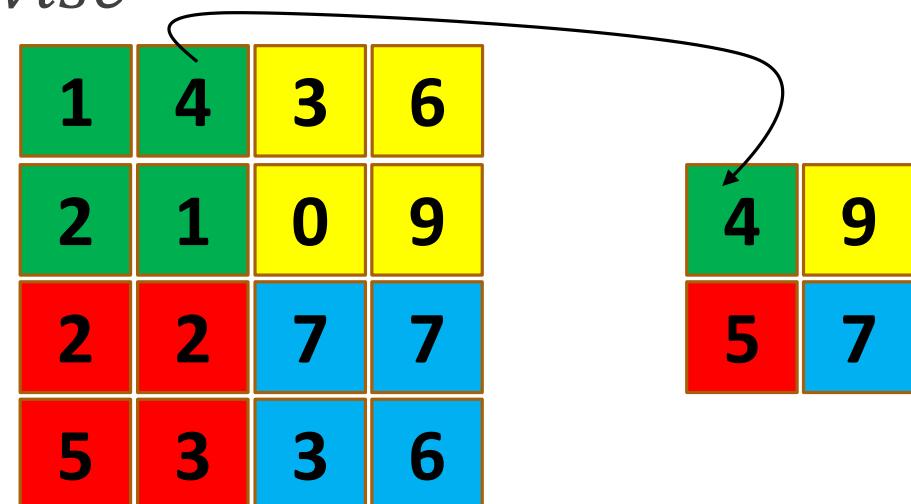
Max pooling (New module!)

- Run a sliding window of size $[h_f, w_f]$
- At each location keep the maximum value
- Activation function: $i_{\max}, j_{\max} = \arg \max_{i,j \in \Omega(r,c)} x_{ij} \rightarrow a_{rc} = x[i_{\max}, j_{\max}]$
- Gradient w.r.t. input $\frac{\partial a_{rc}}{\partial x_{ij}} = \begin{cases} 1, & \text{if } i = i_{\max}, j = j_{\max} \\ 0, & \text{otherwise} \end{cases}$
- The preferred choice of pooling



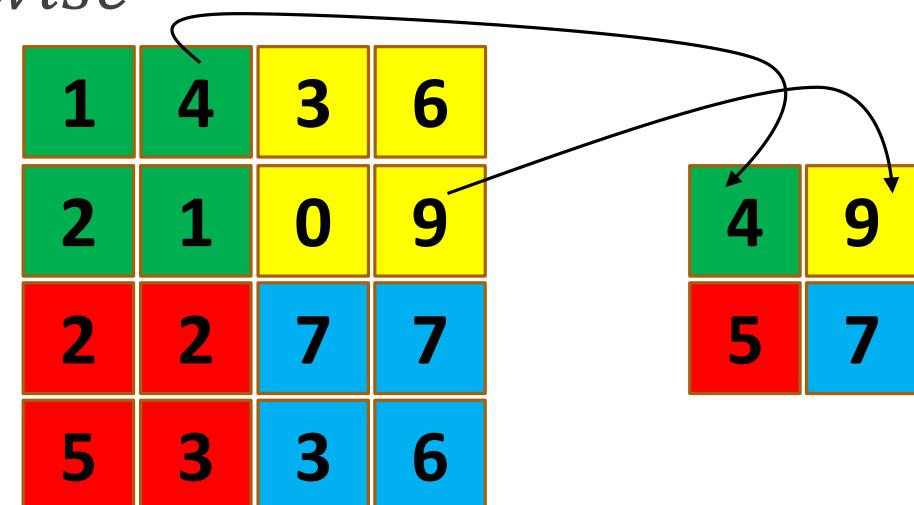
Max pooling (New module!)

- Run a sliding window of size $[h_f, w_f]$
- At each location keep the maximum value
- Activation function: $i_{\max}, j_{\max} = \arg \max_{i,j \in \Omega(r,c)} x_{ij} \rightarrow a_{rc} = x[i_{\max}, j_{\max}]$
- Gradient w.r.t. input $\frac{\partial a_{rc}}{\partial x_{ij}} = \begin{cases} 1, & \text{if } i = i_{\max}, j = j_{\max} \\ 0, & \text{otherwise} \end{cases}$
- The preferred choice of pooling



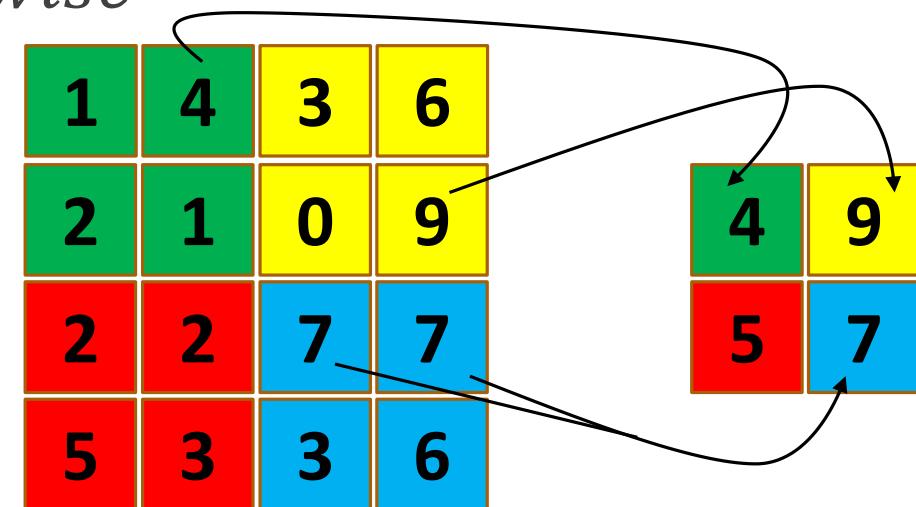
Max pooling (New module!)

- Run a sliding window of size $[h_f, w_f]$
- At each location keep the maximum value
- Activation function: $i_{\max}, j_{\max} = \arg \max_{i,j \in \Omega(r,c)} x_{ij} \rightarrow a_{rc} = x[i_{\max}, j_{\max}]$
- Gradient w.r.t. input $\frac{\partial a_{rc}}{\partial x_{ij}} = \begin{cases} 1, & \text{if } i = i_{\max}, j = j_{\max} \\ 0, & \text{otherwise} \end{cases}$
- The preferred choice of pooling



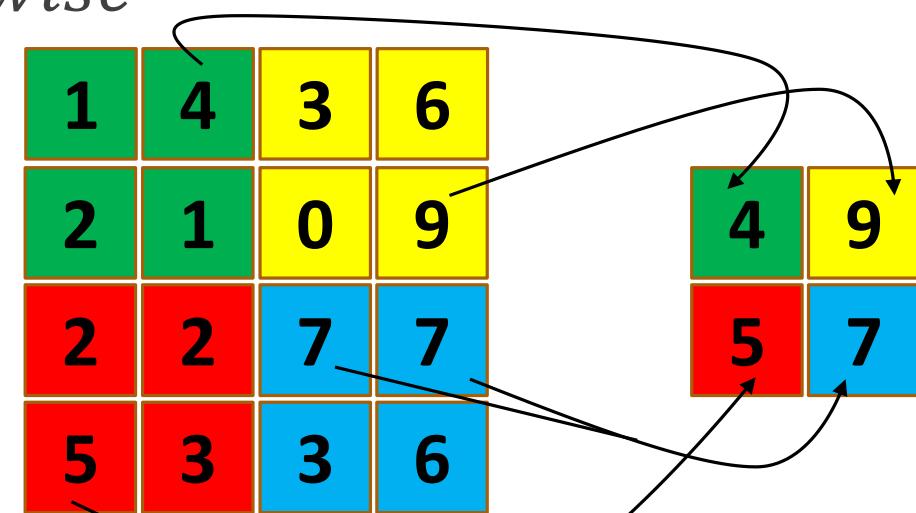
Max pooling (New module!)

- Run a sliding window of size $[h_f, w_f]$
- At each location keep the maximum value
- Activation function: $i_{\max}, j_{\max} = \arg \max_{i,j \in \Omega(r,c)} x_{ij} \rightarrow a_{rc} = x[i_{\max}, j_{\max}]$
- Gradient w.r.t. input $\frac{\partial a_{rc}}{\partial x_{ij}} = \begin{cases} 1, & \text{if } i = i_{\max}, j = j_{\max} \\ 0, & \text{otherwise} \end{cases}$
- The preferred choice of pooling



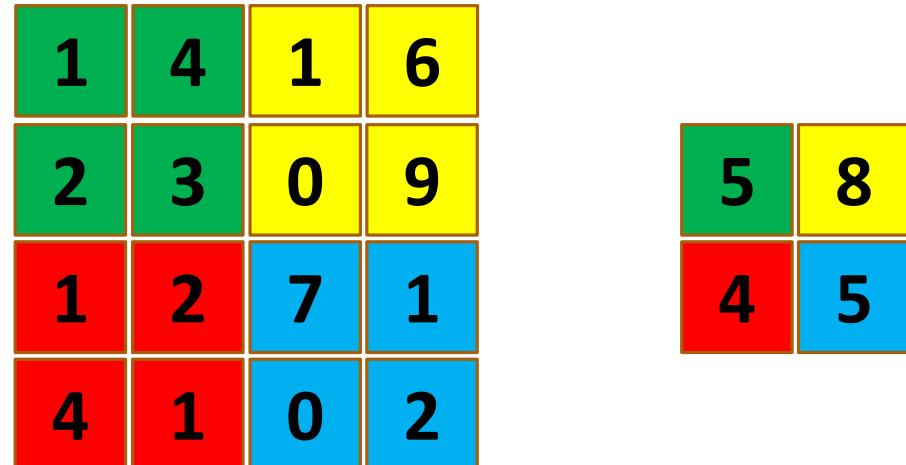
Max pooling (New module!)

- Run a sliding window of size $[h_f, w_f]$
- At each location keep the maximum value
- Activation function: $i_{\max}, j_{\max} = \arg \max_{i,j \in \Omega(r,c)} x_{ij} \rightarrow a_{rc} = x[i_{\max}, j_{\max}]$
- Gradient w.r.t. input $\frac{\partial a_{rc}}{\partial x_{ij}} = \begin{cases} 1, & \text{if } i = i_{\max}, j = j_{\max} \\ 0, & \text{otherwise} \end{cases}$
- The preferred choice of pooling

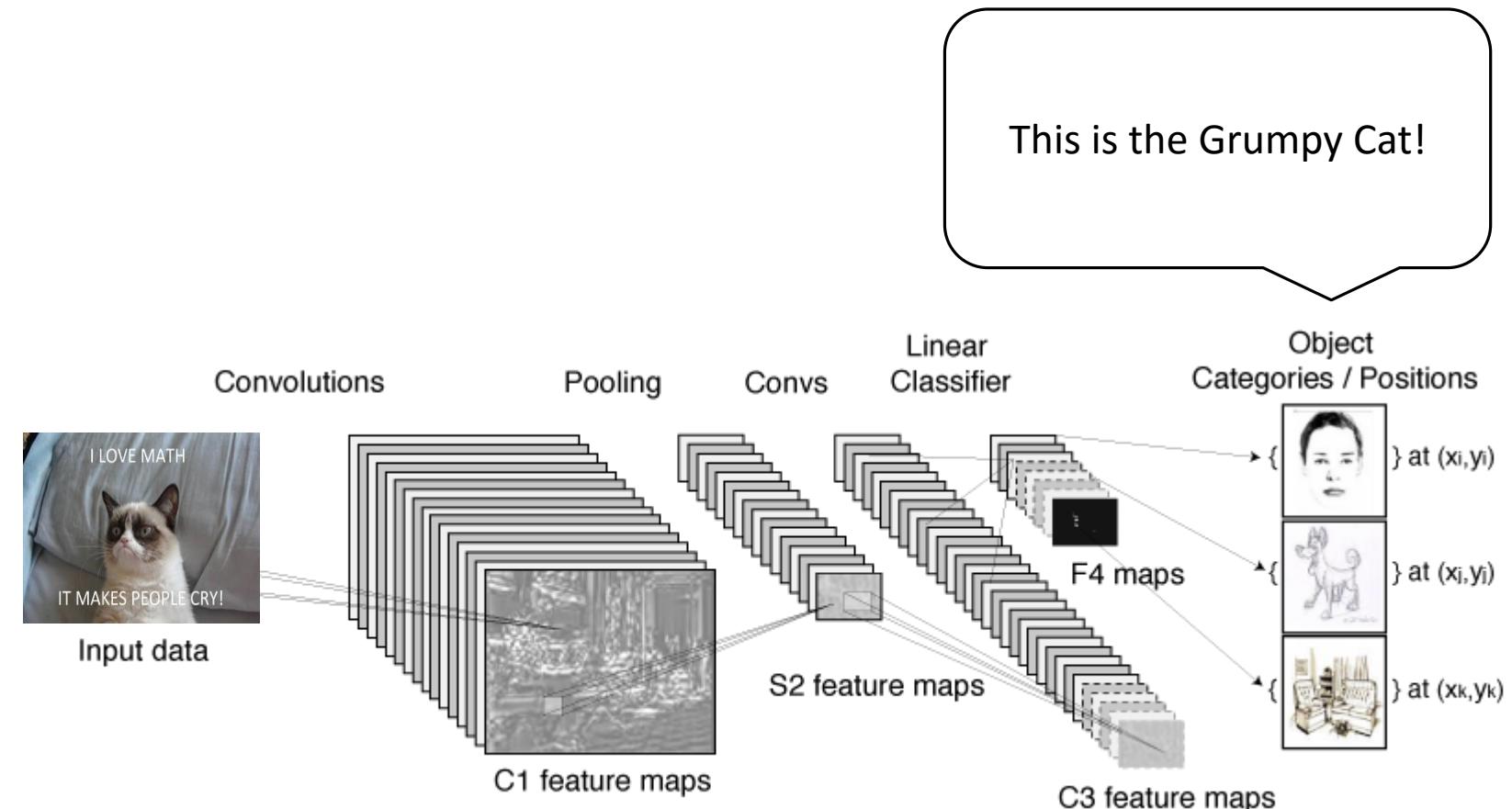


Average pooling (New module!)

- Run a sliding window of size $[h_f, w_f]$
- At each location keep the maximum value
- Activation function: $a_{rc} = \frac{1}{r \cdot c} \sum_{i,j \in \Omega(r,c)} x_{ij}$
- Gradient w.r.t. input $\frac{\partial a_{rc}}{\partial x_{ij}} = \frac{1}{r \cdot c}$

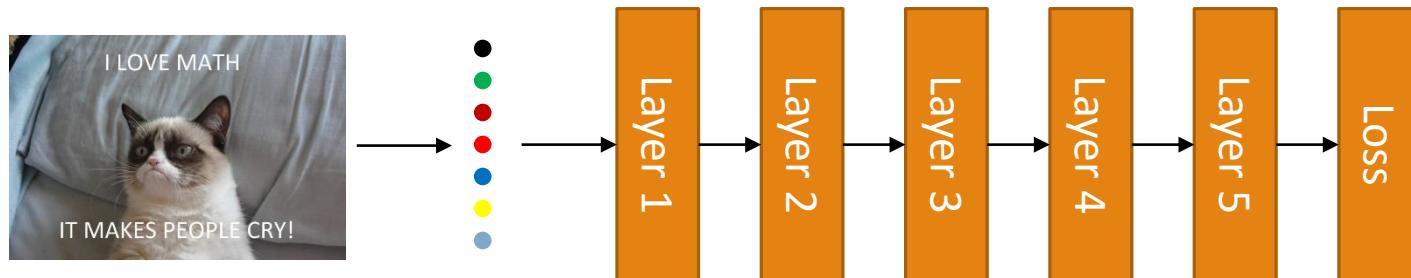


Convnets for Object Recognition

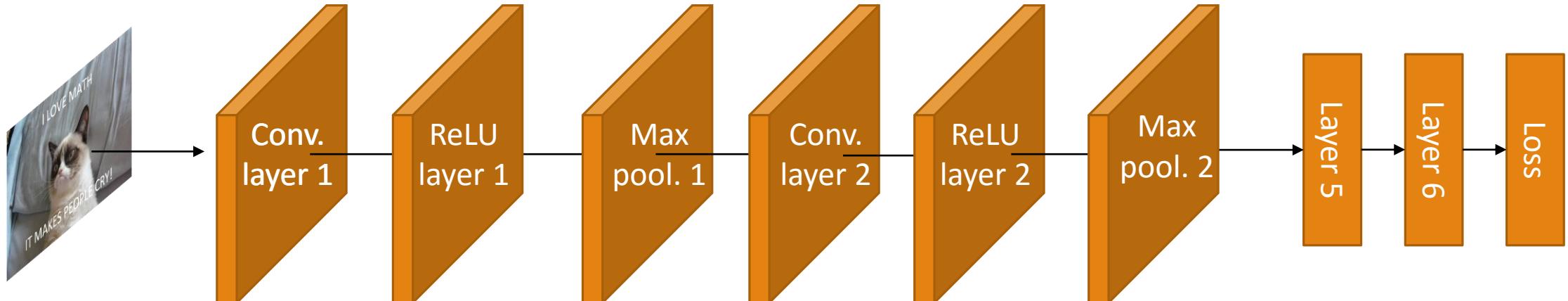


Standard Neural Network vs Convnets

Neural Network



Convolutional Neural Network



Convets in practice

- Several convolutional layers
 - 5 or more
- After the convolutional layers non-linearities are added
 - The most popular one is the ReLU
- After the ReLU usually some pooling
 - Most often max pooling
- After 5 rounds of cascading, vectorize last convolutional layer and connect it to a fully connected layer
- Then proceed as in a usual neural network

CNN Case Study I: Alexnet

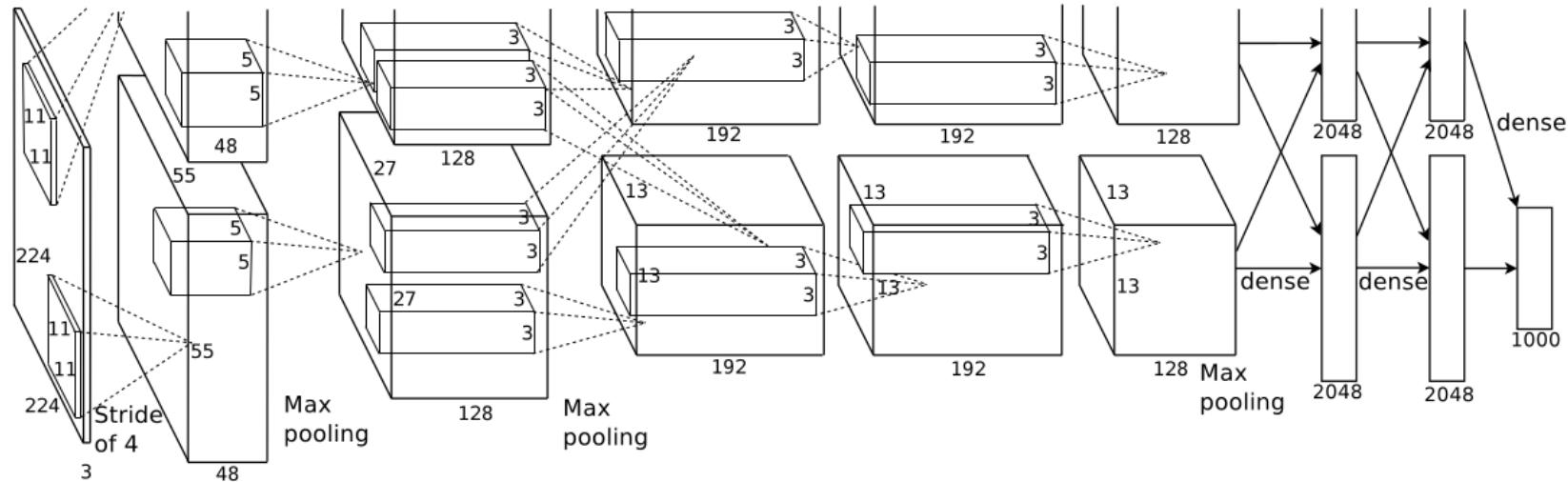
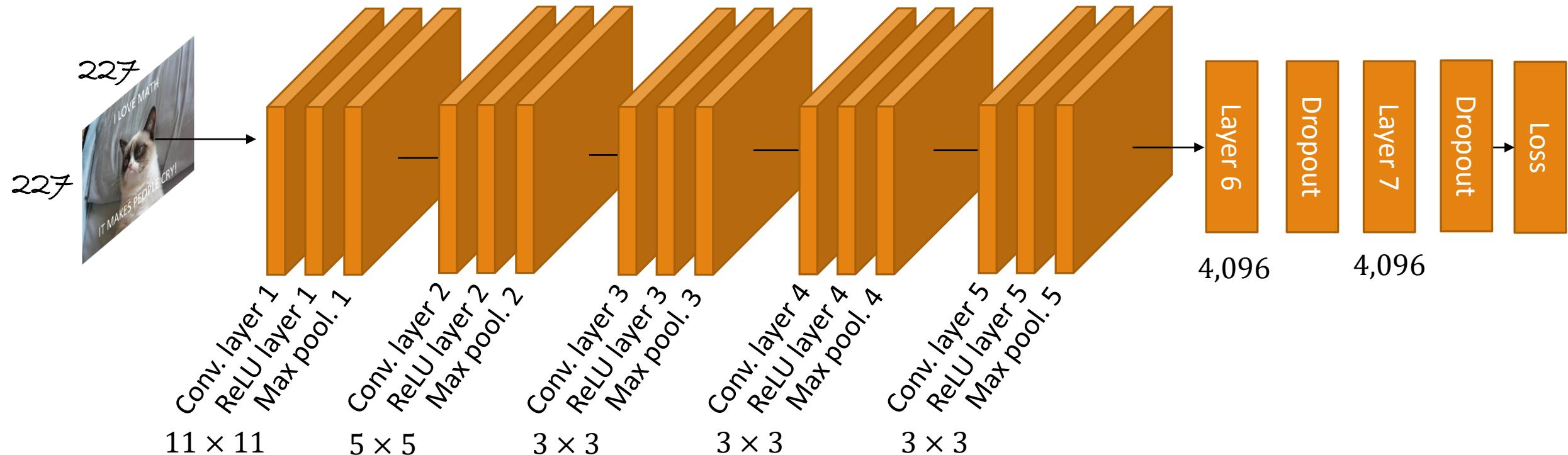


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network's input is 150,528-dimensional, and the number of neurons in the network's remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

Alexnet prototxt (Caffe configuration file)

https://github.com/BVLC/caffe/blob/master/models/bvlc_alexnet/train_val.prototxt

Architectural details



CNN Case Study II: VGNet

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224×224 RGB image)					
conv3-64	conv3-64 LRN	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 conv1-256	conv3-256 conv3-256 conv3-256	conv3-256 conv3-256 conv3-256
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 conv1-512	conv3-512 conv3-512 conv3-512	conv3-512 conv3-512 conv3-512
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 conv1-512	conv3-512 conv3-512 conv3-512	conv3-512 conv3-512 conv3-512
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Table 2: Number of parameters (in millions).

Network	A,A-LRN	B	C	D	E
Number of parameters	133	133	134	138	144

VGNet prototxt

https://gist.github.com/ksimonyan/211839e770f7b538e2d8#file-vgg_ilsvrc_16_layers_deploy-prototxt

More cases

- Two-stream network
 - Moving images (videos)
- Network in Network
- Deep Fried Network
- Resnet
 - Winner of ILSVRC 2016

Summary

- What are the Convolutional Neural Networks?
- Why are they so important for Computer Vision?
- How do they differ from standard Neural Networks?
- How can we train a Convolutional Neural Network?

Next lecture

- What do convolutions look like?
- Build on the visual intuition behind Convnets
- Deep Learning Feature maps
- Transfer Learning